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Author One Author Two

1 Introduction

Introduction

2 Methods

Methods

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2.1 The Model

The Model

Let $s_i \in \mathrm{R}^2$ for each $i = 1, \ldots, n$ denote a spatial location for which we have collected data, and $\boldsymbol{s} = (s_1, \ldots, s_n)^\top$ be the vector of all such locations. Then $Y(\boldsymbol{s})$ are univariate dependent variables and $\boldsymbol{X}(\boldsymbol{s}) = (X_1(\boldsymbol{s}), \ldots, X_p(\boldsymbol{s}))^\top$ are $p \times 1$ vectors of covariates. A linear regression model with spatially varying coefficients assumes $Y(\boldsymbol{s})$ are dependent on $\boldsymbol{X}(\boldsymbol{s})$ as follows:

$$Y(\boldsymbol{s}) = \sum_{j=1}^{p} X_r(\boldsymbol{s}) w_r(\boldsymbol{s}) + w_0(\boldsymbol{s}) + \epsilon(\boldsymbol{s}),$$

where $w_r(\boldsymbol{s})$ are the regression coefficients corresponding to $X_r(\boldsymbol{s})$, $w_0(\boldsymbol{s})$ are spatial random effects, and $\epsilon(\boldsymbol{s})$ are independently and identically distributed multivariate normal (MVN) measurement errors, i.e.

$$\epsilon(\boldsymbol{s}) \sim \mathrm{MVN}(0, \tau^2 I_n).$$

For convenience, we will treat $w_0(\boldsymbol{s})$ as the regression coefficients corresponding to $X_0(\boldsymbol{s}) = \mathbf{1}_n$, a vector of length n with every element equal to 1.

For each $w_r(\boldsymbol{s})$, we assign a Gaussian process (GP) prior with squared exponential covariance, i.e.

$$w_r \sim \mathrm{GP}(0, C(s, s')), \text{ where}$$
$$C(s, s') = \sigma_r^2 \exp \phi_r^{-1} \|s - s'\|^2.$$

We refer to $C(\cdot, \cdot)$ as the covariance function, which calculates the covariance between two locations s and s'. Squared exponential covariance is a commonly used covariance function due to useful properties such as infinite smoothness and gradual decrease in covariance with distance. The parameter σ_r^2 is the spatial variance and ϕ_r is the spatial decay, which indicates how quickly correlation decreases with the squared distance.

We assign inverse-gamma conjugate priors for the variance parameters τ^2 and each σ_r^2. In addition, each phi_r is assigned a uniform prior. In summary,

$$\tau^2 \sim \text{Inv.Gamma}(\alpha_\tau, \beta_\tau),$$
$$\sigma_r^2 \sim \text{Inv.Gamma}(\alpha_r, \beta_r), \text{ and}$$
$$\phi_r \sim \text{Uniform}(l_r, u_r).$$

3 Conclusion

Conclusion