

## 1.1

$$\frac{dN}{dt} = R(T) = \Gamma N(t) - \beta N(t)^2 \quad (1)$$

$$N_0 = 1E5 = 100000$$

**A:**

Taking  $\beta$  to be 0,

$$\frac{dN}{dt} = \Gamma N(t) \quad (2)$$

$$\int \frac{dN}{N(t)} = \int \Gamma dt$$

$$N(t) = Ce^{\Gamma t}$$

Using the baoundary condition  $N(0) = N_0, C = N_0$

$$N(t) = N_0 e^{\Gamma t} = N_0 e^{\frac{t}{\tau}} \quad (3)$$

Finding halflife  $N(T_{1/2}) = 2N_0$

$$2N_0 = N_0 e^{\frac{T_{1/2}}{\tau}}$$

$$\ln(2) = \frac{T_{1/2}}{\tau}$$

The relation between halflife and lifetime then becomes

$$\boxed{T_{1/2} = \tau \ln(2)} \quad (4)$$

**B:**

**C:**

**D:**

Starting at *Eq.1*

$$\int \frac{dN}{\Gamma N(t) - \beta N(t)^2} = \int dt$$

$$\ln\left(\frac{\Gamma}{\Gamma - \beta N}\right) + \ln\left(\frac{\Gamma - \beta N}{\Gamma} - 1\right) = \Gamma t + C$$

Solve for  $N$ ,

$$\ln\left(1 - \frac{\Gamma}{\Gamma - \beta N}\right) = \Gamma t + C$$

$$N(t) = \frac{\Gamma C e^{\Gamma t}}{\beta C e^{\Gamma t} - \beta}$$

Again using the boundary condition that  $N(0) = N_0$ ,  $C = \frac{N_0 \beta}{N_0 \beta - \Gamma}$

$$N(t) = \frac{\Gamma \frac{N_0 \beta}{N_0 \beta + \Gamma} e^{\Gamma t}}{\beta \frac{N_0 \beta}{N_0 \beta + \Gamma} e^{\Gamma t} - \beta} \quad (5)$$

Where  $\beta = 4E - 8$ . The equation for the asymptote is  $R(t) = 0$  or

$$N(t) = \frac{\Gamma}{\beta} \quad (6)$$