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PID autotuning for weighted servo/regulation control operation

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ABSTRACT

This paper analyzes optimal controller settings for controllers with One-Degree-of-Freedom (1-DoF) Proportional-Integral-Derivative (PID) structure. A new analysis is conducted from the point of view of the *operating mode* (either servo or regulation mode) of the control-loop and *tuning mode* of the controller. Performance of the optimal tuning settings can be degraded when the operating mode is different from that selected for tuning and obviously both situations can be present in any control system. In this context, a Weighted Performance Degradation index, that considers the importance and balance between the servo and regulation operation modes, is minimized and based on this minimization, an autotuning procedure as a function of the normalized process dead-time is proposed.

1. Introduction

Proportional-Integrative-Derivative (PID) controllers are with no doubt the most extensive option that can be found on industrial control applications [1]. Their success is mainly due to its simple structure and to the physical meaning of the corresponding three parameters (therefore making manual tuning possible). This fact makes PID control easier to understand by the control engineers than other most advanced control techniques. In addition, the PID controller provides satisfactory performance in a wide range of practical situations.

During the last years, in fact since the initial work of Ziegler and Nichols [2], much work has been done developing methods to determine the PID controller parameters (see for example [3–5]). O'Dwyer [6] presents a collection of tuning rules for PID controllers, which show their abundance.

Within the wide range of approaches to autotuning, optimal methods have received special interest. These methods provide, given a simple model process description – such as a First-Order-Plus-Dead-Time (FOPDT) model- settings for optimal closed-loop responses [7].

For One-Degree-of-Freedom (1-DoF) controllers, it is usual to relate the tuning method to the expected operation mode for the control system, known as *servo* or *regulation*. Therefore, controller settings can be found for optimal set-point or load-disturbance responses. This fact allows better performance of the controller when the control system operates on the selected tuned mode but, a degradation in the performance is expected when the tuning and operation modes are different. Obviously there is always the need to choose one of the two possible ways to tune the controller, for set-point tracking or load-disturbances rejection. In the case of 1-DoF PID, tuning can be optimal just for one of the two operation modes. The main problem, about the Performance Degradation analysis for both tuning modes, was previously formulated in [8] and some approaches related to tuning methods and autotuning have been proposed in [9,10].

What is provided in this paper is a continuation of the these ideas in order to find an *intermediate* tuning for the controller that improves the overall performance of the system, considered as a *trade-off* between servo and regulation operation modes. The settings are determined from the combination of the optimal ones for set-point and load-disturbance, presented in [7], and taking into account the balance between the importance of each one of the operation modes for the control system (servo or regulation). The optimization is here performed using genetic algorithms [11].

The proposed new method considers a 1-DoF PID controller as an alternative when an *explicit* 2-DoF PID controller is not available. It should be remembered that for the Two-Degree-of-Freedom (2-DoF) PID controller, tuning is usually optimal for regulation operation and suboptimal for servo-control, where this suboptimal behavior is achieved using a set-point weighting factor as an extra tuning parameter that gives the second Degree-of-Freedom, to

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improve the tracking action. Also, sometimes is not strictly necessary, or not justified, to increase the number of the tuning parameters in contrast to the benefits that could be obtained. It could be stated that the proposed *intermediate* tuning is a particular case that results in a suboptimal tuning, when both operation modes may happen.

The paper is organized as follows. Next section introduces the general problem formulation, with some related concepts. Section 3 presents the *intermediate* tuning between the parameters of both operation modes in such a way that a Weighted Performance Degradation (*WPD*) is minimized; the results are generalized in terms of an autotuning procedure that is presented in Section 4. Some examples are shown in Section 5 and the conclusions are drawn in Section 6.

2. Problem formulation

2.1. Control system configuration

We consider the unity-feedback system shown in Fig. 1, where P is the process and K is the (1-DoF PID) controller.

The variables of interest can be described as follows:

- y is the process output (controlled variable).
- u is the control signal.
- r is the set-point for the process output.
- d is the disturbance of the system.
- e is the control error $e = r - y$.

Also, the process P is assumed to be modelled by a FOPDT transfer function of the form

$$P(s) = \frac{K}{1+Ts}\mathrm{e}^{-Ls}, \tag{1}$$

where K is the process gain, T is the time constant and L is the dead-time. This model is commonly used in process control because is simple and describes the dynamics of many industrial processes approximately [12].

The availability of FOPDT models in the process industry is a well known fact. The generation of such model just needs for a very simple step-test experiment to be applied to the process. This can be considered as an advantage with respect to other methods that need a more *plant demanding* experiment such as methods based on more complex models or even data-driven methods where a sufficiently rich input needs to be applied to the plant. From this point of view, to maintain the need for plant experimentation to a minimum is a key point when considering industrial application of a technique.

In this context, a common characterization of the process parameters is done in terms of the normalized dead-time $\tau = L/T$ [13]. On the other hand, the ideal 1-DoF PID controller with derivative time filter is considered

$$K(s) = K_p\left(1 + \frac{1}{T_i s} + \frac{T_d s}{1+(T_d/N)s}\right), \tag{2}$$

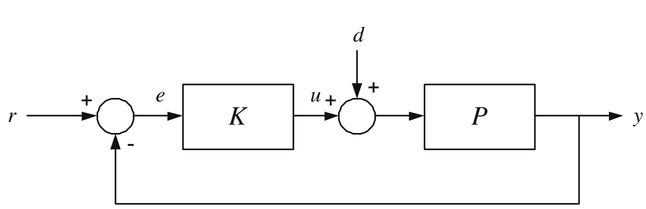

Fig. 1. The considered feedback control system.

where K_p is the proportional gain, T_i is the integral time constant and T_d is the derivative time constant. The derivative time noise filter constant N usually takes values within the range 5–33 [12,13]. Without loss of generality, here we will consider $N = 20$ [7].

2.2. Servo and regulation operation modes

Considering the closed-loop system of Fig. 1 the process output is given by

$$y(s) = \frac{K(s)P(s)}{1+K(s)P(s)}r(s) + \frac{P(s)}{1+K(s)P(s)}d(s). \tag{3}$$

The process output y depends of its two input signals, r and d and from that, the system can operate in two different modes, known as *servo control* or *regulatory control*. In the first case, the control objective is to provide a good tracking of the signal reference r, whereas in the second case is to maintain the output variable at the desired value, despite possible disturbances in d.

For the design of the control system, both operation modes must be considered, however depending on the controller structure (e.g. 1-DoF PID), it is not always possible to specify different performance behaviors for changes in the set-point and load-disturbances.

For the servo operation mode, disturbances are not considered ($d(s) = 0$), then (3) takes the form

$$y_{sp}(s) := \frac{K(s)P(s)}{1+K(s)P(s)}r(s). \tag{4}$$

For regulation operation mode, no changes in the set-point reference are supposed (e.g. $r(s) = 0$), then, process output would be

$$y_{ld}(s) := \frac{P(s)}{1+K(s)P(s)}d(s). \tag{5}$$

2.3. Set-point and load-disturbance tuning modes

Controller tuning is one of the most important aspects in control systems. For the selection of this, it is necessary to take into account some aspects like: the controller structure, the information that is available for the process and the specifications that the output has to fulfill.

The analysis presented in this work is focused on the Integral Square Error (ISE) criteria, $J = \int_0^\infty e(t)^2 dt$, which is one of the most well known and most often used [14], however, the general analysis could be developed in terms of any other performance criterion.

When the settings for optimal set-point (servo control) response are considered, the controller parameters are adjusted according to the following formulae [7]

$$K_p = \frac{a_1}{K}(\tau)^{b_1}, \quad T_i = \frac{T}{a_2 + b_2\tau}, \quad T_d = a_3 T(\tau)^{b_3} \tag{6}$$

and for the optimal load-disturbance (regulatory control) response

$$K_p = \frac{a_1}{K}(\tau)^{b_1}, \qquad \frac{1}{T_i} = \frac{a_2}{T}(\tau)^{b_2}, \qquad T_d = a_3 T(\tau)^{b_3}, \tag{7}$$

where the corresponding values of a_i and b_i given in Table 1.

2.4. Problem statement

If the control-loop has always to operate on one of the two possible operation modes (servo or regulator) the tuning choice will be clear. However, when both situations occur, it may not be so evident which are the most appropriate controller settings.

The analysis to answer the problem, presented previously as an initial stage in [8,9], concentrates on the Performance Degradation index which provides a quantitative evaluation of the controller

Table 1
Optimal PID settings for set-point (sp) and load-disturbance (ld).

τ Range	0.1–1.0		1.1–2.0	
Tuning	SP	LD	SP	LD
a_1	1.048	1.473	1.154	1.524
b_1	−0.897	−0.970	−0.567	−0.735
a_2	1.195	1.115	1.047	1.130
b_2	−0.368	−0.753	−0.220	−0.641
a_3	0.489	0.550	0.490	0.552
b_3	0.888	0.948	0.708	0.851

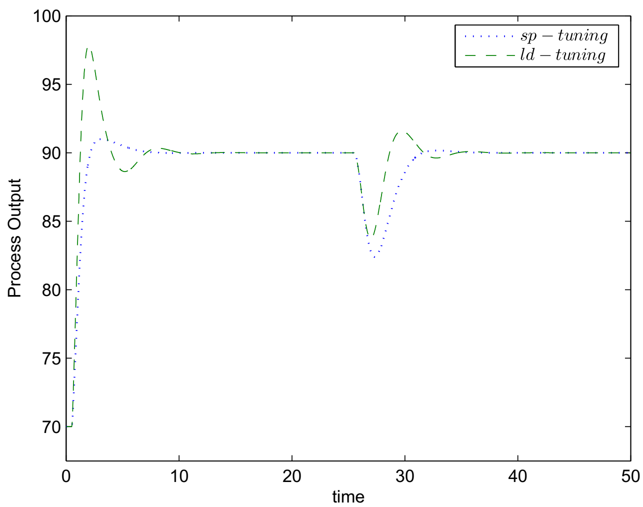

Fig. 2. Performance of the set-point (dotted) and load-disturbance (dashed) settings operating in both servo and regulation modes.

settings with respect to the operation mode and the main objective is to reduce it.

Here, the question "*How to improve the performance when the system operates also in a different mode that it was tuned for?*" is treated by searching an *intermediate* tuning for the controller, between both optimal parameters settings for set-point and load-disturbance, in order to reduce the global Weighted Performance Degradation index.

Also, the selection of the servo/regulation *trade-off* tuning can be made to achieve a balanced performance behavior between the operation modes.

2.5. Motivation example

In order to show the performance of the previously presented settings and how this can degrade when the controller is not operating according to the tuned mode, an example is provided. This motivates the analysis to be presented in the next sections.

Consider the following plant transfer function, taken from [7], and the corresponding FOPDT approximation

$$P_1(s) = \frac{e^{-0.5s}}{(s+1)^2} \approx \frac{e^{-0.99s}}{1+1.65s}. \tag{8}$$

The application of the ISE tuning formulae for optimal set-point response provides: $K_p^{sp} = 1.657, T_i^{sp} = 1.694$ and $T_d^{sp} = 0.513$, whereas the tuning for optimal load-disturbance provides: $K_p^{ld} = 2.418, T_i^{ld} = 1.007$ and $T_d^{ld} = 0.559$.

Fig. 2 shows the performance of both settings when the control system is operating in both, servo and regulation mode. It can be appreciated that the load-disturbance response of the set-point tuning is closer to the optimal regulation one than the load-disturbance tuning to the optimal servo tuning. Therefore the observed Performance Degradation is larger for the load-disturbance tuning. From a global point of view, it will seem better to choose the set-point settings.

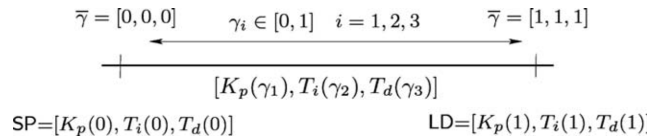

Fig. 3. $\bar{\gamma}$ – *tuning* procedure for the search of the *intermediate* controller.

3. Intermediate tuning for balanced servo/regulation operation

The tuning approaches presented in Section 2.3 can be considered extremal situations. The controller settings are obtained by considering exclusively one mode of operation. This may generate, as it has been shown in the previous section, quite poor performance if the non-considered situation happens. This fact suggests to analyze if, by loosing some degree of optimality with respect to the tuning mode, the Performance Degradation can be reduced when the operation is different to the selected one for tuning.

Based on this observation we suggest to look for an *intermediate* controller. In order to define this exploration, we need to define the search-space and the overall Performance Degradation index to be minimized. Obviously the solution will depend on how this factors are defined.

The search of the controller settings that provide a *trade-off* performance for both operating modes could be stated in terms of a completely new optimization procedure. However, we would like to take advantage of the autotuning formulae (like (6) and (7)), in order to keep the procedure, as well as the resulting controller expression, in similar simple terms. Therefore, the resulting controller settings could be considered as an extension of the optimal ones. On this basis we define a controller settings family parameterized in terms of a vector as

$$\bar{\gamma} = [\gamma_1, \gamma_2, \gamma_3], \tag{9}$$

where γ_i is a variable for each controller parameter (K_p, T_i, T_d) that allows searching for the *intermediate* tuning. The values for this factor are restricted to $\gamma_i \in [0,1]$ $i = 1,2,3$. Also, the set-point tuning will correspond to a contour constraint for each $\gamma_i = 0$, whereas the load-disturbance tuning corresponds to $\gamma_i = 1$. Fig. 3 shows graphically the procedure and the application for the 1-DoF PID controller tuning.

The controller settings family $[K_p(\gamma_1), T_i(\gamma_2), T_d(\gamma_3)]$ will be generated by a linear evolution for the parameters from the set-point tuning to the load-disturbance one and the other way around. Therefore,

$$\begin{aligned} K_p(\gamma_1) &= \gamma_1 K_p^{ld} + (1-\gamma_1)K_p^{sp}, \\ T_i(\gamma_2) &= \gamma_2 T_i^{ld} + (1-\gamma_2)T_i^{sp}, \\ T_d(\gamma_3) &= \gamma_3 T_d^{ld} + (1-\gamma_3)T_d^{sp}, \end{aligned} \tag{10}$$

where $\gamma_i \in [0,1]$ $i = 1,2,3$ and $[K_p^{sp}, T_i^{sp}, T_d^{sp}]$ and $[K_p^{ld}, T_i^{ld}, T_d^{ld}]$ stand for the set-point and load-disturbance settings for $[K_p, T_i, T_d]$, respectively.

The Performance Degradation concept for set-point and load-disturbance tunings depending on the operation mode was previously presented and developed in [8].

There, the performance of the control system is measured in terms of a performance index that takes into account the possibility of an operation mode different from the selected one. This motivates the redefinition of the ISE performance index as

$$J_x(z) = \int_0^\infty e(t,x,z)^2 dt, \tag{11}$$

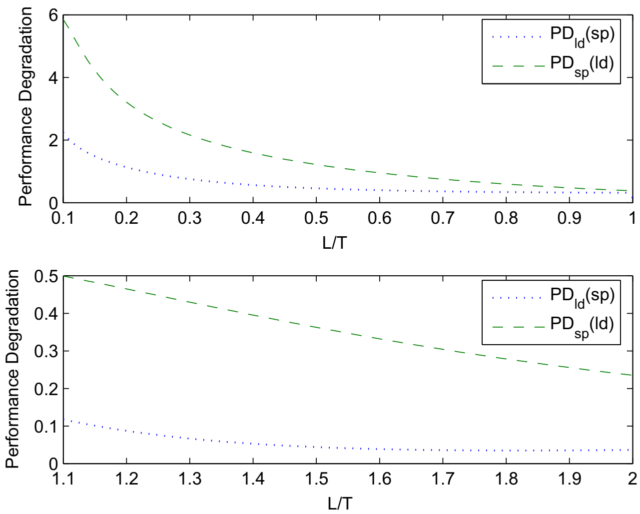

Fig. 4. Performance Degradation of set-point (sp) and load-disturbance (ld) tunings for ISE criteria with respect to the normalized dead-time L/T.

where x denotes the *operating mode* of the control system and z the selected operating mode for tuning, i.e., the *tuning mode*. Thus, we have $x \in \{sp, ld\}$ and $z \in \{sp, ld\}$, where sp states for set-point (servo) tuning and ld for load-disturbance (regulator) tuning.

Performance will not be optimal for both situations. The Performance Degradation measure helps in the evaluation of the loss of performance with respect to their optimal value. Performance Degradation, $PD_x(z)$, will be associated to the *tuning mode* – z – and tested on the, opposite, *operating mode* – x. Now, for every combination of $\overline{\gamma}$ the Performance Degradation needs to be measured with respect to both operating modes (because the corresponding $\overline{\gamma}$ – *tuning* does not necessarily corresponds to an operating mode). Hence,

- $PD_{sp}(\overline{\gamma})$ will represent the Performance Degradation of the $\overline{\gamma}$ – *tuning* on servo operating mode.

$$PD_{sp}(\overline{\gamma}) = \left| \frac{J_{sp}(\overline{\gamma}) - J_{sp}(sp)}{J_{sp}(sp)} \right|. \tag{12}$$

- $PD_{ld}(\overline{\gamma})$ will represent the Performance Degradation of the $\overline{\gamma}$ – *tuning* on regulation operating mode.

$$PD_{ld}(\overline{\gamma}) = \left| \frac{J_{ld}(\overline{\gamma}) - J_{ld}(ld)}{J_{ld}(ld)} \right|. \tag{13}$$

Note that, because the controller settings expressed through (6) and (7) have explicit dependence on the process normalized dead-time τ, it is worth taking into account that, for the PID application, the Performance Degradation will also depend on τ. Fig. 4 shows the performance analysis for the normalized dead-time ranges where PID controller settings (set-point and load-disturbance), are provided by [7].

From the above Performance Degradation definitions, the overall Performance Degradation is introduced and interpreted as a function of $\overline{\gamma}$. There may be different ways to define the $PD(\overline{\gamma})$ function, depending on the importance associated to every operating mode (e.g. applying weighting factors to each component). However, every definition must satisfy the following contour constraints

$$PD(\overline{\gamma}) = \begin{cases} PD_{ld}(sp) & \text{for } \overline{\gamma} = [0,0,0] \\ PD_{sp}(ld) & \text{for } \overline{\gamma} = [1,1,1]. \end{cases}$$

The most simple definition would be

$$PD(\overline{\gamma}) = PD_{ld}(\overline{\gamma}) + PD_{sp}(\overline{\gamma}). \tag{14}$$

This expression represents a compromise, or a balance, between both losses of performance.

As it has been mentioned before, the greatest loss of performance occurs when the load-disturbance tuning operates as a servo mode. Therefore, $PD_{sp}(\overline{\gamma})$ will be the largest component of the global expression of $PD(\overline{\gamma})$ and in the oppositive side $PD_{ld}(\overline{\gamma})$ the smallest one. This causes that the percentage reduction of PD that can be obtained from the PD_{ld} side is lower than the one for the PD_{sp} part. A balanced reduction of $PD(\overline{\gamma})$ from both Performance Degradations is possible by introducing weighting factors associated to each operating mode [8]. This idea can be applied rewriting (14) as

$$WPD(\overline{\gamma};\alpha) = \alpha PD_{ld}(\overline{\gamma}) + (1-\alpha)PD_{sp}(\overline{\gamma}), \tag{15}$$

that we call Weighted Performance Degradation (*WPD*) index, where $\alpha \in [0,1]$ is the weight factor and indicates which of the two possible operation modes is preferred or more important.

One way to express the importance between both operation modes, could be the total time that the system operates in each one of them. For example, a system that operates the 75% of the time as a regulator (or viceversa 25% as a servo), $\alpha = 0.75$. However, the α parameter allows to make a more general choice for the preference of the system operation (not only taking into account the time for each operation mode).

Note also that (15) with $\alpha = 0.50$, represents an equivalent expression obtained previously in (14) that gives the same significance for both operation modes.

The *intermediate* tuning will be determined by proper selection of $\overline{\gamma} = [\gamma_1, \gamma_2, \gamma_3]$. This choice will correspond to the solution of the following optimization problem:

$$\overline{\gamma}_{op} := \left[\gamma_{1op}, \gamma_{2op}, \gamma_{3op}\right] = arg\left[\min_{\overline{\gamma}} WPD(\overline{\gamma};\alpha)\right]. \tag{16}$$

It is obvious that $\alpha = 0$ means

$$WPD(\overline{\gamma};0) = PD_{sp}(\overline{\gamma}) \tag{17}$$

and of course the $\overline{\gamma}_{op}$ that minimizes the Performance Degradation for servo operation mode (17), is the one that corresponds to the set-point tuning ($\overline{\gamma} = [0,0,0]$). On the other side, $\alpha = 1$ is equivalent to

$$WPD(\overline{\gamma};1) = PD_{ld}(\overline{\gamma}) \tag{18}$$

and the tuning that minimizes the Performance Degradation for regulation operation (18) is the load-disturbance tuning that equals to $\overline{\gamma} = [1,1,1]$.

The optimal values (16) jointly with (10), give a tuning formula that provides a worse performance than the optimal settings operating in the same way but also a lower degradation in the performance when the *operating mode* is different from the *tuning mode*.

Remark. It is important to note that the presented procedure has just considered the performance with respect to the proposed Performance Degradation index. Other closed-loop characteristics such as stability robustness, tolerance to parameter variation, etc. are not taken into account. How to consider additional closed-loop characteristics is a subject of current research. However it is clear that in order to include such characteristics into consideration, they need to be part of the original extreme tunings.

4. Optimization and autotuning rules

To provide the possibility to specify any possible combination between both operation modes, the index (15), with an appropriated weight factor α and subjected to the optimization (16), gives the suitable γ_i values that provide the PID tuning according to (10).

However, from a more practical point of view is unusual and very difficult to say for example, that the regulation mode, in a control system, has the 63% of the importance (that means the 37% for the servo). With this respect, we can establish a categorization in order to make the analysis simpler and also to help the choice of the weight factor. Therefore, depending on the operation for the control system, we can identify the following general cases:

- Operation only as a servo that means $\alpha = 0$.
- Operation only as a regulator that means $\alpha = 1$.
- Same importance for both system operation modes, servo and regulation, that is equivalent to $\alpha = 0.50$.
- More importance for the servo than the regulation operation, that can be expressed by $\alpha = 0.25$.
- More importance for regulator than servo, that can be indicated as $\alpha = 0.75$.

This broad classification allows for a qualitative specification of the control system operation.

Here, the optimization was performed using genetic algorithms [11], taking problem (16) as the *fitness function*. The implementation was using MATLAB 7.6.0(R2008a)® for a *population size* of 20 and a maximum number of *generations* of 50.

The optimal solution was found for $\alpha = \{0.25, 0.50, 0.75\}$. As we said before for $\alpha = \{0, 1\}$, as extreme situations, the optimal tunings are the related to set-point and load-disturbance presented previously in Section 2.3.

It is worth to say that at first the optimization was performed by considering an enlarged search space for the $\overline{\gamma}$ vector, however, for the rare cases in which the optimal γ_i parameters were outside the interval $[0, 1]$, the value of the objective function was practically the same and, therefore, it is preferred to constraint the search space in order to provide a bounded controller's family that is easier to understand as are presented of an *intermediate* controller.

Tuning relations (10) allow to select γ_i values on the basis of *trade-off* performance for both operating modes. However, it would be desirable an automatic methodology to choose this set of parameters without the need to run the whole Weighted Performance Degradation analysis.

In order to pursue the previous idea, by repeating the problem optimization posed in (16) for the three weighting factor and different values of the normalized dead-time τ, we can find an optimal set for each γ_i parameter. For each one of these groups, it is possible to approximate a function to determine a general procedure that allows to find the suitable values for the γ_i's, that provide the best *intermediate* tuning. Results are adjusted to the general expression as

$$\gamma_i(\tau) = a + b\tau + c\tau^2 \qquad \tau \in [0.1, 1.0] \cup [1.1, 2.0], \tag{19}$$

where a, b and c are given in Table 2, according to the weighting factor α and for each γ_i and τ range. Fig. 5 shows the followed procedure for $\alpha = 0.50$ and the γ_1 case.

Table 2
$\overline{\gamma}_\alpha$ Settings for autotuning.

	τ Range	0.1–1.0			1.1–2.0		
	Constant	a	b	c	a	b	c
$\alpha = 0.25$	γ_1	0.082	0.074	0.138	0.021	0.040	−0.006
	γ_2	0.896	−1.238	0.854	0.097	−0.723	0.173
	γ_3	0.332	−0.592	0.508	0.323	−0.183	0.033
$\alpha = 0.50$	γ_1	0.093	0.547	−0.106	1.162	−1.258	0.406
	γ_2	0.920	−0.540	0.206	2.222	−2.184	0.639
	γ_3	0.831	−1.197	0.548	−0.436	0.941	−0.334
$\alpha = 0.75$	γ_1	0.108	0.566	0.067	2.197	−2.529	0.774
	γ_2	0.869	−0.271	0.129	1.312	−1.021	0.296
	γ_3	0.211	0.701	−0.683	−0.987	1.791	−0.579

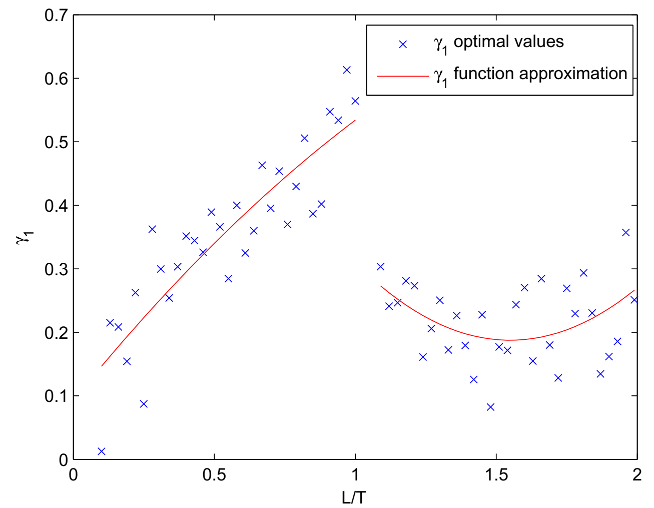

Fig. 5. Optimal set for γ_1 parameter and the corresponding approximated function for $\alpha = 0.50$.

Eq. (19) for each γ_i along with the settings (10) provide what we call here $\overline{\gamma}_\alpha$-*autotuning* for weighted servo/regulation operation, that is the main contribution of this paper.

5. Examples

This section presents several examples to illustrate how the implementation of the $\overline{\gamma}_\alpha$-*autotuning* improves the performance of the closed-loop system respect to the both operation modes.

In all the examples it is supposed that the process output can vary in the 0–100% normalized range and that in the normal operation point, the controlled variable has a value close to 70%.

5.1. Example 1

Let us to consider the system (8), shown before as a *Motivation Example*.

Table 3 shows the PID controller parameters for the system (8) using the [7] method and the proposed $\overline{\gamma}_\alpha$-*autotuning* with $\alpha = \{0.25, 0.50, 0.75\}$. Moreover, the corresponding system outputs responses to a 20% set-point change followed by a −20% load-disturbance change, are shown in Fig. 6 for the following tuning methods: set-point, load-disturbance and $\overline{\gamma}_\alpha$-*autotuning* with its three possible scenarios. The control signal is not shown for the sake of brevity, however it can be easily guessed that it would be smoother when the value of α is lower (see Example 3).

It can be seen that the proposed $\overline{\gamma}_\alpha$-*autotuning* gives lower performance than the optimum settings when the system operates in the same way as it was tuned. However, higher performance can be obtained for the whole system operation (regulatory-control and servo-control), when an *intermediate* controller is used.

Table 4 shows the Performance Degradation values calculated from (12)–(14) and also the *WPD* index (15) for each tuning. The

Table 3
Example 1 – PID controller parameters.

Tuning	K_p	T_i	T_d
set-point(sp)	1.657	1.694	0.513
load-disturbance(ld)	2.418	1.007	0.559
$\overline{\gamma}_{\alpha=0.25}$-*autotuning*	1.791	1.378	0.520
$\overline{\gamma}_{\alpha=0.50}$-*autotuning*	1.949	1.234	0.527
$\overline{\gamma}_{\alpha=0.75}$-*autotuning*	2.016	1.177	0.531

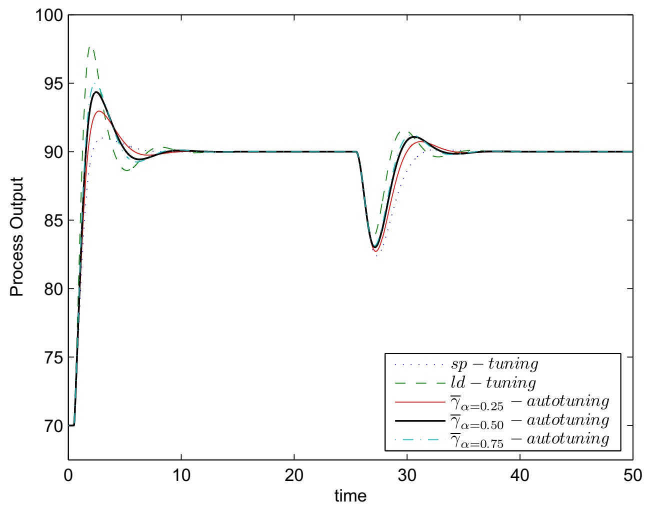

Fig. 6. Example 1 – process output for the control system operating in both servo and regulation modes.

below side of the table presents the improvement in percentage that can be achieved with each one of the $\overline{\gamma}_\alpha$-*autotuning* respect to the extreme tunings (set-point and load-disturbance).

All the values confirm the fact that, in global terms, when both operating modes could appear and taking into account the importance that the control-loop is operating as servo or regulation mode, the proposed $\overline{\gamma}_\alpha$-*autotuning* is the best choice to tune the PID controller in order to get less Performance Degradations.

5.2. Example 2

In order to add completeness to the comparison, a case-study example is provided. We consider the isothermal Continuous Stirred Tank Reactor (CSTR), as the one in Fig. 7, where the isothermal series/parallel Van de Vusse reaction [15,16] is taking place. The reaction can be described by the following scheme:

$$A \xrightarrow{k_1} B \xrightarrow{k_2} C, \qquad 2A \xrightarrow{k_3} D. \tag{20}$$

Doing a mass balance, the system can be described by the following model:

$$\frac{dC_A(t)}{dt} = \frac{F_r(t)}{V}[C_{Ai} - C_A(t)] - k_1 C_A(t) - k_3 C_A^2(t), \qquad \frac{dC_B(t)}{dt} = -\frac{F_r(t)}{V}C_B(t) + k_1 C_A(t) - k_2 C_B(t), \tag{21}$$

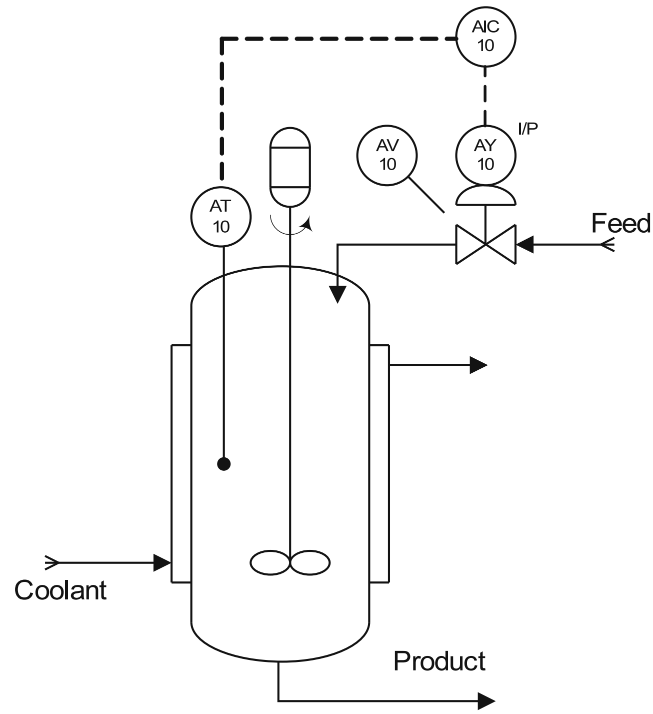

Fig. 7. Example 2 – CSTR system.

where F_r is the feed flow rate of product A, V is the reactor volume which is kept constant during the operation, C_A and C_B are the reactant concentrations in the reactor, and k_i $(i = 1, 2, 3)$ are the reaction rate constants for the three reactions.

In this case, the variables of interest are: the concentration of B in the reactor (C_B as the controlled variable), the flow through the reactor (F_r as the manipulated variable), and the concentration C_{Ai} of A in the feed flow (whose variation can be considered as the disturbance). The kinetic parameters are chosen to be $k_1 = 5/6\ \text{min}^{-1}, k_2 = 5/3\ \text{min}^{-1}$, and $k_3 = 1/6\ \text{l mol}^{-1}\ \text{min}^{-1}$. Also, is assumed that the nominal concentration of A in the feed (C_{Ai}) is 10 mol l^{-1} and the volume $V = 700$ l.

Using (21) and the parameters values, the characterization of the steady-state for the process can be obtained as it is shown in Fig. 8, for three concentrations of C_{Ai}, where is easy to see the non-linearity of the system.

Initially, the system is at the steady-state (therefore the operational point) with $C_{Ao} = 2.9175\ \text{mol l}^{-1}$ and $C_{Bo} = 1.10\ \text{mol l}^{-1}$. From this, it can be selected the measurement range for C_B from

Table 4
Example 1 – *PD* and *WPD* values for the system (8) and the improvement obtained with $\overline{\gamma}_\alpha$-*autotuning*.

Tuning	PD_{sp}	PD_{ld}	$WPD_{\alpha=0.25}$	$WPD_{\alpha=0.50}$	$WPD_{\alpha=0.75}$
set-point(sp)	–	0.3951	0.0988	0.1976	0.2964
load-disturbance(ld)	0.9496	–	0.7123	0.4748	0.2374
$\overline{\gamma}_{\alpha=0.25}$-*autotuning*	0.0336	0.1662	0.0668	–	–
$\overline{\gamma}_{\alpha=0.50}$-*autotuning*	0.1088	0.0376	–	0.0732	–
$\overline{\gamma}_{\alpha=0.75}$-*autotuning*	0.1578	0.0009	–	–	0.0401
Improvement in % of					
$\overline{\gamma}_{\alpha=0.25}$-*autotuning*	96.46%(ld)	57.93%(sp)	32.39%(sp)	–	–
			90.62%(ld)	–	–
$\overline{\gamma}_{\alpha=0.50}$-*autotuning*	88.54%(ld)	90.49%(sp)	–	62.95%(sp)	–
			–	85.58%(ld)	–
$\overline{\gamma}_{\alpha=0.75}$-*autotuning*	83.38%(ld)	99.77%(sp)	–	–	86.47%(sp)
			–	–	83.11%(ld)
(Respect to)					

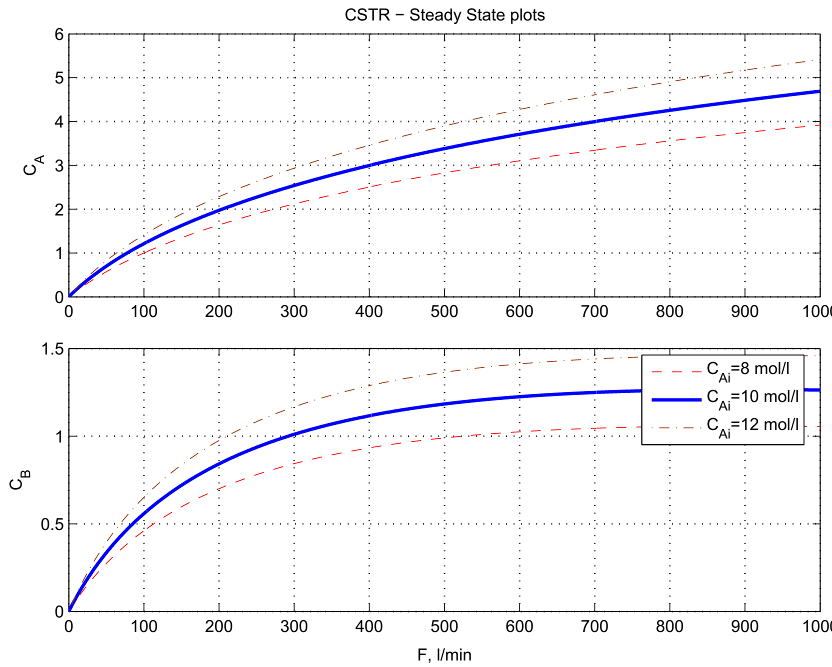

Fig. 8. Example 2 – steady-state characterization for the reactor.

0 to 1.5714 mol/l and the capacity for the control valve with a maximum flow of 634.1719 l/min (variation range of the flow from 0 to 634.1719 l/min) [17]. The signals (y, u, r) will be in percentage (0–100%).

The sensor–transmitter element takes the form

$$y(t)_{\%} = \left(\frac{100}{1.5714}\right) C_B(t) \tag{22}$$

and the control valve with a linear flow characteristic,

$$F_r(t) = \left(\frac{634.1719}{100}\right) u(t)_{\%}. \tag{23}$$

Fig. 9 shows the steady-state characterization, taking into account elements represented by (22) and (23). This is calling *set actuator-process-sensor* and from this is clearly that for the selected steady-state, $r_o = 70\%$ and $u_o = 60\%$.

It is assumed that changes in the set-point would be not bigger than 10% and the possible disturbance in C_{Ai}, can variate around $\pm 10\%$. In Fig. 10 can be seen the process output (including the sensor and the control valve) and also the FOPDT model for a step change in the process input $(y_u(t))$.

Using the identification method [18], the determined FOPDT model is

$$P_3(s) \approx \frac{0.3199e^{-0.5289s}}{0.6238s + 1}. \tag{24}$$

From (24), the application of the ISE tuning formulae for optimal set-point and load-disturbance responses and also the *intermediate* $\overline{\gamma}_\alpha$*-autotuning* provide the parameters for the PID controller that are shown in Table 5.

Process outputs of the closed-loop system are shown in Fig. 11, first for a set-point step change of -10%, follows by a disturbance

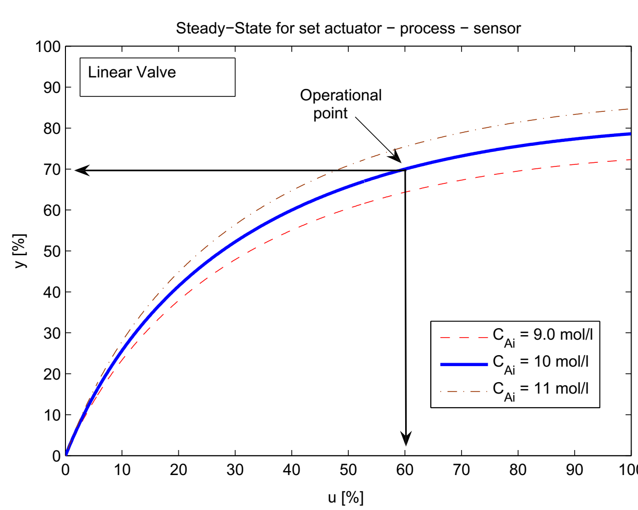

Fig. 9. Example 2 – steady-state characterization for the set actuator-process-sensor.

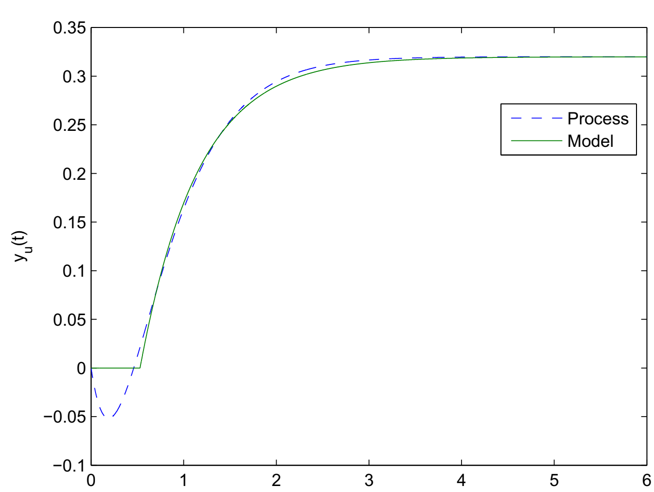

Fig. 10. Example 2 – reaction curve for process and FOPDT model.

Table 5
Example 2 – PID controller parameters.

Tuning	K_p	T_i	T_d
set-point(*sp*)	3.799	0.707	0.264
load-disturbance(*ld*)	5.404	0.494	0.293
$\overline{\gamma}_{\alpha=0.25}$-*autotuning*	4.190	0.609	0.269
$\overline{\gamma}_{\alpha=0.50}$-*autotuning*	4.570	0.577	0.270
$\overline{\gamma}_{\alpha=0.75}$-*autotuning*	4.820	0.551	0.273

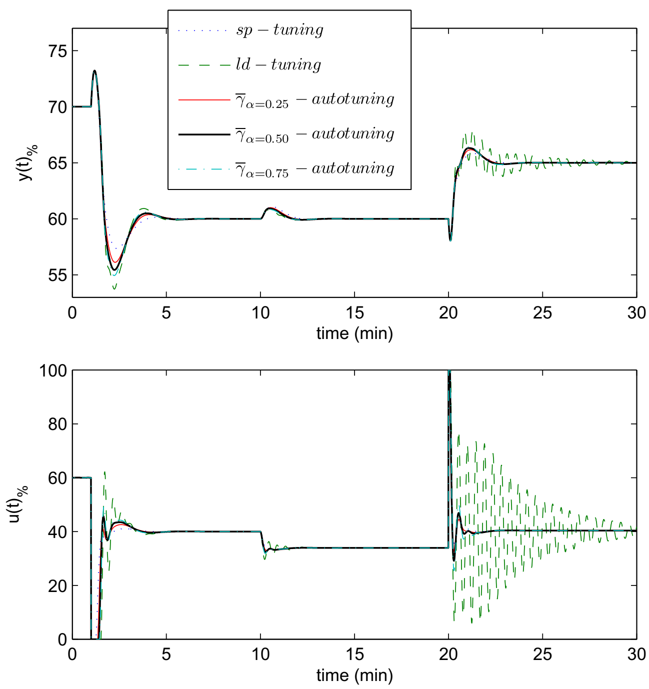

Fig. 11. Example 2 – process output for the non-linear control system operating in both servo and regulation modes.

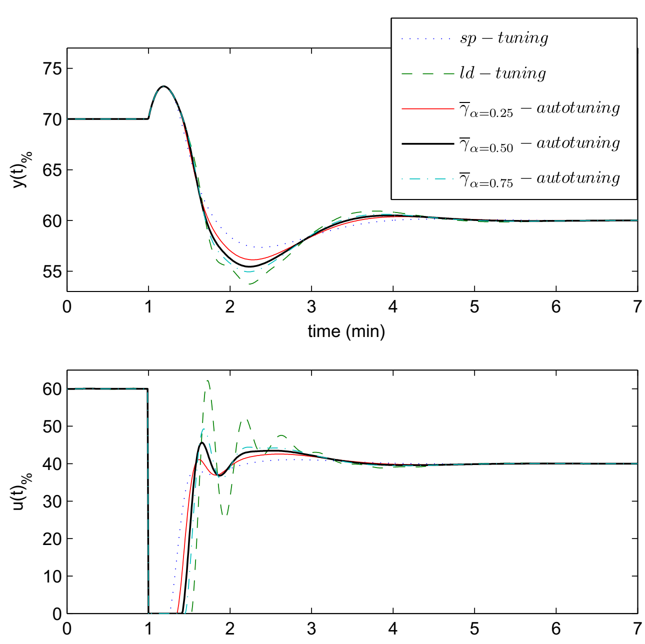

Fig. 12. Example 2 – process output for the non-linear control system operating as servo.

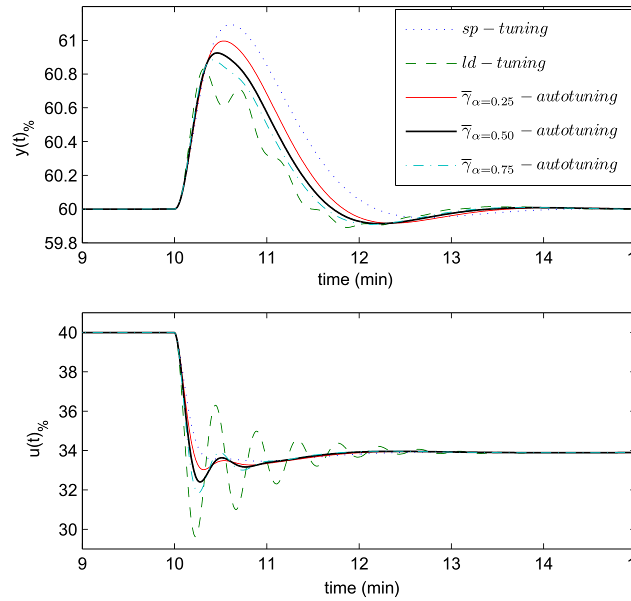

Fig. 13. Example 2 – process output for the non-linear control system operating as regulator.

of +10% and finally a new change in the set-point of +5%, all these situations using the three tuning modes (set-point, load-disturbance and $\overline{\gamma}_{\alpha}$-*autotuning*). Also, the control signal ($u(t)$) can be seen. It appears that, as expected, the control signal is smoother for lower values of α.

A more comprehensive picture of the set-point change is shown in Fig. 12. In this case, it can be seen that, as expected, the set-point tuning gives the better performance for servo operation mode. Furthermore, the $\overline{\gamma}_{\alpha}$-*autotuning* provides a lower degradation, respect to the optimal, than the load-disturbance tuning.

The detail of load-disturbance attenuation is in Fig. 13. Similarly to the previous case, the load-disturbance tuning is the one that gives better performance for regulation operation and the performance degradation of the set-point tuning is greater than the three cases for $\overline{\gamma}_{\alpha}$-*autotuning*.

In general terms, it can be confirmed that the $\overline{\gamma}_{\alpha}$-*autotuning* gives a better performance when the system operates in both servo and regulation modes. Also, the control signal for the *intermediate* tuning seems to be *smoother* than that provided by the optimal regulation settings.

Table 6 shows the *PD* and *WPD* indices and the improvement that can be achieve for each case of the $\overline{\gamma}_{\alpha}$-*autotuning*.

6. Conclusions

In process control it is very usual to have changes in the set-point, as well as in the disturbance. This causes the need to face with both servo and regulatory control problems. For 1-DoF PID controllers, when the tuning objective is different to the real system operation, a degradation in the performance is expected and it can be evaluated. A reduction in the overall Performance Degradation can be obtained by searching an *intermediate* controller between the optimal ones proposed for set-point and load-disturbance tunings.

Autotuning formulae have been presented with the aim to minimize the Weighted Performance Degradation, expressed as a combination depending of the balance between the total time that the system operates in servo and regulation modes. This is the main contribution of this paper because it is a novel feature that allows

Table 6
Example 2 – *PD* and *WPD* values for the non-linear CSTR system and the improvement obtained with $\overline{\gamma}_{\alpha}$-*autotuning*.

Tuning	PD_{sp}	PD_{ld}	$WPD_{\alpha=0.25}$	$WPD_{\alpha=0.50}$	$WPD_{\alpha=0.75}$
set-point(*sp*)	–	0.3284	0.0821	0.1642	0.2463
load-disturbance(*ld*)	0.5316	–	0.3987	0.2658	0.1329
$\overline{\gamma}_{\alpha=0.25}$-*autotuning*	0.0192	0.1398	0.0493	–	–
$\overline{\gamma}_{\alpha=0.50}$-*autotuning*	0.0706	0.0470	–	0.0588	–
$\overline{\gamma}_{\alpha=0.75}$-*autotuning*	0.1334	0.0038	–	–	0.0362
Improvement in % of					
$\overline{\gamma}_{\alpha=0.25}$-*autotuning*	96.39%(ld)	57.73%(sp)	39.95%(sp)	–	–
			87.63%(ld)	–	–
$\overline{\gamma}_{\alpha=0.50}$-*autotuning*	86.72%(ld)	85.99%(sp)	–	64.19%(sp)	–
			–	77.88%(ld)	–
$\overline{\gamma}_{\alpha=0.75}$-*autotuning*	74.91%(ld)	99.15%(sp)	–	–	85.30%(sp)
			–	–	72.76%(ld)
(Respect to)					

to select the tuning according to a general qualitative specification of the control system operation.

Results are given for PID controllers, in order to get results closer to industrial applications. The examples have shown the improvement obtained with each one of the $\overline{\gamma}_{\alpha}$-*autotuning* cases.

Even if the results were presented and exemplified using the ISE performance criteria, it could be possible to reproduce a similar methodology to other PID controllers, like the one that uses derivative action is applied just to process output, or to other PID tunings with different performance objectives.

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