Jane Downer

Math 484: Homework 5

Problem 1

9.1 The eigenvalues of the correlation matrix of a set of predictor variables are

4.603, 1.175, 0.203, 0.015, 0.003, 0.001

and the corresponding eigenvectors are given in Table 9.15.

Table 9.15 Six Eigenvectors of the Correlation Matrix of the Predictors.

	V_1	V_2	V_3	V_4	V_5	V_6
X_1	–0.462	0.058	–0.149	–0.793	0.338	–0.135
X_2	–0.462	0.053	–0.278	0.122	–0.150	0.818
X_3	–0.321	–0.596	0.728	–0.008	0.009	0.107
X_4	–0.202	0.798	0.562	0.077	0.024	0.018
X_5	–0.462	–0.046	–0.196	0.590	0.549	–0.312
X_6	–0.465	0.001	–0.128	0.052	–0.750	–0.450

(a) How many sets of collinearity are there in this data set. Explain.
(b) What are the variables involved in each set? Explain.

(a)

eigenvalues:	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6
	4.603	1.175	0.203	0.015	0.003	0.001
	K_1	K_2	K_3	K_4	K_5	K_6
	$\sqrt{\frac{\lambda_1}{\lambda_1}}$	$\sqrt{\frac{\lambda_1}{\lambda_2}}$	$\sqrt{\frac{\lambda_1}{\lambda_3}}$	$\sqrt{\frac{\lambda_1}{\lambda_4}}$	$\sqrt{\frac{\lambda_1}{\lambda_5}}$	$\sqrt{\frac{\lambda_1}{\lambda_6}}$
	1	1.979	4.762	17.518	39.171	67.845

For $j = 1 \ldots 6$, there is one value K_j between 15 and 30, and 2 values of K_j over 30.
i.e., there are 3 sets of collinearity, and 2 of them are extreme cases.

(b)

For $p = 1 \ldots 6$, we can write the equation:

$$V_{p1}\tilde{X}_1 + V_{p2}\tilde{X}_2 + V_{p3}\tilde{X}_3 + V_{p4}\tilde{X}_4 + V_{p5}\tilde{X}_5 + V_{p6}\tilde{X}_6 = \lambda_p$$

If we do this for each value of p, we have 6 equations with 6 unknown variables and can solve for each value $\tilde{X}_1 \ldots \tilde{X}_p$.

↳ using numpy.linalg ⟶

$\tilde{X}_1 = -2.099$ $\tilde{X}_4 = 0.124$
$\tilde{X}_2 = -2.117$ $\tilde{X}_5 = -2.211$
$\tilde{X}_3 = -2.031$ $\tilde{X}_6 = -2.165$

Except for $\tilde{X}_4$, all variables fall between -2.031 and -2.211.

↓

$\tilde{X}_1, \tilde{X}_2, \tilde{X}_3, \tilde{X}_5$, and $\tilde{X}_6$ are highly correlated.

Problem 2

9.3 Gasoline Consumption: To study the factors that determine the gasoline consumption of cars, data were collected on 30 models of cars. Besides the gasoline consumption (Y), measured in miles per gallon for each car, 11 other measurements representing physical and mechanical characteristics are made. Definitions of variables are given in Table 9.16. The source of the data in Table 9.17 is *Motor Trend* magazine for the year 1975. We wish to determine whether the data set is collinear.

(a) Compute the correlation matrix of the predictor variables $X_1, \cdots, X_{11}$ and the corresponding pairwise scatter plots. Identify any evidence of collinearity.

(b) Compute the eigenvalues, eigenvectors, and the condition number of the correlation matrix. Is collinearity present in the data?

(c) Identify the variables involved in collinearity by examining the eigenvectors corresponding to small eigenvalues.

(d) Regress Y on the 11 predictor variables and compute the VIF for each of the predictors. Which predictors are affected by the presence of collinearity?

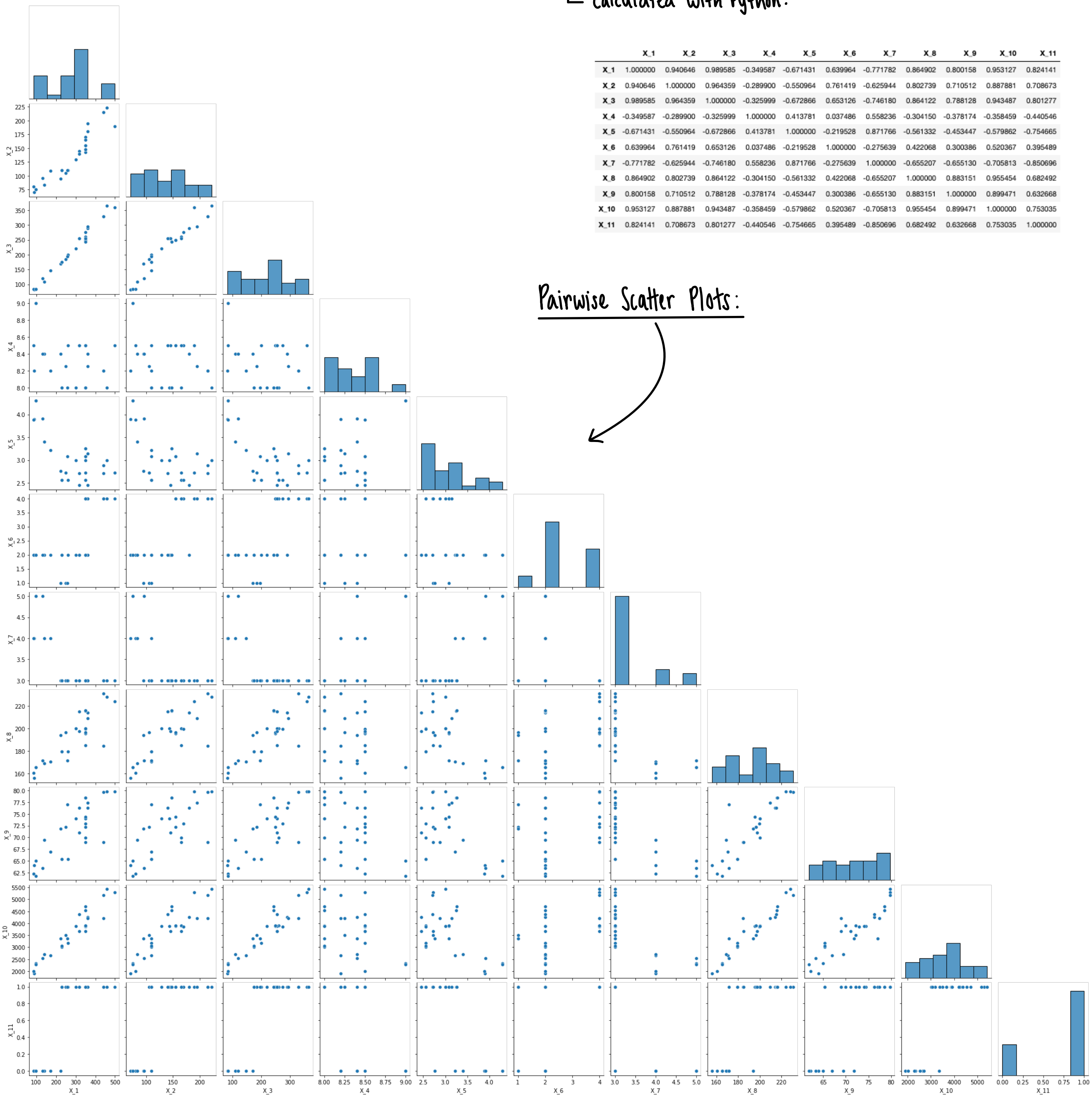
(a) define $$z_{ij} = \frac{x_{ij} - \bar{x}_j}{\sqrt{\sum_{i=1}^{n} (x_{ij} - \bar{x}_j)^2}}$$

Correlation Matrix $A = Z^T Z$

calculated with Python:

	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_10	X_11
X_1	1.000000	0.940646	0.989585	-0.349587	-0.671431	0.639964	-0.771782	0.864902	0.800158	0.953127	0.824141
X_2	0.940646	1.000000	0.964359	-0.289900	-0.550964	0.761419	-0.625944	0.802739	0.710512	0.887881	0.708673
X_3	0.989585	0.964359	1.000000	-0.325999	-0.672866	0.653126	-0.746180	0.864122	0.788128	0.943487	0.801277
X_4	-0.349587	-0.289900	-0.325999	1.000000	0.413781	0.037486	0.558236	-0.304150	-0.378174	-0.358459	-0.440546
X_5	-0.671431	-0.550964	-0.672866	0.413781	1.000000	-0.219528	0.871766	-0.561332	-0.453447	-0.579862	-0.754665
X_6	0.639964	0.761419	0.653126	0.037486	-0.219528	1.000000	-0.275639	0.422068	0.300386	0.520367	0.395489
X_7	-0.771782	-0.625944	-0.746180	0.558236	0.871766	-0.275639	1.000000	-0.655207	-0.655130	-0.705813	-0.850696
X_8	0.864902	0.802739	0.864122	-0.304150	-0.561332	0.422068	-0.655207	1.000000	0.883151	0.955454	0.682492
X_9	0.800158	0.710512	0.788128	-0.378174	-0.453447	0.300386	-0.655130	0.883151	1.000000	0.899471	0.632668
X_10	0.953127	0.887881	0.943487	-0.358459	-0.579862	0.520367	-0.705813	0.955454	0.899471	1.000000	0.753035
X_11	0.824141	0.708673	0.801277	-0.440546	-0.754665	0.395489	-0.850696	0.682492	0.632668	0.753035	1.000000

Pairwise Scatter Plots:



(b)

$$\lambda_j A - \lambda_j V_j = 0 \quad \text{for } j = 1 \dots p$$

λ_j : j^{th} eigenvalue

V_j : j^{th} eigenvector

$$\text{Critical Number} = \sqrt{\frac{\lambda_1}{\lambda_p}} = \sqrt{\frac{7.702}{0.003}} = \boxed{46.931}$$

The critical number is over 30, suggesting severe collinearity.

calculated with Python:

Eigenvalues:

Eigenvalue 1:	7.703
Eigenvalue 2:	1.403
Eigenvalue 3:	0.773
Eigenvalue 4:	0.577
Eigenvalue 5:	0.211
Eigenvalue 6:	0.142
Eigenvalue 7:	0.095
Eigenvalue 8:	0.050
Eigenvalue 9:	0.033
Eigenvalue 10:	0.008
Eigenvalue 11:	0.003

Eigenvectors:

	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}	X_{11}
Eigenvector 1:	[-0.353,	-0.330,	-0.351,	0.161,	0.266,	-0.205,	0.304,	-0.323,	-0.303,	-0.345,	-0.312]
Eigenvector 2:	[0.112,	0.261,	0.140,	0.553,	0.347,	0.548,	0.352,	0.078,	-0.006,	0.100,	-0.182]
Eigenvector 3:	[-0.031,	-0.078,	-0.043,	-0.119,	0.433,	-0.418,	0.221,	0.370,	0.546,	0.267,	-0.243]
Eigenvector 4:	[0.007,	0.195,	0.004,	-0.786,	0.352,	0.381,	0.134,	-0.180,	-0.095,	-0.041,	-0.119]
Eigenvector 5:	[-0.026,	0.143,	0.085,	-0.097,	-0.516,	0.007,	0.050,	0.200,	-0.106,	0.029,	-0.800]
Eigenvector 6:	[-0.095,	-0.239,	-0.185,	0.091,	0.072,	0.383,	-0.577,	-0.204,	0.520,	-0.140,	-0.275]
Eigenvector 7:	[-0.268,	-0.349,	-0.355,	-0.093,	-0.064,	0.377,	0.021,	0.675,	-0.197,	0.063,	0.164
Eigenvector 8:	[-0.259,	0.051,	-0.068,	-0.062,	-0.439,	0.166,	0.559,	-0.155,	0.524,	-0.203,	0.222
Eigenvector 9:	[0.497,	-0.652,	0.033,	-0.063,	-0.138,	0.134,	0.250,	-0.253,	-0.015,	0.394,	-0.063
Eigenvector 10:	[-0.291,	0.291,	-0.466,	0.051,	-0.086,	-0.005,	-0.056,	-0.294,	-0.055,	0.714,	0.017
Eigenvector 11:	[0.618,	0.258,	-0.682,	0.013,	-0.045,	-0.060,	0.049,	0.091,	0.053,	-0.260,	-0.010

small eigenvalues V_{ij} close to zero

(c)

When inspecting the eigenvectors for evidence of collinearity, I considered those corresponding to the 2 smallest eigenvalues. For values V_{ij} $(i = 1 \dots 11,\ j = 1 \dots 11)$ within the eigenvectors, I took note when the absolute value of V_{ij} was close to zero. For example, the 11^{th} eigenvector suggests the equation:

$$\tilde{X}_1 V_{1,11} + \tilde{X}_2 V_{2,11} + \tilde{X}_3 V_{3,11} + \tilde{X}_4 V_{4,11} + \tilde{X}_5 V_{5,11} + \tilde{X}_6 V_{6,11} + \tilde{X}_7 V_{7,11} + \tilde{X}_8 V_{8,11} + \tilde{X}_9 V_{9,11} + \tilde{X}_{10} V_{10,11} + \tilde{X}_{11} V_{11,11} = 0$$

We can see that $V_{4,11}$, $V_{5,11}$, $V_{6,11}$, $V_{7,11}$, $V_{8,11}$, $V_{9,11}$, and $V_{11,11}$ are all approximately zero. This implies:

$$\tilde{X}_1 V_{1,11} + \tilde{X}_2 V_{2,11} + \tilde{X}_3 V_{3,11} + \cancelto{0}{\tilde{X}_4 V_{4,11}} + \cancelto{0}{\tilde{X}_5 V_{5,11}} + \cancelto{0}{\tilde{X}_6 V_{6,11}} + \cancelto{0}{\tilde{X}_7 V_{7,11}} + \cancelto{0}{\tilde{X}_8 V_{8,11}} + \cancelto{0}{\tilde{X}_9 V_{9,11}} + \tilde{X}_{10} V_{10,11} + \cancelto{0}{\tilde{X}_{11} V_{11,11}} \doteq 0$$

i.e., it implies a linear relationship between $\tilde{X}_1$, $\tilde{X}_2$, $\tilde{X}_3$, and $\tilde{X}_{10}$.

Applying this method to the eigenvectors corresponding to the 5 smallest eigenvalues yields:

$$\tilde{X}_1 V_{1,7} + \tilde{X}_2 V_{2,7} + \tilde{X}_3 V_{3,7} + 0 + 0 + \tilde{X}_6 V_{6,7} + 0 + \tilde{X}_8 V_{8,7} + \tilde{X}_9 V_{9,7} + 0 + \tilde{X}_{11} V_{11,7} \doteq 0$$

$$\tilde{X}_1 V_{1,8} + 0 + 0 + 0 + \tilde{X}_5 V_{5,8} + \tilde{X}_6 V_{6,8} + \tilde{X}_7 V_{7,8} + \tilde{X}_8 V_{8,8} + \tilde{X}_9 V_{9,8} + \tilde{X}_{10} V_{10,8} + \tilde{X}_{11} V_{11,8} \doteq 0$$

$$\tilde{X}_1 V_{1,9} + \tilde{X}_2 V_{2,9} + 0 + 0 + \tilde{X}_5 V_{5,9} + \tilde{X}_6 V_{6,9} + \tilde{X}_7 V_{7,9} + \tilde{X}_8 V_{8,9} + 0 + \tilde{X}_{10} V_{10,9} + 0 \doteq 0$$

$$\tilde{X}_1 V_{1,10} + \tilde{X}_2 V_{2,10} + \tilde{X}_3 V_{3,10} + 0 + 0 + 0 + 0 + \tilde{X}_8 V_{8,10} + 0 + \tilde{X}_{10} V_{10,10} + 0 \doteq 0$$

$$\tilde{X}_1 V_{1,11} + \tilde{X}_2 V_{2,11} + \tilde{X}_3 V_{3,11} + 0 + 0 + 0 + 0 + 0 + 0 + \tilde{X}_{10} V_{10,11} + 0 \doteq 0$$

In these equations, $\tilde{X}_1$ appears 5/5 times, and $\tilde{X}_8$ and $\tilde{X}_{10}$ each appear 4/5 times. This suggests X_1, X_8, and X_{10} are the biggest contributors to collinearity.

(d)

$$VIF_j = \frac{1}{1-R_j^2}$$

$\hookrightarrow R_j^2 = 1 - \frac{SSE_j}{SST_j}$

SSE_j and SST_j :
the values of SSE and SST obtained by regressing the j^{th} predictor on the other variables

j	1	2	3	4	5	6	7	8	9	10	11
R_j^2	0.991	0.975	0.993	0.223	0.871	0.802	0.913	0.946	0.852	0.985	0.795
VIF_j	114.58	49.430	145.126	1.287	7.762	5.057	11.542	18.409	6.743	68.362	4.884

$VIF_j = 1 \longrightarrow$ best possible case

$4 < VIF_j < 10 \longrightarrow$ warrants further investigation

$VIF_j \geq 10 \longrightarrow$ serious collinearity

The results show that X_5, X_6, X_9 and X_{11} are moderately affected by collinearity, and that X_1, X_2, X_3, X_7, X_8, and X_{10} are severely affected by collinearity.

Problem 3

1.93 1.06 0.01

10.2 Suppose we fit the model

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \varepsilon, \qquad (10.44)$$

to a set of data, where each of the three variables has a mean of 0 and a variance of 1. The three eigenvalues of the correlation matrix of the three predictor variables are 1.93, 1.06, and 0.01. The corresponding eigenvectors are given in Table 10.17. Table 10.18 shows a computer output obtained when regressing Y on the principal components C_1, C_2, and C_3.

(a) Compute the least squares estimate of β_0 when fitting the model in (10.44) to the data?

(b) Is there evidence of collinearity in the predictor variables? Explain.

(c) What is R^2 when Y is regressed on X_1, X_2, and X_3?

(d) What is the formula used to obtain C_1?

(e) Derive the principal components predicted equation $\hat{Y}_{PC}$ of the model in (10.44).

Table 10.17 Three Eigenvectors of the Correlation Matrix of the Three Predictors in Model (10.44)

	V_1	V_2	V_3
X_1	0.500	–0.697	0.514
X_2	0.484	0.717	0.501
X_3	0.718	0.002	–0.696

Table 10.18 Regression Output from the Regression of Y on the Principal Components C_1, C_2, and C_3

ANOVA Table

Source	Sum of Squares	df	Mean Square	F-Test
Regression	86.6542	3	28.8847	225
Residual	12.3458	96	0.128602	

Coefficients Table

Variable	Coefficient	s.e.	t-Test	p-value
C_1	0.67	0.03	25.9	0.0001
C_2	–0.02	0.03	–0.56	0.5782
C_3	–0.56	0.37	–1.53	0.1291

$\alpha_1 = 0.67$

$\alpha_2 = -0.02$

$\alpha_3 = -0.56$

(a)

get $\hat{\theta}$ from $\hat{\alpha}$:

$$\hat{\theta} = V\hat{\alpha} = \boxed{V}_{p\times p}\boxed{\hat{\alpha}}_{p\times 1}$$

$$\begin{bmatrix} 0.500 & -0.697 & 0.514 \\ 0.484 & 0.717 & 0.501 \\ 0.718 & 0.002 & -0.696 \end{bmatrix} \times \begin{bmatrix} 0.67 \\ -0.02 \\ -0.56 \end{bmatrix} = \begin{array}{ccc} \hat{\theta}_1 & \hat{\theta}_2 & \hat{\theta}_3 \\ (0.187 & -0.0997 & 0.870) \end{array}$$

get $\hat{\beta}$ from $\hat{\theta}$: $\hat{\beta}_j = \frac{S_y}{S_j}\hat{\theta}_j \quad j = 1..p$

$S_j = 1$ for $j = 1, 2, 3$

$S_y = \sqrt{\frac{\Sigma(y_i - \bar{y})^2}{n-1}}$

$SST = SSR - SSE$

$= 86.6542 - 12.3458$

$= 74.3084$

$= \sqrt{\frac{74.3084}{99}}$

$= 0.866$

$\frac{S_y}{S_j} = 0.866$

$\hat{\beta}_1 = 0.866 \times 0.187 = 0.162$

$\hat{\beta}_2 = 0.866 \times (-0.0997) = -0.086$

$\hat{\beta}_3 = 0.866 \times 0.870 = -0.753$

I have tried and cannot figure out how to get $\hat{\beta}_0$ from the given information. I know that the relationship between $\hat{\beta}_0$ and $\bar{y}$ is:

$$\hat{\beta}_0 = \bar{y} - \sum_{j=1}^{p} \hat{\beta}_j \bar{X}_j$$

$$\bar{y} = \hat{\beta}_0 + \hat{\beta}_1 \cancel{\bar{X}_1}^{0} + \hat{\beta}_2 \cancel{\bar{X}_2}^{0} + \hat{\beta}_3 \cancel{\bar{X}_3}^{0}$$

$$= \hat{\beta}_0$$

Yet I do not see how I can obtain $\bar{y}$ with the given information.

(b)

$$\begin{bmatrix} cov(C_1,C_1) & cov(C_1,C_2) & cov(C_1,C_3) \\ cov(C_2,C_1) & cov(C_2,C_2) & cov(C_2,C_3) \\ cov(C_3,C_1) & cov(C_3,C_2) & cov(C_3,C_3) \end{bmatrix} = \begin{bmatrix} var(C_1) & 0 & 0 \\ 0 & var(C_2) & 0 \\ 0 & 0 & var(C_3) \end{bmatrix} = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix} = \begin{bmatrix} 1.93 & 0 & 0 \\ 0 & 1.06 & 0 \\ 0 & 0 & 0.01 \end{bmatrix}$$

$\lambda_3 = var(C_3) \doteq 0 \longrightarrow C_3$ is approximately constant

$$C_3 = V_{31}\tilde{X}_1 + V_{32}\tilde{X}_2 + V_{33}\tilde{X}_3 \doteq 0$$
$$0.514\tilde{X}_1 + 0.501\tilde{X}_2 - 0.696\tilde{X}_3 \doteq 0$$
$$[\tilde{X}_3 \doteq 0.739\tilde{X}_1 + 0.720\tilde{X}_2]$$

The fact that λ_3 is so small suggests that there is a linear relationship between the 3 predictors. λ_3 is the only eigenvalue close to $\emptyset$, so we don't need to consider the others.

(c)

$$SST = SSR - SSE$$
$$= 86.6542 - 12.3458$$
$$= 74.3084$$

$$R^2 = 1 - SSE/SST$$
$$= 1 - 12.3458/74.3084$$
$$= \boxed{0.834}$$

(d)

$$C_1 = V_{11}\tilde{X}_1 + V_{21}\tilde{X}_2 + V_{31}\tilde{X}_3$$
$$= 0.500\tilde{X}_1 + 0.484\tilde{X}_2 + 0.718\tilde{X}_3$$

Since we've determined

$$\tilde{X}_3 \doteq 0.739\tilde{X}_1 + 0.720\tilde{X}_2$$

We can rewrite the equation for C_1 as:

$$C_1 = 0.500\tilde{X}_1 + 0.484\tilde{X}_2 + 0.718(0.739\tilde{X}_1 + 0.720\tilde{X}_2)$$
$$= 1.031\tilde{X}_1 + 1.017\tilde{X}_2$$
$$\boxed{C_1 \doteq \tilde{X}_1 + \tilde{X}_2}$$

(e)

$$a_2 = V_{12}\theta_1 + V_{22}\theta_2 + V_{32}\theta_3$$

$$\cancel{-0.02} \doteq 0 = -0.697\theta_1 + 0.717\theta_2 + \cancel{0.002}\theta_3 \doteq 0$$

$$0.697\theta_1 \doteq 0.717\theta_2$$

$$\theta_1 \doteq \theta_2$$

$$a_1 = V_{11}\theta_1 + V_{21}\theta_2 + V_{31}\theta_3$$

$$0.67 = 0.500\theta_1 + 0.484\theta_2 + 0.718\theta_3$$

$$0.67 \doteq 0.500\theta_1 + 0.484\theta_1 + 0.718\theta_3$$

$$0.67 \doteq \cancel{0.984}\theta_1 + 0.718\theta_3 \quad \doteq 1$$

$$\theta_1 \doteq 0.67 - 0.718\theta_3$$

$$a_3 = V_{13}\theta_1 + V_{23}\theta_2 + V_{33}\theta_3$$

$$-0.56 = 0.514\theta_1 + 0.501\theta_2 - 0.696\theta_3$$

$$-0.56 \doteq \theta_1(0.514 + 0.501) - 0.696\theta_3$$

$$-0.56 \doteq (0.67 - 0.718\theta_3)(0.514 + 0.501) - 0.696\theta_3$$

$$-0.56 \doteq 0.680 - 0.729\theta_3$$

$$\theta_3 \doteq 0.12$$

$$\theta_1 \doteq 0.67 - 0.718\theta_3$$
$$= 0.67 - 0.718 \times 0.12$$
$$= 0.584$$

$$\theta_2 \doteq \theta_1$$
$$\doteq 0.584$$

$$\boxed{\begin{aligned} \tilde{Y} &= \tilde{X}\theta + \varepsilon' \\ &= \tilde{X}_1\theta_1 + \tilde{X}_2\theta_2 + \tilde{X}_3\theta_3 + \varepsilon \\ &\doteq 0.584\tilde{X}_1 + 0.584\tilde{X}_2 + 0.12\tilde{X}_3 + \varepsilon' \end{aligned}}$$

Problem 4

Problem 4 Using the Longley's data in Table 10.19:

1) Compute the correlation matrix of the six predictors. Do you see any evidence of collinearity? How many different sets of collinearity exist in the data? What are the variables involved in each set of collinearity?

2) Transform the original model ($Y = \beta_0 + \beta_1 X_1 + \ldots + \beta_6 X_6 + \epsilon$) to the standardized model ($\tilde{Y} = \theta_1 \tilde{X}_1 + \ldots + \theta_6 \tilde{X}_6 + \epsilon'$).

3) Compute the principle components. Using Principle Component Regression, first regress $\tilde{Y}$ on all six principle components, provide the estimates of the coefficients θ_j in the standardized model; then decide which principle components you choose to retain and provide the estimates of the coefficients in the standardized model again.

4) Using Ridge Regression, construct the ridge trace. Then use the ridge trace and VIF values, determine the recommended value for k. Provide the estimates of the regression coefficients θ_j in the standardized model using the chosen k value.

(1) Correlation matrix: $Z^T Z \longrightarrow Z_{ij} = \dfrac{X_{ij} - \bar{X}_j}{\sqrt{\Sigma (X_{ij} - \bar{X}_j)^2}}$

$$\begin{bmatrix} 1.000, & 0.992, & 0.621, & 0.465, & 0.979, & 0.991 \\ 0.992, & 1.000, & 0.604, & 0.446, & 0.991, & 0.995 \\ 0.621, & 0.604, & 1.000, & -0.177, & 0.687, & 0.668 \\ 0.465, & 0.446, & -0.177, & 1.000, & 0.364, & 0.417 \\ 0.979, & 0.991, & 0.687, & 0.364, & 1.000, & 0.994 \\ 0.991, & 0.995, & 0.668, & 0.417, & 0.994, & 1.000 \end{bmatrix}$$

(2) $\hat{\theta}_j = \dfrac{S_j}{S_y} \hat{\beta}_j$

$S_j = \sqrt{\dfrac{\Sigma (X_{ij} - \bar{x}_j)^2}{n-1}}$

$S_y = \sqrt{\dfrac{\Sigma (y_i - \bar{y})^2}{n-1}}$

computed in python

$\downarrow$

S_y : 3511.968		$\hat{\beta}_0 = -3.482 \times 10^6$	$\hat{\theta}_0 = 0$
S_1 : 107.916	$S_1/S_y =$ 0.031	$\hat{\beta}_1 =$ 1.506	$\hat{\theta}_1 = 0.046$
S_2 : 99394.94	$S_2/S_y =$ 28.302	$\hat{\beta}_2 = -3.582 \times 10^{-2}$	$\hat{\theta}_2 = -1.014$
S_3 : 934.46	$S_3/S_y =$ 0.266	$\hat{\beta}_3 = -2.02$	$\hat{\theta}_3 = -0.538$
S_4 : 695.920	$S_4/S_y =$ 0.198	$\hat{\beta}_4 = -1.03$	$\hat{\theta}_4 = -0.205$
S_5 : 6956.102	$S_5/S_y =$ 1.981	$\hat{\beta}_5 = -5.110 \times 10^{-2}$	$\hat{\theta}_5 = -0.101$
S_6 : 4.761	$S_6/S_y =$ 0.001	$\hat{\beta}_6 = 1.829 \times 10^3$	$\hat{\theta}_6 = 2.48$

$\tilde{Y} = \tilde{X}\theta + \varepsilon'$

$$\boxed{\tilde{Y} = 0.046\,\tilde{X}_1 - 1.014\,\tilde{X}_2 - 0.538\,\tilde{X}_3 - 0.205\,\tilde{X}_4 - 0.101\,\tilde{X}_5 + 2.48\,\tilde{X}_6 + \varepsilon'}$$

(3)

$$\tilde{X}_{ij} = \frac{X_{ij} - \bar{X}_j}{S_j}$$

	X1~	X2~	X3~	X4~	X5~	X6~
0	-1.731100	-1.543433	-0.896035	-1.460927	-1.411135	-1.575299
1	-1.221442	-1.290533	-0.929209	-1.653478	-1.263926	-1.365259
2	-1.249241	-1.304336	0.522960	-1.423566	-1.099898	-1.155219
3	-1.128777	-1.037271	0.168746	-1.374710	-0.933713	-0.945180
4	-0.507921	-0.590809	-1.171059	0.707427	-0.768965	-0.735140
5	-0.331857	-0.409472	-1.349771	1.418716	-0.597174	-0.525100
6	-0.248458	-0.224493	-1.416119	1.351180	-0.334958	-0.315060
7	-0.155793	-0.247361	0.411666	1.068101	-0.173229	-0.105020
8	-0.044595	0.098300	-0.309603	0.634143	-0.005175	0.105020
9	0.270466	0.316732	-0.397353	0.359686	0.188324	0.315060
10	0.622594	0.554058	-0.275358	0.274906	0.434295	0.525100
11	0.844990	0.571936	1.592022	0.043557	0.650652	0.735140
12	1.011787	0.955839	0.663147	-0.078583	0.854214	0.945180
13	1.160051	1.156020	0.789423	-0.133187	1.142019	1.155219
14	1.299049	1.312688	1.725788	-0.049844	1.499116	1.365259
15	1.410247	1.682134	0.870753	0.316578	1.819554	1.575299

$$\begin{array}{cccccc} \lambda_1 & \lambda_2 & \lambda_3 & \lambda_4 & \lambda_5 & \lambda_6 \\ 4.6034 & 1.1753 & 0.2034 & 0.0149 & 0.0026 & 0.0004 \\ V_1 & V_2 & V_3 & V_4 & V_5 & V_6 \end{array}$$

$$\begin{bmatrix} 0.462 & 0.058 & 0.149 & -0.793 & 0.338 & -0.135 \\ 0.462 & 0.053 & 0.278 & 0.122 & -0.150 & 0.818 \\ 0.321 & -0.596 & -0.728 & -0.008 & 0.009 & 0.107 \\ 0.202 & 0.798 & -0.562 & 0.077 & 0.024 & 0.018 \\ 0.462 & -0.046 & 0.196 & 0.590 & 0.549 & -0.312 \\ 0.465 & 0.001 & 0.128 & 0.052 & -0.750 & -0.450 \end{bmatrix}$$

$$C_j = V_{1j}\tilde{X}_1 + \dots + V_{pj}\tilde{X}_p$$

	C1	C2	C3	C4	C5	C6
0	-3.47993942494881	-0.750580572442948	0.308158466414131	0.164666140768712	-0.00177611277109346	0.0100783375261086
1	-3.01151741739423	-0.848074294544778	0.642506894569963	-0.125417723279966	-0.0111830331388457	0.0627761640471501
2	-2.34472061605862	-1.53970737925661	-0.492868798981170	0.00864602924795397	-0.00490741849636411	0.00659465645455315
3	-2.09507077364535	-1.27593143291000	-0.110795245010434	0.0612848983786532	0.0138943453737017	-0.0610770211341694
4	-1.43768517692614	1.23629421608880	-0.0297672269559677	-0.0972770988962581	0.0434881706489869	0.0525614303554333
5	-1.00922250862055	1.92250840812568	-0.162210062246990	-0.0462663831235201	0.0136137805642869	0.0370944521202123
6	-0.701365081251758	1.91093592809721	0.0661833750886370	0.0711209530274476	-0.0309848889381482	0.0217310697614993
7	0.0327778867443915	0.592713170108866	-1.03936808880231	0.0646572098187673	-0.0167181372752623	-0.00254685431629528
8	0.0999740686132466	0.693502096580758	-0.0978844466858240	0.101102675094166	0.0190760209506710	-0.0990105339042351
9	0.449895700675214	0.547934616882896	0.292727920994652	-0.0174366198709219	-0.0140107760339722	-0.0839653681959285
10	0.955549362519553	0.429462403270996	0.445099338824307	-0.119186192319856	-0.0271613897929253	-0.0239765515494268
11	1.81684425253618	-0.863910630132284	-0.676986899193066	-0.187657391834481	-0.00895601983198335	0.0210712726461955
12	1.94014418510606	-0.386961797181598	0.266269491636304	-0.143995485052796	0.0229355899081425	-0.0372386911433105
13	2.36125375235751	-0.499619170590485	0.366065658916747	-0.0616490410371217	-0.00512696614006491	-0.0168595688797963
14	3.07788190772572	-0.991000741196824	-0.201329317227249	0.0677157699124837	7.21034495432260e-6	0.0555783822371660
15	3.34519988256759	-0.177564820899678	0.424198938658267	0.259692259166738	0.00780962462791679	0.0571888239748437

Regressing $\tilde{Y}$ on $C_1, \dots, C_6$:

$$\hat{\theta} = \begin{bmatrix} 0.0 \\ 0.444 \\ 0.112 \\ 0.530 \\ -1.203 \\ -1.757 \\ -1.984 \end{bmatrix}$$

Which to drop? λ_4, λ_5, and λ_6 are all very small.

$$\begin{array}{c|ccc|l} & \lambda_4 & \lambda_5 & \lambda_6 & \\ & 0.0149 & 0.0026 & 0.0004 & \text{\# with} \\ & V_4 & V_5 & V_6 & \\ \hline \tilde{X}_1 & -0.793 & 0.338 & -0.135 & 2/3 \text{ have large magnitude} \\ \tilde{X}_2 & 0.122 & -0.150 & 0.818 & 1/3 \text{ " "} \\ \tilde{X}_3 & -0.008 & 0.009 & 0.107 & \\ \tilde{X}_4 & 0.077 & 0.024 & 0.018 & \\ \tilde{X}_5 & 0.590 & 0.549 & -0.312 & 3/3 \text{ " "} \\ \tilde{X}_6 & 0.052 & -0.750 & -0.450 & 3/3 \text{ " "} \end{array}$$

$\leftarrow$ values further from $\emptyset$ suggest collinearity.

drop C_4, C_5, C_6

(3, continued)

Reduced Model: $\tilde{Y} \sim C_1 + C_2 + C_3$

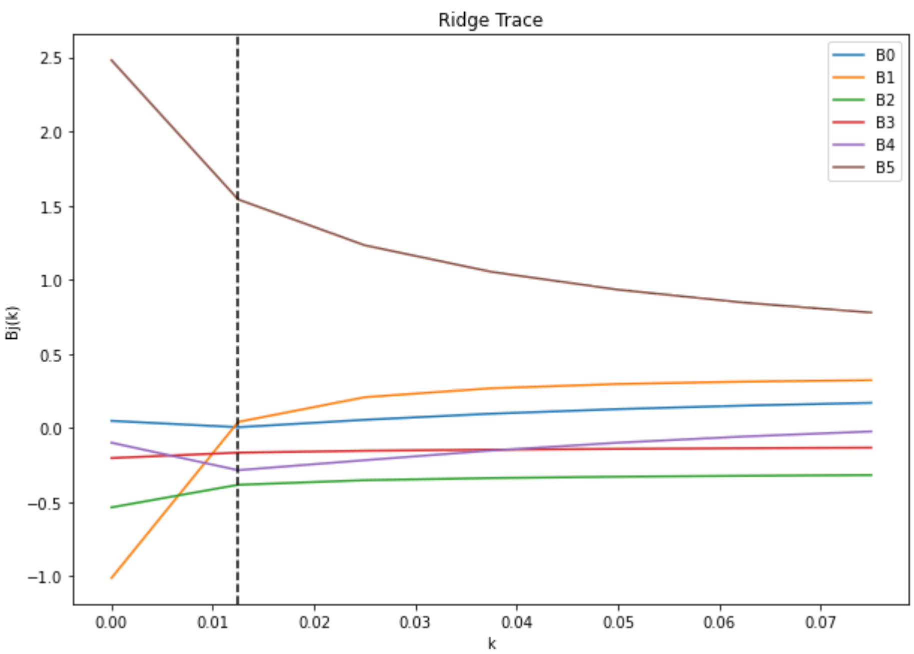
Regressing $\tilde{Y}$ on C_1, C_2, and C_3: $\hat{\theta} = \begin{bmatrix} 0.0 \\ 0.444 \\ 0.112 \\ 0.530 \end{bmatrix}$

Full model: $\hat{\theta} = \begin{bmatrix} 0.0 \\ 0.444 \\ 0.112 \\ 0.530 \\ -1.203 \\ -1.757 \\ -1.984 \end{bmatrix}$ Reduced model: $\hat{\theta} = \begin{bmatrix} 0.0 \\ 0.444 \\ 0.112 \\ 0.530 \end{bmatrix}$

(4) $\hat{\theta}^{ridge} = (\tilde{X}^T\tilde{X} + kI)^{-1}\tilde{X}^T\tilde{Y}$

$k_i = \dfrac{p \cdot \hat{\sigma}^2}{\sum_{i=1}^{p} \hat{\theta}_j^2(k_{i-1})}$

for values $k = \begin{bmatrix} 0.025 \\ 0.0375 \\ 0.05 \\ 0.0625 \\ 0.075 \end{bmatrix}$



levels off around $k = 0.0125$

$\beta(k = 0.0125) = \begin{bmatrix} 0.003 \\ 0.039 \\ -0.385 \\ -0.168 \\ -0.287 \\ 1.54 \end{bmatrix}$