

# Happy 12<sup>th</sup> Birthday Arduino!



**Saturday, April 2  
at Robot Garden,  
11AM to 3PM**



**Robot Garden**



**[day.arduino.cc](http://day.arduino.cc)**

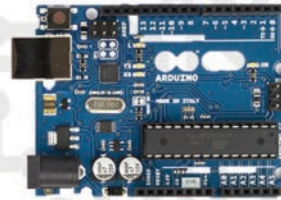
# The Maker-verse



**computer**



**microcontroller**



**motors**



**servos**



**sensors**



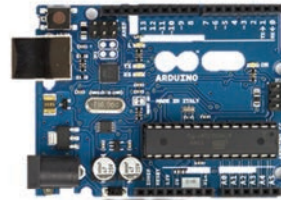
**Electricity × Magnetism**

# The Maker-verse plus a Village

computer



microcontroller



motors



servos



sensors



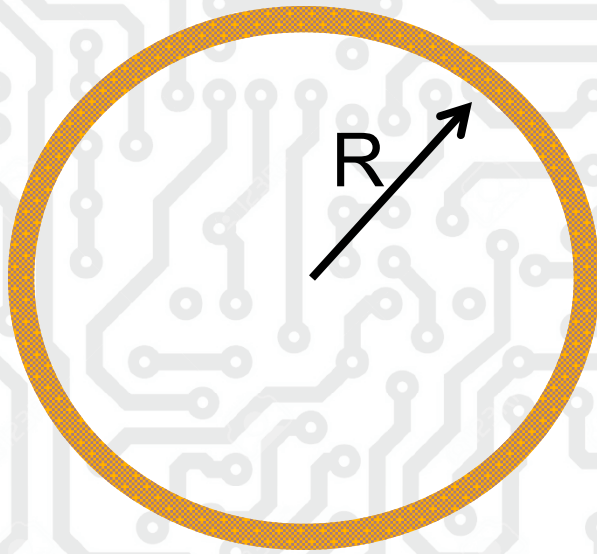
**Electricity × Magnetism**

# Review

- **Ohm's Law  $V = I \times R$**
- **Power  $P = I \times V$**
- **Electromagnetics underpins our world and this course**
- **Introduction of Magnetic field**
- **The Homopolar Motor**
  - **Magnetic Field Picture**
  - **Current Picture**



# Simple Motor



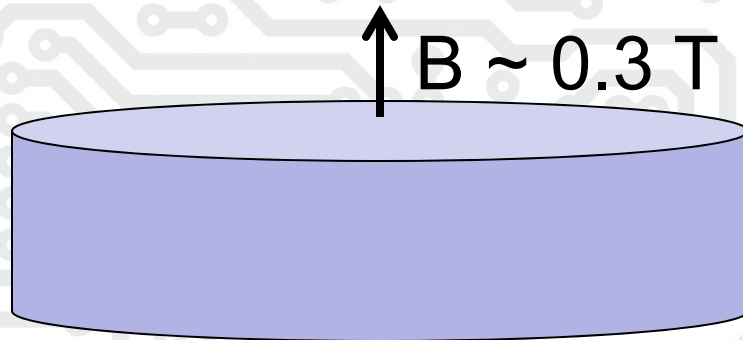
Wire loop:

$N \sim 10$  loops

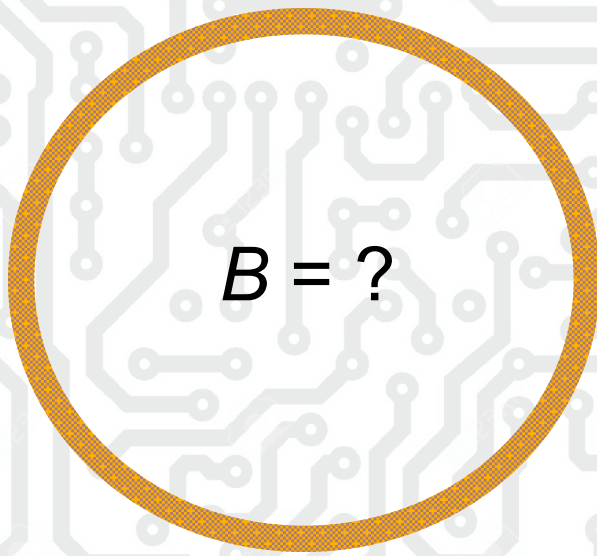
$R = 7.5 \text{ mm} = 7.5 \times 10^{-3} \text{ m}$

$M \sim 0.5 \text{ gram}$

$I \sim 4 \text{ Amps}$



# What is the field generated by this loop?



We need the  
*Biot-Savart Law*  
to find out:

$$B = \frac{\mu_0}{4\pi} \frac{I_1 dl_1 \times \hat{r}}{r^2}$$

$$B \sim \frac{\mu_0}{4\pi} \frac{I_1 2\pi R}{R^2} = \frac{\mu_0 I_1}{2R} \leftarrow \text{This turns out to be the exact answer}$$

$$\sim \mu_0 (4 \text{ A}) / (2 \times 7.5 \text{ mm}) \sim 270 \mu_0$$

# So how big is $\mu_0$ ?

**Recall that  $\mu_0$  is the coupling constant for the magnetic force:**

$$F_B = \frac{\mu_0}{4\pi} \frac{I_1 dl_1 \bullet I_2 dl_2}{r^2}$$

# So how big is $\mu_0$ ?

**Recall that  $\mu_0$  is the coupling constant for the magnetic force (also called the permeability of free space):**

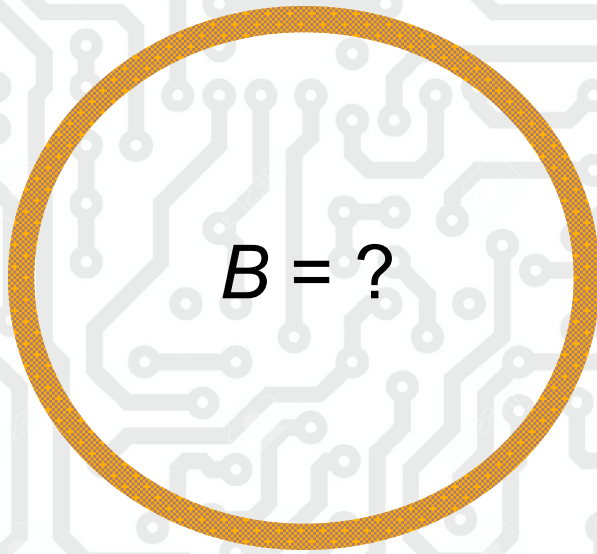
$$F_B = \frac{\mu_0}{4\pi} \frac{I_1 dl_1 \bullet I_2 dl_2}{r^2}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ T m / A}$$

**That's a small number!**



# What is the field generated by this loop?



We need the  
*Biot-Savart Law*  
to find out:

$$B = \frac{\mu_0}{4\pi} \frac{I_1 dl_1 \times \hat{r}}{r^2}$$

$$B \sim \frac{\mu_0}{4\pi} \frac{I_1 2\pi R}{R^2} = \frac{\mu_0 I_1}{2R} \leftarrow \text{This turns out to be the exact answer}$$

$$\begin{aligned} B &\sim \mu_0 (4 \text{ A}) / (2 \times 7.5 \text{ mm}) \sim 270 \mu_0 \\ &\sim 3 \times 10^{-5} \text{ T} = 0.3 \text{ Gauss} \end{aligned}$$

# What can we do about this?

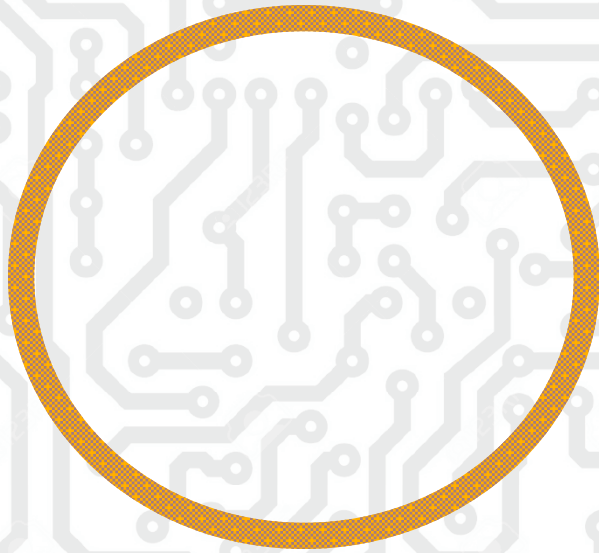

$$B = ?$$

$B \sim 3 \times 10^{-5} \text{ T} = 0.3 \text{ Gauss}$   
similar to that of the Earth

**This is much smaller than the 0.3 T from our magnet. One trick we can play is wind the current around the loop  $N$  times, multiplying the field by  $N$ . If  $N=10$ :**

$$B \sim 3 \text{ G} = 3 \times 10^{-4} \text{ T}$$

# Simple Motor

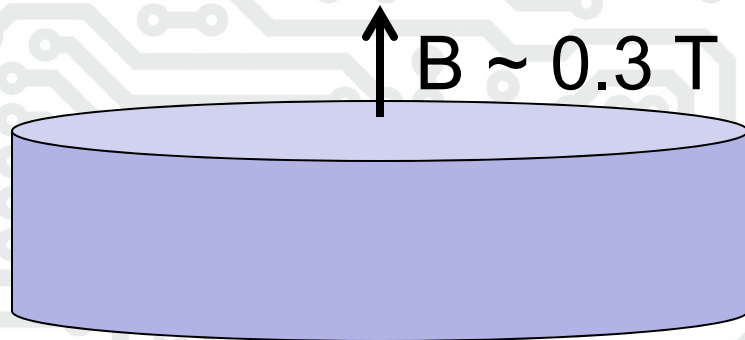


Magnetic moment of the wire loop:

$$\mu_m = N I A$$

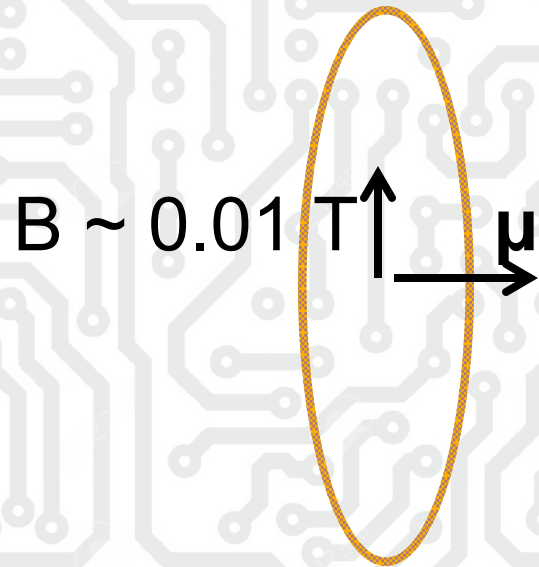
$$\sim 10 * (4 \text{ A}) * (\pi (7.5\text{e-}3 \text{ m})^2)$$

$$\sim 7 \times 10^{-3} \text{ A m}^2$$



# Simple Motor

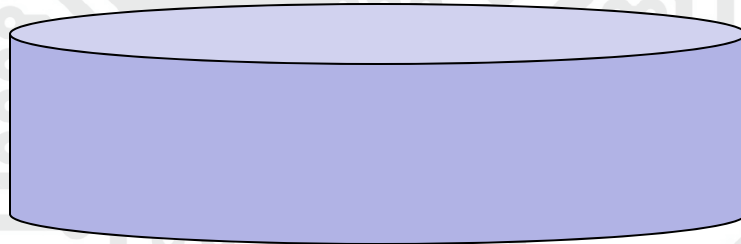
$$\text{Max Torque } \tau_{mx} = \mu_m \times B \\ \sim 7 \times 10^{-5} \text{ N m}$$



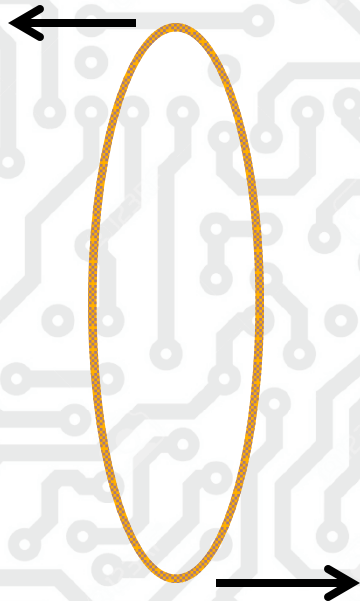
But no torque over remainder of rotation, so average torque:

$$\tau \sim \tau_{mx} / 10 \sim 10^{-5} \text{ N m}$$

Similar to the torque on the homopolar motor



# Simple Motor

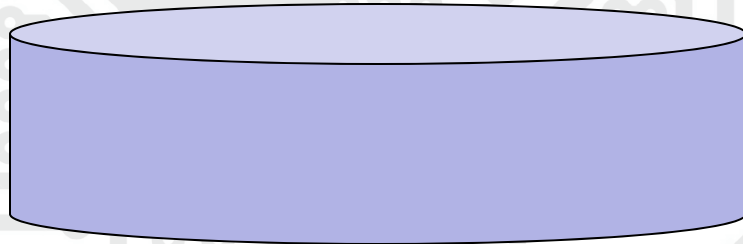


$$\text{Max Torque } \tau_{mx} = \mu_m \times B \\ \sim 7 \times 10^{-5} \text{ N m}$$

But no torque over remainder of rotation, so average torque:

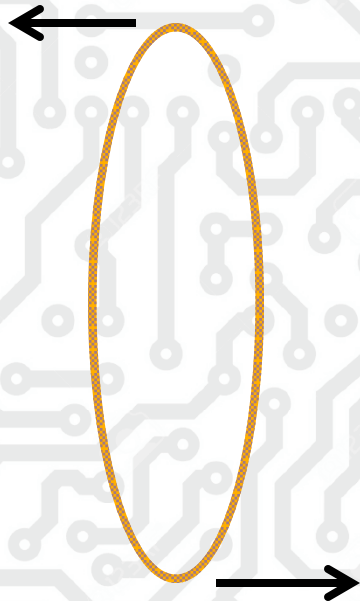
$$\tau \sim \tau_{mx} / 10 \sim 7 \times 10^{-6} \text{ N m}$$

Similar to the torque on the homopolar motor



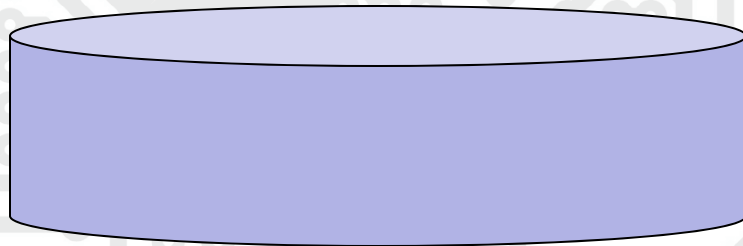


# Simple Motor



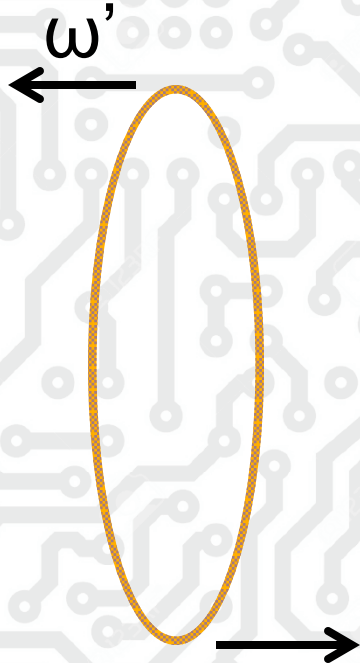
0.5 m of 26 gauge  
copper wire has a mass:  
 $M \sim 0.5 \times 10^{-3} \text{ kg}$

and a moment of inertia:  
 $I = \frac{1}{2} MR^2 \sim 1.5 \times 10^{-8}$



This wire ring is much  
lighter than the magnet in  
the homopolar motor

# Simple Motor



Newton's force law for rotation:

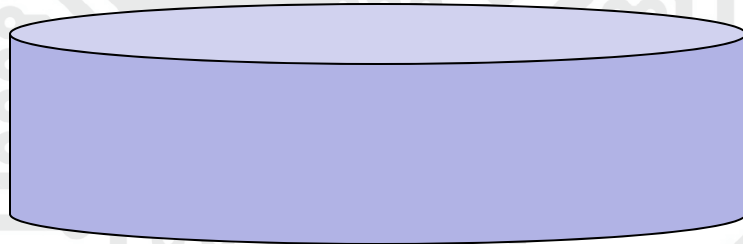
$$\tau = I\omega'$$

So

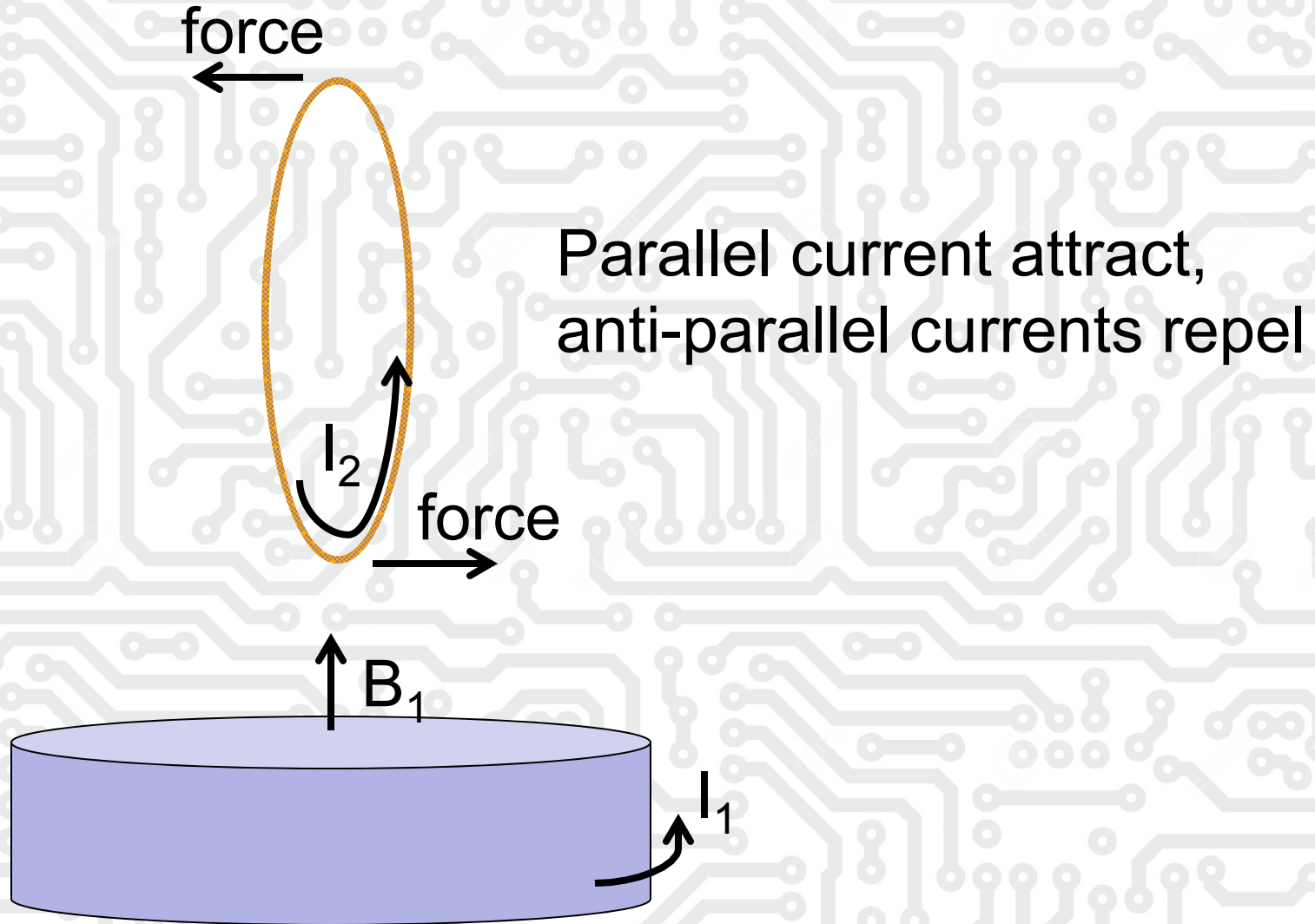
$$\omega' = \tau/I \approx 500 \text{ s}^{-2}$$

$$f' = \omega'/2\pi \approx 75 \text{ s}^{-2} \\ = 75 \text{ Hz / s}$$

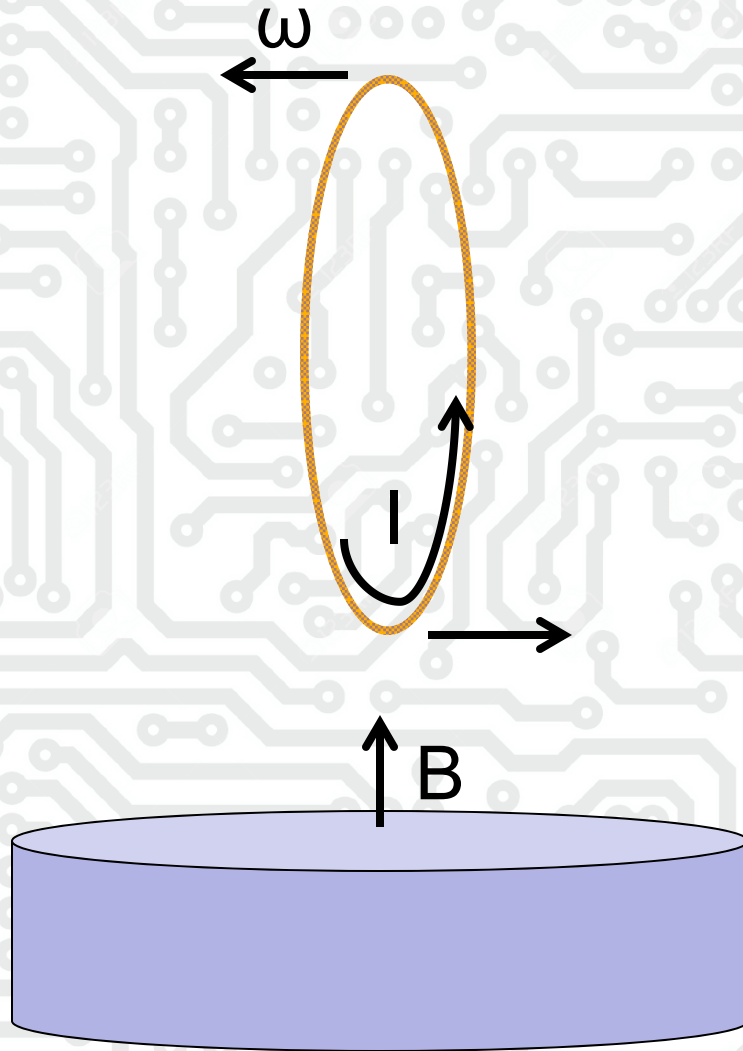
This is a large acceleration due to the small mass of the wire.



# The Current Picture



# Back EMF



The motion of the coil through the magnets  $B$  field (w/o a battery) induces a current in the wire...

...a so-called Back Electromotive Force ... or voltage ... drives this current.

**Our coil pulled a large current (4 Amps) to produce a small field (3 Gauss, similar to the Earth's). Luckily it was light (0.5 gram).**

**How can we do anything useful with magnetic fields with such a small permeability of free space?**

$$\mu_0 = 4\pi \times 10^{-7} \text{ T m / A}$$





# Relative permeability of materials

Material	$\mu / \mu_0$ (relative permeability)
Iron	200,000
Nickel or Carbon Steel	100
Wood	1.00000043
Air	1.0
Copper	0.999994
Superconductors	0

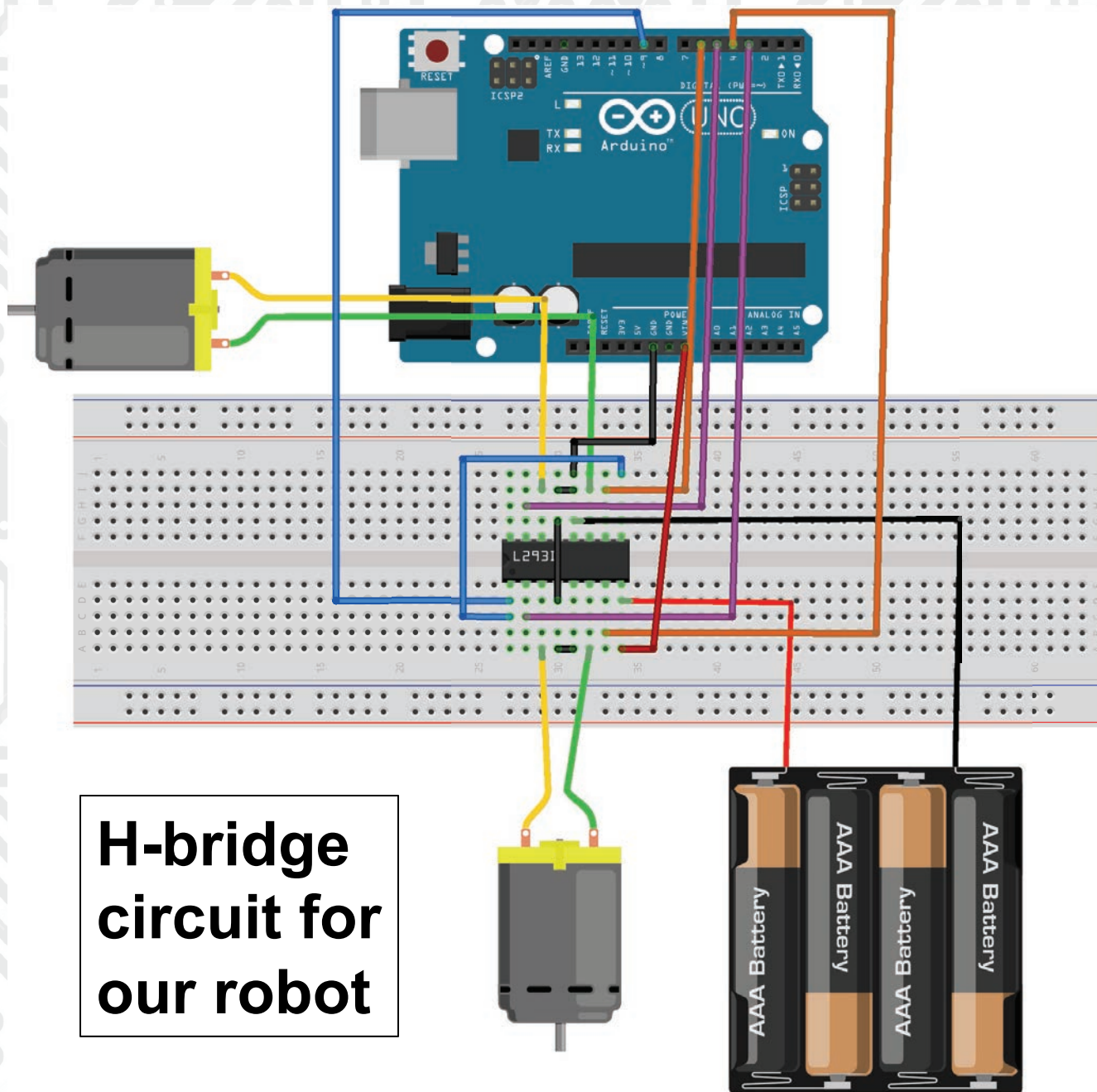
**Ferromagnetic**

**Paramagnetic**

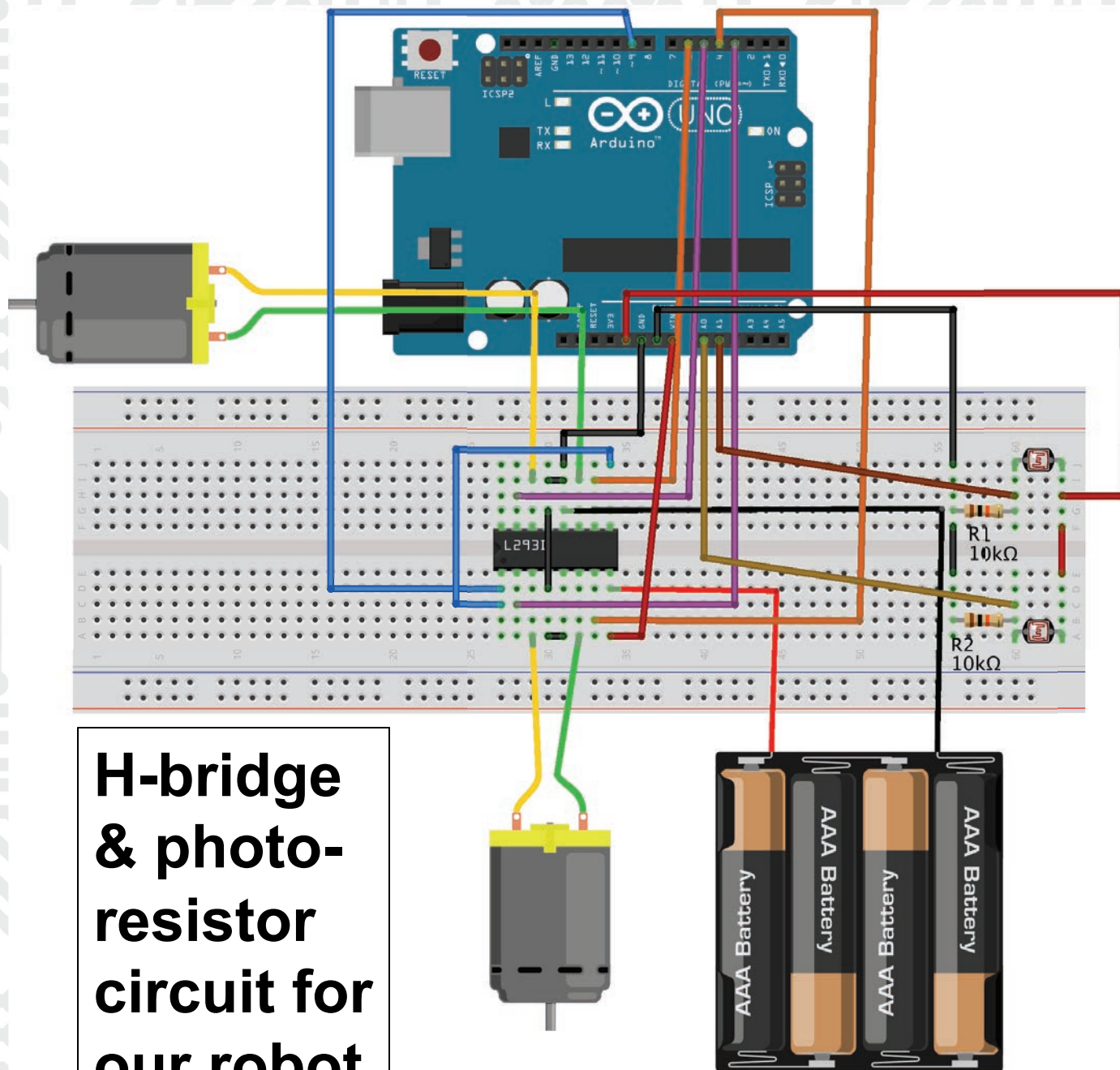
**Diamagnetic**

Ferromagnetism is the key that  
unlocks subatomic magnetism  
(spin) to the macro-world





**H-bridge  
circuit for  
our robot**



**H-bridge  
& photo-  
resistor  
circuit for  
our robot**



# Summary

- **Simple Wire Motor**
  - **Magnetic Field Picture**
  - **Current Picture**
- **Permeability (magnetic force constant) is small!**
- **Magnetic permeability of matter:**
  - **Diamagnetism**
  - **Paramagnetism**
  - **Ferromagnetism (a miracle!)**





# Units

**Distance – meters (m)**

**Mass – kilograms (kg)**

**Time – seconds (s)**

**Charge – Coulomb (C)**

**Current – Amp (A)**

**Resistance – Ohm ( $\Omega$ )**

**Magnetic Field – Tesla (T)**

**f – femto ( $10^{-15}$ )**

**p – pico ( $10^{-12}$ )**

**n – nano ( $10^{-9}$ )**

**$\mu$  – micro ( $10^{-6}$ )**

**m – milli ( $10^{-3}$ )**

**k – kilo ( $10^3$ )**

**M – mega ( $10^6$ )**

**G – giga ( $10^9$ )**

**T – tera ( $10^{12}$ )**

**P – peta ( $10^{15}$ )**