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AMS 595 Project #2

https://github.com/jdsipala/ams595-project2

This project uses MATLAB to explore the Mandelbrot set. The idea of the Mandelbrot set comes from fractal geometry, which studies never-ending patterns found in nature or other complex examples, such as the coastline paradox. The idea is that these patterns repeat at different scales and measuring more closely makes the length of the measurement grow infinitely. I created 5 different functions based on the assignment I was given. The contents given in the assignment show how a series of functions can all be used in sequence to recreate and analyze the Mandelbrot set. Each function plays a role in creating the boundary and structure of the fractal.

The first function I created was "fractal". This function uses a complex number \mathbf{c} and the formula $z = z^2 + c$ and loops until the function either diverges (absolute value of z > 2) or reaches 100 iterations (per the assignment). This tells us whether the point is inside or outside the Mandelbrot set and how quickly it diverges.

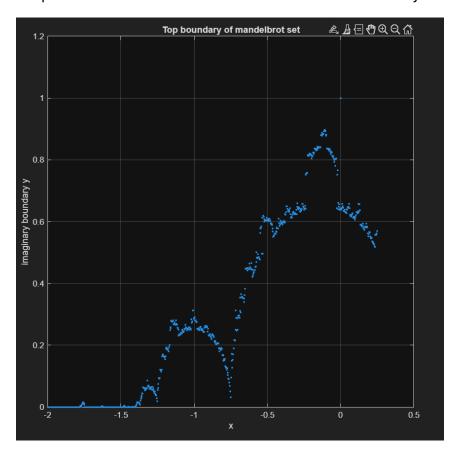
The indicator function is a simple function that basically tests whether each complex point is inside or outside the Mandelbrot set. The indicator function uses the fractal function and returns a -1 or +1 based on whether the number of iterations run from the fractal function was 100 or not. This function essentially checks along a vertical line at a given x-value and tests multiple y-values to see where the sign changes from inside to outside (-1 to +1).

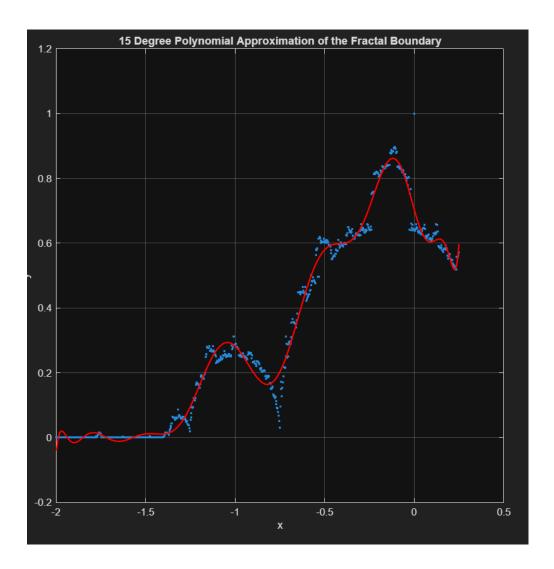
The bisection function is used to find the boundary between the inside and outside of the Mandelbrot set. It takes the indicator function and two starting points (one inside the Mandelbrot set and one outside) and repeatedly splits the interval in half until it finds the midpoint where the sign changes from -1 to +1. This midpoint is essentially a point on the boundary. Repeating this process for around 1000 points is what creates the curve that you see on the graphs below that outlines the shape of the fractal.

The final function "poly_len" is used to calculate the total curve length of the fitted polynomial that represents the Mandelbrot boundary. Using the formulas given in the assignment and MATLAB's built-in integral function, this function gives an estimate of how long the curve is based on a particular range. In my main script, I have used the result from the "poly_len" function and doubled it. Doubling the length of the curve gives an estimate of the length of the full outline of the fractal.

Overall, this was a very interesting project, this is not a topic I was familiar with before I started this assignment. It was a great introduction to the Mandelbrot set by using simple

functions to create complex shapes. Also, measuring and visualizing these complex shapes. Below are some visualizations and results. Thank you.





>> combined_code
Full outline length of the fractal boundary: 6.1042