HW 1

1.3.6

Give an English sentence in the form "if ... then ..." that is equivalent to each sentence.

(a) Maintaining a B average is sufficient for Joe to be eligible for the honors.

A – If Joe maintaining a B average, then he is eligible for the honors program.

(b) ~~I~~ ... is necessary ...

A – If Joe is eligible ~~for~~ the honors program, then he has maintain a B average.

(c) Rajiv can go on the roller coaster only if he is at least four feet tall.

A – If Rajiv can go on the roller coaster, then he is at least four feet tall.

(d) ... if ...

A – If Rajiv is four feet tall, then he can go on the roller coaster.

1.5.2

~~(a)~~ Use the laws of propositional logic to prove:

(f) $\neg(p \vee (\neg p \wedge q)) \equiv \neg p \wedge \neg q$

$\neg p \wedge \neg(\neg p \wedge q)$	De Morgan laws
$\neg p \wedge (p \vee \neg q)$	De Morgan laws
$(\neg p \wedge p) \vee (\neg p \wedge \neg q)$	Distributive laws
$F \vee (\neg p \wedge \neg q)$	Complement laws
$\neg p \wedge \neg q$	Identity laws

1.6.1

(f) $(p \vee q) \rightarrow r$

$\therefore (p \wedge q) \rightarrow r$

$\boxed{\text{Valid}}$

p	q	r	$(p \vee q) \rightarrow r$	$(p \wedge q) \rightarrow r$
T	T	T	T	T
T	T	F	F	F
T	F	T	T	T
T	F	F	F	T
F	T	T	T	T
F	T	F	F	T
F	F	T	T	T
F	F	F	T	T

(j) $q \rightarrow p$

p

$\therefore \neg(p \rightarrow q)$

$\boxed{\text{Valid}}$

p	q	$q \rightarrow p$	$\neg(p \rightarrow q)$
T	T	T	F
T	F	T	T
F	T	F	F
F	F	T	F

1.6.3

(c) If $\sqrt{2}$ is an irrational number, then $2\sqrt{2}$ is an irrational number.

$2\sqrt{2}$ is an irrational number.

$\therefore \sqrt{2}$ is an irrational number.

s	f	s→f
T	T	T
T	F	F
F	T	T
F	F	T

$\boxed{\text{Valid}}$ Assign : s: $\sqrt{2}$ is an irrational number

f: $2\sqrt{2}$ is an irrational number

The form of argument :

$$\begin{array}{l} s \to f \\ f \\ \hline \therefore s \end{array}$$

the hypothese $s \to f$ and f are both true in first line, conclusion is true on first line of truth table.

1.7.2

(c) $(p \wedge q) \to r$

$\neg r$

q

$\therefore \neg p$

1.	$\neg r$	hypothesis
2.	$(p \wedge q) \to r$	hypothesis
3.	$\neg(p \wedge q)$	Modus tollens, 1, 2
4.	$\neg p \vee \neg q$	De Morgans law, 3,
5.	q	hypothesis
6.	$\neg q \vee \neg p$	Commutatives law
7.	$\neg p$	Disjunctive syllogism, 6, 5

1.7.4.

(b) If it was not foggy or it didn't rain (or both), then the race was ~~not~~ held and there was a trophy ceremony

The trophy ceremony was not held.

$\therefore$ It rained

$\neg(w \wedge f) \to p \wedge s$

$$\begin{array}{l} \neg(w \wedge f) \to s \\ \neg s \\ \hline \therefore f \end{array}$$

$$\begin{array}{l} \neg s \\ \hline \therefore f \end{array}$$

w: It was foggy

f: It was rained

s: The trophy ceremony was held

p: The race was held

1. $\neg s$ hypothesis
2. $\neg(w \wedge f) \to \neg s$ hypothesis
3. $\neg\neg(w \wedge f)$ Modus tollens 1,2
4. $w \wedge f$ Double Negation law, 3
5. $f \wedge w$ Commutative law, 4
6. f Simplification, 5

01.8.5

P(x) is a predicate and the ~~xxx~~ domain for the variable x is {1, 2, 3, 4}. For each of the logical expressions given, give an equivalent logical expression that does not use quantifiers.

(b) $\exists x\, P(x)$

$P(1) \vee P(2) \vee P(3) \vee P(4)$

1.9.4

(b) $\forall x\, W(x)$

(f) $\forall x (\neg W(x) \to S(x) \vee V(x))$

(g) $Ex (\neg W(x) \to \neg(S(x) \vee V(x)))$

(i) $Ex (S(x) \wedge W(x))$

1.10.4

(b) $\neg\forall x (\neg P(x) \to Q(x)) \equiv \exists x (\neg P(x) \wedge \neg Q(x))$

$\exists x \neg(\neg P(x) \to Q(x))$ De Morgan's

$\exists x \neg(\neg\neg P(x) \vee Q(x))$ Conditional Identity

$\exists x \neg(P(x) \vee Q(x))$ Double Negation

$\exists x (\neg P(x) \wedge \neg Q(x))$ De Morgan's

1.9.9

(b) False

(c) True

(f) True

o 2.1.2

(a) True (b) False (c) False (e) False (i) True

2.2.3

(a) what is the cardinality of $P(\{1,2,3,4,5,6\})$?

$\boxed{\text{Cardinality} = 1}$

2.2.6

(c) $P(P(P(\emptyset)))$ $\{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}\}$

$|P(P(P(\emptyset)))| = 3$

$A_1 = \{1,2,3 \ldots\}$ $A_2 = \{2,4,6,8\}$

$A_3 = \{3,6,9,12\}$ $A_4 = \{4,8,12,16\}$

$A_5 = \{5,10,15,20\}$

o 2.3.2

(a) $A_i \{i \in Z^+ : x$ is multiple of $60\}$

(b) $A_i \{2,3,4,5,6,8,9,10,12,15,16,20\}$

$A \cap B = \emptyset$ $\emptyset \cap C$

$\{\cdot\}$

o 2.4.4

(b) True (c) True (d) False (e) True (f) True

2.5.3

(d) If $A = \{a\}$, and $B = \{b\}$, then $(B - A) \cup A = \{a, b\}$, and $A = \{a\}$

2.6.7

(f) $P(A \times B)$

2.7.3

(b)