Assignment 2 - Search

In this assignment, you are going to complete the implementation of a state representation for the Traveling Salesman Problem and perform several simulations of simulated annealing.

What you need to do:
1. Follow the instructions and complete the parts with **# TODO**.
2. Complete the implementations.
3. Run experiments to search for the best setting of parameters **k** and **lam**.
4. Report the results using tables.
5. Discuss your findings.

Your Information

TODO: Enter your information. Name: Jane Downer CWID: A20452471 Section: 02

Note -- my own functions and methods are in `helper.py`.

In [1]:
```
import helper
from   helper import *
```

Implementations

TODO: Complete the implementation of `TSPNode` and `read_TSP_from_file`. See below.

In [2]:
```
class Node:
    def __init__(self, state, parent = None):
        self.state = state
        self.parent = parent

    def __repr__(self):
        return "Node: {}".format(self.state)

    def path(self):
        current = self
        path_back = [current]
        while current.parent is not None:
            path_back.append(current.parent)
            current = current.parent
        return reversed(path_back)

    def expand(self):
        raise NotImplementedError

    def value(self):
        raise NotImplementedError
```

In [3]:
```
class TSPNode(Node):
    _random_state = None
```

```
    _distances    = None

    def __init__(self, state, parent = None):
        """
        A state is an ordered list of cities. For e.g., ["A", "C", "D", "B"].
        This represents the solution of A - C - D - B - A.
        """
        super().__init__(state, parent)

    def __repr__(self):
        return "Node: <{}> {:.1f}".format(" ".join(self.state), self.value())

    def expand(self):
        """
        Generate one random neighbor using the TSPNode._random_state.

        The random neighbor should be generated as follows. Pick two cities at r

        return:

        [neighbor_node]: a list of one TSPNode whose parent is this node.
        """''
        two_random    = random.sample(self.state, 2)
        current_state = list(self.state)
        new_state     = current_state.copy()
        index_1 = current_state.index(two_random[0])
        index_2 = current_state.index(two_random[1])
        new_state[index_1] = two_random[1]
        new_state[index_2] = two_random[0]
        neigbhor_node = TSPNode(state = new_state)
        return [neigbhor_node]

    def value(self):
        """
        Calculate the total cost.

        return:
        -1*total_distance: the total cost of current state

        """
        total_distance = 0
        n = len(self.state)

        for i in range(n):
            from_c = self.state[i]
            if i == n-1:
                to_c = self.state[0]
            else:
                to_c = self.state[i+1]

            total_distance += TSPNode._distances[from_c][to_c]

        return -1*total_distance
```

In [4]:

```
class Graph:
    def __init__(self):
        self.distances = defaultdict(dict)

    def add_edge(self, from_c, to_c, dist):
```

```
        self.distances[from_c][to_c] = dist
        self.distances[to_c][from_c] = dist
```

In [5]:

```
def make_graph(city_coords, dist_mat):
    """
    Create a graph for the given TSP

    param:
    city_coords: dictionary of cities
    dist_mat: distance matrix

    return:
    graph: an instance of Graph class that saved all necessary edges and costs

    """
    graph = Graph()

    cities = list(city_coords.keys())

    for i in range(len(cities)-1):
        from_c = cities[i]
        for j in range(i+1, len(cities)):
            to_c = cities[j]
            dist = dist_mat[i][j]
            graph.add_edge(from_c, to_c, dist)
    return graph
```

In [6]:

```
def init_state(seed, city_coords):
    """
    Create an initial state node

    param:
    seed: random seed
    city_coords: dictionary of cities

    return:
    initial_state: an instance of TSPNode class with a randomly generated state

    """
    rand_state = np.random.RandomState(seed=seed)

    cities         = list(city_coords.keys())
    shuffle_cities = list(rand_state.permutation(cities))
    initial_state  = TSPNode(shuffle_cities)
    return initial_state
```

In [7]:

```
def exp_schedule(k, lam):
    """
    The exponential schedule function for simulated annealing

    param:
    k: initial temperature
    lam: cooling factor lam

    return:
    a function that accepts the current number of iteration as input and outputs
```

```python
    """
    return lambda t: k * np.exp(-lam * t)

def linear_schedule(k, lam):
    return lambda t: max(0, k - lam*t)


def log_schedule(k, lam):
    return lambda t: k / (1+lam*np.log(t+1))
```

In [8]:

```python
def simulated_annealing(initial_n, temp_schedule, max_iter, random_state):
    """
    Simulated annealing algorithm

    param:
    initial_n: initial state
    temp_schedule: temperature schedule function
    max_iter: the max number of iterations
    random_state: random state used to select random node or generate probabilit

    return:
    current_n: an instance of TSPNode as solution state

    """
    current_n = initial_n
    for t in range(max_iter):

        T = temp_schedule(t)
        next_nodes = current_n.expand()

        if len(next_nodes) == 0:
            return current_n
        else:
            next_n  = random_state.choice(next_nodes)

            delta_e =  next_n.value() - current_n.value()

            if delta_e > 0:
                current_n = next_n
            else:
                p = np.exp(delta_e/T)
                if random_state.random() < p:
                    current_n = next_n
    return current_n
```

In [9]:

```python
def check_margin(x, y, x_inner_lim, y_inner_lim):
    if x < x_inner_lim[0]:
        return True
    elif x > x_inner_lim[1]:
        return True
    elif x_inner_lim[0] <= x <= x_inner_lim[1] and (y > y_inner_lim[1] or y < y_
        return True
    else:
        return False
```

In [10]:

```
def TSP_generator(seed, x_inner_lim, x_outer_lim, y_inner_lim, y_outer_lim, num_
    i = 0
    cities = set()
    dist_mat = np.zeros((num_city, num_city))

    # Generate cities
    while len(cities) < num_city:
        rand_state = np.random.RandomState(seed=seed + i)
        x_coord = rand_state.uniform(x_outer_lim[0], x_outer_lim[1], num_city)
        y_coord = rand_state.uniform(y_outer_lim[0], y_outer_lim[1], num_city)

        # Check if the generated coordinates are in the inner area
        new_set = [(x, y) for x, y in zip(x_coord, y_coord) if check_margin(x, y
        cities.update(new_set)
        i += 1

    cities = list(cities)[:num_city]
    cities_dict = dict(zip(string.ascii_uppercase, cities))

    # Generate edge cost
    coordinates = np.asarray(cities)
    for i in range(num_city):
        for j in range(i + 1, num_city):
            dist_mat[i][j] = np.sqrt(np.sum((coordinates[i] - coordinates[j]) **
    return cities_dict, dist_mat
```

In [11]:

```
def TSP_plot(city_coords):
    pylab.rcParams['figure.figsize'] = (10.0, 8.0)

    for k_name, v_coord in city_coords.items():
        x, y = v_coord
        plt.scatter(x, y, marker='x', c='r', s=100)
        plt.text(x, y + 0.04, k_name, fontsize='xx-large')
    plt.show()
```

In [12]:

```
def solution_visualization(solution, city_coords):
    """
    Visualize the final solution

    param:
    solution: a TSPNode of final state
    city_coords: dictionary of cities

    """
    pylab.rcParams['figure.figsize'] = (10.0, 8.0)

    for k_name, v_coord in city_coords.items():
        x, y = v_coord
        plt.scatter(x, y, marker='x', c='r', s=100)
        plt.text(x, y+0.04, k_name, fontsize='xx-large')

    # Draw the line between two cities
    for i, c in enumerate(solution.state):
        x_start, y_start = city_coords[c]
        if i != len(solution.state) - 1:
            x_end, y_end = city_coords[solution.state[i+1]]
        else:
```

```
            x_end, y_end = city_coords[solution.state[0]]

        x, y = [x_start, x_end], [y_start, y_end]
        x_mid, y_mid = (x_start + x_end)/2, (y_start + y_end)/2
        plt.plot(x, y, 'ro-')

    plt.show()
```

In [13]:

```
def read_TSP_from_file():
    """
    Read cities from file

    return:
    city_coords: a Dictionary as {city_name: (x_coordinate, y_coordinate)}
    dist_mat: a matrix of euclidean distance between each pair of cities

    """
    keep = []
    f = open('berlin52.tsp', 'r')
    keep_bool = False
    x = f.readlines()
    city_coords = {}
    for item in x:
        if 'EOF' in item:
            keep_bool = False
        if keep_bool == True:
            [city, coord_1, coord_2] = item.strip().split(' ')
            city_coords[city] = (float(coord_1),float(coord_2))
            keep.append(item)
        if 'NODE_COORD_SECTION' in item:
            keep_bool = True
    f.close()
    coords = list(city_coords.values())
    dist_mat = np.zeros((len(coords), len(coords)))
    for i in range(len(coords)):
        for j in range(len(coords)):
            c1, c2 = coords[i], coords[j]
            dist_mat[i][j] = np.sqrt((c1[0]-c2[0])**2 + (c1[1]-c2[1])**2)


    return city_coords, dist_mat
```

Testing your implementation

Generate or read a TSP problem

In [14]:

```
rand_seed  = 11
rand_state = np.random.RandomState(rand_seed)
```

In [15]:

```
x_inner_range, x_outer_range = (-99, 99), (-100, 100)
y_inner_range, y_outer_range = (-99, 99), (-100, 100)

num_city     = 10
```

```python
max_iter     = 1000
schedule_k   = 500
schedule_lam = 0.25
```

In [16]:

```python
cities_coords_1, dist_mat_1 = TSP_generator(rand_seed, x_inner_range, x_outer_ra
                                                      y_inner_range, y_outer_range,
```

In [17]:

```python
cities_coords_1
```

Out[17]:

```
{'A': (99.13890417289295, 90.71933136600211),
 'B': (99.29114501781939, 34.36131251541505),
 'C': (-99.29089063805382, 76.46810625333362),
 'D': (99.31484603092986, 11.991420744425014),
 'E': (-94.06859823501537, 99.39149259753762),
 'F': (-27.35708069579421, -99.52680499115773),
 'G': (99.97345852477585, 41.93031251762548),
 'H': (69.21701945617659, 99.22516370932331),
 'I': (-97.08500750291607, -99.54815329629729),
 'J': (3.4595767693178487, -99.50702375982277)}
```

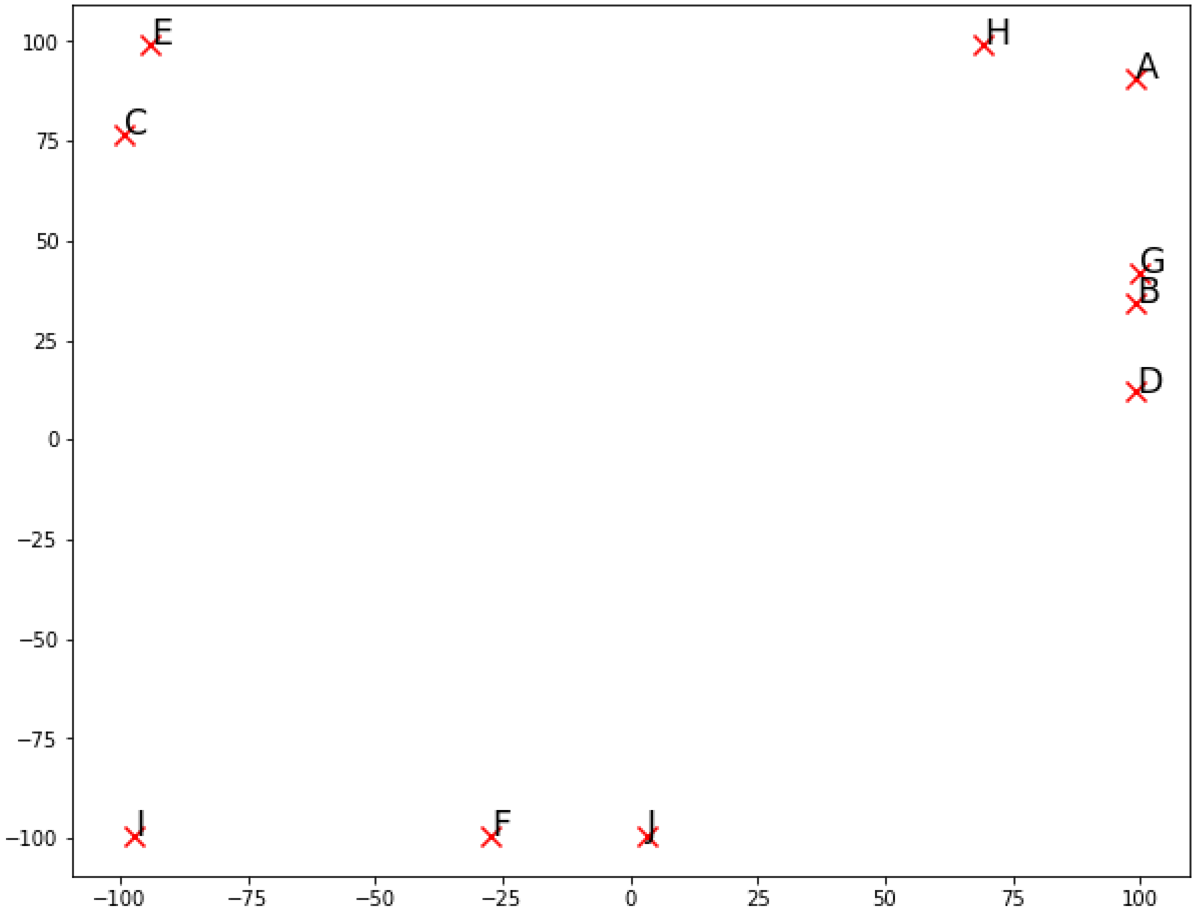
In [18]:

```python
display(dist_mat_1)
```

```
array([[  0.        ,  56.35822448, 198.94089797,  78.72810722,
        193.40203041, 228.46187119,  48.79615601,  31.1073684 ,
        273.32314068, 212.93332261],
       [  0.        ,   0.        , 202.99706147,  22.36990433,
        204.00224173, 184.29813099,   7.59969162,  71.4966588 ,
        237.68748035, 164.6342034 ],
       [  0.        ,   0.        ,   0.        , 208.80967797,
         23.51072052, 190.12806683, 202.23535804, 170.03764121,
        176.03008137, 203.77660547],
       [  0.        ,   0.        ,   0.        ,   0.        ,
        212.2167031 , 168.766382  ,  29.94613516,  92.28003617,
        225.86274382, 147.0378719 ],
       [  0.        ,   0.        ,   0.        ,   0.        ,
          0.        , 209.80685329, 202.37121092, 163.28570241,
        198.96251263, 221.52283117],
       [  0.        ,   0.        ,   0.        ,   0.        ,
          0.        ,   0.        , 190.32388792, 220.972627  ,
         69.72793008,  30.81666381],
       [  0.        ,   0.        ,   0.        ,   0.        ,
          0.        ,   0.        ,   0.        ,  65.02813635,
        242.58646978, 171.22923075],
       [  0.        ,   0.        ,   0.        ,   0.        ,
          0.        ,   0.        ,   0.        ,   0.        ,
        259.16634759, 209.32874529],
       [  0.        ,   0.        ,   0.        ,   0.        ,
          0.        ,   0.        ,   0.        ,   0.        ,
          0.        , 100.54459268],
       [  0.        ,   0.        ,   0.        ,   0.        ,
          0.        ,   0.        ,   0.        ,   0.        ,
          0.        ,   0.        ]])
```

In [19]:

```python
TSP_plot(cities_coords_1)
```



In [20]:

```
tsp_graph                  = make_graph(cities_coords_1, dist_mat_1)
TSPNode._distances         = tsp_graph.distances
TSPNode._random_state = rand_state
```

2. Generate initial state

In [21]:

```
initial_n_1 = init_state(rand_seed, cities_coords_1)
initial_n_1.state
```

Out[21]:

```
['H', 'I', 'C', 'G', 'E', 'F', 'B', 'D', 'A', 'J']
```

3. Run Simulated Annealing

In [22]:

```
t_schedule = exp_schedule(k=schedule_k, lam=schedule_lam)
```

In [23]:

```
solution_n = simulated_annealing(initial_n_1, t_schedule, max_iter, rand_state)
solution_n.state
```

Out[23]:

```
['F', 'J', 'D', 'B', 'G', 'A', 'H', 'E', 'C', 'I']
```

In [24]:

```
sol_path = list(solution_n.path())
for node in sol_path:
```

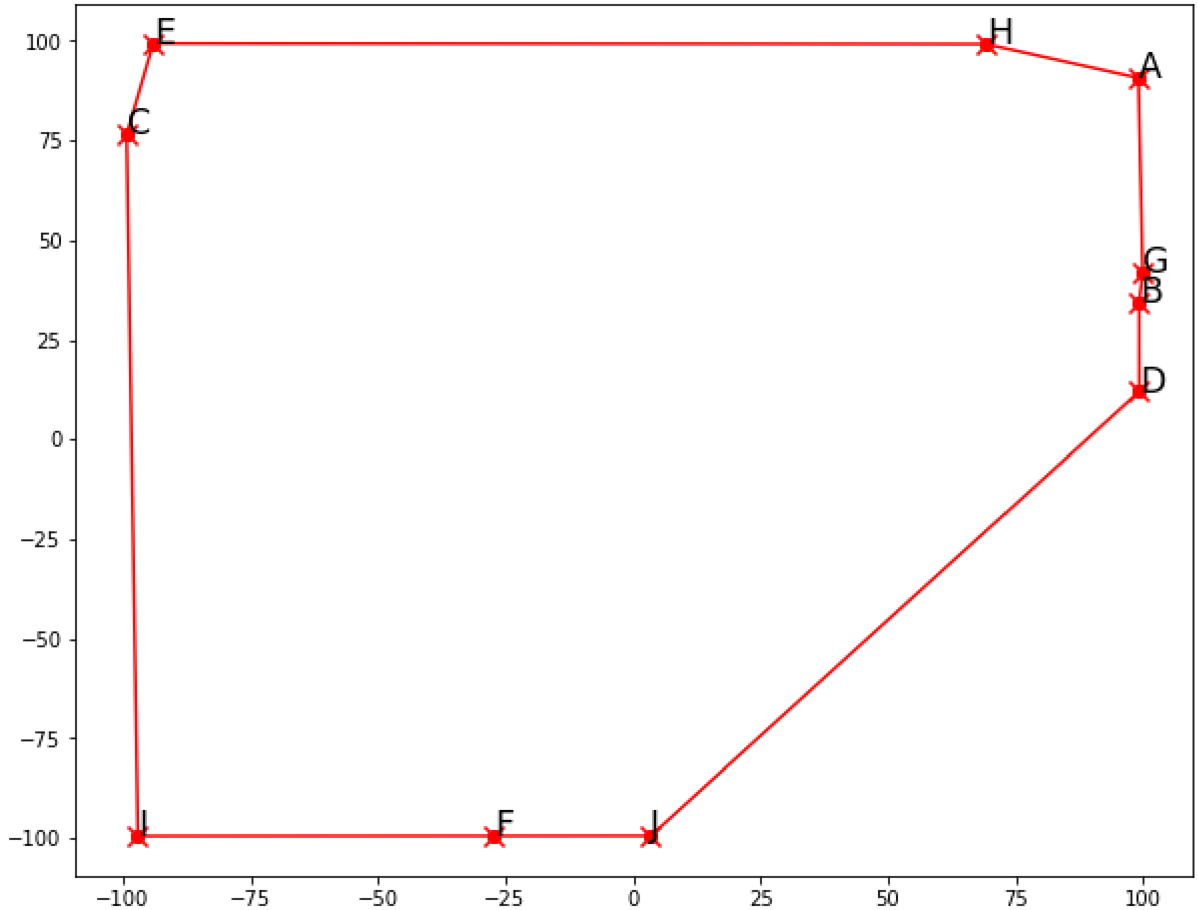
```
    print(node)
```

```
Node: <F J D B G A H E C I> -720.3
```

4. Visualize final solution

In [25]:

```
solution_visualization(solution_n, cities_coords_1)
```



Run Simulations

TSP-1 (Large cost)

The TSP problem generated with large costs. Please use the given random seed.

In [26]:

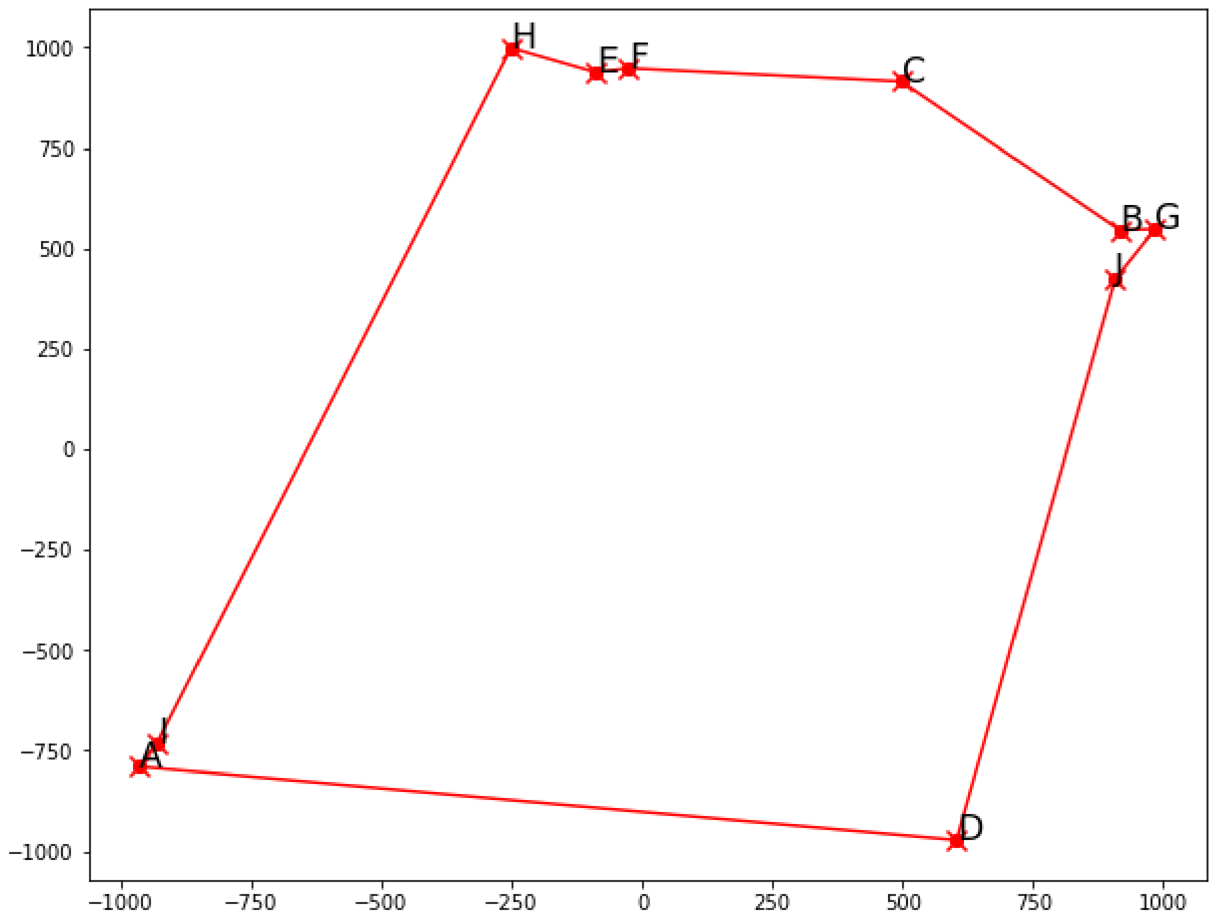
```
num_city = 10
x_inner_range, x_outer_range = (-900, 900), (-1000, 1000)
y_inner_range, y_outer_range = (-900, 900), (-1000, 1000)

cities_coords_2, dist_mat_2 = TSP_generator(1234, x_inner_range, x_outer_range,
                                            y_inner_range, y_outer_range, num_ci
tsp_graph_2 = make_graph(cities_coords_2, dist_mat_2)
TSPNode._distances = tsp_graph_2.distances
```

Brute Force

In [27]:

```
cities_no_A = list(cities_coords_2.keys())
cities_no_A.remove('A')
possible_paths = [list(L) for L in list(permutations(cities_no_A))]
possible_paths = [['A'] + L for L in possible_paths]

as_TSPNodes    = [TSPNode(state=p) for p in possible_paths]
total_dists    = [n.value() for n in as_TSPNodes]
path_dist_dict = dict(zip(total_dists,possible_paths))

min_dist = max(path_dist_dict.keys())
min_path = path_dist_dict[min_dist]

solutionTSP = TSPNode(state=min_path)
solutionTSP._distances=total_dists

print('Optimal result:',solutionTSP)
solution_visualization(solutionTSP, cities_coords_2)
```

```
Optimal result: Node: <A I H E F C B G J D> -6467.9
```

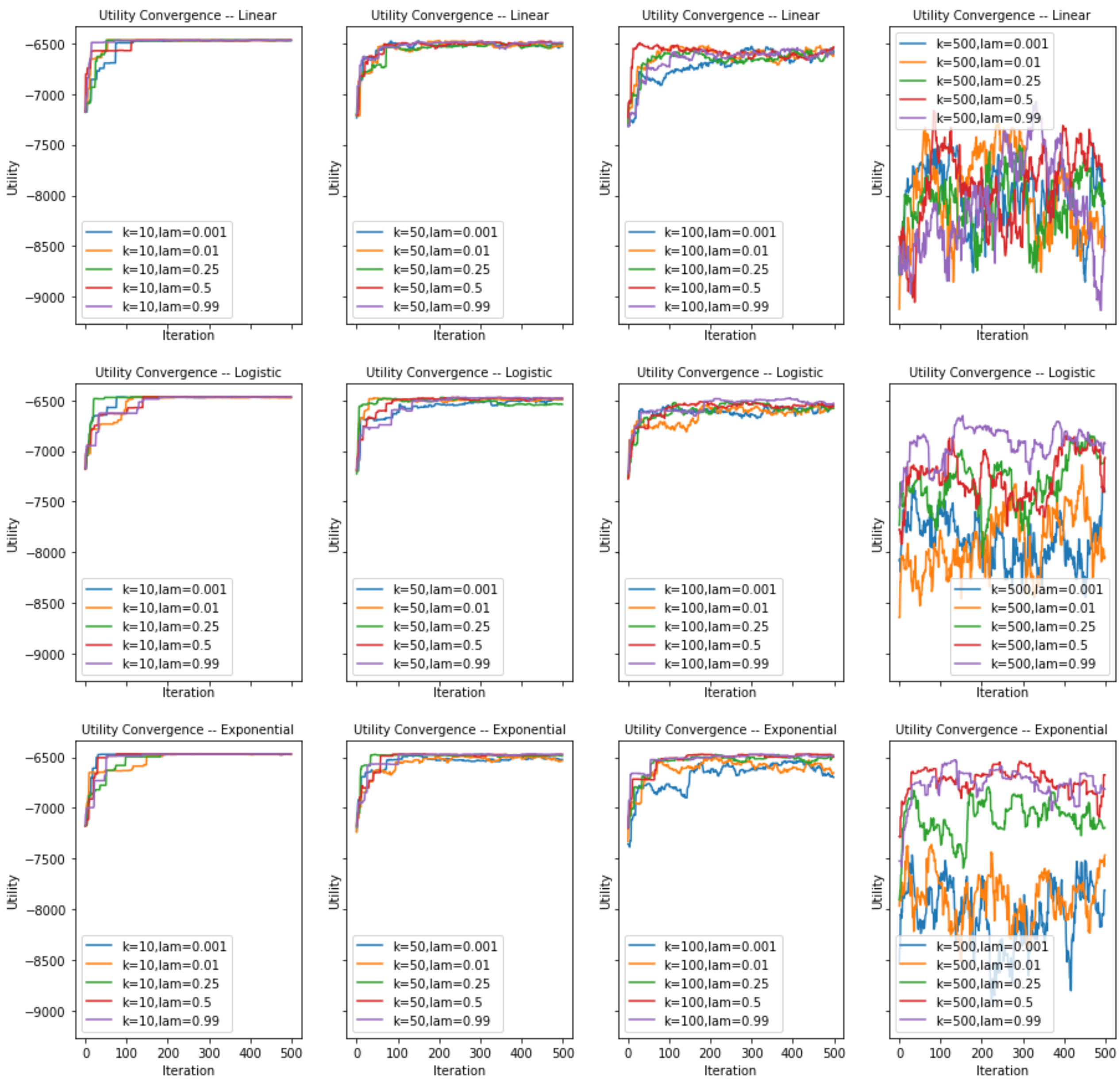


Detour -- Finding the best k and lambda

In [28]:

```
simulated_annealing_test([10,50,100,500],
                         [0.001,0.01,0.25,0.5,0.99],
                         initial_n_1,
                         1234,
                         trials=10,
                         name_sched_pairs=[('Linear',linear_schedule),
```

```
                                ('Logistic',log_schedule),
                                ('Exponential',exp_schedule)])
```



Utility converges most quickly when k=10 or k=50 -- with slightly more stability when k=50 for the linear and exponential schedules. With larger values of lambda -- particularly with the Logistic and Exponential schedules -- the patterns become more distinct, and we can observe that lower values of lambda outperform higher values.

Exponential Schedule

In [29]:

```
max_iter,        num_trials,    rand_seed = 500, 15, 1234
k_set,           lam_set                  = [10,100], [0.1, 0.99]
res_dict_exp_1, data_exp_1,     sol_exp_1 = results(k_set,     lam_set,     exp_sche
                                                    max_iter, num_trials, cities_c
display(dict(zip(res_dict_exp_1.keys(),[v[0] for v in res_dict_exp_1.values()])))
```

```
{(10, 0.1): -6684.546622451529,
 (10, 0.99): -6467.877708018376,
```

```
 (100, 0.1): -6573.144732433278,
 (100, 0.99): -6576.2121652349515}
```

Linear Schedule

In [30]:

```
# TODO: you need to decide the value set of k and lam by yourself

# I modified this code and instead used a res_dict function I created earlier.
# I also used the values of k and lambda determiend by the tests above.

max_iter,      num_trials, rand_seed      = 500, 15, 1234
k_set,         lam_set                    = [10,100], [0.1, 0.99]
res_dict_lin_1, data_lin_1,    sol_lin_1 = results(k_set,     lam_set,     linear_s
                                                   max_iter, num_trials, cities_c

display(dict(zip(res_dict_lin_1.keys(),[v[0] for v in res_dict_lin_1.values()]))
```

```
{(10, 0.1): -6576.212165234953,
 (10, 0.99): -6467.877708018377,
 (100, 0.1): -6533.482072203343,
 (100, 0.99): -6573.144732433276}
```

Log Schedule

In [31]:

```
# TODO: you need to decide the value set of k and lam by yourself

# I modified this code and instead used a res_dict function I created earlier.
# I also used the values of k and lambda determiend by the tests above.

max_iter,      num_trials, rand_seed = 500, 15, 1234
k_set,         lam_set               = [10,100], [0.1, 0.99]
res_dict_log_1, data_log_1,    sol_log_1 = results(k_set,     lam_set,     log_sche
                                                   max_iter, num_trials, cities_c
display(dict(zip(res_dict_log_1.keys(),[v[0] for v in res_dict_log_1.values()]))
```

```
{(10, 0.1): -6467.877708018376,
 (10, 0.99): -6683.391969805659,
 (100, 0.1): -6553.181747384116,
 (100, 0.99): -6477.694136684019}
```

In [32]:

```
random_state = np.random.RandomState(seed=1234)
```

In [33]:

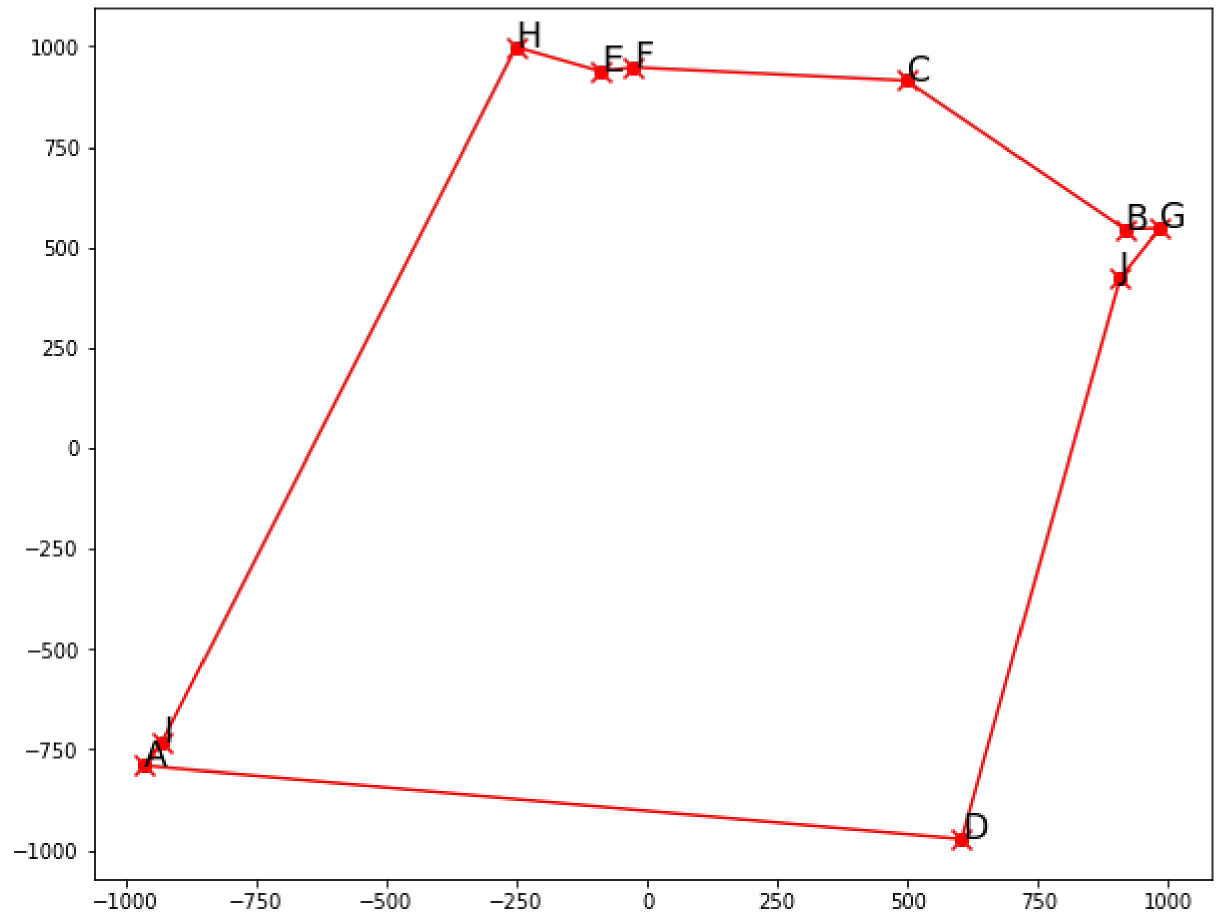
```
datas_1, dicts_1, solutions_1 = [data_exp_1,      data_lin_1,      data_log_1],\
                                [res_dict_exp_1, res_dict_lin_1, res_dict_log_1]
                                [sol_exp_1,       sol_lin_1,       sol_log_1]

pretty_table_1, best_path_1, k_lam_list_1, first_iters_1, close_iters_1 = pretty

display(pretty_table_1)
```

```
Exponential:
   Solution: [A, I, H, E, F, C, B, G, J, D]
   Optimal path found by at least one configuration of parameters: 136 iteration
s
   Approximate convergence by at least one configuration of parameters: 77 itera
tions
        k (approx. convergence):       10
```

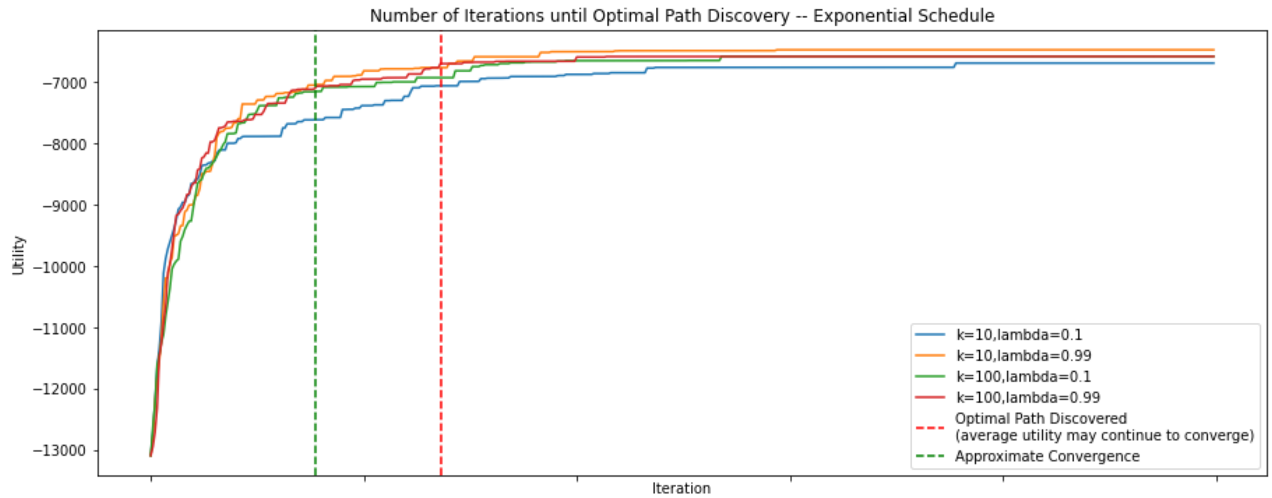
```
        lambda (approx. convergence): 0.99
Linear:
   Solution: [A, I, H, E, F, C, B, G, J, D]
   Optimal path found by at least one configuration of parameters: 134 iteration
s
   Approximate convergence by at least one configuration of parameters: 65 itera
tions
        k (approx. convergence):      10
        lambda (approx. convergence): 0.1
Logarithmic:
   Solution: [A, I, H, E, F, C, B, G, J, D]
   Optimal path found by at least one configuration of parameters: 151 iteration
s
   Approximate convergence by at least one configuration of parameters: 101 iter
ations
        k (approx. convergence):      100
        lambda (approx. convergence): 0.1
```

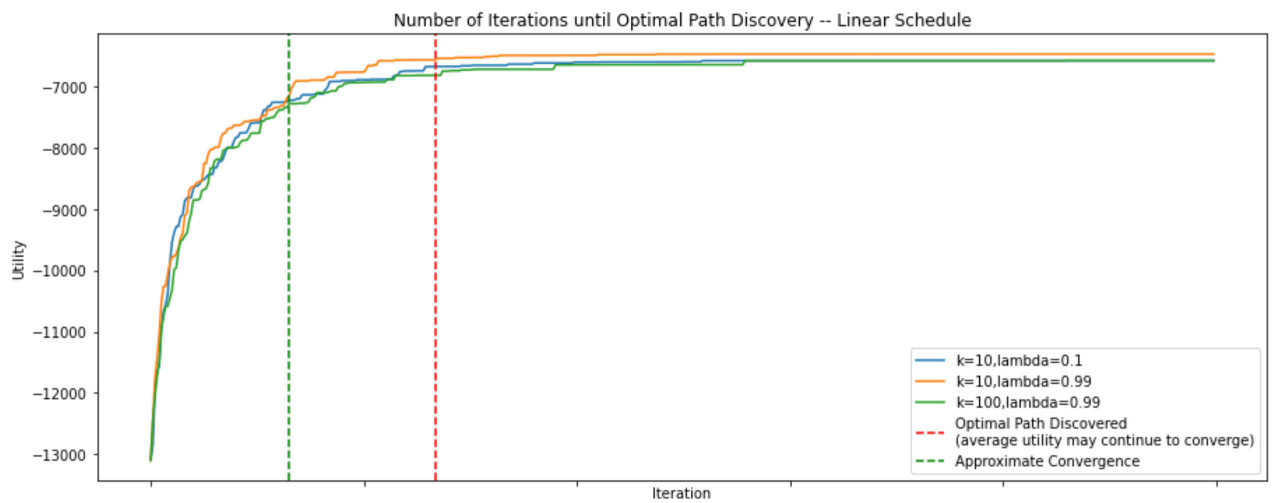
Exponential					Linear					
(k, lambda)	Average Value	Most Frequent Path	Avg Iter Until Max Value	Avg Iter Until Approx Max Value	(k, lambda)	Average Value	Most Frequent Path	Avg Iter Until Max Value	Avg Iter Until Approx Max Value	(k, lambda
(10, 0.1)	-6684.55	[A, I, H, E, F, C, B, G, J, D]	238	115	(10, 0.1)	-6576.21	[A, I, H, E, F, C, B, G, J, D]	171	65	(10, 0.1)
(10, 0.99)	-6467.88	[A, I, H, E, F, C, B, G, J, D]	136	77	(10, 0.99)	-6467.88	[A, I, H, E, F, C, B, G, J, D]	134	65	(10, 0.99
(100, 0.1)	-6573.14	[A, I, H, E, F, C, B, G, J, D]	147	88	(100, 0.1)	-6533.48	[A, I, H, E, F, C, G, B, J, D]	271	75	(100, 0.1
(100, 0.99)	-6576.21	[A, I, H, E, F, C, B, G, J, D]	209	79	(100, 0.99)	-6573.14	[A, I, H, E, F, C, B, G, J, D]	154	89	(100, 0.9

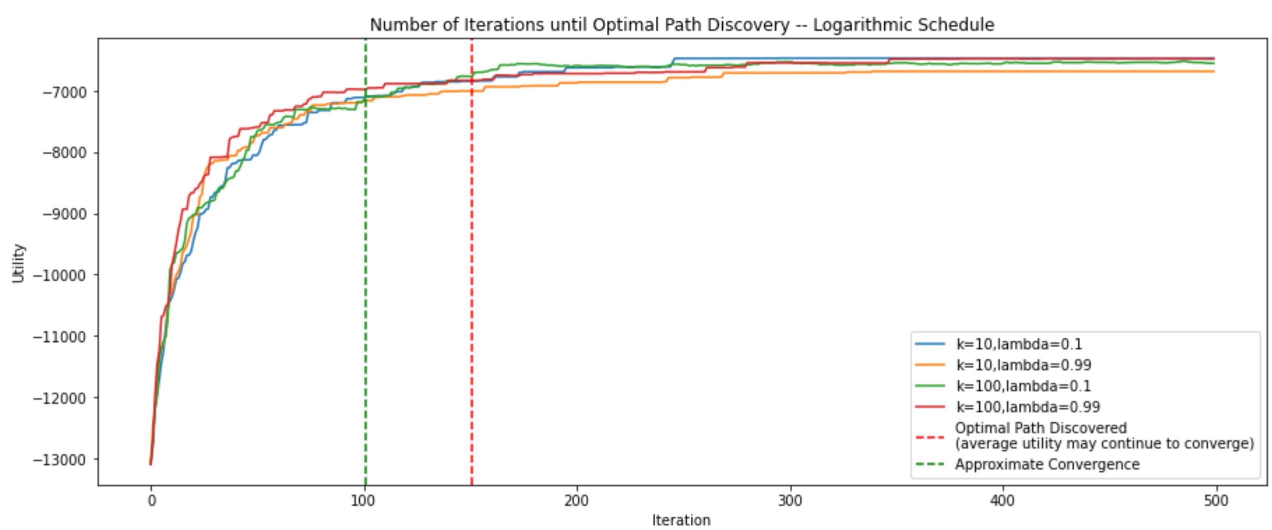
```
<class 'list'>
```



H
E
F
C
B
G
I
J
A
D
1000
750
500
250
0
−250
−500
−750
−1000
−1000
−750
−500
−250
0
250
500
750
1000







TSP-2 (Small cost)

The TSP problem generated with small costs. Please use your CWID as the random seed.

In [34]:

```
num_city = 10
x_inner_range, x_outer_range = (-9, 9), (-15, 15)
y_inner_range, y_outer_range = (-9, 9), (-15, 15)
```

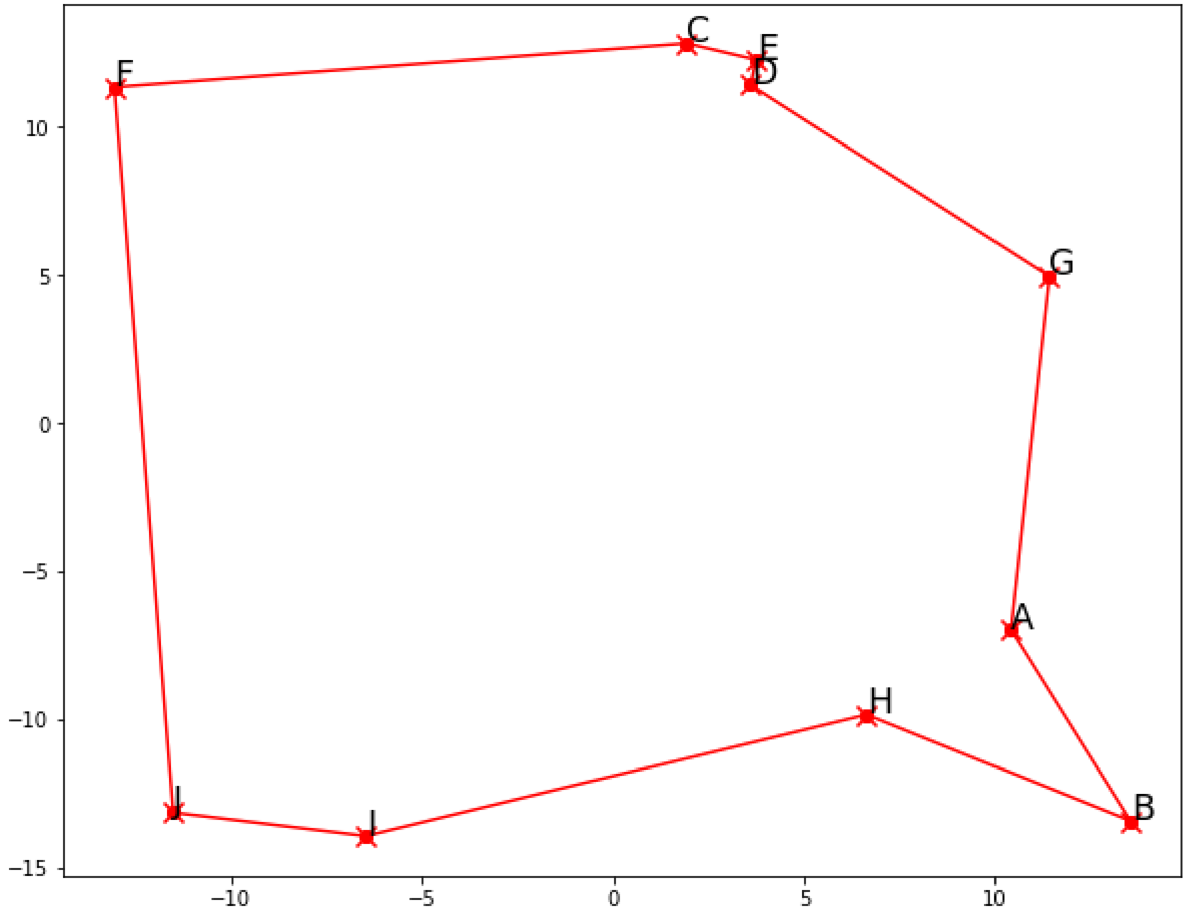
```python
# TODO: Please replace "4321" with your own CWID number after "A"
your_own_seed = int("20452471")

cities_coords_3, dist_mat_3 = TSP_generator(your_own_seed, x_inner_range, x_oute
tsp_graph_3 = make_graph(cities_coords_3, dist_mat_3)
TSPNode._distances = tsp_graph_3.distances
```

Brute Force

In [35]:

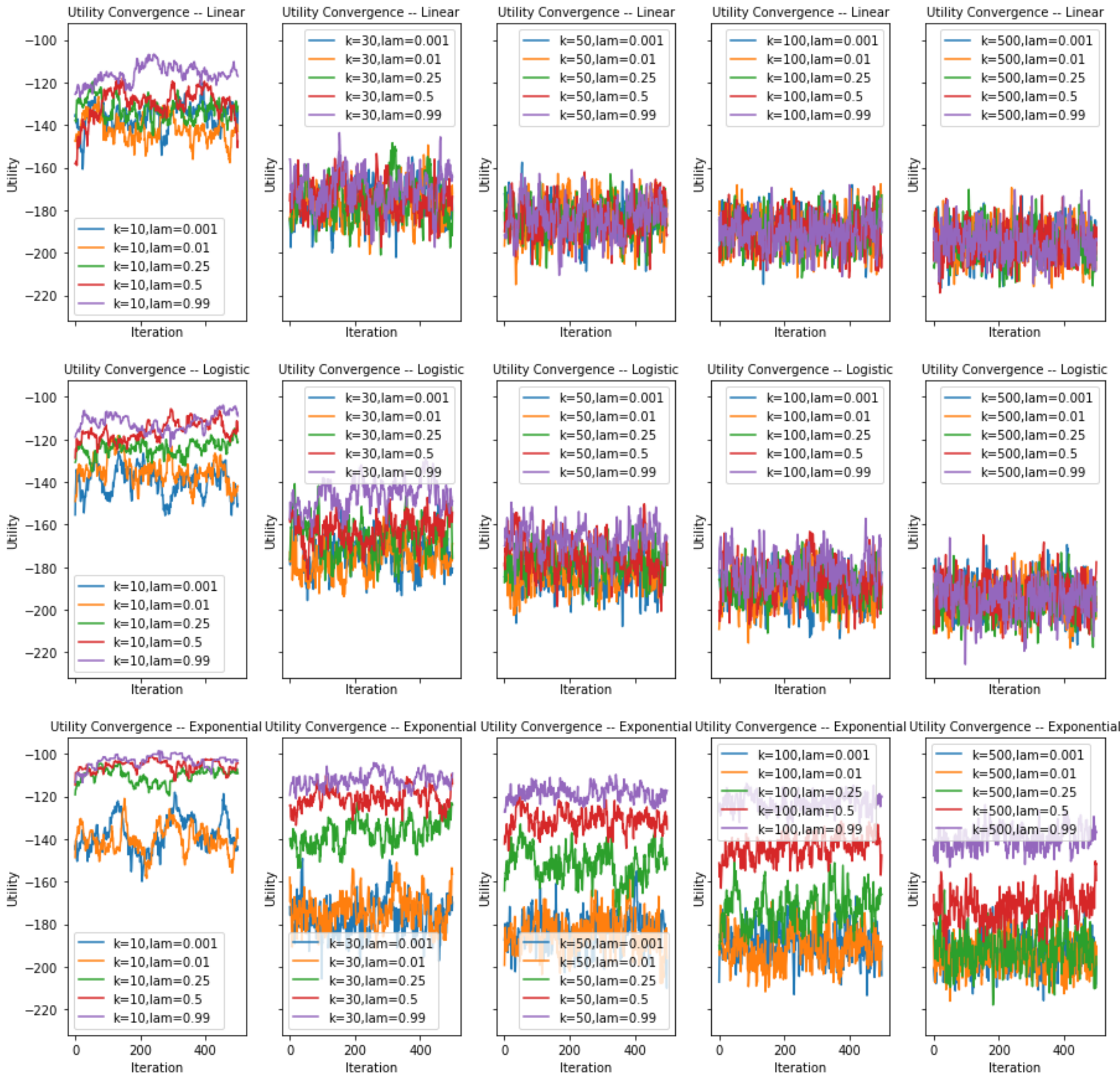
```python
# TODO
# TODO Implement brute force search and record the optimal result and visualize

### All possible paths

cities_no_A = list(cities_coords_3.keys())
cities_no_A.remove('A')
possible_paths = [list(L) for L in list(permutations(cities_no_A))]
possible_paths = [['A'] + L for L in possible_paths]

as_TSPNodes    = [TSPNode(state=p) for p in possible_paths]
total_dists    = [n.value() for n in as_TSPNodes]
path_dist_dict = dict(zip(total_dists,possible_paths))

min_dist = max(path_dist_dict.keys())
min_path = path_dist_dict[min_dist]

solutionTSP = TSPNode(state=min_path)
solutionTSP._distances=total_dists

print('Optimal result:',solutionTSP)
solution_visualization(solutionTSP, cities_coords_3)
```

```
Optimal result: Node: <A G D E C F J I H B> -98.2
```



Choosing k & lambda

In [37]:
```
### Choosing k and lam
initial_n_2 = init_state(20452471,cities_coords_3)
simulated_annealing_test([10,30,50,100,500],[0.001,0.01,0.25,0.5,0.99],initial_n
```



Once again, the algorithm performs best with lowers values of and higher values of lambda. However, compared to the larger data, the cost stays relatively constant within any given configuration of temperature schedule, k, and lambda -- i.e., it doesn't converge.

Exponential Schedule

In [38]:

```
# I modified this code and instead used a res_dict function from the helper file

max_iter, num_trials, rand_seed        = 500, 10, your_own_seed
k_set,     lam_set                     = [10,100], [0.1, 0.99]
res_dict_exp_2, data_exp_2, sol_exp_2 = results(k_set,     lam_set,     exp_schedu
                                                max_iter, num_trials, cities_coo
display(dict(zip(res_dict_exp_2.keys(),[v[0] for v in res_dict_exp_2.values()]))
```

```
{(10, 0.1): -98.23639866968088,
 (10, 0.99): -98.23639866968092,
 (100, 0.1): -101.84673745211214,
 (100, 0.99): -98.23639866968092}
```

Linear Schedule

In [39]:

```
# I modified this code and instead used a res_dict function from the helper file

max_iter, num_trials, rand_seed = 500, 10, your_own_seed
k_set,         lam_set           = [10,100], [0.1, 0.99]
res_dict_lin_2, data_lin_2, sol_lin_2 = results(k_set,    lam_set,    linear_sch
                                                max_iter, num_trials, cities_coo
display(dict(zip(res_dict_lin_2.keys(),[v[0] for v in res_dict_lin_2.values()]))
```

```
{(10, 0.1): -98.2363986696809,
 (10, 0.99): -98.23639866968091,
 (100, 0.1): -177.01244117331524,
 (100, 0.99): -98.2363986696809}
```

Log Schedule

In [40]:

```
# I modified this code and instead used a res_dict function from the helper file

max_iter, num_trials, rand_seed = 500, 10, your_own_seed
k_set,         lam_set           = [10,100], [0.1,0.99]
res_dict_log_2, data_log_2, sol_log_2 = results(k_set,    lam_set,    log_schedu
                                                max_iter, num_trials, cities_coo
display(dict(zip(res_dict_log_2.keys(),[v[0] for v in res_dict_log_2.values()]))
```

```
{(10, 0.1): -122.20080912799067,
 (10, 0.99): -99.48382256667973,
 (100, 0.1): -187.5454302931661,
 (100, 0.99): -154.34354579088603}
```

In [41]:

```
# TODO: Present a table of your results for TSP-2. Consider using pandas.
```

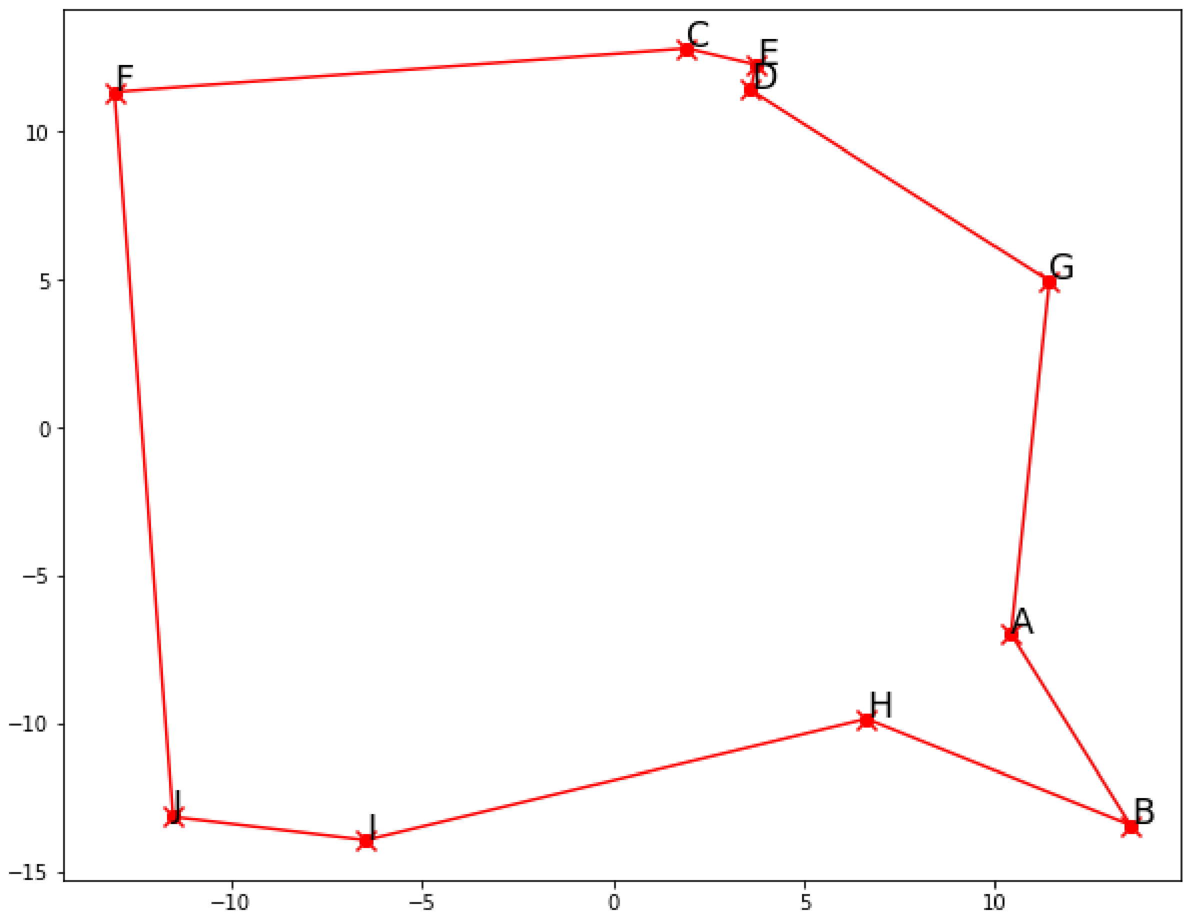
In [42]:

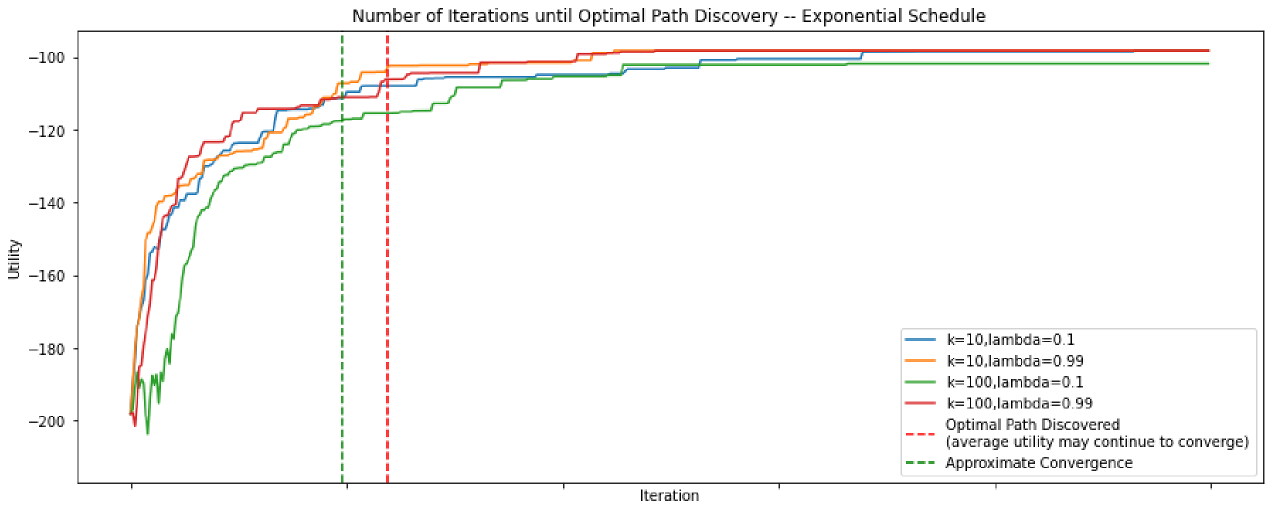
```
datas_2, dicts_2, solutions_2 = [data_exp_2,     data_lin_2,     data_log_2],\
                                [res_dict_exp_2, res_dict_lin_2, res_dict_log_2]
                                [sol_exp_2,      sol_lin_2,      sol_log_2]

pretty_table_2, best_path_2, k_lam_list_2, first_iters_2, close_iters_2 = pretty

display(pretty_table_2)
```

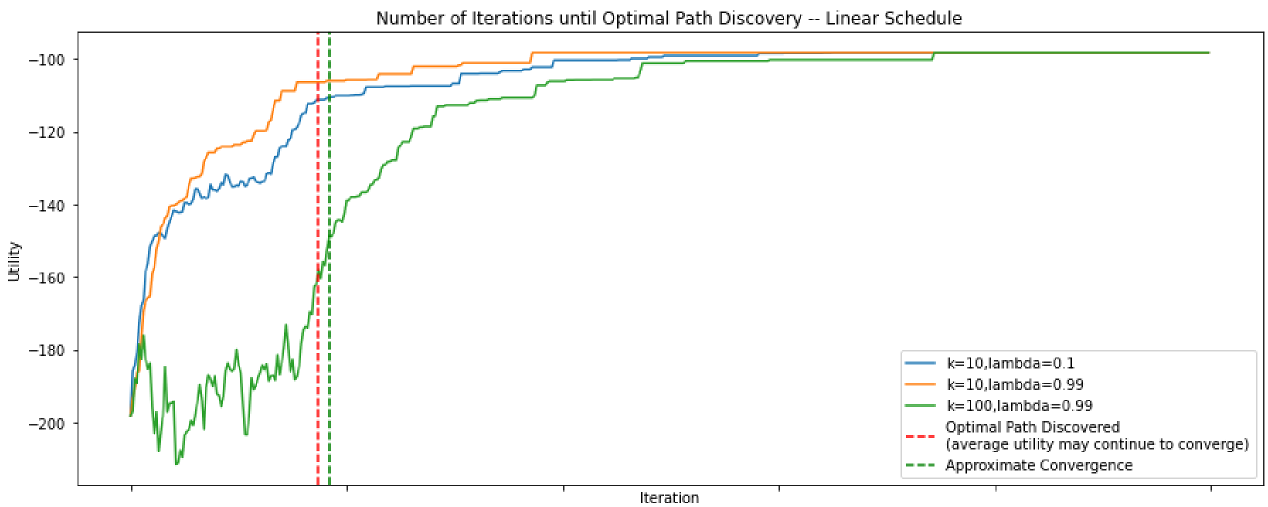
```
Exponential:
   Solution: [A, G, D, E, C, F, J, I, H, B]
   Optimal path found by at least one configuration of parameters: 119 iteration
s
   Approximate convergence by at least one configuration of parameters: 98 itera
tions
        k (approx. convergence):      100
        lambda (approx. convergence): 0.99
Linear:
   Solution: [A, G, D, E, C, F, J, I, H, B]
   Optimal path found by at least one configuration of parameters: 87 iterations
   Approximate convergence by at least one configuration of parameters: 92 itera
tions
        k (approx. convergence):      10
        lambda (approx. convergence): 0.99
Logarithmic:
   Solution: [A, G, E, D, C, F, J, I, H, B]
   Optimal path found by at least one configuration of parameters: 174 iteration
```

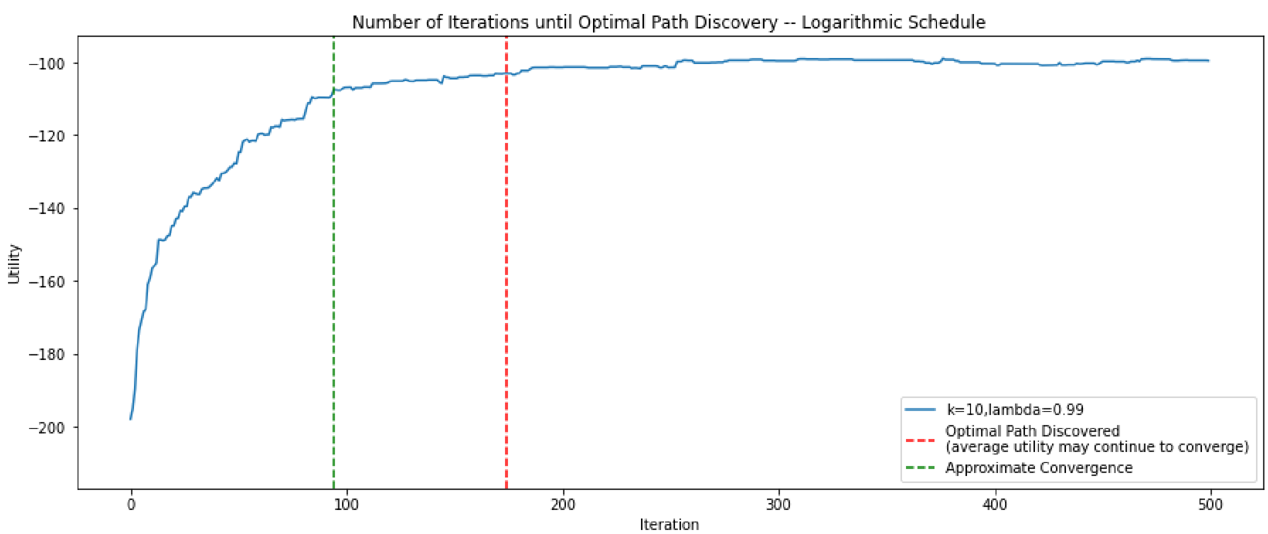
```
s
   Approximate convergence by at least one configuration of parameters: 94 itera
tions
        k (approx. convergence):       10
        lambda (approx. convergence): 0.99
```

	Exponential					Linear				
(k, lambda)	Average Value	Most Frequent Path	Avg Iter Until Max Value	Avg Iter Until Approx Max Value	(k, lambda)	Average Value	Most Frequent Path	Avg Iter Until Max Value	Avg Iter Until Approx Max Value	(k, lambda
(10, 0.1)	-98.24	[A, G, D, E, C, F, J, I, H, B]	149	164	(10, 0.1)	-98.24	[A, G, D, E, C, F, J, I, H, B]	189	126	(10, 0.1)
(10, 0.99)	-98.24	[A, G, D, E, C, F, J, I, H, B]	119	122	(10, 0.99)	-98.24	[A, G, D, E, C, F, J, I, H, B]	87	92	(10, 0.99
(100, 0.1)	-101.85	[A, G, D, E, C, F, J, I, H, B]	192	124	(100, 0.1)	-177.01	[A, J, E, F, B, D, G, C, H, I]	500	15	(100, 0.1
(100, 0.99)	-98.24	[A, G, D, E, C, F, J, I, H, B]	150	98	(100, 0.99)	-98.24	[A, G, D, E, C, F, J, I, H, B]	234	197	(100, 0.9









TSP-3 (berlin52)

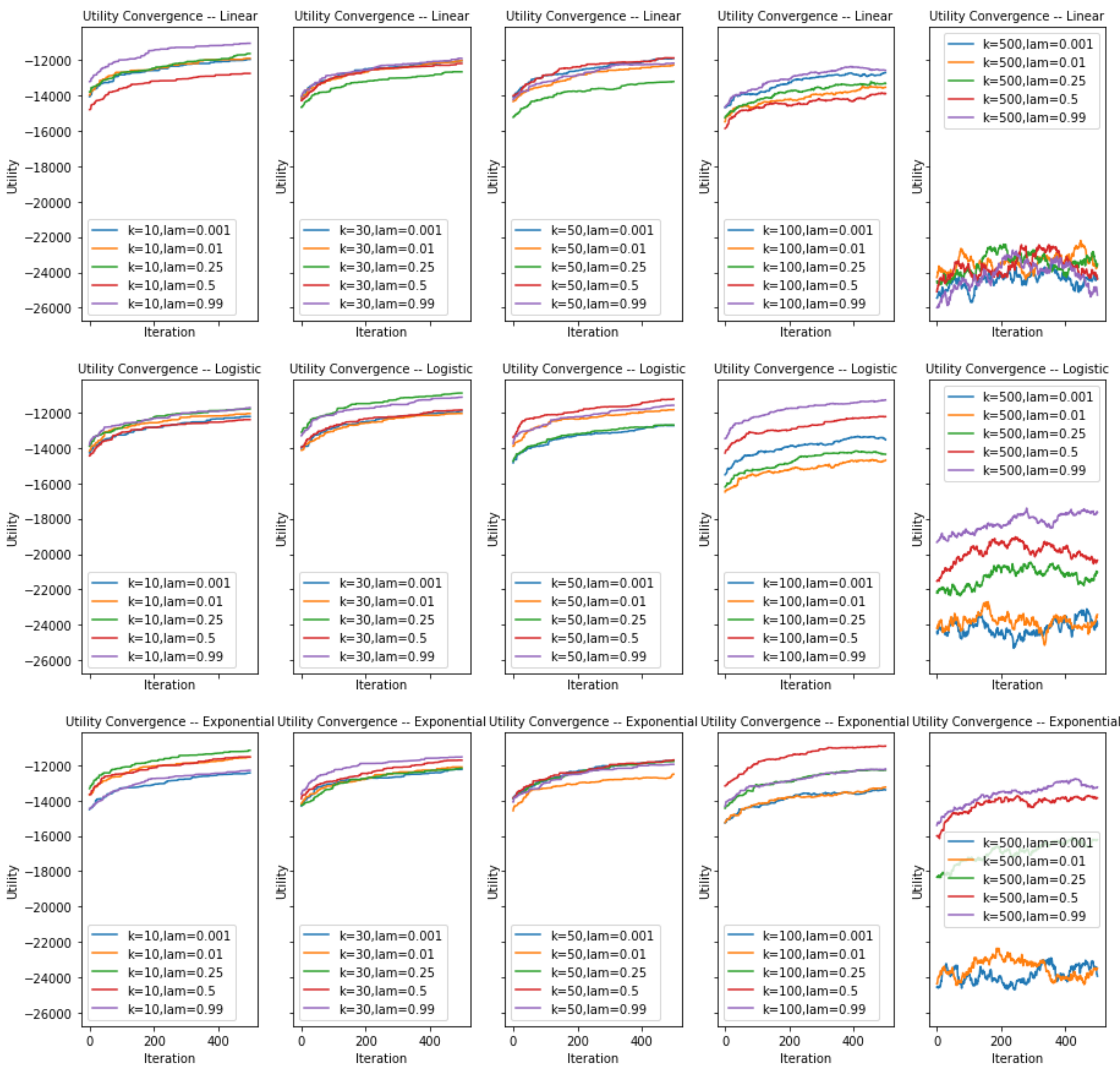
The TSP problem loaded from the file "berlin52.tsp" with 52 cities.

In [44]:

```
cities_coords_4, dist_mat_4 = read_TSP_from_file()
tsp_graph_4 = make_graph(cities_coords_4, dist_mat_4)
TSPNode._distances = tsp_graph_4.distances
```

In [46]:

```
### Choosing k and lambda
initial_n_3 = init_state(1234,cities_coords_4)
simulated_annealing_test([10,30,50,100,500],[0.001,0.01,0.25,0.5,0.99],initial_n
```



With this data, the algorithm seems to perform best at k=10 and lambda=0.5, so we will keep the range of values around those numbers.

Exponential Schedule

In [47]:

```
# I modified this code and instead used a res_dict function from the helper file

max_iter,        num_trials, rand_seed = 500, 10, 1234
k_set,           lam_set               = [10,20], [0.25, 0.5, 0.75]
res_dict_exp_3, data_exp_3, sol_exp_3 = results(k_set,     lam_set,     exp_schedu
                                                max_iter, num_trials, cities_coo
display(dict(zip(res_dict_exp_3.keys(),[v[0] for v in res_dict_exp_3.values()]))
```

```
{(10, 0.25): -15319.3104441367,
 (10, 0.5): -15470.437179440947,
```

```
 (10, 0.75): -15237.91202847534,
 (20, 0.25): -15520.121032171863,
 (20, 0.5): -15292.956360487247,
 (20, 0.75): -15365.148483619696}
```

Linear Schedule

In [48]:

```
# I modified this code and instead used a res_dict function from the helper file

max_iter,        num_trials, rand_seed = 500, 10, 1234
k_set,           lam_set               = [10,20], [0.25, 0.5, 0.75]
res_dict_lin_3, data_lin_3, sol_lin_3 = results(k_set,     lam_set,     linear_sch
                                                 max_iter, num_trials, cities_coo
display(dict(zip(res_dict_lin_3.keys(),[v[0] for v in res_dict_lin_3.values()]))
```

```
{(10, 0.25): -15303.713233959375,
 (10, 0.5): -15910.298391288143,
 (10, 0.75): -15385.933230050598,
 (20, 0.25): -15502.052336074883,
 (20, 0.5): -16119.381743795791,
 (20, 0.75): -15054.571008915265}
```

Log Schedule

In [49]:

```
# I modified this code and instead used a res_dict function from the helper file

max_iter,        num_trials, rand_seed = 500, 10, 1234
k_set,           lam_set               = [10,20], [0.25, 0.5, 0.75]
res_dict_log_3, data_log_3, sol_log_3 = results(k_set,     lam_set,     log_schedu
                                                 max_iter, num_trials, cities_coo
display(dict(zip(res_dict_log_3.keys(),[v[0] for v in res_dict_log_3.values()]))
```

```
{(10, 0.25): -15421.424711250427,
 (10, 0.5): -15142.599986842024,
 (10, 0.75): -15973.704358661882,
 (20, 0.25): -15416.948781959329,
 (20, 0.5): -15408.35305528594,
 (20, 0.75): -15723.294504568936}
```

In [50]:

```
# TODO: Present a table of your results for TSP-3. Consider using pandas.
```

In [53]:

```
datas_3, dicts_3, solutions_3 = [data_exp_3,     data_lin_3,     data_log_3],\
                                [res_dict_exp_3, res_dict_lin_3, res_dict_log_3]
                                [sol_exp_3,      sol_lin_3,      sol_log_3]

pretty_table_3, best_path_3, k_lam_list_3, first_iters_3, close_iters_3 = pretty
display(pretty_table_3)
```

```
Exponential:
   Solution: [1, 4, 11, 12, 15, 6, 38, 24, 40, 42, 7, 2, 30, 44, 46, 51, 33, 35,
31, 22, 45, 3, 17, 21, 34, 18, 23, 39, 37, 50, 16, 20, 43, 10, 9, 8, 41, 19, 29,
26, 47, 13, 48, 5, 28, 27, 14, 52, 25, 36, 49, 32]
   Optimal path found by at least one configuration of parameters: 500 iteration
s
   Approximate convergence by at least one configuration of parameters: 394 iter
ations
        k (approx. convergence):      10
        lambda (approx. convergence): 0.75
```

```
Linear:
   Solution: [1, 22, 35, 37, 32, 18, 3, 7, 2, 30, 42, 17, 21, 23, 31, 45, 8, 28,
27, 4, 12, 13, 52, 14, 47, 9, 10, 41, 33, 25, 51, 11, 6, 15, 5, 39, 48, 38, 43,
24, 46, 26, 29, 50, 34, 40, 36, 16, 20, 19, 49, 44]
   Optimal path found by at least one configuration of parameters: 500 iteration
s
   Approximate convergence by at least one configuration of parameters: 395 iter
ations
        k (approx. convergence):      20
        lambda (approx. convergence): 0.75
Logarithmic:
   Solution: [1, 32, 22, 2, 7, 30, 21, 18, 3, 49, 34, 44, 12, 28, 29, 16, 20, 4
6, 47, 26, 13, 27, 48, 10, 17, 42, 14, 52, 11, 25, 6, 4, 51, 33, 43, 39, 37, 50,
40, 35, 36, 45, 9, 24, 5, 15, 38, 19, 8, 41, 31, 23]
   Optimal path found by at least one configuration of parameters: 500 iteration
s
   Approximate convergence by at least one configuration of parameters: 379 iter
ations
        k (approx. convergence):      10
        lambda (approx. convergence): 0.5
```

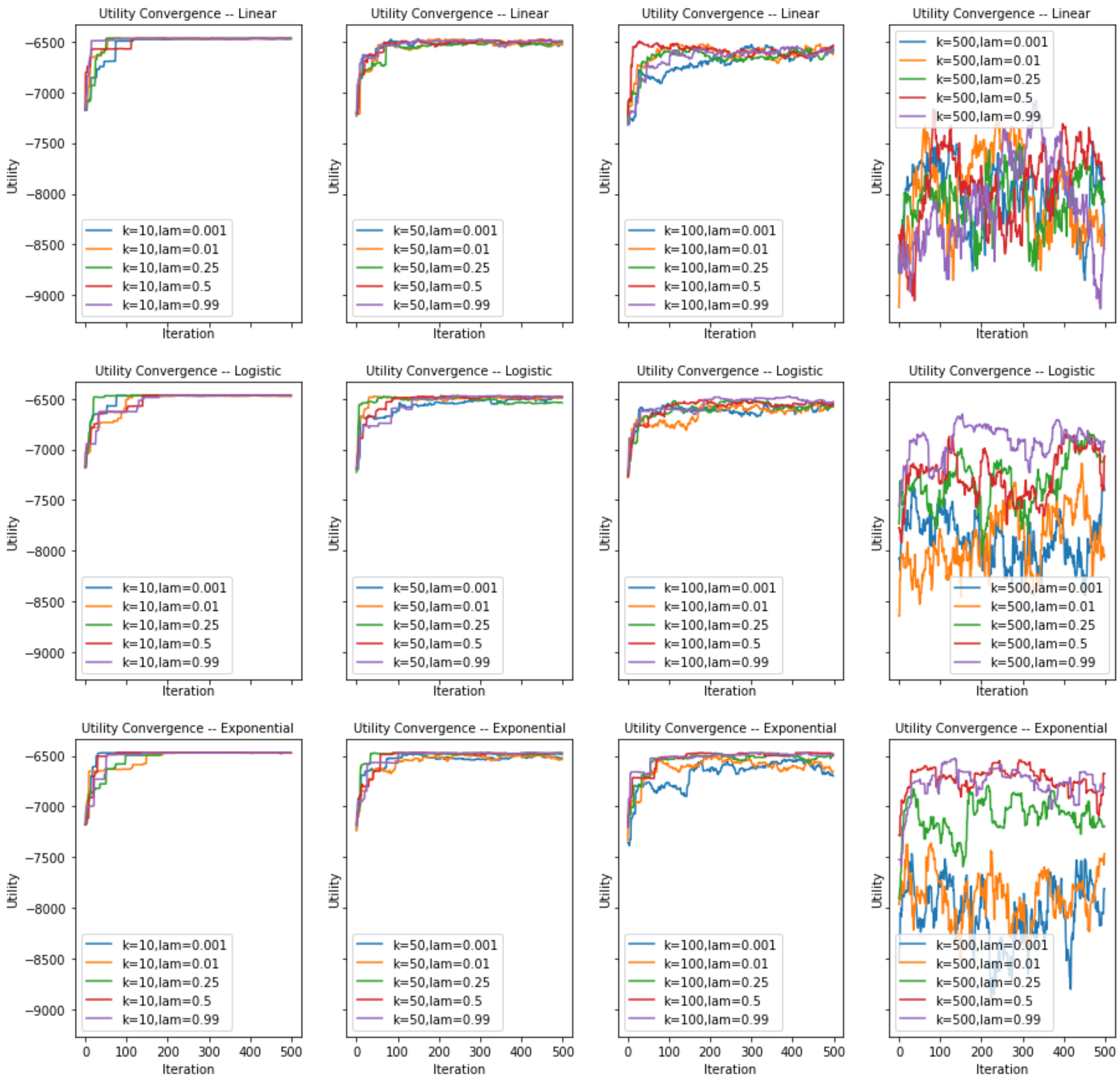
Exponential					Linear					
(k, lambda)	**Average Value**	**Most Frequent Path**	**Avg Iter Until Max Value**	**Avg Iter Until Approx Max Value**	**(k, lambda)**	**Average Value**	**Most Frequent Path**	**Avg Iter Until Max Value**	**Avg Iter Until Approx Max Value**	**(k, lambda**
(10, 0.25)	-15319.31	[1, 23, 20, 24, 44, 50, 35, 18, 7, 17, 21, 31, 46, 47, 26, 27, 28, 40, 15, 39, 43, 36, 34, 29, 30, 2, 42, 22, 32, 16, 12, 25, 5, 33, 11, 13, 14, 52, 51, 4, 37, 8, 10, 9, 19, 6, 48, 38, 49, 45, 41, 3]	500	400	(10, 0.25)	-15303.71	[1, 35, 32, 37, 34, 40, 15, 14, 13, 47, 48, 38, 44, 50, 20, 2, 7, 22, 18, 8, 41, 45, 49, 23, 30, 27, 52, 26, 24, 21, 42, 31, 43, 33, 10, 9, 16, 29, 12, 11, 51, 19, 3, 17, 36, 39, 6, 5, 46, 25, 28, 4]	500	393	(10, 0.25
(10, 0.5)	-15470.44	[1, 35, 34, 28, 51, 24, 37, 39, 48, 38, 43, 40, 10, 9, 8, 29, 16, 47, 26, 46, 30, 42, 7, 2, 18, 17, 3, 36, 31, 21, 15, 32, 45, 49, 33, 14, 52, 11, 13, 50, 20, 44, 5, 25, 4, 6, 12, 27, 23, 19, 41, 22]	500	358	(10, 0.5)	-15910.30	[1, 36, 37, 35, 34, 25, 11, 52, 51, 12, 43, 33, 40, 18, 50, 20, 16, 38, 4, 26, 27, 6, 15, 5, 48, 39, 49, 23, 30, 2, 10, 9, 22, 31, 19, 7, 42, 21, 45, 8, 41, 32, 44, 29, 13, 14, 28, 24, 46, 47, 3, 17]	500	372	(10, 0.5)

Exponential					Linear					
(k, lambda)	Average Value	Most Frequent Path	Avg Iter Until Max Value	Avg Iter Until Approx Max Value	(k, lambda)	Average Value	Most Frequent Path	Avg Iter Until Max Value	Avg Iter Until Approx Max Value	(k, lambda
(10, 0.75)	-15237.91	[1, 4, 11, 12, 15, 6, 38, 24, 40, 42, 7, 2, 30, 44, 46, 51, 33, 35, 31, 22, 45, 3, 17, 21, 34, 18, 23, 39, 37, 50, 16, 20, 43, 10, 9, 8, 41, 19, 29, 26, 47, 13, 48, 5, 28, 27, 14, 52, 25, 36, 49, 32]	500	394	(10, 0.75)	-15385.93	[1, 35, 49, 21, 7, 2, 42, 29, 39, 46, 47, 14, 52, 11, 28, 51, 6, 48, 40, 36, 19, 8, 9, 10, 23, 30, 20, 22, 3, 17, 41, 45, 37, 24, 15, 33, 43, 38, 27, 13, 26, 50, 16, 32, 18, 31, 44, 25, 12, 4, 5, 34]	500	375	(10, 0.75
(20, 0.25)	-15520.12	[1, 20, 50, 16, 39, 32, 49, 5, 12, 28, 24, 4, 11, 14, 13, 25, 6, 31, 17, 41, 19, 45, 40, 37, 48, 38, 43, 22, 23, 42, 30, 2, 7, 29, 47, 44, 26, 52, 27, 46, 51, 33, 15, 34, 35, 36, 21, 18, 3, 8, 9, 10]	500	346	(20, 0.25)	-15502.05	[1, 45, 38, 36, 34, 24, 5, 9, 41, 3, 44, 35, 49, 31, 19, 15, 28, 26, 29, 23, 30, 47, 52, 14, 33, 10, 8, 32, 39, 13, 27, 6, 4, 12, 11, 51, 43, 25, 50, 16, 37, 40, 48, 46, 20, 21, 17, 18, 22, 42, 2, 7]	500	372	(20, 0.25
(20, 0.5)	-15292.96	[1, 7, 2, 22, 31, 10, 41, 33, 4, 15, 5, 35, 44, 30, 42, 21, 25, 26, 27, 12, 48, 38, 40, 13, 52, 14, 47, 28, 39, 45, 36, 29, 16, 37, 24, 6, 51, 11, 43, 9, 49, 46, 3, 17, 18, 23, 20, 50, 32, 8, 19, 34]	500	388	(20, 0.5)	-16119.38	[1, 19, 8, 10, 41, 51, 33, 4, 24, 40, 22, 23, 29, 7, 42, 50, 12, 9, 17, 18, 34, 44, 16, 46, 13, 11, 28, 6, 47, 14, 52, 27, 26, 35, 32, 49, 36, 39, 38, 37, 5, 15, 45, 3, 43, 25, 48, 21, 2, 30, 31, 20]	500	345	(20, 0.5)
(20, 0.75)	-15365.15	[1, 16, 37, 48, 15, 43, 6, 5, 25, 33, 9, 23, 20, 44, 31, 3, 18, 17, 21, 2, 7, 42, 30, 29, 50, 35, 39, 36, 27, 11, 4, 38, 34, 22, 19, 41, 40, 49, 32, 45, 24, 51, 13, 28, 12, 52, 14, 47, 26, 46, 8, 10]	500	378	(20, 0.75)	-15054.57	[1, 22, 35, 37, 32, 18, 3, 7, 2, 30, 42, 17, 21, 23, 31, 45, 8, 28, 27, 4, 12, 13, 52, 14, 47, 9, 10, 41, 33, 25, 51, 11, 6, 15, 5, 39, 48, 38, 43, 24, 46, 26, 29, 50, 34, 40, 36, 16, 20, 19, 49, 44]	500	395	(20, 0.75

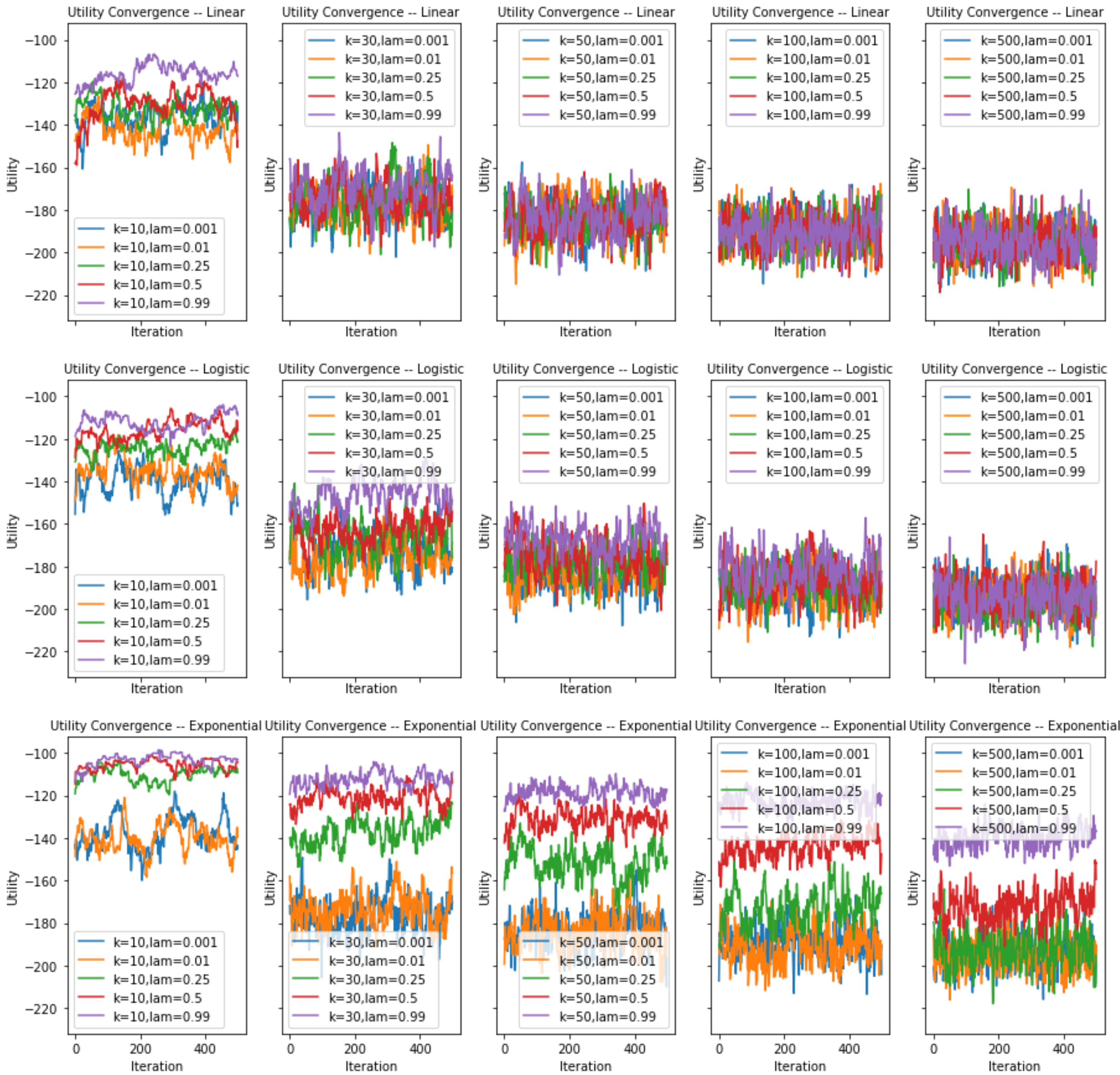
Report

Across datasets and cost structures, the algorithm performed best with lower levels of k and higher levels of lambda.

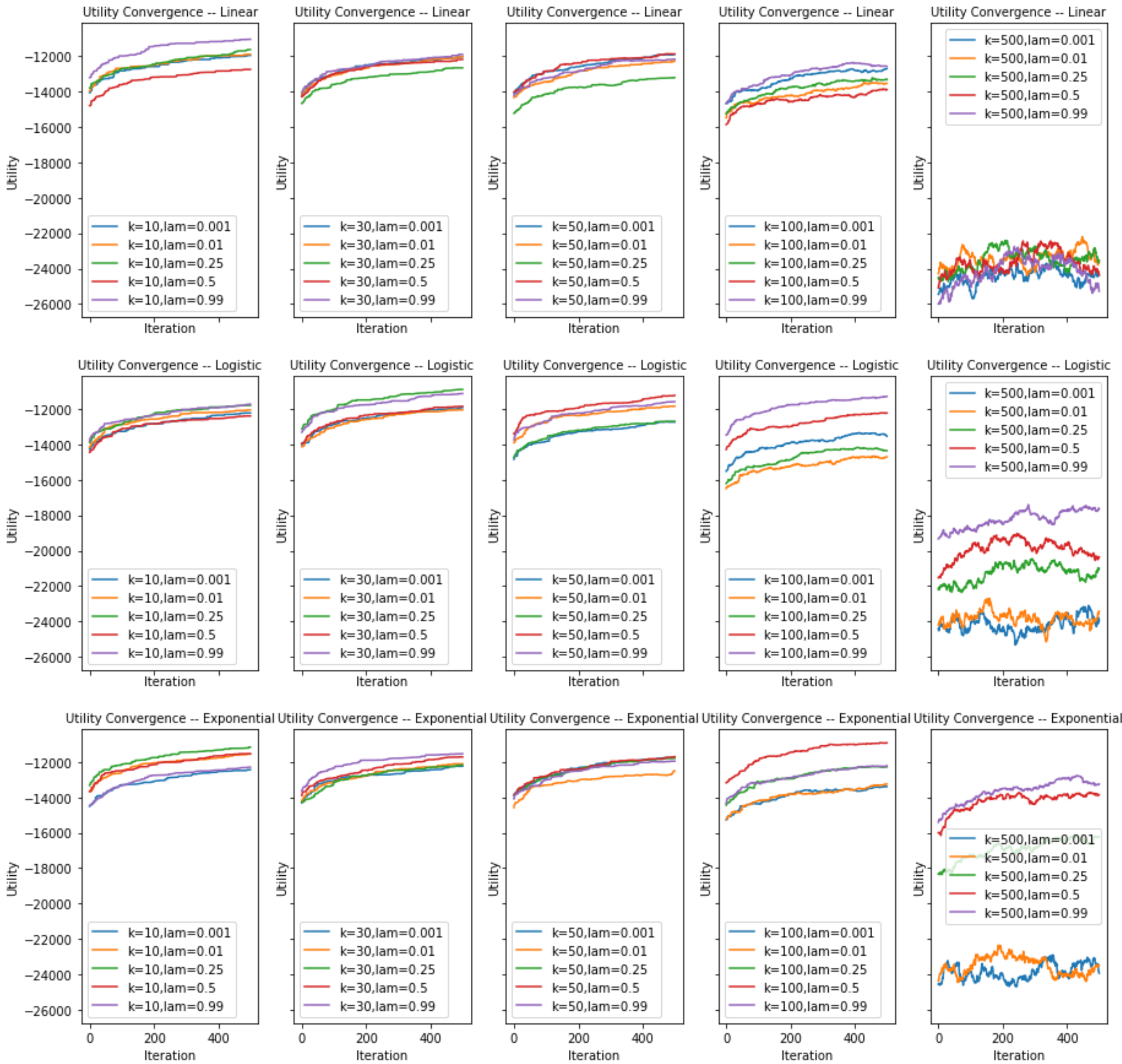
Large Costs:



Small Costs:



Berlin Dataset:

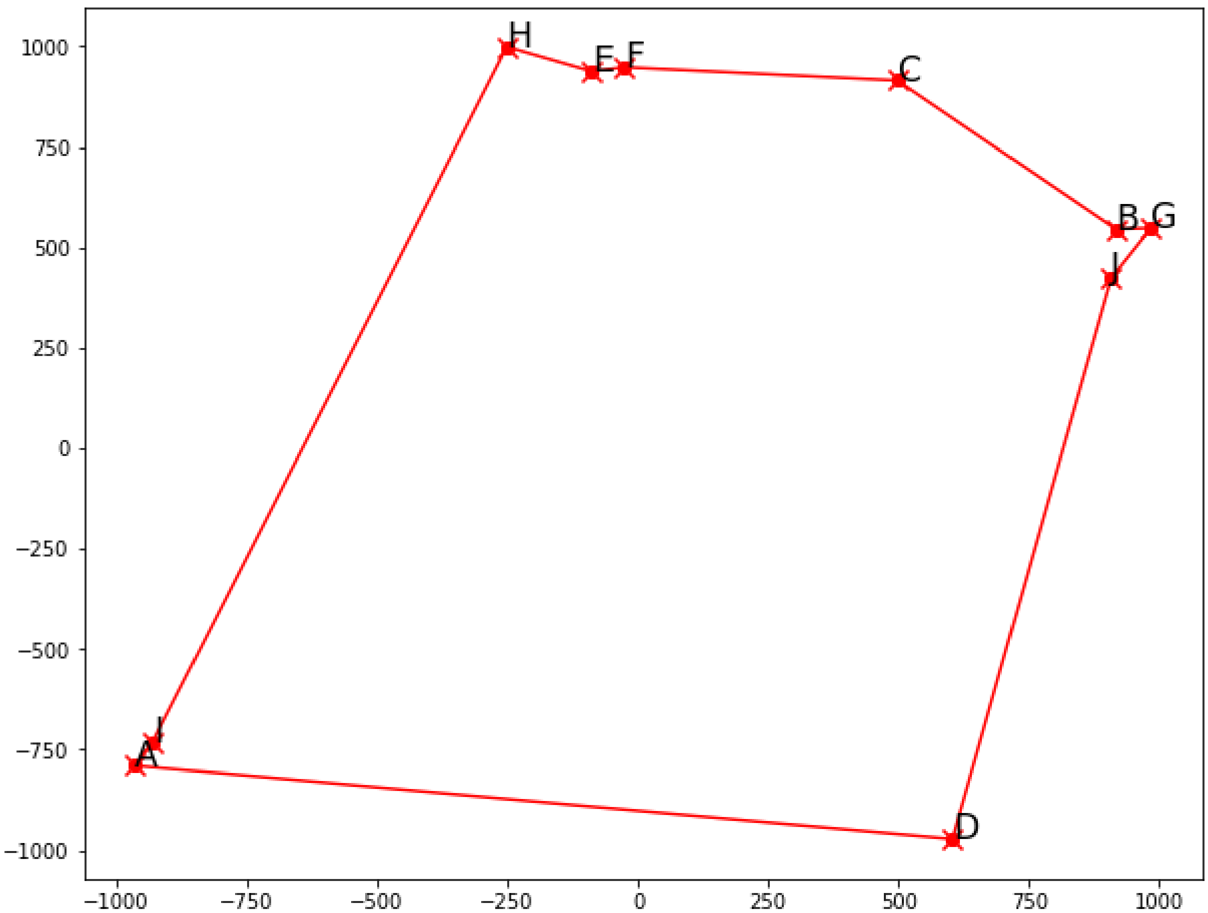


The algorithm succeeded at finding reasonable best paths when there were fewer cities. The results were more chaotic with the Berlin dataset.

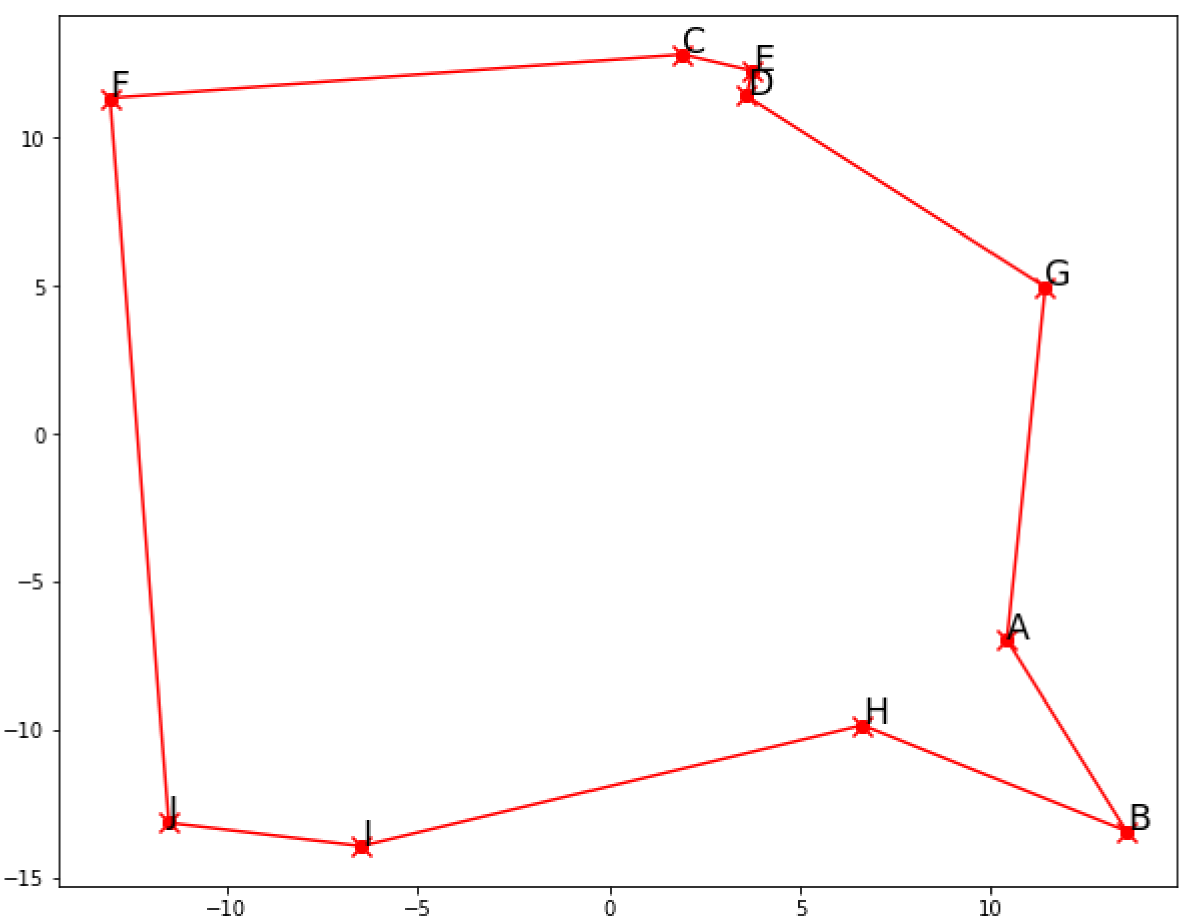
In [58]:

```
print('Large costs')
solution_visualization(TSPNode(state = best_path_1),cities_coords_2)
plt.show()
print('Small costs')
solution_visualization(TSPNode(state = best_path_2),cities_coords_3)
plt.show()
print('Berlin data')
solution_visualization(TSPNode(state = best_path_3),cities_coords_4)
plt.show()
```

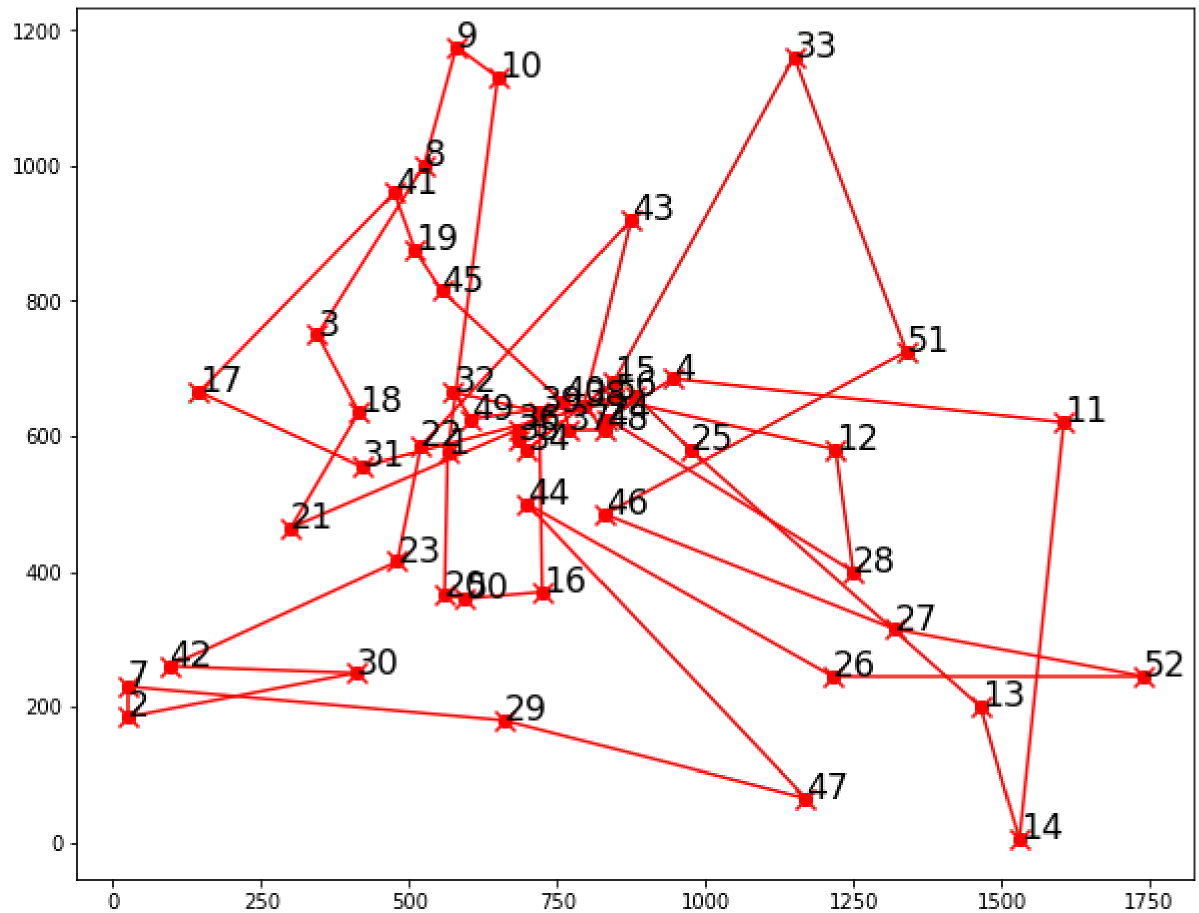
```
Large costs
```



Small costs



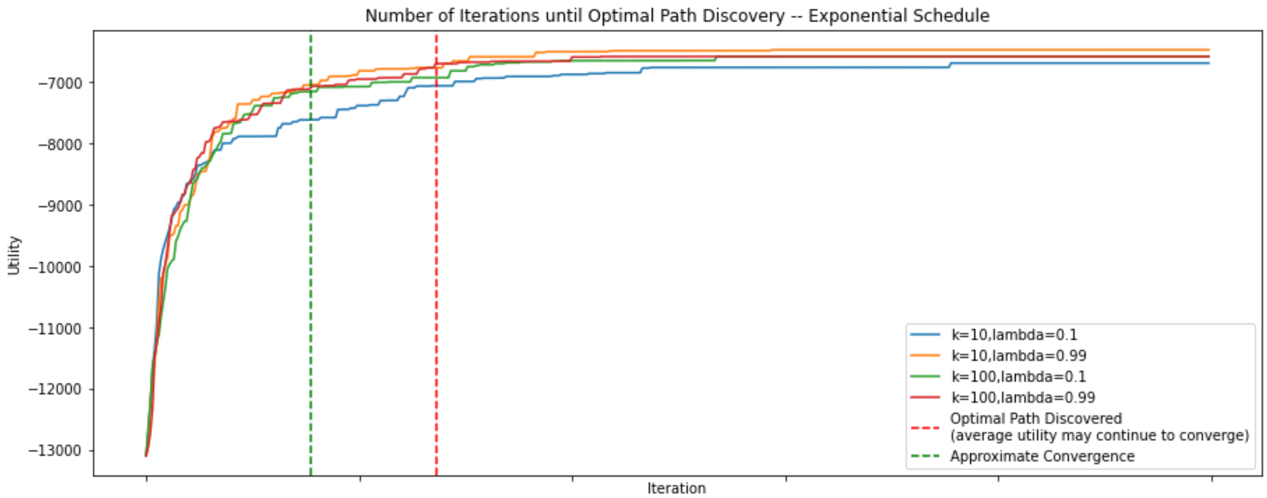
Berlin data

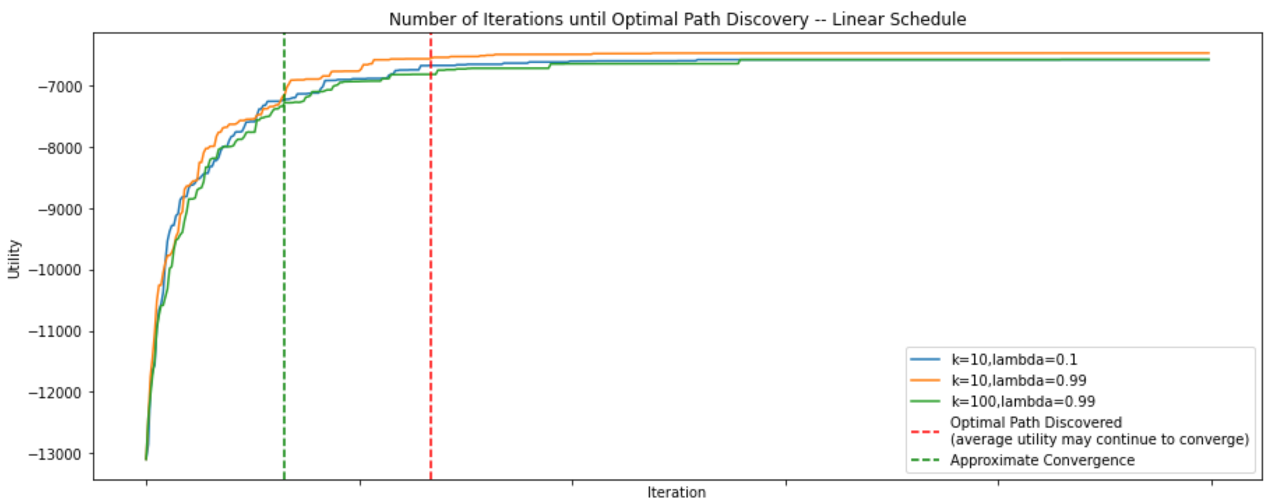


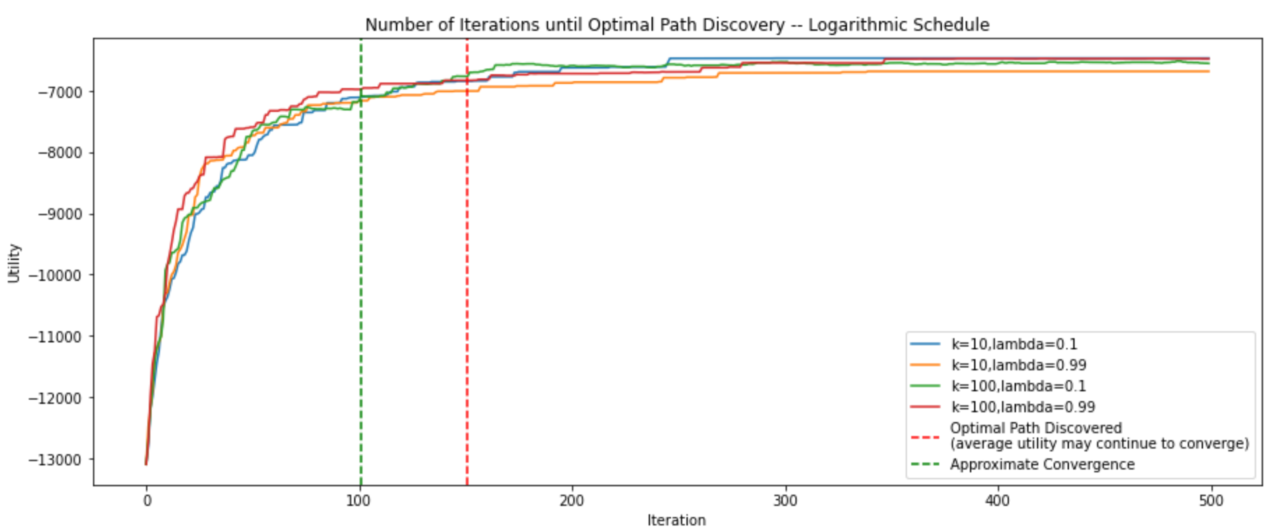
The data converged reasonably well with the datasets with fewer cities, but not with the berlin dataset. Plotting utility confirms my findings that small k and large lambas performed best.

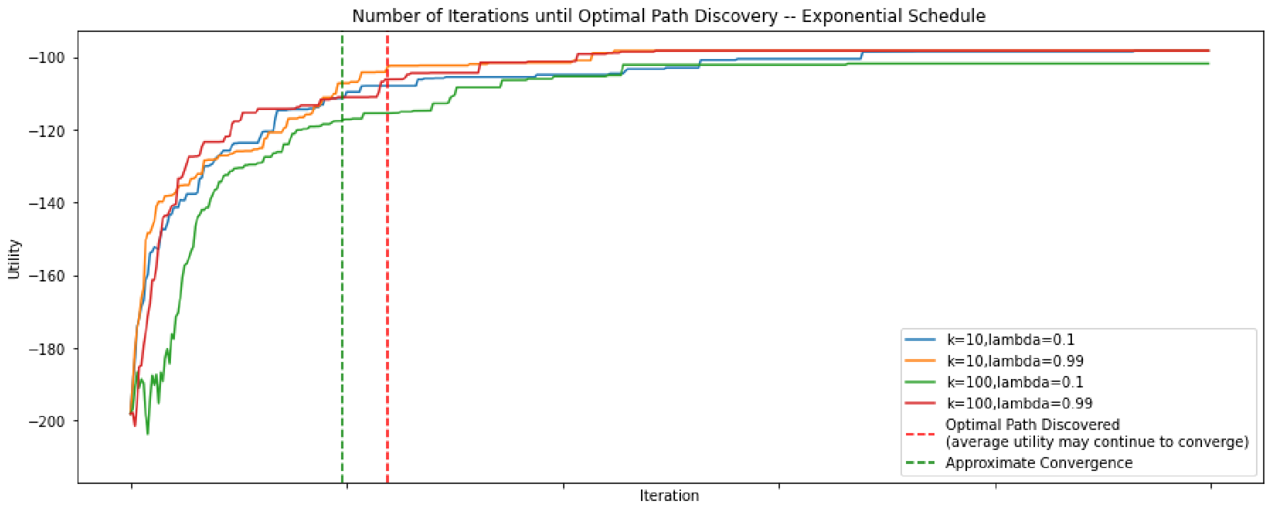
In [59]:

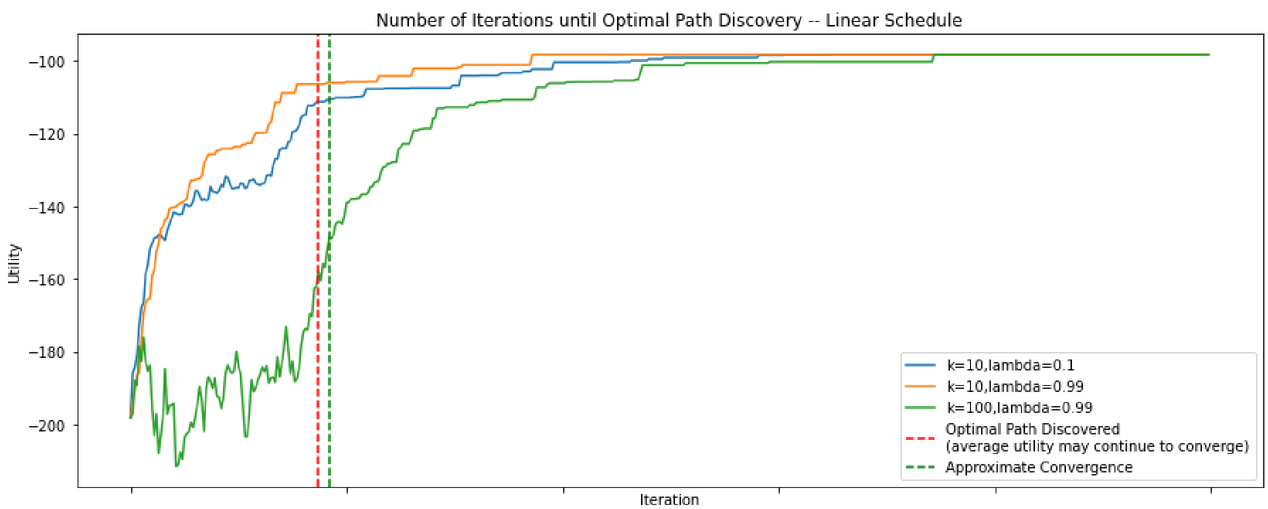
```
plot_utility(dicts_1,k_lam_list_1,first_iters_1,close_iters_1,initial_n_1)
plt.show()
plot_utility(dicts_2,k_lam_list_2,first_iters_2,close_iters_2,initial_n_2)
plt.show()
plot_utility(dicts_3,k_lam_list_3,first_iters_3,close_iters_3,initial_n_3)
plt.show()
```

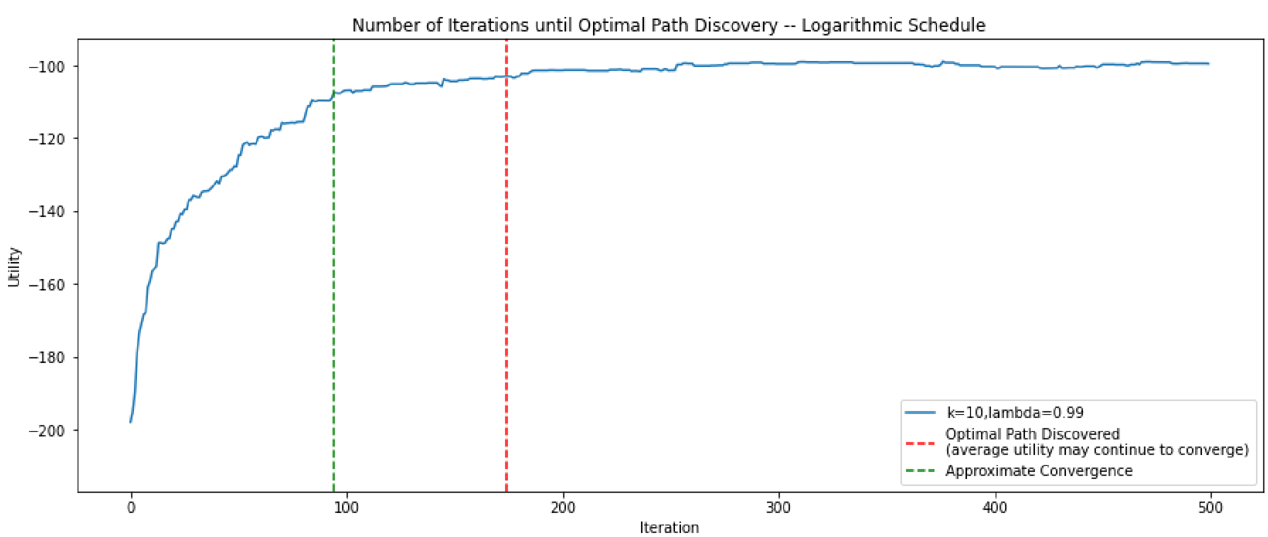

Number of Iterations until Optimal Path Discovery -- Exponential Schedule
Utility
Iteration
−7000
−8000
−9000
−10000
−11000
−12000
−13000
k=10,lambda=0.1
k=10,lambda=0.99
k=100,lambda=0.1
k=100,lambda=0.99
Optimal Path Discovered
(average utility may continue to converge)
Approximate Convergence

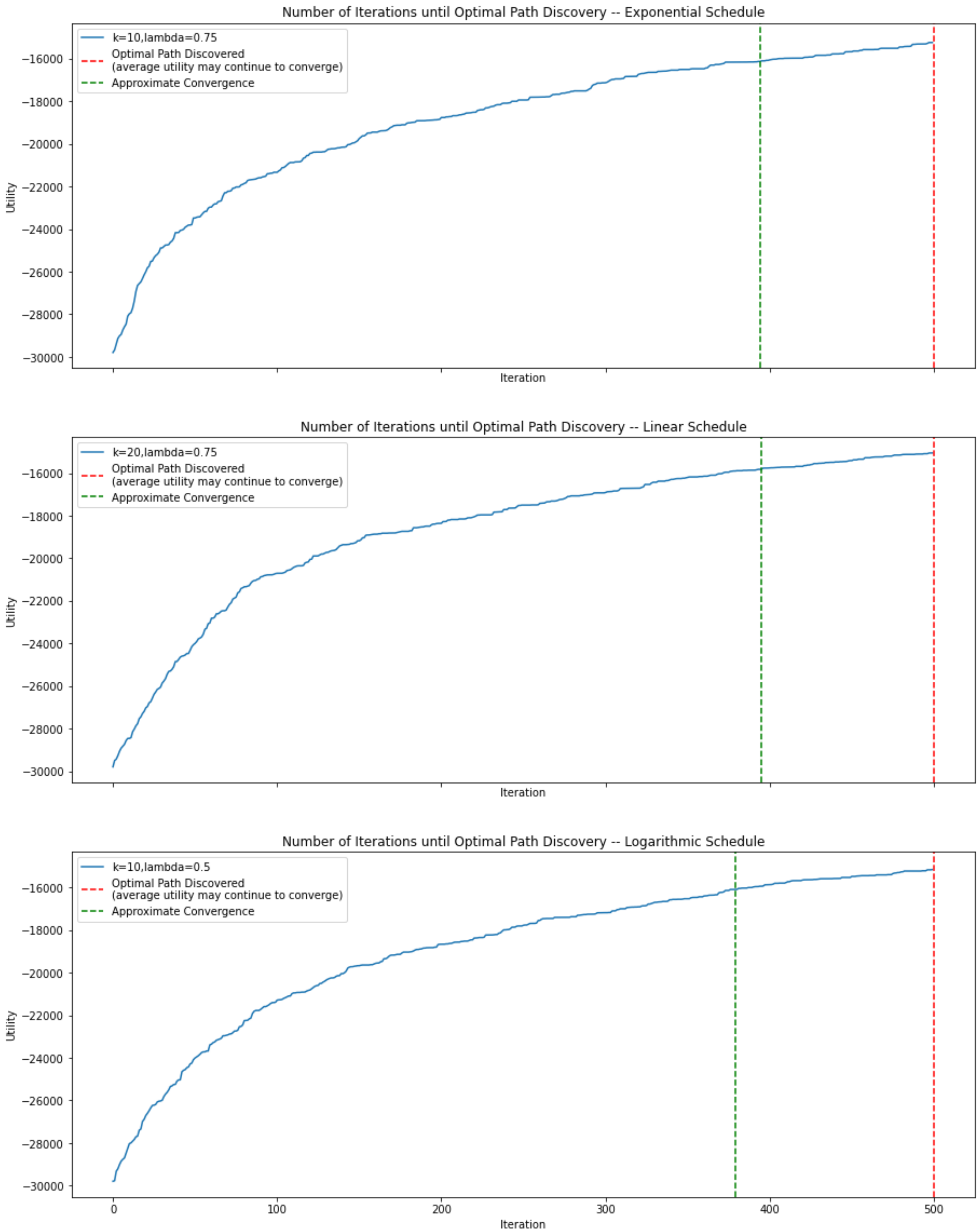

Number of Iterations until Optimal Path Discovery -- Linear Schedule
Utility
Iteration
−7000
−8000
−9000
−10000
−11000
−12000
−13000
k=10,lambda=0.1
k=10,lambda=0.99
k=100,lambda=0.99
Optimal Path Discovered
(average utility may continue to converge)
Approximate Convergence


Number of Iterations until Optimal Path Discovery -- Logarithmic Schedule
Utility
Iteration
−7000
−8000
−9000
−10000
−11000
−12000
−13000
0
100
200
300
400
500
k=10,lambda=0.1
k=10,lambda=0.99
k=100,lambda=0.1
k=100,lambda=0.99
Optimal Path Discovered
(average utility may continue to converge)
Approximate Convergence


Number of Iterations until Optimal Path Discovery -- Exponential Schedule
Utility
Iteration
−100
−120
−140
−160
−180
−200
k=10,lambda=0.1
k=10,lambda=0.99
k=100,lambda=0.1
k=100,lambda=0.99
Optimal Path Discovered
(average utility may continue to converge)
Approximate Convergence


Number of Iterations until Optimal Path Discovery -- Linear Schedule
Utility
Iteration
−100
−120
−140
−160
−180
−200
k=10,lambda=0.1
k=10,lambda=0.99
k=100,lambda=0.99
Optimal Path Discovered
(average utility may continue to converge)
Approximate Convergence


Number of Iterations until Optimal Path Discovery -- Logarithmic Schedule
Utility
Iteration
−100
−120
−140
−160
−180
−200
0
100
200
300
400
500
k=10,lambda=0.99
Optimal Path Discovered
(average utility may continue to converge)
Approximate Convergence



Number of Iterations until Optimal Path Discovery -- Exponential Schedule
k=10,lambda=0.75
Optimal Path Discovered
(average utility may continue to converge)
Approximate Convergence
Utility
Iteration
Number of Iterations until Optimal Path Discovery -- Linear Schedule
k=20,lambda=0.75
Optimal Path Discovered
(average utility may continue to converge)
Approximate Convergence
Utility
Iteration
Number of Iterations until Optimal Path Discovery -- Logarithmic Schedule
k=10,lambda=0.5
Optimal Path Discovered
(average utility may continue to converge)
Approximate Convergence
Utility
Iteration
−16000
−18000
−20000
−22000
−24000
−26000
−28000
−30000
0
100
200
300
400
500