Hw 6

7.4.3 (a) $\boxed{10!}$. Because there are ten members

(b) $\boxed{2 \cdot 8!}$ vp × president × 8!

(c) $10-4=6$, $(6+2)! = 8!$

total $= 8!(2\times2) = \boxed{4 \times 8!}$

7.5.8 (c) h = hearts d = diamonds

ways to select = {(5h, 0d), (4h, 1d), (3h, 2d), (2h, 3d), (1h, 4d), (0h, 5d)}

$\boxed{\binom{26}{5}}$ $= \boxed{\binom{13}{5}\binom{13}{0} + \binom{13}{4}\binom{13}{1} + \binom{13}{3}\binom{13}{2} + \binom{13}{2}\binom{13}{3} + \binom{13}{1}\binom{13}{4} + \binom{13}{0}\binom{13}{5} = \binom{26}{5}}$

(d) number of five card hands of same rank = ways to select 4 cards of same rank and 1 other card

$= \boxed{\binom{13}{1}\binom{39}{1}}$

(e) rank of cards in the hand = 2, 3, 4, 5, 6, 7, 8, 9, 10, jack, queen, kins. ace, $\binom{13}{2}$

rank for 3-card combination = 2 rank in full house, $\binom{2}{1}$

suits for 3-card combination = 4 suits, choose 3 $\binom{4}{3}$

suites for 2-card combination = 4 suites, choose 2 $\binom{4}{2}$

$\boxed{\binom{13}{2}\cdot\binom{2}{1}\cdot\binom{4}{3}\cdot\binom{4}{2}}$

(f) select 5 different ranks from 13 $\binom{13}{5}$

select suits for each 5 cards hand: $\binom{4}{1}$ · 5 times

$\boxed{\binom{13}{5}\cdot\binom{4}{1}\cdot\binom{4}{1}\cdot\binom{4}{1}\cdot\binom{4}{1}\cdot\binom{4}{1}}$

7.6.5 (a) 20 main dishes, 5 friends,

the number of different possibles, for the waiter need to place all 5 dishes in the center of the table.

$\boxed{\binom{20}{5}}$

(b) the number of different possible for waiters must remember who ordered which dish as 5 times of 20 main dish which is

$20 \cdot 20 \cdot 20 \cdot 20 \cdot 20 = \boxed{20^5}$

(c) the number of different possible for waiters remember the dish, no two people will order the same dish.

$\boxed{P(20, 5)}$

7.7.2 (a) # of card $= 52$

# of club $= 13$

$C(n,r) = \frac{n!}{(r!(n-r)!)}$

$C(52, 5) = \frac{52!}{5!(52-5)!} = \frac{52!}{5!\,47!}$

5 card-hand $= 5$

# of 5 card hand combination formed without at least one club $= 52 - 13 = 39$

$C(n,r) = C(39,5) = \frac{39!}{5!\,(39-5)!} = \frac{39!}{5!\,34!}$

together, # of 5 card hand have one club

$= \boxed{\frac{52!}{5!\,47!} - \frac{39!}{5!\,34!}}$ $\binom{52}{5} - \binom{39}{5}$

(b) # of card = 52 5 card hand in 13 rank

# of rank = 13 $= \frac{13!}{5!(13-5)!} = \frac{13!}{5!8!}$

4 suits on desk, possible ways $= 4^5$

# of 5 card-hands with no card in same rank

$= \frac{13!}{5!8!} \times 4^5$

total # of combination forming 5 card hand using 52 cards

$\frac{52!}{5!(52-5)!} = \frac{52!}{5!47!}$

# of 5 card hand with at least 2 card with same rank =

# of 5 card hand form using 52 card − # of combination with no card in same rank

$\boxed{\frac{52!}{5!47!} - \frac{13!}{5!8!} \times 4^5}$ $\boxed{\binom{52}{5} - \binom{13}{5} 4^5}$

7.8.4 (a) # of comic book = 20 $n=5, r=20$

# of children = 5

# of ways to distribute the comic books if there are no restrictions on how many go to each kid other than fact that all 20 will be given out is obtained by substituting the n value and r value

$= n^r = \boxed{5^{20}}$

(b) each kid get 4 books, $n=20$, $k_1=4$, $k_2=4$, $k_3=4$, $k_4=4$, $k_5=4$

# of ways $= \boxed{\binom{20}{4}\binom{16}{4}\binom{12}{4}\binom{8}{4}}$

evenly divided that 4 go to each kid

7.3.4 (a) # of ways choose 25 coins from piles, 4 piles in total

total coin choose 25 coins $= 25+4-1 = 28$

of selected coin from 4 piles $= 4-1 = 3$

# of way select 25 coin $= \boxed{\binom{28}{3}} = \frac{28!}{3!\,25!}$

(b) # of way total coin that at least 5 coins chosen must be quarters

$= 25+4-1-(5\times1) = 28-5 = 23$

# of selected coin from 4 pile $= 4-1 = 3$

# of total $= \boxed{\binom{23}{3}} = \frac{23!}{3!\,20!}$

(c) # of way for 25 coin that only 10 quarters, at most 10 quarters can be selected

$= 25+3-1-10 = 17$

# of selected coin from 4 piles $= 4-1 = 3$

# of total from the default − # of way for 10 quarters

$= \boxed{\binom{28}{3} - \binom{17}{3}} = \frac{28!}{3!\,25!} - \frac{17!}{3!\,14!}$

7.10.2 (a) 35 coins, 5 grandchildren,

# of total coin to total grandchildren $= 35+5-1 = 39$

# of chose way $= 5-1 = 4$

$\boxed{\binom{39}{4}}$

(b) # of distribution that all coin are distinct $\boxed{5^{35}}$

$\frac{39!}{4!}$

(c) same number of coins

$35 / 5 = 7$

$\boxed{\text{\# of ways} = 1}$

each will get 7 coins

(d) distinct coins, same numbers

$$\boxed{\underbrace{\binom{35}{7}\binom{28}{7}\binom{21}{7}\binom{14}{7}\binom{7}{7}}_{5 \text{ kid}}}$$

7.11.5 (b) Daughter 1, 2, 3, 4 = D1, D2, D3, D4

\# of chance next to D1 $= 2 \cdot 8!$

\# of chance next to D2 $= 2 \cdot 8!$

\# of chance next to D3 $= 2 \cdot 8!$

\# D1 and D2 $= 2 \cdot 7!$

\# D2 and D3 $= 2 \cdot 7!$

\# D1 and D3 $= 2 \cdot 7!$

\# D1, D2, D3, D4 $= 2 \cdot 6!$

\# total ways $= 2 \cdot 8! + 2 \cdot 8! + 2 \cdot 8! - 2 \cdot 7! - 2 \cdot 7! - 2 \cdot 7! + 2 \cdot 6!$

$= \boxed{6 \cdot 8! - 6 \cdot 7! + 2 \cdot 6!}$

7.13.2 (a) # total population = 121.4 million

1000000 - 1 = 999999

10000 - 1 = 9999

\# income between 10000 and 1000000

$N = 121.4 \times 1000000 = 121.4 \times 10^6$

$R = 999999 - 9999 = 990000$

\# pigeonhole principle N put into m with $(N > m)$ then at least one combination must contain more than one.

\# people earning same annual income $= \left\lceil \frac{N}{R} \right\rceil = \left\lceil \frac{121.4 \times 10^6}{990000} \right\rceil = \boxed{123}$

7.13.3 (b) Pigeonhole principle converse :

Suppose function f maps a set of n element to target set with k elements, n and k are positive, To guarantee there is element y in the target to which f maps at least b elements from domain, then n must be at least $k(b-1)+1$ or $k+1$

\# of month = 12 $\therefore n = 12$

\# of people born in same month = 20

$\therefore k = 20-1 = 19$ one less than # of born in one month

$k+1 = \boxed{12 \times 19 + 1}$

7.12.23. (c) # snicker + # of twix + and 6 other varieties = 13

As stated, 2 snicker

$13-2=11$

\# of ways $= \binom{11+7-1}{7-1} = \boxed{\binom{17}{6}}$

(d) # of twix ≥ 4

total # $= 13-4 = 9$

\# of ways $= \binom{9+8-1}{8-1} = \binom{16}{7}$

total $= \binom{20}{7}$

for twix $\leq X$

$= \boxed{\binom{20}{7} - \binom{16}{7}}$

(e) snicker ≥ 2, twix ≤ 3

First, snicker ≥ 2, twix ≥ 4

$13-6=7$

$|A \cap B^c|$ # of ways $\binom{7+8-1}{8-1} = \binom{13}{7}$

$\because |A \cap B| = |A| - |A \cap B^c|$

$\therefore A \cap B =$ snicker ≥ 2, twix ≤ 3.

$A =$ snicker ≥ 2, $13-2=11$

\# of ways $\binom{11+8-1}{8-1} = \binom{18}{7} = |A|$

$|A \cap B| = |A| - |A \cap B^c|$

$= \boxed{\binom{18}{7} - \binom{13}{7}}$