

CS 278 Lab: Disproof by Counterexample

Nowadays some proofs are generated by computer. In 1976, the [four color theorem](#) was the first major theorem to be verified using a computer program.

In this assignment you will write a computer program to disprove a statement by finding a counterexample. The statement is the following:

Every odd number from 3 onwards can be written as the sum of a prime number and twice a square.

Some odd numbers (that are greater than or equal to 3) can be written as the sum of a prime number and twice a square. For example, $3=3+2(0^2)$, $5=3+2(1^2)$, $7=5+2(1^2)$, $9=7+2(1^2)$, $11=3+2(2^2)$, $13=5+2(2^2)$, $15=7+2(2^2)$, etc. But not all of the odd numbers can be written like that. You need to write a program that would find the smallest counterexample, that is, the smallest odd number (≥ 3) that cannot be written as the sum of a prime number and twice a square. (Hint: The smallest counterexample lies between 1000 and 10000.)

One way to solve it is the following: generate a list of squares and check odd numbers to see if the number minus twice a square is a prime.

You need to do the following:

- (1) Write a method that takes a natural number n as a parameter and determines if n is a prime number.
- (2) Generate a list of the first 100 squares (from 0 to 99) and put them in an array of size 100. (100 is enough because $100^2=10000$ and we know that there is a counterexample between 1000 and 10000.)
- (3) For every odd number n starting from 3 check whether it can be written as the sum of twice a square and a prime number. To do it, go through the squares in the squares array. For each square, multiply it by 2 and subtract the result from the number n . Use your method from (1) to check whether the difference is a prime number or not. If it is not a prime number then do the same for the next square in the squares array. Otherwise, if the difference is prime, the number n can be written in the desired form so you need to check the next odd number. Continue until you find the smallest counterexample.
- (4) Output the smallest counterexample which you find.

What to submit:

- Submit the source code of your program using Canvas.
- If you write your program in a programming language other than Java, then submit instructions on how to compile and run your program on CS machines.