# qLogLogistic likelihood

#### Parametrisation

The LogLogistic distribution has cumulative distribution function

$$F_0(y) = \frac{1}{1 + \lambda y^{-\alpha}}, \quad y > 0$$

if variant=0, or

$$F_1(y) = \frac{1}{1 + (\lambda y)^{-\alpha}}, \quad y > 0$$

if variant=1, where

 $\alpha > 0$  is a shape parameter, and

 $\lambda > 0$  is a scale parameter.

The  $\lambda$  is defined implicitely through the quantile, as

$$F_0(y_q) = q$$
, or  $F_q(y_q) = q$ ,  $0 < q < 1$ 

and the linear predictor is defined on  $y_q$ .

#### **Link-functions**

The parameter  $\lambda$  is linked to the linear predictor, implicitely through

$$y_q = \exp(\eta)$$

## Hyperparameters

The  $\alpha$  parameter is represented as

$$\theta = \log \alpha$$

and the prior is defined on  $\theta$ .

### **Specification**

- family equals qloglogistic (regression) or qloglogisticsurv (survival)
- variant=0 (default) or 1, chosing between parameterisation  $F_0$  or  $F_1$ .
- Required arguments: y (regression) or an inla.surv-object using inla.surv() (for survival data), and quantile= q.

#### Hyperparameter spesification and default values

#### Regression:

doc A quantile loglogistic likelihood

### hyper

theta

hyperid 60011 name log alpha

```
short.name alpha
         initial 1
         fixed FALSE
         prior loggamma
         param 25 25
         to.theta function(x) log(x)
         from.theta function(x) exp(x)
survival FALSE
discrete FALSE
link default log neglog
pdf qloglogistic
   Survival:
doc A quantile loglogistic likelihood (survival)
hyper
     theta
         hyperid 60021
         name log alpha
         short.name alpha
         initial 1
         fixed FALSE
         prior loggamma
         param 25 25
         to.theta function(x) log(x)
         from.theta function(x) exp(x)
survival TRUE
discrete FALSE
link default log neglog
pdf qloglogistic
Example
In the following example we estimate the parameters in a simulated case
lam_loglogistic = function(yq, alpha, q, variant = 0)
{
    if (variant == 0) {
        lambda = yq^alpha * (1/q-1)
    } else if (variant == 1) {
        lambda = 1/yq * (1/(1/q-1))^(1/alpha)
    } else
        stop("ERR")
    return (lambda)
```

```
}
rloglogistic = function(n, lambda, alpha, variant=0)
    u = runif(n)
    if (variant == 0) {
        y = (lambda/(1.0/u - 1.0))^(1.0/alpha)
    } else if (variant == 1) {
        y = (1.0/(1.0/u -1.0))^(1.0/alpha) / lambda
    } else {
        stop("ERROR")
    }
}
n = 500
alpha = 2.1
x = c(scale(runif(n)))
eta = 1.1+2.2*x
yq = exp(eta)
for(variant in 0:1) {
    for(q in c(0.2, 0.8)) {
        print(paste("variant=", variant, "quantile=", q))
        lambda = lam_loglogistic(yq, alpha, q, variant=variant)
        y = rloglogistic(n,
                         lambda = lambda,
                         alpha = alpha,
                         variant = variant)
        formula = y \sim 1 + x
        rr=inla(formula,
               family ="qloglogistic",
               data=data.frame(y, x),
               control.family = list(list(variant = variant, control.link = list(quantile = q)
        print("REGRESSION")
        print(summary(rr))
        event = rep(1,n)
        formula=inla.surv(y,event) ~ 1 + x
        r=inla(formula,
               family ="qloglogisticsurv",
               data = list(y=y, event=event, x=x),
               control.family = list(list(variant = variant, control.link = list(quantile = q)
        print("SURVIVAL")
        print(summary(r))
    }
}
```

## Notes

• Loglogistic surv model can be used for right censored, left censored, interval censored data. If the observed times y are large/huge, then this can cause numerical overflow in the likelihood routine. If you encounter this problem, try to scale the observatios, time = time / max(time) or similar.