Direct Proofs and Proofs by Contrapositive Solution

Part One. Prove each of the following statements:

1. For all integers n, if n is odd than 3n+5 is even.
2. If a is any odd integer and b is any even integer, then 2a + 3b is even.

(Note: The definition of an **odd** integer is an integer that can be expressed as 2k + 1, where k is an integer. The definition of an **even** integer is an integer that can be expressed as 2k, where k is an integer.)

1. For all integers n, if n is odd than 3n+5 is even.
   Proof.
   Assume n is an odd integer. By definition of odd, n = 2k+1 for some integer k.
   By substitution,
   $$\begin{aligned} 3n+5 &= 3(2k+1) +5 \\ &= 6k+3 +5 \\ &= 6k+8 \\ &= 2(3k+4) \end{aligned}$$
   Let t=3k+4. t is an integer because it is a sum of products of integers.
   We have that 3n+5 = 2t, where t is an integer. By definition of even, 3n+5 is even.

2. If a is any odd integer and b is any even integer, then 2a + 3b is even.
   Proof.
   Assume a is an odd integer and b is an even integer.
   By definition of odd, a = 2n+1 for some integer n. By definition of even, b = 2m for some integer m.
   By substitution,
   $$\begin{aligned} 2a + 3b &= 2(2n+1) + 3(2m) \\ &= 4n + 2 + 6m \\ &= 2(2n + 1 + 3m). \end{aligned}$$
   Let x = 2n + 1 + 3m. Then, x is an integer because it is a sum of products of integers.
   We have that 2a + 3b = 2x where x is an integer. By definition of even, 2a + 3b is even.

Part Two. Fill in the blanks in the following proof by contrapositive that for all integers n, if 5 does not divide n^2 then 5 does not divide n. (Note: $5 \mid n$ means that 5 divides n or n is divisible by 5.)
Proof (by contrapositive): [The contrapositive is: For all integers n, if $5 \mid n$ then $5 \mid n^2$.]
Suppose n is any integer such that (a). [We must show that (b).]
By definition of divisibility, $n =$ (c) for some integer k. By substitution, $n^2 =$ (d) $= 5(5k^2)$. But $5k^2$ is an integer because it is a product of integers. Hence $n^2 = 5\cdot$ (an integer), and so (e) [as was to be shown].

(a) $5 \mid n$

(b) $5 \mid n^2$

(c) 5k

(d) $(5k)^2$

(e) $5 \mid n^2$