

CSCE 221

Problem Set 20

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I.

Compute XY

$$X = 117937, Y = 404783$$

$$X = 117 \times 10^3 + 937, Y = 404 \times 10^3 + 783$$

$$\begin{aligned} Z &= (0117)(0404)10^3 + (0937)(0783) + \dots \\ &\dots + (0937)(0404 \times 10^3) + (0783)(0117 \times 10^3) \end{aligned}$$

$$\begin{aligned} Z &= (01 \times 10^2 + 17)(04 \times 10^2 + 04) \times 10^3 + \dots \\ &\dots + (09 \times 10^2 + 37)(07 \times 10^2 + 83) + \dots \\ &\dots + (09 \times 10^2 + 37)(04 \times 10^5 + 04 \times 10^3) + \dots \\ &\dots + (07 \times 10^2 + 83)(01 \times 10^2 + 17 \times 10^3) \end{aligned}$$

$$\begin{aligned} Z &= (04 \times 10^4 + 68 \times 10^2 + 4 \times 10^2 + 68) \times 10^3 + \dots \\ &\dots + (63 \times 10^4 + 259 \times 10^2 + 747 \times 10^2 + 3071) + \dots \\ &\dots + (36 \times 10^7 + 148 \times 10^5 + 36 \times 10^5 + 148 \times 10^3) + \dots \\ &\dots + (07 \times 10^4 + 83 \times 10^2 + 1119 \times 10^5) \end{aligned}$$

$$\begin{aligned} Z &= (40 \times 10^7 + 1475 \times 10^5 + 216 \times 10^3) + \dots \\ &\dots + (70 \times 10^4 + 1089 \times 10^2 + 3071) \end{aligned}$$

$$\begin{aligned} Z &= (400000000 + 147500000 + 216000 + \dots \\ &\dots + 700000 + 108900 + 3071) \end{aligned}$$

$$Z = \boxed{5478027971}$$

II.

Where n is the number of digits, a multiplication of $n = 4$ will be,

$$Z = (A_1 \times 10^2 + A_2)(A_1 \times 10^2 + A_2)$$

$$Z = ((A_3 + A_4) \times 10^2 + A_2)((A_3 + A_4) \times 10^2 + A_2)$$

which requires 9 multiplications and k recursive breakdowns.

$$4^k = 9$$

solving for k

$$k \log_2(4) = \log_2(9)$$

$$2k = \log_2(9)$$

$$k = \log_2(9^{\frac{1}{2}})$$

so

$$\boxed{O(n^{\log_2(3)})}$$