Proof by contrapositive, proof by contradiction, proof by cases

1. To prove a statement by contradiction, you suppose that __________________________

and you show that ____.

2. A proof by contrapositive of a statement of the form "*For all x in a domain,* if *P(x)* then *Q(x)*" is a direct proof of

____. (Note: Domain is a collection of possible values for *x* . For example, integers, real numbers, etc.)

3. a) To prove a statement of the form "*For all x in a domain*, if *P(x)* then *Q(x)*" by contrapositive, you suppose that

____ and you show that ________________________.

b) To prove "For every integer n, if n^3 is even, then n is even" by contrapositive, you suppose that

____ and you show that ________________________.

c) To prove "For every real number x, if x is irrational, then -x is also irrational" by contrapositive, you suppose that

____ and you show that ________________________.

d) To prove "If x and y are real numbers and $x + y > 20$, then $x > 10$ or $y > 10$" by contrapositive, you suppose that

____ and you show that ________________________.

e) To prove "If x and y are integers and xy is even, then x is even or y is even" by contrapositive, you suppose that

____ and you show that ________________________.

4. Fill in the blanks in the following proof by contradiction that there is no least positive real number.
Proof: Suppose not. That is, suppose that there is a least positive real number x. [We must deduce _(a)_] Consider the number x/2. Since x is a positive real number, x/2 is also _(b)_ . In addition, we can deduce that $x/2 < x$ by multiplying both sides of the inequality $1 < 2$ by _(c)_ and dividing _(d)_ . Hence x/2 is a positive real number that is less than the least positive real number. This is a _(e)_[Thus the supposition is false, and so there is no least positive real number.]

(a)

(b)

(c)

(d)

(e)

5. Fill in the blanks in the following proof by contrapositive that for all integers n, if 5 does not divide n^2 then 5 does not divide n. (Note: $5 \mid n$ means that 5 divides n or n is divisible by 5.)

Proof (by contrapositive): [The contrapositive is: For all integers n, if $5 \mid n$ then $5 \mid n^2$.]
Suppose n is any integer such that <u>(a)</u>. [We must show that <u>(b)</u>.]
By definition of divisibility, $n =$ <u>(c)</u> for some integer k. By substitution, $n^2 =$ <u>(d)</u> $= 5(5k^2)$. But $5k^2$ is an integer because it is a product of integers. Hence $n^2 = 5\cdot$ (an integer), and so <u>(e)</u> [as was to be shown].

(a)

(b)

(c)

(d)

(e)

6. Prove that the product of any two consecutive integers has the form 3k or 3k+2 for some integer k.

(Use the fact that any integer can be written in one of the 3 forms: 3t, 3t+1, 3t+2 for some integer t.)