# CSCE 222
# Homework 3

Jacob Purcell, Texas A&M, Student

2.3

*12*

For a function to be one-one, the following condition must hold;

$$f(n_1) = f(n_2) \tag{1}$$

$$\therefore n_1 = n_2$$

*a):*

$$f(n) = n - 1 \tag{2}$$

$$n_1 - 1 = n_2 - 1 \tag{3}$$

Add 1 to both sizes;

$$n_1 = n_2 \tag{4}$$

$$\boxed{\therefore one - one}$$

*b):*

$$f(n) = n^2 + 1 \tag{5}$$

$$n_1^2 + 1 = n_2^2 + 1 \tag{6}$$

Subtract 1 from both sides;

$$n_1^2 = n_2^2 \tag{7}$$

Square root both sides;

$$\pm n_1 = \pm n_2 \tag{8}$$

This creates the situation that

$$n_1 = \pm n_2 \tag{9}$$

Where two inputs are mapped to one output,

$$\boxed{\therefore not\ one - one} \tag{10}$$

*c):*

$$f(n) = n^3 \tag{11}$$

$$n_1^3 = n_2^3 \tag{12}$$

Take the cube root,

$$n_1 = n_2 \tag{13}$$

$$\boxed{\therefore one - one} \tag{14}$$

*d):*

$$f(n) = \frac{n}{2} \tag{15}$$

$$\frac{n_1}{2} = \frac{n_2}{2} \tag{16}$$

Multiply each side by 2,

$$\boxed{\therefore one - one} \tag{17}$$

*14*

To check whether the functions are surjective, an inverse will be found for each function, then plugged back into the original equation to verify freedom from contradiction. The condition that $f(m, n) = y$ is understood.

*a):*

$$y = 2m - n \tag{18}$$

Solve for $n$,

$$n = 2m - y \tag{19}$$

Plug in $n$,

$$y = 2m - (2m - y) \tag{20}$$

Remove parenthases and distribute negation,

$$y = 2m - 2m + y \tag{21}$$

Add like terms

$$y = y \tag{22}$$

$$\boxed{\therefore surjective} \tag{23}$$

*b):*

$$y = m^2 - n^2 \tag{24}$$

Solve for $n$,

$$n = \pm\sqrt{m^2 - y} \tag{25}$$

From here, it is apparent that part of the domain lies within $\mathbb{I}$ outside of $\mathbb{Z}$ in the case that $y > m^2$.

$$\boxed{\therefore not\ surjective} \tag{26}$$

*c):*

$$y = m + n + 1 \tag{27}$$

Solve for $n$,

$$n = y - m - 1 \tag{28}$$

Plug in $n$,

$$y = m + y - m - 1 + 1 \tag{29}$$

Combine like terms,

$$y = y \tag{30}$$

$$\boxed{\therefore surjective} \tag{31}$$

*d):*

$$y = \mid m \mid - \mid n \mid \tag{32}$$

This creates the situation that $(m > n) \exists \mathbb{Z}^+$,

$$y^+ = m - n \tag{33}$$

or $(m < n) \exists \mathbb{Z}^-$

$$y^- = n - m \tag{34}$$

Solving for $n$,

$$n = m - y^+, \ n = m + y^- \tag{35}$$

Plug in $n$,

$$y^+ = m - m + y^+, \ y^- = m + y^- - m \tag{36}$$

Combine like terms,

$$y^+ = y^+, \ y^- = y^- \tag{37}$$

Where $y^+ \exists \mathbb{Z}^+$ and $y^- \exists \mathbb{Z}^-$. The solution $f(0,0) = 0$ is understood. This creates a situation where $y$ can be any element of $\mathbb{Z}$.

$$\boxed{\therefore surjective} \tag{38}$$

*e):*

$$y = m^2 - 4 \tag{39}$$

Solve for m,

$$m = \pm\sqrt{y + 4} \tag{40}$$

If $y < -4, m \exists \mathbb{I}$ which is outside of the domain of $\mathbb{Z}$.

$$\boxed{\therefore not\ surjective} \tag{41}$$

*20*

*a):* Injective but not surjective,

$$\boxed{f(x) = \frac{1}{x}} \tag{42}$$

Every input has one output, however, $y = 0$ is not mapped by an input.

*b):* Surjective not injective,

$$\boxed{f(x, z) = x^2 - z^2} \tag{43}$$

This covers the entire domain from $N - N$ however certain values are mapped to multiple times($e.g.\ x = \pm N, z = 0$).

*c):* Bijective,

$$\boxed{f(x) = x} \tag{44}$$

In this case, every output corresponds to one output, and every output is mapped.

*d):* General function,

$$\boxed{f(x) = x^2} \tag{45}$$

Two inputs correspond to one output and negative outputs are not covered in the domain $x \exists \mathbb{R}$.

*22*

For bijection, a function must be injective and surjective. The following proofs will reuse methods from previous parts.

*a):*

$$f(x) = -3x + 4 \tag{46}$$

Injective: $f(y) = f(x)$

$$-3x + 4 = -3y + 4 \tag{47}$$

After adding 4 and dividing by $-3$, the expession becomes

$$x = y \tag{48}$$

$$\therefore injective$$

Surjective: $f(f^{-1}(x)) \exists R, y = f(x)$

$$y = -3x + 4 \tag{49}$$

Solve for $x$,

$$x = \frac{4 - y}{3} \tag{50}$$

Plug $x$ into original function,

$$y = -3(\frac{4 - y}{3}) + 4 \tag{51}$$

Distribute 3,

$$y = -(4 - y) + 4 \tag{52}$$

Distribute negative and combine like terms,

$$y = y \tag{53}$$

$$\therefore surjective$$

This function is surjective and injective,

$$\boxed{\therefore bijective}$$

*b):*

$$f(x) = -3x^2 + 7 \tag{54}$$

Injective: $f(y) = f(x)$

$$-3x^2 + 7 = -3y^2 + 7 \tag{55}$$

Subtract 7 from both sides and divide $-3$,

$$x^2 = y^2 \tag{56}$$

take root 2 of both sides,

$$\pm x = \pm y \tag{57}$$

$$x = \pm y \tag{58}$$

Since two inputs map to the same output, this function is not injective.

$$\boxed{\therefore not\ bijective}$$

*c):*

$$f(x) = \frac{x-1}{x+2} \tag{59}$$

Surjective: $f(f^{-1}(x)) \exists R, y = f(x)$

$$y = \frac{x-1}{x+2} \tag{60}$$

Solving for $x$, multiply by $x+2$

$$(x+2)y = x - 1 \tag{61}$$

Distribute $y$,

$$xy + 2y = x - 1 \tag{62}$$

Collect like variables,

$$xy - x = -2y - 1 \tag{63}$$

Factor $x$, divide $y+1$

$$x = \frac{-2y-1}{y-1} \tag{64}$$

There is a discontinuity at $y = 1$, and therefore does not have an input mapping and is not an element of $\mathbb{R}$, making this function not surjective.

$$\boxed{\therefore not\ bijective}$$

*d):*

$$f(x) = x^5 + 1 \tag{65}$$

Injective: $f(y) = f(x)$

$$y^5 + 1 = x^5 + 1 \tag{66}$$

subtract 1, take root 5,

$$y = x \tag{67}$$

$$\therefore injective$$

Surjective: $f(f^{-1}(x)) \exists R, y = f(x)$

$$y = x^5 + 1 \tag{68}$$

Solve for y,

$$x = (y-1)^{\frac{1}{5}} \tag{69}$$

plug x into original function,

$$y = ((y-1)^{\frac{1}{5}})^5 + 1 \tag{70}$$

Multiply explonential terms per rules of exponents,

$$y = y - 1 + 1 \tag{71}$$

Add constants,

$$y = y \tag{72}$$

$$\therefore surjective$$

Both injective and surjective,

$$\boxed{\therefore bijective}$$

*60*

There are 8 bits per byte, dimensional analysis says that the total number of bits divided by the equivalent corresponding number of bytes (as determined by how many bytes are in a bit) is 1, which gives rise to the equation

$$\frac{x\ bits}{8y\ bytes} = 1,\ this\ says\ that\ if\ we\ have\ 8\ bits,\ we\ have\ 1\ byte \tag{73}$$

We want a form that outputs a number of bytes given a number of bits, multiplying both sides by number of bytes yields

$$x\ bits = 8y\ bytes \tag{74}$$

It is noted that $y$ is a function of $x$ and $x$ is a function of $y$, which means that we can attain a bit count for any number of bytes or a byte count for any number of bits. For this excersize we will consider only the following case,

$$y(x) = \frac{x}{8} \tag{75}$$

Since bytes are discrete packages of 8 bits, any decimal greater than 0 will result in a new byte.

*a):*

$$y(4) = \frac{4}{8} \tag{76}$$

divide,

$$y = \frac{1}{2} = 0.5 \approx \boxed{1} \tag{77}$$

*b):*

$$y(10) = \frac{10}{8} \tag{78}$$

divide,

$$y = \frac{5}{4} = 1.25 \approx \boxed{2} \tag{79}$$

*c):*

$$y(500) = \frac{500}{8} \tag{80}$$

divide,

$$y = \frac{125}{2} = 62.5 \approx \boxed{63} \tag{81}$$

*d):*

$$y(x) = \frac{3000}{8} \tag{82}$$

divide,

$$y = \boxed{375} \tag{83}$$