Solutions to Propositions, Compound propositions, and Conditional statements worksheet

A. Indicate which of the following statements are propositions.

1. $2 + 3 = 7$ Proposition
2. Julius Caesar was president of the United States. Proposition
3. What time is it? No
4. Be quiet ! No
5. The difference of two primes. No
6. $2 + 2 = 4$. Proposition
7. Washington D.C. is the capital of New York. Proposition
8. How are you? No

B.

- Let
  - m = Juan is a math major
  - c = Juan is a computer science major
  - How would we write "Juan is a math major but not a computer science major"

    $m \wedge \neg c$

- Let s = stocks are increasing, i = interest rates are steady. How would we write
  - Stocks are increasing but interest rates are steady

    $s \wedge i$

  - Neither are stocks increasing nor are interest rates steady

    $\neg s \wedge \neg i$

C. Let h = John is healthy, w = John is wealthy, s = John is wise.
Write compound propositions representing

1. John is healthy and wealthy but not wise $h \wedge w \wedge \neg s$

2. John is not wealthy but he is healthy and wise $\neg w \wedge h \wedge s$

3. John is neither healthy, wealthy, nor wise $\neg h \wedge \neg w \wedge \neg s$

4. John is neither wealthy nor wise, but he is healthy. $\neg w \wedge \neg s \wedge h$

5. John is wealthy, but he is not both healthy and wise. $w \wedge \neg(h \wedge s)$ or $w \wedge (\neg h \vee \neg s)$ (both are correct)

   Wrong answer: $w \wedge \neg h \wedge \neg s$ Wrong answer: $w \wedge (h \oplus s)$. These wrong answers have different meaning than the original sentence.

D. Indicate whether each statement is true or false, assuming that the "or" in the sentence means the inclusive or. Then indicate whether the statement is true or false if the "or" means the exclusive or.

1. February has 31 days or the number 5 is an integer. Inclusive or: T. Exclusive or: T

2. January has 31 days or the number 5 is an integer. Inclusive or: T. Exclusive or: F

Note: When translating English sentences into logic we will always translate "or" as inclusive or unless it is explicitly stated that or is exclusive.

E. Write each of the following into implication form (put in the form p → q and specify what is p and what is q)

1. If you study in this course you will get an A.
   you study in this course → you will get an A

2. Tomorrow is Friday if today is Thanksgiving.
   today is Thanksgiving → tomorrow is Friday

3. n is prime implies n is odd or n is 2.
   n is prime → (n is odd or n is 2)

4. A sufficient condition for Jon's team to win the championship is that it win the rest of its games.
   it wins the rest of its games → Jon's team wins the championship

5. A necessary condition for this computer program to be correct is that it not produce error messages during translation.
   this computer program is correct → it does not produce error messages during translation

6. n is divisible by 6 only if n is divisible by 2 and n is divisible by 3.
   n is divisible by 6 → (n is divisible by 2 and n is divisible by 3)

7. P being a rectangle is necessary for P being a square.
   P is a square → P is a rectangle

F. Taking the long view on your education, you go to the Prestige Corporation and ask what you should do in college to be hired when you graduate. The personnel director replies that you will be hired only if you major in mathematics or computer science, get a B average or better, and take accounting. You do, in fact, become a math major, get a B+ average, and take accounting. You return to Prestige Corporation, make a formal application, and are turned down. Did the personnel director lie to you?

The Personnel Director did not lie. By using the phrase "only if," the Personnel Director set forth conditions that were necessary but not sufficient for being hired: if you did not satisfy those conditions then you would not be hired. The Personnel Director's statement said nothing about what would happen if you did satisfy those conditions.

G. Complete the following truth tables. Recall the precedence rules: ¬ has higher precedence than ∧, ∧ higher than ∨, ∨ higher than ⟶, and ⟶ higher than ⟷.

1)

p	q	¬p	¬p∨q	¬q	¬p∨q ⟶ ¬q
T	T	F	T	F	F
T	F	F	F	T	T
F	T	T	T	F	F
F	F	T	T	T	T

2)

p	q	p⟶q	q⟶p (converse)	¬q⟶¬p (contrapositive)	¬p⟶¬q (inverse)	¬(q⟶p) (negation)
T	T	T	T	T	T	F
T	F	F	T	F	T	F
F	T	T	F	T	F	T
F	F	T	T	T	T	F

Compare truth values of original statement, its converse, its contrapositive, its inverse, and its negation. What do you notice?

H. For the following statement (p⟶q): If Sara lives in Athens, then she lives in Greece.
Write the following in English

1. Converse (q⟶p)
   If Sara lives in Greece, then she lives in Athens.
2. Contrapositive (¬q⟶¬p)
   If Sara does not live in Greece, then she does not live in Athens.
3. Inverse (¬p⟶¬q)
   If Sara does not live in Athens, then she does not live in Greece.
4. Negation The following are possible answers, all of them are logically equivalent:
   It is not the case that if Sara lives in Athens, then she lives in Greece.
   Sara lives in Athens and she does not live in Greece.

I. "If compound X is boiling, then its temperature must be at least 150◦C." Assuming that this statement is true, which of the following must also be true?

a. If the temperature of compound X is at least 150◦C, then compound X is boiling.

This statement is the converse of the given statement, and so it is not necessarily true. For instance, if the actual boiling point of compound X were 200 ◦ C, then the given statement would be true but this statement would be false.
For any statement, its truth table is different from the truth table of its converse (see the last truth table in part C of this worksheet). Therefore, the truth value of a statement may be different from the truth value of its converse.

b. If the temperature of compound X is less than 150◦C, then compound X is not boiling.

This statement must be true. It is the contrapositive of the given statement.
For any statement, its contrapositive has the same truth value (see the last truth table in part C of this worksheet.