MATHEMATICAL FORMULATION

Sets

$$I: Set\ of\ Dishes$$

$$J: Set\ of\ meals\ (breakfast,\ lunch,\ dinner)$$

$$K: Set\ of\ days\ of\ the\ week$$

Decision Variables

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$\boldsymbol{x_{ijk}}$	Quantity of portions of 100gr from dish i for meal j and in day k
$\boldsymbol{y_{ijk}}$	Auxiliary binary variable to determinate if dish i for meal j and in day k will be used

Objective Functions

Minimize the environmental impact or maximize the protein value of the meal plan; this choice will be decided by the user. Depending on the priority, the optimization will vary solving first the top priority and then, the second priority. For the footprint, the model has a score of sustainability that combines the CO_2 emissions and water usage, both with the same weight. However, these metrics have been standardized before, using the method of Min-Max scale, to avoid problems of scale. In the case of Protein Maximization, the model is multiplied by -1 to convert it into a minimization problem. The objective is to minimize the total environmental impact or maximize the protein value of the meal plan for a week.

$$Sustainability\ Score_i = 0.5\ \times CO2_i + \ 0.5\ \times H2O_i$$

$$Min \sum_{\mathrm{i}=1}^{502} \sum_{j=1}^{3} \sum_{k=1}^{7} X_{ijk} \times Sustainability\ Score_i$$

$$Min \sum_{i=1}^{502} \sum_{j=1}^{3} \sum_{k=1}^{7} - X_{ijk} \times \ P_i$$

Constraints

1. The model has to ensure to select only one dish per meal. If y=1 then X can take a value above 0; but if y=0 then X is zero, therefore only one dish will be possible per meal. M=1000 helps ensure this.

$$X_{ijk} \leq M \times y_{ijk} \quad \forall \text{ i} \in \text{I} ; \forall j \in J ; \forall k \in K$$

$$\sum_{i=1}^{512} y_{ijk} = 1 \quad \forall j \in J ; \forall k \in K$$

2. For each day, the meal plan should provide 3 dishes.

$$y_{ijk} \leq X_{ijk} \quad \forall i \in I ; \forall j \in J ; \forall k \in K$$

3. All dishes must be different for the weekly mean plan, to avoid repeating dishes.

$$\sum_{j=1}^{3} \sum_{k=1}^{7} y_{ijk} \leq 1 \quad \forall i \in I$$

4. Selected dishes should belong to their meal category (breakfast, lunch, dinner). This is because some dishes are only available at breakfast, while others are only for lunch and dinner.

$$X_{ijk} \leq M \cdot B_i \quad \forall i \in I; j = 1; \forall k \in K$$
$$X_{ijk} \leq M \cdot (1 - B_i) \quad \forall i \in I; \ j = \{2,3\}; \forall k \in K$$

5. Vegetarians should receive suggestions of only vegetarian dishes in their meal plan.

$$\text{if vegetarian option is selected: } X_{ijk} \leq M \times V_i \quad \forall\, i \in I;\ \forall\, j \in J;\ \forall\, k \in K$$

6. The maximum number of portions (100gr) in each meal will depend on two choices:

- **Vegetarian preference:** vegetarian plans require more portions to fulfill minimum protein requirements.
- **Fitness Goal:** a person who wants to lose weight should restrict more the number of portions than someone who wants to gain muscle.

$$\sum_{i=1}^{512} X_{ijk} \leq \max \text{P ortions}_{\text{i}} \quad \forall\, j \in J;\ \forall\, k \in K$$

$$\text{if vegetarian option and muscle gain is selected: } \text{max\_portions}_1 = 4$$

$$\text{if vegetarian option and weight loss is selected: } \text{max\_portions}_2 = 3$$

$$\text{if only weight loss is selected: } \text{max\_portions}_3 = 2$$

$$\text{if only muscle gain is selected: } \text{max\_portions}_4 = 3$$

7. Minimum (Maximum) daily requirements (suggestion): The project will only consider two important factors (protein and sugar, but the same logic can expand to other micronutrients and micronutrients). These thresholds have been chosen according to the user's goal (weight loss or muscle gain). Also, the thresholds depend on the user's weight, for the case of the protein requirement, and on the user's sex, for the case of the sugar suggestion.

$$\text{min\_protein}_1 = 1 \times \text{weight\_kg} \quad \text{if the target is weight loss}$$

$$\text{min\_protein}_2 = 1.7 \times \text{weight\_kg} \quad \text{if the target is muscle gain}$$

$$\sum_{j=1}^{3}\sum_{i=1}^{502} X_{ijk} \times P_i \geq \text{minProtein} \quad \forall\, k \in K$$

max_sugar $= 25$, if sex = women; else, max_sugar= 36

$$\sum_{j=1}^{3}\sum_{i=1}^{502} X_{ijk} \times S_i \leq \text{max\_}sugar \quad \forall\, k \in K$$

8. Budget constraint: The user will provide its budget to ensure that the cost of the meal plan is below or equal to its budget

$$\sum_{i=1}^{502}\sum_{j=1}^{3}\sum_{k=1}^{7} X_{ijk} \times C_i \leq Budget$$