

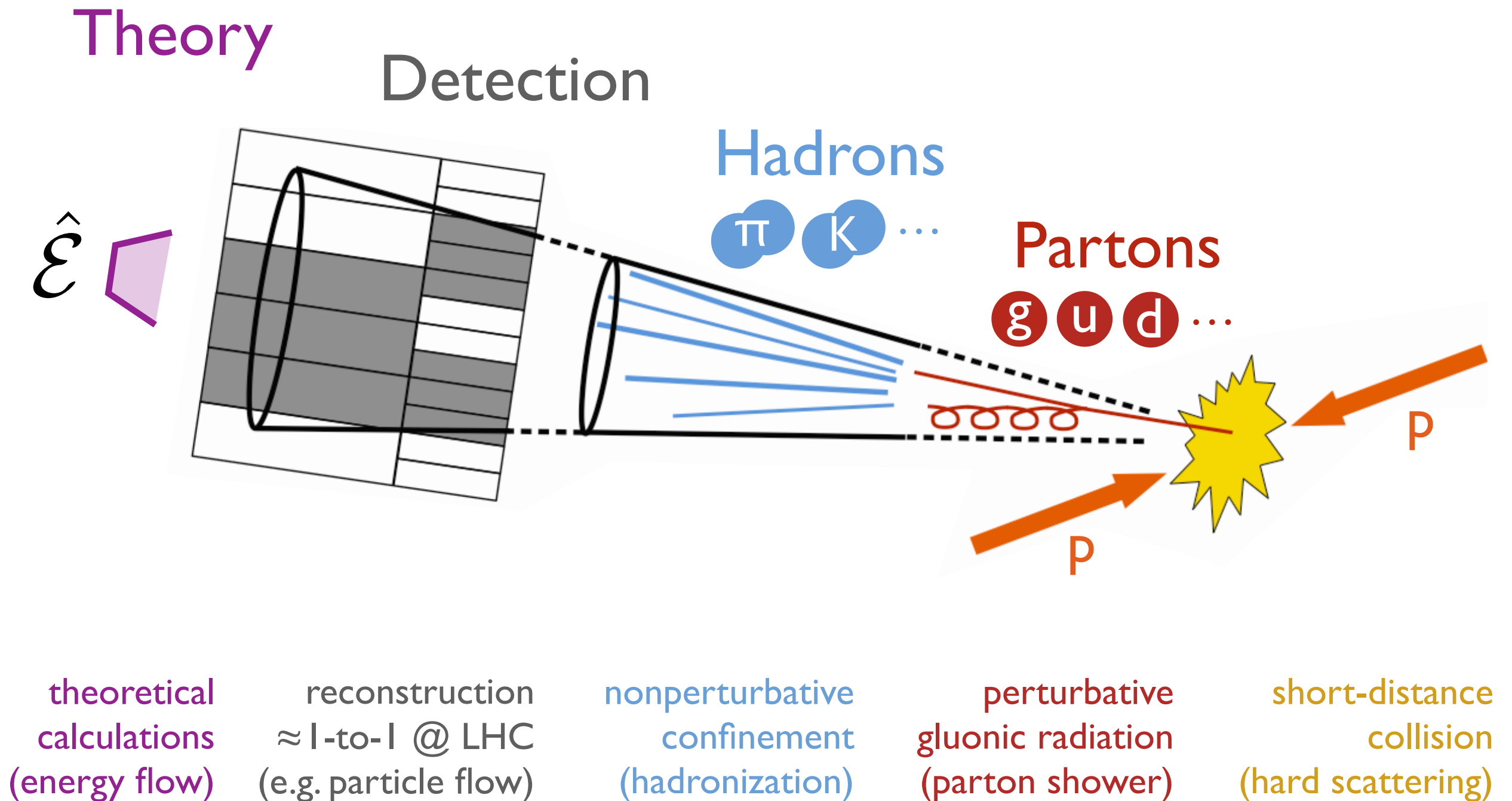
# Introduction to Jet Physics from QCD

Jesse Thaler



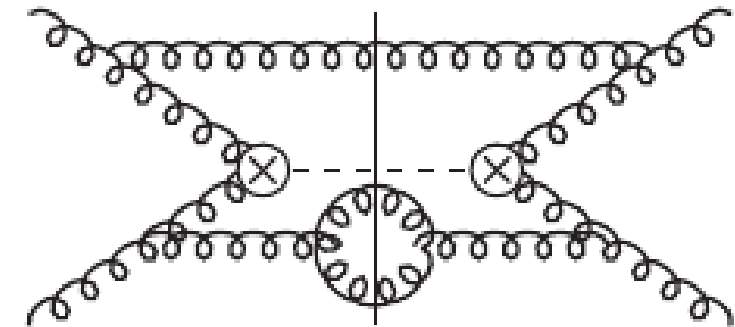
KEK — January 16, 2017

# From Last Week



# QCD Renaissance

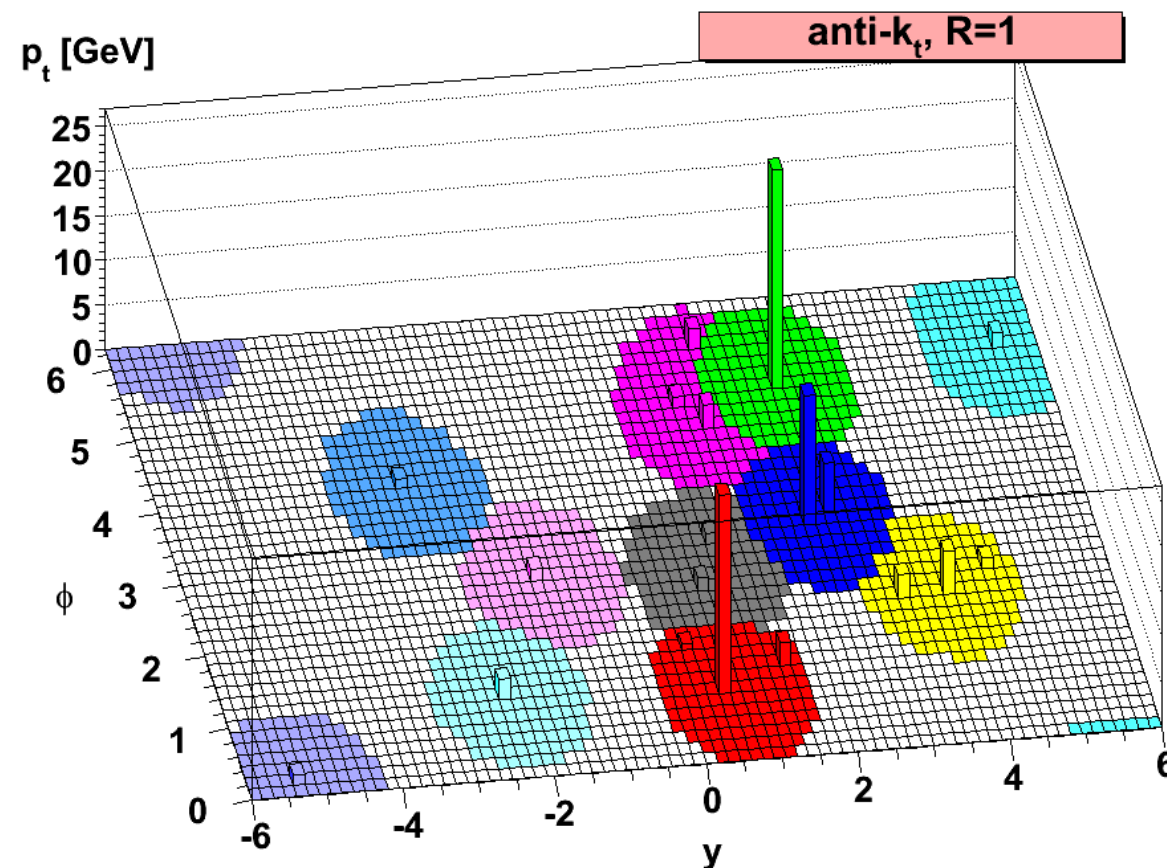
Theory c. 2008–present



Loop/Leg/Log Explosion



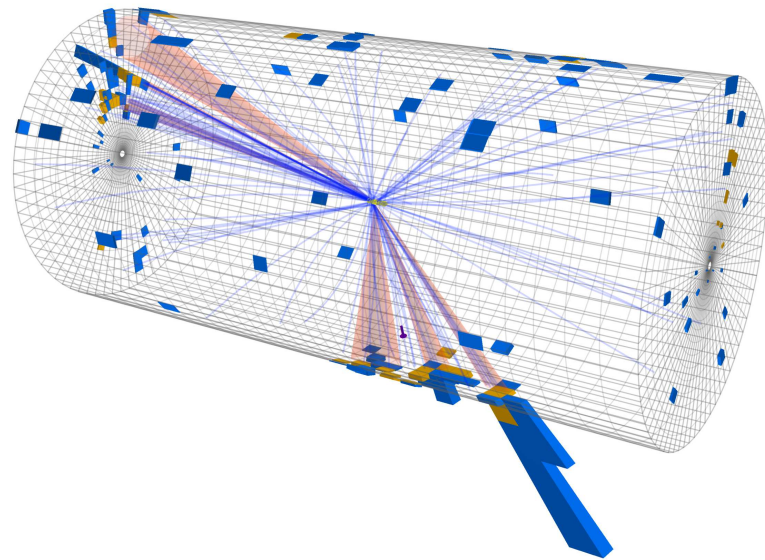
Jet Substructure



New Jet Algorithms

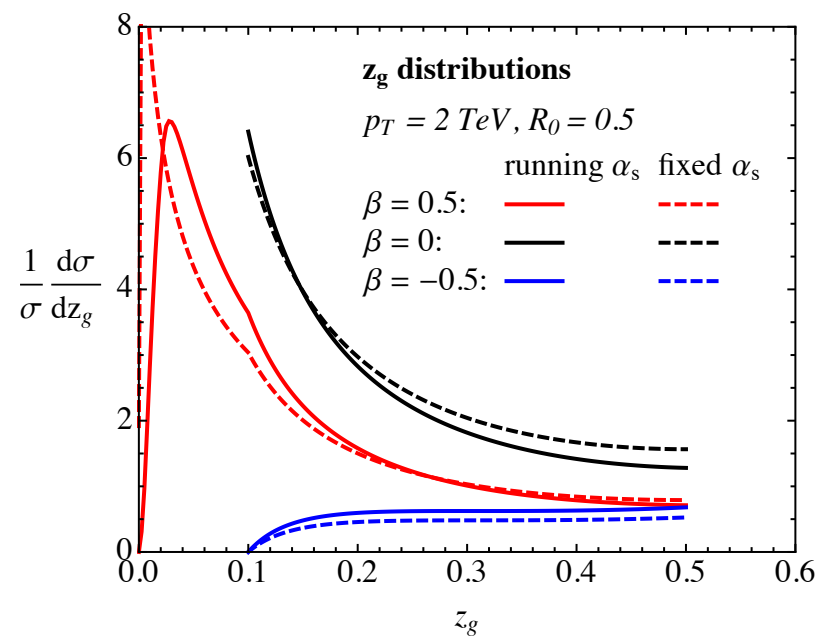
[Anti-k<sub>t</sub>: Cacciari, Salam, Soyez, 2008; see also Delsart, 2006] [N<sup>3</sup>LO: Anastasiou, Duhr, Dulat, Herzog, Mistlberger, 2015]  
[BDRS: Butterworth, Davison, Rubin, Salam, 2008; see also Seymour, 1991, 1994]

# Jet Substructure



## *Boosting the Search for New Phenomena*

[Last Thursday & Friday]



## *Pushing the Boundaries of Quantum Field Theory*

[Today & Tuesday]

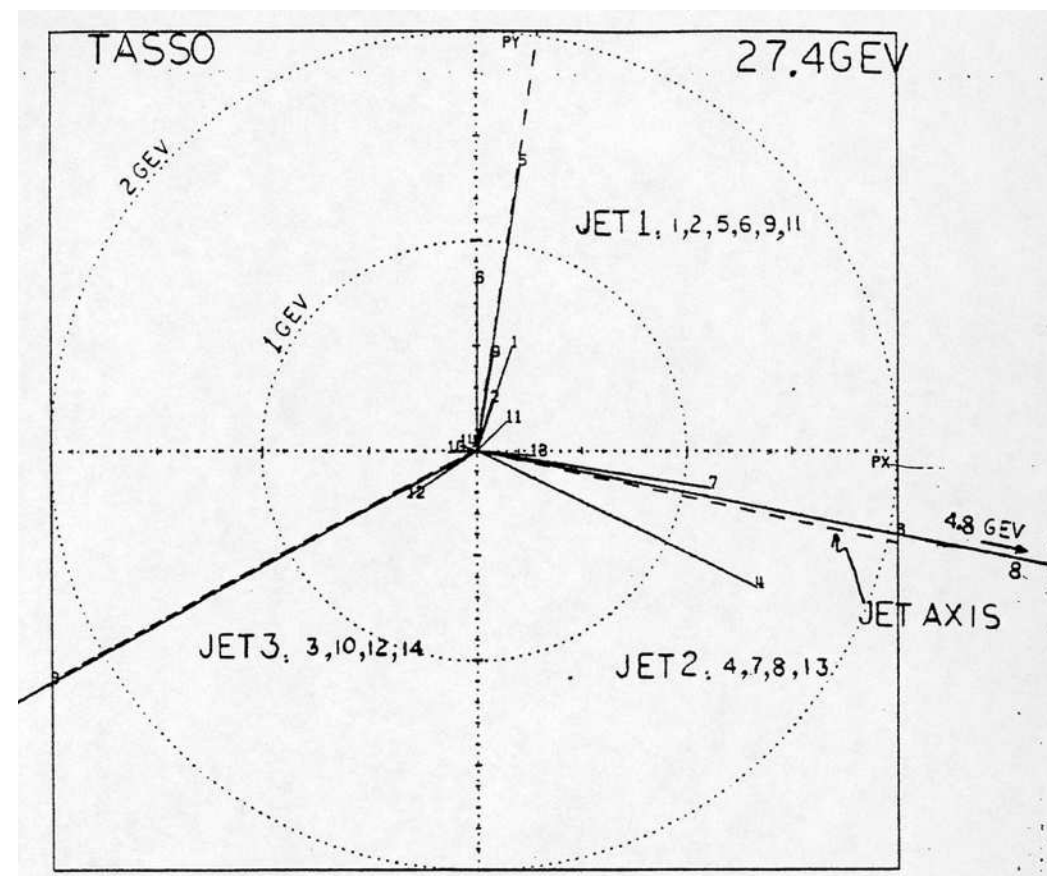
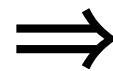
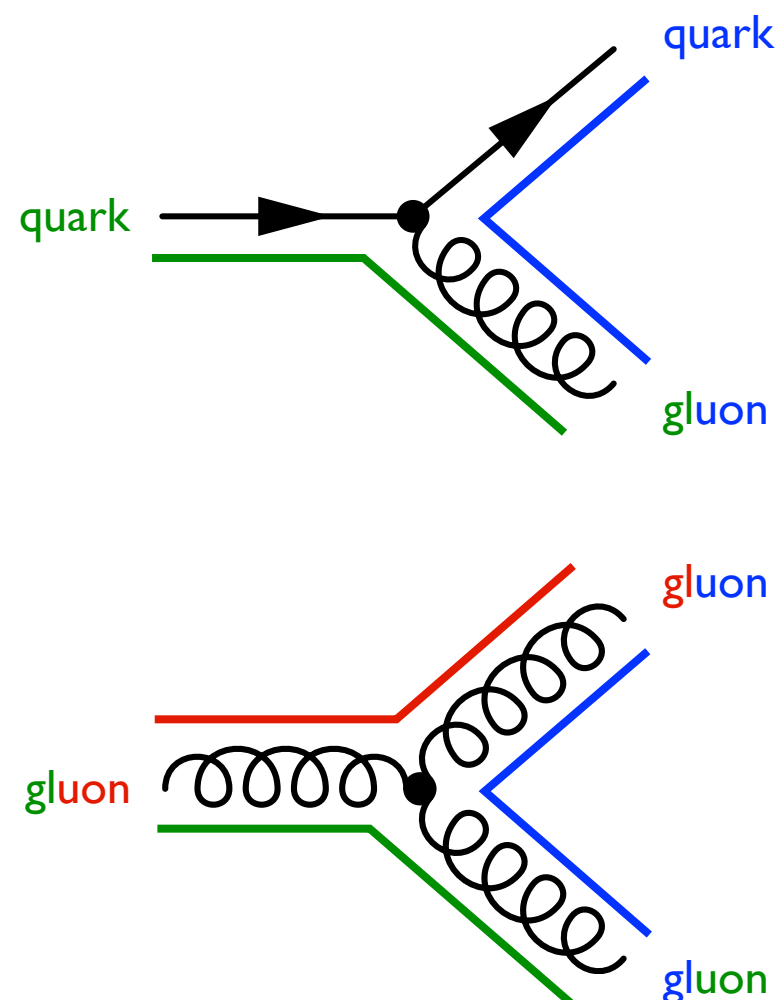
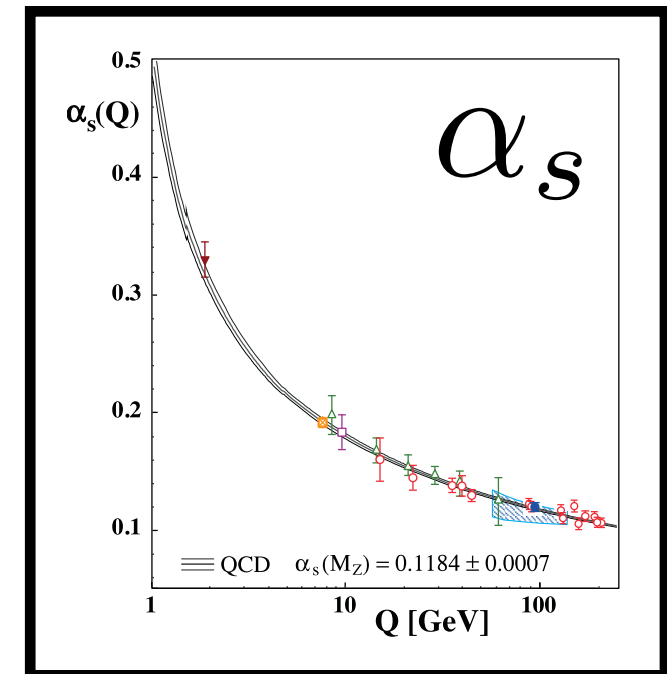
# *Dynamics of Jet Formation*

$$\mathcal{L} = \frac{1}{4g^2} G_{\mu\nu}^a G_{\mu\nu}^a + \sum_j \bar{q}_j (i\gamma^\mu D_\mu + m_j) q_j$$

where  $G_{\mu\nu}^a \equiv \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + if_{bc}^a A_\mu^b A_\nu^c$

and  $D_\mu \equiv \partial_\mu + it^a A_\mu^a$

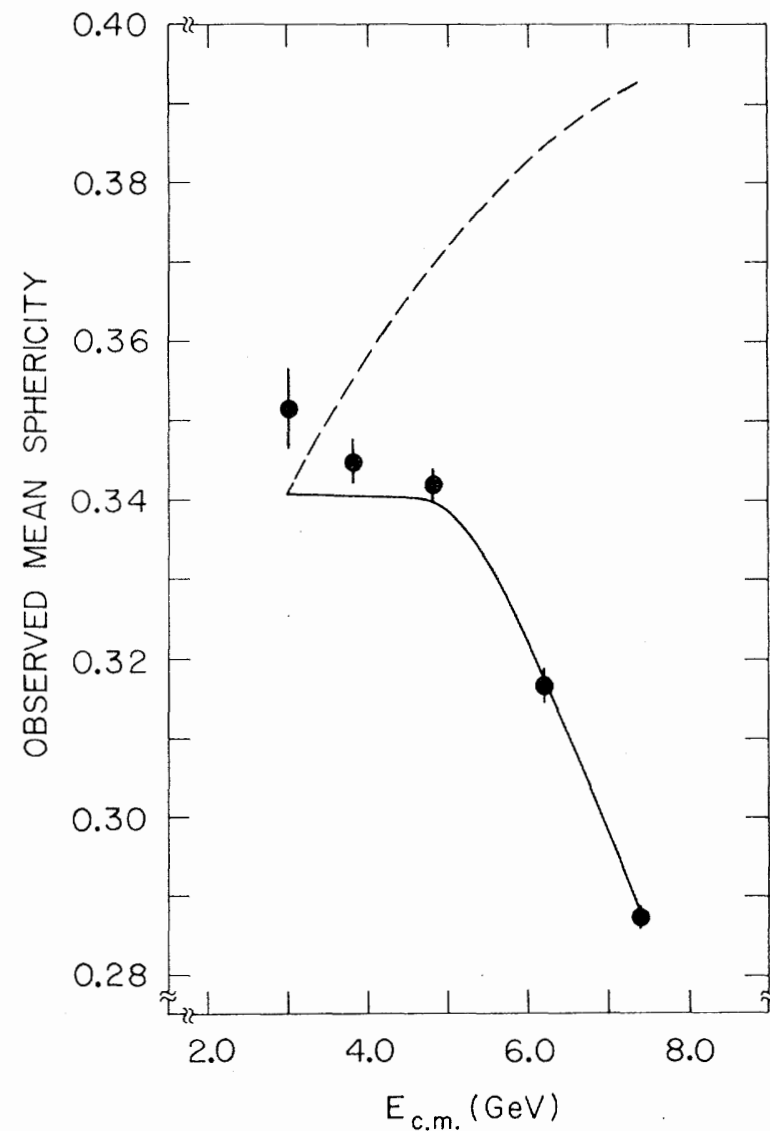
That's it!



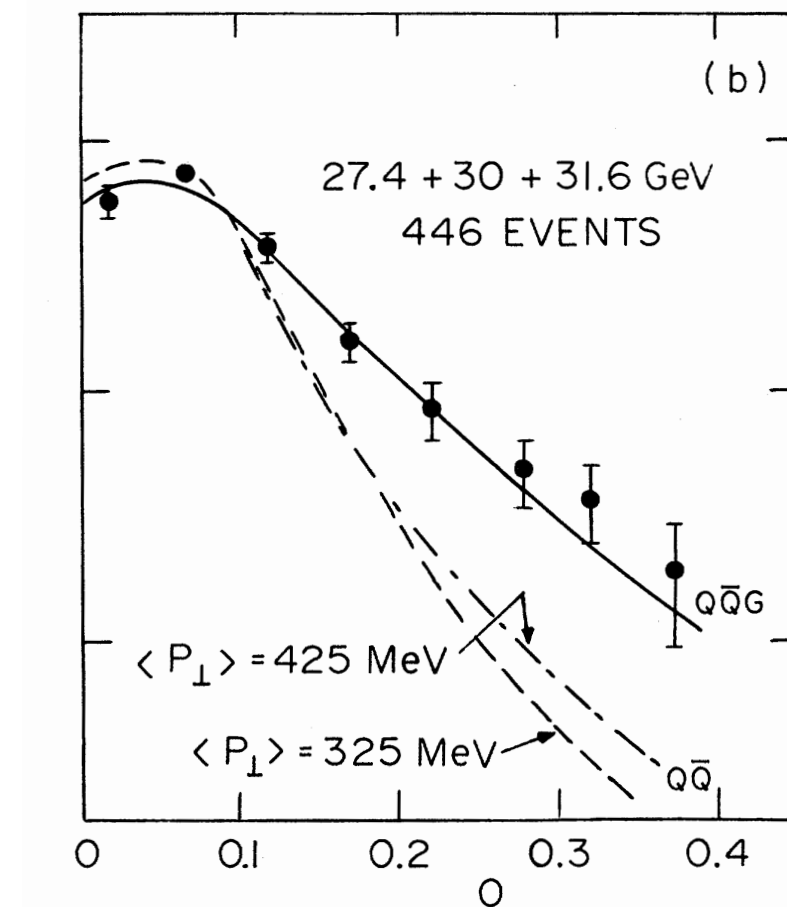
[TASSO @ PETRA @ DESY, 1979]

# First Light on Jets

*Jets @ SPEAR, 1975*



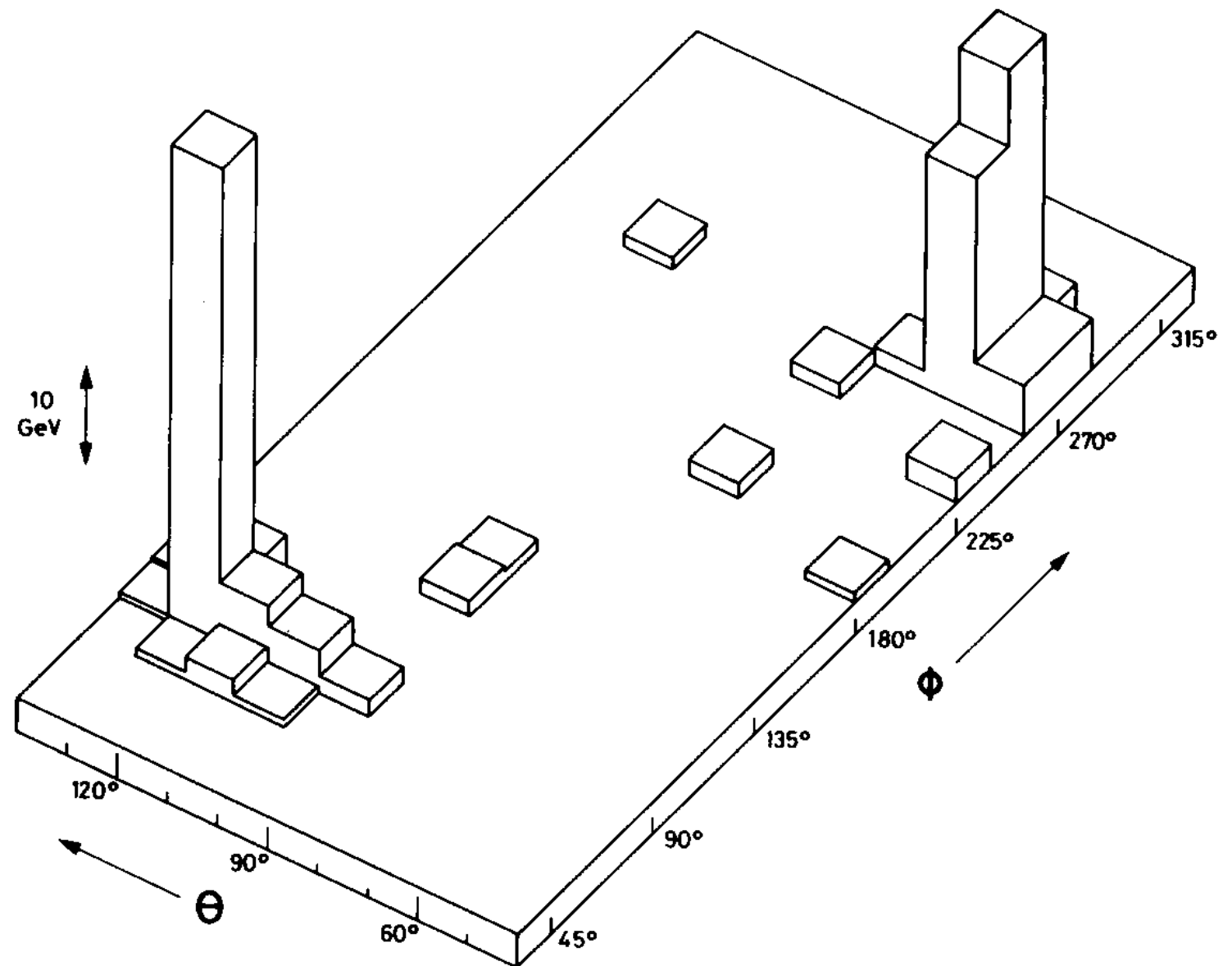
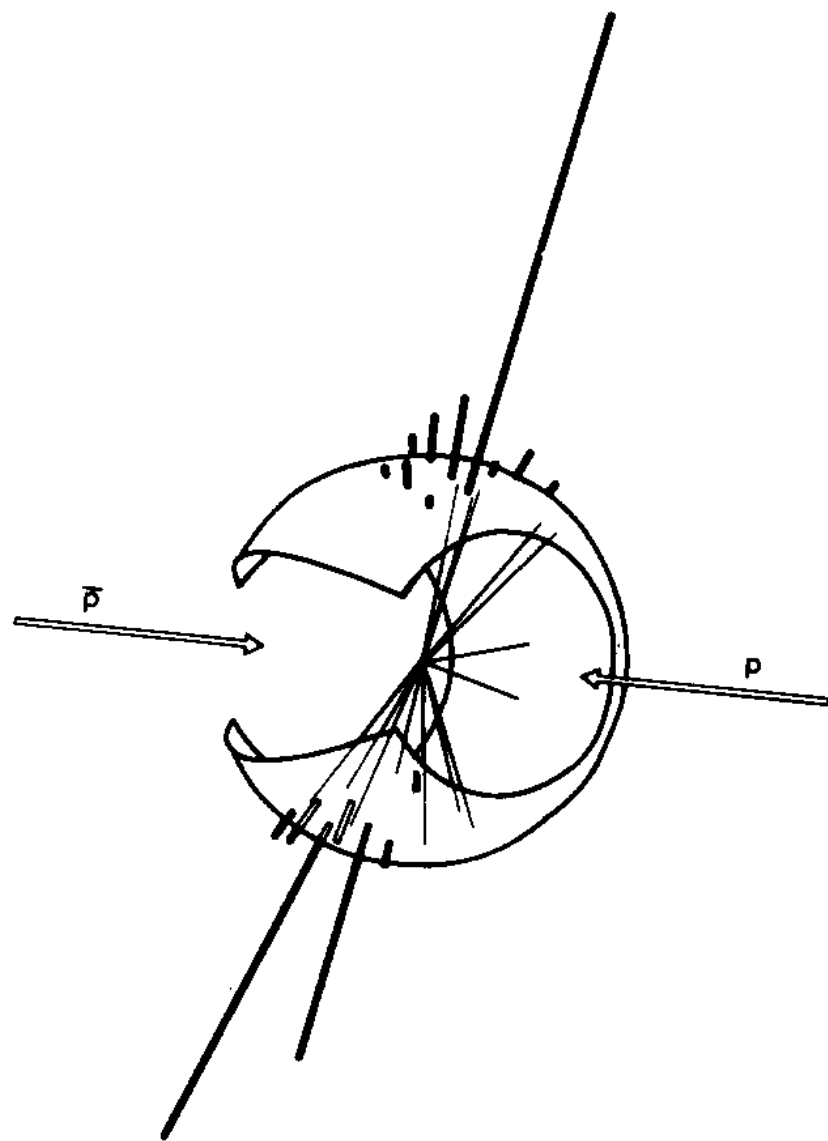
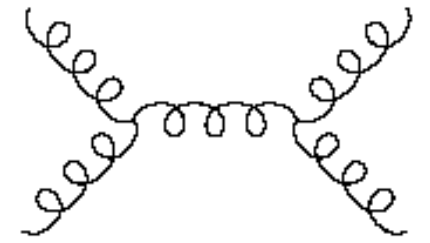
*Gluons @ PETRA, 1979*



Event shapes to probe jet formation

# Four Decades of Jets and QCD

UA2, 1982

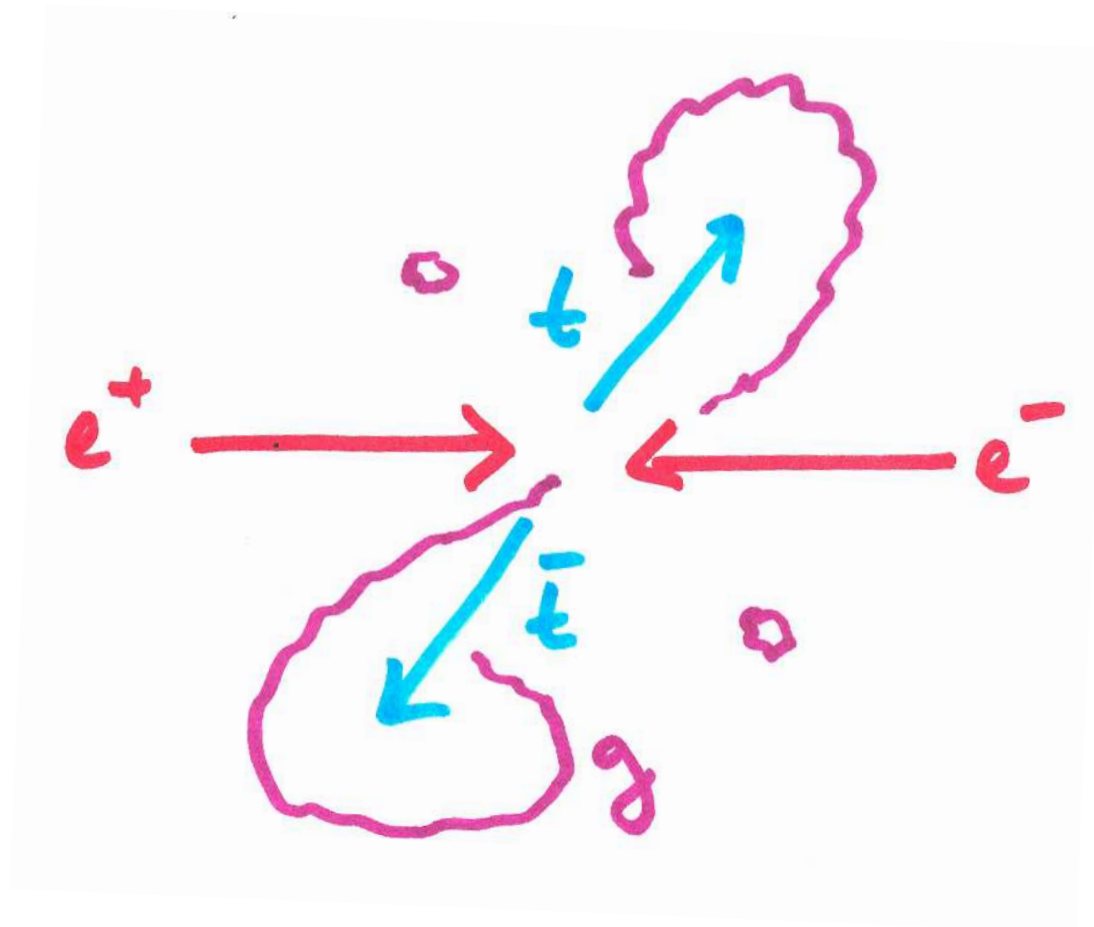


Jet algorithms: interpret cluster of hadrons as quasi-parton



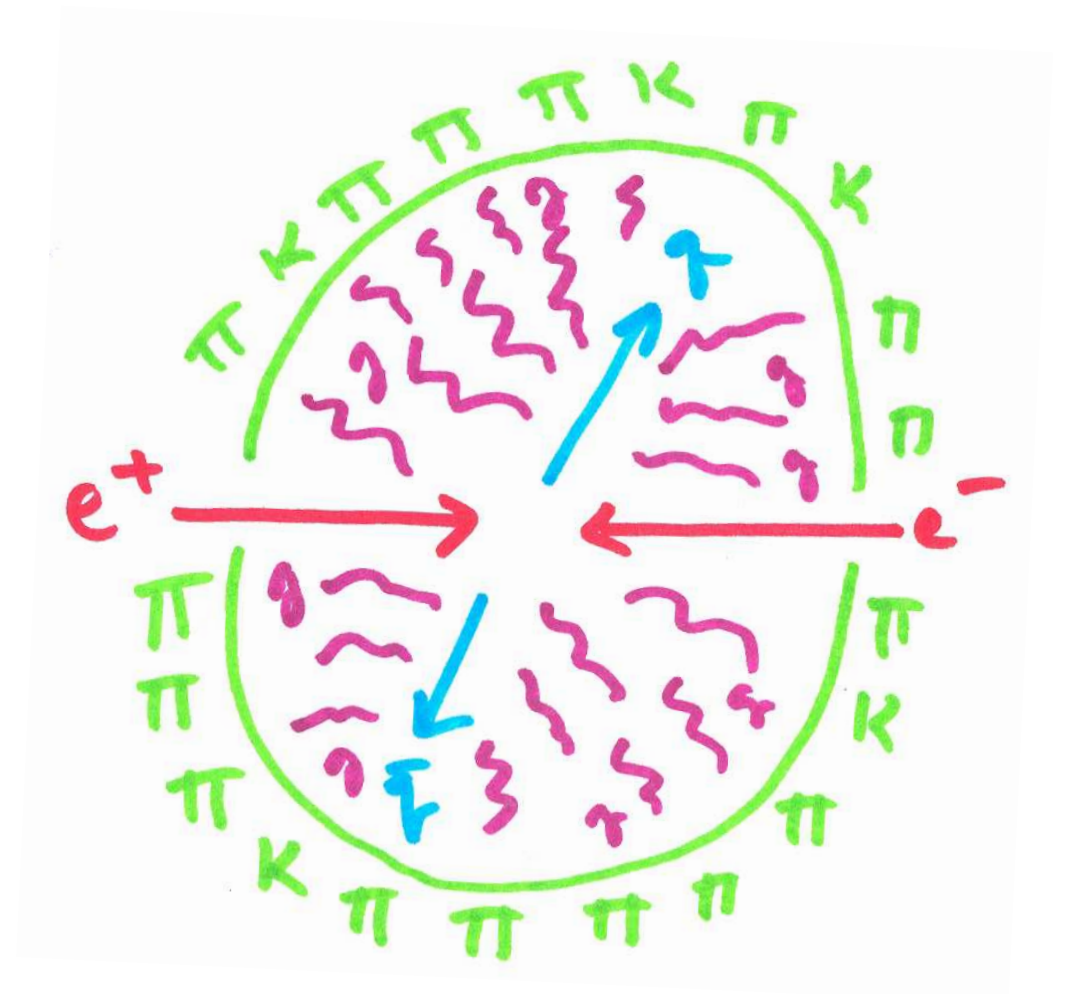
# Jets are not automatic!

## Quirky World (QCD with only top quark)



Can't break color flux tubes!  
Just “toponium” and glueballs!

## Quasi-Conformal World ( $\beta \approx 0, g \approx 4\pi$ )

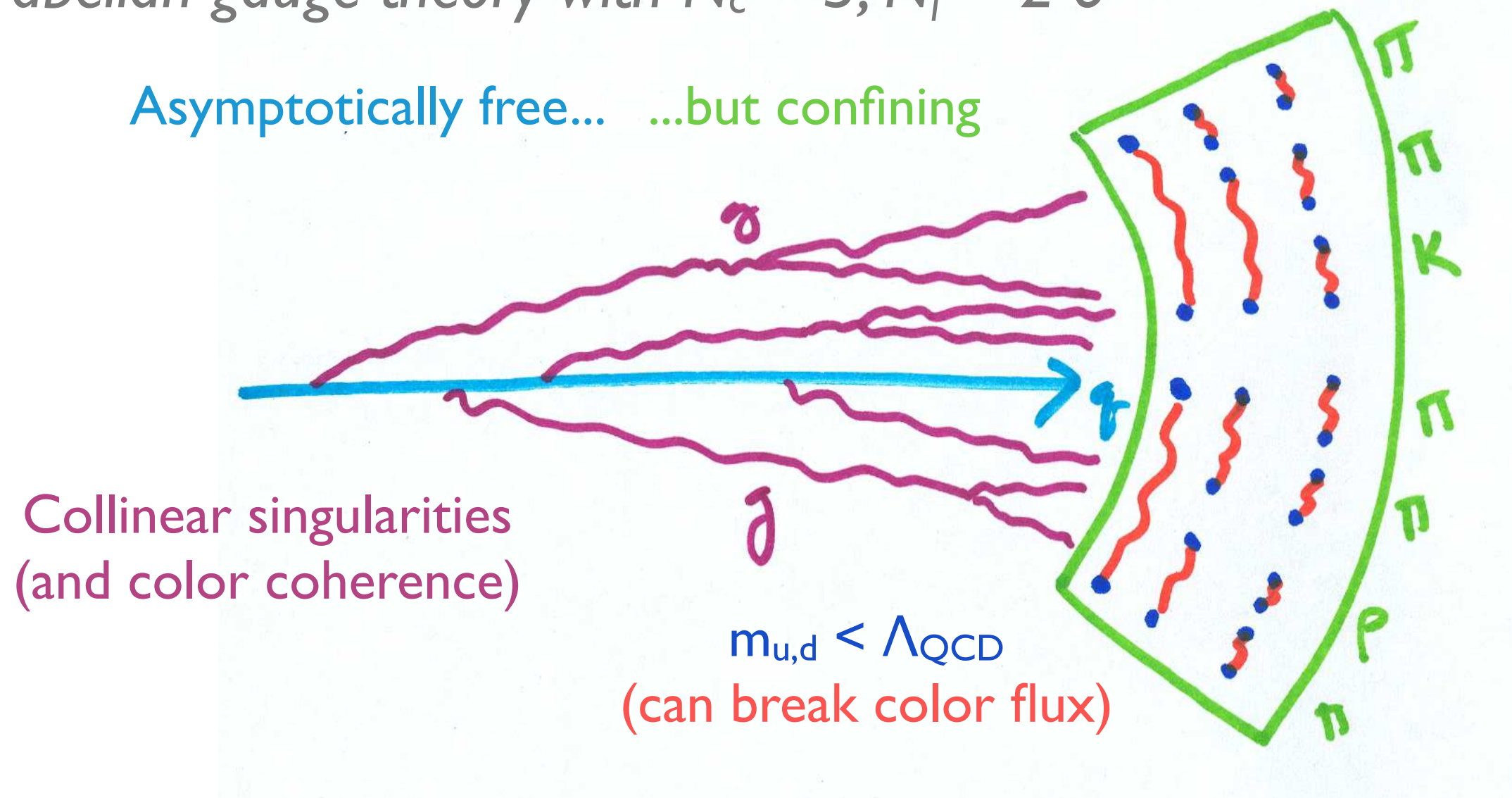


No hierarchy of scales!  
All “spherical” events!

# Jets are emergent property of QCD

# Non-abelian gauge theory with $N_c = 3$ , $N_f = 2-6$

# Asymptotically free... ..but confining



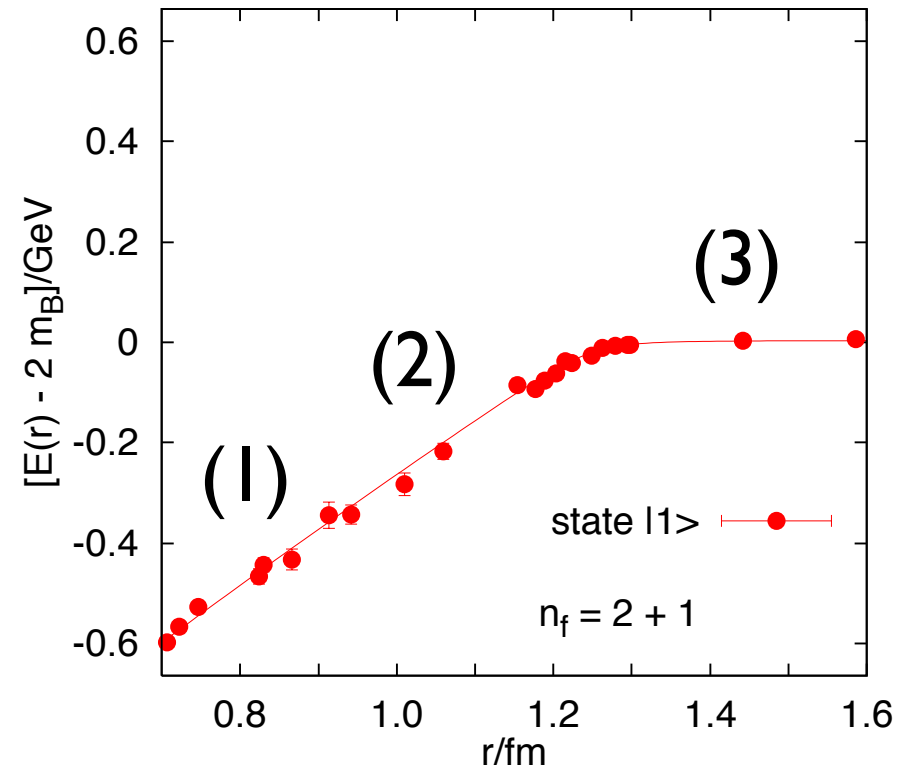
## Energy flow in UV (where partons go)



## Energy flow in IR (where hadrons go)

Jet = quark/gluon + radiation + ambiguities ( $m_J \approx 10\% p_{TJ}$ )

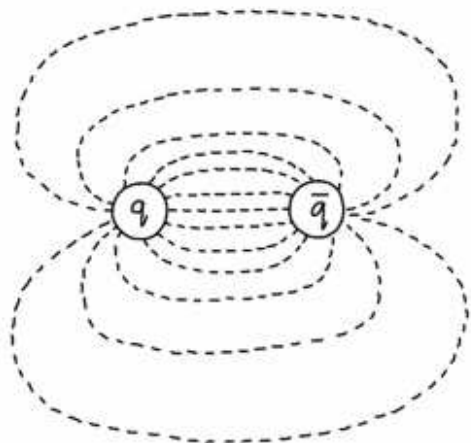
# Confinement/Liberation



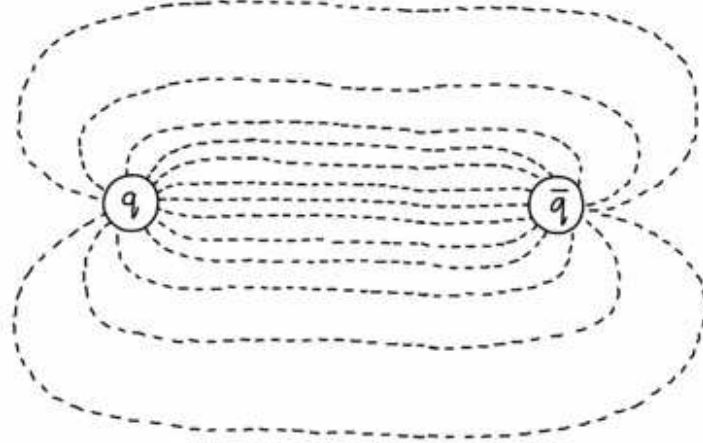
Potential between two heavy quarks (from lattice calculation)

[SESAM, 2005]

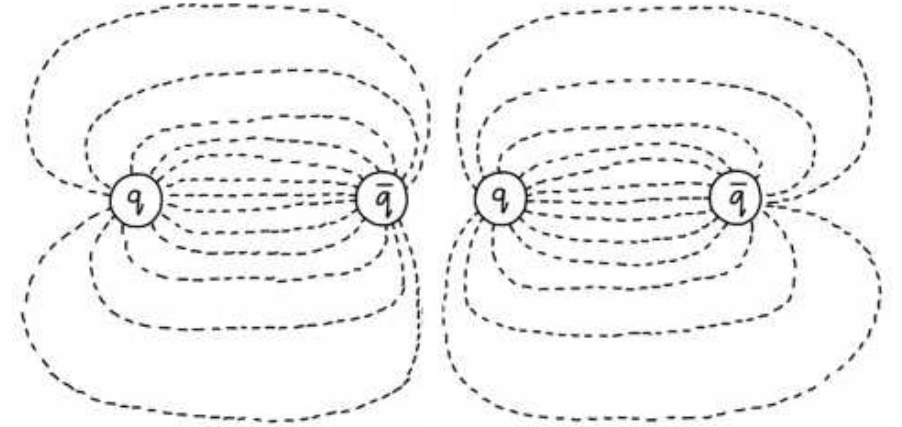
(1)



(2) = linear confinement



(3) = string breaking



[pictures from [coffeeshopphysics.com](http://coffeeshopphysics.com)]

# String picture gives jet basics

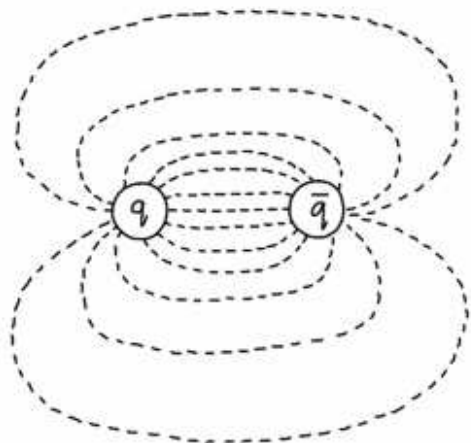
String breaks easily: Quark/gluon direction  $\approx$  Jet direction

String has energy density: Massless quarks/gluons  $\rightarrow$  Massive jet

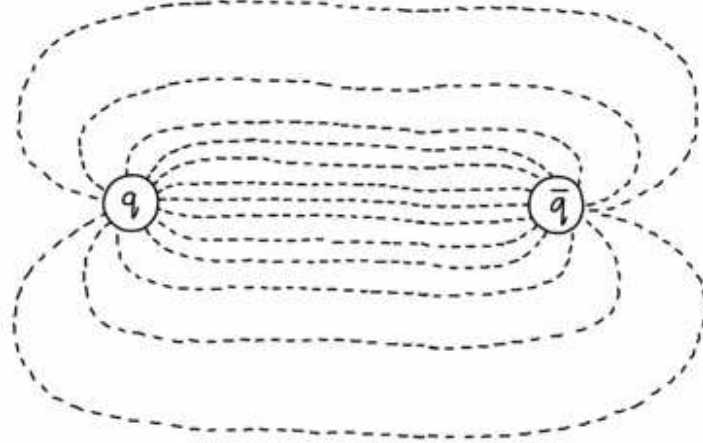
String breaks by popping quarks: Jets are mostly  $q \bar{q}$  bound states (mesons)

String is color singlet: Jets are fundamentally ambiguous

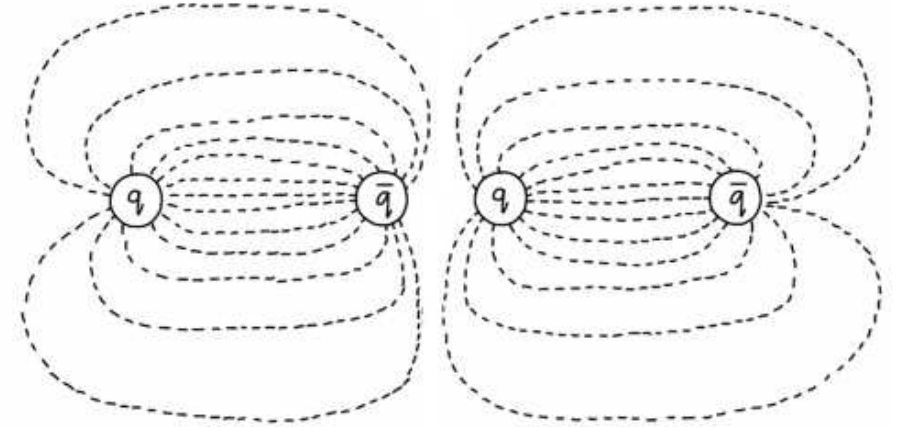
(1)



(2) = linear confinement



(3) = string breaking



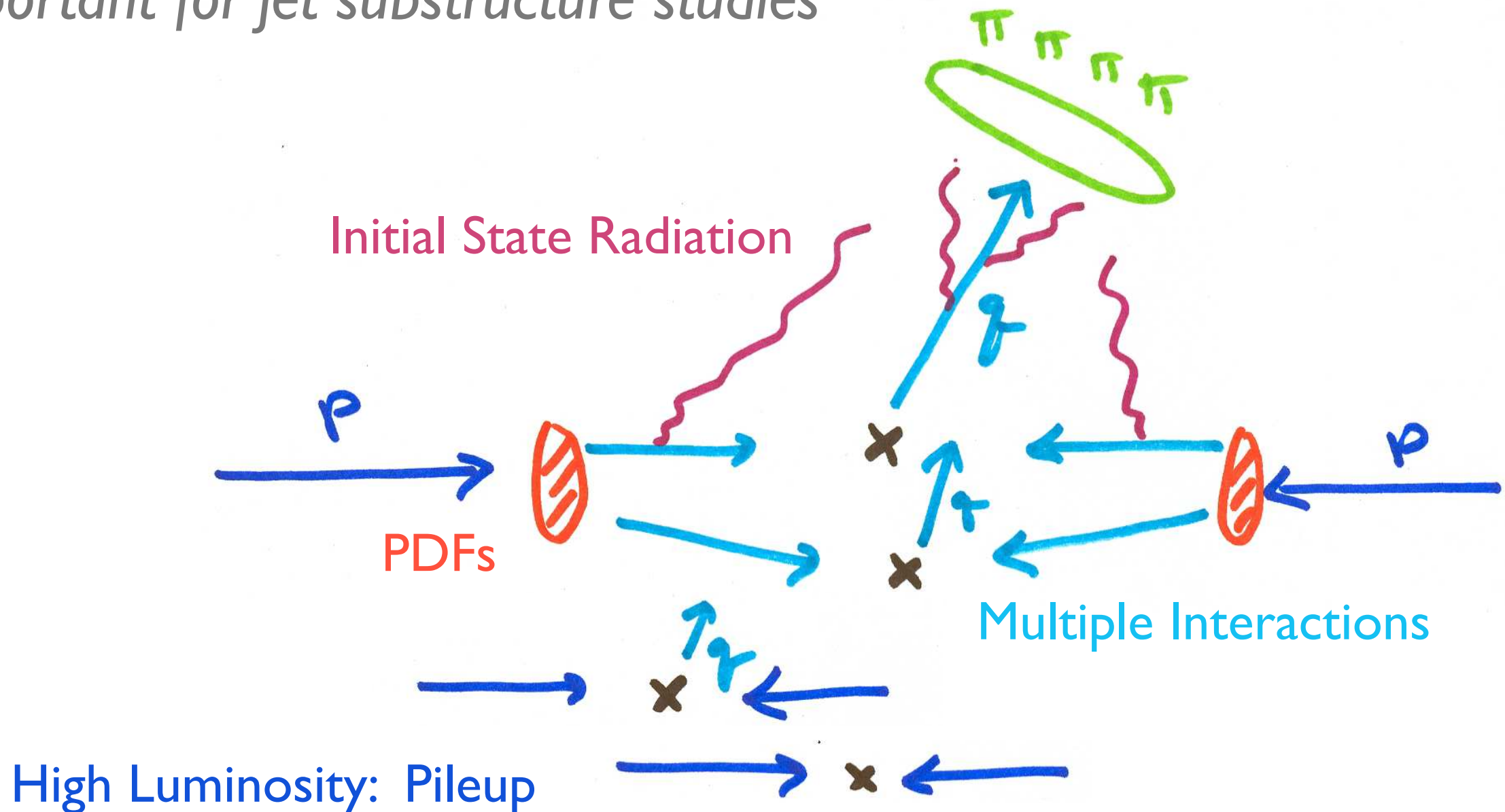
[pictures from [coffeeshopphysics.com](http://coffeeshopphysics.com)]





# Messiness is also a property of QCD

*Important for jet substructure studies*



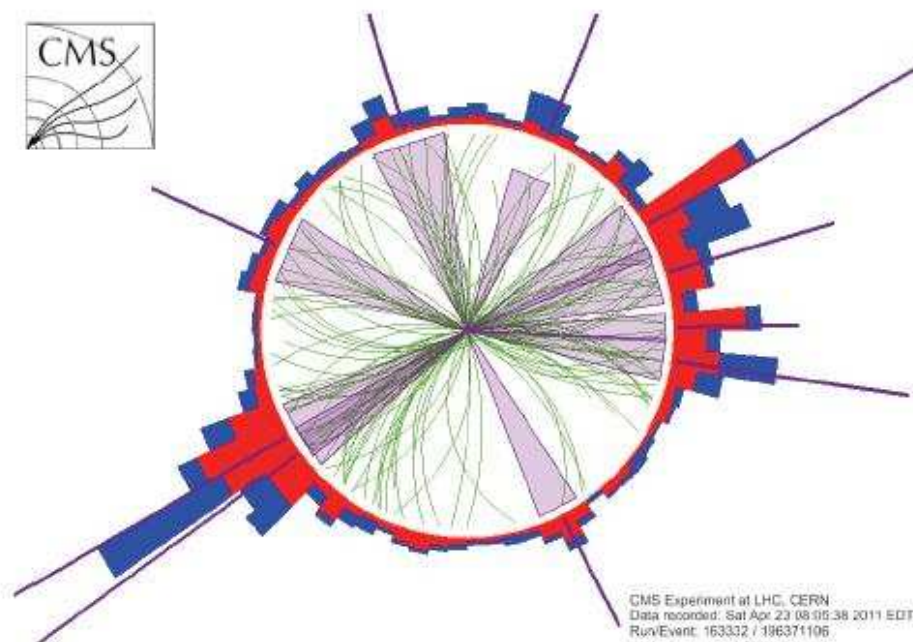
Jet = desired radiation from hard quark/gluon  
+ additional contamination

# *Identifying Jets*

# What is a Jet?

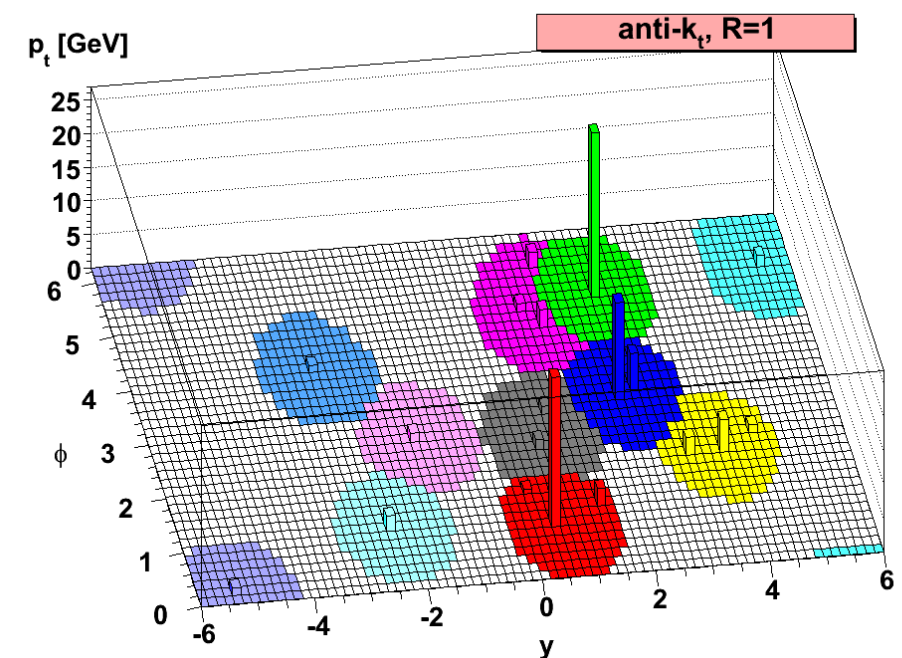
## A physical phenomena:

*Emergent feature of  
confining gauge theories*



## An analysis technique:

*Method to interpret  
hadronic final states*



*Freedom to use  
different analysis strategies for  
different physical questions*



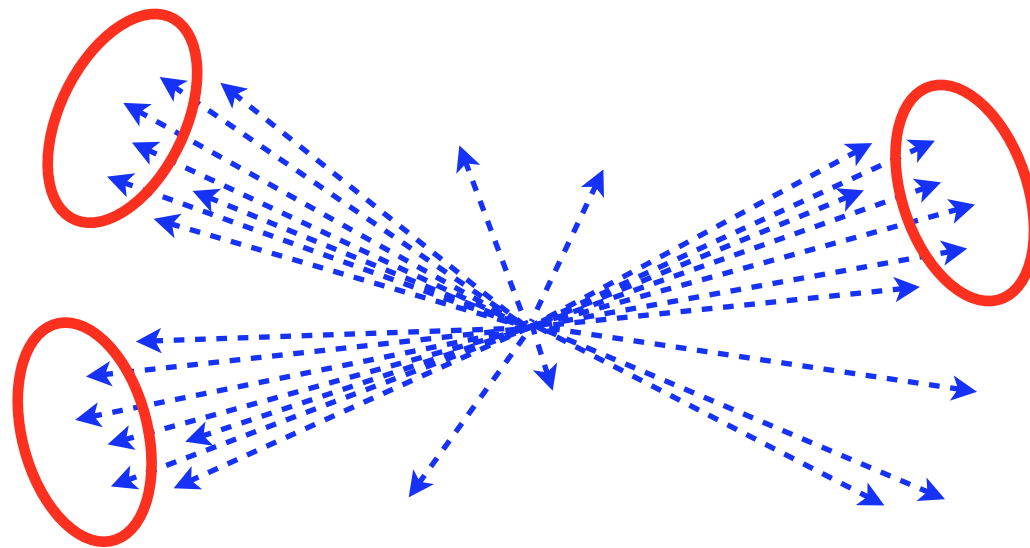
# Generic Jet Algorithm

Inputs:

$$\{p_1, p_2, \dots, p_k\}_{\text{hadrons}} \Rightarrow \{p_1, p_2, \dots, p_N\}_{\text{jets}}$$

Outputs:

Unless otherwise stated:  $\sum_{i \in \text{jet}} p_i = p_{\text{jet}}$  (aka “E-scheme recombination”, other schemes also plausible)



Remember, jets are massive:

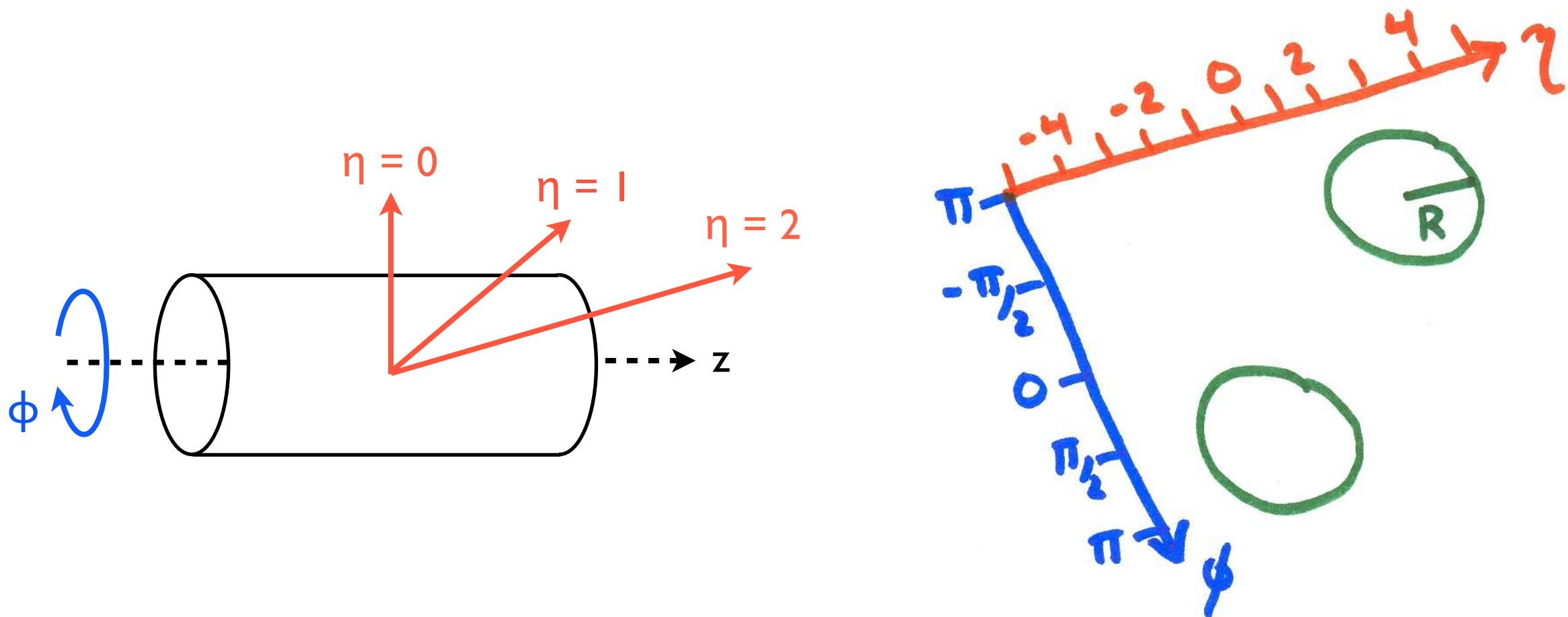
$$p_{\text{jet}}^2 = \left( \sum_{i \in \text{jet}} p_i \right)^2 \geq \sum_{i \in \text{jet}} m_i^2 \geq m_{\text{quark/gluon}}^2$$

# Coordinate System for Jets

*Invariant to longitudinal boosts along beam direction*

Better to use  
true rapidity  $y$

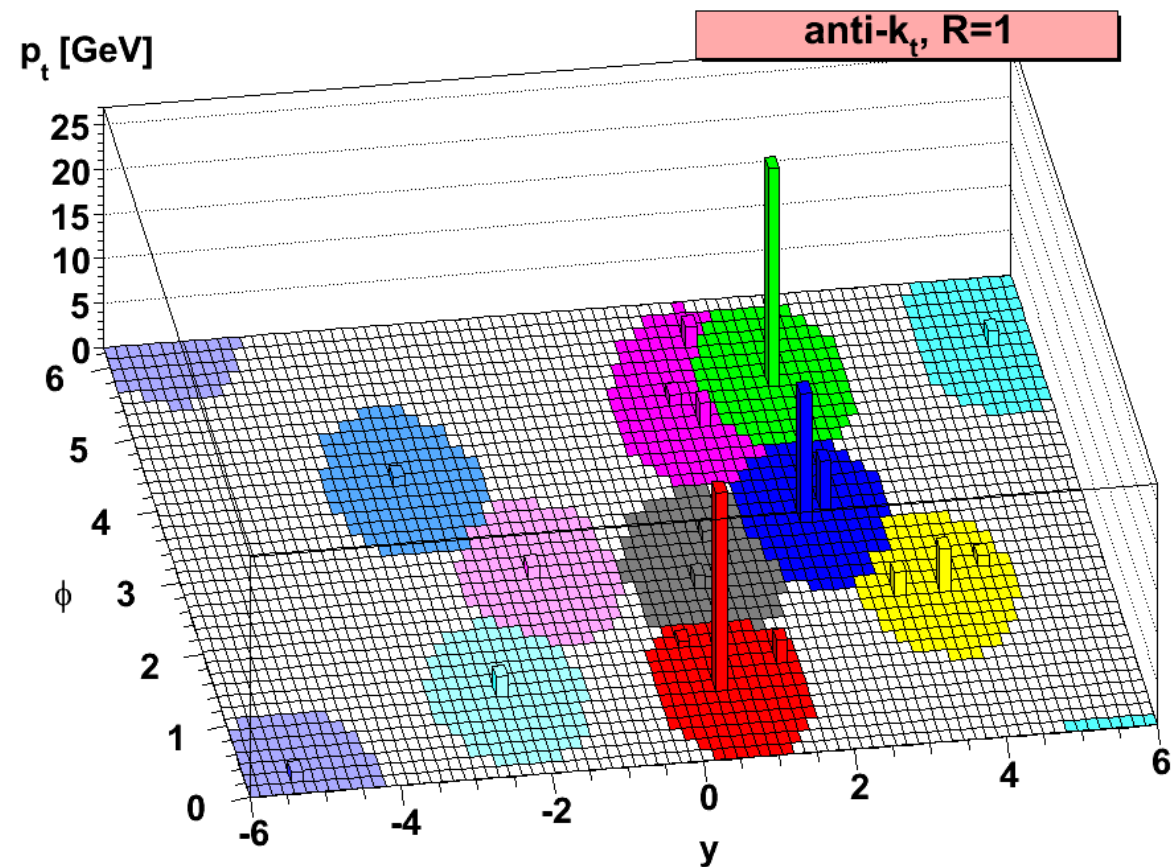
$$p_T = \sqrt{p_x^2 + p_y^2} \quad \Delta R = \sqrt{(\Delta\phi)^2 + (\Delta\eta)^2}$$



Typically: Cluster hadrons within characteristic radius  $R$

# Anti- $k_t$ Sequential Recombination

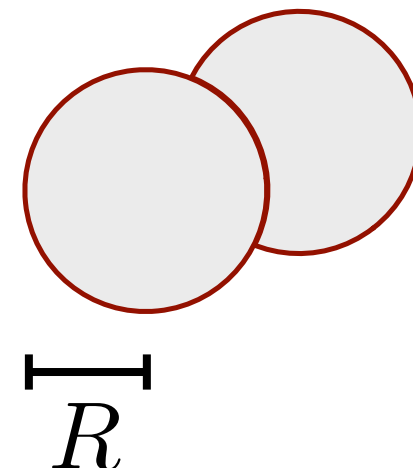
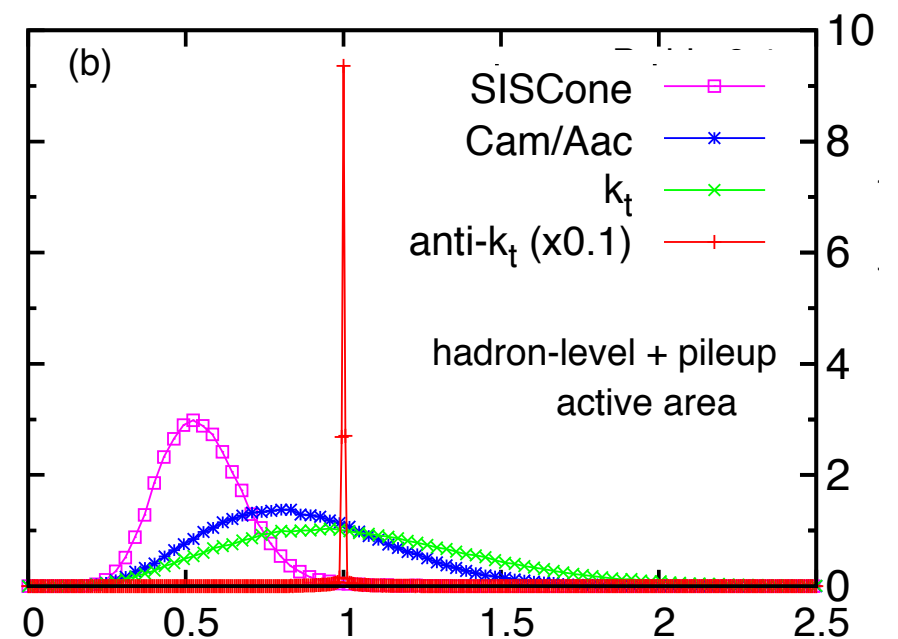
*Ask me offline if you want to learn more*



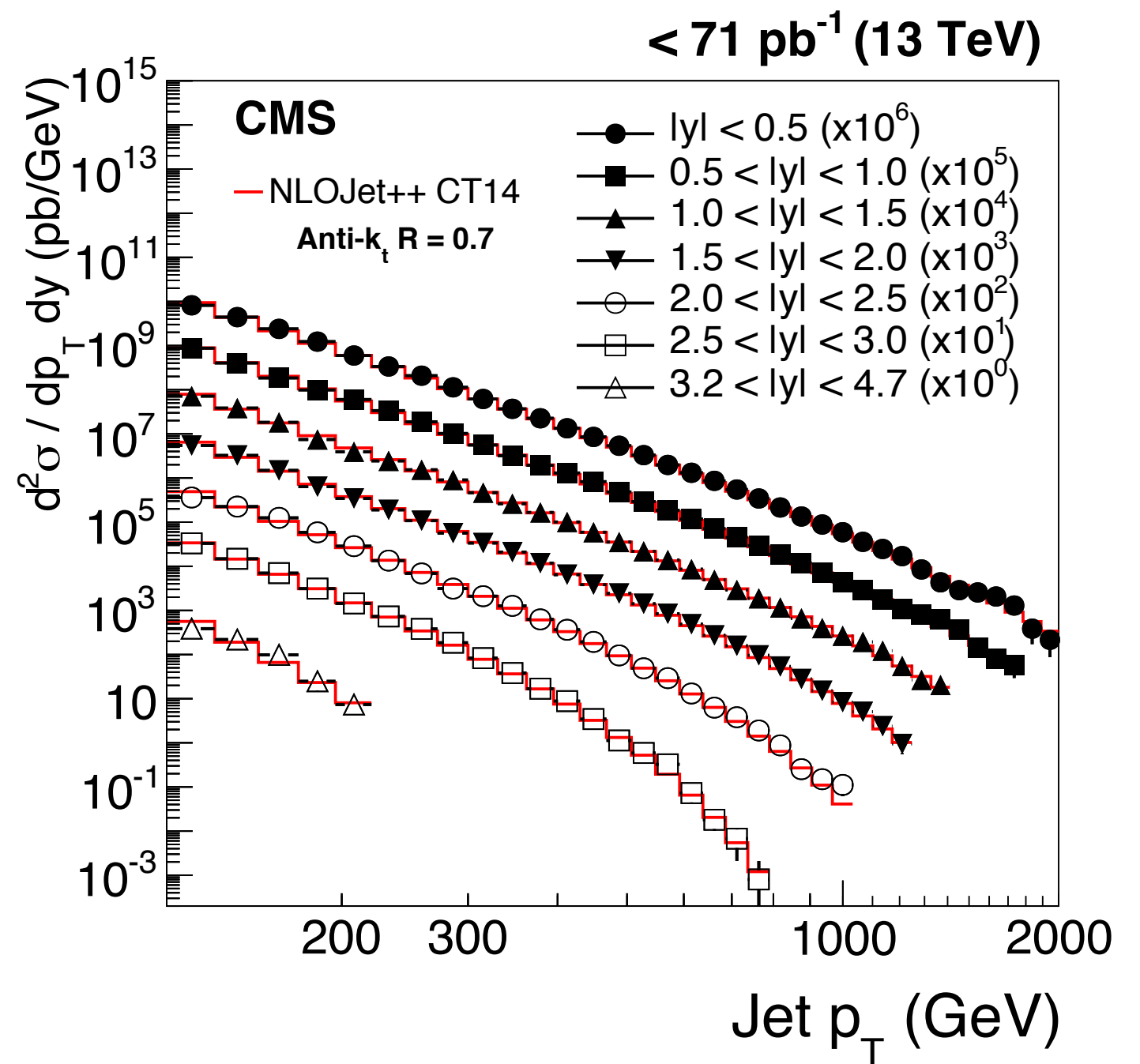
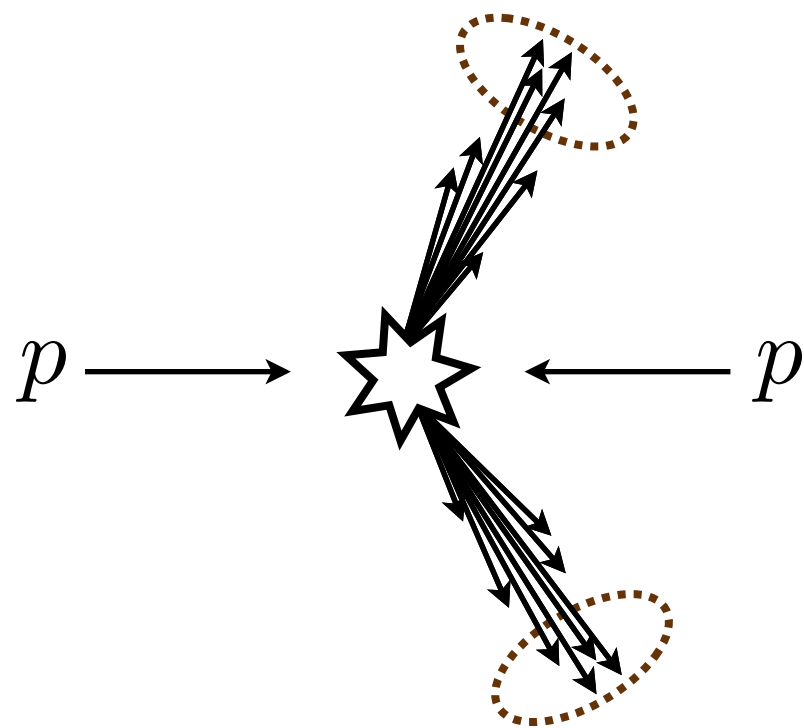
[Cacciari, Salam, Soyez, 2008]

At LHC:  $R = 0.4$  for standard jets  
 $R = 0.8$  for “fat” jets

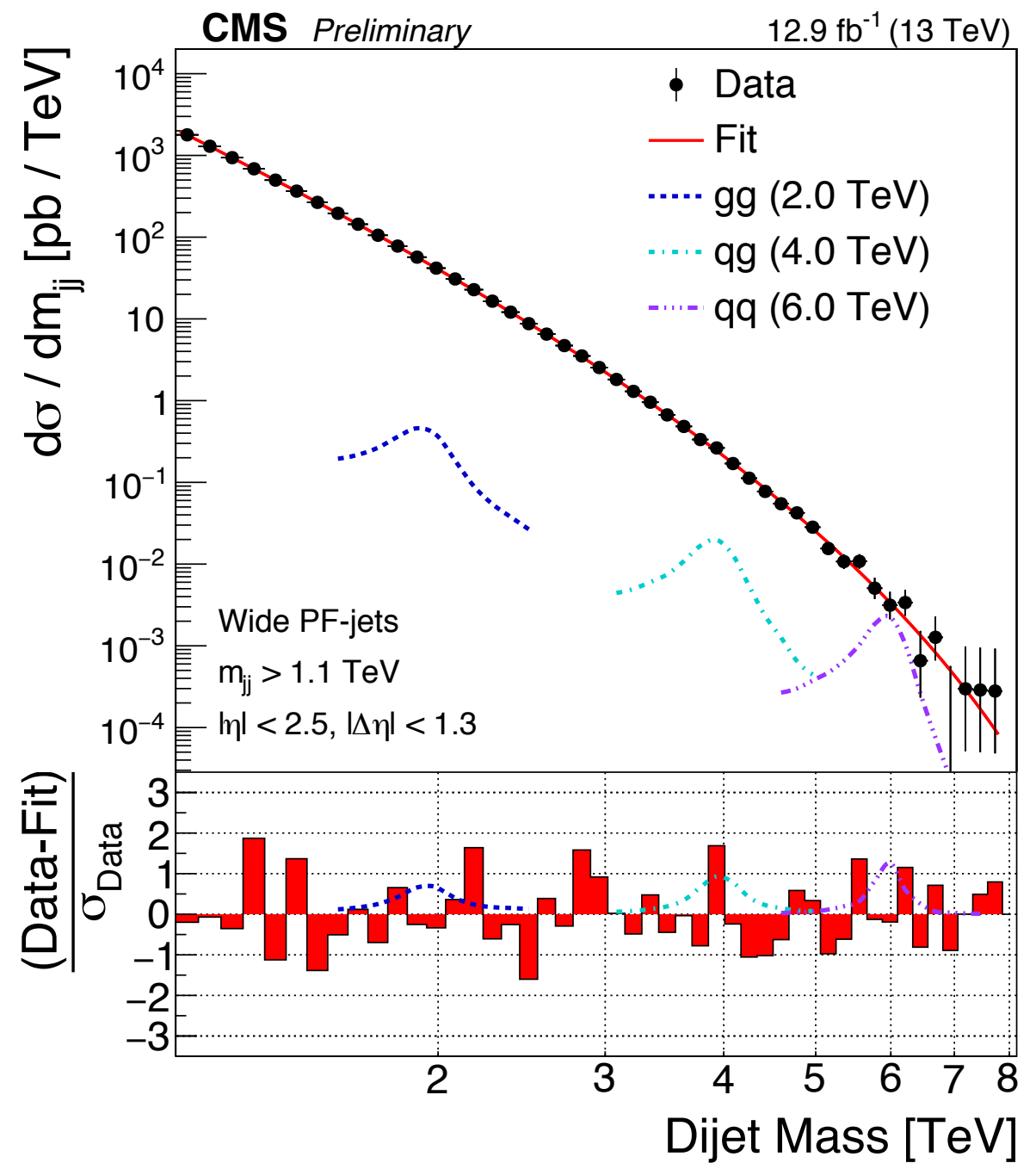
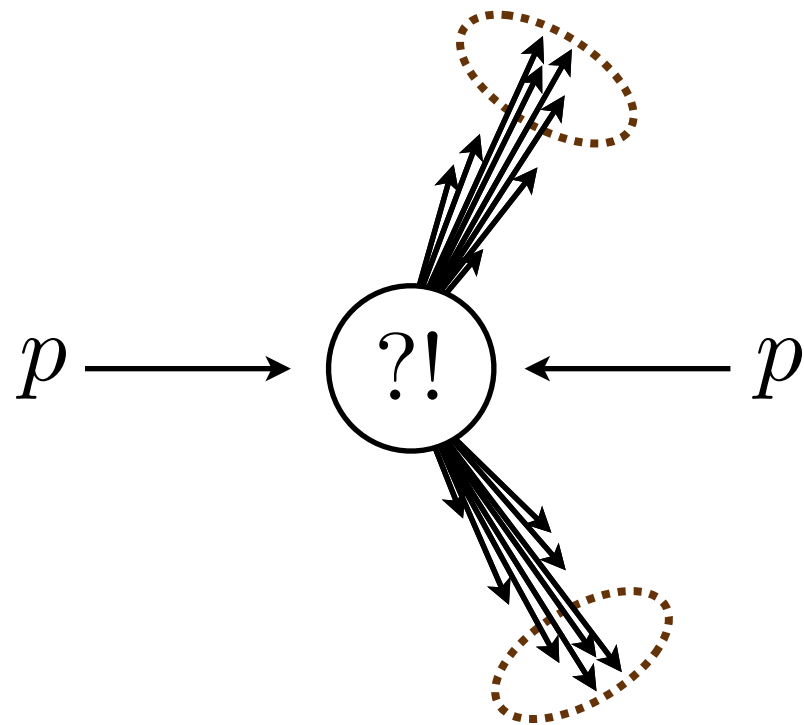
Uniform catchment area  
(for hardest jet)



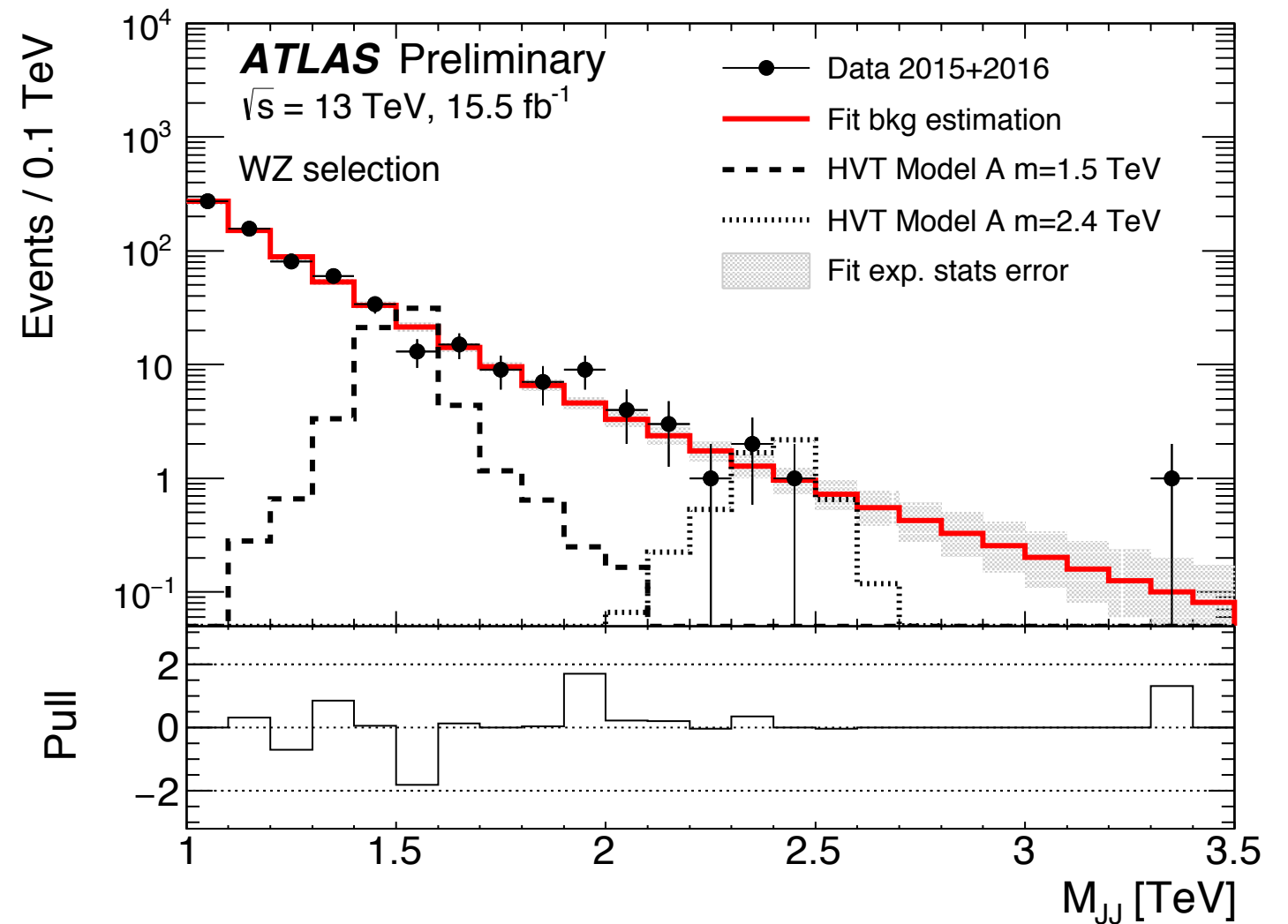
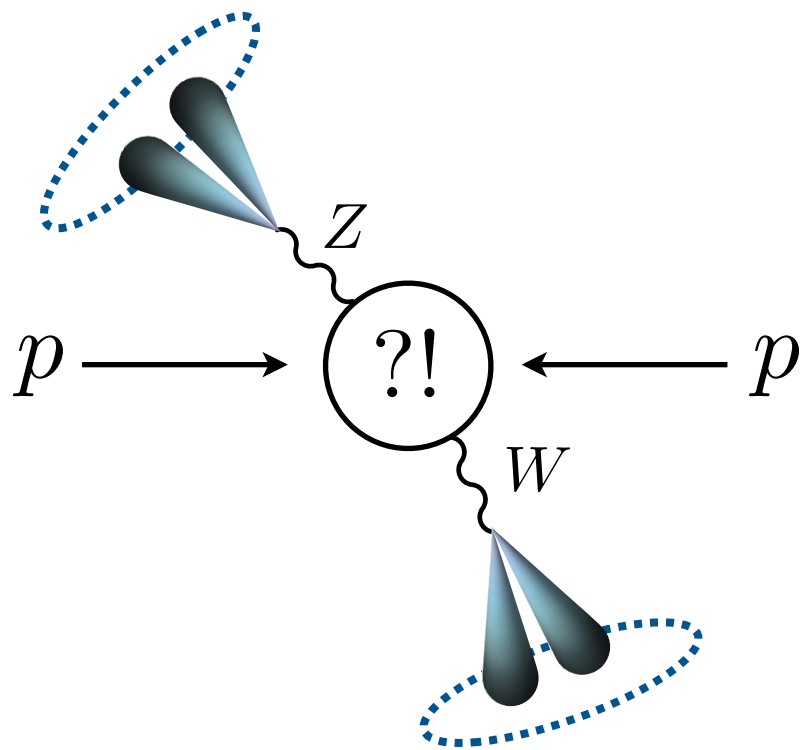
# Dijet Spectrum



# Dijet Resonance?



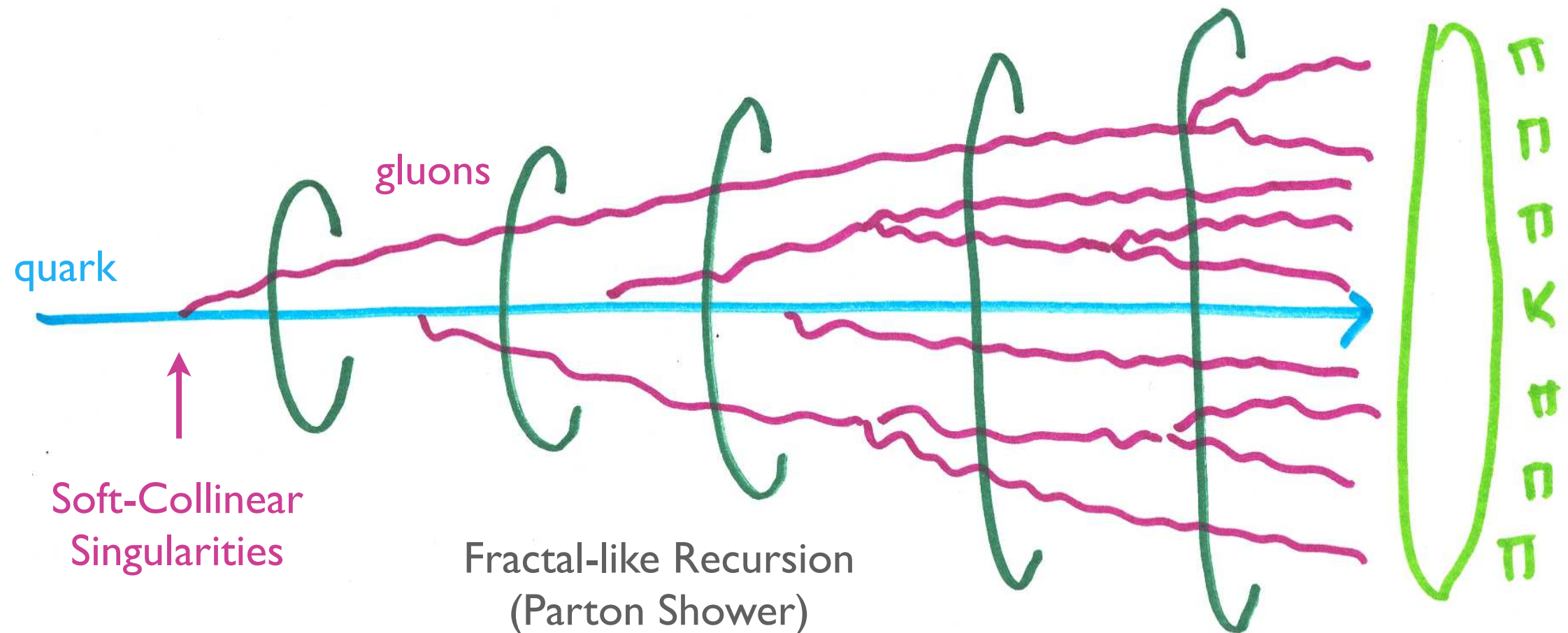
# Diboson Resonance? (all hadronic channel)



Inspiration to study the substructure of jets

# *The Soft/Collinear Limit of QCD*

# Computational Control



Energy flow of partons

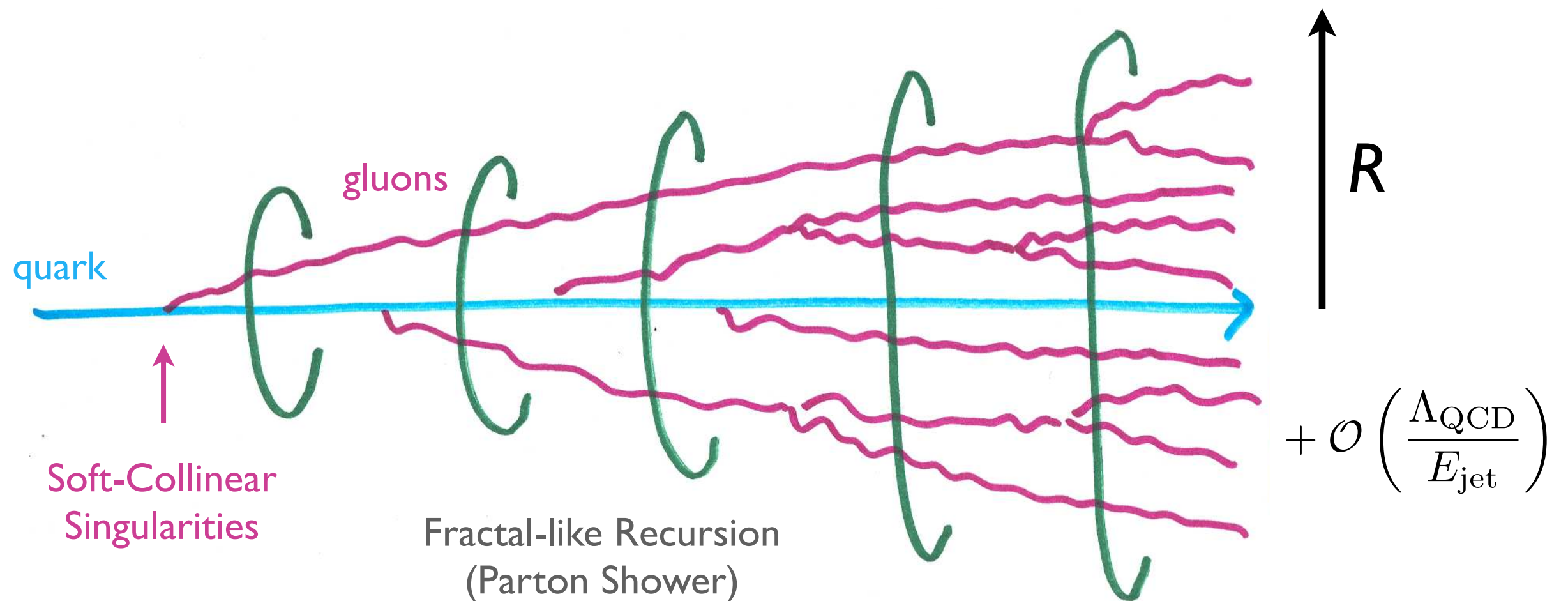


Energy flow of hadrons

Relies crucially on string breaking



# Perturbative Jet Calculations



Energy flow of partons



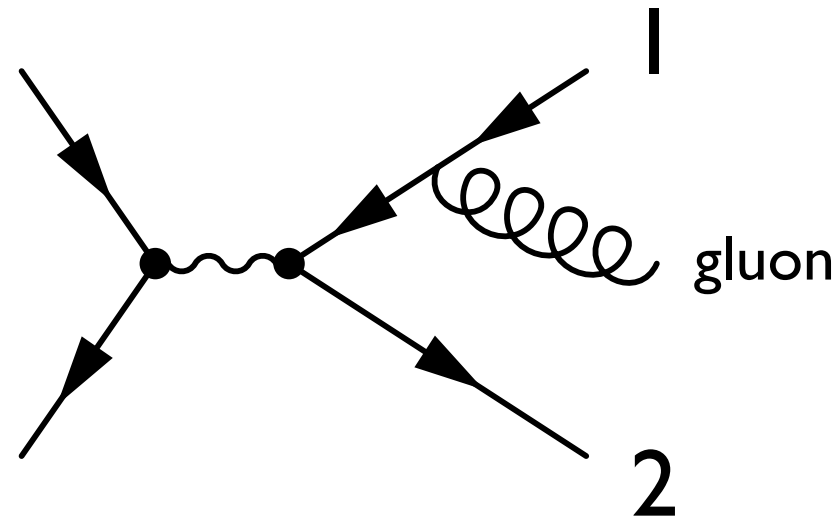
Good approximation  
to jet structure

For jet substructure: multiple soft/collinear emissions (resummation)  
typically more important than single hard emission (fixed order)

# An Instructive Calculation

*Every theorist should do this once*

$$e^+ e^- \rightarrow q \bar{q} g$$



$$t_1 = \frac{m_{1,\text{gluon}}^2}{E_{\text{CM}}^2}$$

$$t_2 = \frac{m_{2,\text{gluon}}^2}{E_{\text{CM}}^2}$$

$$\frac{d^2\sigma}{dt_1 dt_2} = \sigma_0 \frac{\alpha_s}{2\pi} C_F \frac{(1-t_1)^2 + (1-t_2)^2}{t_1 t_2}$$

Born  $2 \rightarrow 2$   
cross section

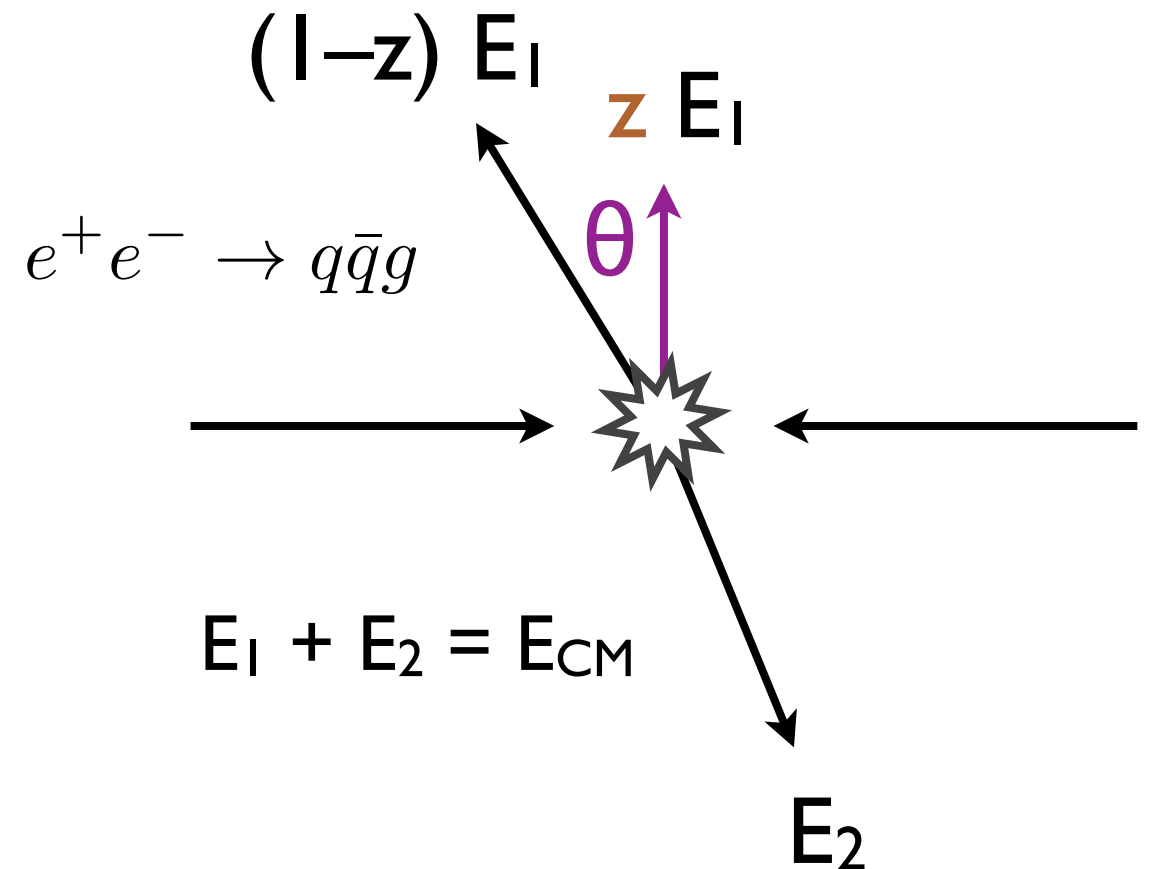
Casimir  
factor

$$\sum_a T_{ij}^a T_{jk}^a = C_F \delta_{ik}$$

# Collinear Limit

*Gluon close to quark*

2 → 3 process  
 5 total phase space variables  
 3 Euler angles  
 5 − 3 = 2 relevant variables



As  $\theta \rightarrow 0$ :  $t_1 \simeq \frac{1}{4}z(1-z)\theta^2$      $t_2 \simeq z$

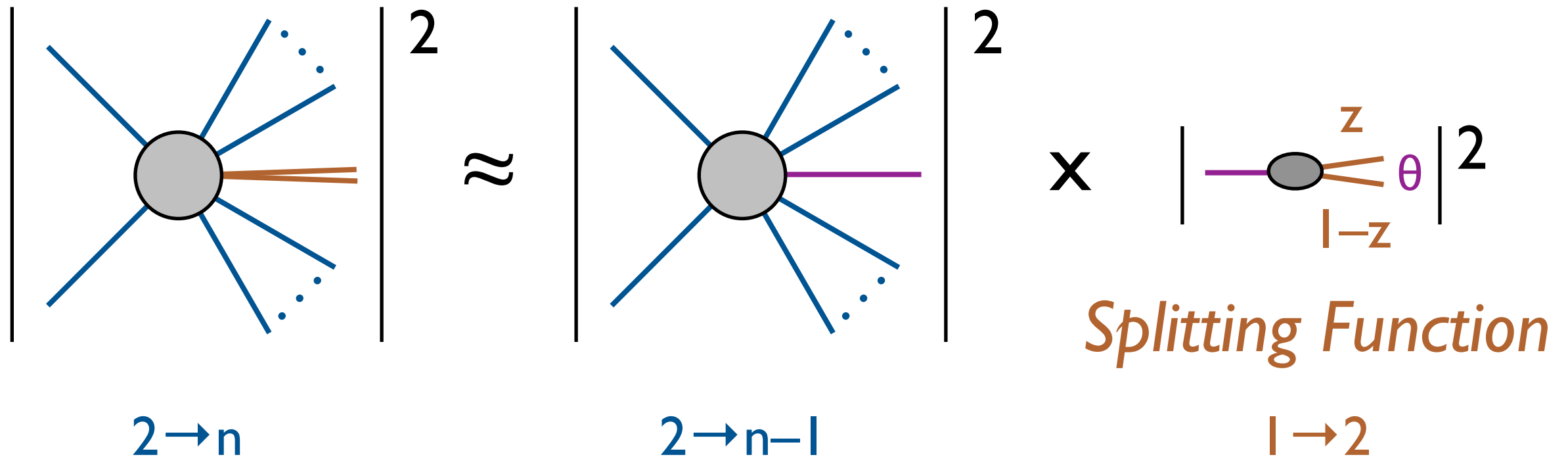
$$\frac{d^2\sigma}{dz d\theta} \simeq \sigma_0 \frac{\alpha_s}{\pi} C_F \frac{1}{\theta} \frac{1 + (1-z)^2}{z}$$

(remember the Jacobian!)

┌───┐  
 Collinear  
 singularity

# Key: Collinear Limit is Universal

*Collinear splittings are process independent*



$$d\sigma_{2 \rightarrow n} = d\sigma_{2 \rightarrow n-1} dP_{i \rightarrow jk}$$

$$dP_{i \rightarrow jk} = \underbrace{\frac{d\theta}{\theta}}_{\text{Collinear singularity}} \underbrace{dz P_{i \rightarrow jk}(z)}_{\text{Altarelli-Parisi splitting function}}$$

# Soft & collinear limit particularly simple

$$\theta \rightarrow 0 \text{ \& } z \rightarrow 0: \quad \left| \text{---} \text{---} \text{---} \right|^2$$

Splitting  
Probability:

$$dP_{i \rightarrow ig} = \frac{2\alpha_s}{\pi} C_i \underbrace{\frac{d\theta}{\theta}}_{\text{Collinear singularity}} \underbrace{\frac{dz}{z}}_{\text{Soft singularity}}$$

In this limit, only difference between hard quark and hard gluon is  $C_i$   
(i.e. both emit soft gluons)

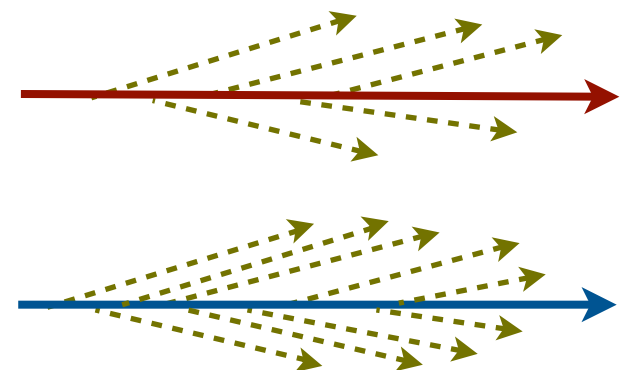
Color Factors:

SU(N)

SU(3)

Quark:  $C_F$   $(N^2 - 1)/(2N)$   $4/3$

Gluon:  $C_A$   $N$   $3$



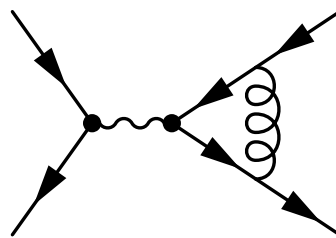
# What about soft/collinear singularities?

Splitting  
Probability:

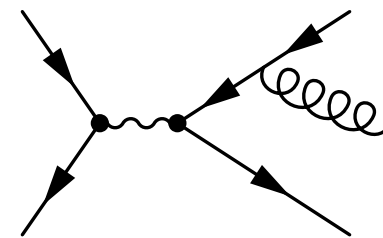
$$dP_{i \rightarrow ig} = \frac{2\alpha_s}{\pi} C_i \frac{d\theta}{\theta} \frac{dz}{z}$$

KLN Theorem:

Virtual Diagrams



$\Sigma$  Real Emissions



IRC divergences cancel by  
order-by-order in  $\alpha_s$  expansion

Effectively:

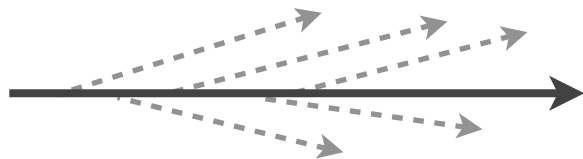
$$\text{virtual} + \int \frac{d\theta}{\theta} \frac{dz}{z} = \text{finite}$$

Note:

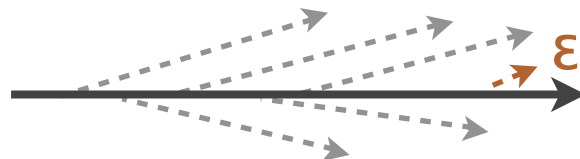
Restricting integration range gives  
logarithms (possibly large)

# Infrared/Collinear Safety

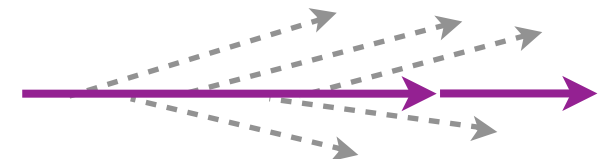
Original Jet



Infrared



Collinear



IRC Safe Observable: Insensitive to **IR** or **C** emissions

IRC Safe    IRC Unsafe

Standard Lore:

Calculable in pQCD?



Controlled  $\Lambda_{\text{QCD}}$  Effects?



[Tomorrow's talk will challenge this lore!]

# Examples from Jet Substructure

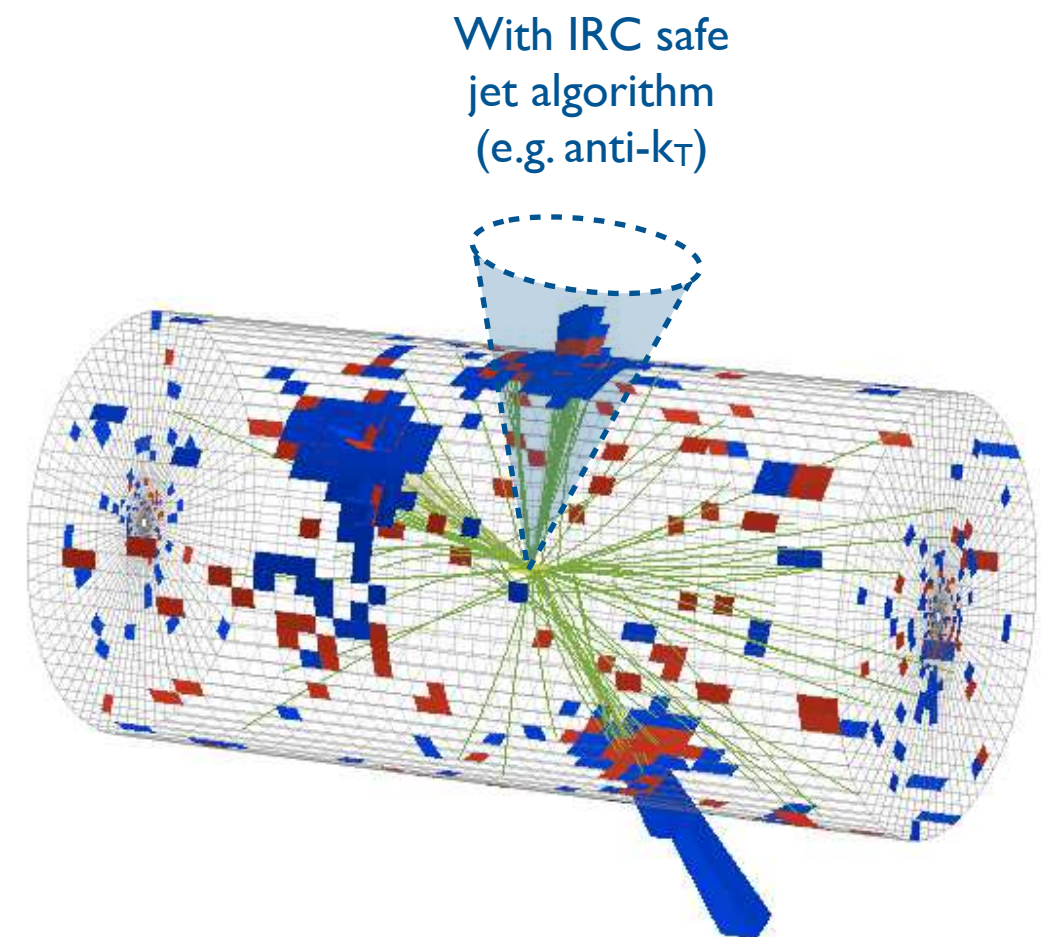
Jet  $p_T$ :  $\sum_{i \in \text{jet}} p_{T,i}$  IRC Safe

$p_T^D$ :  $\sum_{i \in \text{jet}} \frac{p_{T,i}^2}{p_{T,\text{jet}}^2}$  IR Safe  
C Unsafe  
[CMS HIG-11-027]

Multiplicity:  $\sum_{i \in \text{jet}} 1$  IRC Unsafe

Jet Mass:  $\sum_{i,j \in \text{jet}} p_i \cdot p_j$  IRC Safe

N-subjettiness:  $\sum_{i \in \text{jet}} p_{T,i} \min \{ \Delta R_{i,1}, \Delta R_{i,2}, \dots, \Delta R_{i,N} \}^\beta$  IRC Safe  
[JDT, Van Tilburg, 1011.2268, 1108.2701]

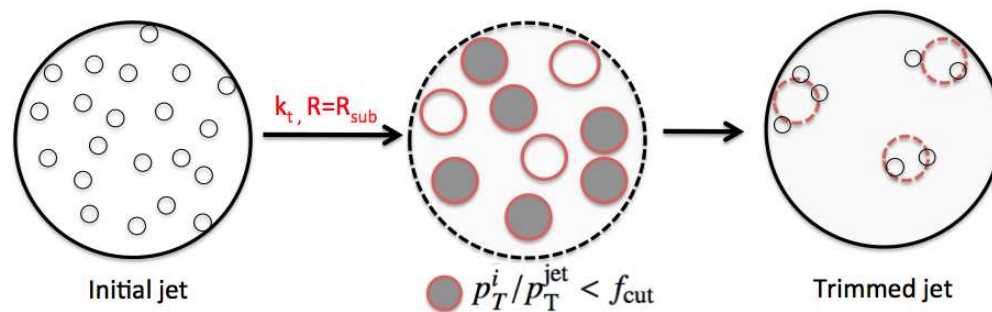




# *Grooming from First Principles*

# From Last Week

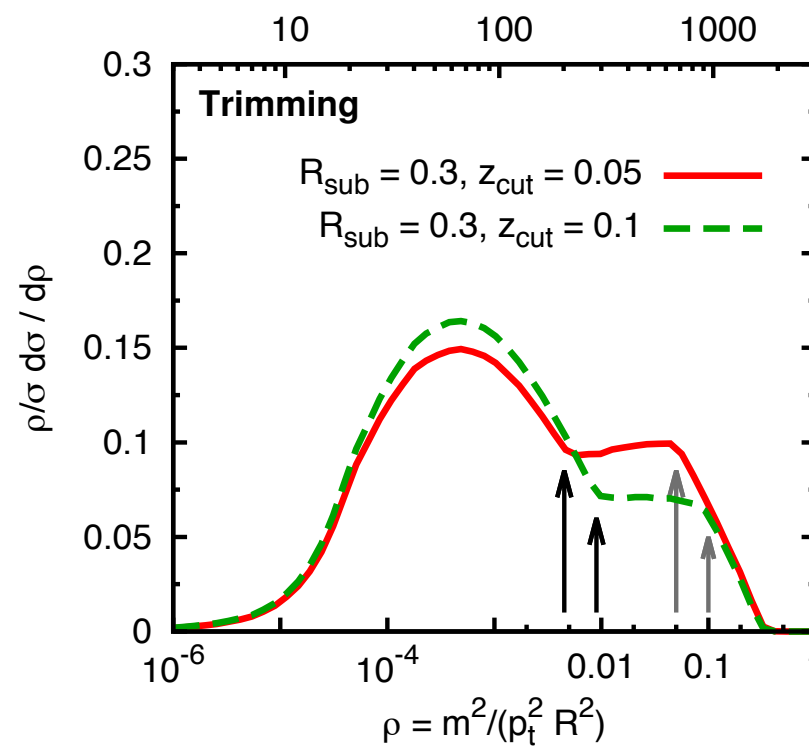
## Jet Trimming



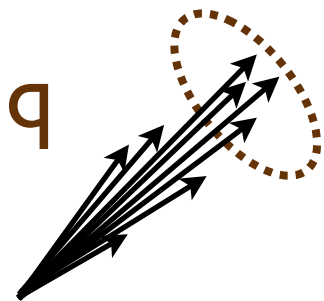
[Krohn, JDT, Wang, 0912.1342; diagram from ATLAS, 1306.4945]

$R_{\text{sub}}$ : subjet radius  
 $z_{\text{cut}}$ : fractional energy threshold

## Pythia 6 Simulation



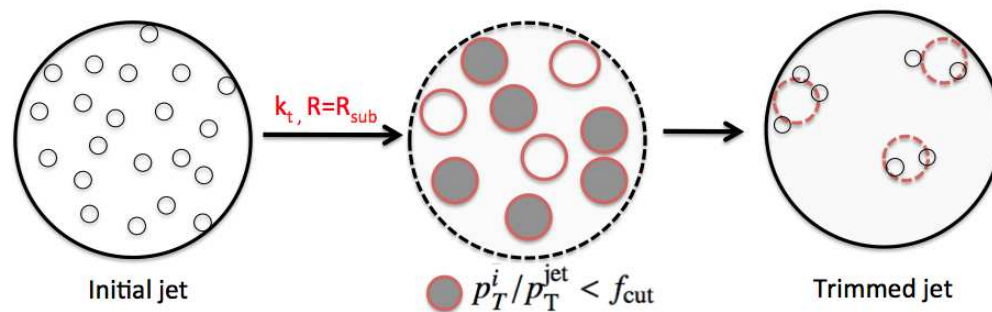
Trimmed  
 Jet Mass:  
 3 TeV quark jets



[Dasgupta, Fregoso, Marzani, Salam, 1307.0007]

# From Last Week

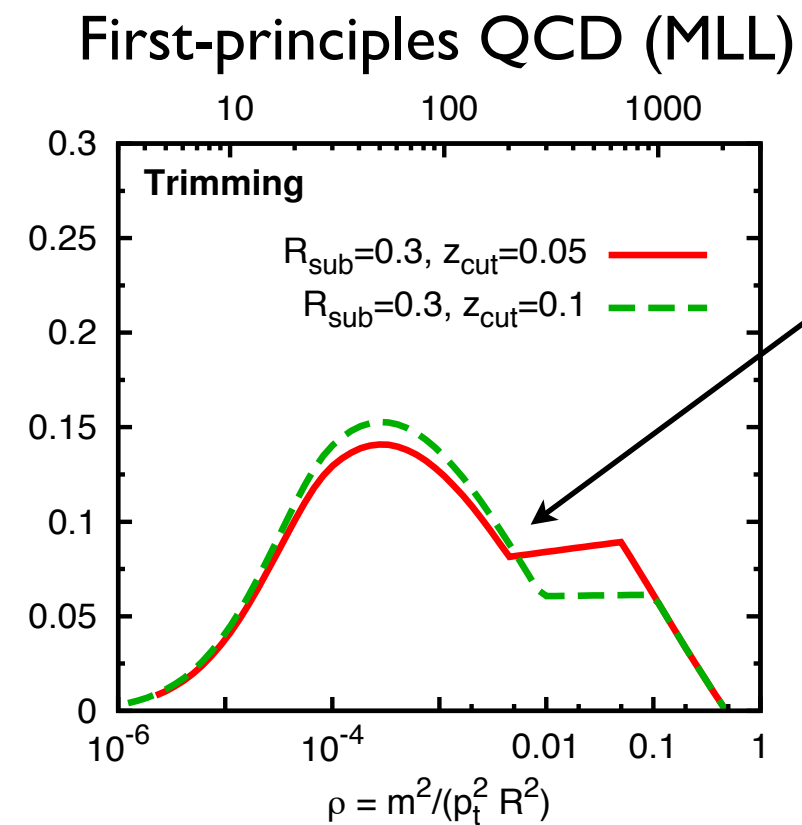
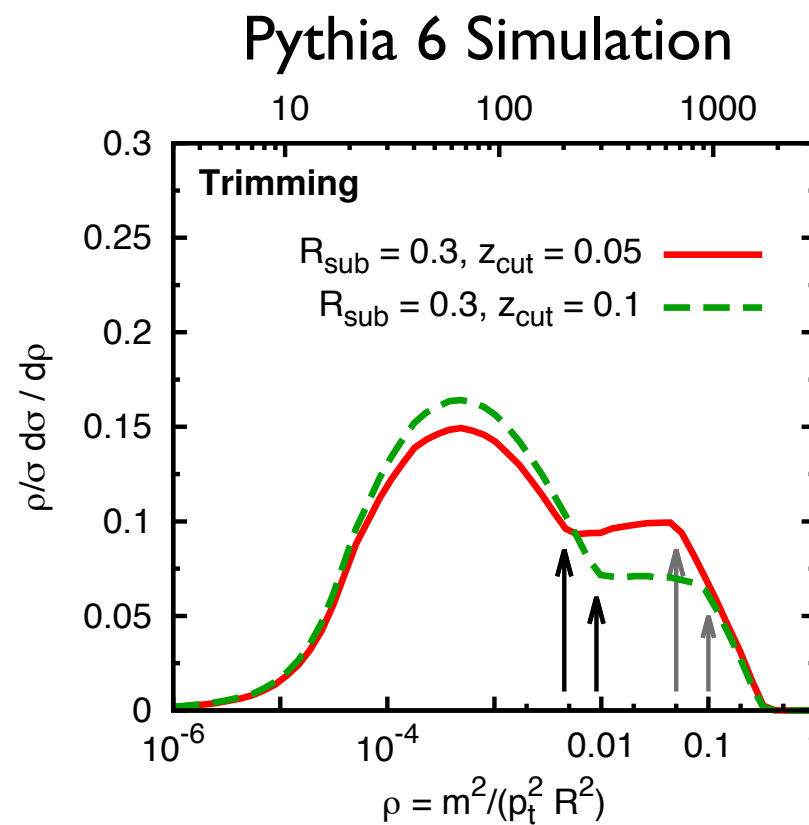
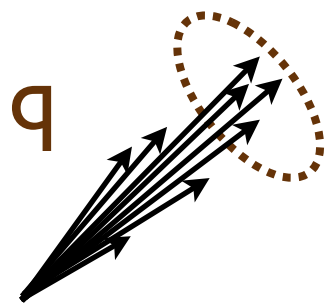
## Jet Trimming



[Krohn, JDT, Wang, 0912.1342; diagram from ATLAS, 1306.4945]

$R_{\text{sub}}$ : subjet radius  
 $z_{\text{cut}}$ : fractional energy threshold

Trimmed  
 Jet Mass:  
 3 TeV quark jets



[Dasgupta, Fregoso, Marzani, Salam, 1307.0007]

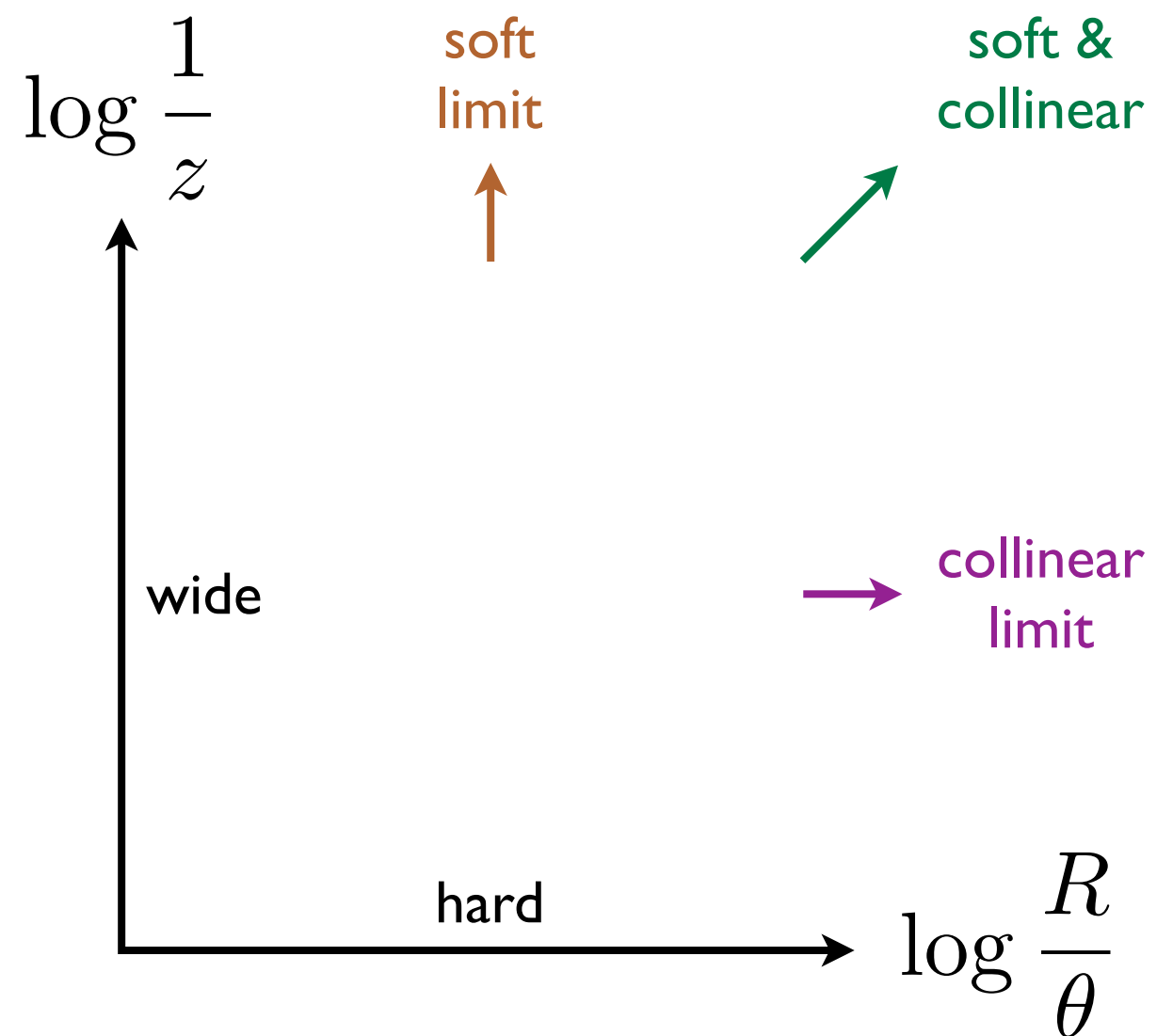
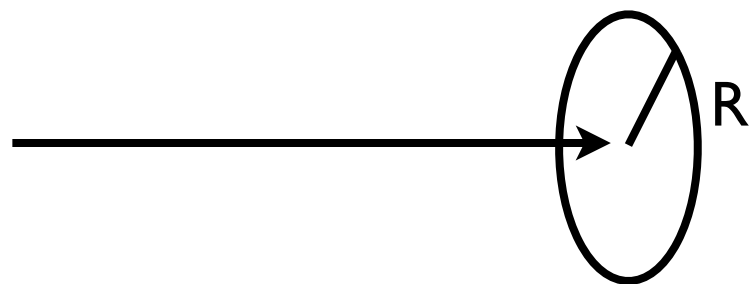
# Soft/Collinear Phase Space

*Basis for parton shower*

$$dP_{i \rightarrow ig} = \frac{2\alpha_s}{\pi} C_i \underbrace{\frac{d\theta}{\theta} \frac{dz}{z}}$$

Uniform in logarithmic plane

Eikonal Hard Quark/Gluon...

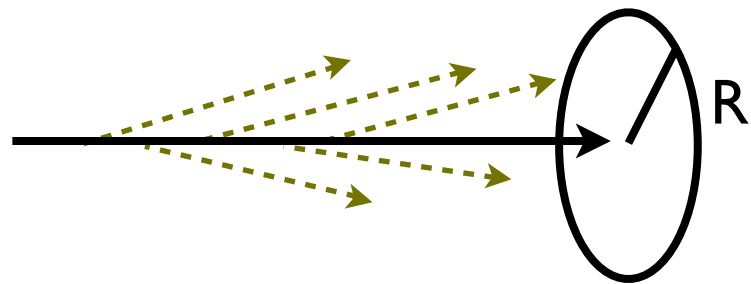


# Soft/Collinear Phase Space

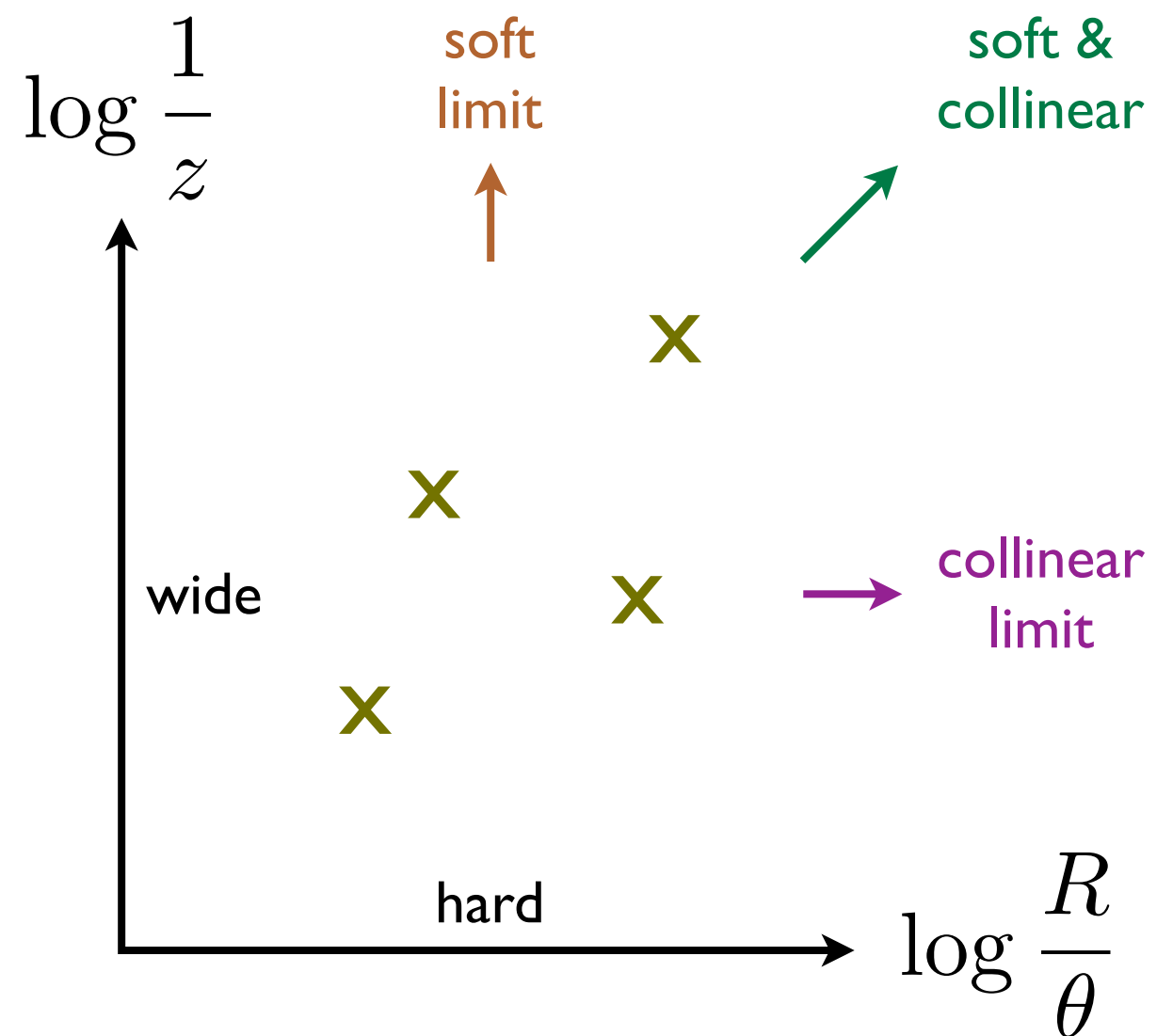
*Basis for parton shower*

$$dP_{i \rightarrow ig} = \frac{2\alpha_s}{\pi} C_i \underbrace{\frac{d\theta}{\theta} \frac{dz}{z}}_{\text{Uniform in logarithmic plane}}$$

Eikonal Hard Quark/Gluon...

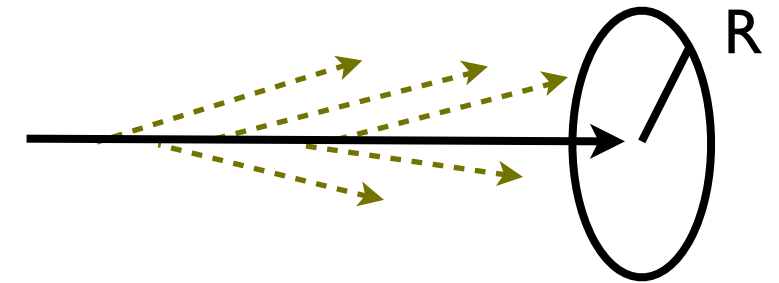


...Surrounded by Soft Gluon Haze



# Soft/Collinear Phase Space

*Basis for parton shower*



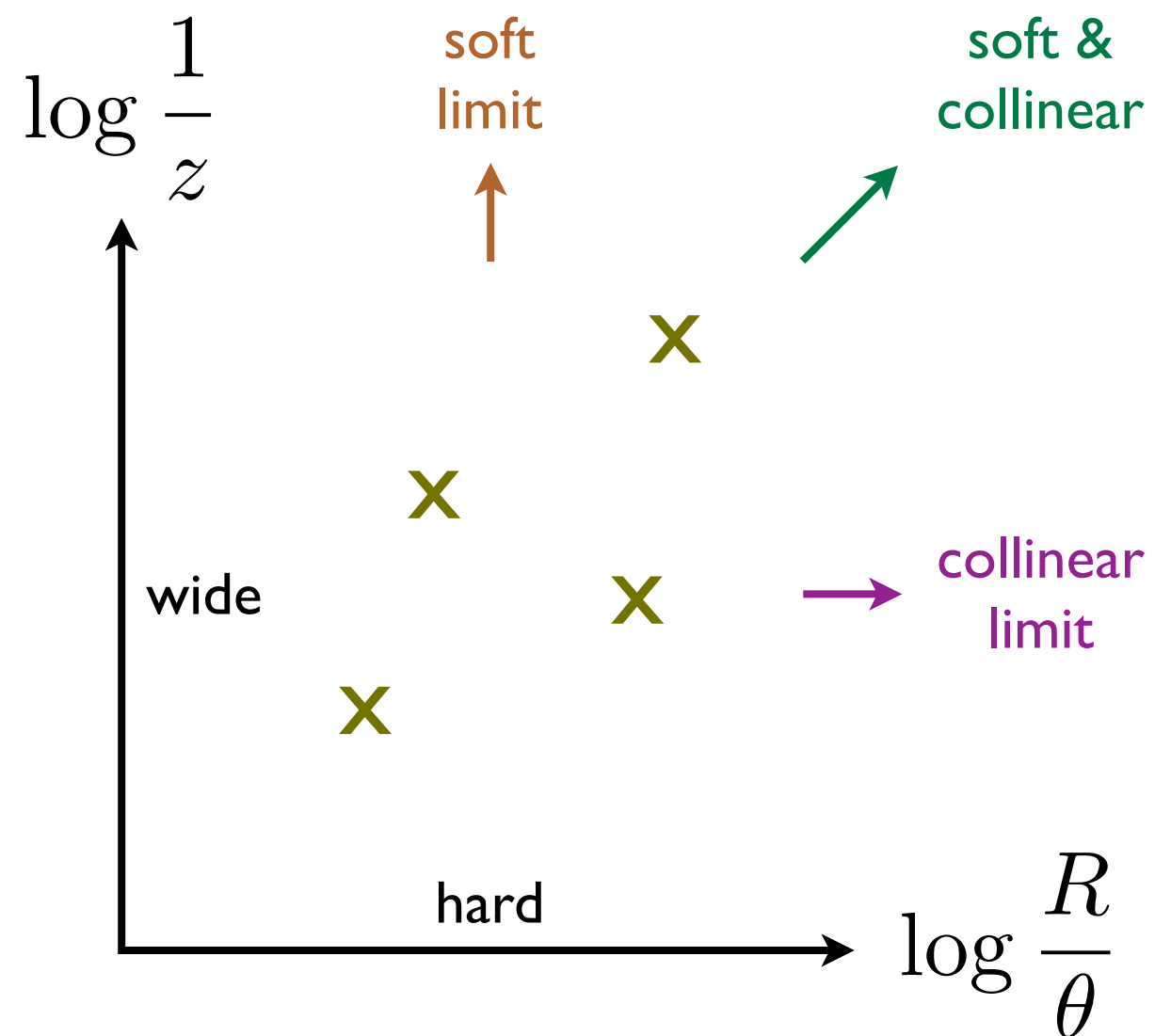
$$dP_{i \rightarrow ig} = \frac{2\alpha_s}{\pi} C_i \frac{d\theta}{\theta} \frac{dz}{z}$$

## Immediate Observations:

Arbitrary emissions?  
Captures (some) physics  
at all orders in  $\alpha_s$

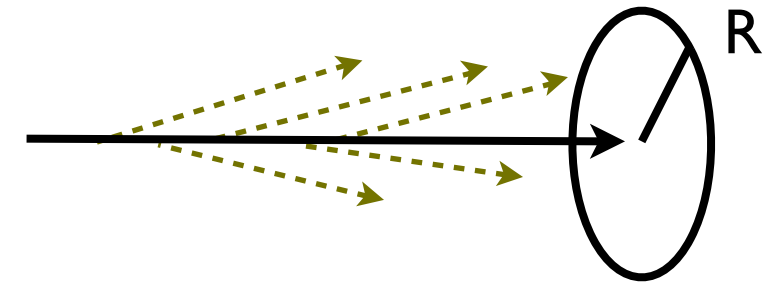
Soft/collinear singularities?  
Logarithmic plane extends  
up and to the right

IRC safe observables?  
Smooth behavior in singular limit  
(virtual contributions at infinity)



# Soft/Collinear Phase Space

*Basis for parton shower*



$$dP_{i \rightarrow ig} = \frac{2\alpha_s}{\pi} C_i \frac{d\theta}{\theta} \frac{dz}{z}$$

## Other Known Effects:

Color coherence?

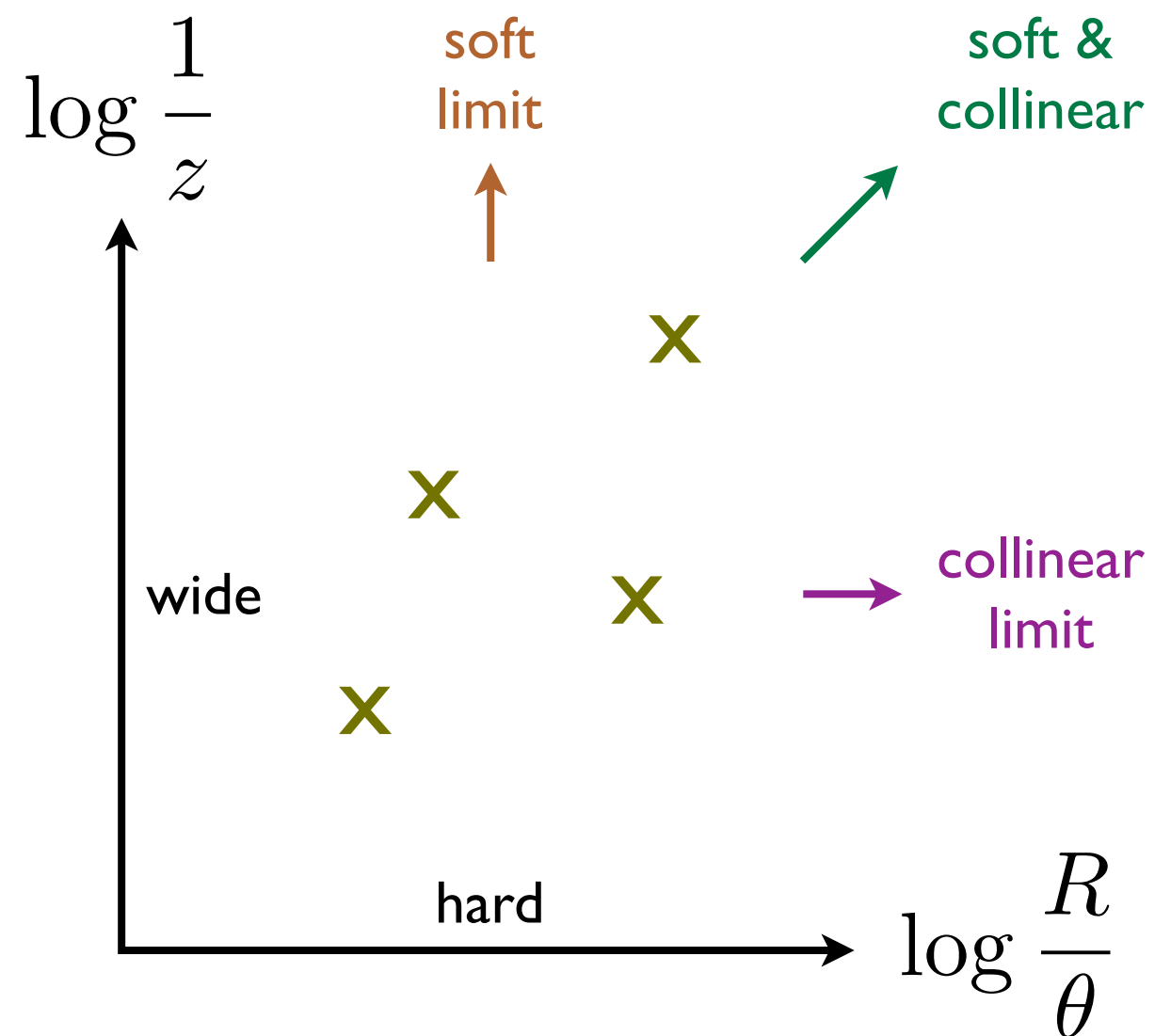
At large N, emissions  
effectively ordered by angle

Multiple emissions?

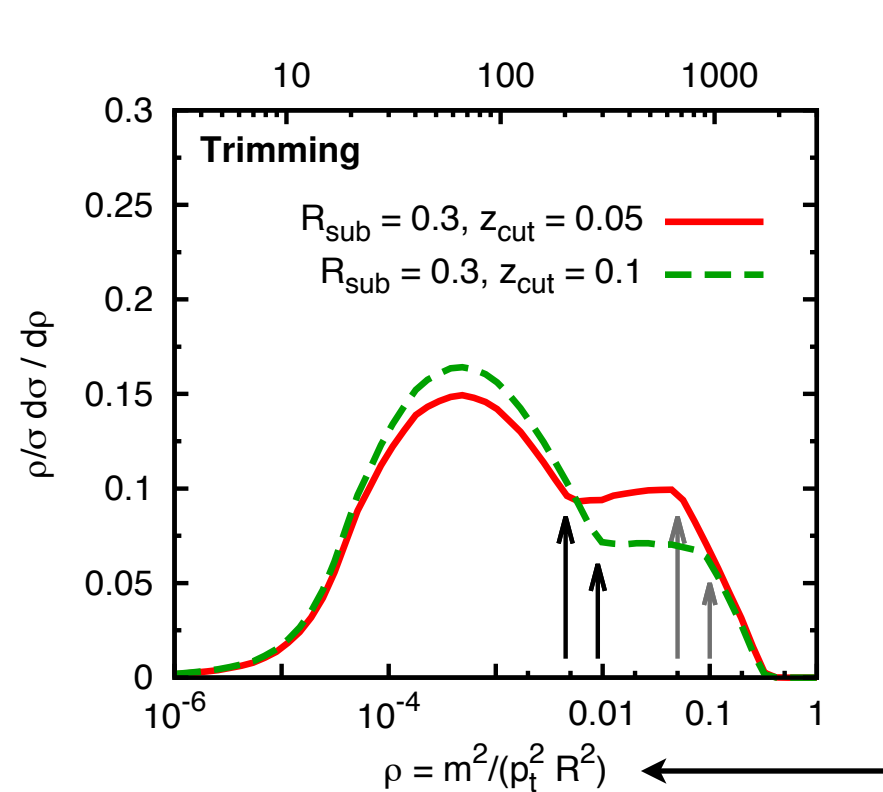
Calculations below assume  
one emission sets observable

Matrix element corrections?

Should supplement semi-classical  
picture with quantum effects



# Predicting Trimmed Jet Mass



Straightforward to replace  $E_{\text{jet}} \rightarrow p_{T\text{jet}}$

When strongly ordered,  
one emission dominates:

Restrictions from  
trimming:

$$\rho = \frac{m^2}{E_{\text{jet}}^2 R^2} \simeq \sum_{i \in \text{jet}} z_i \left( \frac{\theta_i}{R} \right)^2$$

soft/collinear  
limit

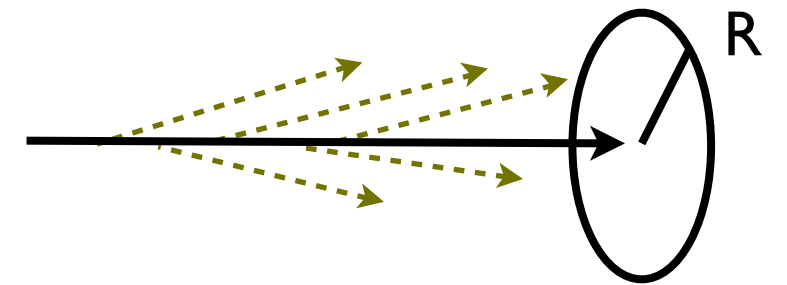
$$\log \frac{1}{\rho} \simeq \log \frac{1}{z_{\text{dom}}} + 2 \log \frac{R}{\theta_{\text{dom}}}$$

$$\theta_{\text{dom}} < R_{\text{sub}} \Rightarrow \text{No restriction}$$

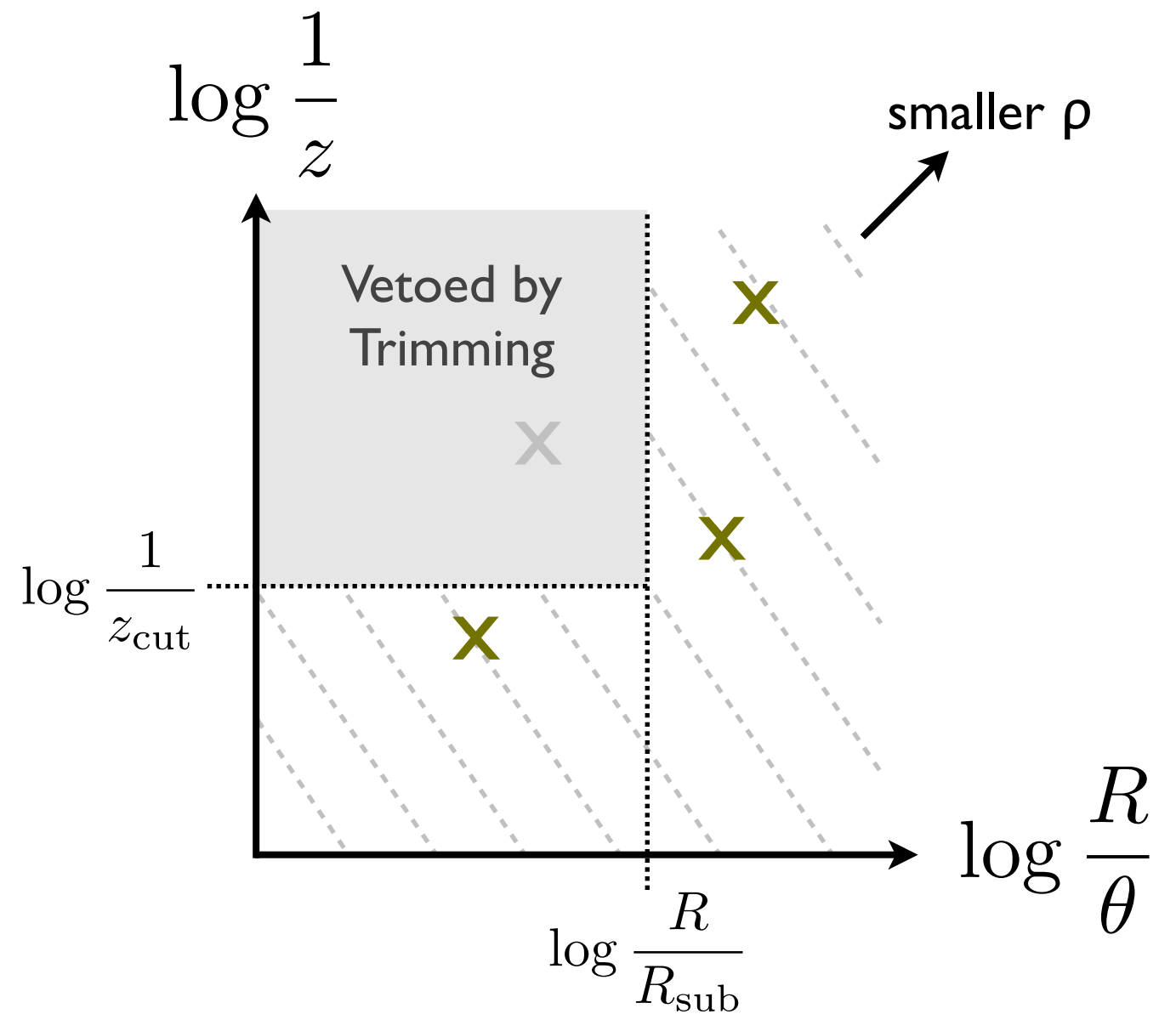
$$\theta_{\text{dom}} > R_{\text{sub}} \Rightarrow z_{\text{dom}} > z_{\text{cut}}$$



# Trimmed Phase Space

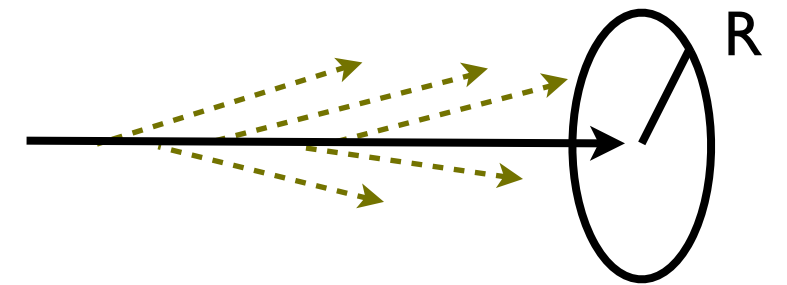


$$dP_{i \rightarrow ig} = \frac{2\alpha_s}{\pi} C_i \frac{d\theta}{\theta} \frac{dz}{z}$$

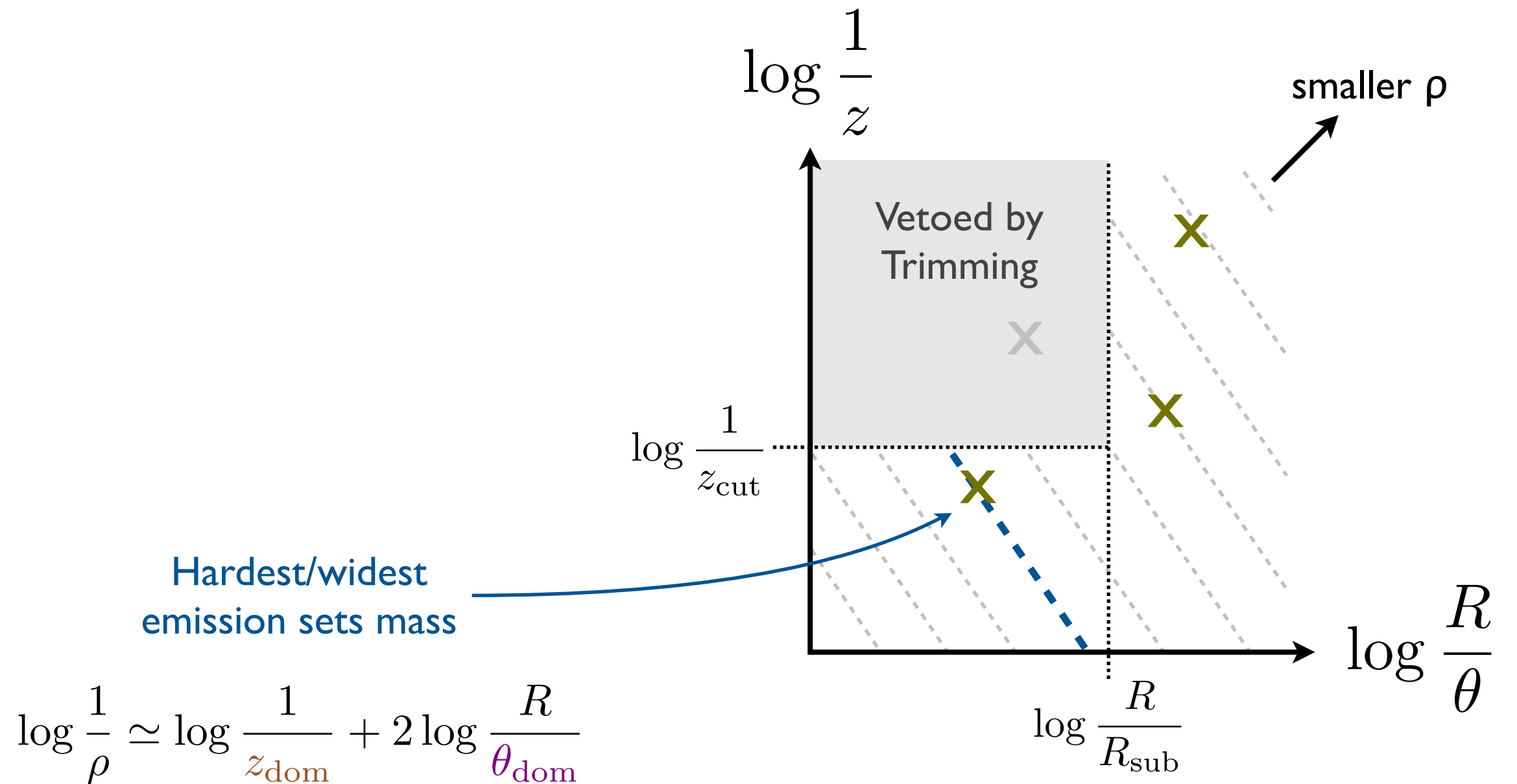


$$\log \frac{1}{\rho} \simeq \log \frac{1}{z_{\text{dom}}} + 2 \log \frac{R}{\theta_{\text{dom}}}$$

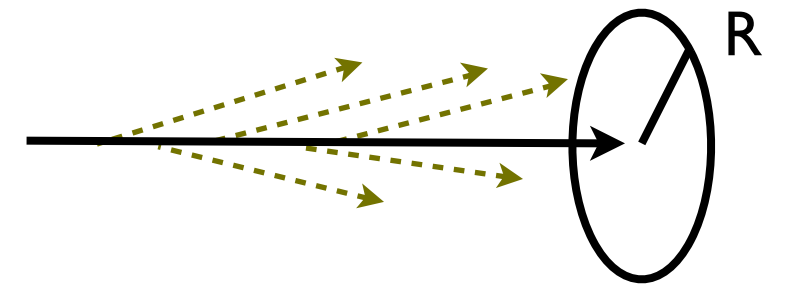
# Trimmed Phase Space



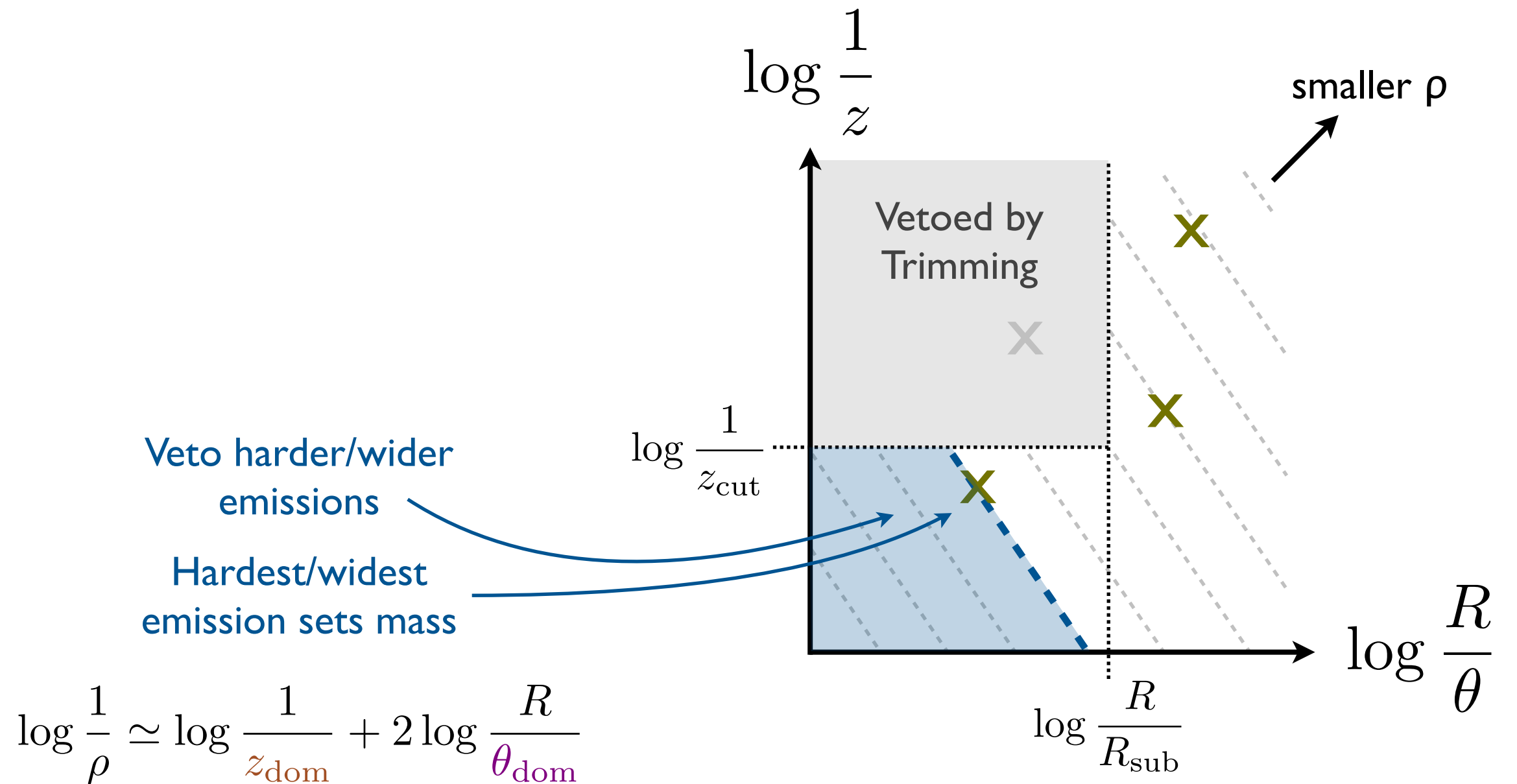
$$dP_{i \rightarrow ig} = \frac{2\alpha_s}{\pi} C_i \frac{d\theta}{\theta} \frac{dz}{z}$$



# Trimmed Phase Space



$$dP_{i \rightarrow ig} = \frac{2\alpha_s}{\pi} C_i \frac{d\theta}{\theta} \frac{dz}{z}$$



# Trimmed Phase Space

## Cumulative Probability:

i.e. trimmed mass is below some maximum value

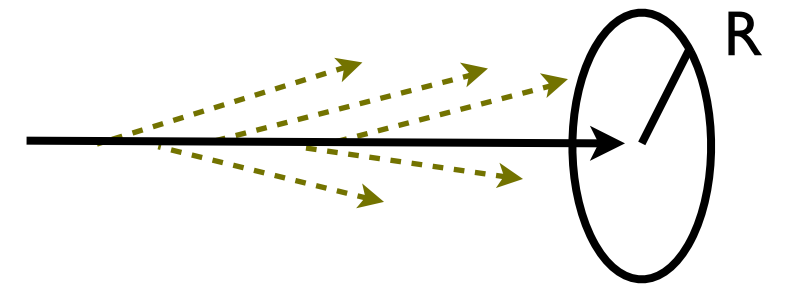
$$\Delta(\rho^{\max}) = \exp \left[ -\frac{2\alpha_s}{\pi} C_i \int_{\rho^{\max}}^{\infty} \frac{d\rho}{\rho^3} \right]$$

Differentiate to find cross section

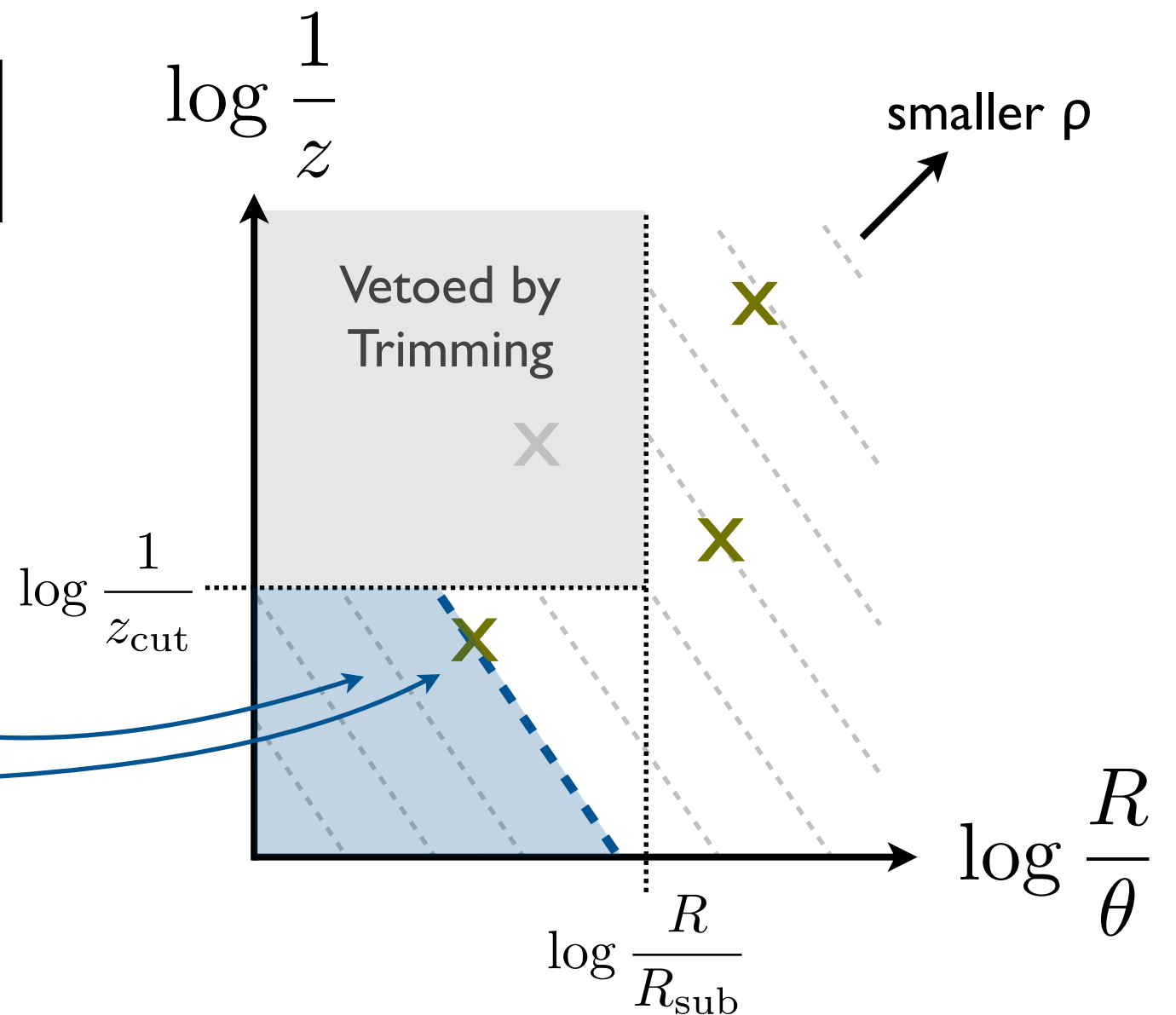
Veto harder/wider emissions

Hardest/widest emission sets mass

$$\log \frac{1}{\rho} \simeq \log \frac{1}{z_{\text{dom}}} + 2 \log \frac{R}{\theta_{\text{dom}}}$$

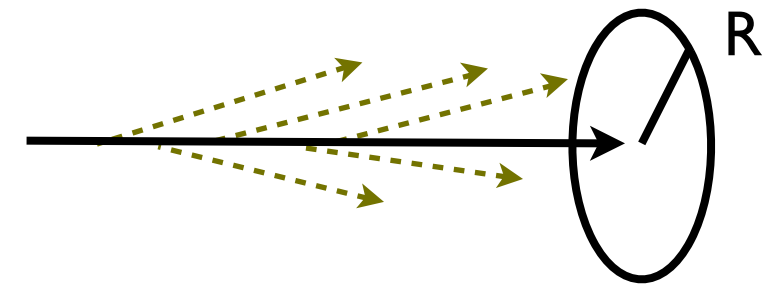


$$dP_{i \rightarrow ig} = \frac{2\alpha_s}{\pi} C_i \frac{d\theta}{\theta} \frac{dz}{z}$$

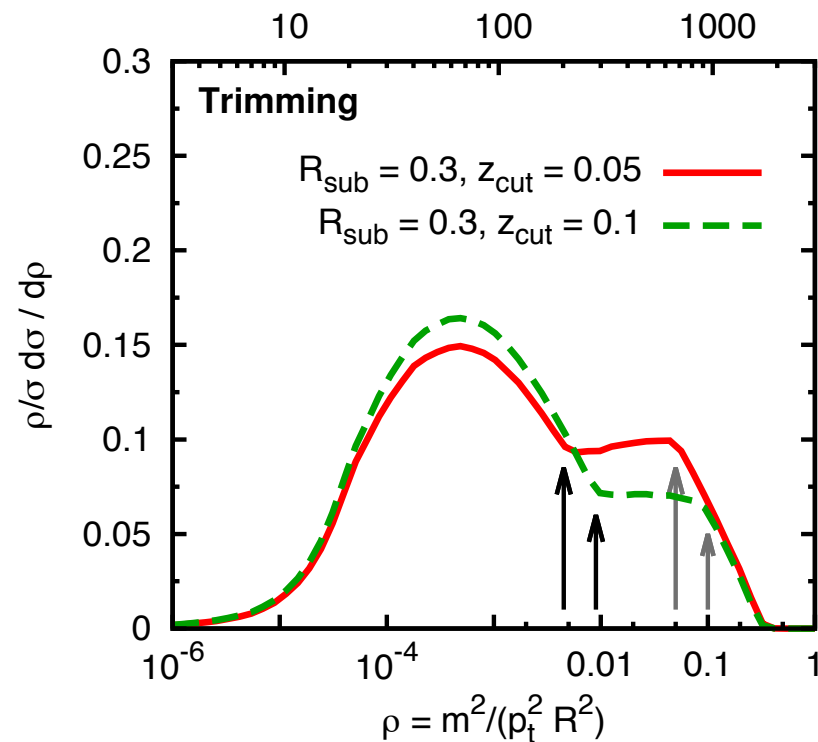


# Trimmed Phase Space

Immediately understand kink locations

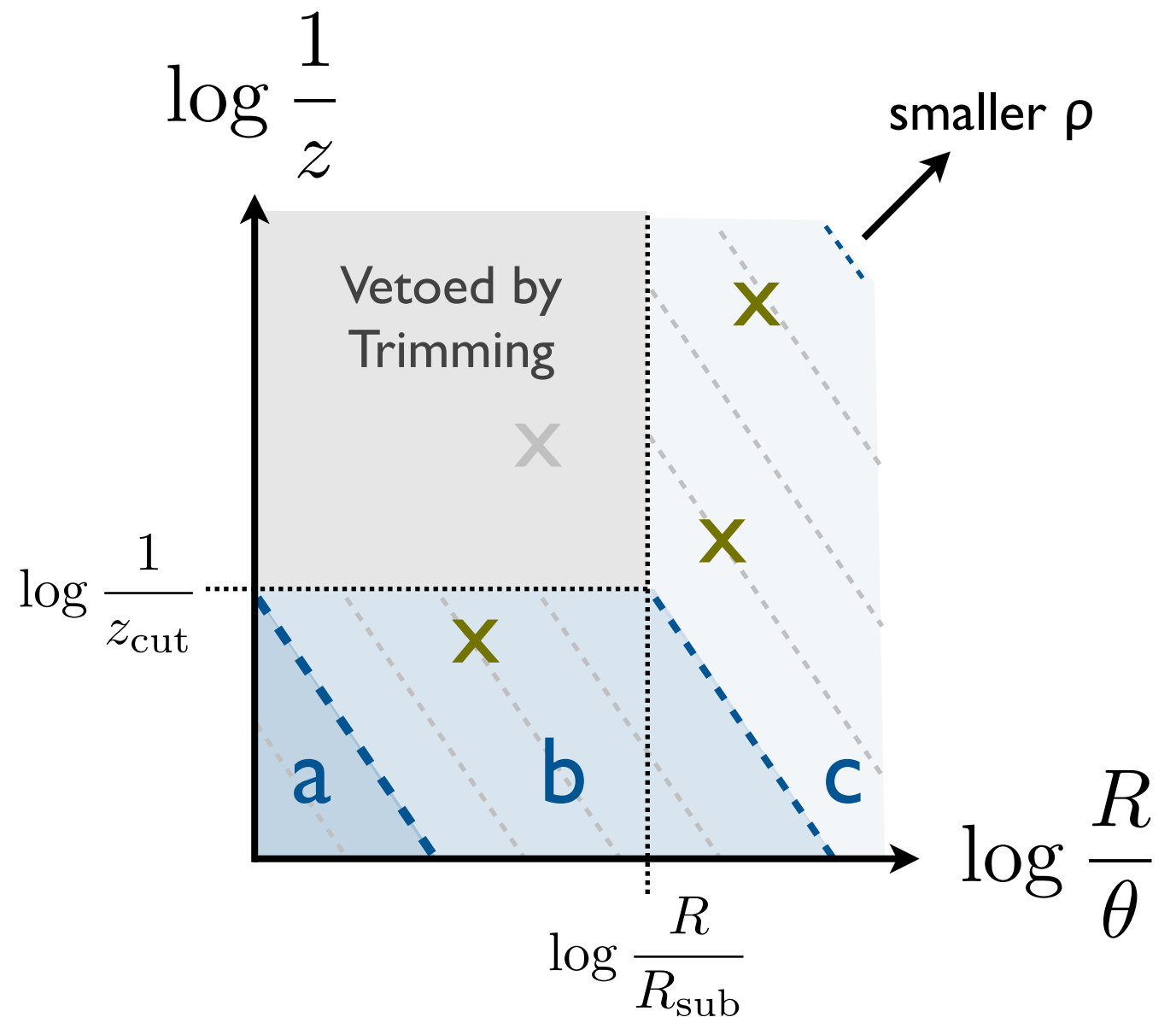


$$dP_{i \rightarrow ig} = \frac{2\alpha_s}{\pi} C_i \frac{d\theta}{\theta} \frac{dz}{z}$$



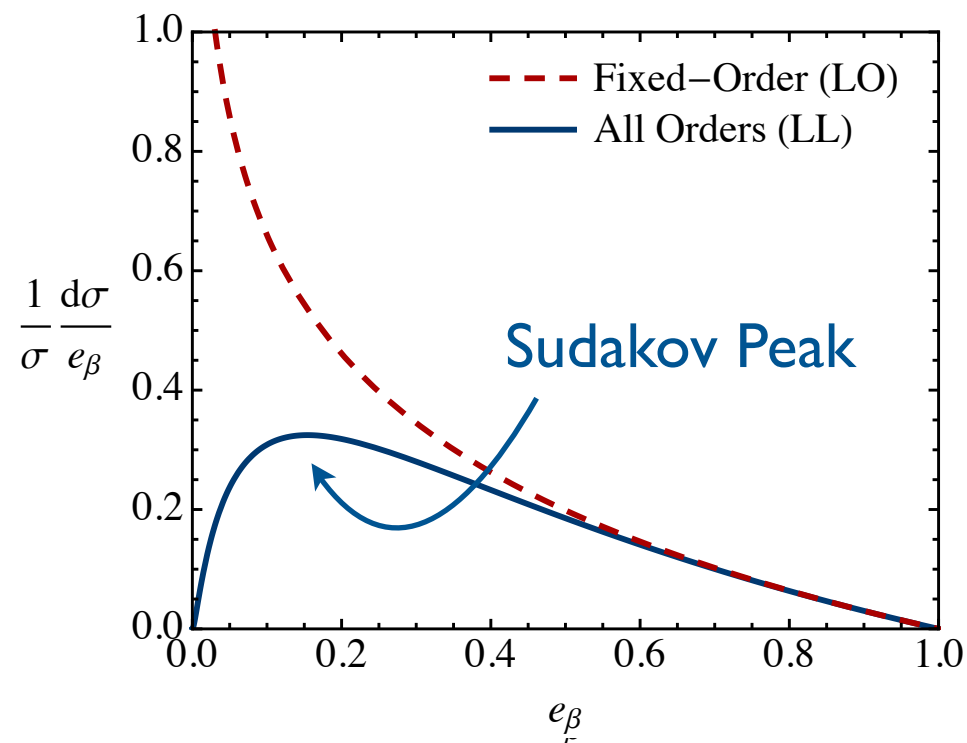
c b a

$$\Delta = e^{-\frac{2\alpha_s}{\pi} C_i}$$



# Resummation vs. Fixed-Order

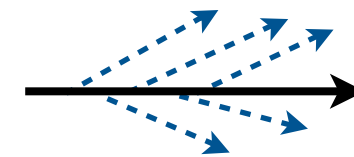
## Ordinary jet mass



$$\Delta = \exp \left[ -\frac{2\alpha_s}{\pi} C_i \left( \frac{1}{4} \log^2 \frac{m^2}{p_T^2 R^2} \right) \right]$$

Sudakov double logs  
dominate distribution

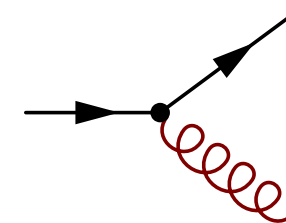
All  $\alpha_s$  Orders  
(resummed logs)



$m=0$  suppressed

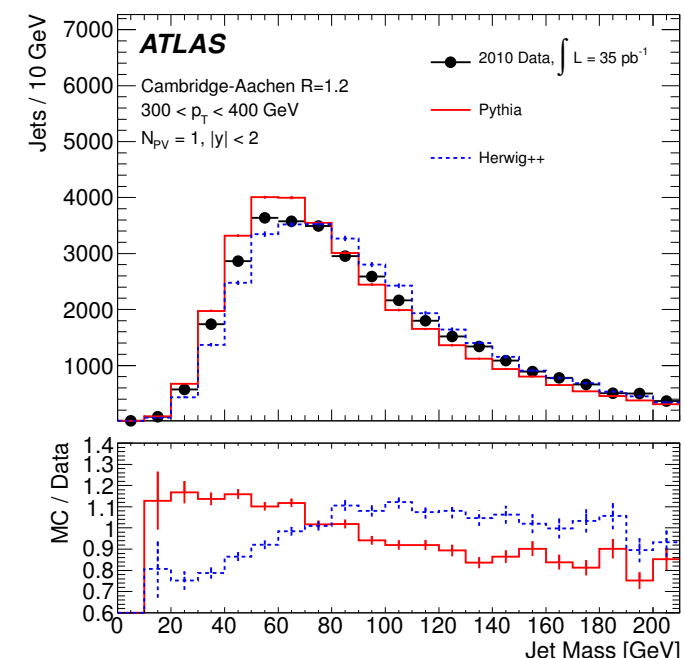
VS.

Fixed  $O(\alpha_s)$



$m=0$  singular

Recall  
Jet Mass  
Data

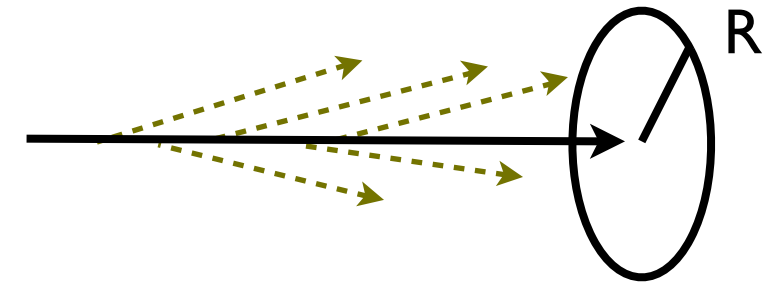


# Systematically Improvable

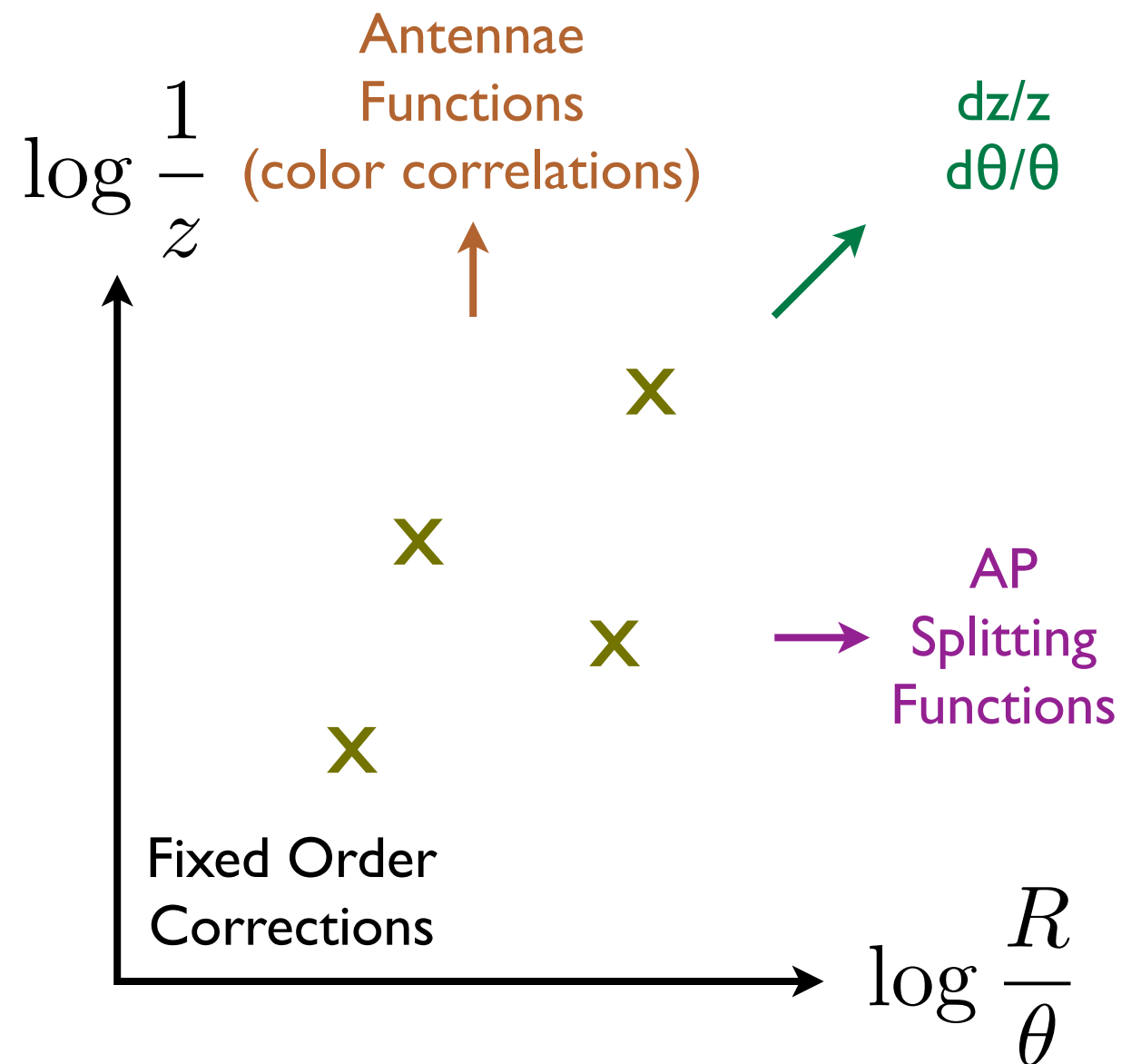
Strongly-Ordered Limit:  
Leading logarithmic terms  
(i.e.  $\alpha_s \log^2 \rho$ )

Higher-Order Effects:  
Running  $\alpha_s$ ,  
Multiple Emissions,  
Energy/Momentum Recoil  
Full Splitting Functions,  
Soft Color Correlations,  
Fixed-Order Corrections,  
Non-global Logarithms, ...

(Many effects already included in  
realistic parton showers)



$$dP_{i \rightarrow ig} = \frac{2\alpha_s}{\pi} C_i \frac{d\theta}{\theta} \frac{dz}{z}$$





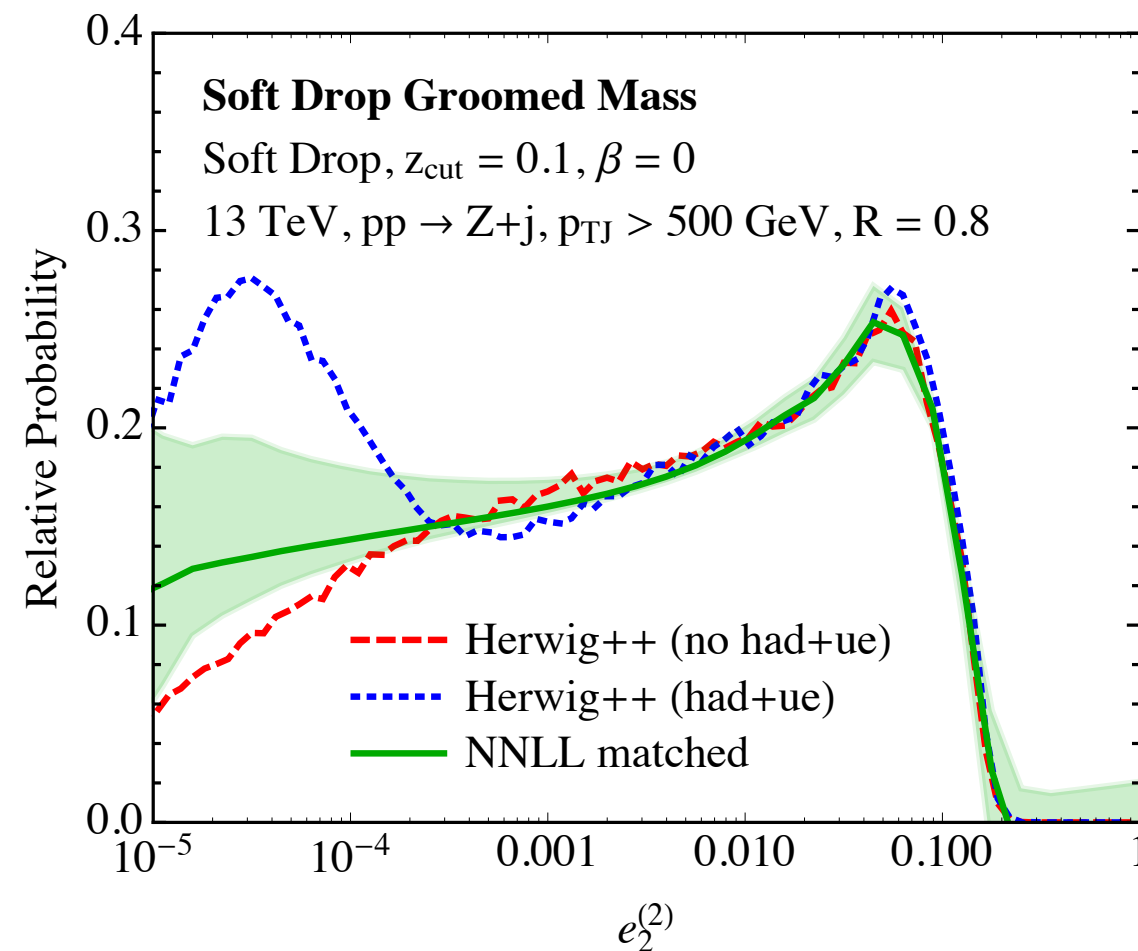
# Systematically Improvable

Strongly-  
Leading log  
(i.e.

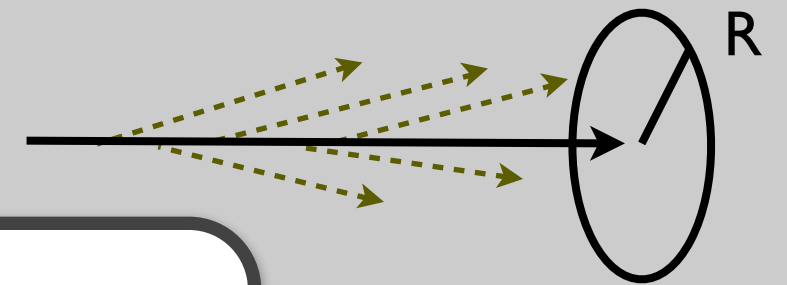
Higher-  
Ru  
Multip  
Energy/Mo  
Full Split  
Soft Colo  
Fixed-Ord  
Non-globa

(Many effects  
realistic p

## First NNLL + $O(\alpha_s^2)$ calculation for substructure in pp



[Frye, Larkoski, Schwartz, Yan, 1603.06375, 1603.09338]



$$= \frac{2\alpha_s}{\pi} C_i \frac{d\theta}{\theta} \frac{dz}{z}$$

$$\frac{dz}{z} \frac{d\theta}{\theta}$$

(ns)

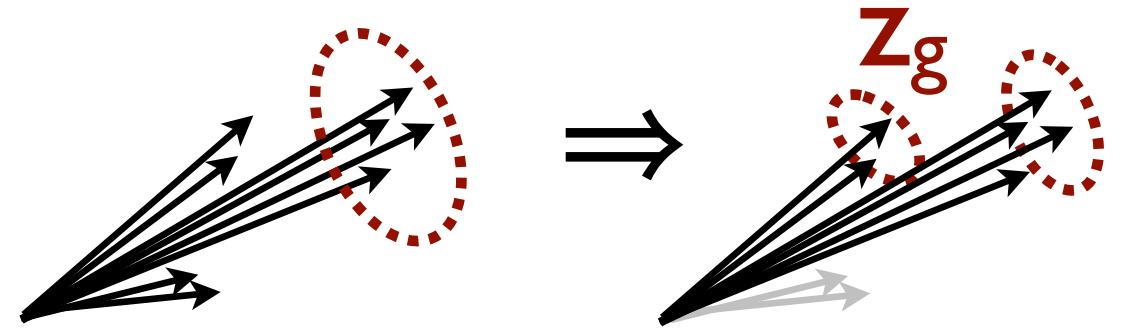
X

AP  
Splitting  
Functions

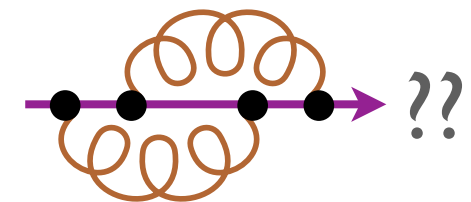
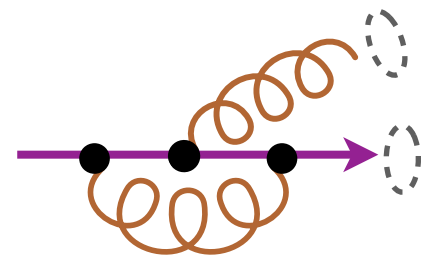
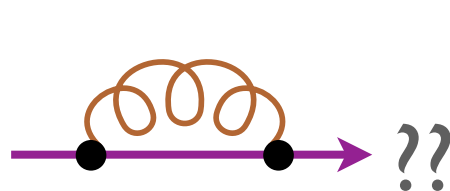
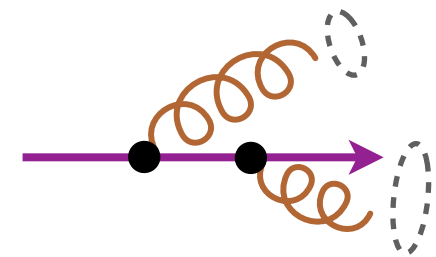
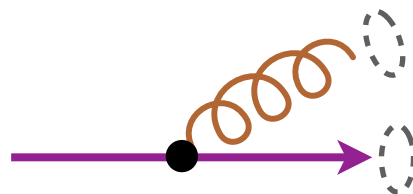
$$\log \frac{R}{\theta}$$

# *Preview of Tomorrow*

# Unsafe Calculations?

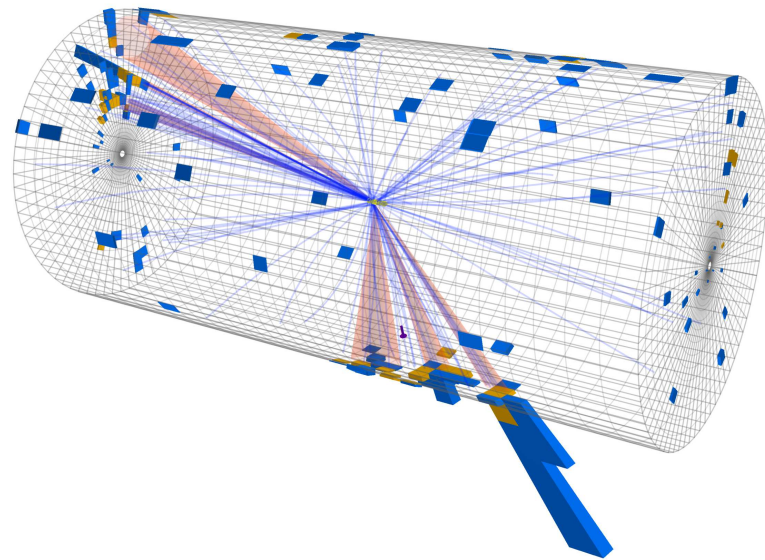


$$p(z_g) = \left( \text{undefined} \right) + \alpha_s \left( \text{infinity} \right) + \alpha_s^2 \left( \text{infinity}^2 \right) + \dots$$



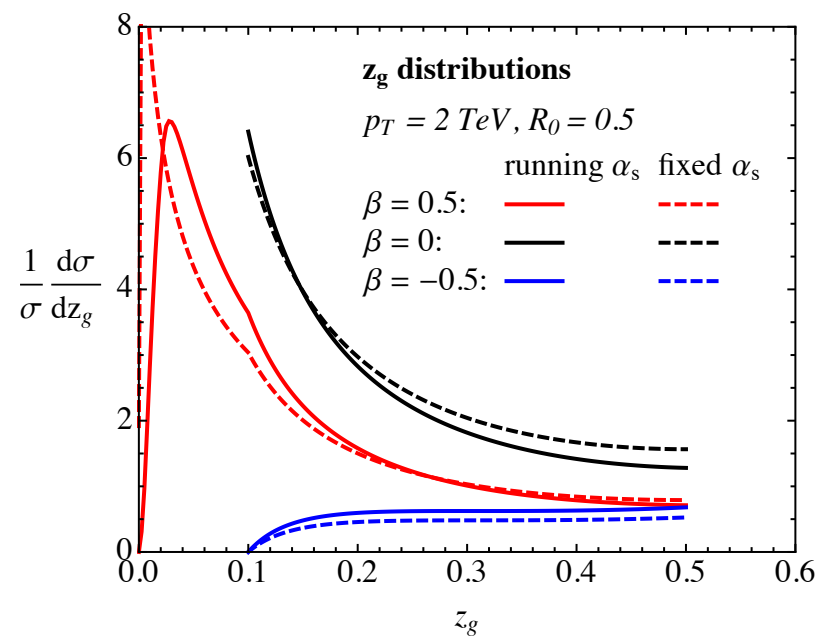
Can you still make  
perturbative predictions for  
IRC unsafe observables?

# Jet Substructure



## *Boosting the Search for New Phenomena*

[Last Thursday & Friday]



## *Pushing the Boundaries of Quantum Field Theory*

[Today & Tuesday]