

Nombre Juan Diego Rozo Alvarez

Fecha día mes año

Profesor

Materia M. Numérica.

Institución

Curso

Nota

1.) Parábola $y = A + Bx + Cx^2$ que pasa por los puntos $(1, 1)$, $(2, -1)$, $(3, 1)$. A, b y c son:

$$\left. \begin{aligned} A + B(1) + C(1)^2 &= 1 \\ A + B(2) + C(2)^2 &= -1 \\ A + B(3) + C(3)^2 &= 1 \end{aligned} \right\} \quad \left. \begin{aligned} 1A + 1B + 1C &= 1 \\ 1A + 2B + 4C &= -1 \\ 1A + 3B + 9C &= 1 \end{aligned} \right\} \quad \left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & -1 \\ 1 & 3 & 9 & 1 \end{array} \right) \xrightarrow{F_2 - F_1, F_3 - F_1} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & -2 \\ 0 & 2 & 8 & 0 \end{array} \right)$$

$$\xrightarrow{F_3 - 2F_2} \left(\begin{array}{ccc|c} 1 & 0 & -2 & 3 \\ 0 & 1 & 3 & -2 \\ 0 & 0 & 2 & 4 \end{array} \right) \xrightarrow{F_3/2} \left(\begin{array}{ccc|c} 1 & 0 & -2 & 3 \\ 0 & 1 & 3 & -2 \\ 0 & 0 & 1 & 2 \end{array} \right) \xrightarrow{F_1 + 2F_3, F_2 - 3F_3} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 7 \\ 0 & 1 & 0 & -8 \\ 0 & 0 & 1 & 2 \end{array} \right) \quad \begin{aligned} A &= 7 \\ B &= -8 \\ C &= 2 \end{aligned}$$

2.) A es la matriz de elementos: $a_{11}=4, a_{12}=3, a_{13}=-1, a_{21}=-2, a_{22}=-4$
 $a_{31}=5, a_{32}=1, a_{33}=2, a_{34}=6$

Factorizar $A=20$.

elementos de L:

$$A = \begin{pmatrix} 4 & 3 & -1 \\ -2 & 4 & 5 \\ 1 & 2 & 6 \end{pmatrix}, \quad \begin{aligned} m_{21} &= -2/4 = -1/2 \\ m_{31} &= 1/4 = 1/4 \end{aligned} \quad \xrightarrow{F_2 + \frac{1}{2}F_1, F_3 - \frac{1}{4}F_1} \begin{pmatrix} 4 & 3 & -1 \\ 0 & 5/2 & 9/2 \\ 0 & 5/4 & 25/4 \end{pmatrix}, \quad m_{32} = 2/5 = 2/5$$

$$\xrightarrow{F_3 - \frac{1}{2}F_2} \begin{pmatrix} 4 & 3 & -1 \\ 0 & 5/2 & 9/2 \\ 0 & 0 & 1/2 \end{pmatrix} = U, \quad L = \begin{pmatrix} 1 & 0 & 0 \\ -1/2 & 1 & 0 \\ 1/4 & -1/2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ -0,5 & 1 & 0 \\ 0,25 & -0,5 & 1 \end{pmatrix} //$$

3.) Raíz de la ecuación por método de la secante:

$$f(x) = x^3 - 2x^2 + 10x - 20 = 0, \quad x_0 = 0, \quad x_1 = 1, \quad x_{n+1} = x_n - \frac{f(x_n)(x_n - x_{n-1})}{f(x_n) - f(x_{n-1})}$$

$$\begin{aligned} 1. \quad x_0 &= 0, \quad f(x_0) = -20 \\ x_1 &= 1, \quad f(x_1) = -7 \\ x_1 - x_0 &= 1 \\ f(x_1) - f(x_0) &= 13 \end{aligned}$$

$$x_2 = 1 - (-7) \frac{1}{13} = 1 + \frac{7}{13} = \frac{20}{13}$$

$$x_2 \approx 1,53846$$

$$\begin{aligned} 2. \quad x_1 &= 1, \quad f(x_1) = -7 \\ x_2 &= 1,53846, \quad f(x_2) = 3,76963 \\ x_2 - x_1 &= 0,53846 \\ f(x_2) - f(x_1) &= 10,75963 \end{aligned}$$

$$x_3 = 1,53846 - \frac{3,76963(0,53846)}{10,75963}$$

$$x_3 \approx 1,35031 //$$

4.) para obter x_3 . $f(x) = x^3 + 2x^2 + 10x - 20 = 0$ por Newton Raphson.
 $x_0 = 1$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \quad f'(x) = 3x^2 + 4x + 10$$

1. $x_0 = 1$
 $f(x_0) = -7$
 $f'(x_0) = 17$

$$x_1 = 1 - \frac{(-7)}{17}$$

$$x_1 = 1 + \frac{7}{17} = \frac{24}{17}$$

$$x_1 \approx 1,41176$$

2. $x_1 = 1,41176$
 $f(x_1) = 0,91746$
 $f'(x_1) = 21,62623$

$$x_2 = 1,41176 - \frac{0,91746}{21,62623}$$

$$x_2 = 1,36933$$

3. $x_2 = 1,36933$
 $f(x_2) = 0,01101$
 $f'(x_2) = 21,10251$

$$x_3 = 1,36933 - \frac{0,01101}{21,10251}$$

$$x_3 = 1,368808$$

$$x_3 = 1,36881 //$$

5) $y = f(x) = \cos(x)$, $x_0 = 0,2$, $x_1 = 1,0$. obter polinômios de Lagrange.

$$P_n(x) = \sum_{j=0}^n y_j \frac{\prod_{i=0, i \neq j}^n (x - x_i)}{\prod_{i=0, i \neq j}^n (x_j - x_i)}, \quad N = 1$$

$$P_1(x) = y_0 \frac{(x - x_1)}{(x_0 - x_1)} + y_1 \frac{(x - x_0)}{(x_1 - x_0)}, \quad y_0 = \cos(0,2) = 0,98007$$

$$y_1 = \cos(1) = 0,54030$$

$$P_1(x) = 0,98007 \frac{(x - 1)}{-0,8} + 0,54030 \frac{(x - 0,2)}{0,8}$$

$$P_1(x) = -1,22508(x - 1) + 0,675378(x - 0,2) //$$

6) obtenção de distâncias:

x_0	x_1	x_2	x_3
1	5	20	40
56,5	113	181	214,5

ter PIN grade 3:

$$P_n(x) = \sum_{i=0}^n a_i \prod_{j=0, j \neq i}^{n-1} (x - x_j)$$

x	$f(x)$	Δf	$\Delta^2 f$	$\Delta^3 f$
1	56,5			
5	113	14,25		
20	181	45,33	-0,50482	
40	214,5	1,675	-0,01666	0,012856

$$P_3(x) = 56,5 + 14,25(x - 1) - 0,50482(x - 1)(x - 5) + 0,012856(x - 1)(x - 5)(x - 20) //$$

2.) mezcla para construir aceras.

cemento, arena, grava: proporciones.

x_1 : 1/8	3/8	4/8	m^3	P_1
x_2 : 2/10	5/10	3/10	m^3	P_2
x_3 : 2/5	3/5	0/5	m^3	P_3
x_1	x_2	x_3		

Total:

2,3 4,8 2,9 m^3

$$P_1: \begin{cases} 1/8 x_1 + 2/10 x_2 + 2/5 x_3 = 2,3 \\ 3/8 x_1 + 5/10 x_2 + 3/5 x_3 = 4,8 \\ 4/8 x_1 + 3/10 x_2 + 0/5 x_3 = 2,9 \end{cases} \quad \left(\begin{array}{ccc|c} 1/8 & 1/5 & 2/5 & 2,3 \\ 3/8 & 1/2 & 3/5 & 4,8 \\ 1/2 & 3/10 & 0 & 2,9 \end{array} \right) F_1(8)$$

$$\left(\begin{array}{ccc|c} 1 & 8/5 & 16/5 & 92/5 \\ 3/8 & 1/2 & 3/5 & 24/5 \\ 1/2 & 3/10 & 0 & 29/10 \end{array} \right) \begin{matrix} F_2 - 3F_1 \\ F_3 - \frac{1}{2}F_1 \end{matrix} \quad \left(\begin{array}{ccc|c} 1 & 8/5 & 16/5 & 92/5 \\ 0 & -1/10 & -3/5 & -21/10 \\ 0 & -1/2 & -8/5 & -63/10 \end{array} \right) F_2(10) \quad \left(\begin{array}{ccc|c} 1 & 8/5 & 16/5 & 92/5 \\ 0 & 1 & 6 & 21 \\ 0 & -1/2 & -8/5 & -63/10 \end{array} \right)$$

$$F_3 + \frac{1}{2}F_2 \quad \left(\begin{array}{ccc|c} 1 & 8/5 & 16/5 & 92/5 \\ 0 & 1 & 6 & 21 \\ 0 & 0 & 7/5 & 21/5 \end{array} \right) F_3(5/7) \quad \left(\begin{array}{ccc|c} 1 & 8/5 & 16/5 & 92/5 \\ 0 & 1 & 6 & 21 \\ 0 & 0 & 1 & 3 \end{array} \right) F_2 - 6F_3 \quad \left(\begin{array}{ccc|c} 1 & 8/5 & 16/5 & 92/5 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 3 \end{array} \right)$$

$$F_1 - \frac{16}{5}F_3 \quad \left(\begin{array}{ccc|c} 1 & 8/5 & 0 & 44/5 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 3 \end{array} \right) F_1 - 8/5 F_2 \quad \left(\begin{array}{ccc|c} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 3 \end{array} \right) \therefore \begin{matrix} x_1 = 4 \\ x_2 = 3 \\ x_3 = 3 \end{matrix}$$