Estadística III para In

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Agenda

- anuncios varios
  - Parcial, revisión del parcial
  - Tarea se enviará este Sábado con fecha de entr
- modelos de analitica (machine learning-ML) Supe
  - Regresión logística
    - Matemática de la regresión logística
      - Gradiente descendiente
    - K-fold
    - Regularización
    - Feature selection o selección de features
- Práctica de regresión en Python

Supervisado, Regresión y regres

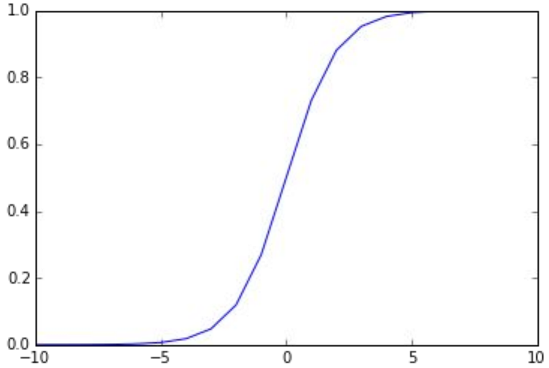
Regresión logística

en donde ... W los coeficientes de una función, objetivo de reducir los errores de clasificación.

$$\tilde{y} = \frac{1}{1 + e^{-\sum_{i=0}^{p} w_i x_i + b}}$$

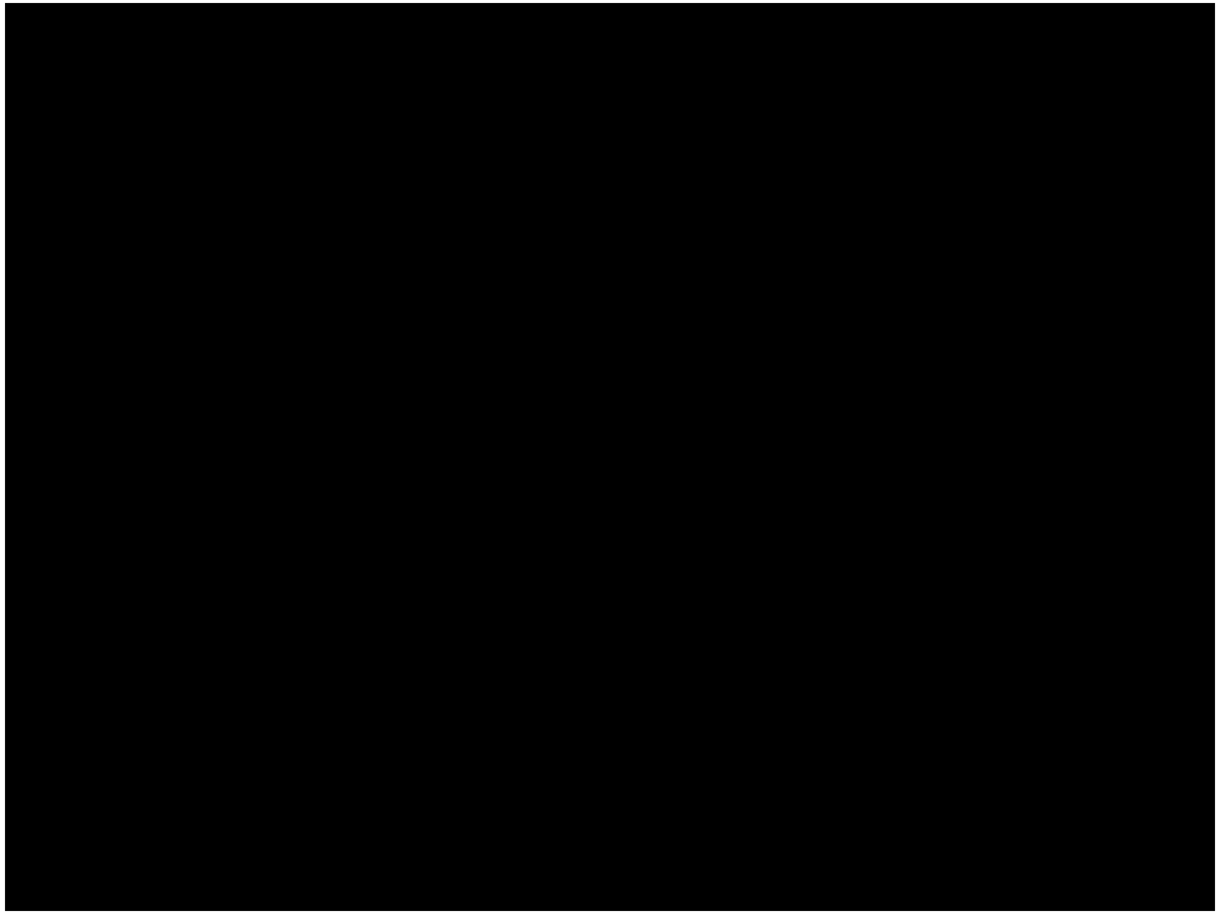
Sigmoid

La función tiene forma de ... entre [0,1]. Por defecto $\tilde{y} > 0.5$



Ejemplo

utilizando datos históricos ... si un alumno pasará el curso ... personas que el modelo pred...



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Supervisado, Regresión y regres

$$y = \frac{1}{(1+e^{-(b1x_1+b2x_2+b0)})}$$

Cada dato : $p(y =" c " | experimentos) = p(y_1 =" C ") * p(y_2 =" C ")$

$$p(y = 1|d) = 1/n \prod_{i=1}^{n} (1 - p(y))^{1-y} . (p(y))^{y}$$

$$l = \prod_{i=1}^{n} (1 - \frac{1}{(1+e^{-(b1x+b2x_2+b_0)})})^{(1-y)} * (\frac{1}{(1+e^{-(b1x+b2x_2+b_0)})})^{y}$$

$$\log(l) = \sum_{i=1}^{n} +(1-y)\log(1 - \frac{1}{(1+e^{-(b_1x+b_2x_2+b_0)})}) + y\log(\frac{1}{(1+e^{-(b_1x+b_2x_2+b_0)})})$$

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Supervisado, Regresión y regres

$$\frac{dl}{db_1} = \frac{dl}{du}\frac{du}{db_1}$$

$$\frac{dl}{db_1} = y\log([1+e^{-(b_1x+b_2x_2+b_0)}]^{-1}) + (1-y)\log(\frac{e^{-(b_1x+b_2x_2+b_0)}}{1+e^{-(b_1x+b_2x_2+b_0)}})$$

$$\frac{dl}{db_1} = -y\log(1+e^{-(b_1x+b_2x_2+b_0)}) + (1-y)[\log(e^{-(b_1x+b_2x_2+b_0)}) - \log(1+e^{-(b_1x+b_2x_2+b_0)})]$$

$$\frac{dl}{db_1} = \log(e^{-(b_1x+b_2x_2+b0)}) - \log(1+e^{-(b_1x+b_2x_2+b_0)}) - y\log(e^{-(b_1x+b_2x_2+b_0)})$$

$$\frac{dl}{db_1} = \frac{-(b_1x_1+b_2x_2+b0)}{db_1} - \frac{\log(1+e^{-(b_1x_1+b_2x_2+b_0)})}{db_1} - \frac{y(-(b_1x_1+b_2x_2+b_0))}{db_1}$$

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$$= -x_1 - (1 + e^{-(b_1x_1+b_2x_2+c)})^{-1}(e^{-(b_1x_1+b_2x_2+c)})(-x_1)) + yx_1$$

$$= -x_1 - (-x_1)(\frac{1}{1+e^{(b_1x_1+b_2x_2+c)}}) + yx_1$$

$$= -x_1 - (-x_1)(\frac{1}{1+e^{(b_1x_1+b_2x_2+c)}}) + yx_1$$

$$= \frac{-x_1+x_1+-x_1e^{(b_1x_1+b_2x_2+c)}}{1+e^{(b_1x_1+b_2x_2+c)}}) + yx_1$$

$$= x_1(y - \frac{e^{(b_1x_1+b_2x_2+c)}}{1+e^{(b_1x_1+b_2x_2+c)}})$$

$$\frac{dl}{db_1} = x_1(y - \frac{1}{1+e^{-(b_1x_1+b_2x_2+c)}})$$

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Calcular b_0. ¶

$$\frac{dl}{db0} = \log(e^{-(b_1x_1+b_2x_2+b_0)}) - \log(1+e^{-(b_1x_1+b_2x_2+b_0)}) - y\log(e^{-(b_1x+b_2x_2+b_0)})$$

$$\frac{dl}{d0} = \frac{-(b_1x_1+b_2x_2+b_0)}{dc} - \frac{\log(1+e^{-(b_1x_1+b_2x_2+b_0)})}{db0} - \frac{y(-(b_1x_1+b_2x_2+b_0))}{db0}$$

$$\frac{dl}{d0} = -1 - (-1)(\frac{1}{1+e^{(b_1x_1+b_2x_2+b_0)}}) +y$$

$$\frac{dl}{d0} = y - \frac{e^{(b_1x_1+b_2x_2+b_0)}}{1+e^{(b_1x_1+b_2x_2+b_0)}})$$

$$\frac{dl}{d0} = y - \frac{1}{1+e^{-(b_1x_1+b_2x_2+b_0)}}$$

Aplicar el gradiente descendien

Algorithm 21 GRADIENTDESCENT($\mathcal{F}, K, \eta_1, \dots$)

```
z^(0) ← ⟨0, 0, ..., 0⟩                    // initialize variable we are optimizing
for k = 1 ... K do
    g^(k) ← ∇_z F |_{z^(k-1)}              // compute gradient at current location
    z^(k) ← z^(k-1) − η^(k) g^(k)          // take a step down the gradient
end for
return z^(K)
```

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Aplicar el gradiente descendiente

```
def train(self, x , y):
    # Copiamos las variables y hacemos la actualziacion despues de calcular
    b = self.b[:]
    c = self.c
    if np.array(x).ndim < 2 :
        b[0] = b[0] + self.lr * x[0]*(y - self.sigmoid(x))
        b[1] = b[1] + self.lr * x[1]*(y - self.sigmoid(x))
        if not self.is_norm:
            c = c + self.lr * (y - self.sigmoid(x))

    else:
        n_row = np.array(x).shape[0]
        b[0] = b[0] + self.lr *(sum([x[i][0]*(y[i] -self.sigmoid(x[i]))
                        for i in range(n_row)])/float(n_row))
        b[1] = b[1] + self.lr *(sum([x[i][1]*(y[i] -self.sigmoid(x[i]))
                        for i in range(n_row)])/float(n_row))
        if not self.is_norm:
            c = c + self.lr *(sum([y[i] -self.sigmoid(x[i])
                            for i in range(n_row)])/float(n_row))
    self.b = b
    self.c = c
```

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K-fold

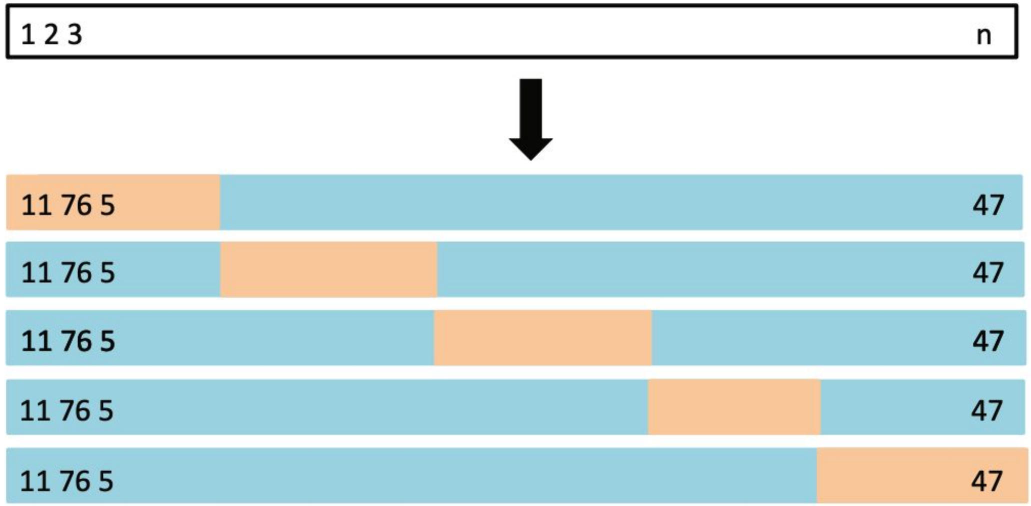

FIGURE 5.5. *A schematic display of 5-fold CV. A set of n observations is randomly split into five non-overlapping groups. Each of these fifths acts as a validation set (shown in beige), and the remainder as a training set (shown in blue). The test error is estimated by averaging the five resulting MSE estimates.*

or linear discriminant analysis, or any of the methods discussed in later chapters. The magic formula (5.2) does not hold in general, in which case the model has to be refit n times.

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Regularización

La suma de los cuadrados de los coeficientes Ridge y La suma de los

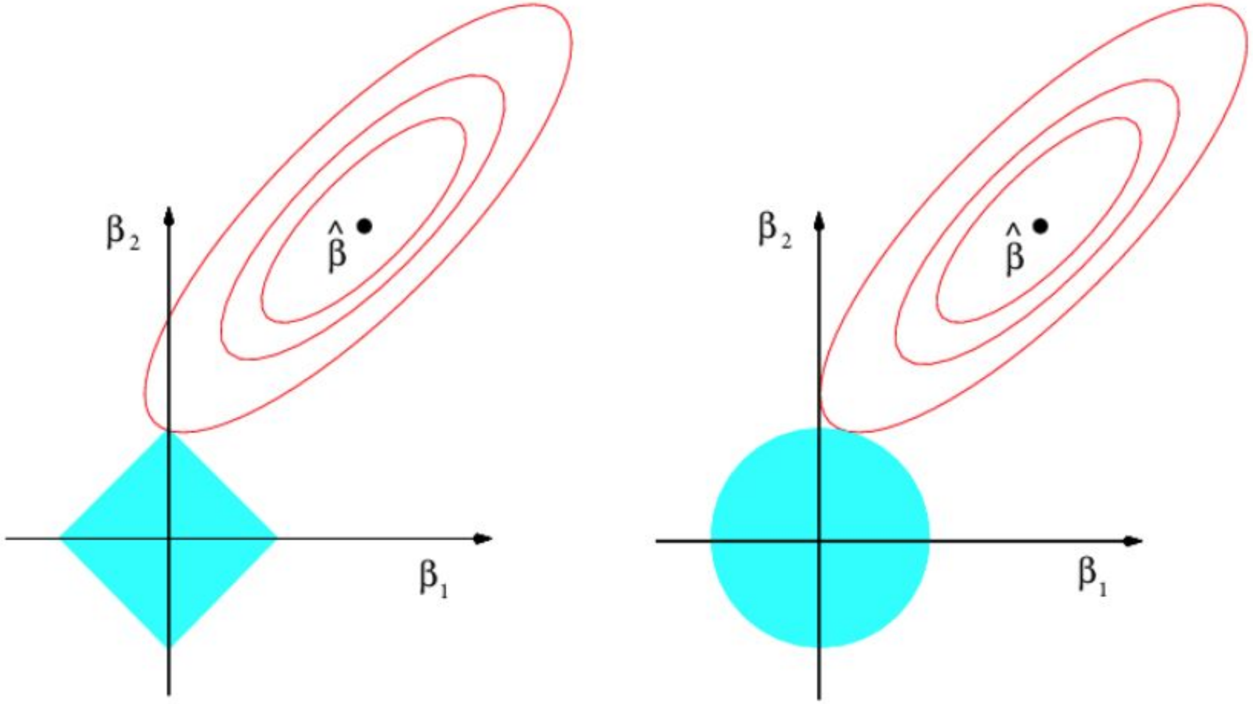

FIGURE 6.7. *Contours of the error and constraint functions for the lasso* (left) *and ridge regression* (right). *The solid blue areas are the constraint regions,* $|\beta_1| + |\beta_2| \leq s$ *and* $\beta_1^2 + \beta_2^2 \leq s$, *while the red ellipses are the contours of the RSS.*

Con ecuaciones simples

$$\underset{\beta}{\text{minimize}} \left\{ \sum_{i=1}^{n} \left(y_i - \beta_0 - \sum_{j=1}^{p} \beta_j x_{ij} \right)^2 \right\} \quad \text{subject to} \quad \sum_{j=1}^{p} |\beta_j| \leq s \tag{6.8}$$

and

$$\underset{\beta}{\text{minimize}} \left\{ \sum_{i=1}^{n} \left(y_i - \beta_0 - \sum_{j=1}^{p} \beta_j x_{ij} \right)^2 \right\} \quad \text{subject to} \quad \sum_{j=1}^{p} \beta_j^2 \leq s, \tag{6.9}$$

Forma matricial

$$\mathcal{L}(\boldsymbol{w}, b) = \sum_n \exp\left[-y_n(\boldsymbol{w} \cdot \boldsymbol{x}_n + b) \right] + \frac{\lambda}{2} ||\boldsymbol{w}||^2$$

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Feature Selection

Algorithm 6.2 *Forward stepwise selection*

1. Let $\mathcal{M}_0$ denote the *null* model, which contains no predictors.
2. For $k = 0, \ldots, p - 1$:
   (a) Consider all $p - k$ models that augment the predictors in $\mathcal{M}_k$ with one additional predictor.
   (b) Choose the *best* among these $p - k$ models, and call it $\mathcal{M}_{k+1}$. Here *best* is defined as having smallest RSS or highest R^2.
3. Select a single best model from among $\mathcal{M}_0, \ldots, \mathcal{M}_p$ using cross-validated prediction error, C_p (AIC), BIC, or adjusted R^2.

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Feature Selection

Algorithm 6.3 *Backward stepwise selection*

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1. Let $\mathcal{M}_p$ denote the *full* model, which contains all p predictors.

2. For $k = p, p-1, \ldots, 1$:

   (a) Consider all k models that contain all but one of the predictors in $\mathcal{M}_k$, for a total of $k-1$ predictors.

   (b) Choose the *best* among these k models, and call it $\mathcal{M}_{k-1}$. Here *best* is defined as having smallest RSS or highest R^2.

3. Select a single best model from among $\mathcal{M}_0, \ldots, \mathcal{M}_p$ using cross-validated prediction error, C_p (AIC), BIC, or adjusted R^2.

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