Table 1.5.1: Laws of propositional logic.

Idempotent laws:	$p \vee p \equiv p$	$p \wedge p \equiv p$
Associative laws:	$(p \vee q) \vee r \equiv p \vee (q \vee r)$	$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$
Commutative laws:	$p \vee q \equiv q \vee p$	$p \wedge q \equiv q \wedge p$
Distributive laws:	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
Identity laws:	$p \vee F \equiv p$	$p \wedge T \equiv p$
Domination laws:	$p \wedge F \equiv F$	$p \vee T \equiv T$
Double negation law:	$\neg\neg p \equiv p$	
Complement laws:	$p \wedge \neg p \equiv F$ $\neg T \equiv F$	$p \vee \neg p \equiv T$ $\neg F \equiv T$
De Morgan's laws:	$\neg(p \vee q) \equiv \neg p \wedge \neg q$	$\neg(p \wedge q) \equiv \neg p \vee \neg q$
Absorption laws:	$p \vee (p \wedge q) \equiv p$	$p \wedge (p \vee q) \equiv p$
Conditional identities:	$p \rightarrow q \equiv \neg p \vee q$	$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$

Part 1 Supply a reason for each step below (name a corresponding propositional logic law) :

$(p \wedge \neg q) \vee (p \wedge q)$ $\equiv p \wedge (\neg q \vee q)$ by (a)____distributive____________________

$\equiv p \wedge (q \vee \neg q)$ by (b)____commutative__________________

$\equiv p \wedge T$ by (c)____complement__________________

$\equiv p$ by (d)____identity______________________

Therefore, $(p \wedge \sim q) \vee (p \wedge q) \equiv p$.

Part 2

Use logical equivalences from Table 1.5.1 to simplify the following expression (find a simple logically equivalent expression): $\neg((\neg p \wedge q) \vee (\neg p \wedge \neg q)) \vee (p \wedge q)$ Note: There are many different ways to simplify. One of them is below. The final result (p) should be the same no matter how you choose to simplify.

$\equiv (\neg(\neg p \wedge q) \wedge \neg(\neg p \wedge \neg q)) \vee (p \wedge q)$ by De Morgan's law

$\equiv ((\neg\neg p \vee \neg q) \wedge (\neg\neg p \vee \neg\neg q)) \vee (p \wedge q)$ by De Morgan's laws

$\equiv ((p \vee \neg q) \wedge (p \vee q)) \vee (p \wedge q)$ by Double negation law

$\equiv (p \vee (\neg q \wedge q)) \vee (p \wedge q)$ by Distributive law

$\equiv (p \vee F) \vee (p \wedge q)$ by Commutative and Complement laws

$\equiv p \vee (p \wedge q)$ by Identity law

$\equiv p$ by Absorption law

Table 1.7.1: Rules of inference known to be valid arguments.

Rule of inference	Name		Rule of inference	Name
p $\underline{p \rightarrow q}$ $\therefore q$	Modus ponens		p $\underline{q}$ $\therefore p \wedge q$	Conjunction
$\neg q$ $\underline{p \rightarrow q}$ $\therefore \neg p$	Modus tollens		$p \rightarrow q$ $\underline{q \rightarrow r}$ $\therefore p \rightarrow r$	Hypothetical syllogism
$\underline{p}$ $\therefore p \vee q$	Addition		$p \vee q$ $\underline{\neg p}$ $\therefore q$	Disjunctive syllogism
$\underline{p \wedge q}$ $\therefore p$	Simplification		$p \vee q$ $\underline{\neg p \vee r}$ $\therefore q \vee r$	Resolution

Part 3. Justify each step in the following proof sequences:

1.

Prove $B \wedge [(B \wedge C) \rightarrow \neg A] \wedge (B \rightarrow C) \rightarrow \neg A$ 1. B hypothesis 2. $(B \wedge C) \rightarrow \neg A$ hypothesis 3. $B \rightarrow C$ hypothesis 4. C Modus ponens with 1 and 3 5. $B \wedge C$ Conjunction with 1 and 4 6. $\neg A$ Modus ponens with 5 and 2	B $(B \wedge C) \rightarrow \neg A$ $\underline{B \rightarrow C}$ $\therefore \neg A$

2.

Prove $\neg A \wedge B \wedge [B \rightarrow (A \vee C)] \rightarrow C$ 1. $\neg A$ hypothesis 2. B hypothesis 3. $B \rightarrow (A \vee C)$ hypothesis 4. $A \vee C$ Modus ponens with 2 and 3 5. $\neg(\neg A) \vee C$ double negation law from 4 6. $(\neg A) \rightarrow C$ conditional identity law from 5 7. C Modus ponens with 1 and 6	$\neg A$ B $\underline{B \rightarrow (A \vee C)}$ $\therefore C$

3.

Prove $[A \rightarrow (B \vee C)] \wedge \neg B \wedge \neg C \rightarrow \neg A$ 1. $A \rightarrow (B \vee C)$ hypothesis 2. $\neg B$ hypothesis 3. $\neg C$ hypothesis 4. $\neg B \wedge \neg C$ Conjunction with 2 and 3 5. $\neg (B \vee C)$ De Morgan law from 4 6. $\neg A$ Modus Tollens with 5 and 1	$A \rightarrow (B \vee C)$ $\neg B$ $\neg C$ ——— $\therefore \neg A$

Part 4. Use truth table to determine whether each of the following arguments is valid or not.

- $\neg (A \vee \neg B)$
  $\underline{C \rightarrow B}$
  $\therefore (\neg A \wedge C)$

			hypotheses		conclusion
A	B	C	$\neg (A \vee \neg B)$	$C \rightarrow B$	$\neg A \wedge C$
T	T	T	F	T	F
T	T	F	F	T	F
T	F	T	F	F	F
T	F	F	F	T	F
F	T	T	T	T	T
F	T	F	T	T	F
F	F	T	F	F	T
F	F	F	F	T	F

There are 2 rows where both hypotheses are true (rows 5 and 6). The conclusion in row 6 is F; therefore, the argument is not valid.
In other words, when A=F, B=T, and C=F, both hypotheses are true but the conclusion is false. The argument is not valid.

- $\neg A \wedge C$
  $\underline{B \rightarrow A}$
  $\therefore (\neg B \wedge C)$

			hypotheses		conclusion
A	B	C	$\neg A \wedge C$	$B \rightarrow A$	$\neg B \wedge C$
T	T	T	F	T	F
T	T	F	F	T	F
T	F	T	F	T	T
T	F	F	F	T	F
F	T	T	T	F	F
F	T	F	F	F	F
F	F	T	T	T	T
F	F	F	F	T	F

There is one row where both hypotheses are true – row 7 (A=F, B=F, C=F). In this row, the conclusion is also true. Therefore, the argument is valid.

Part 5. For each exercise, decide what conclusion, if any, can be reached from the given hypotheses.

1. If the car was involved in the hit-and-run, then the paint would be chipped. But the paint is not chipped.

Let p= the car was involved in the hit-and-run, q= the paint is chipped. The hypotheses that we have are $p \longrightarrow q$ and $\neg q$. We can use Modus Tollens. The conclusion is that the car was not involved in the hit-and-run ($\neg p$).

2. Either the weather will turn bad or we will leave on time. If the weather turns bad, then the flight will be cancelled.

Let p= the weather turns bad, q= we will leave on time. Then, "the flight is cancelled" is $\neg q$. The hypotheses that we have are $p \vee q$ and $p \longrightarrow \neg q$. No conclusion can be reached about individual truth values of p and q because both hypotheses are true in two cases: 1) when p=T and q=F, and 2) when p=F and q=T. Therefore, p can be either T or F, and q can be either T or F.

If we want to be precise, we can conclude $p \oplus q$ (exclusive or). It can be shown using truth tables.

3. If the bill was sent today, then you will be paid tomorrow. You will be paid tomorrow.

Let p= the bill was sent today, q= you will be paid tomorrow, Then, hypotheses are $p \longrightarrow q$ and q. No conclusion can be reached about p because it could be either T or F when both hypotheses are true.

4. The grass needs mowing and the trees need trimming. If the grass needs mowing then we need to rake the leaves.

Let p=the grass needs mowing, q= the trees need trimming, r= we need to rake the leaves. Then, the hypotheses are $p \wedge q$, $p \longrightarrow r$. We can conclude that we need to rake the leaves (r). This can be obtained in two steps. First, use simplification to get p from $p \wedge q$. Second, use Modus ponens to get r from p and $p \longrightarrow r$.

Part 6. Use rules of inference to prove that the following argument is valid.

- $(A \rightarrow B)$
  $\underline{A \rightarrow (B \rightarrow C)}$
  $\therefore (A \rightarrow C)$

1. $A \rightarrow B$ hypothesis
2. $A \rightarrow (B \rightarrow C)$ hypothesis
3. $\neg A \vee \neg B \vee C$ conditional identity applied twice to 2
4. $\neg B \vee \neg A \vee C$ commutative law from 3
5. $B \rightarrow (A \rightarrow C)$ conditional identity applied twice to 4
6. $A \rightarrow (A \rightarrow C)$ hypothetical syllogism from 1 and 5
7. $\neg A \vee \neg A \vee C$ conditional identity applied twice to 6
8. $\neg A \vee C$ idempotent law from 7
9. $A \rightarrow C$ conditional identity from 8