CSCE 222
Homework 7

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D):

$$x^n = 2x^{n-1} - x^{n-2}$$
$$x^2 = 2x - 1$$
$$x^2 - 2x + 1 = 0$$
$$(x-1)(x-1) = 0$$
$$x = 1, 1$$

$$a_n = c_1x^n + nc_2x^n$$
$$a_0 = 4 = c_1$$
$$a_1 = 1 = 4 + c_2$$
$$\boxed{a_n = 4 - 3n}$$

E):

$$x^n = x^{n-2}$$
$$x^2 - 1 = 0$$
$$x = \pm 1$$

$$a_n = c_1x^n + c_2x^n$$
$$a_0 = c_1 + c_2 = 5$$
$$a_1 = c_1 - c_2 = -1$$
$$2c_1 = 4, c_1 = 2, c_2 = 3$$
$$\boxed{a_n = 2 + 3(-1)^n}$$

G):

$$x^{n+2} = -4x^{n+1} + 5x^n$$
$$x^2 + 4x - 5 = 0$$
$$x = -5, 1$$

$$a_n = c_1(-5)^n + c_2$$
$$a_0 = c_1 + c_2 = 2$$
$$a_1 = -c_15 + c_2 = 8$$
$$-c_16 = 6, c_1 = -1, c_2 = 3$$
$$a_n = 3 - (-5)^n$$

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A):

$$\boxed{L_n = \frac{L_{n-1} + L_{n-2}}{2}}$$

B):

$$x^n = \frac{x^{n-1} + x^{n-2}}{2}$$
$$2x^2 - x^1 - 1 = 0$$
$$(2x+1)(x-1) = 0$$

$$L_n = c_1(-\frac{1}{2})^n + c_2$$
$$L_1 = 100,000 = c_1(-\frac{1}{2}) + c_2$$
$$L_2 = 300,000 = c_1(\frac{1}{4}) + c_2$$
$$200,000 = c_1(\frac{3}{4})$$
$$c_1 = 266,666\frac{2}{3}, c_2 = 233,333\frac{1}{3}$$
$$\boxed{L_n = 266,666\frac{2}{3}(-\frac{1}{2})^n + 233,333\frac{1}{3}}$$

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$$x^n = 2x^{n-1} + x^{n-2} - 2x^{n-3}$$
$$x^3 = 2x^2 + x - 2$$

Using Mathematica,

$$x = -1, 1, 2$$

$$a_n = c_1(-1)^n + c_2 + c_32^n$$
$$a_0 = 3 = c_1 + c_2 + c_3$$
$$a_1 = 6 = c_1(-1) + c_2 + c_32$$
$$a_2 = 0 = c_1 + c_2 + c_34$$

$$9 = 2c_2 + 3c_3$$

$$6 = 2c_2 + 6c_3$$

$$3 = -3c_2$$

$$c_3 = -1, c_2 = 6, c_1 = -2$$

$$\boxed{a_n = -2(-1)^n + 6 - 2^n}$$

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$$x^n = 5x^{n-2} - 4x^{n-4}$$

$$x^4 - 5x^2 + 4 = 0$$

$$(x^2 - 4)(x^2 - 1) = 0$$

$$x = 1, -1, 2, -2$$

$$a_n = c_1 + c_2(-1)^n + c_3 2^n + c_4(-2)^n$$

$$a_0 = 3 = c_1 + c_2 + c_3 + c_4$$

$$a_1 = 2 = c_1 - c_2 + c_3 2 - c_4 2$$

$$a_2 = 6 = c_1 + c_2 + c_3 4 + c_4 4$$

$$a_3 = 8 = c_1 - c_2 + c_3 8 - c_4 8$$

$$5 = 2c_1 + 3c_3 - c_4$$

$$14 = 2c_1 + 12c_3 - 4c_4$$

$$8 = 2c_1 + 6c_3 + 2c_4$$

$$9 = 9c_3 - 3c_4$$

$$3 = 3c_3 + 3c_4$$

$$12 = 12c_3$$

$$c_3 = 1, c_4 = 0, c_1 = 1, c_2 = 1$$

$$\boxed{a_n = 1 + (-1)^n + 2^n}$$

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A): First, adjust closed form for a_{n-1},

$$a_{n-1} = (n-1)2^{n-1}$$

Plug into recurrence relation,

$$a_n = 2((n-1)2^{n-1}) + 2^n$$

Simplify,

$$a_n = (n-1)2^n + 2^n$$

$$a_n = n2^n$$

Which is the original expression.

$$\boxed{\therefore n2^n = 2a_{n-1} + 2^n}$$

B):

$$x^n = 2x^{n-1} + F(n)$$

$$x = 2 + F(n)$$

$$x = 2, F(n) = 2^n$$

$$a_n = \alpha 2^n + \alpha^(p)$$

$$\alpha^(p) = n^m p_0 s^n$$

$$\alpha^(p) = n(np_1 + p_0)2^n$$

$$\boxed{a_n = \alpha 2^n + n(np_1 + p_0)2^n}$$

9.1

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A):

$$x + y = 0$$

Reflexive:

$$x = 1$$

$$xRx = 2, 2 \neq 1, \therefore \boxed{Not\ Reflexive}$$

Symmetric: Because addition is commutative, $x+y = y+x$

$$\therefore \boxed{Symmetric}$$

Antisymmetric:

$$x = -y, y = -x, x = y\ iff\ x, y = 0$$

$$\therefore \boxed{Not\ Antisymmetric}$$

Transitive:

$$x = 1, y = -1, z = 1$$

$$xRy \in R, yRz \in R, xRz \notin R$$

$$\therefore \boxed{Not\ Transitive}$$

C):

$$R : x - y \in \mathbb{Q}, x, y \in \mathbb{R}$$

Reflexive:

$$x = x$$

$$xRx = 0 \; \forall \; x$$

$$\because 0 \in \mathbb{Q}, \boxed{Reflexive}$$

Symmetric: Due to the properties of subtraction, if xRy is on $\mathbb{Q}$, so is yRx.

$$\therefore \boxed{Symmetric}$$

Antisymmetric:

$$x = -y \in \mathbb{Q}, y = -x \in \mathbb{Q}, x = y \; iff \; x, y = 0$$

$$\therefore \boxed{Not \; Antisymmetric}$$

Transitive:

$$x \in Q, y \in Q, z \in Q$$

$$xRy \in Q, yRz \in Q, xRz \in Q$$

$$\therefore \boxed{Transitive}$$

D): Reflexive:

$$xRx = 2x, 2x \neq x$$

$$\therefore \boxed{Not \; Reflexive}$$

Symmetric:

$$xRy \in \mathbb{R}, yRx \in \mathbb{R}$$

$$\therefore \boxed{Symmetric}$$

Antisymmetric:

$$x = 2y, y = 2x, x = y \; iff \; x, y = 0$$

$$\therefore \boxed{Not \; Antisymmetric}$$

Transitive:

$$x = 4, y = 2, z = 1$$

$$xRy \in R, yRz \in R, xRz \notin R$$

$$\therefore \boxed{Not \; Transitive}$$

E): Reflexive:

$$xx \geq 0 \because x^2 \geq 0 \forall x \in \mathbb{R}, x = x$$

$$\therefore \boxed{Reflexive}$$

Symmetric:

$$xRy \in \mathbb{R}, yRx \in \mathbb{R}$$

$$\therefore \boxed{Symmetric}$$

Antisymmetric:

$$xy \geq 0 \rightarrow yx \geq 0$$

$$\therefore \boxed{Antisymmetric}$$

Transitive:

$$x = -1, y = 0, z = 1$$

$$xy \in R, yz \in R, xz \notin R$$

$$\therefore \boxed{Not \; Transitive}$$