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Three body problem in the spherical geometry

by

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A thesis submitted in partial fulfillment for the
degree of Doctor of Philosophy

in the
Faculty Name
Department or School Name

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Abstract

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The Thesis Abstract is written here (and usually kept to just this page). The page is kept centered vertically so can expand into the blank space above the title too...

Acknowledgements

The acknowledgements and the people to thank go here, don't forget to include your project advisor…

Contents

List of Figures

List of Tables

Abbreviations

LAH **L**ist **A**bbreviations **H**ere

Physical Constants

Speed of Light $c = 2.997\ 924\ 58 \times 10^{8}\ \mathrm{ms}^{-\mathrm{S}}$ (exact)

Symbols

a	distance	m
P	power	W (Js^{-1})
ω	angular frequency	$rads^{-1}$

For/Dedicated to/To my…

Chapter 1

Introduction

In this chapter a classic approach of a somehow general case of the three body problem in 2 dimensions is going to be presented. This will give some necessary intuition to develop the analogous problem in the spherical geometry. To begin with, the problem is going to be described in great detail; then its integrability is going to be proven; and finally, a formalism to describe the movement of the particles is going to be presented.

1.1 The three body problem in the plane

The three body problem presented here is that of three particles of electrical charge e and mass m confined to a plane, under the influence of a strong magnetic field perpendicular to it, and forces whose potentials satisfy translational and rotational symmetries in the plane.

Given this information, the Hamitonian associated with this system has the form:

$$H = \sum_{i=1}^{3} \frac{1}{2m_i} \left\| \vec{p_i} - e\vec{A}\left(\vec{q_i}\right) \right\|^2 + V\left(\vec{q_1}, \vec{q_2}, \vec{q_3}\right) + \frac{\omega_c^2}{2} \sum_{i=1}^{3} \|\vec{q_i}\|^2 \tag{1.1}$$

Where $\vec{q_i} = x_i\hat{\imath} + y_i\hat{\jmath}$, $\vec{p_i} = p_{x_i}\hat{\imath} + p_{y_i}\hat{\jmath}$ and $\vec{A}\left(\vec{q}\right)$ is the magnetic vector potential, which satisfies $\nabla \times \vec{A} = B\hat{k}$.

Besides, the potential $V\left(\vec{q_1}, \vec{q_2}, \vec{q_3}\right)$ satisfies the symmetries:

$$V\left(R\vec{q_1}+\vec{a},R\vec{q_2}+\vec{a},R\vec{q_3}+\vec{a}\right)=V\left(\vec{q_1},\vec{q_2},\vec{q_3}\right) \tag{1.2}$$

For any rotation R and translation $\vec{a}$ in the plane.

1.1.1 Integrability of the system

For the proof of integrability for this system, and for further analysis of the trajectories of the particles, let us perform the well known canonical transformation of the guiding centres.

This transformation is defined by the following two equations:

$$\vec{\pi_i}=\vec{p_i}-e\vec{A}\left(\vec{q_i}\right) \tag{1.3}$$

$$\vec{R_i}=\vec{q_i}-\frac{\hat{k}\times\vec{\pi_i}}{eB} \tag{1.4}$$

The equation 1.3 passes from the canonical momentum $\vec{p_i}$ to the linear momentum $\vec{\pi_i}$, which is much more intuitive and understandable; while the equation 1.4 transforms the general position $\vec{q_i}$ to the position of the instantaneous guiding centre $\vec{R_i}$.

In a system without the interaction potentials, the electrically charged particles are known to perform the circular motion of the cyclotron with radii that depends on the initial linear momenta. In this case, the guiding centres would be constant in time as would be the linear momenta. However, with the introduction of an interacting potential, the momentum of each particle may vary making the guiding centre change too, which is why the instantaneous interpretation of the guiding centres is necessary.

Now, let us calculate the Poisson brackets for this new set of coordinates in a specific particle.

$$\begin{aligned}
\{\pi_1, \pi_2\} &= \frac{\partial \pi_1}{\partial q_\alpha}\frac{\partial \pi_2}{\partial p_\alpha} - \frac{\partial \pi_2}{\partial q_\alpha}\frac{\partial \pi_1}{\partial p_\alpha} \\
&= -e\delta_{\alpha 2}\frac{\partial A_1}{\partial q_\alpha} + e\delta_{\alpha 1}\frac{\partial A_2}{\partial q_\alpha} \\
&= -e\frac{\partial A_1}{\partial q_2} + e\frac{\partial A_2}{\partial q_1} \\
&= e(\nabla \times \vec{A})_3 = eB
\end{aligned}$$

$$\begin{aligned}
\{R_1, R_2\} &= \{q_1, q_2\} + \left\{q_1, -\frac{\pi_1}{eB}\right\} + \left\{\frac{\pi_2}{eB}, q_2\right\} + \left\{\frac{\pi_2}{eB}, -\frac{\pi_1}{eB}\right\} \\
&= \frac{1}{eB}\left(\cancelto{-1}{\{p_1, q_1\}} - \cancelto{0}{e\{A_1, q_1\}} + \cancelto{-1}{\{p_2, q_2\}} \cancelto{0}{e\{A_2, q_2\}}\right) + \frac{eB}{(eB)^2} \\
&= \frac{-2}{eB} + \frac{1}{eB} = -(eB)^{-1}
\end{aligned}$$

1.2 Another Section

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Appendix A

An Appendix

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