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Three body problem in the spherical geometry

by

Jesus David Prada Gonzalez

A thesis submitted in partial fulfillment for the degree of Doctor of Philosophy

in the

Faculty Name
Department or School Name

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Abstract

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The Thesis Abstract is written here (and usually kept to just this page). The page is kept centered vertically so can expand into the blank space above the title too…

Acknowledgements

The acknowledgements and the people to thank go here, don't forget to include your project advisor…

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Abbreviations

LAH **L**ist **A**bbreviations **H**ere

Physical Constants

Speed of Light	c	$=$	$2.997\ 924\ 58 \times 10^{8}\ \mathrm{ms}^{-\mathrm{S}}$ (exact)

Symbols

a	distance	m
P	power	W (Js^{-1})
ω	angular frequency	rads^{-1}

For/Dedicated to/To my…

Chapter 1

Introduction

In this chapter a classic approach of a somehow general case of the three body problem in 2 dimensions is going to be presented. This will give some necessary intuition to develop the analogous problem in the spherical geometry. To begin with, the problem is going to be described in great detail; then its integrability is going to be proven; and finally, a formalism to describe the movement of the particles is going to be presented.

1.1 The three body problem in the plane

The three body problem presented here is that of three particles of electrical charge $-e$ and mass m confined to a plane, under the influence of a strong magnetic field perpendicular to it, and forces whose potentials satisfy translational and rotational symmetries in the plane.

Given this information, the Hamitonian associated with this system has the form:

$$H = \sum_{i=1}^{3} \frac{1}{2m_i} \left\| \vec{p_i} + e\vec{A}\left(\vec{q_i}\right) \right\|^2 + V\left(\vec{q_1}, \vec{q_2}, \vec{q_3}\right) + \frac{\omega_c^2}{2} \sum_{i=1}^{3} \|\vec{q_i}\|^2 \tag{1.1}$$

Where $\vec{q_i} = x_i\hat{\imath} + y_i\hat{\jmath}$, $\vec{p_i} = p_{x_i}\hat{\imath} + p_{y_i}\hat{\jmath}$ and $\vec{A}\left(\vec{q}\right)$ is the magnetic vector potential, which satisfies $\nabla \times \vec{A} = B\hat{k}$.

Besides, the potential $V\left(\vec{q_1}, \vec{q_2}, \vec{q_3}\right)$ satisfies the symmetries:

$$V\left(R\vec{q_1}+\vec{a},R\vec{q_2}+\vec{a},R\vec{q_3}+\vec{a}\right)=V\left(\vec{q_1},\vec{q_2},\vec{q_3}\right) \tag{1.2}$$

For any rotation R and translation $\vec{a}$ in the plane.

1.1.1 Integrability of the system

For the proof of integrability for this system, and for further analysis of the trajectories of the particles, let us perform the well known canonical transformation of the guiding centres.

This transformation is defined by the following two equations:

$$\vec{\pi_i}=\vec{p_i}+e\vec{A}\left(\vec{q_i}\right) \tag{1.3}$$

$$\vec{R_i}=\vec{q_i}+\frac{\hat{k}\times\vec{\pi_i}}{eB} \tag{1.4}$$

The equation 1.3 passes from the canonical momenta $\vec{p_i}$ to the linear momenta $\vec{\pi_i}$, which is much more intuitive and understandable; while the equation 1.4 passes the general position $\vec{q_i}$ to the position of the instantaneous guiding centre $\vec{R_i}$.

In a system without the interaction potentials, the electrically charged particles are known to perform circular motion (cyclotron) with radius depending on the initial linear momentum. In this case, the guiding centres would be constant in time as would be the linear momenta. However, with the introduction of an interacting potential, the momenta of each particle may vary, making the guiding centres change too, making it necessary the interpretation of the guiding centres as the centres of the cyclotron that the particles would describe given no interactions, and the instantaneous linear momenta.

1.2 Another Section

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Appendix A

An Appendix

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