HW 3

1. 4.1.3 (c) $f(x) = \sqrt{x^2}$

Yes, a function. $\{x : x \in \mathbb{R}\}$

2. 4.1.5 Express the range using roster notation

(j) Let $A = \{1, 2, 3\}$ $f : P(A) \to \mathbb{Z}$. For $X \subseteq A$, $f(x) = |X|$

$P(A) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$

$f(\emptyset) = 0$ $f(\{1\}) = f(\{2\}) = f(\{3\}) = 1$

$f(\{1,2\}) = f(\{1,3\}) = f(\{2,3\}) = 2$

$f(\{1,2,3\}) = 3$

$\boxed{\text{Range in } f = \{0, 1, 2, 3\}}$

(k) Let $A = \{1, 2, 3\}$, $f : P(A) \to P(A)$. For $X \subseteq A$, $f(X) = X \cup \{1\}$.

$P(A) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$

$f(\emptyset) = \{1\}$ $f(\{1\}) = \{1\}$ $f(\{2\}) = \{1,2\}$ $f(\{3\}) = \{1,3\}$

$f(\{1,2\}) = \{1,2\}$ $f(\{1,3\}) = \{1,3\}$ $f(\{2,3\}) = \{1,2,3\}$

$f(\{1,2,3\}) = \{1,2,3\}$

$\boxed{\text{Range in } f = \{\{1\}, \{1,2\}, \{1,3\}, \{1,2,3\}\}}$

3. 4.2.3 Compute the value of each expression

(a) $\lfloor -3.7 \rfloor = \boxed{-4}$

(b) $\lceil -4.2 \rceil = \boxed{-4}$

(d) $\lfloor \lfloor 3.5 \rfloor - 4.3 \rfloor = \lfloor 3 - 4.3 \rfloor = \boxed{-1}$

(e) $\lfloor \frac{3}{2} + \lceil \frac{1}{3} \rceil \rfloor = \lfloor 1.5 + 1 \rfloor = \boxed{2}$

4. 4.3.4

(d) $f : \{0,1\}^3 \to \{0,1\}^4$. The output of f is obtained by taking the input string and adding an extra copy of the first bit to the end of the string. So $f(100) = 1001$

$f(000)=0000$, $f(001)=0010$ $f(010)=0100$ $f(011)=0110$

$f(100)=1001$, $f(101)=1011$ $f(110)=1101$ $f(111)=1111$

One-to-one, not onto. For example, 1000 is missing.

(f) $A\{1,2,3,4,5,6,7,8\}$ $f: P(A)\to P(A)$, $X\subseteq A$, $f(x)=A-X$

Onto and one-to-one. For example, let $Z\in P(A)$, $Z\in P(A)\Rightarrow Z\subseteq A$

$\therefore A-Z\subseteq A$ $f(A-Z)=A-(A-Z)=Z$ $\therefore$

$A-Z\in P(A)$ $\therefore$ Onto

Let $X, Y\in P(A)$, so $f(x)=f(y)$

$f(x)=f(y)\Rightarrow A-X=A-Y$

$X=Y$

$\therefore$ if $X\neq Y$, $f(x)\neq f(y)$, one-to-one

(g) $A\{1,2,3,4,5,6,7,8\}$, $B=\{1\}$, $f: P(A)\to P(A)$, $X\subseteq A$, $f(x)=X-B$

Not one-to-one, not onto. For example, $\{1,2,3\}\in P(A)$

$\{1,2,3\}-\{1\}=\{2,3\}$, $\{2,3\}\in P(A)$

$\{2,3\}-1=\{2,3\}$ not one-to-one.

$\{1,2,3\}\in P(A)$ but it is not the image of any $X\in P(A)$, not onto

(i) $A=\{a,b,c\}$, $h: P(A)\to P(A)$, For $X\subseteq A$, $h(X)=X\cup\{a\}$.

Not one-to-one, not onto. For example, let $X_1=\{b,c\}$, $X_2=\{a,b,c\}$

$h(X_1)=\{b,c\}\cup\{a\}=\{a,b,c\}$; $h(x_2)=\{a,b,c\}\cup\{a\}=\{a,b,c\}$

$X_1\neq X_2$ but $h(x_1)=h(x_2)$, not one-to-one

Also, let $\{b\}\in P(A)$

$\{b\}\subseteq A=\{a,b,c\}$

$\nexists\, h(x)=\{b\}=X\cup\{a\}=\{b\}$

$a\notin\{b\}$, not onto

[illegible] sum & Decision Tree
Adaline

5. 4.4.2 (b) $f: Z \to Z$, $f(x) = 2x + 34$, Let $f(x) = y$

$y = 2x + 34$

$2x = y - 34$

$x = \frac{y-34}{2}$

$f^{-1}(y) = \frac{y-34}{2}$

if $x = 1$, $f(1) = 36$

$f^{-1}(x) = \frac{x-2}{3} = f^{-1}(36) = \frac{x-2}{2} = \frac{34}{3}$

Therefore, $\boxed{\text{not well-defined}}$

(h) $f: \{0,1\}^3 \to \{0,1\}^3$

$\boxed{\text{Well defined}}$, because it is one-to-one, $f^{-1}: \{0,1\}^3 \to \{0,1\}^3$, $f^{-1}(xyz) = zxy$

such like remove the last digit to first

(j) $f: Z \times Z \to Z \times Z$, $f(x,y) = (1-y, x)$

$\boxed{f^{-1}(x,y) = (y, 1-x)}$, well defined inverse.

6. 4.5.7 (a) what is $d^{-1}(1001)$?

$\boxed{d^{-1}(1001) = 1100}$

(b) which function is not well-defined, $f \circ g$ or $g \circ f$?

$\boxed{f \circ g \text{ is not well defined}}$ $\because g\ \{0,1\}^3$ & $f\ \{0,1\}^4$

$g \circ f$ is $\{0,1\}^4$ to $\{0,1\}^4$

(c) what is the range of $g \circ f$?

$g \circ f = \{0,1\}^4 \to \{0,1\}^3$

$\boxed{\text{Range} = \{001, 011, 101, 111\}}$

(d) what is $(f \circ d)(1011)$?

$(f \circ d)(1011) = f(d(1011)) = f(1110) = \boxed{1111}$

$d(1011) = 1110$

7. 4.6.1 (d) $\frac{6^{2k-1}}{6^{-k}} = 6^{2k-1} \cdot 6^{k} = \boxed{6^{3k-1}}$

(e) $\frac{6^{2k}}{6} = 6^{2k} \cdot 6^{-1} = \boxed{6^{2k-1}}$

(f) $36 \cdot 6^{k} = 6^{2} \cdot 6^{k} = \boxed{6^{k+2}}$

(g) $6^{k^2} \cdot 6 = 6^{k^2} \cdot 6^{1} = \boxed{6^{k^2+1}}$

8. 4.6.2 (b) $2 \cdot \log_5 k = \boxed{\log_5 k^2}$

(c) $\log_5 k - \log_5 7 = \boxed{\log_5 \left(\frac{k}{7}\right)}$

(d) $(\log_3 k)(\log_3 5) = \frac{\log_3 k}{\log_3 5} = \boxed{\log_5 k}$

(e) $(\log_3 (k^2))/(\log_3 25) = \frac{\log_5 k^2}{\log_5 3} / \frac{\log_5 25}{\log_5 3} = \frac{\log_5 k^2}{\log_5 25} = \frac{2\log_5 k}{2\log_5 5} = \boxed{\log_5 k}$

9. 4.6.3 (b) $4^{\log_2 x} = y$

$\log_2 4^{\log_2 x} = \log_2 y$

$(\log_2 x)(\log_2 4) = \log_2 y$

$(\log_2 x)(\log_2 2^2) = \log_2 y$

$(\log_2 x) 2 = \log_2 y$

$\log_2 x^2 = \log y$

$\boxed{x^2} = y$

(c) $2^{\log_4 x} = y$

$\log_4 2^{\log_4 x} = \log_4 y$

$(\log_4 x)(\log_4 2) = \log_4 y$

$(\log_4 x)(\log_4 4^{1/2}) = \log_4 y$

$(\log_4 x) \cdot \frac{1}{2} = \log_4 y$

$\log_4 x^{\frac{1}{2}} = \log_4 y$

$\boxed{x^{\frac{1}{2}}} = y$

10. 4.7.1 (d) $[77 \cdot (-65) + 147] \bmod 7$

$= (7 \times 11)(-65) \bmod 7 + (7 \times 21) \bmod 7$

$= 0 (-65) \bmod 7 + 0 \bmod 7$

$= \boxed{0}$

(e) $44^{12} \bmod 6$

$= (44 \bmod 6)^{12} \bmod 6$

$= (1)^{6} \bmod 6 \cdot (2^2 \bmod 6) \bmod 6$

$= (1^{6} \bmod 6)(2^2 \bmod 6) \bmod 6$

$= (1 \bmod 6)(4 \bmod 6) \bmod 6$

$= (1 \cdot 4) \bmod 6$

$= \boxed{4}$

11. 4.7.5.(b) mod 13, {-63, -54, -41, 11, 13, 76, 80, 130, 132, 137}

Congruent to 0 mod 13 = 13, 130
Congruent to 2 mod 13 = -63, 80, 132
Congruent to 7 mod 13 = 137
Congruent to 11 mod 13 = -54, -41, 11, 76

12. 5.1.2 (c) $\overline{(x+y)(uv+\bar{x}\bar{y})}$

$= \overline{(x+y)} + \overline{(uv+\bar{x}\bar{y})}$

$= \bar{x}\bar{y} + (\overline{uv})(\overline{\bar{x}\bar{y}})$

$= \bar{x}\bar{y} + (\bar{u}+\bar{v})(\bar{\bar{x}}+\bar{\bar{y}})$

$= \boxed{\bar{x}\bar{y} + (\bar{u}+\bar{v})(x+y)}$

13. 5.2.3 (d) $f(x,y,z) = \bar{x}\bar{y}\bar{z} + \bar{x}yz + x\bar{y}z + xy\bar{z}$

x	y	z	$f(x,y,z)$	$\bar{x}$	$\bar{y}$	$\bar{z}$	$\overline{xyz}$	$\bar{x}yz$	$x\bar{y}z$	$xy\bar{z}$
0	0	0	1	1	1	1	1	0	0	0
0	0	1	0	1	1	0	0	0	0	0
0	1	0	0	1	0	1	0	0	0	0
0	1	1	1	1	0	0	0	1	0	0
1	0	0	0	0	1	1	0	0	0	0
1	0	1	1	0	1	0	0	0	1	0
1	1	0	1	0	0	1	0	0	0	1
1	1	1	0	0	0	0	0	0	0	0

Answers →

14. 5.3.5 (a) $\overline{\bar{x}\bar{y}\bar{z} + \bar{x}yz + x\bar{y}z + xyz} = (\overline{\bar{x}\bar{y}\bar{z}})(\overline{\bar{x}yz})(\overline{x\bar{y}z})(\overline{xyz})$

$= (\bar{\bar{x}}+\bar{\bar{y}}+\bar{\bar{z}})(\bar{\bar{x}}+\bar{y}+\bar{z})(\bar{x}+\bar{\bar{y}}+\bar{z})\cdot(\bar{x}+\bar{y}+\bar{z})$

$= \boxed{(x+y+z)(x+\bar{y}+\bar{z})(\bar{x}+y+\bar{z})(\bar{x}+\bar{y}+\bar{z})}$ in CNF

(b)

x	y	z	f(x, y, z)	$\overline{f}(x,y,z)$
0	0	0	0	1
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	1	0
1	1	1	0	1

$\overline{f}(x,y,z) = \boxed{\bar{x}\bar{y}\bar{z} + \bar{x}yz + x\bar{y}z + xyz}$ in DNF

(c) $f(x,y,z) = \overline{\text{DNF}(\overline{f}(x,y,z))}$

$= \overline{(\bar{x}\bar{y}\bar{z} + \bar{x}yz + x\bar{y}z + xyz)}$

$= \boxed{(x+y+z)(x+\bar{y}+\bar{z})(\bar{x}+y+\bar{z})(\bar{x}+\bar{y}+\bar{z})}$ in CNF

15. 5.4.4 (a) $X_{A1}, X_{A2}, X_{B1}, X_{B2}, X_{C1}, X_{C2}, X_{D1}, X_{D2}, X_{E1}, X_{E2}$ in Boolean expression

$X_{A1} = 1$ is class A is scheduled during time slot 1

Course to be scheduled in exactly one of the two periods are:

$(X_{A1}+X_{A2})(\overline{X_{A1}}+\overline{X_{A2}})(X_{B1}+X_{B2})(\overline{X_{B1}}+\overline{X_{B2}})(X_{C1}+X_{C2})(\overline{X_{C1}}+\overline{X_{C2}})(X_{D1}+X_{D2})$
$(\overline{X_{D1}}+\overline{X_{D2}})$

Pairs of courses cannot scheduled at the same time:

$(\overline{X_{B1}}+\overline{X_{C1}})(\overline{X_{B2}}+\overline{X_{C2}})(\overline{X_{A1}}+\overline{X_{D1}})(\overline{X_{A2}}+\overline{X_{D2}})(\overline{X_{C1}}+\overline{X_{D1}})(\overline{X_{C2}}+\overline{X_{D2}})$
$(\overline{X_{B1}}+\overline{X_{A1}})(\overline{X_{B2}}+\overline{X_{A2}})$

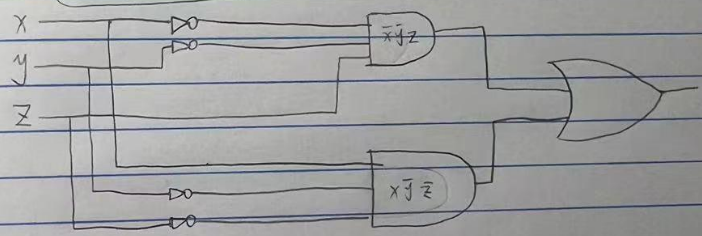
Combine together:

$(X_{A1}+X_{A2})(\overline{X_{A1}}+\overline{X_{A2}})(X_{B1}+X_{B2})(\overline{X_{B1}}+\overline{X_{B2}})(X_{C1}+X_{C2})(\overline{X_{C1}}+\overline{X_{C2}})(X_{D1}+X_{D2})(\overline{X_{D1}}+\overline{X_{D2}})$
$(\overline{X_{B1}}+\overline{X_{C1}})(\overline{X_{B2}}+\overline{X_{C2}})(\overline{X_{A1}}+\overline{X_{D1}})(\overline{X_{A2}}+\overline{X_{D2}})(\overline{X_{C1}}+\overline{X_{D1}})(\overline{X_{C2}}+\overline{X_{D2}})(\overline{X_{B1}}+\overline{X_{A1}})$
$(\overline{X_{B2}}+\overline{X_{A2}})$

(b) $(x_{A1} + x_{A2})(\bar{x}_{A1} + \bar{x}_{A2})(x_{B1} + x_{B2})(\bar{x}_{B1} + \bar{x}_{B2})(x_{C1} + x_{C2})(\bar{x}_{C1} + \bar{x}_{C2})(x_{D1} + x_{D2})$
$(\bar{x}_{D1} + \bar{x}_{D2})(\bar{x}_{B1} + \bar{x}_{C1})(\bar{x}_{B2} + x_{C2})(\bar{x}_{A1} + \bar{x}_{D1})(\bar{x}_{A2} + \bar{x}_{D2})(\bar{x}_{C1} + \bar{x}_{D1})(\bar{x}_{C2} + \bar{x}_{D2})$
$(\bar{x}_{B1} + \bar{x}_{A1})(\bar{x}_{B2} + \bar{x}_{A2})$

(c) Not satisfiable, because no matter which value assign to the boolean variable, the expression results to false,

16, 5.5.2 (b) $\boxed{f(x, y, z) = \bar{x}\bar{y}z + x\bar{y}\bar{z}}$



17, 5.5.4 (b) $f(x, y, z) = \bar{x}yz + x\bar{y}z + xy\bar{z} + xyz$