CSCE 222
Homework 5

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2.4

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$$a_n = -3a_{n-1} + 4a_{n-2} \tag{1}$$

a):

$$a_n = 0$$

$$a_{n-1} = 0$$

$$a_{n-2} = 0$$

plugging in, the expression becomes

$$a_n = 0 + 0 = 0$$

$\boxed{\therefore \textit{ is a solution}}$

b):

$$a_n = 1$$

$$a_{n-1} = 1$$

$$a_{n-2} = 1$$

$$a_n = -3 + 4 = 1$$

$\boxed{\therefore \textit{ is a solution}}$

c):

$$a_n = (-4)^n$$

$$a_{n-1} = (-4)^{n-1}$$

$$a_{n-2} = (-4)^{n-2}$$

$$a_n = -3(-4)^{n-1} + 4(-4)^{n-2}$$

$$a_n = (-4)^{n-2}(12 + 4) = 16(-4)^{n-2}$$

$$16(-4)^{n-2} = (-4)(-4)(-4)^{n-2} = (-4)^{n-2}$$

$\boxed{\therefore \textit{ is a solution}}$

d):

$$a_n = 2(-4)^n + 3$$

$$a_{n-1} = 2(-4)^{n-1} + 3$$

$$a_{n-2} = 2(-4)^{n-2} + 3$$

$$a_n = -6(-4)^{n-1} - 9 + 8(-4)^{n-2} + 12$$

$$a_n = (-4)^{n-2}(-6 + 8) - 3 = 2(-4)^n + 3$$

$\boxed{\therefore \textit{ is a solution}}$

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a):

$$a_n = 3$$

$$a_n - a_{n-1} = 3 - 3$$

$$\boxed{a_n = a_{n-1}, a_0 = 3}$$

b):

$$a_n = 2n$$

$$a_n - a_n = 2n - 2n + 2$$

$$\boxed{a_n = a_{n-1} + 2, a_0 = 0}$$

c):

$$a_n = 2n + 3$$

$$a_n - a_{n-1} = 2n + 3 - 2n + 2 - 3$$

$$\boxed{a_n = a_{n-1} + 2, a_0 = 3}$$

d):

$$a_n = 5^n$$

$$\frac{a_n}{a_{n-1}} = \frac{5^n}{5^{n-1}}$$

$$\boxed{a_n = 5a_{n-1}, a_0 = 1}$$

e):

$$a_n - a_n n - 1 = n^2 - (n-1)^2$$

$$a_n = a_{n-1} + n^2 - (n-1)^2$$

$$\boxed{a_n = a_n + 2n - 1, a_0 = 0}$$

f):

$$a_n - a_{n-1} = n^2 + n - (n-1)^2 - (n-1)$$

$$\boxed{a_n = a_{n-1} + 2n, a_0 = 0}$$

g):

$$a_n + a_{n-1} = n + (-1)^n + n - 1 + (-1)^{n-1}$$

$$\because 1^n = -(-1)^{n-1}$$

$$a_n + a_{n-1} = n + (-1)^n + n - 1 - (-1)^n$$

$$\boxed{a_n = -a_{n-1} + 2n - 1, a_0 = 1}$$

h):

$$\frac{a_n}{a_{n-1}} = \frac{n!}{(n-1)!}$$

$$\boxed{a_n = a_{n-1}n, a_0 = 1}$$

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a):

$$\boxed{a_n = 1.05a_{n-1} + 1000, a_0 = 50000}$$

b): compute through iteration,

$$a_0 = 50000$$

$$a_1 = 53500$$

$$...$$

$$a_8 = 83418$$

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a):

$$1 + (-1) = 0$$

$$1 + 1 = 2$$

Since there are 4 0's and 4 2's, the sum becomes

$$4 * 2 = \boxed{8}$$

b):

$$\Sigma 3^j - \Sigma 2^j$$

clsoed form of both sums

$$\frac{3^9 - 1}{3 - 1} - \frac{2^9 - 1}{2 - 1}$$

$$9841 - 511 = \boxed{9330}$$

c):

$$2\Sigma 3^j + 3\Sigma 2^j$$

closed form becomes

$$2\frac{3^9 - 1}{3 - 1} + 3\frac{2^9 - 1}{2 - 1}$$

$$19682 + 1533 = \boxed{21215}$$

d):

$$\Sigma 2^{j+1} - \Sigma 2^j$$

$$\frac{2^9 - 2}{2 - 1} - \frac{2^9 - 1}{2 - 1}$$

$$510 - 511 = \boxed{-1}$$

5.1

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Prove:

$$2\Sigma(-7)^n = \frac{1 - (-7)^{n+1}}{4}$$

By induction, base case: n = 1

$$2 = \frac{1 + 7}{4} = 2$$

holds for k, now, n = k+1;

$$2\Sigma(-7)^k + 2(-7)^{k+1} = \frac{1 - (-7)^{k+2}}{4}$$

$$\frac{1 - (-7)^{k+1}}{4} + 2(-7)^{k+1} = \frac{1 - (-7)^{k+2}}{4}$$

$$\frac{1 - (-7)^{k+1}(7)}{4} = \frac{1 - (-7)^{k+2}}{4}$$

$$\frac{1 - (-7)^{k+2}}{4} = \frac{1 - (-7)^{k+2}}{4}$$

$$\boxed{\therefore 2\Sigma(-7)^n = \frac{1 - (-7)^{n+1}}{4}}$$

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By induction, n = 0;

$$1 = 1$$

holds for k, now, n = k+1;

$$\Sigma(-\frac{1}{2})^j + (-\frac{1}{2})^{k+1} = \frac{2^{k+2} + (-1)^{k+1}}{3(2)^{k+1}}$$

$$\frac{2}{2} * \frac{2^{k+1} + \frac{3}{3} * (-1)^k}{3(2)^k} + (-\frac{1}{2})^{k+1} = \frac{2^{k+2} + (-1)^{k+1}}{3(2)^{k+1}}$$

$$\frac{2^{k+2} + 2(-1)^k + 3(-1)^{k+1}}{3(2)^{k+1}} = \frac{2^{k+2} + (-1)^{k+1}}{3(2)^{k+1}}$$

$$\frac{2^{k+2} + (-1)^{k+1}}{3(2)^{k+1}} = \frac{2^{k+2} + (-1)^{k+1}}{3(2)^{k+1}}$$

$$\boxed{\therefore \Sigma(-\frac{1}{2})^j = \frac{2^{n+1} + (-1)^n}{3(2)^n}}$$

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by induction, n = 1;

$$2 = 2$$

holds for k, now, n = k+1;

$$\Sigma j2^j + (k+1)2^{k+1} = k2^{k+2} + 2$$

$$(k-1)2^{k+1} + 2 + (k+1)2^{k+1} = k2^{k+2} + 2$$

$$2^k(k - 1 + k + 1) + 2 = k2^{k+2} + 2$$

$$2k2^k + 2 = k2^{k+2} + 2$$

$$k2^{k+2} + 2 = k2^{k+2} + 2$$

$$\boxed{\therefore \Sigma j2^j = (n-1)2^{n+1} + 2}$$

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a):

$$\boxed{P(2) \rightarrow 2! < 2^2}$$

b):

$$\boxed{2 < 4}$$

c):

$$\boxed{P(k) \rightarrow k! < k^k}$$

d):

$$\boxed{(k+1)! < (k+1)^{k+1}}$$

e):

$$(k+1)(k!) < (k+1)(k+1)^k$$

$$k! < k^k < (k+1)^k$$

$$\boxed{\therefore n! < n^n}$$

f): Since the inductive hypothesis extends the base case, if the hypothesis is true, then $P(k+1)$ is true since $k! < (k+1)^k$ as shown above. Finally, since it was shown that $P(n)$ holds for k and $k+1$, $P(n)$ holds for all n within the specified domain.