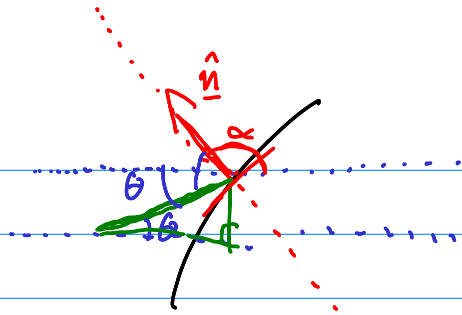
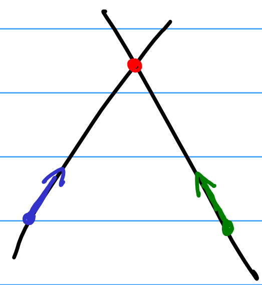


$$180^\circ - \alpha = \gamma - \theta$$
$$\gamma = 180^\circ + \theta - \gamma$$



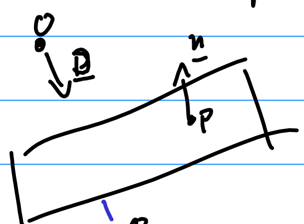
$$\underline{r}_1 + d_1 \underline{\hat{n}}_1 = \underline{r}_2 + d_2 \underline{\hat{n}}_2$$

Find d_1

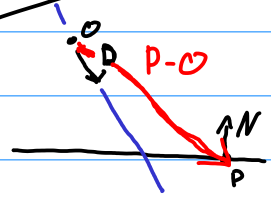
$$\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} + d_1 \begin{pmatrix} a_1 \\ b_1 \end{pmatrix} = \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} + d_2 \begin{pmatrix} a_2 \\ b_2 \end{pmatrix}$$

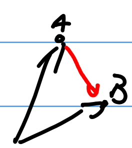
$$\begin{pmatrix} x_1 - x_2 \\ y_1 - y_2 \end{pmatrix} = d_2 \begin{pmatrix} a_2 \\ b_2 \end{pmatrix} - d_1 \begin{pmatrix} a_1 \\ b_1 \end{pmatrix}$$

Line and a plane



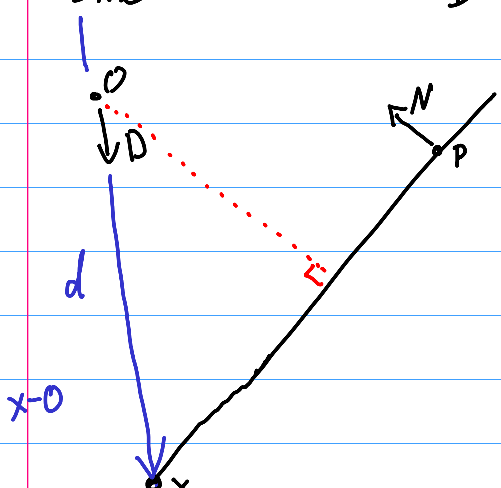
$$\frac{(\vec{P} - \vec{O}) \cdot \vec{N}}{\vec{D} \cdot \vec{N}}$$





B-A

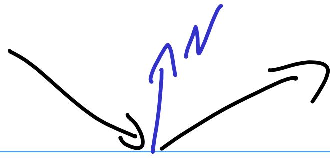
Line and a 2D 'plane'



$$(X - O) \cdot N = (P - O) \cdot N \leftarrow$$
$$X \cdot N = P \cdot N$$

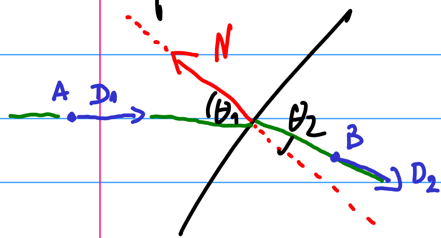
$$(d \times D) \cdot N = (P - O) N$$
$$d \; D \cdot N = (P - O) \cdot N$$
$$d = \frac{(P - O) \cdot N}{D \cdot N}$$

Reflection



$$\vec{D} - 2\vec{N}(\vec{N}\cdot\vec{D})$$

Refraction



$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

Refraction calculations (2006 document)

$$\vec{t} = \frac{n_1}{n_2}\vec{i} + \left(\frac{n_1}{n_2}\cos\theta_i - \sqrt{1-\sin^2\theta_t}\right)\vec{n}$$

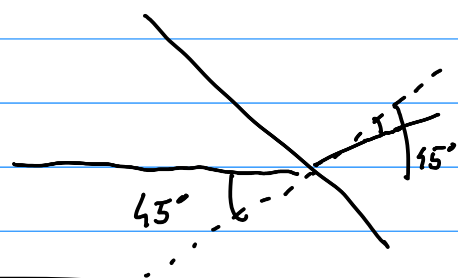
$$= \frac{n_1}{n_2}\vec{i} + \left(\frac{n_1}{n_2}\cos\theta_i - \sqrt{1-\left(\frac{n_1}{n_2}\right)^2\left(1-\cos^2\theta_i\right)}\right)\vec{n}$$

Snell:

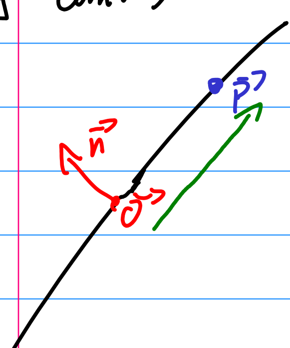
$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$$\sin\theta_2 = \frac{n_1}{n_2}\sin\theta_1$$

$$\theta_2 = \arcsin\left(\frac{n_1}{n_2}\sin\theta_1\right)$$



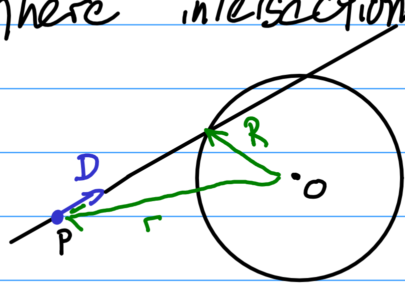
Ray canvas



$$|\vec{P}-\vec{O}|$$

Sphere intersections

ABCDEFGHIJKLMNOPQRSTUVWXYZ
abcdefghijklmnopqrstuvwxyz



$$\vec{r} + L\vec{D} = \vec{R}$$

$$|\vec{r} + L\vec{D}|^2 = R^2$$

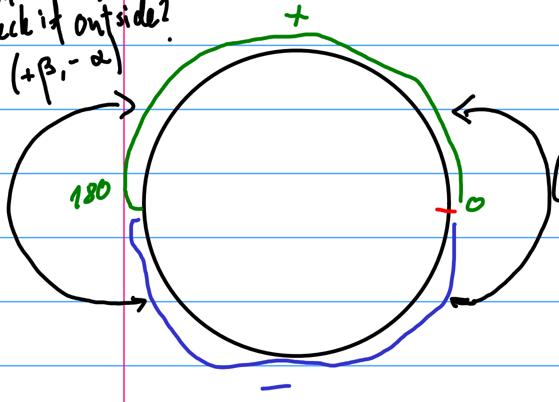
$$r^2 + L^2D^2 + 2L\vec{r}\cdot\vec{D} = R^2$$

$$L^2D^2 + L\cdot 2\vec{r}\cdot\vec{D} + r^2 - R^2 = 0$$

$$c = \frac{-\cancel{2}\vec{r}\cdot\vec{D} \pm \sqrt{\cancel{4}|\vec{r}\cdot\vec{D}|^2 - \cancel{4}D^2(r^2-R^2)}}{\cancel{2}\cancel{D^2}} = -\vec{r}\cdot\vec{D} \pm \sqrt{|\vec{r}\cdot\vec{D}|^2 - (r^2-R^2)}$$

Angles in a circle

can check if outside?
$(+\beta, -\alpha)$



$(-\alpha, +\beta)$
can just check if in the range

Lensmaker example

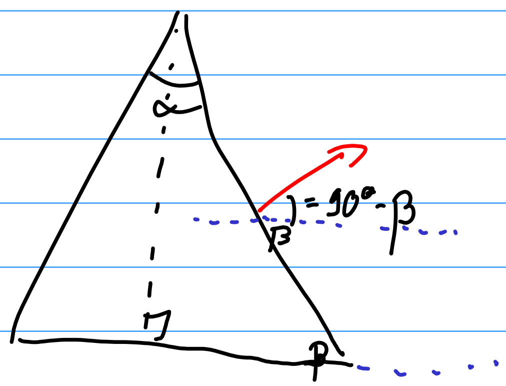
$R_1 = 1,\quad R_2 = -1,\quad d = 1,\quad n = 2$

$$\frac{1}{f} = (2-1)\left(\frac{1}{1} + \frac{1}{1} + \frac{(2-1)\cdot 1}{2\times 1\times(-1)}\right)$$

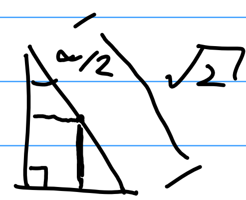
$$= 2 + \frac{1}{-2} = \frac{4}{2} - \frac{1}{2} = \frac{3}{2}$$

$$f = 2/3$$

Prism



$$\beta = 180° - 90° - \frac{\alpha}{2}$$
$$= 90° - \frac{\alpha}{2}$$
$$90° - \beta = \alpha/2$$



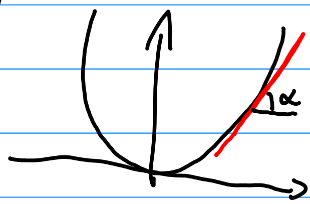
Photon momentum

$k = \frac{n\omega}{c}$ in vacuum $k_v = \frac{\omega}{c} = \frac{2\pi}{\lambda_v}$

$k = n k_v$

$p = \hbar k = \hbar n k_v = \frac{2\pi \hbar n}{\lambda_v} = \frac{hn}{\lambda_v}$

Parabola



$y = ax^2 + bx + c$

$y' = 2ax + b$

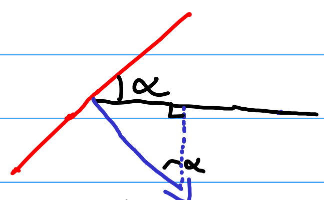
$\tan\alpha = y' = 2ax + b$

$\frac{\sin\alpha}{\cos\alpha} = y' = 2ax + b$

$\sin\alpha = (2ax+b)\cos\alpha$

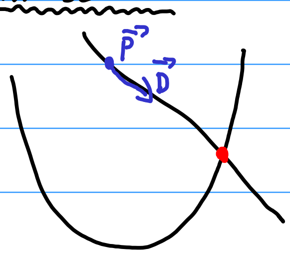
$\sin^2\alpha = (2ax+b)\cos^2\alpha$

$\sin^2\alpha = (2ax+b)(1 - \sin^2\alpha)$



$(\sin\alpha, -\cos\alpha)$

Intersection



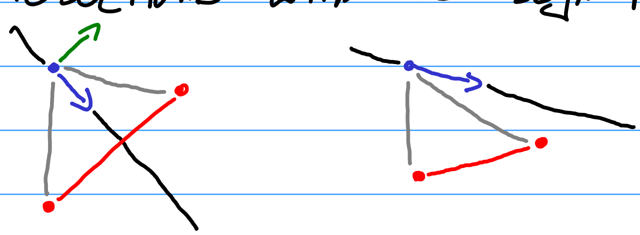
$$\begin{cases} ax^2 + bx + c = P_y + t D_y \\ x = P_x + t D_x \end{cases}$$

$$a(P_x + tD_x)^2 + b(P_x + tD_x) + c = P_y + tD_y$$

Special case: $D_x = 0$, eq. no longer quadratic:

$$aP_x^2 + bP_x + c = P_y + tD_y$$

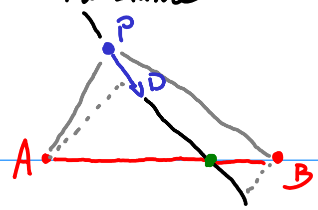
Intersections with line segments



Cross products have neg. signs for both PA and PB

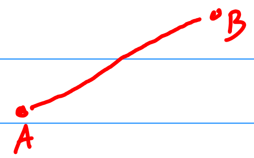
$\longrightarrow$ there's an intersection

Now, distance

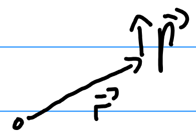


Actually distance calculation can be done just as for planes

Normal to a segment



Angular momentum in 2D



$$\vec{r} \times \vec{p} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ r_1 & r_2 & r_3 \\ p_1 & p_2 & p_3 \end{vmatrix} = \begin{pmatrix} \\ \\ r_1 p_2 - r_2 p_1 \end{pmatrix}$$

$$x p_y - y p_x$$

Gaussian beam

$$I(r,z) = I_0 \left(\frac{w_0}{w(z)}\right)^2 \exp\left(\frac{-2r^2}{w(z)^2}\right)$$

P_0 - total power

$$= 2\frac{P_0}{\pi \cancel{w_0^2}} \frac{\cancel{w_0^2}}{w^2} \exp\left(\frac{-2r^2}{w^2}\right)$$

$$= \frac{2P_0}{\pi w^2} \exp\left(\frac{-2r^2}{w^2}\right) = P(y)$$

Um, actually. This should be normalised

$$a = \frac{2}{w^2}, \quad \frac{2P_0}{\pi w^2}\sqrt{\frac{\pi w^2}{2}} = \frac{P_0}{\sqrt{2\pi}\, w}$$

$$F(r) = \int_0^r P(\tilde{r})\, d\tilde{r} = \frac{2P_0}{\pi w^2} \int_0^r \exp\left(\frac{-2\tilde{r}^2}{w^2}\right) d\tilde{r}$$

$$= \frac{2P_0}{\pi w^2} \sqrt{\frac{\pi}{4c}}\, \mathrm{erf}(\sqrt{c}\, r) \qquad // \; c = \frac{2}{w^2}$$

$$= \frac{2P_0}{\pi w^2} \sqrt{\frac{\pi w^2}{8}}\, \mathrm{erf}\left(\frac{\sqrt{2}\, r}{w}\right) = \frac{P_0}{w} \sqrt{\frac{1}{2\pi}}\, \mathrm{erf}\left(\frac{\sqrt{2}\, r}{w}\right)$$

$$= x$$

$$x = F(r), \quad r = F^{-1}(x)$$

$$\frac{\sqrt{2\pi}\, w}{P_0} x = \mathrm{erf}\left(\frac{\sqrt{2}\, r}{w}\right)$$

$$\frac{\sqrt{2}}{w} r = \mathrm{erf}^{-1}\left(\frac{\sqrt{2\pi}}{P_0} x\right)$$

$$\cancel{x = \frac{P_0}{\sqrt{2\pi}\, w}} : \cancel{\frac{P_0}{\sqrt{2\pi}}}\, w\, \mathrm{erf}\left(\frac{\sqrt{2}\, r}{w}\right)$$

$$x = \mathrm{erf}\left(\frac{\sqrt{2}\, r}{w}\right)$$

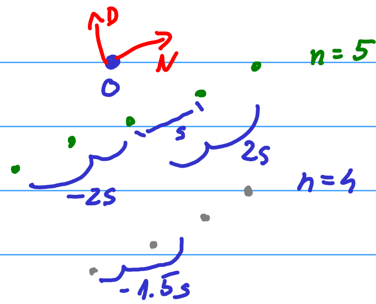
$$r = \frac{w}{\sqrt{2}}\, \mathrm{erf}(x)$$

Correctly normalised

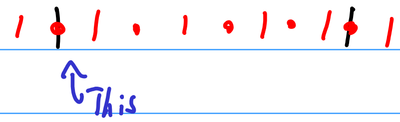
$$\boxed{\Gamma = \frac{w}{\sqrt{2}}\,\mathrm{erf}^{-1}\left(\frac{\sqrt{2\pi}}{P_0}x\right)}$$

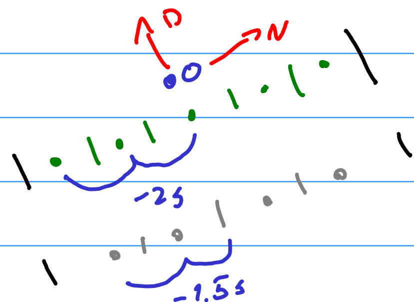
Test!

Ray distribution



Change of plans - if each ray has a 'surrounding area' it's these that should cover the given 'radius'

so  instead of 



Re-normalising the Gaussian beam for 2D

$$I = \frac{2P_0}{\pi w^2}\exp\left(\frac{-2r^2}{w^2}\right)$$

$$\iint \frac{2P_0}{\pi w^2}\, r \exp\left(\frac{-2r^2}{w^2}\right) dr\, d\theta$$

$$= 2\pi \int \frac{2P_0}{\pi w^2}\, r \exp\left(\frac{-2r^2}{w^2}\right) dr$$

$a = 2/w^2$

$$\int_{-\infty}^{\infty} \frac{2P_0}{\pi w^2}\exp\left(\frac{-2r^2}{w^2}\right) = \frac{2P_0}{\pi w^2}\sqrt{\frac{\pi}{a}} = \frac{2P_0}{\pi w^2}\sqrt{\frac{\pi w^2}{2}}$$

$$= \sqrt{2\pi}\,\frac{1}{w}\,P_0 \;/\; \times \frac{w}{\sqrt{2\pi}}$$

$$\frac{2P_0}{\pi w^2} \times \frac{w}{\sqrt{2\pi}} : \; = \frac{\sqrt{2}\,P_0}{\pi^{3/2} w} = \underline{\sqrt{\frac{2}{\pi^3}}\,\frac{P_0}{w}}$$

$$\int \exp\left(\frac{-2r^2}{w^2}\right) = \sqrt{\frac{\pi w^2}{2}} = \sqrt{\frac{\pi}{2}}\,w$$

R and T coefficients

$\cos\theta_i$ known already

$$\cos\theta_t = \sqrt{1-\sin^2\theta_t} = \sqrt{1-\left(\frac{n_1}{n_2}\right)^2(1-\cos^2\theta_i)}$$

$R_\perp + R_\parallel =$

$$= \frac{(n_1\cos\theta_i - n_2\cos\theta_t)^2}{(n_1\cos\theta_i + n_2\cos\theta_t)^2} + \frac{(n_2\cos\theta_i - n_1\cos\theta_t)^2}{(n_2\cos\theta_i + n_1\cos\theta_t)^2}$$

No obvious simplification

Total momentum transfer for a simple mirror

For a single photon

$$p = \hbar k = \frac{h}{2\pi}\frac{2\pi}{\lambda} = h/\lambda \qquad E = pc$$

In general

$$\frac{dE}{dt} = \frac{dp}{dt}c \qquad P = Fc \qquad F = P/c$$

But say the light gets reflected

$$F = 2P/c$$

Differential equations

$$F = ma \qquad F = m\frac{dv}{dt} \qquad v = \frac{dx}{dt}$$

$$\begin{cases} \frac{dx_i}{dt} = v_i \\ \frac{dv_i}{dt} = \frac{F}{m} \end{cases}$$

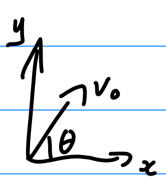
This is given each step by a function

$$F = \frac{dp}{dt}$$

Euler method

$$y_{n+1} = y_n + h\underline{f(t_n, y_n)}$$

Projectile motion



$$\begin{cases} v_x = v_0\cos\theta \\ v_y = v_0\sin\theta - gt \end{cases} \qquad x = v_x t \qquad y = v_0\sin\theta\, t - \frac{gt^2}{2}$$

Projectile motion + drag

https://demonstrations.wolfram.com/ProjectileWithAirDrag

$$F = ma$$

$$F_x = m\ddot{x} = -\alpha\dot{x}$$

$$F_y = m\ddot{y} = -mg - \alpha\dot{y}$$

$$\begin{cases} \dfrac{d^2x}{dt^2} = -\dfrac{\alpha}{m}\dfrac{dx}{dt} \\ \dfrac{d^2y}{dt^2} = -g - \dfrac{\alpha}{m}\dfrac{dy}{dt} \end{cases}$$

$$\cancel{\begin{cases} \dfrac{dx}{dt} = -\dfrac{\alpha}{m}x + v_{0x} \\ \dfrac{dy}{dt} = -gt - \dfrac{\alpha}{m}y + v_{0y} \end{cases}}$$

Ansatz : $x = A + Be^{-\beta x}$

$$\dot{x} = -\beta Be^{-\beta x}$$

$$\ddot{x} = \beta^2 Be^{-\beta x}$$

$$\dot{x} = -\frac{\alpha}{m}B = v_{0x}$$

$$B = -\frac{mv_{0x}}{\alpha}$$

$$\ddot{x} = -\frac{\alpha}{m}\dot{x}$$

$$\beta^2 \cancel{Be^{-\beta x}} = +\beta \cancel{Be^{-\beta x}}\frac{\alpha}{m}$$

$$\beta = \frac{\alpha}{m}$$

All the exponents should be $e^{-\beta t}$

$$x = A - \frac{mv_{0x}}{\alpha}e^{-\alpha x/m}$$

at $x=0$: $A - \dfrac{mv_{0x}}{\alpha} = 0$

$$A = \frac{mv_{0x}}{\alpha}$$

$$\boxed{x = \frac{mv_{0x}}{\alpha}\left(1 - e^{-\alpha x/m}\right)}$$

Ansatz $y = A + Be^{-\beta x} + Ct$

$$\dot{y} = -\beta Be^{-\beta x} + C$$

$$\ddot{y} = +\beta^2 Be^{-\beta x}$$

$$\dot{y}(0) = \frac{\alpha}{m}B - \frac{mg}{\alpha} = v_0$$

$$\boxed{B = -\frac{m}{\alpha}\left(v_0 + \frac{mg}{\alpha}\right)}$$

$$\ddot{y} = -g - \frac{\alpha}{m}\dot{y}$$

$$\beta^2 Be^{-\beta x} = -g + \frac{\alpha}{m}\beta Be^{-\beta x} - \frac{\alpha}{m}C$$

$$\boxed{C = -\frac{mg}{\alpha}} \qquad \boxed{\beta = \frac{\alpha}{m}}$$

$$y(0) = A + \frac{m}{\alpha}\left(v_0 + \frac{mg}{\alpha}\right) = 0$$

$$A = -B$$

$$y = -\frac{m}{\alpha}\left(v_0 + \frac{mg}{\alpha}\right)\left(e^{-\alpha x/m} - 1\right) - \frac{mg}{\alpha}t$$

$$\boxed{y = \frac{m}{\alpha}\left(v_0 + \frac{mg}{\alpha}\right)\left(1 - e^{-\alpha x/m}\right) - \frac{mg}{\alpha}t}$$

Damped harmonic oscillator

https://brilliant.org/wiki/damped-harmonic-oscillators/

general solution:

$$x(t) = e^{-\frac{c}{2}t}\left(A\cos\left(\sqrt{1-\tfrac{c^2}{4}}\,t\right) + B\sin\left(\sqrt{1-\tfrac{c^2}{4}}\,t\right)\right)$$

$$x(0) = A = 1$$

$$x' = A\cdot\left(-\frac{c}{2}\right)e^{-\frac{c}{2}t}\cos\left(\sqrt{1-\tfrac{c^2}{4}}\,t\right)$$

$$- Ae^{-\frac{c}{2}t}\sin\left(\sqrt{1-\tfrac{c^2}{4}}\,t\right)\sqrt{1-\tfrac{c^2}{4}}$$

$$- \frac{c}{2}Be^{-\frac{c}{2}t}\sin(\ldots)$$

$$+ Be^{-c/2\,t}\cos\left(\sqrt{1-\tfrac{c^2}{4}}\,t\right)\sqrt{1-\tfrac{c^2}{4}}\,t$$

$$x'(0) = -\frac{c}{2}A + \sqrt{1-\tfrac{c^2}{4}}\,B = 0$$

$A := 1,50$

$$\sqrt{1-\frac{c^2}{4}}\,B = \frac{c}{2}$$

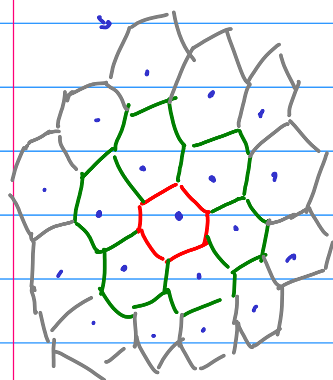
$$B = \frac{c}{2\sqrt{1-\frac{c^2}{4}}}$$

Gaussian beam width

$$w = w_0\sqrt{1+\left(\frac{z}{z_R}\right)^2}$$

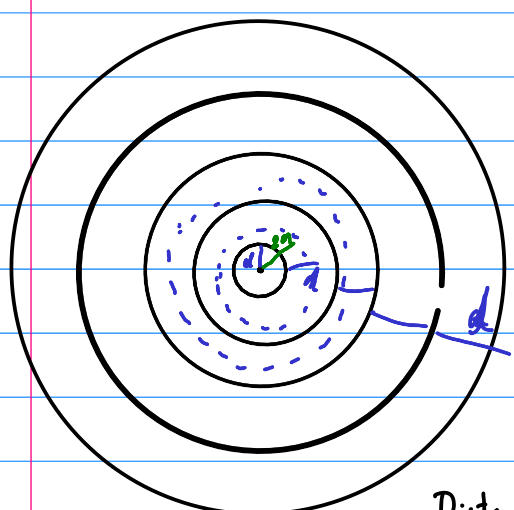
$$= w_0\sqrt{1+\left(\frac{z\lambda}{\pi w_0^2 n}\right)^2}$$

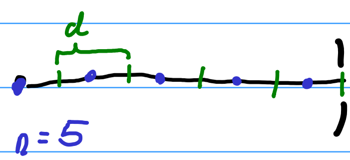
2D ray distribution



1, 6, 12

Circle dividing





$R = (n-\frac{1}{2})d$, $d = \frac{R}{n-1/2}$

$L = 2\pi r = 2\pi(nd)$

So, total length is: $2\pi d + 2\pi(2d) + 2\pi(3d) + 2\pi(4d) =$

$= 2\pi d(1+2+3+4)$

$= 2\pi d\frac{n(n-1)}{2} = \pi d n(n-1) \approx 20\pi d$ in this example

Distance between points should be equal

$\pi d n \frac{n-1}{N-1}$

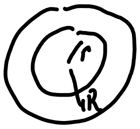
Say we aim for this to be equal to d — N is given

$\pi \cancel{d} n\frac{n-1}{N-1} = \cancel{d}$ $\quad n\frac{n-1}{N-1} = \frac{1}{\pi}$

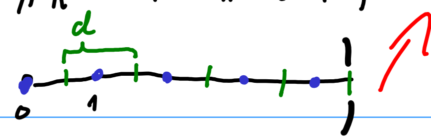
$n(n-1) = \frac{N-1}{\pi}$

$n^2 - n - \frac{N-1}{\pi} = 0$ $\quad n = \frac{1 \pm \sqrt{1+\frac{4(N-1)}{\pi}}}{2}$ reject negative solution

$n = \frac{1}{2} + \sqrt{1+\frac{4(N-1)}{\pi}}/2$



$\pi R^2 - \pi r^2 = \pi(R^2 - r^2)$



$R = (n + \frac{1}{2})d$

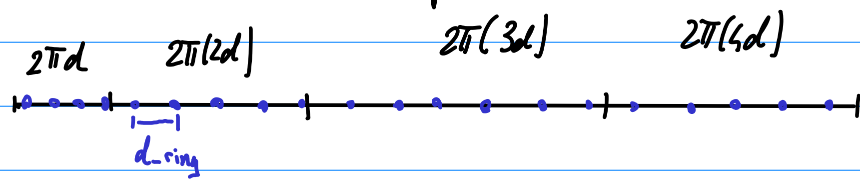
$r = (n - \frac{1}{2})d$

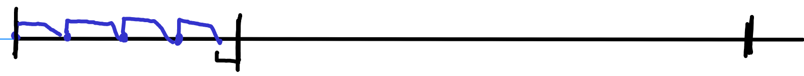
$$A = \pi d^2\left((n+\tfrac{1}{2})^2 - (n-\tfrac{1}{2})^2\right)$$
$$= \pi d^2\left((n+\tfrac{1}{2}+n-\tfrac{1}{2})(n+\tfrac{1}{2}-n+\tfrac{1}{2})\right)$$
$$= \pi d^2 \times 2n = \underline{2n\pi d^2}$$

$n^2 + n + \cancel{\tfrac{1}{4}} - (n^2 - n + \cancel{\tfrac{1}{4}})$

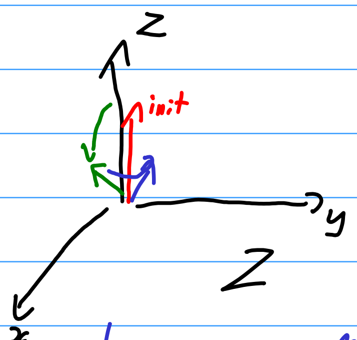
$= 2n$

How do we ensure correct point distribution?





3D Rotations



X

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix}$$

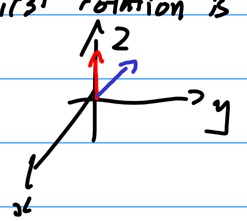
Z

$$\begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

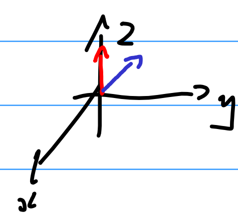
Y

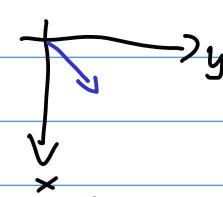
$$\begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix}$$

First rotation is around y

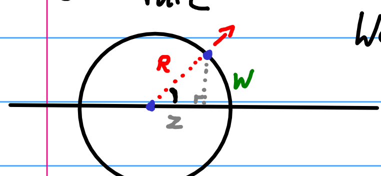


Then rotate it along z





Curvature



We know z and R.

Correction: we don't actually know z. We know w. z from Pythagoreas?

We'd like to calculate the normal vector at that point.

$\cos\theta = \frac{z}{R}$

$\sin\theta = \frac{w}{R}$

$$\frac{1}{R} = \frac{z}{z^2+z_R^2} \qquad z_R = \frac{\pi w_0^2 (n)}{\lambda}$$

$$z^2 + w^2 = R^2$$
$$z = \sqrt{R^2 - w^2}$$

$$\cos\theta = \frac{\sqrt{R^2-w^2}}{R} = \sqrt{1-\left(\frac{w}{R}\right)^2} = \sqrt{1-\sin\theta}$$ of course...

So, we can easily compute $\sin\theta$, and from there $\cos\theta$.

$$\sin\theta = \frac{w}{R} = \frac{wz}{z^2+z_R^2}$$

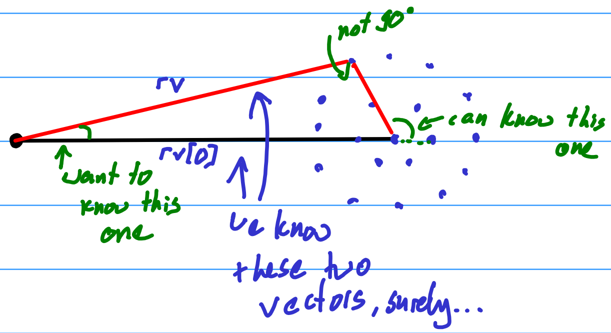
Rename $w \to r$ to avoid confusion

$$\sin\theta = \underline{\frac{rz}{z^2+z_R^2}} = \frac{rz}{z^2 + \frac{\pi^2 w_0^4}{\lambda^2}}$$

$$\begin{pmatrix} \cos & -\sin & 0 \\ \sin & \cos & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} a \\ 0 \\ b \end{pmatrix} = \begin{pmatrix} a\cos\varphi \\ a\sin\varphi \\ b \end{pmatrix}$$

$$a = \frac{rz}{z^2+z_R^2}$$
$$b = \sqrt{1-a^2}$$

Getting φ:



Ray-triangle intersection

Two alternative forms

$$\begin{pmatrix} t \\ u \\ v \end{pmatrix} = \frac{1}{(D\times E_2)\cdot E_1} \begin{pmatrix} (T\times E_1)\cdot E_2 \\ (D\times E_2)\cdot T \\ (T\times E_1)\cdot D \end{pmatrix}$$

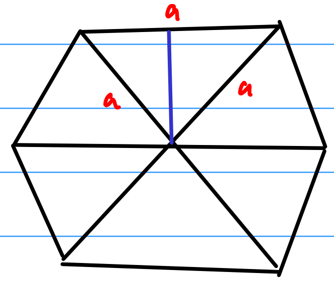
$$\begin{pmatrix} t \\ u \\ v \end{pmatrix} = \frac{1}{P\cdot E_1} \begin{pmatrix} Q\cdot E_2 \\ P\cdot T \\ Q\cdot D \end{pmatrix}$$

$P = D\times E_2$, $Q = T\times E_1$

ray direction

$T = O - V_0$

Hexagonal ray distribution

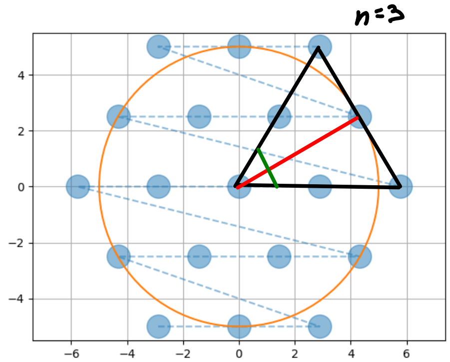


$$a^2 = \left(\frac{a}{2}\right)^2 + r^2$$

$$r^2 = a^2 - a^2/4$$

$$r^2 = \frac{3}{4}a^2$$

$$\boxed{r = \sqrt{3}\,\frac{a}{2}} \qquad a = \frac{2r}{\sqrt{3}}$$



$$(n-1)b = \frac{2r}{\sqrt{3}}$$

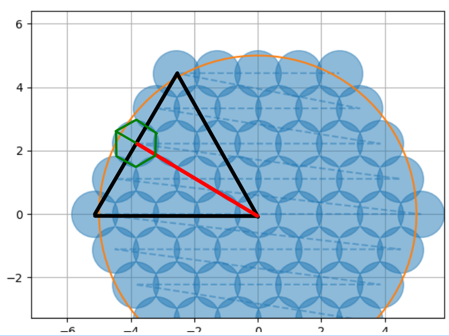
$$b = \frac{2r}{\sqrt{3}}\,\frac{1}{n-1}$$

Say we want to keep the rays inside

$$\frac{\sqrt{3}\,b}{2}(n-1) = r - \frac{1}{2}\cdot\frac{\sqrt{3}}{2}b$$

$$r = \frac{\sqrt{3}}{2}b\left(n-1+\frac{1}{2}\right)$$

$$r = \frac{\sqrt{3}}{2}b\left(n-1/2\right) \qquad b = \frac{2}{\sqrt{3}}r\,\frac{1}{n-1/2}$$



$$\frac{b}{2} = \frac{\sqrt{3}}{2}x \qquad x = b/\sqrt{3}$$

$$\frac{\sqrt{3}}{2}b(n-1) = r - \frac{b}{\sqrt{3}} = r - \frac{b\sqrt{3}}{3}$$

$$r = \sqrt{3}\,b\left(\frac{n}{2} - \frac{1}{2} + \frac{1}{3}\right) = \sqrt{3}\,b\left(\frac{n}{2} - \frac{1}{6}\right)$$

$$= \frac{\sqrt{3}\,b}{2}\left(n - 1/3\right)$$

Hexagonal number

$$3n(n-1) + 1 = N$$

$$3n^2 - 3n + 1 - N = 0$$

$$n = \frac{3 \pm \sqrt{9 - 12(1-N)}}{6} \quad \text{Choose pos. sol.}$$

$$n = \frac{3 + \sqrt{12N - 3}}{6}$$

Hexagon area

$$6 \times \frac{1}{2}a \times \frac{\sqrt{3}}{2}a = \frac{6}{4}\sqrt{3}\,a^2 = \frac{3}{2}\sqrt{3}\,a^2$$

A unitcell

height: $\frac{b}{2}$

$$\frac{b}{2} = \frac{\sqrt{3}\,a}{2} \qquad a = \frac{b}{\sqrt{3}}$$

area: $6 \times \frac{1}{2} \times \frac{b}{2} \times \frac{b}{\sqrt{3}} = \frac{\sqrt{3}\,b^2}{2}$

3D Rotations

$\frac{d\vec{L}}{dt} = \vec{\tau}$, analogous to $\frac{d\vec{p}}{dt} = \vec{F}$

$\vec{L} = \underline{\underline{I}}\,\vec{\omega}$, or (Einstein): $L_i = I_{ij}\,\omega^j$

The equations we have been using so far:

$\dot{x} = v_x$
$\dot{y} = v_y$
$\dot{z} = v_z$
$\dot{v}_x = F_x/m$
$\dot{v}_y = F_y/m$
$\dot{v}_z = F_z/m$

These come from $F=ma$, or (more precisely) $F = m\dot{v}$, which uses the fact that m is constant

Euler's equations

In the rotating frame

$$\begin{cases} I_1\dot{\omega}_1 + (I_3 - I_2)\,\omega_2\omega_3 = M_1 \\ I_2\dot{\omega}_2 + (I_1 - I_3)\,\omega_3\omega_1 = M_2 \\ I_3\dot{\omega}_3 + (I_2 - I_1)\,\omega_1\omega_2 = M_3 \end{cases}$$

Store orientation as a quaternion,
and $\vec{\omega}$ as a vector?

possibly even in the object's coords?
Then we only need to rotate $\vec{M}$.

Then we just need the rate of change for the quaternion - did we just arrive at the paper's ODEs?

I think we might have...

$\begin{pmatrix} 0 \\ \omega \end{pmatrix} = q \begin{pmatrix} 0 \\ W \end{pmatrix} q^{-1}$, $\begin{pmatrix} 0 \\ W \end{pmatrix} = q^{-1} \begin{pmatrix} 0 \\ \omega \end{pmatrix} q$

I in body frame

$\dot{\omega}$ in body frame

$$J\,\dot{W} = M - W \wedge J\,W,$$

this is the same as the equation above

$$\dot{q} = \frac{dq}{dt} = \frac{1}{2} q \otimes \begin{pmatrix} 0 \\ W \end{pmatrix}.$$