CSCE 222
Homework 2

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1.6

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1) a: Kangaroos(p) live in($\rightarrow$) Australia(q) and($\cdot$) are marsupials(r).
Therefore, kangaroos(p) are($\rightarrow$) marsupials(r). $\boxed{HypotheticalSyllogism}$

2) b: It is either hotter than 100 degrees today(p) or($+$) the pollution is dangerous(q).
It is less than 100 degrees outside today(p').
Therefore, the pollution is dangerous($\therefore q$). $\boxed{DisjunctiveSyllogism}$

3) c: Linda is an excellent swimmer(p).
If Linda is an excellent swimmer(p), then($\rightarrow$) she can work as a lifeguard(q).
Therefore, Linda can work as a lifeguard($\therefore q$). $\boxed{ModusPonens}$

4) d: Steve will work at a computer company this summer(p).
Therefore, this summer Steve will work at a computer company or he will be a beach bum($\therefore p + q$). $\boxed{Addition}$

5) e: If I work all night on this homework(p), then($\rightarrow$) I can answer all the exercises(q).
If I answer all the exercises(q),($\rightarrow$) I will understand the material(r).
Therefore, if I work all night on this homework, then I will understand the material($\boxed{p \rightarrow r}$). $\boxed{HypotheticalSyllogism}$

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6) a: If I play hockey(p), then($\rightarrow$) I am sore the next day(q).
I use the whirlpool(r) if($\leftarrow$) I am sore(q).
I did not use the whirlpool(r').
$\boxed{\therefore p'}$
Using Hypothetical Syllogism the expression becomes $p \rightarrow r$. The boxed conclusion was drawn using Modus Tollens.

7) b: If I work(p), it($\rightarrow$) is either sunny(q) or($+$) partly sunny(r).
I worked(p) ($\rightarrow$)last Monday(s) or($+$) I worked last Friday(t).
It was not sunny(q') on($\cdot$) Tuesday(u).
It was not partly(r') sunny on Friday(t).
$\boxed{\therefore s \cdot q}$
Every combination evaluates to not working except if it is sunny on monday. Through simplification, it is deduced that since p is true, $\therefore sq \equiv T$.

8) c: All insects(o) have($\equiv$) six legs(p).
Dragonflies(m) are($\rightarrow$) insects(o).
Spiders(l) do not ($\rightarrow$)have six legs(p').
Spiders(l) eat($\rightarrow$) dragonflies(m').
Line 3 says $l \rightarrow p'o'$, $\boxed{\therefore l \equiv m'}$

9) d: Every student(p) has($\rightarrow$) an Internet account(q).
Homer(r) does($\rightarrow$) not have an Internet account(q').
Maggie(s) has($\rightarrow$) an Internet account(q).
Using Disjunctive Syllogism, we can conclude that homer is not a student, $\therefore$ maggie is a student.

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10) a: Everyone enrolled in the university($T \equiv$) has lived in a dormitory(p).
Mia(q) has($\rightarrow$) never lived in a dormitory(p').
Therefore, Mia is not enrolled in the university($\therefore q'$).
This statement is $\boxed{true}$ since line 2 implies that a tautology is false if q is true.

11) b: A convertible car(p) is($\rightarrow$) fun to drive(q).
Isaacs car is not a convertible(p').
Therefore, Isaacs car is not fun to drive($\therefore q'$).
This statement is $\boxed{false}$ since $p' \rightarrow q$ evaluates to true or false when line 3 states that it evaluates to false. Another way to think of it would be that it is never declared that convertibles are the only fun car.

1.7

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In the case that x, y, x are all an even number(n),

$$3 * n\%2 = 0$$

The only possibility left is that at least one of $x, y,$ or z is odd.

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If m and n are odd,

$$m\%2 = 1, n\%2 = 1, (m\%2)(n\%2) = 1, \therefore mn\%2 = 1$$

In order for $mn\%2 = 0$, $m\%2 = 0$, or $n\%2 = 0$.

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12) a: Suppose n is odd,

$$3n\%2 = 1, 2\%2 = 0, \therefore (3n + 2)\%2 = 1$$

13) b: Suppose $(3n+2)\%2 = 1$ and $n\%2 = 0$. $3n\%2$ must be 1, however, $3n\%2 = 0$, $\therefore (3n + 2)\%2 = 0$.

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$$(7n+4)\%2 = 0$$
$$(7n\%2) + (4\%2) = 0$$
$$(7\%2)(n\%2) + 0 = 0$$
$$(1)(n\%2) = 0$$
$$\therefore n\%2 = 0$$

2.1

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14) a: There exists a number x on the set of real numbers such that $x^3 = -1$. True since $(-1)^{1/3} = -1$ and $-1\epsilon\mathbb{R}$.

15) b: There exists a number x on the set of integers such that $x + 1 > x$. True since 1 is a positive integer, increasing the observed value $\forall x$.

16) c: For all numbers x on the set of integers, x minus 1 is also on the set of integers. True since x and 1 are on the set of integers, and they are performing a $\mathbb{Z}$ operation that is defined to return a value on the same set(subtraction).

17) d: For all numbers x on the set of integers, x^3 is also on the set of integers. True since x is on the set of integers, and it is performing a $\mathbb{Z}$ operation that is defined to return a value on the same set(multiplication).

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$$x\epsilon\mathbb{Z}$$

18) a: $x^3 \geq 1, \boxed{T} \forall x > 0$.

19) b: $x^2 = 2, \boxed{F}, 0^2 = 0, 1^2 = 1, 2^2 = 4$.

20) c: $x < x^2, \boxed{T} \forall x > 1$.

2.2

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21) a: The set of sophomores(A) taking($\cap$) discrete mathematics in your school(B). $\boxed{A \cap B}$

22) b: The set of sophomores(A) at your school who are not($-$) taking discrete mathematics(B). $\boxed{A - B}$

23) c: The set of students at your school who either are sophomores(A) or($+$) are taking discrete mathematics(B). $\boxed{A + B}$

24) d: The set of students at your school who either($+$) are not sophomores(A') or are not taking discrete mathematics(B') $\boxed{A' + B'}$