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CS 430-02
Homework 1

1. (a)

$n^{\frac{1}{\lg n}}$
n
n^2
$(\lg n)^{\lg n}$
1
$2^{\lg n}$
$n^2 + n$
$n^{\lg \lg n}$

- $1 \le n$ by definition
- $n \le n^2$ - both positive integers
- $n^{1/\lg n} = ?$

↳ change of base: $\log_b m = \frac{\log_a m}{\log_a b}$

$$\frac{1}{\lg(n)} = \frac{1}{\log_2(n)}$$
$$= \frac{\log_2(2)}{\log_2(n)}$$
$$= \log_n(2)$$

$$n^{\log_n(2)} = 2 \longrightarrow n^{1/\lg(n)} = 2 \text{ for all } n$$

- $\lg(n)^{\lg(n)}$

exponent will increase with n

$$\lg(n) \longrightarrow 2^{?} = n$$

$$\lim_{n \to \infty} = \infty \qquad \lg(n)^{\lg(n)} = \text{increasing \#}^{\text{increasing \#}}$$

- $2^{\lg(n)} = 2^{\log_2(n)} = n$

- $n^2 + n \longrightarrow$ only keeping higher order term

$$\longrightarrow O(n^2 + n) = O(n^2)$$

• $n^{\lg(\lg n)}$

$\hookrightarrow n = 2^{\log_2(n)}$

$\hookrightarrow \lg(\lg(n)) = \log_2(\log_2(n))$

$$\Rightarrow \quad n^{\lg(\lg(n))} = \left(2^{\log_2 n}\right)^{\log_2(\log_2(n))}$$

$$= 2^{\log_2 n \times \log_2(\log_2(n))}$$

$$= \left(2^{\log_2(\log_2(n))}\right)^{\log_2(n)}$$

$$= \log_2(n)^{\log_2(n)}$$

$$\boxed{1,\ n^{1/\lg(n)},\ n,\ 2^{\lg(n)},\ n^2,\ n^2+n,\ n^{\lg(\lg(n))},\ \lg(n)^{\lg(n)}}$$

(b) $n^2 + n = \Theta(n^2)$

$$n^{\lg(\lg(n))} = \lg(n)^{\lg(n)} = \Theta\left(\lg(n)^{\lg(n)}\right)$$

$$\boxed{1,\ n^{1/\lg(n)},\ n,\ 2^{\lg(n)},\ (n^2,\ n^2+n),\ \left(n^{\lg(\lg(n))},\ \lg(n)^{\lg(n)}\right)}$$

2.

Insertion Sort (A)

```
for j ← 2 to (length(A)):
    key ← A_j
    i ← j - 1
    while i > 0 and key > A_i :
        A_{i+1} ← A_i
        i ← i - 1
    A_{i+1} ← key
```

3.

	n	n^2	2^n
example: n^2	Ω	θ	O
$\frac{n^3}{10} + 3n$	Ω	Ω	O
$n\log n^4$	Ω	O	O
$(n-2)!$	Ω	Ω	Ω
$2n + 2n^{-1}$	Θ	O	O

(Work shown below)

- $\frac{n^3}{10} + 3n = \Theta(n^3)$

$$\lim_{n\to\infty} \frac{n^3}{n} = \infty$$

$$\lim_{n\to\infty} \frac{n^3}{n^2} = \infty$$

$$\lim_{n\to\infty} \frac{n^3}{2^n} = 0$$

- $n \cdot \log(n^4) = ?$

$$\frac{d}{dx} \log_a x^b = \frac{b \cdot \log_a e}{x}$$

$$\lim_{n\to\infty} \frac{n \cdot \log(n^4)}{n} = \lim_{n\to\infty} (\log(n^4))$$

$$= \infty$$

$$\lim_{n\to\infty} \frac{n \cdot \log(n^4)}{n^2} = \lim_{n\to\infty} \frac{\log(n^4)}{n}$$

$$= \lim_{n\to\infty} \frac{\frac{d}{dn} \log(n^4)}{\frac{d}{dn} n}$$

$$= \lim_{n\to\infty} \frac{\left(\frac{4 \cdot \log_2(e)}{n}\right)}{1}$$

$$= 0$$

- $2n + 2n^{-1} = \Theta(n)$

$$\lim_{n\to\infty} \frac{n\cdot\log(n^4)}{2^n} = \lim_{n\to\infty} \frac{\frac{d}{dn} n\cdot\log(n^4)}{\frac{d}{dn} 2^n}$$

$$= \lim_{n\to\infty} \frac{\log(x^4) + 4\cdot\log(e)}{\ln(2)\cdot 2^n}$$

$$= \lim_{n\to\infty} \frac{\frac{4\cdot\log(e)}{n} + 0}{\ln(2)\cdot\ln(2)\cdot 2^n}$$

$$= \lim_{n\to\infty} \frac{4\cdot\log(e)}{n\cdot\ln(2)\cdot\ln(2)\cdot 2^n}$$

$$= 0$$

- $(n-2)! = n^{n-2} + \ldots = \Theta(n^{n-2})$

$$\lim_{n\to\infty} \frac{n^{n-2}}{n} = \infty$$

$$\lim_{n\to\infty} \frac{n^{n-2}}{n^2} = \infty$$

$$\lim_{n\to\infty} \frac{n^{n-2}}{2^n} = \infty$$

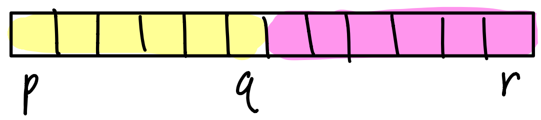
- $2n + 2n^{-1} = \Theta(n)$

$$\lim_{n\to\infty} \frac{n}{n} = 1$$

$$\lim_{n\to\infty} \frac{n}{n^2} = 0$$

$$\lim_{n\to\infty} \frac{n}{2^n} = 0$$

4. A(p, q, r)



merge(A, p, q, r)

```
L ← A[p..q+1]
R ← A[q+1..r+1]
sorted ← []
while length(L) ≥ 1 and length(R) ≥ 1:
    if R_1 ≤ L_1:
        sorted.append(R_1)
        R ← R[2.. length(R)]
    else:
        sorted.append(L_1)
        L ← L[2.. length(L)]

if length(L) == 0 :
    for i ← 1 to length (R)
        sorted.append R_i

else if length(R) == 0:
    for j ← 1 to length (L)
        sorted.append L_j
```