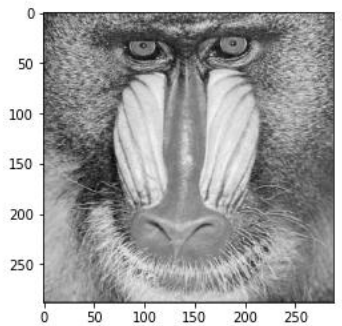
[27]
```python
import cv2
from google.colab import files
import numpy as np
from numpy import *
import matplotlib
import matplotlib.pyplot as plt
import pandas as pd
import re
import scipy
import scipy.stats as stats
from scipy.io import loadmat
import seaborn as sb
import plotly.graph_objects as go

import sklearn
from sklearn import manifold
from sklearn.manifold import Isomap
from sklearn.manifold import SpectralEmbedding as LaplacianEigenmap
from sklearn.neighbors import kneighbors_graph


import sys
import time


from google.colab import drive
drive.mount('/content/drive')
```

```
Drive already mounted at /content/drive; to attempt to forcibly remount, call drive.mount("/content/drive", force_remount=True).
```

Problem 2

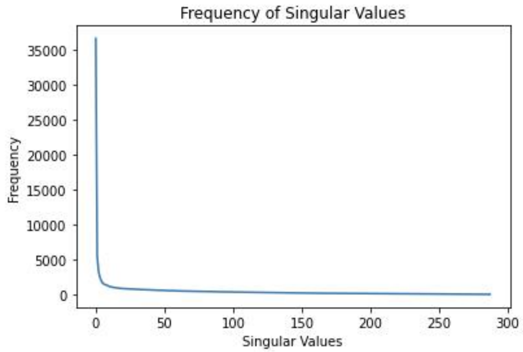
1.

[49]
```python
img = cv2.imread('/content/drive/MyDrive/2022 files/CS 584/CS584 - Assignment 4/mandrill_color.png')
X   = np.mean(img, axis=2)
plt.imshow(X, cmap='gray')
plt.show()
```



2.

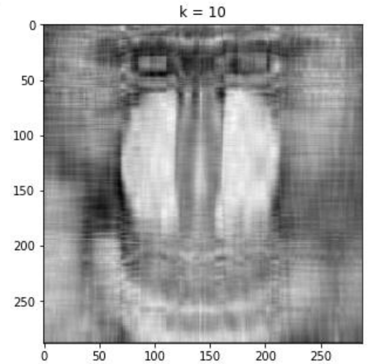
```
[29] XtX = np.asmatrix(np.matmul(X.T, X))

e_vals,e_vecs = np.linalg.eig(XtX)
idx           = e_vals.argsort()[::-1]
singulars     = np.sqrt(e_vals[idx])
e_vecs        = e_vecs[:,idx]
e_vals        = sorted(e_vals,reverse=True)

V = np.asmatrix(e_vecs)
S = np.diag(singulars)
U = np.matmul(np.matmul(X,V),np.linalg.inv(S))

plt.plot(singulars)
plt.xlabel('Singular Values')
plt.ylabel('Frequency')
plt.title('Frequency of Singular Values')
plt.show()
```



3.

```
# def compression_ratio(k,compressed):
#     m,n = X.shape
#     compression_ratio = np.round((k*(m+n+1))/(m*n),3)
#     return compression_ratio


k=10
compressed = U[:, :k] @ np.diag(list(singulars)[:k]) @ V.T[:k,:]
m,n = compressed.shape
compression_ratio = np.round((k*(m+n+1))/(m*n),3)

plt.figure(figsize=(5,5))
plt.imshow(compressed, cmap='gray'),
plt.title(f'k = {k}')
plt.show()
k=10
print('Compression ratio when k={}: {}'.format(k,compression_ratio))
```

[30]

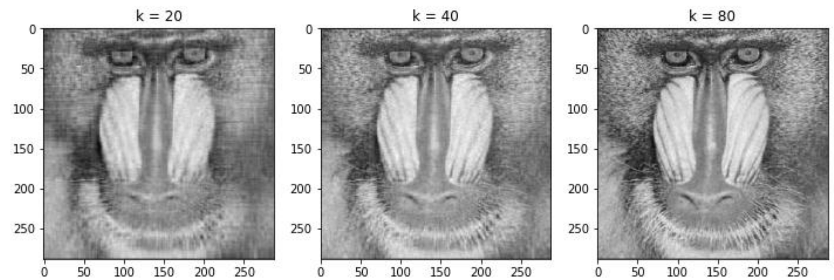



```
Compression ratio when k=10: 0.07
```

4.

[31]

```
k = [20,40,80]
plt.figure(figsize=(12, 12))

crs = []
for i in range(len(k)):
    compressed = U[:, :k[i]] @ np.diag(list(singulars)[:k[i]]) @ V.T[:k[i],:]#V[:k[i], :]
    m,n        = compressed.shape
    cr         = np.round((k[i]*(m+n+1))/(m*n),3)
    crs.append(cr)

    plt.subplot(2, 3, i+1),
    plt.imshow(compressed, cmap='gray'),
    plt.title(f'k = {k[i]}')
plt.show()
print('\n\n')

dict_ = {'':['Compression Ratio'], 'k=20':crs[0], 'k=40':crs[1], 'k=80':crs[2]}
display(pd.DataFrame(dict_).set_index(''))
print("\n\nThe reconstructed image looks pretty close to the original when k = 80, so there is no need to increase k any more.")
```



	k=20	k=40	k=80
Compression Ratio	0.139	0.278	0.557

```
The reconstructed image looks pretty close to the original when k = 80, so there is no need to increase k any more.
```

Problem 3

1.

[32]
```python
places_file = '/content/drive/MyDrive/2022 files/CS 584/CS584 - Assignment 4/places.rtf'

columns  = ['climate', 'housing', 'healthcare', 'crime', 'transportation', 'education',
            'arts', 'recreation', 'economic welfare', 'c1', 'c2', 'c3', 'c4', 'c5']
columns_ = ['City'] + columns
col_data = [[]]*10
places_dict = dict(zip(columns_,col_data))

with open(places_file) as f:
  lines = f.readlines()
  lines = [l for l in lines if l[0].lower() in 'abcdefghijklmnopqrstuvwxyz']

for line in lines:
  line = re.sub(r'[\s\n\t\\]+',' ',line).split(' ')
  line = [int(line[i]) if i > 0 else line[i] for i in range(10)]
  for j in range(10):
    places_dict[columns_[j]] = places_dict[columns_[j]] + [line[j]]


places_data = pd.DataFrame(places_dict).set_index('City')
display(places_data.head())
```

City	climate	housing	healthcare	crime	transportation	education	arts	recreation	economic welfare
Akron,OH	575	8138	1656	886	4883	2438	5564	2632	4350
Albany,GA	468	7339	618	970	2531	2560	237	859	5250
Albany-Schenectady-Troy,NY	476	7908	1431	610	6883	3399	4655	1617	5864
Albuquerque,NM	659	8393	1853	1483	6558	3026	4496	2612	5727
Alexandria,LA	520	5819	640	727	2444	2972	334	1018	5254

2.

[33]
```python
X_log = np.log10(places_data)
display(X_log.head())
```

City	climate	housing	healthcare	crime	transportation	education	arts	recreation	economic welfare
Akron,OH	2.759668	3.910518	3.219060	2.947434	3.688687	3.387034	3.745387	3.420286	3.638489
Albany,GA	2.670246	3.865637	2.790988	2.986772	3.403292	3.408240	2.374748	2.933993	3.720159
Albany-Schenectady-Troy,NY	2.677607	3.898067	3.155640	2.785330	3.837778	3.531351	3.667920	3.208710	3.768194
Albuquerque,NM	2.818885	3.923917	3.267875	3.171141	3.816771	3.480869	3.652826	3.416973	3.757927
Alexandria,LA	2.716003	3.764848	2.806180	2.861534	3.388101	3.473049	2.523746	3.007748	3.720490

3.

```
[34] X_mat = np.asarray(X_log)
     x_bar = np.array([np.mean(c) for c in X_mat.T])
     X_std = X_mat - np.mean(X_mat, axis = 0)

     C = np.matmul(X_std.T,X_std)
     e_vals,e_vecs = np.linalg.eig(C)
     idx           = e_vals.argsort()[::-1]
     e_vecs        = np.round(e_vecs[:,idx],3)
     e_vals        = np.round(sorted(e_vals,reverse=True),3)
```

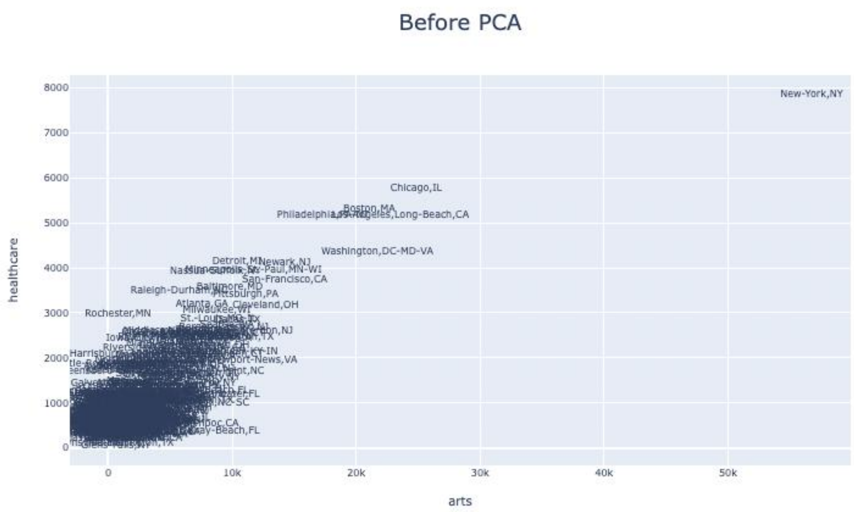
4.

```
[35] PC1,PC2 = e_vecs[:,0],e_vecs[:,1]
     print('PC1:\n{}\n\nPC2:\n{}\n\n'.format(PC1,PC2))
     i1,i2 = list(np.abs(PC1)).index(max(np.abs(PC1))),list(np.abs(PC2)).index(max(np.abs(PC2)))
     f1,f2 = columns[i1],columns[i2]
     print(('The most important features correspond to the largest absolute values in'
         + ' the principal\ncomponents.In PC1 and PC2, the largest absolute values'
         + ' are at indices {} and {}, respectively,\nwhere the first index is 0.'
         + ' These correspond to features {} and {}, i.e., {}\nand {}.').
         format(i1,i2,i1,i2,f1,f2))
```

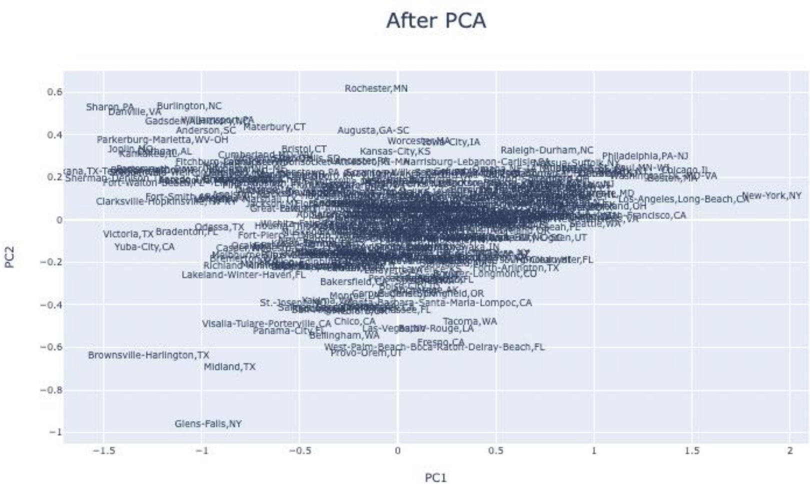
```
PC1:
[0.035 0.093 0.406 0.101 0.15  0.032 0.875 0.159 0.02 ]

PC2:
[-0.009 -0.012  0.857 -0.222 -0.058  0.062 -0.3   -0.341 -0.053]


The most important features correspond to the largest absolute values in the principal
components.In PC1 and PC2, the largest absolute values are at indices 6 and 2, respectively,
where the first index is 0. These correspond to features 6 and 2, i.e., arts
and healthcare.
```

5.

```python
x_scores = [[x[i1] for x in X_std.tolist()],[x[i2] for x in X_std.tolist()]]
PCs      = e_vecs[:,:2]
PC_mat   = np.dot(PCs.T, X_std.T).T
PC_df    = pd.DataFrame(PC_mat, columns=['PC1','PC2'])

fig = go.Figure(data=go.Scatter(x=places_data.iloc[:,i1],
                                y=places_data.iloc[:,i2],
                                mode='text',
                                text=places_data.index))
fig.update_layout(title={'text': "Before PCA",
                         'x':0.5,
                         'xanchor': 'center',
                         'yanchor': 'top',
                         'font':dict(size=18)},
                  xaxis_title=columns[i1],
                  yaxis_title=columns[i2],
                  font=dict(size=8),
                  width=800,height=500)
fig.show()


fig = go.Figure(data=go.Scatter(x=PC_df['PC1'],
                                y=PC_df['PC2'],
                                mode='text',
                                text=places_data.index))
fig.update_layout(title={'text': "After PCA",
                         'x':0.5,
                         'xanchor': 'center',
                         'yanchor': 'top',
                         'font':dict(size=18)},
                  xaxis_title='PC1',
                  yaxis_title='PC2',
                  font=dict(size=8),
                  width=800,height=500)
fig.show()
```



[36]



Before applying PCA, New York City is a clear outlier. This isn't the case after PCA.

6.

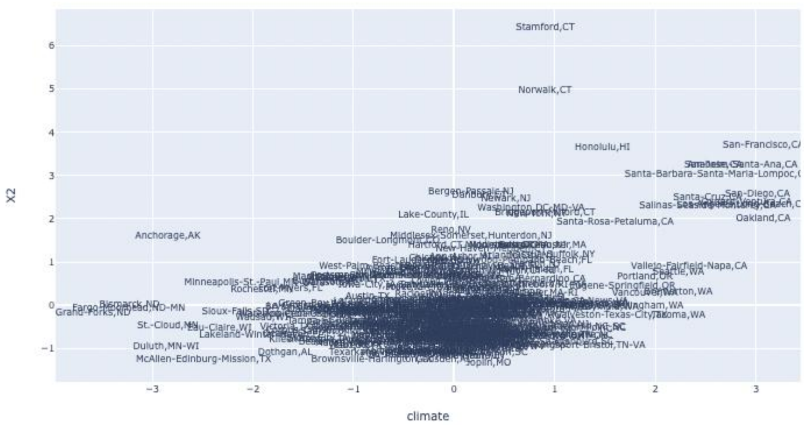
```
X = pd.DataFrame(places_data.apply(stats.zscore,axis=0)).iloc[:,:9]
display(X.head())

X_mat = np.asarray(X)
C     = np.matmul(X_mat.T,X_mat)

e_vals,e_vecs = np.linalg.eig(C)
idx           = e_vals.argsort()[::-1]
e_vecs        = np.round(e_vecs[:,idx],3)
e_vals        = np.round(sorted(e_vals,reverse=True),3)

PC1,PC2 = e_vecs[:,0],e_vecs[:,1]
i1,i2   = list(np.abs(PC1)).index(max(np.abs(PC1))),list(np.abs(PC2)).index(max(np.abs(PC2)))
f1,f2   = columns[i1],columns[i2]
print('PC1:\n{}\n(Associated with {})\nPC2:\n{}\n(Associated with {})\n'.format(PC1,f1,PC2,f2))
```

	climate	housing	healthcare	crime	transportation	education	arts	recreation	economic welfare
City									
Akron,OH	0.299769	-0.090292	0.466609	-0.210475	0.463339	-1.175471	0.518568	0.971738	-1.084171
Albany,GA	-0.585961	-0.425681	-0.569703	0.024722	-1.157450	-0.795144	-0.629306	-1.223873	-0.249436
Albany-Schenectady-Troy,NY	-0.519738	-0.186837	0.241975	-0.983264	1.841560	1.820380	0.322694	-0.285196	0.320038
Albuquerque,NM	0.995109	0.016747	0.663288	1.461101	1.617599	0.657578	0.288433	0.946971	0.192973
Alexandria,LA	-0.155513	-1.063718	-0.547739	-0.655669	-1.217402	0.489237	-0.608404	-1.026974	-0.245726

```
PC1:
[-0.206 -0.356 -0.46  -0.281 -0.351 -0.275 -0.463 -0.328 -0.139]
(Associated with arts)
PC2:
[ 0.218  0.254 -0.3    0.351 -0.182 -0.484 -0.196  0.384  0.47 ]
(Associated with education)
```

```python
z_data = pd.DataFrame(places_data.apply(stats.zscore,axis=0))

#####

fig = go.Figure(data=go.Scatter(x=z_data.iloc[:,0],
                                y=z_data.iloc[:,1],
                                mode='text',
                                text=z_data.index))
fig.update_layout(title={'text': "Before PCA",
                         'x':0.5,
                         'xanchor': 'center',
                         'yanchor': 'top',
                         'font':dict(size=18)},
                  xaxis_title=columns[0],
                  yaxis_title="X2",
                  font=dict(size=8),
                  width=800,
                  height=500)
fig.show()

#####

X_z    = np.asmatrix(z_data)
C      = np.matmul(X_z.T,X_z)

e_vals, e_vecs   = np.linalg.eig(C)
idx              = np.argsort(e_vals)[::-1]
e_vals_, e_vecs_ = e_vals[idx], e_vecs[:,idx]

PCs      = e_vecs[:,:2]
PC1, PC2 = PCs[:,0],PCs[:,1]
print('PC1:\n{}\n\nPC2:\n{}\n\n'.format(PC1,PC2))

PC_mat = np.dot(PCs.T , X_z.T).T
PC_df  = pd.DataFrame(PC_mat, columns=['PC1','PC2'])

#####

fig = go.Figure(data=go.Scatter(x=PC_df.iloc[:,0],
                                y=PC_df.iloc[:,1],
                                mode='text',
                                text=places_data.index))
fig.update_layout(title={'text': "After PCA",
                         'x':0.5,
                         'xanchor': 'center',
                         'yanchor': 'top',
                         'font':dict(size=18)},
                  xaxis_title="PC1",
                  yaxis_title="PC2",
                  font=dict(size=8),
                  width=800,
                  height=500)
fig.show()
```

Before PCA




```
PC1:
[[-0.20597624]
 [-0.35640757]
 [-0.45991451]
 [-0.28148136]
 [-0.35096481]
 [-0.27519246]
 [-0.46271437]
 [-0.32763525]
 [-0.13941911]]

PC2:
[[ 0.2179069 ]
 [ 0.254412  ]
 [-0.29980915]
 [ 0.35118173]
 [-0.18240488]
 [-0.48383877]
 [-0.19631688]
 [ 0.3844578 ]
 [ 0.46995465]]
```

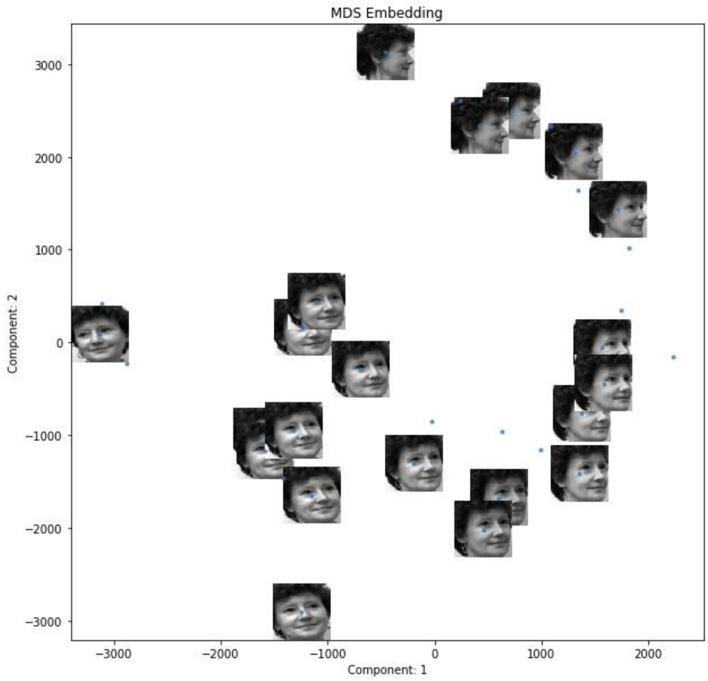
After PCA

When I computed the base-10 logarithm, the data initially showed NYC as an outlier before PCA, but not after. Here, it is the opposite.

Problem 4

1.

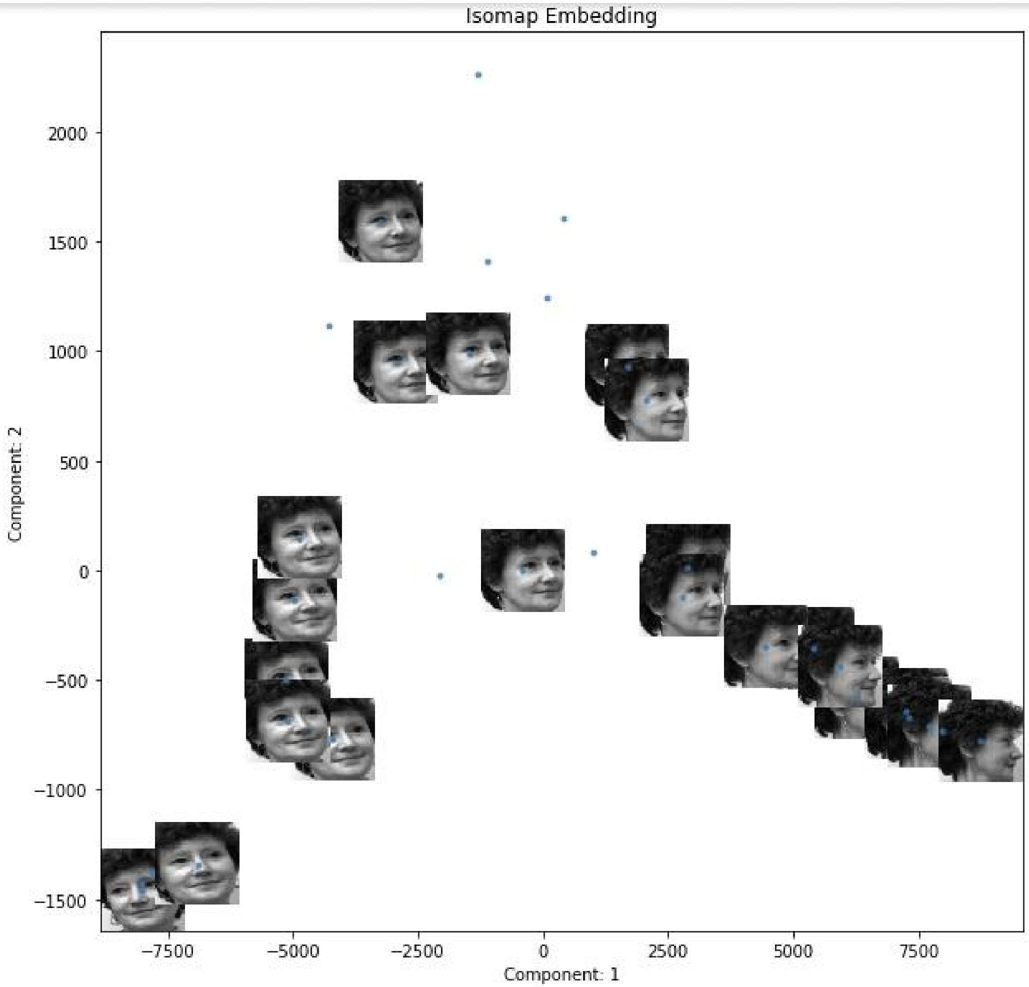
```
faces    = loadmat('/content/drive/MyDrive/2022 files/CS 584/CS584 - Assignment 4/face.mat')
faces_   = faces['Y']
s1,s2,s3 = faces_.shape
faces_   = faces_.reshape(s1*s2,s3)
df       = pd.DataFrame(faces_).T

mds               = manifold.MDS(n_components=2)
manifold_mat_mds = mds.fit_transform(df)
manifold_df_mds  = pd.DataFrame(manifold_mat_mds, columns=['Component 1', 'Component 2'])

fig = plt.figure()
fig.set_size_inches(10, 10)
ax = fig.add_subplot(111)
ax.set_title('MDS Embedding')
ax.set_xlabel('Component: 1')
ax.set_ylabel('Component: 2')
x_size = (max(manifold_df_mds['Component 1']) - min(manifold_df_mds['Component 1']))*0.1
y_size = (max(manifold_df_mds['Component 2']) - min(manifold_df_mds['Component 2']))*0.1
for i in range(num_images):
    img_num = np.random.randint(0, num_images)
    x0 = manifold_df_mds.loc[img_num, 'Component 1'] - (x_size / 2.)
    y0 = manifold_df_mds.loc[img_num, 'Component 2'] - (y_size / 2.)
    x1 = manifold_df_mds.loc[img_num, 'Component 1'] + (x_size / 2.)
    y1 = manifold_df_mds.loc[img_num, 'Component 2'] + (y_size / 2.)
    img = df.iloc[img_num,:].values.reshape(s1, s2)
    ax.imshow(img,
              aspect='auto',
              cmap=plt.cm.gray,
              interpolation='nearest',
              extent=(x0, x1, y0, y1))

ax.scatter(manifold_df_mds['Component 1'], manifold_df_mds['Component 2'], marker='.',alpha=0.7)
plt.show()
```



2.

```
[44] iso = manifold.Isomap(n_neighbors=5, n_components=2)
     iso.fit(df)
     manifold_mat_iso = iso.transform(df)
     manifold_df_lle  = pd.DataFrame(manifold_mat_iso, columns=['Component 1', 'Component 2'])

     num_images, num_px = df.shape

     fig = plt.figure()
     fig.set_size_inches(10, 10)
     ax = fig.add_subplot(111)
     ax.set_title('Isomap Embedding')
     ax.set_xlabel('Component: 1')
     ax.set_ylabel('Component: 2')
     x_size = (max(manifold_df_lle['Component 1']) - min(manifold_df_lle['Component 1']))*0.1
     y_size = (max(manifold_df_lle['Component 2']) - min(manifold_df_lle['Component 2']))*0.1
     for i in range(num_images):
         img_num = np.random.randint(0, num_images)
         x0 = manifold_df_lle.loc[img_num, 'Component 1'] - (x_size / 2.)
         y0 = manifold_df_lle.loc[img_num, 'Component 2'] - (y_size / 2.)
         x1 = manifold_df_lle.loc[img_num, 'Component 1'] + (x_size / 2.)
         y1 = manifold_df_lle.loc[img_num, 'Component 2'] + (y_size / 2.)
         img = df.iloc[img_num,:].values.reshape(s1, s2)
         ax.imshow(img,
                   aspect='auto',
                   cmap=plt.cm.gray,
                   interpolation='nearest',
                   extent=(x0, x1, y0, y1))

     ax.scatter(manifold_df_lle['Component 1'], manifold_df_lle['Component 2'], marker='.',alpha=0.7)
     plt.show()
     print("The Isomap Embedding results in a trend that is \'unrolled\' compared\n"
          +"to the MDS Embedding.")
```

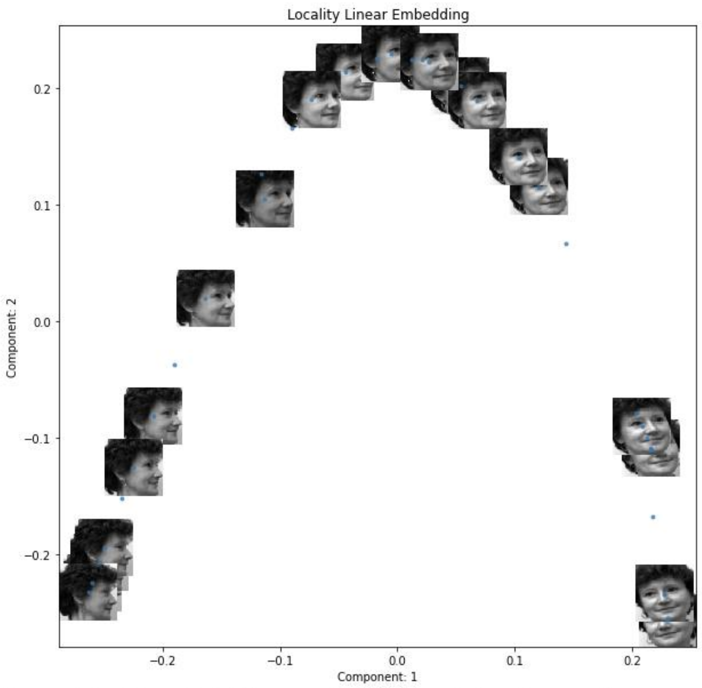
[44]



The Isomap Embedding results in a trend that is 'unrolled' compared to the MDS Embedding.

3.

```python
lle = manifold.LocallyLinearEmbedding(n_neighbors=5, n_components=2)
lle.fit(df)
manifold_mat_lle = lle.transform(df)
manifold_df_lle  = pd.DataFrame(manifold_mat_lle, columns=['Component 1', 'Component 2'])

num_images, num_px = df.shape

fig = plt.figure()
fig.set_size_inches(10, 10)
ax = fig.add_subplot(111)
ax.set_title('Locality Linear Embedding')
ax.set_xlabel('Component: 1')
ax.set_ylabel('Component: 2')
x_size = (max(manifold_df_lle['Component 1']) - min(manifold_df_lle['Component 1']))*0.1
y_size = (max(manifold_df_lle['Component 2']) - min(manifold_df_lle['Component 2']))*0.1
for i in range(num_images):
    img_num = np.random.randint(0, num_images)
    x0 = manifold_df_lle.loc[img_num, 'Component 1'] - (x_size / 2.)
    y0 = manifold_df_lle.loc[img_num, 'Component 2'] - (y_size / 2.)
    x1 = manifold_df_lle.loc[img_num, 'Component 1'] + (x_size / 2.)
    y1 = manifold_df_lle.loc[img_num, 'Component 2'] + (y_size / 2.)
    img = df.iloc[img_num,:].values.reshape(s1, s2)
    ax.imshow(img,
              aspect='auto',
              cmap=plt.cm.gray,
              interpolation='nearest',
              extent=(x0, x1, y0, y1))

ax.scatter(manifold_df_lle['Component 1'], manifold_df_lle['Component 2'], marker='.',alpha=0.7)
plt.show()
print("The Locality Linear Embedding results in a trend clearly resembles a\n"
    + "parabola, exhibiting the clearest trend yet.")
```

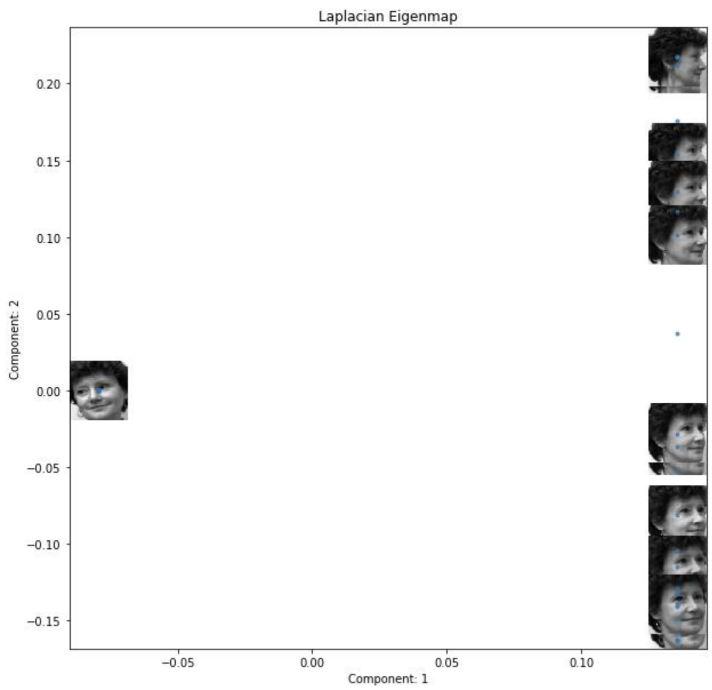



```
The Locality Linear Embedding results in a trend clearly resembles a
parabola, exhibiting the clearest trend yet.
```

4.

```python
#adjacency = kneighbors_graph(faces_.T, 5, mode='connectivity', include_self=True)
le              = LaplacianEigenmap(affinity='nearest_neighbors',n_components=2)
manifold_mat_le = le.fit_transform(faces_.T)
manifold_df_le  = pd.DataFrame(manifold_mat_le, columns=['Component 1', 'Component 2'])

num_images, num_px = df.shape

fig = plt.figure()
fig.set_size_inches(10, 10)
ax = fig.add_subplot(111)
ax.set_title('Laplacian Eigenmap')
ax.set_xlabel('Component: 1')
ax.set_ylabel('Component: 2')
x_size = (max(manifold_df_le['Component 1']) - min(manifold_df_le['Component 1']))*0.1
y_size = (max(manifold_df_le['Component 2']) - min(manifold_df_le['Component 2']))*0.1
for i in range(num_images):
    img_num = np.random.randint(0, num_images)
    x0 = manifold_df_le.loc[img_num, 'Component 1'] - (x_size / 2.)
    y0 = manifold_df_le.loc[img_num, 'Component 2'] - (y_size / 2.)
    x1 = manifold_df_le.loc[img_num, 'Component 1'] + (x_size / 2.)
    y1 = manifold_df_le.loc[img_num, 'Component 2'] + (y_size / 2.)
    img = df.iloc[img_num,:].values.reshape(s1, s2)
    ax.imshow(img,
              aspect='auto',
              cmap=plt.cm.gray,
              interpolation='nearest',
              extent=(x0, x1, y0, y1))

ax.scatter(manifold_df_le['Component 1'], manifold_df_le['Component 2'], marker='.',alpha=0.7)
plt.show()
print("The Laplacian Eigenmap results in the only (almost) linear trend.")
```

```
/usr/local/lib/python3.7/dist-packages/sklearn/manifold/_spectral_embedding.py:261: UserWarning:

Graph is not fully connected, spectral embedding may not work as expected.
```




```
The Laplacian Eigenmap results in the only (almost) linear trend.
```

Problem 5

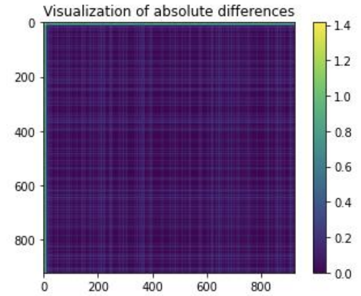
1.

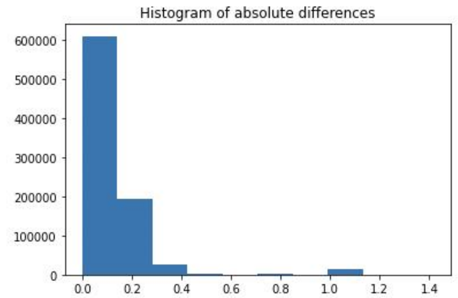
[47]
```
n=10
d=5000
X   = np.identity(d)[:,:10]
eps = 0.1
m   = int(np.ceil(4*math.log(n)/(eps**2)))
print("m = {}".format(m))
print("Rounded to the next integer: m = {}".format(m))
```

```
m = 922
Rounded to the next integer: m = 922
```

2.

[18]
```
from sklearn.metrics.pairwise import euclidean_distances
from sklearn.random_projection import SparseRandomProjection

projector = SparseRandomProjection(n_components=m,eps=0.1)
A = projector.fit_transform(X.T)
dist_og = euclidean_distances(X[:m,:])
dist_new = euclidean_distances(A.T)
diff = np.abs(dist_og-dist_new)


plt.imshow(diff)
plt.colorbar()
plt.title('Visualization of absolute differences')
plt.show()

plt.hist(diff.flatten())
plt.title('Histogram of absolute differences')
plt.show()

print("\n\nThe Lemma seems to hold, mostly.")
```






```
The Lemma seems to hold, mostly.
```

3.

```
[25] def JL(n,d,m,eps,plot=True):
       X = np.identity(d)[:,:n]

       projector = SparseRandomProjection(n_components=m,eps=eps)
       A = projector.fit_transform(X.T)
       dist_og = euclidean_distances(X[:m,:])
       dist_new = euclidean_distances(A.T)
       diff = np.abs(dist_og-dist_new)

       if plot==True:
         plt.imshow(diff)
         plt.colorbar()
         plt.title('Visualization of absolute differences')
         plt.show()

         plt.hist(diff.flatten())
         plt.title('Histogram of absolute differences')
         plt.show()

       diff = diff.flatten()
       good = len(diff[diff<=0.2])
       total = len(diff)
       p = np.round(100*good/total,3)
       if p > 70:
         return False
       else:
         print('\n\nThe lemma does not hold for n={}, d={}, eps={}.'.format(p,n,d,eps))
         return True
```

```
for i in range(500):
  d = 5000*(2**(i+1))
  s = 'Made it to d = {} before crashing.'.format(d)
  sys.stdout.write('\r' + s)
  sys.stdout.flush()
  time.sleep(0.01)
```

```
Made it to d = 40000 before crashing.
```

+ Code + Text

4.

```
[26] for i in range(500):
       n = 10*(2**(i+1))
       s = 'Made it to n = {} before crashing.'.format(n)
       sys.stdout.write('\r' + s)
       sys.stdout.flush()
       time.sleep(0.01)
```

```
Made it to n = 32733906078961418700131896968275991522166420460430647894832913680961337964046745548832700923259041571508866841275600710092172565 45885
```

```
[50] !jupyter nbconvert —to html '/content/drive/MyDrive/2022 files/CS 584/CS584 - Assignment 4/CS584 - Assignment 4 - Python.ipynb'
```

```
[NbConvertApp] WARNING | pattern '—to' matched no files
[NbConvertApp] WARNING | pattern 'html' matched no files
[NbConvertApp] Converting notebook /content/drive/MyDrive/2022 files/CS 584/CS584 - Assignment 4/CS584 - Assignment 4 - Python.ipynb to html
[NbConvertApp] Writing 625041 bytes to /content/drive/MyDrive/2022 files/CS 584/CS584 - Assignment 4/CS584 - Assignment 4 - Python.html
```

```
[ ]
```