# Random walk model of order 1 (RW1)

#### Parametrization

The random walk model of order 1 (RW1) for the Gaussian vector  $\mathbf{x} = (x_1, \dots, x_n)$  is constructed assuming independent increments:

$$\Delta x_i = x_i - x_{i-1} \sim \mathcal{N}(0, \tau^{-1})$$

The density for  $\mathbf{x}$  is derived from its n-1 increments as

$$\pi(\mathbf{x}|\tau) \propto \tau^{(n-1)/2} \exp\left\{-\frac{\tau}{2} \sum (\Delta x_i)^2\right\}$$

$$= \tau^{(n-1)/2} \exp\left\{-\frac{1}{2} \mathbf{x}^T \mathbf{Q} \mathbf{x}\right\}$$

where  $\mathbf{Q} = \tau \mathbf{R}$  and  $\mathbf{R}$  is the structure matrix reflecting the neighbourhood structure of the model.

It is also possible to define a *cyclic* version of the RW1 model, in this case the graph is modified so that last node  $x_n$  is neighbour of  $x_{n-1}$  and  $x_1$ .

# **Hyperparameters**

The precision parameter  $\tau$  is represented as

$$\theta = \log \tau$$

and the prior is defined on  $\theta$ .

# Specification

The RW1 model is specified inside the f() function as

The (optional) argument values is a numeric or factor vector giving the values assumed by the covariate for which we want the effect to be estimated. See next example for an application.

The logical option scale.model determine if the model should be scaled to have an average variance (the diagonal of the generalized inverse) equal to 1. This makes prior spesification much easier. Default is FALSE so that the model is not scaled.

#### Current recommended prior

If you do not know which prior to use, the current recommendation is

where u should be set to a value appropriate for your case:

Gaussian likelihood (no link) Set u to be the empirical standard deviation of your data

Poisson likelihood and log link Set u to 1

Binomial and logit link Set u to 0.5

Binomial and probit link Set u to 0.33

Increasing u gives a weaker prior, decreasing u gives a stronger prior.

#### Hyperparameter spesification and default values

```
doc Random walk of order 1
hyper
    theta
         hyperid 4001
         name log precision
         short.name prec
         prior loggamma
         param 1 5e-05
         initial 4
         fixed FALSE
         to.theta function(x) log(x)
         from.theta function(x) exp(x)
constr TRUE
nrow.ncol FALSE
augmented FALSE
aug.factor 1
aug.constr
n.div.by
n.required FALSE
set.default.values FALSE
min.diff 1e-05
pdf rw1
Example
n=100
z=seq(0,6,length.out=n)
y=sin(z)+rnorm(n,mean=0,sd=0.5)
data=data.frame(y=y,z=z)
formula=y~f(z,model="rw1",
            hyper = list(prec = list(prior="loggamma",param=c(1,0.01))))
result=inla(formula,data=data,family="gaussian")
```

### Notes

- The RW1 is intrinsic with rank deficiency 1.
- The RW1 model for irregular locations are supported although not described here.
- The term  $\frac{1}{2}\log(|R|^*)$  of the normalisation constant is not computed, hence you need to add this part to the log marginal likelihood estimate, if you need it.