$\boxed{\text{Problem 1.8}}$

$t \to$ non negative random var

(a) $\alpha > 0$, Prove $\mathbb{P}[t \geq \alpha] \leq \mathbb{E}(t)/\alpha$

If we take $\mathbb{I}(t \geq \alpha) \in \{0, 1\}$

Then this will always be true:

$$\alpha \mathbb{I}(t \geq \alpha) \leq t$$

Taking the expected value of both sides:

$$E[\alpha \mathbb{I}(t \geq \alpha)] \leq E[t]$$

$$\alpha E[\mathbb{I}(t \geq \alpha)] \leq E[t]$$

The expected value of $\mathbb{I}$ is just the probability, rearrange to get:

$$\underline{Pr[t \geq \alpha] \leq E[t]/\alpha}$$

(b) If v is any random variable with μ, σ^2, prove

$$\mathbb{P}[(v-\mu)^2 \geq \alpha] \leq \sigma^2/\alpha$$

- $(v-\mu)^2$ is always positive
- subsistute $t = (v-\mu)^2$:

$$\mathbb{P}[(v-\mu)^2 \geq \alpha] \leq \frac{E[(v-\mu)^2]}{\alpha} \longrightarrow \text{this is the variance!}$$

$$\underline{\mathbb{P}[(v-\mu)^2 \geq \alpha] \leq \frac{\sigma^2}{\alpha}}$$

(c) $v_1 \ldots v_N$ are iid with μ, σ^2. $v = \frac{1}{N}\sum_{n=1}^{N} v_N$, prove

$$\mathbb{P}[(v-\mu)^2 \geq \alpha] \leq \frac{\sigma^2}{N\alpha}$$

- $E[v] = \frac{1}{N}\sum_{n=1}^{N} \mu = \mu$
- $Var[v] = \frac{1}{N^2}\sum_{n=1}^{N} \sigma^2 = \frac{\sigma^2}{N}$
- Subsistute $Var[v]$ for σ^2 in (b) gets us

$$\underline{\mathbb{P}[(v-\mu)^2 \geq \alpha] \leq \frac{\sigma^2}{N\alpha}}$$

$\boxed{\text{Problem 1.12}}$

N data points $y_1 \le \dots \le y_N$

(a) Algorithm uses $E_{in}(h) = \sum_{n=1}^{N} (h - y_n)^2$
Show this equals $h_{mean} = \frac{1}{N} \sum_{n=1}^{N} y_n$

We want to minimize $E_{in}(h)$, so take derative

$$\frac{\partial E_{in}(h)}{\partial h} = \sum_{n=1}^{N} 2(h - y_n) \cdot 1$$

Setting equal to 0:

$$\sum_{n=1}^{N} (h - y_n) = 0$$

$$\sum_{n=1}^{N} h - \sum_{n=1}^{N} y_n = 0$$

$$N(h_{mean}) = \sum_{n=1}^{N} y_n$$

$$\boxed{h_{mean} = \frac{1}{N} \sum_{n=1}^{N} y_n}$$

(b) $E_{in}(h) = \sum_{n=1}^{N} |h - y_n| = \sum_{n=1}^{N} \sqrt{(h - y_n)^2}$

$$\frac{\partial E_{in}(h)}{\partial h} = \sum_{n=1}^{N} \frac{h - y_n}{|h - y_n|} = \sum_{n=1}^{N} \text{sign}(h - y_n) = 0$$

→ to accomplish this, exactly half of h must be positive and half of h must be negative. This means h must equal h_{med}.

(c) $\boxed{y_N = y_N + \varepsilon \text{ where } \varepsilon \to \infty}$

$h_{mean} \to \infty$ because $h_{mean} = \frac{1}{N} \sum_{n=1}^{N} y_N + \underbrace{\varepsilon}_{\text{goes to } \infty}$

h_{med} stays the same so long as half $> y$, and half $< y$

Problem 2.3

Compute max number of dichotomies $m_H(N)$ for models:

(a) $\boxed{\text{Positive/Negative ray}}$

Positive: $m_H(N) = N+1$

Negative: $m_H(N) = N+1$

But, all + and all − are always repeated

so $m_H(N) = \underbrace{N+1}_{\text{Positive ray}} + \underbrace{N+1}_{\text{Negative ray}} \underbrace{-2}_{\text{repeats}} = \boxed{2N}$

$\boxed{d_{vc} = 2}$

(b) $\boxed{\text{Positive/Negative Interval}}$

Positive/Negative intervals: $m_H(N) = \frac{1}{2}N^2 + N/2 + 1 = \binom{N+1}{2} + 1$

What is repeated? all +

all −

But can get $N-2$ new dichotomies w/ negative

so $m_H(N) = \binom{N+1}{2} + 1 + \binom{N-1}{2}$

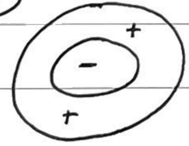
$= \frac{1}{2}N^2 + \frac{1}{2}N + 1 + \frac{1}{2}N^2 - \frac{3}{2}N + 1$

$= \boxed{N^2 - N + 2}$

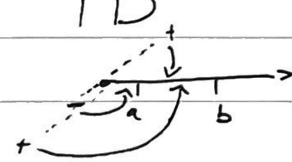
$\boxed{d_{vc} = 3}$ as $m_H(4) = 14$

(c) $\boxed{\text{Spheres in } \mathbb{R}^d, +1 \text{ for } a \le \sqrt{x_1^2 + \dots x_d^2} \le b}$

2D



1D



Mapping from ND → 2D → 1D shows that this is just like a positive interval. So long you can map the interval back to ND, we use the same growth function $m_H(N) = \binom{N+1}{2} + 1$ and $\boxed{d_{vc} = 3}$

$\boxed{\text{Problem 2.8}}$

Which are possible growth functions?

✓ $\boxed{1+N}$: Formula 2.10 from LFD gives us $m_H(N) \leq N^{d_{vc}}+1$

$d_{vc}=1$ here so gives us:

$$1+N \leq N^1+1$$

This will always hold, so $1+N$ is a possible function.

✓ $\boxed{1+N+\frac{N(N-1)}{2}}$: $d_{vc}=2$

so $1+N+\frac{N^2}{2}+N \leq N^2+1$

The N^2 coefficient issmaller on ~~the~~ the left so this is possible.

✓ $\boxed{2^N}$: $d_{vc}=\infty$, but this is the growth function if d_{vc} is ∞ by definition so this is possible.

X $\boxed{2^{\lfloor\sqrt{N}\rfloor}}$: $d_{vc}=1$ so:

$$2^{\lfloor\sqrt{N}\rfloor} \leq N^1+1$$

This is fine at $N=10, 20$, but at $N=30$

$2^5 \not\leq 31$ so this is impossible.

X $\boxed{2^{\lfloor N/2\rfloor}}$: $d_{vc}=0$ so bounded by $N^0+1=2$. This will never be true above 4. so not possible

X $\boxed{1+N+\frac{N(N-1)(N-2)}{6}}$: $d_{vc}=1$

can't be bounded by N^1+1, so not possible.