

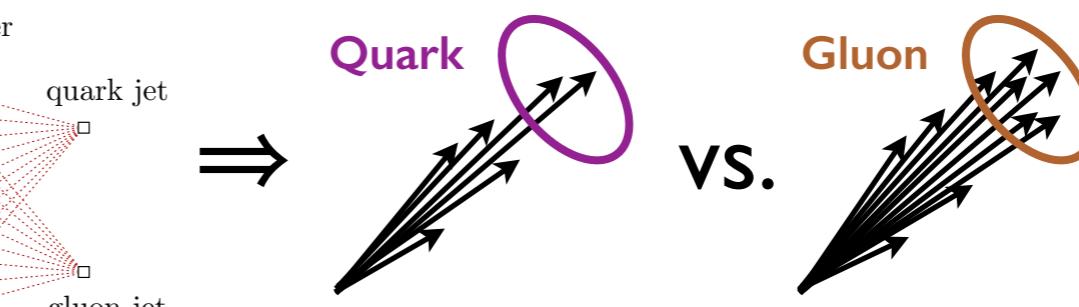
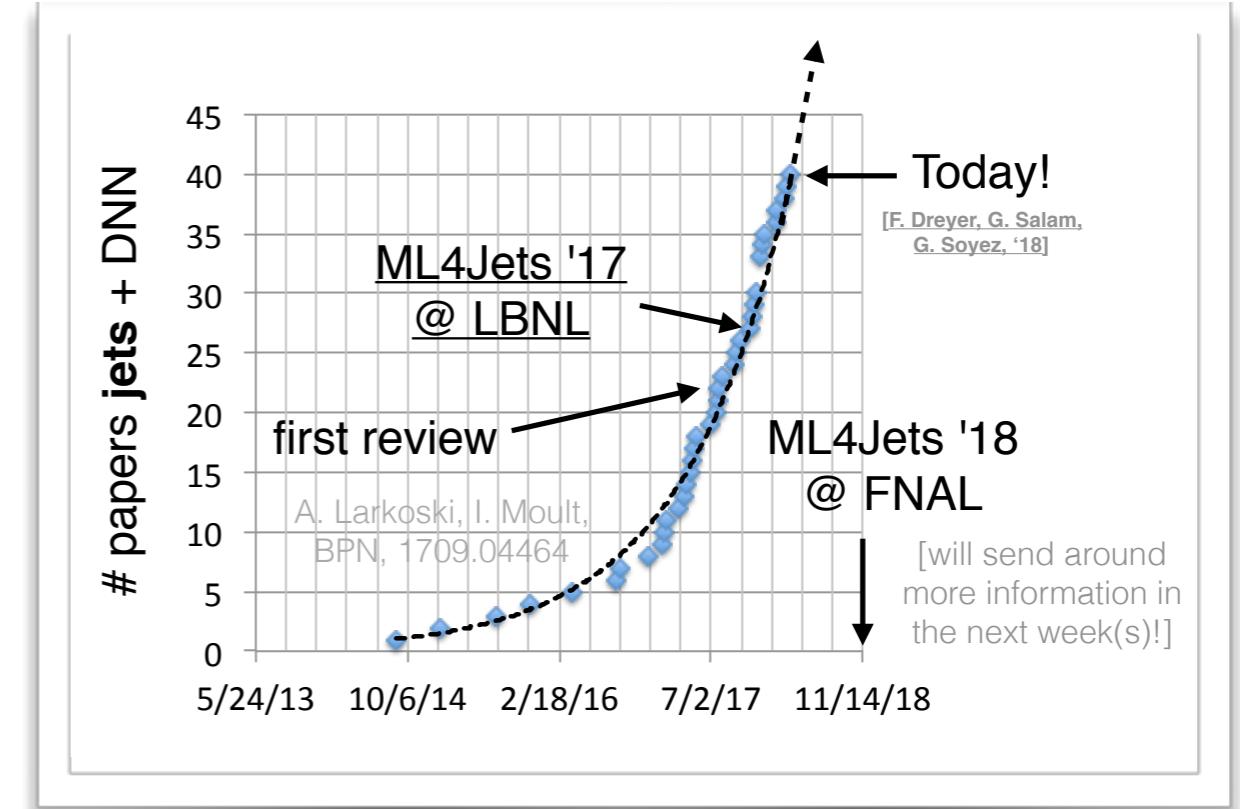
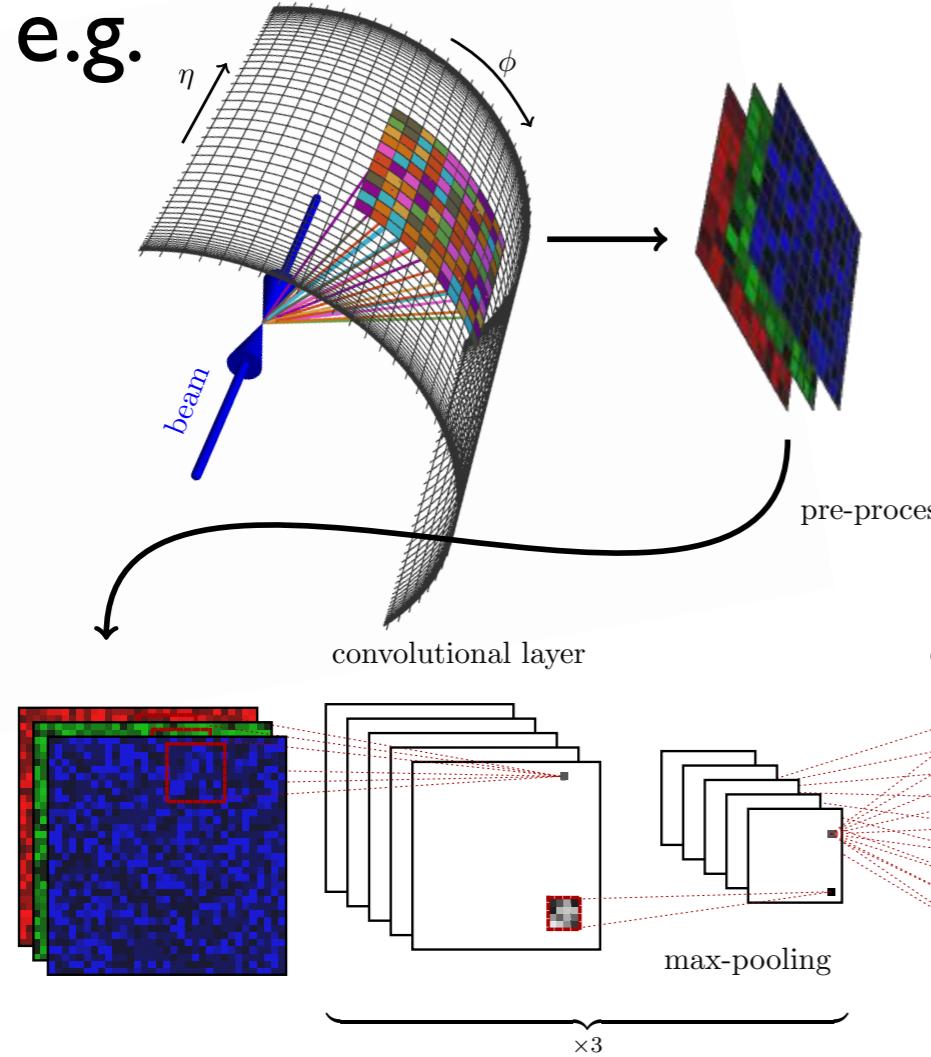
Deep Sets for Particle Jets

Jesse Thaler



NYU CCPP Seminar — October 17, 2018

The Rise of Machine Learning for Jets



[e.g. Komiske, Metodiev, Schwartz, 1612.01551; Nachman, Boost 2018 Talk, July 20, 2018; reviews in Larkoski, Moult, Nachman, 1709.04464; Guest, Cranmer, Whiteson, 1806.11484]

My Perspective c. 2016



“Deep Learning” vs. “Deep Thinking”

My Perspective c. 2018

BOOST 2018

10th International Workshop on Boosted Objects
Phenomenology, Reconstruction and Searches

“Deep Learning”

&

~~vs.~~

“Deep Thinking”

*New first-principles studies of QCD
facilitated by advances in
mathematics, statistics, and computer science*

Desired Outcomes \Leftrightarrow Algorithms/Observables

Proximate Reasons for My Conversion



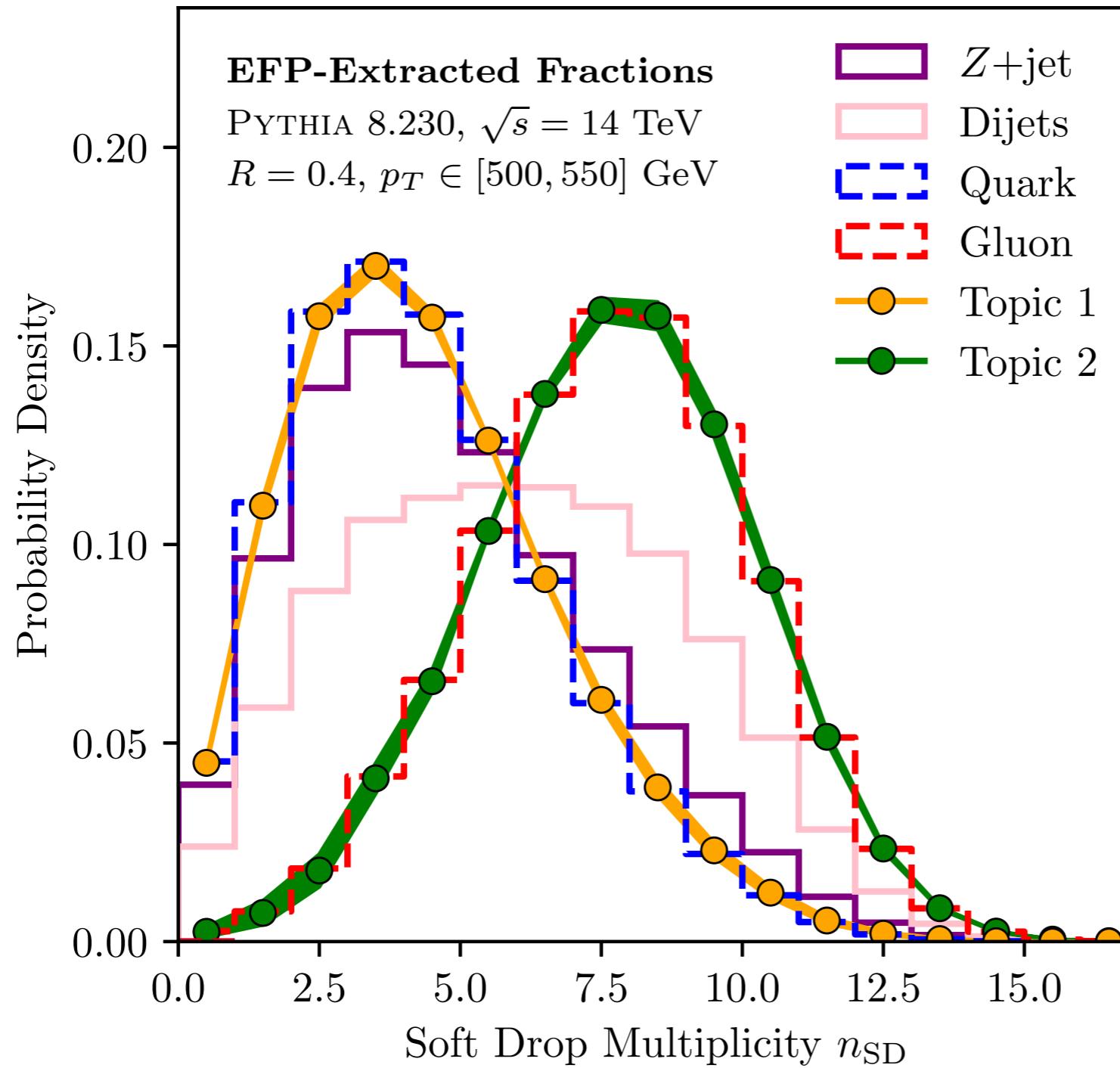
Patrick Komiske



Eric Metodiev

plus Ben Nachman, Kyle Cranmer, Daniel Whiteson, Mike Williams, Matt Schwartz, Dan Roberts, Phiala Shanahan, ...

Physics Reasons for My Conversion (I)



For Offline Discussions

First-principles QCD meets blind source separation with same underlying structure as...

[Komiske, Metodiev, JDT, 1809.01140;
see also Metodiev, JDT, 1802.00008; Metodiev, Nachman, JDT, 1708.02949]

Topic Modeling

Topics

gene	0.04
dna	0.02
genetic	0.01
...	

life	0.02
evolve	0.01
organism	0.01
...	

brain	0.04
neuron	0.02
nerve	0.01
...	

data	0.02
number	0.02
computer	0.01
...	

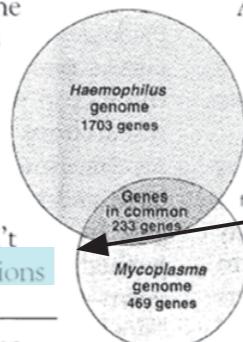
Documents

Seeking Life's Bare (Genetic) Necessities

COLD SPRING HARBOR, NEW YORK—How many genes does an organism need to survive? Last week at the genome meeting here,* two genome researchers with radically different approaches presented complementary views of the basic genes needed for life. One research team, using computer analyses to compare known genomes, concluded that today's organisms can be sustained with just 250 genes, and that the earliest life forms required a mere 128 genes. The other researcher mapped genes in a simple parasite and estimated that for this organism, 800 genes are plenty to do the job—but that anything short of 100 wouldn't be enough.

Although the numbers don't match precisely, those predictions

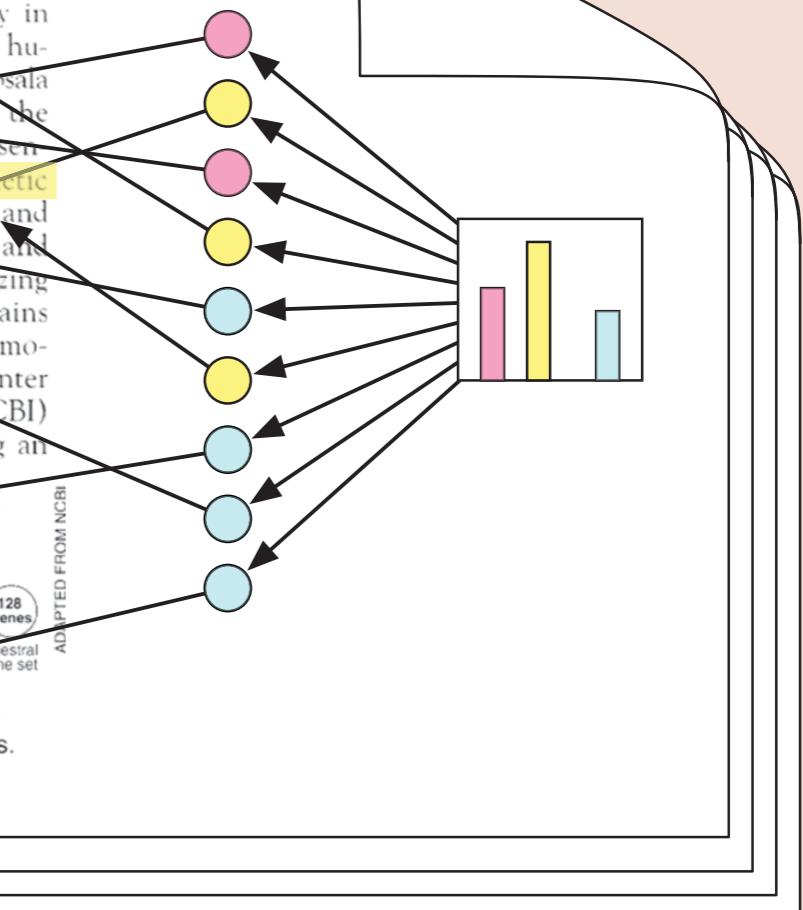
"are not all that far apart," especially in comparison to the 75,000 genes in the human genome, notes Siv Andersson of Uppsala University in Sweden, who arrived at the 800 number. But coming up with a consensus answer may be more than just a genetic numbers game, particularly as more and more genomes are completely mapped and sequenced. "It may be a way of organizing any newly sequenced genome," explains Arcady Mushegian, a computational molecular biologist at the National Center for Biotechnology Information (NCBI) in Bethesda, Maryland. Comparing an



Stripping down. Computer analysis yields an estimate of the minimum modern and ancient genomes.

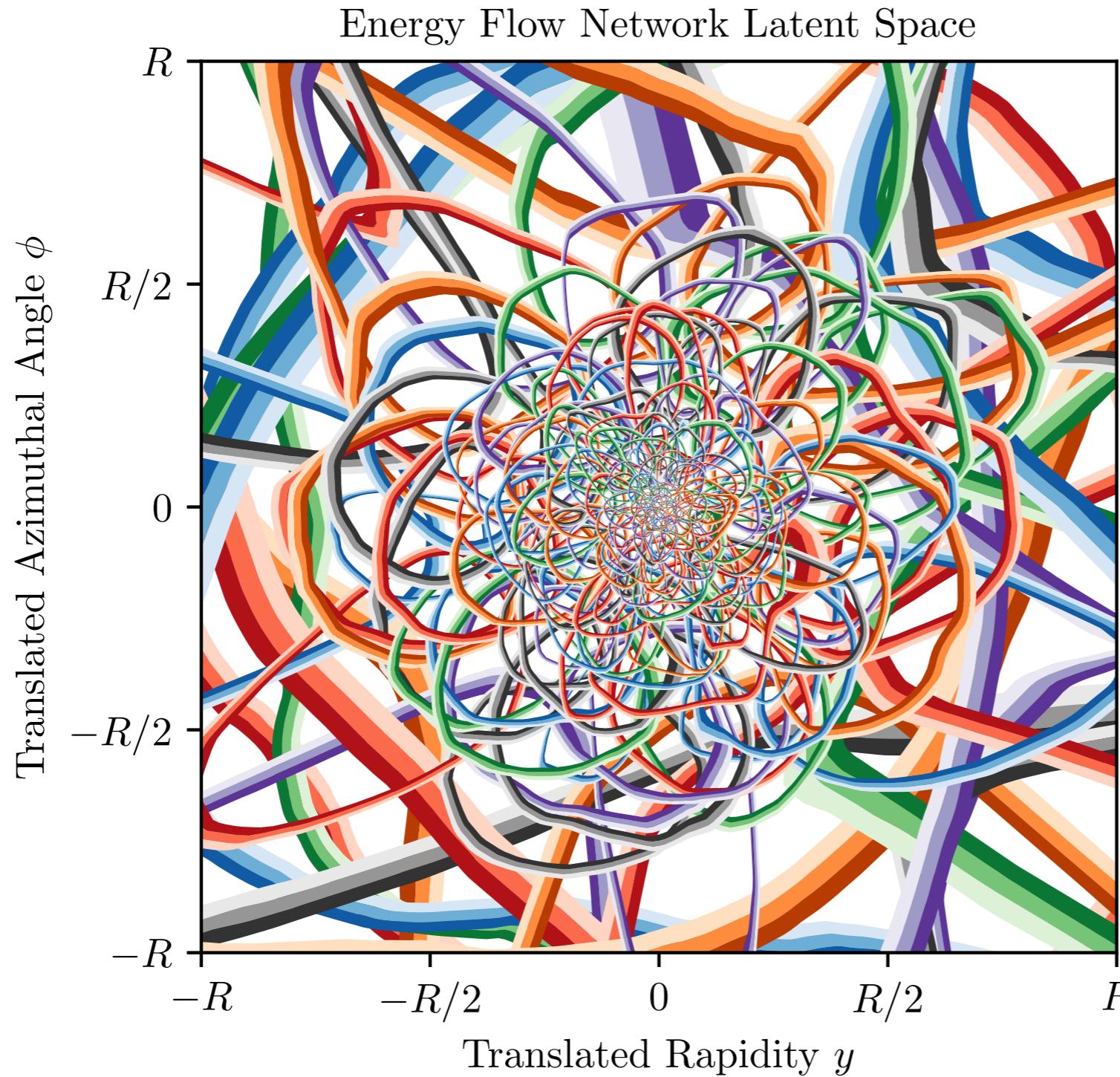
SCIENCE • VOL. 272 • 24 MAY 1996

Topic proportions and assignments



[Blei, 2012]

Physics Reasons for My Conversion (2)



Today's Talk

First-principles QCD
meets neural networks
with same underlying
symmetries as...

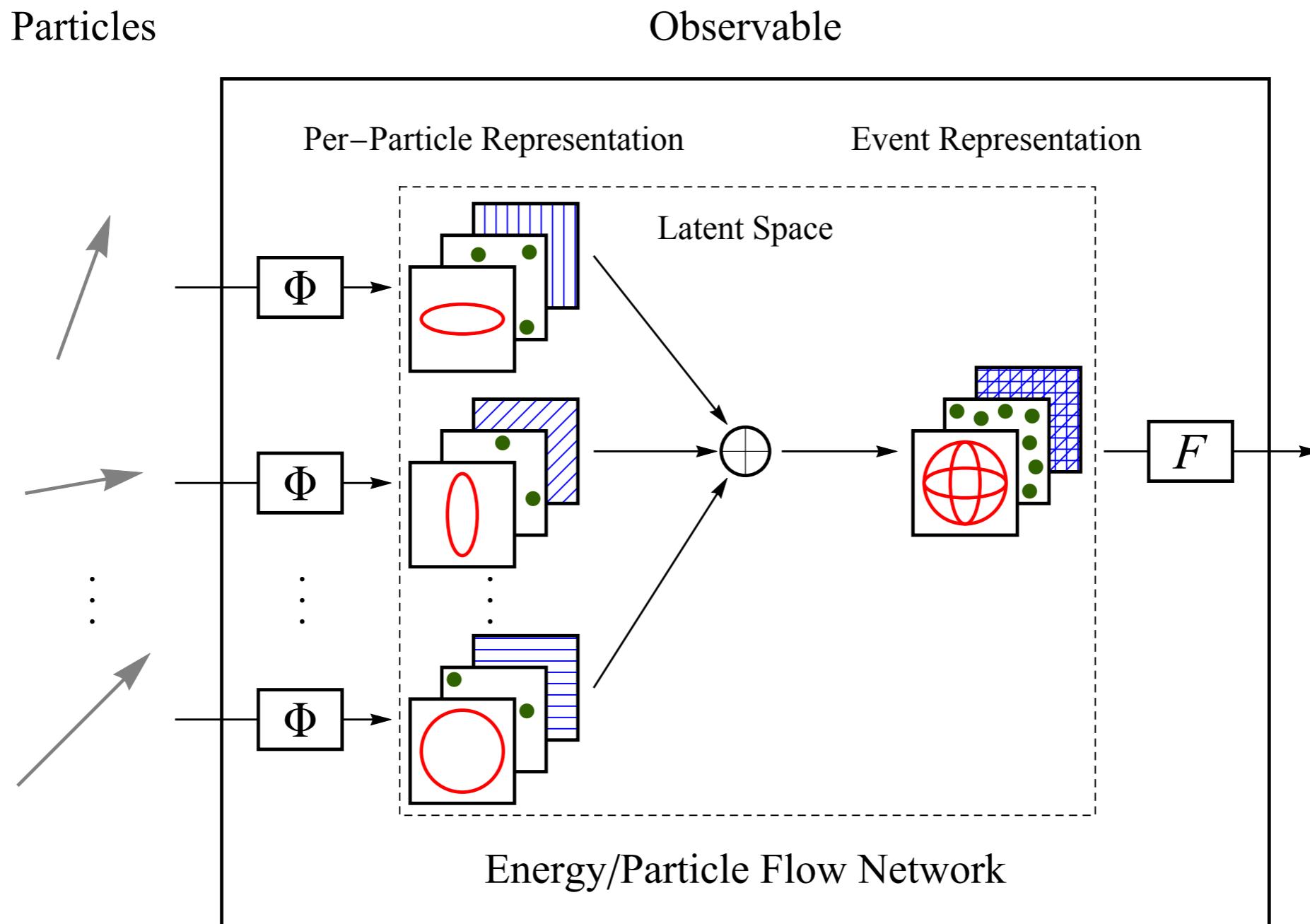
[Komiske, Metodiev, JDT, 1810.05165;
see also Komiske, Metodiev, JDT, 1712.07124]

Point Clouds



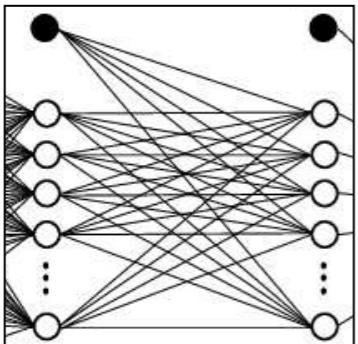
[Popular Science, 2013]

Introducing Energy Flow Networks

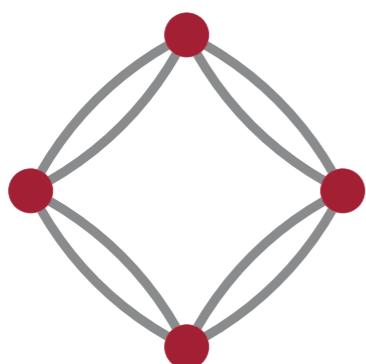


[Komiske, Metodiev, JDT, 1810.05.165]

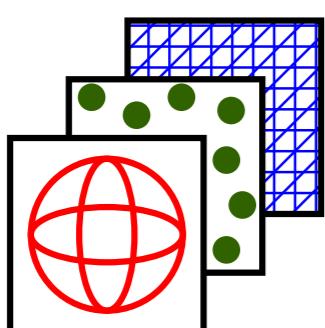
Outline



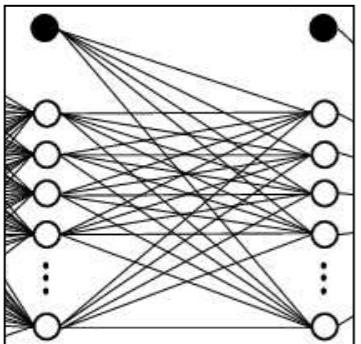
Into the Network



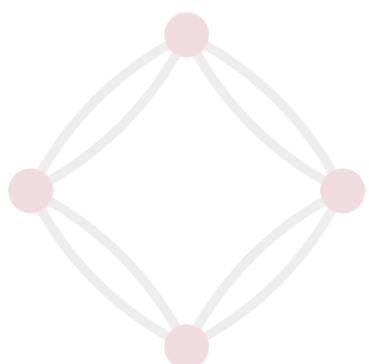
Symmetries & Safety



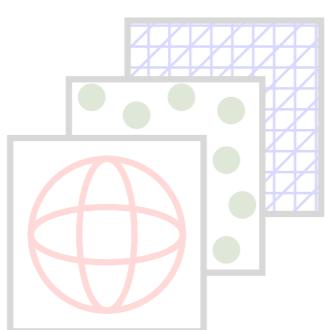
Deep Sets for Particle Jets



Into the Network



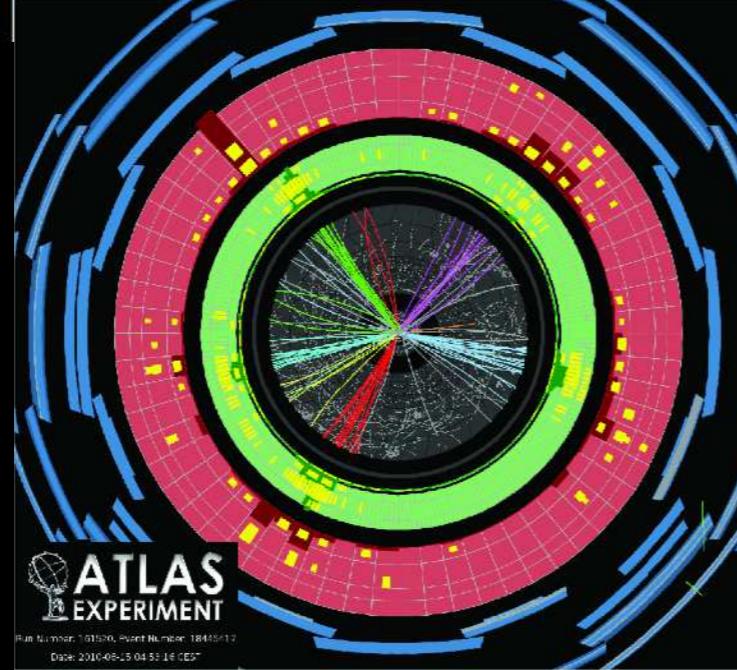
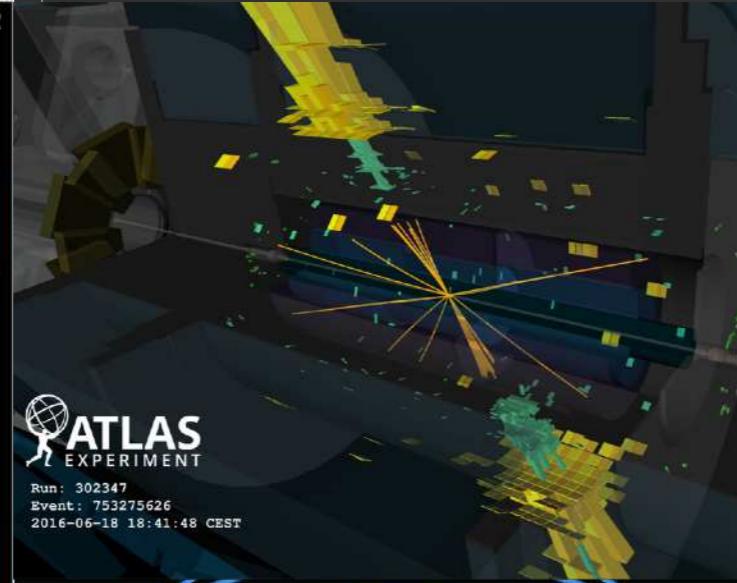
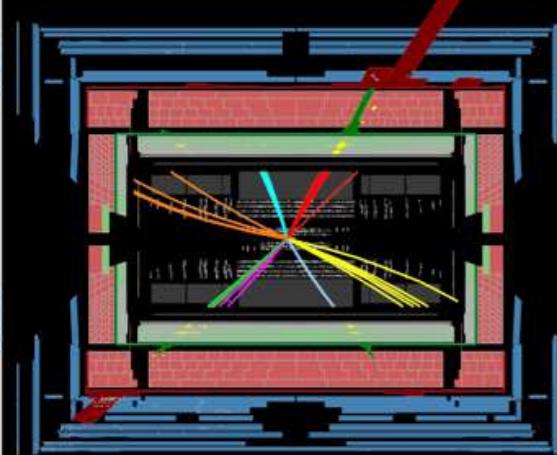
Symmetries & Safety



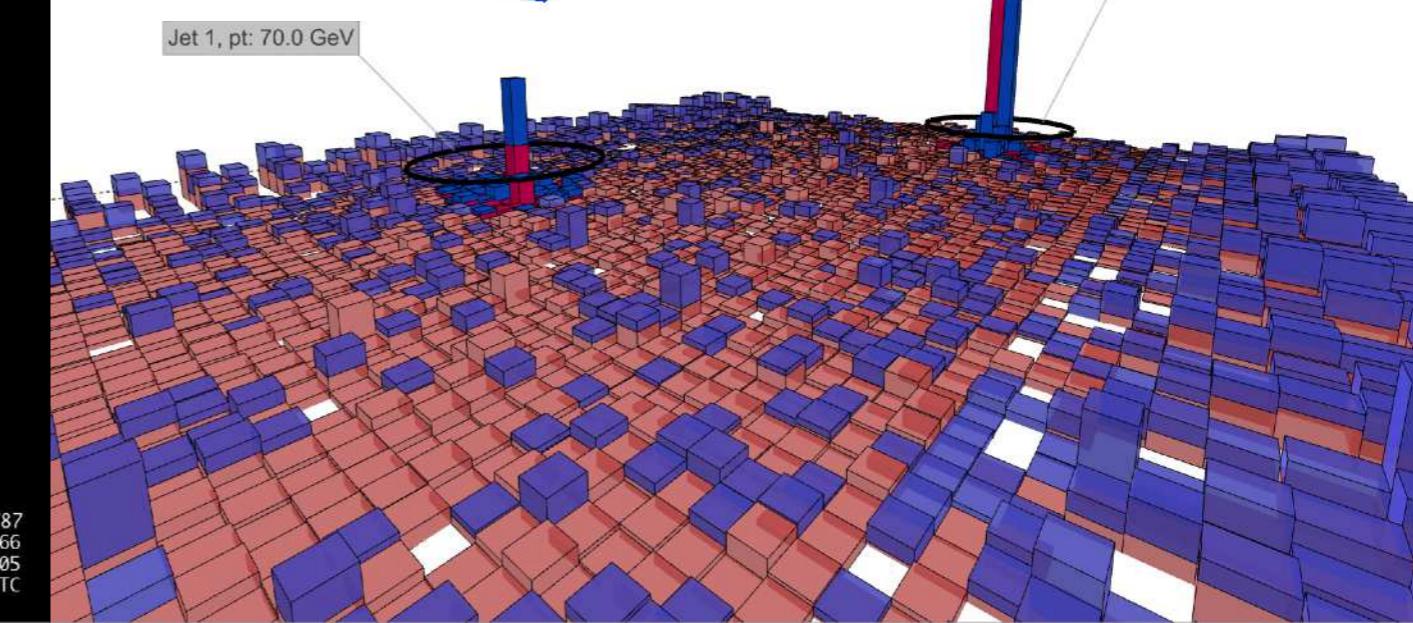
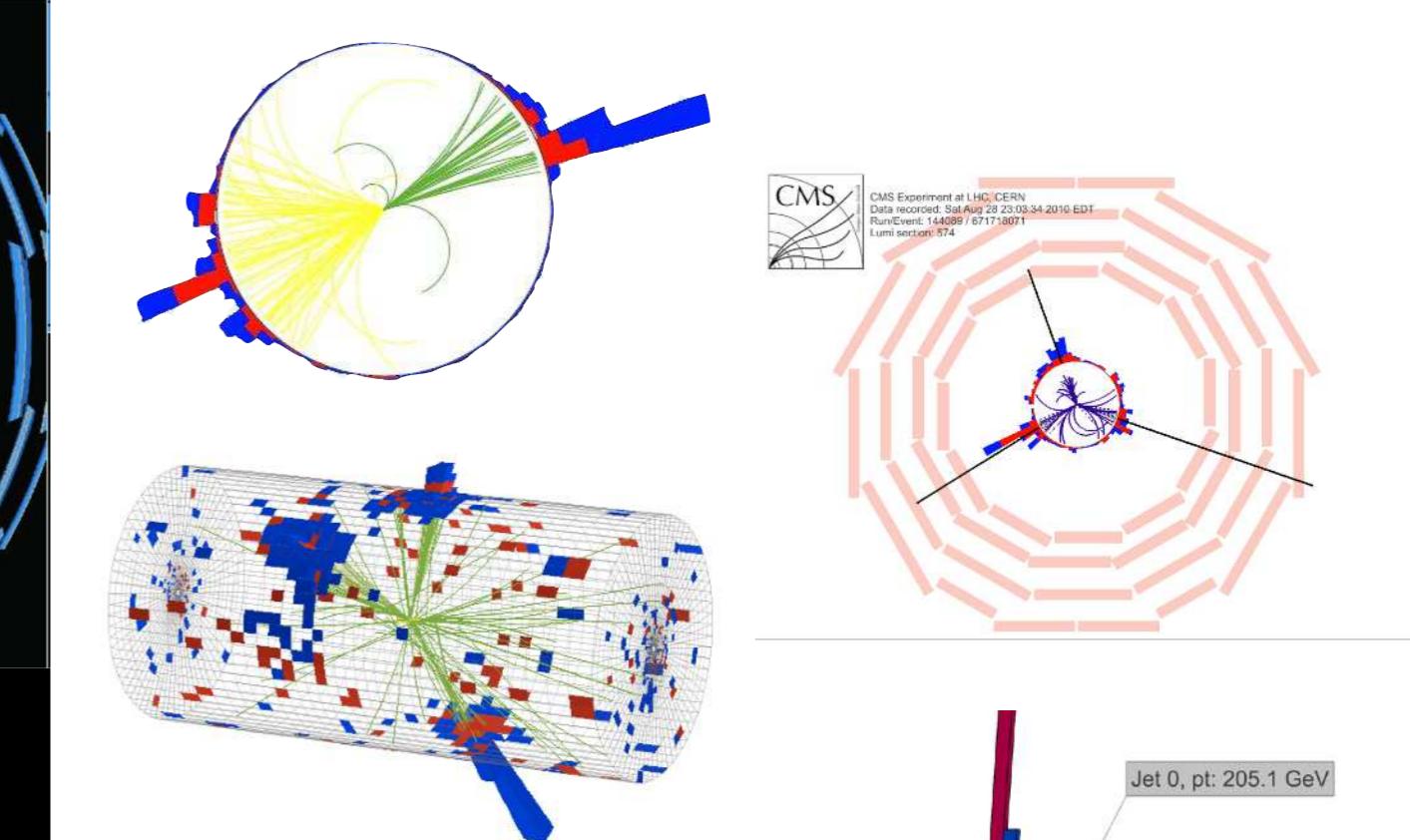
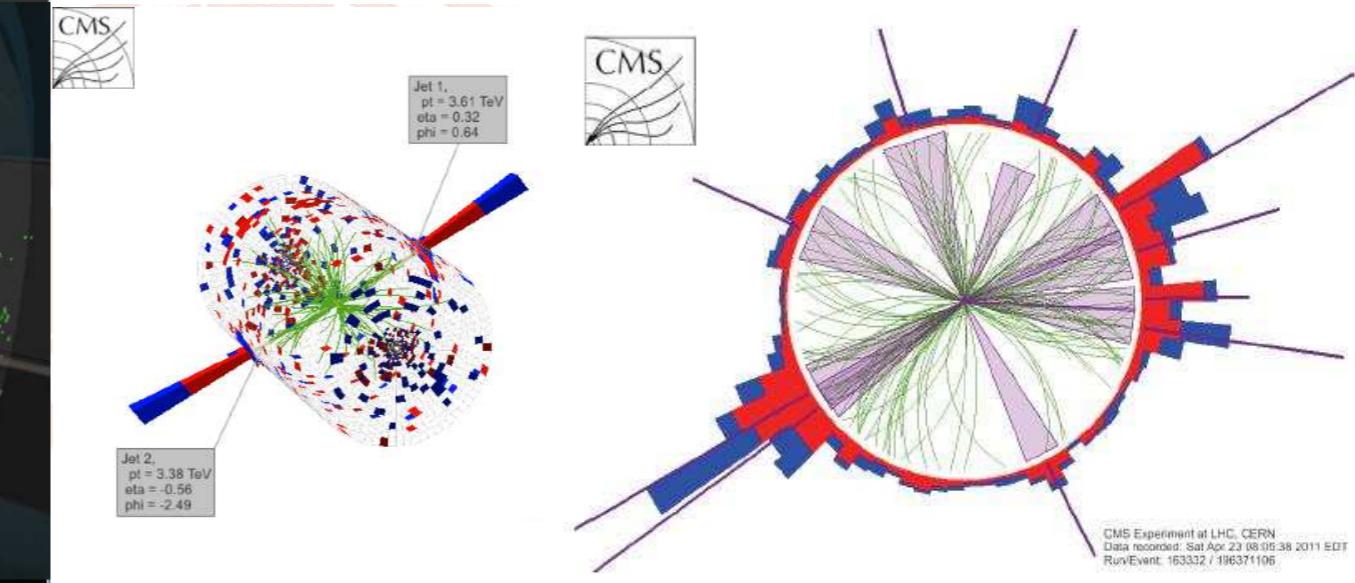
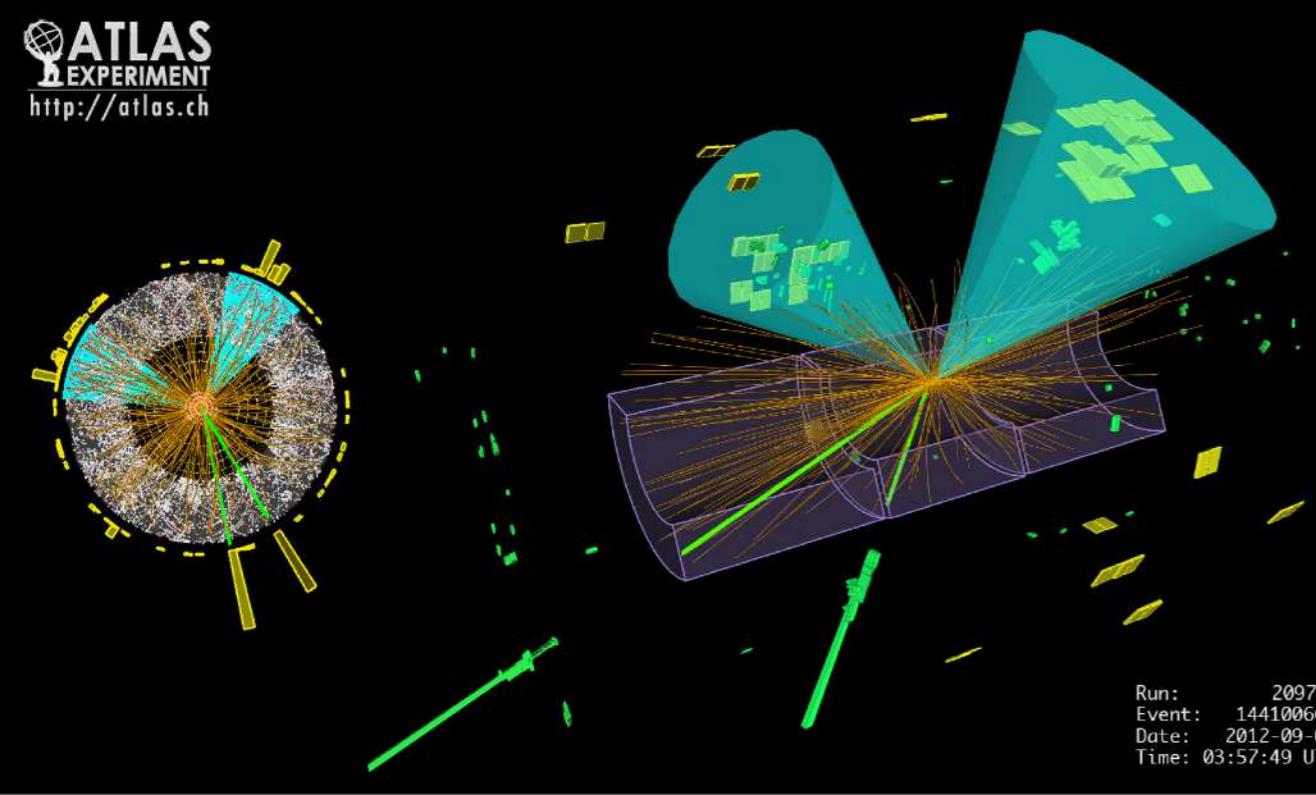
Deep Sets for Particle Jets

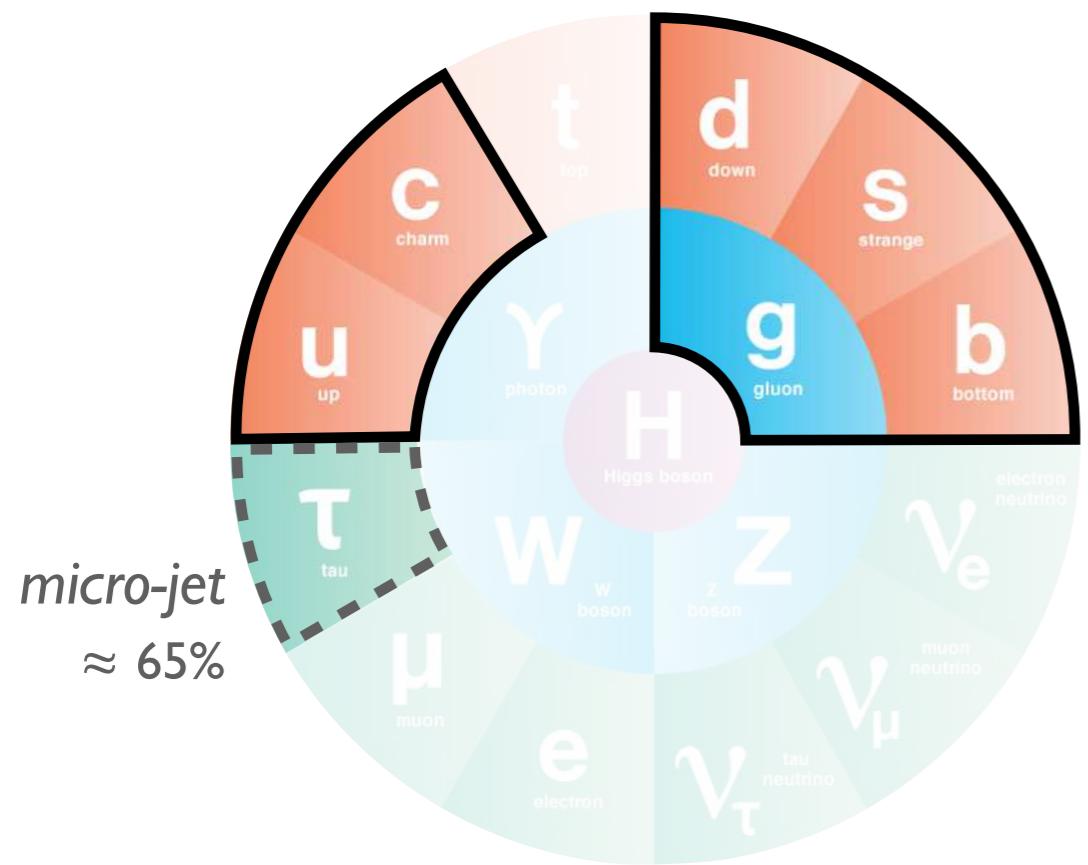
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Date: 2010-07-18 11:05:54 CEST



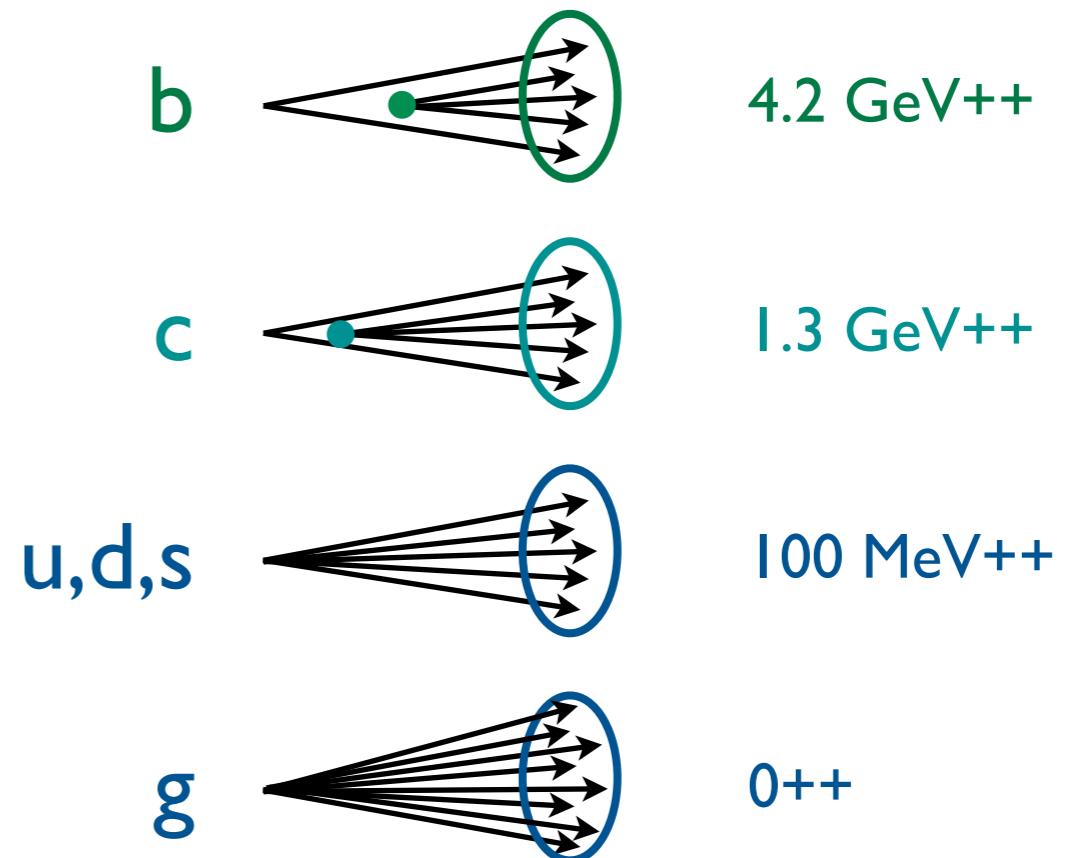
ATLAS
EXPERIMENT
<http://atlas.ch>

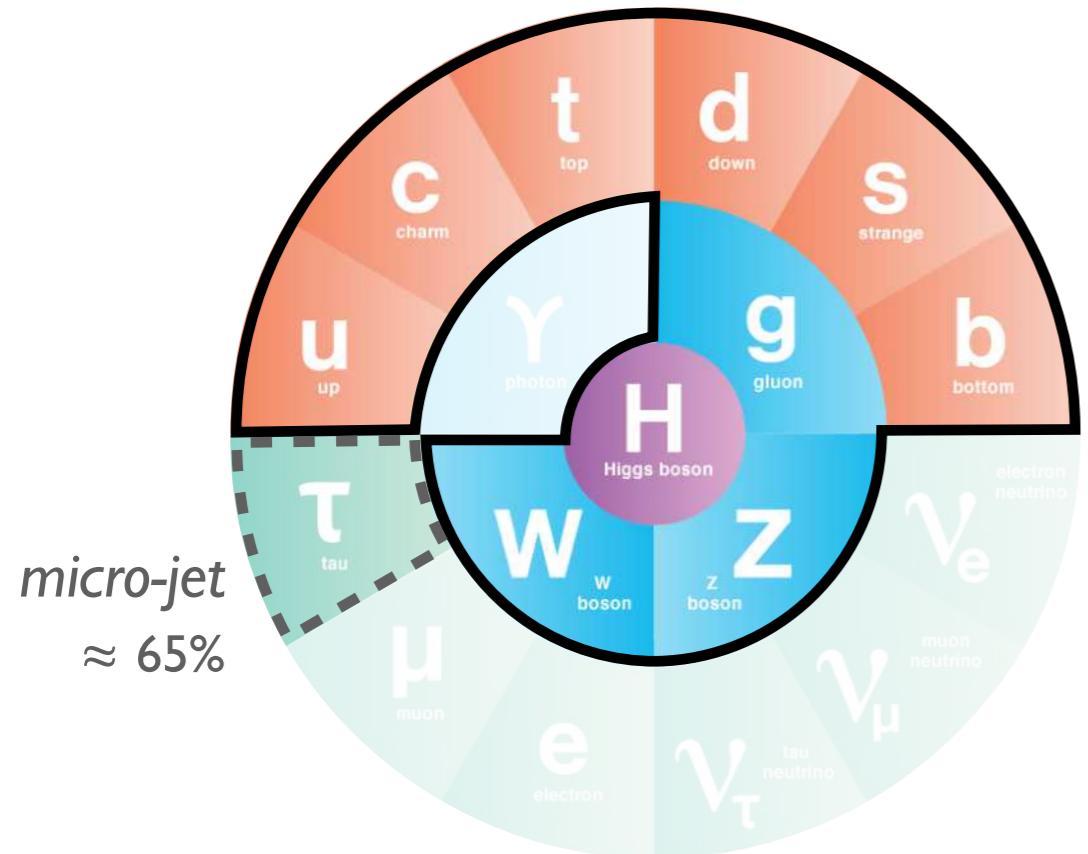




Jets from the Standard Model

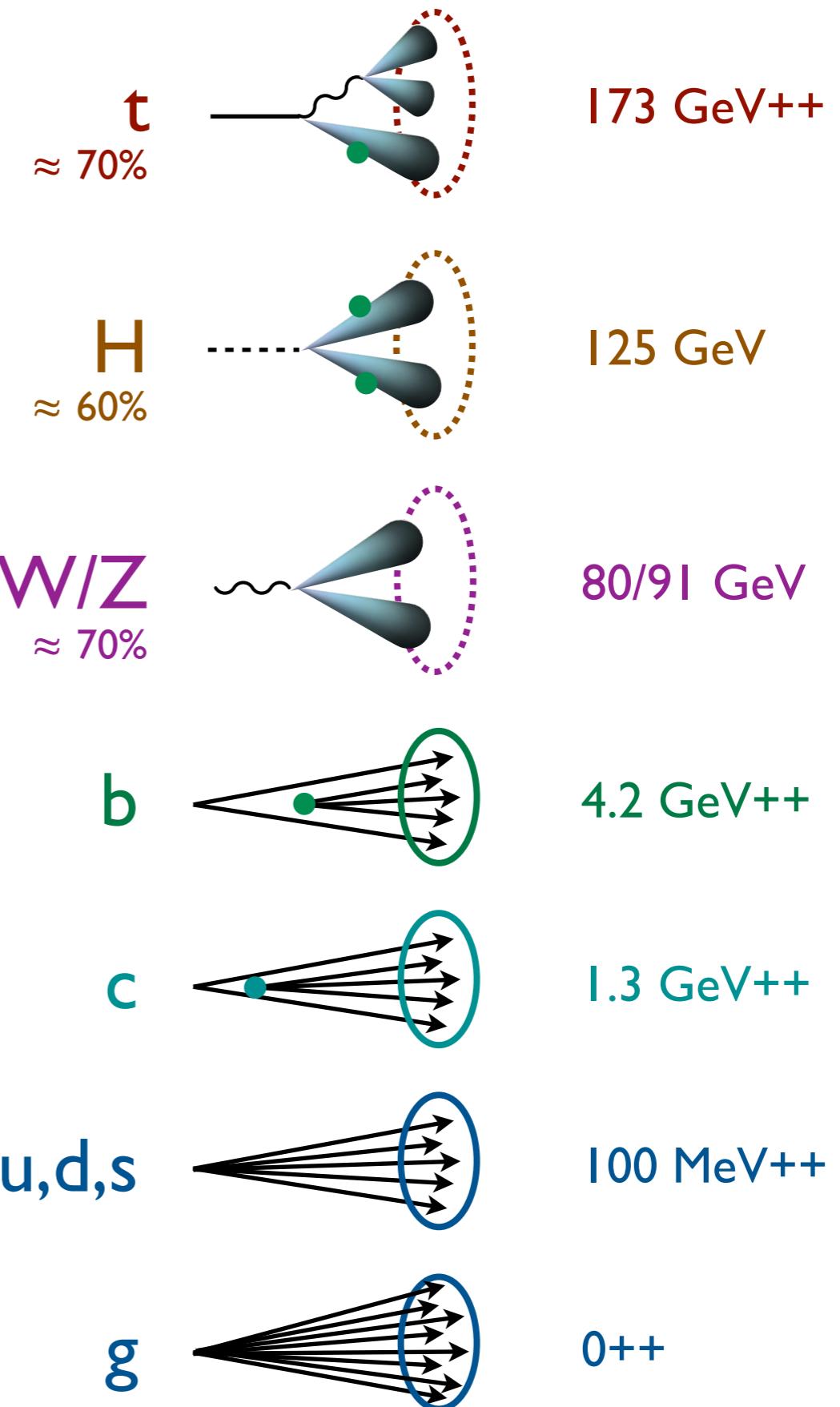
++ = Mass from QCD Radiation

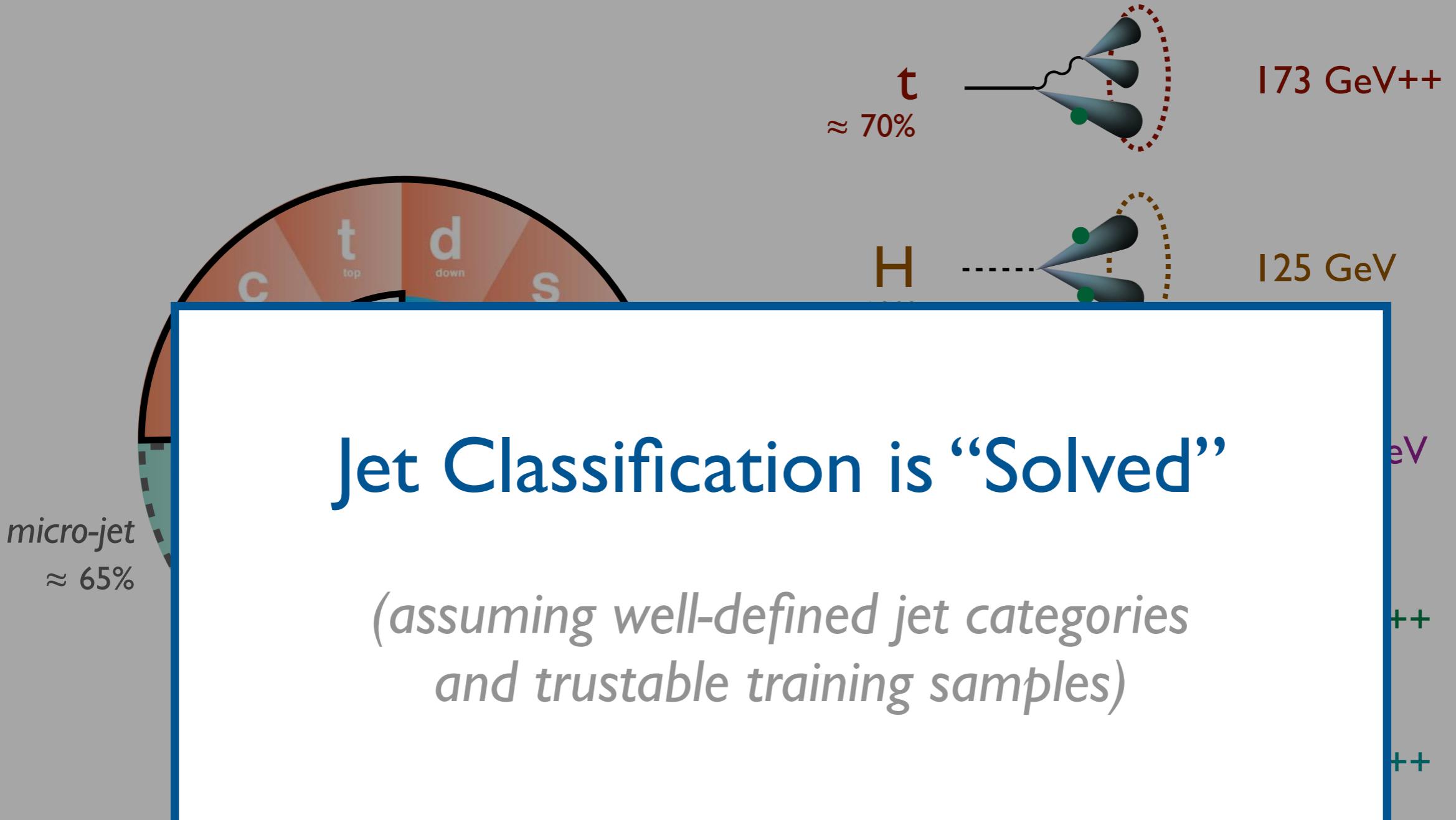




Jets from the Standard Model

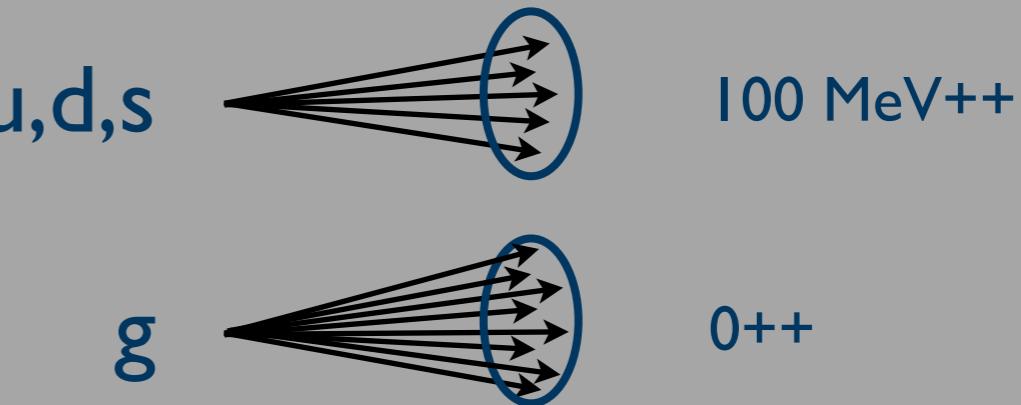
$++$ = Mass from QCD Radiation

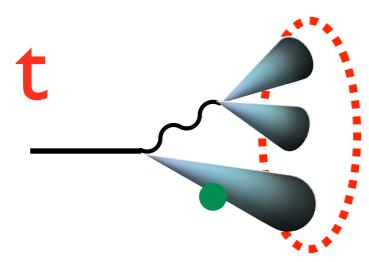




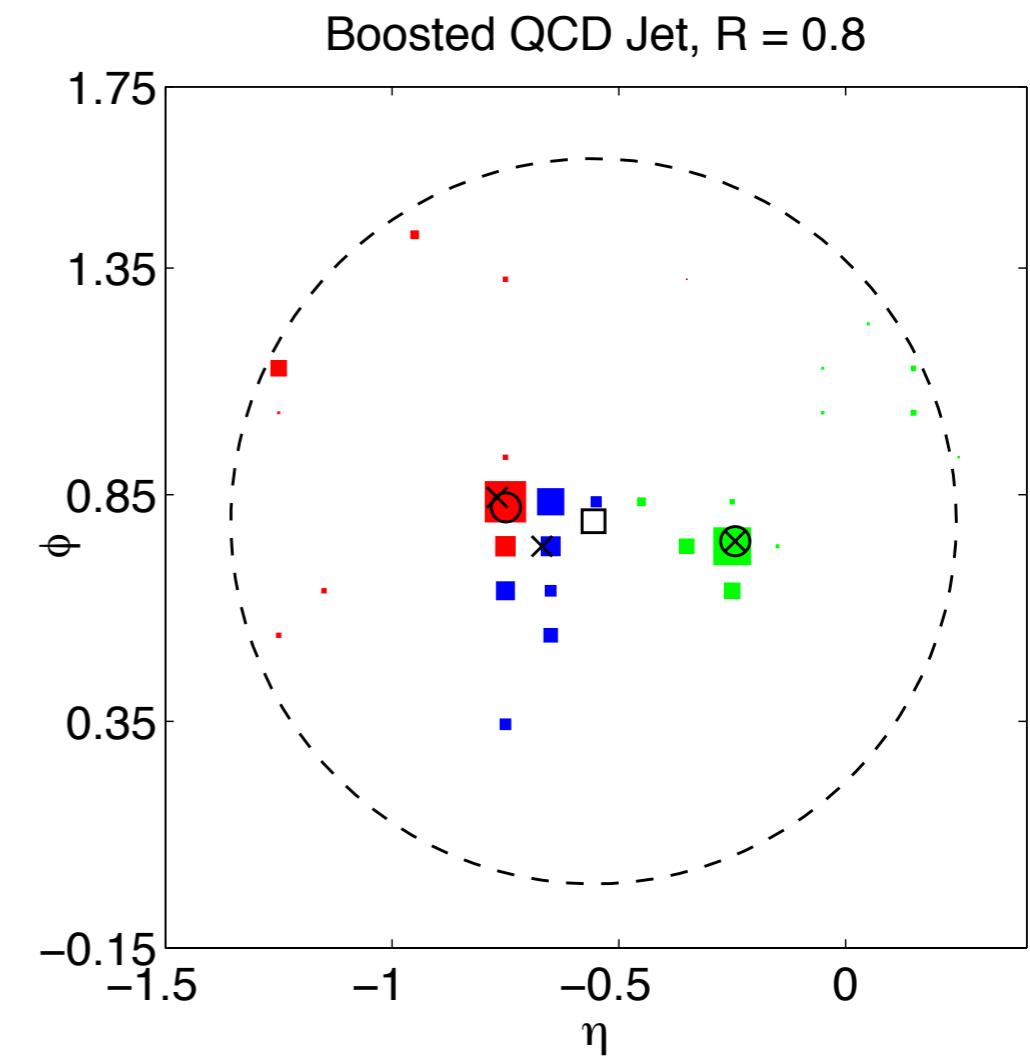
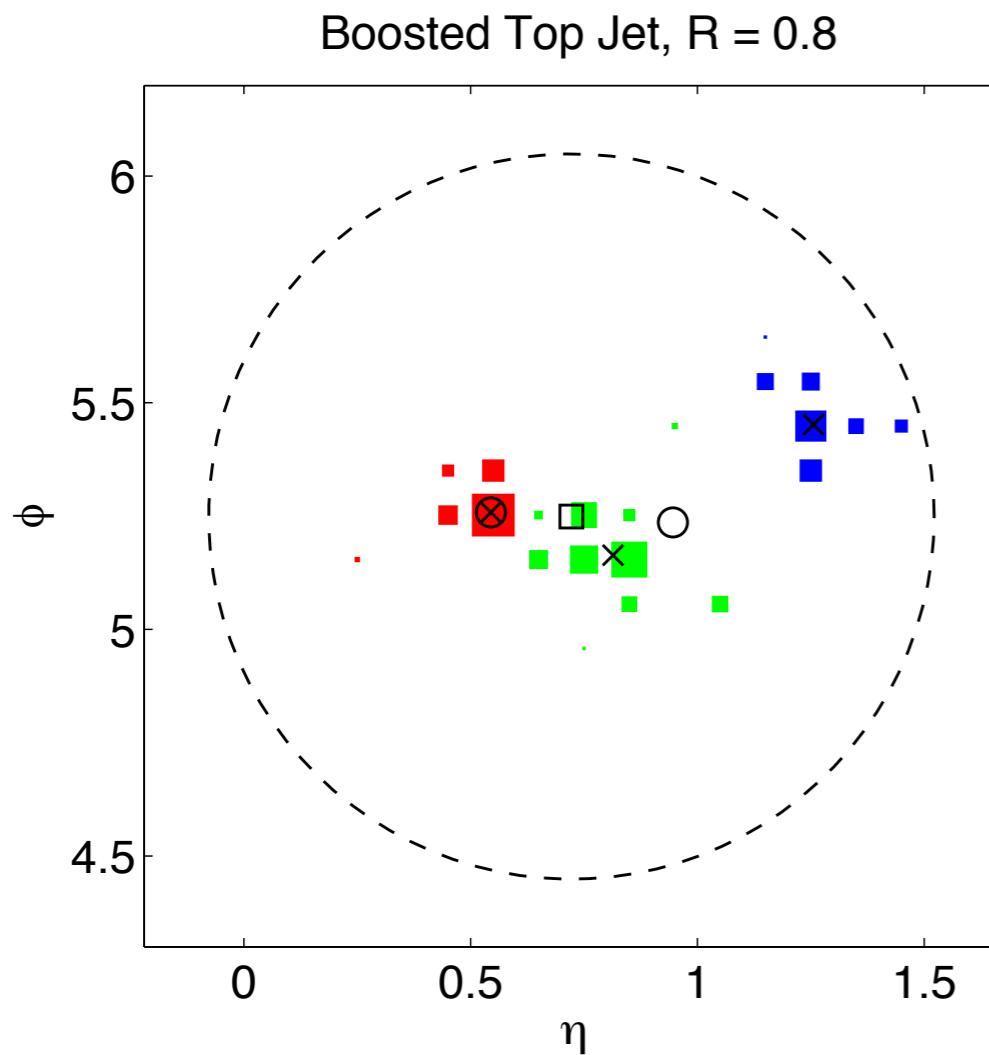
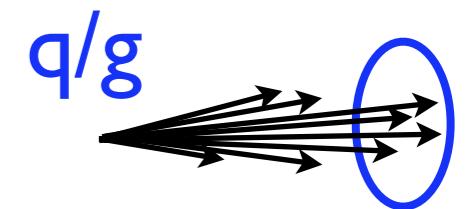
Standard Model

++ = Mass from QCD Radiation

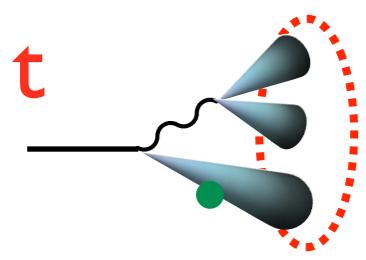




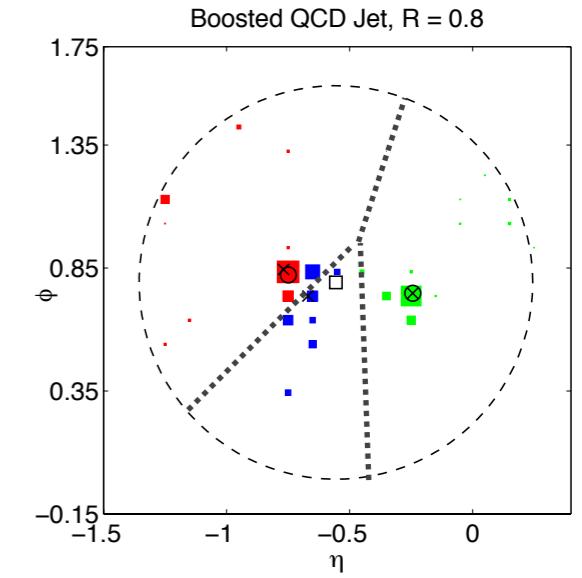
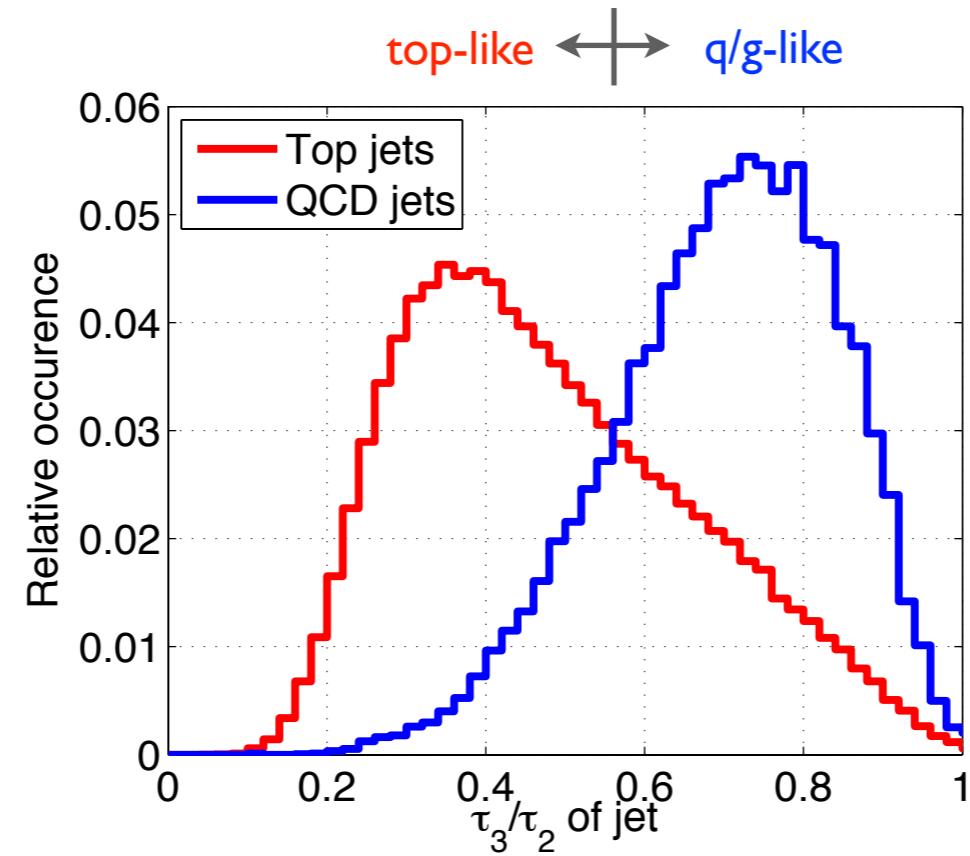
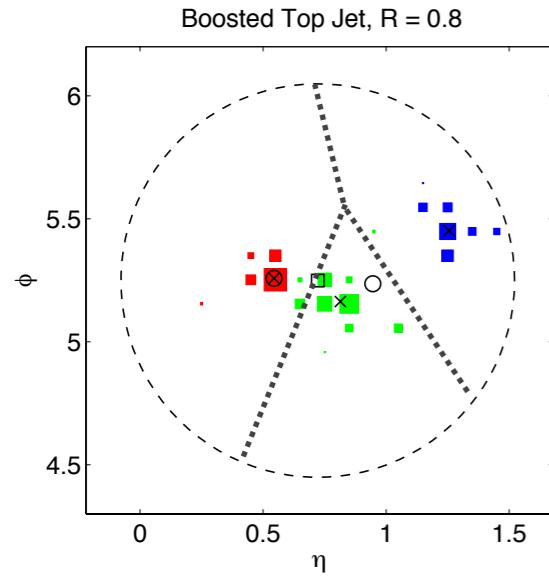
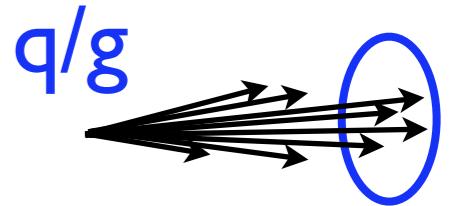
3-Prong vs. 1-Prong



If your eyes can do it...



3-Prong vs. 1-Prong

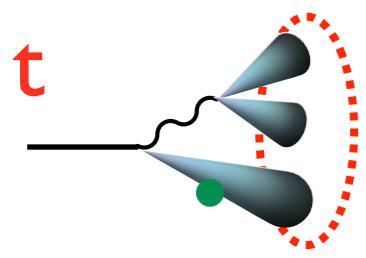


N-subjettiness

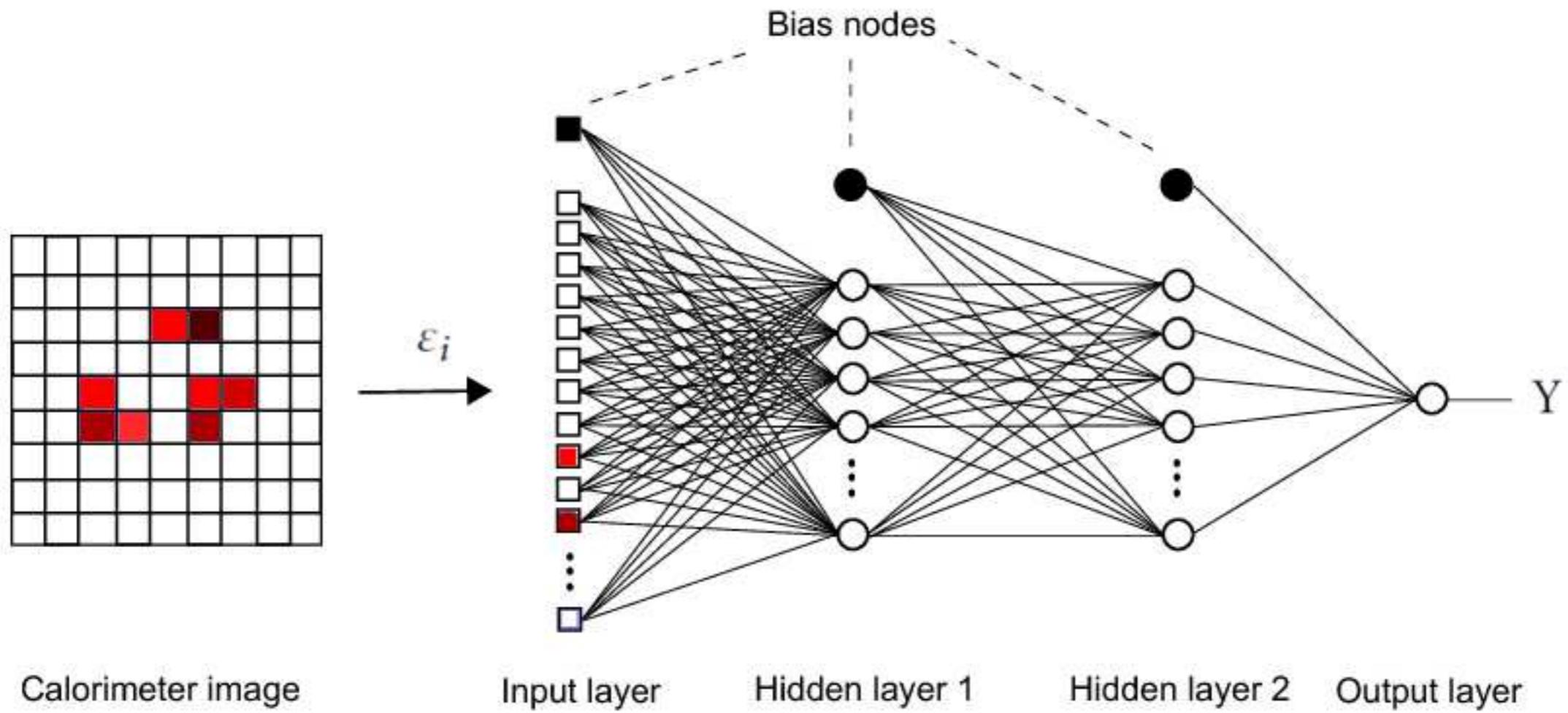
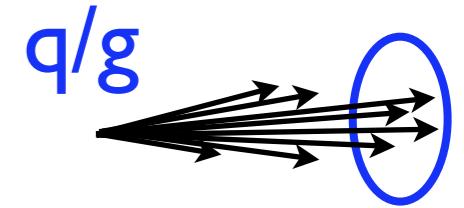
“Deep Thinking”:

$$\tau_N = \sum_k p_{T,k} \min \{ \Delta R_{k,1}, \Delta R_{k,2}, \dots, \Delta R_{k,N} \}$$

[e.g. JDT, Van Tilburg, 1011.2268, 1108.2701]



3-Prong vs. 1-Prong



“Deep Learning”: BDTs, FLDs, DNNs, CNNs, RNNs, ...

[e.g. Almeida, Backović, Cliche, Lee, Perelstein, 1501.05968]

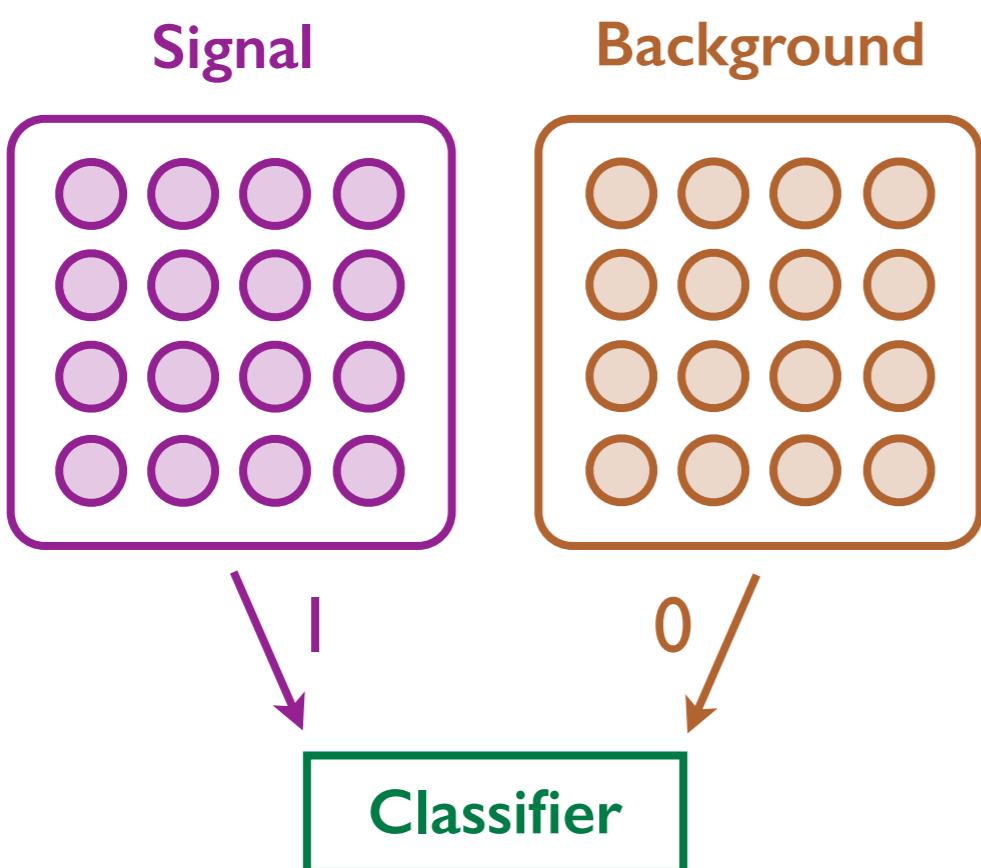
A Cartoon of Machine Learning

For fully-supervised jet classification

(see backup for regression, generation, modeling)

$$\ell_{\text{MSE}} = \left\langle (\textcolor{violet}{h}(\vec{x}) - 1)^2 \right\rangle_{\text{signal}} + \left\langle (\textcolor{violet}{h}(\vec{x}) - 0)^2 \right\rangle_{\text{background}}$$

Classifier Inputs



Minimize Loss Function

(assuming infinite training sets,
and flexible enough functional form)

$$h(\vec{x}) = \frac{p_{\text{sig}}(\vec{x})}{p_{\text{sig}}(\vec{x}) + p_{\text{bkgd}}(\vec{x})}$$

Optimal Classifier (Neyman–Pearson)

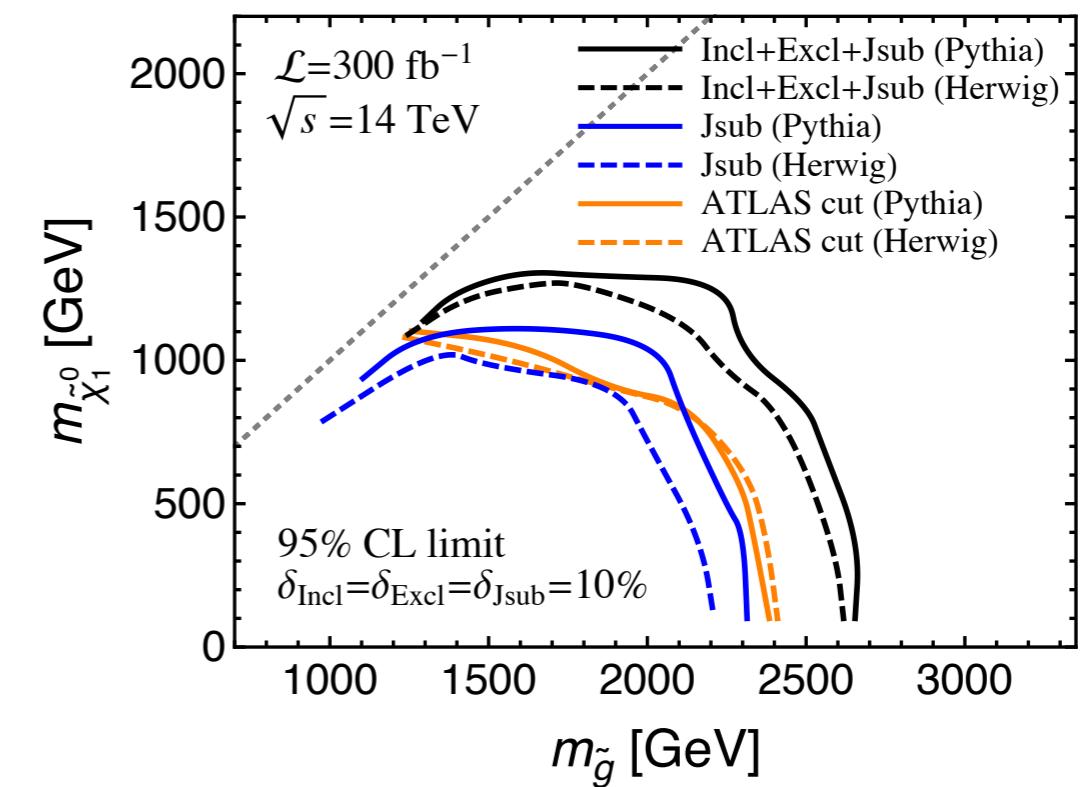
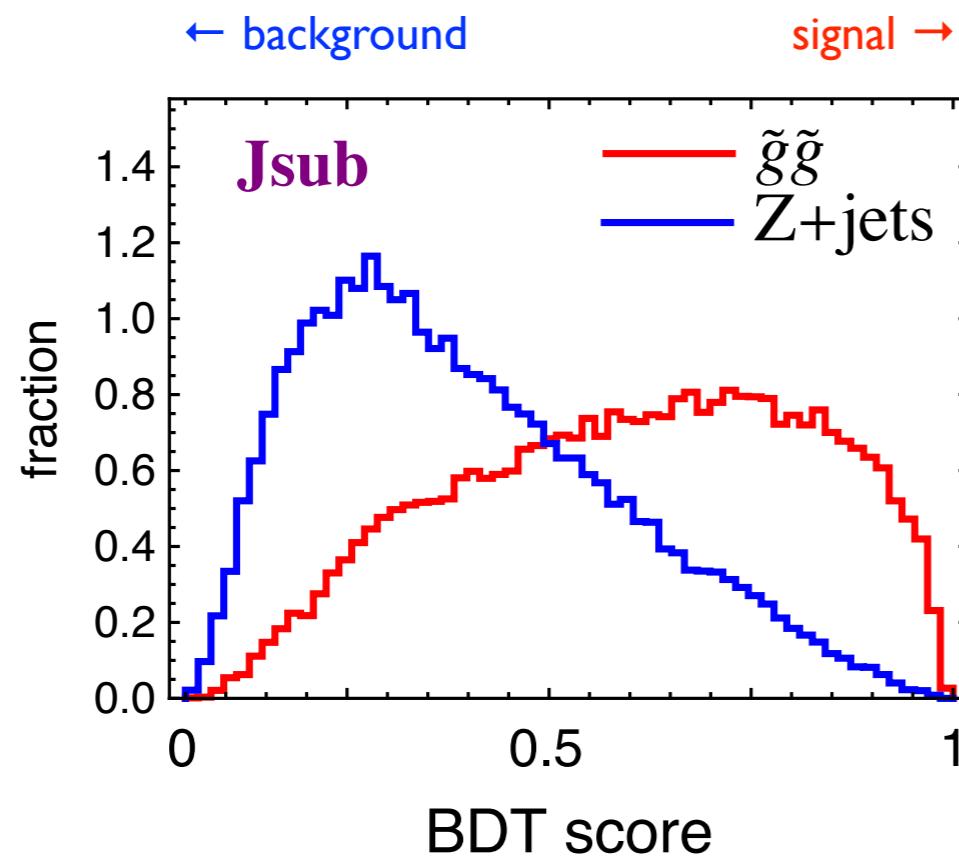
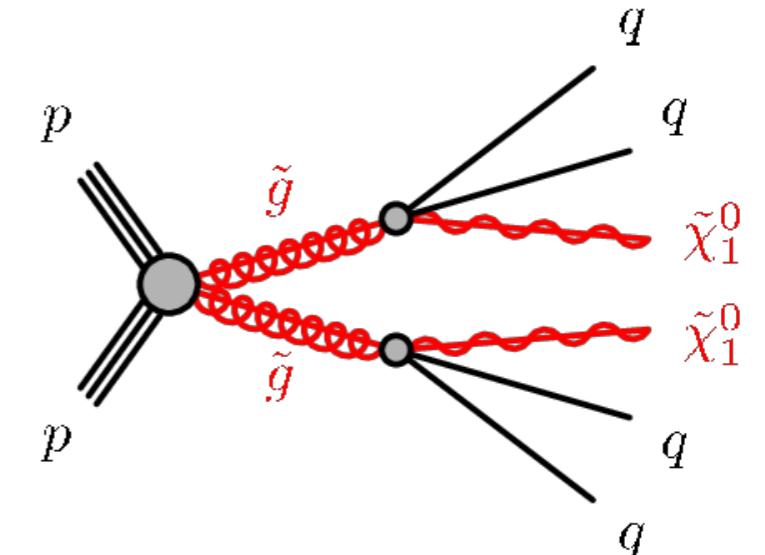
E.g. SUSY Search for Gluino Pairs

Classifier: Boosted decision tree (for each of 4 jets)

Inputs: Jet mass, width, track multiplicity

Signal: Quark enriched ($C_F = 4/3$)

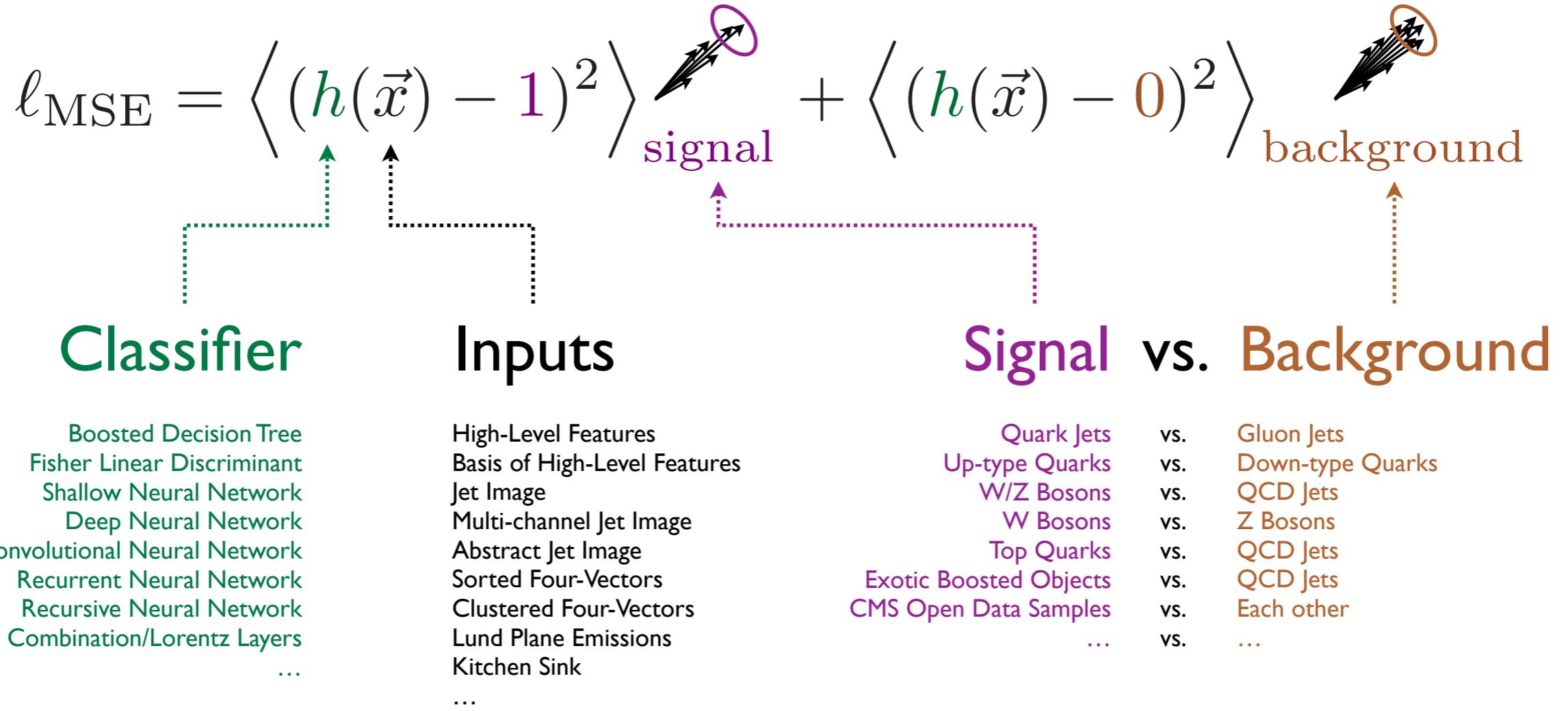
Background: Gluon enriched ($C_A = 3$)



[Bhattacherjee, Mukhopadhyay, Nojiri, Sakakie, Webber, 1609.08781]

Jet Classification Studies

Mix and match



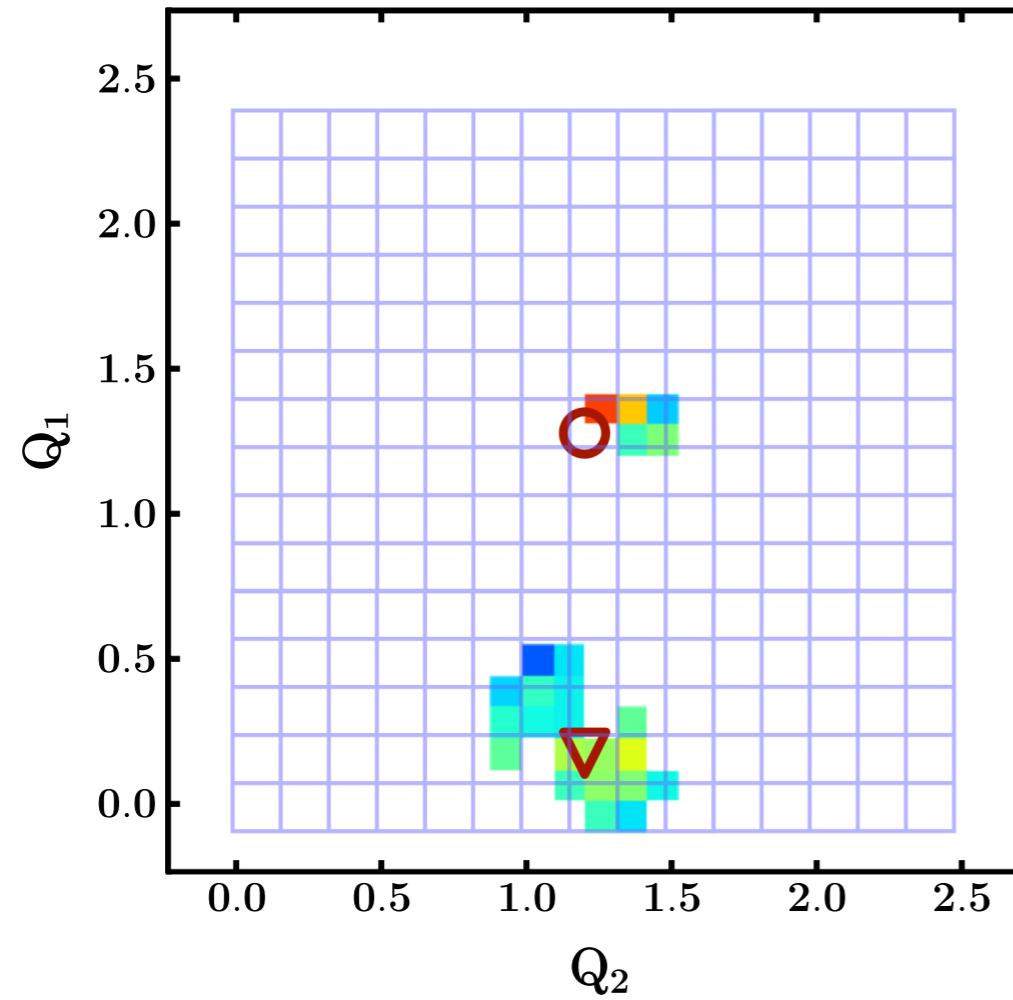
[Lönnblad, Peterson, Rögnvaldsson, 1990, ..., Cogan, Kagan, Strauss, Schwartzman, 1407.5675; Almeida, Backović, Cliche, Lee, Perelstein, 1501.05968; de Oliveira, Kagan, Mackey, Nachman, Schwartzman, 1511.05190; Baldi, Bauer, Eng, Sadowski, Whiteson, 1603.09349; Conway, Bhaskar, Erbacher, Pilot, 1606.06859; Guest, Collado, Baldi, Hsu, Urban, Whiteson, 1607.08633; Barnard, Dawe, Dolan, Rajcic, 1609.00607; Komiske, Metodiev, Schwartz, 1612.01551; Kasieczka, Plehn, Russell, Schell, 1701.08784; Loupe, Cho, Becot, Cranmer, 1702.00748; Pearkes, Fedorko, Lister, Gay, 1704.02124; Datta, Larkoski, 1704.08249, 1710.01305; Butter, Kasieczka, Plehn, Russell, 1707.08966; Fernández Madrazo, Heredia Cacha, Lloret Iglesias, Marco de Lucas, 1708.07034; Aguilar Saavedra, Collin, Mishra, 1709.01087; Cheng, 1711.02633; Luo, Luo, Wang, Xu, Zhu, 1712.03634; Komiske, Metodiev, JDT, 1712.07124; Macaluso, Shih, 1803.00107; Fraser, Schwartz, 1803.08066; Choi, Lee, Perelstein, 1806.01263; Lim, Nojiri, 1807.03312; Dreyer, Salam, Soyez, 1807.04758; Moore, Nordström, Varma, Fairbairn, 1807.04769; plus my friends who will scold me for forgetting their paper; plus many ATLAS/CMS performance studies]

Jet Classification Studies

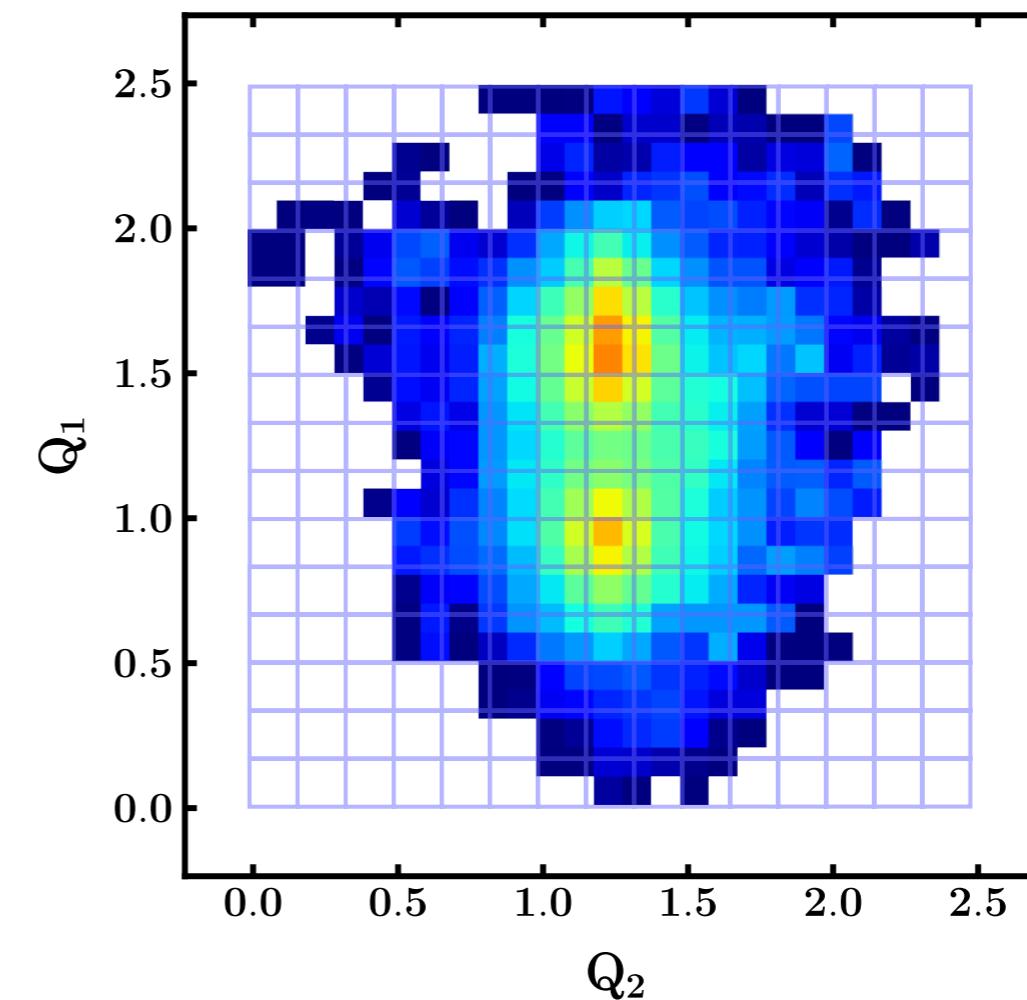
Mix and match

Standard CNN input: Jet images

Individual W jet



Ensemble average

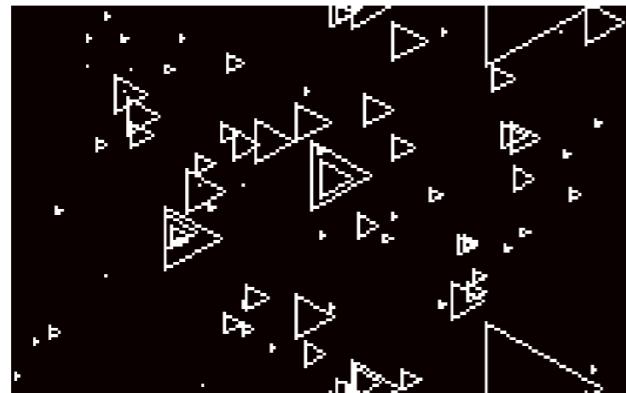


[Cogan, Kagan, Strauss, Schwartzman, 1407.5675]

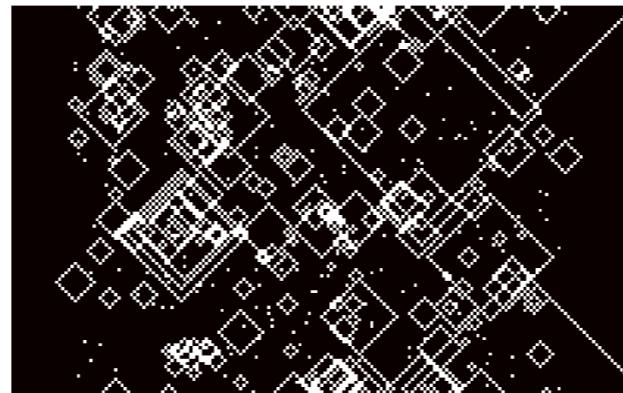
Jet Classification Studies

Mix and match

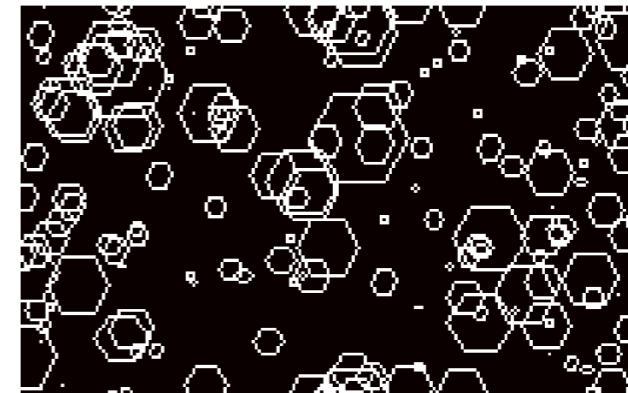
Novel CNN input: Abstract event images



(a) Photons



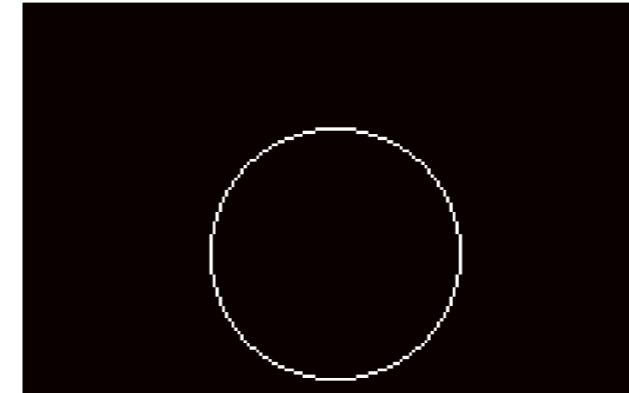
(b) Charged Particles



(c) Neutral Hadrons



(d) Lepton

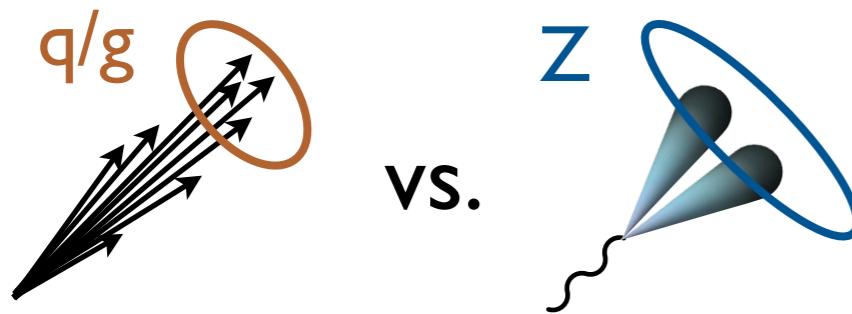


(e) E_T^{miss}

Addresses sparsity problem of standard energy-to-intensity mapping

[Nguyen, Weitekamp, Anderson, Castello, Cerri, Pierini, Spiropulu, Vlimant, 1807.00083;
using Fernández Madrazo, Heredia Cacha, Lloret Iglesias, Marco de Lucas, 1708.07034]

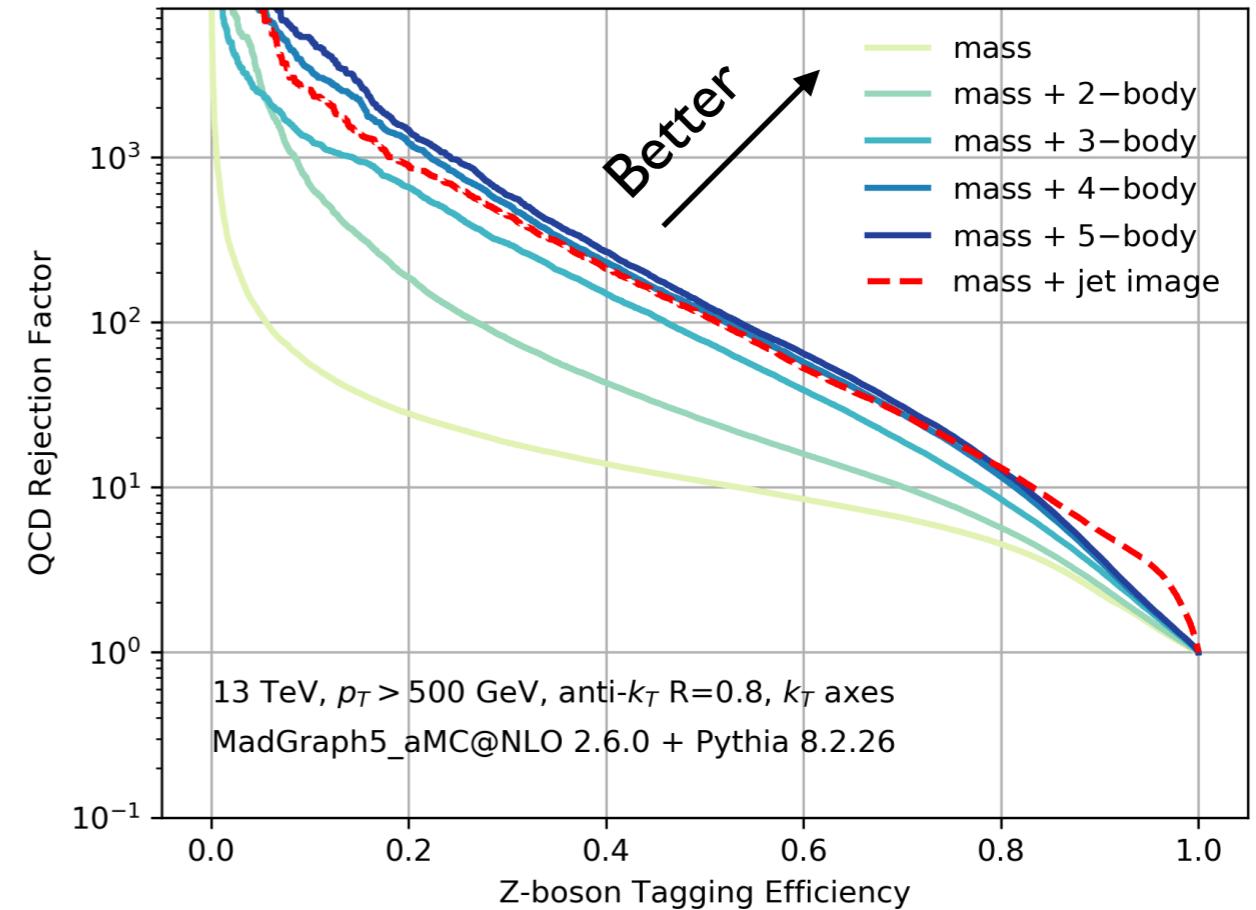
Evidence for Performance Saturation



vs.

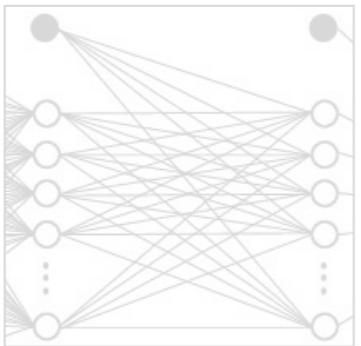
*“Any sufficiently advanced technology
is indistinguishable from magic”*

Jet Images CNN \approx “Expert” BDT

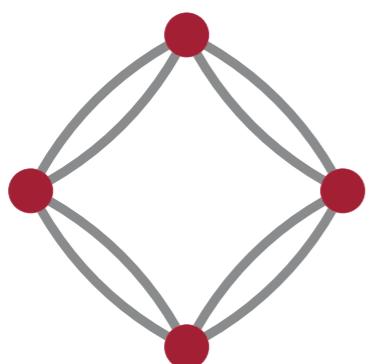


Next frontier is robustness, versatility & transparency

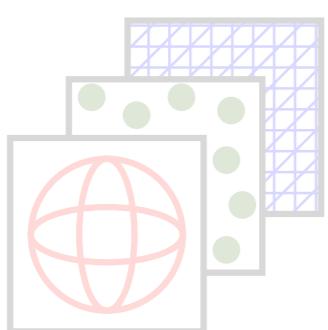
[plot from Moore, Nordström, Varma, Fairbairn, 1807.04769; see also Datta, Larkoski, 1704.08249]



Into the Network



Symmetries & Safety



Deep Sets for Particle Jets

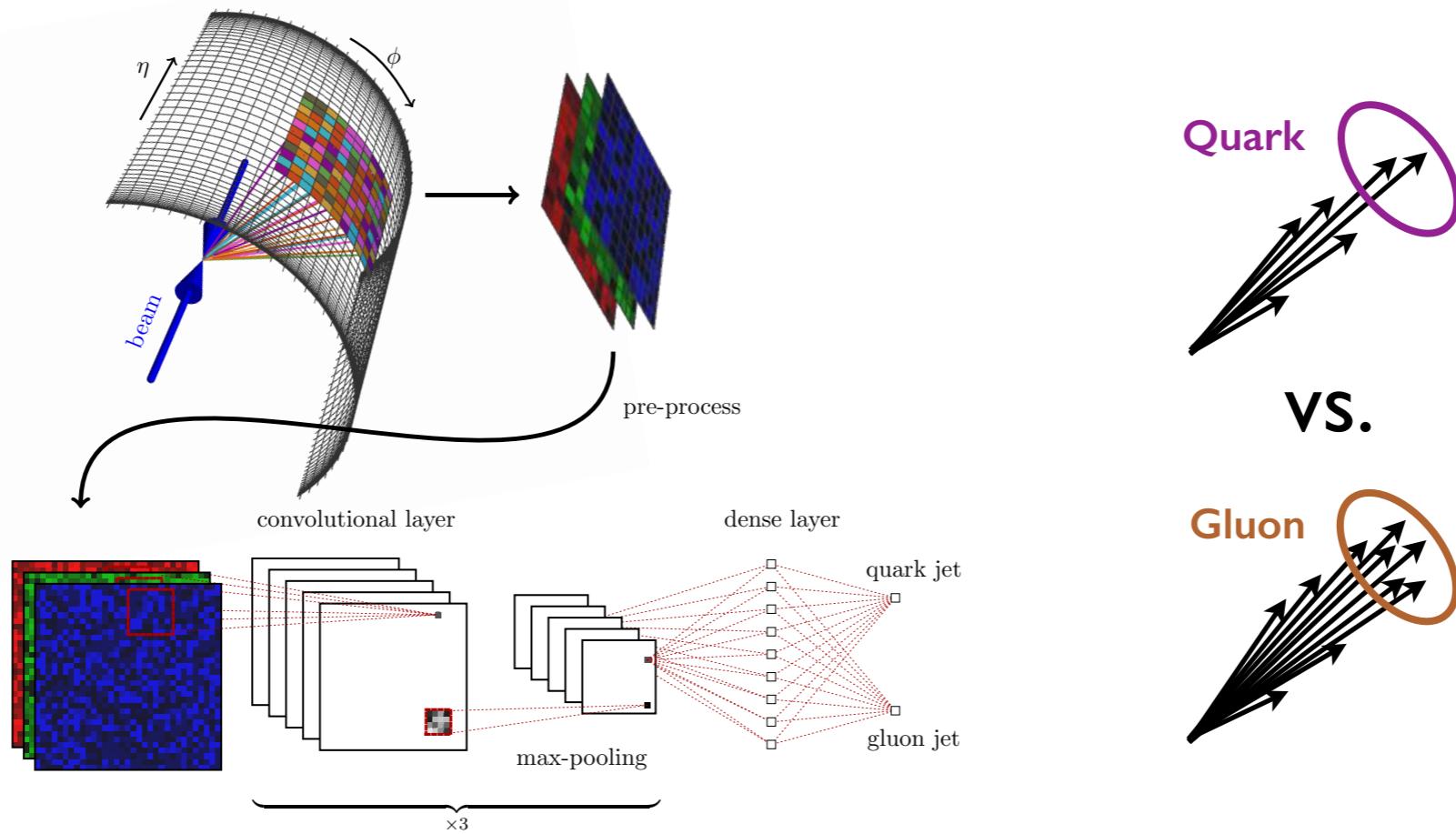


Patrick Komiske

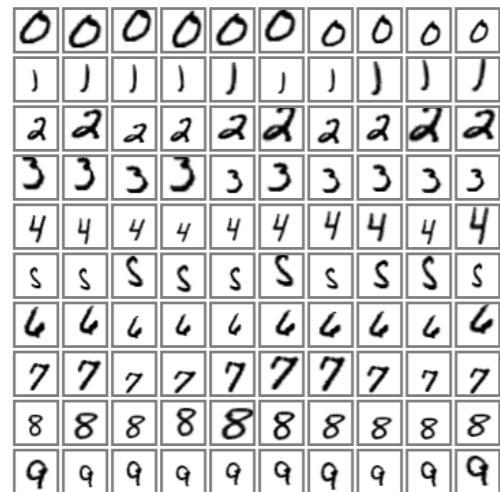
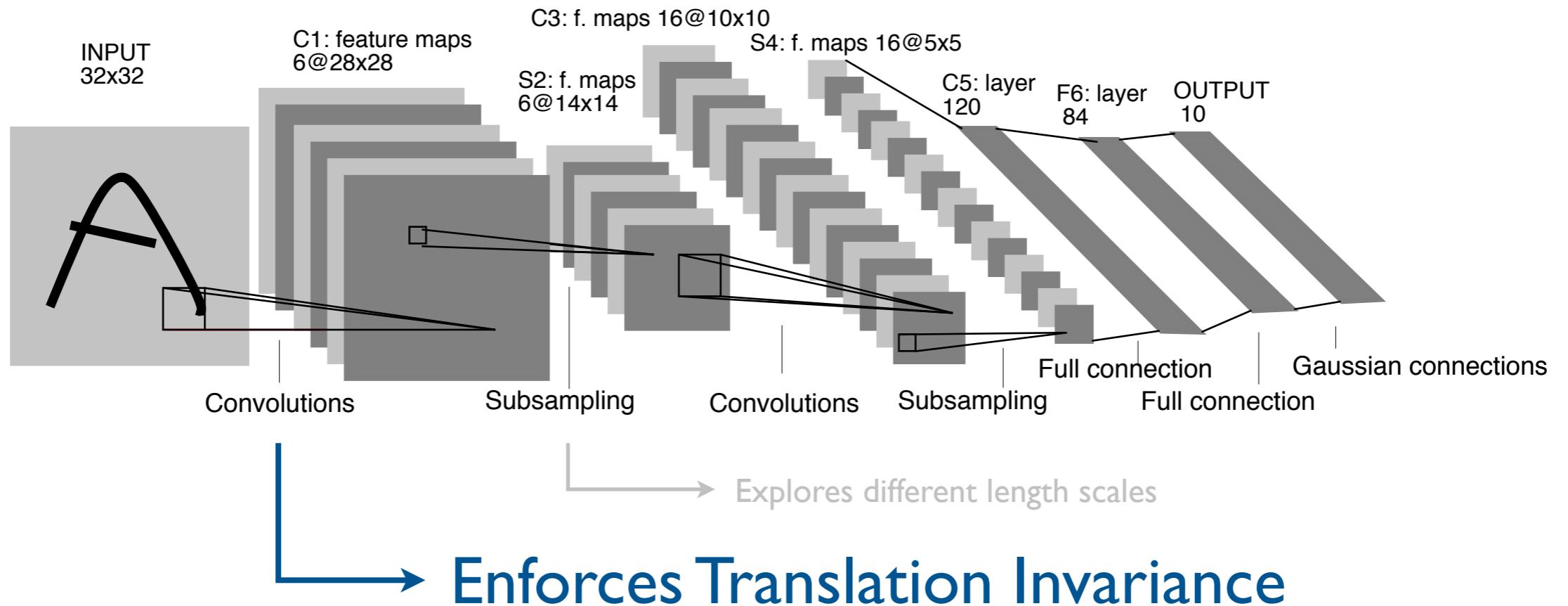


Eric Metodiev

Two grad students walk into my office with their CNN...



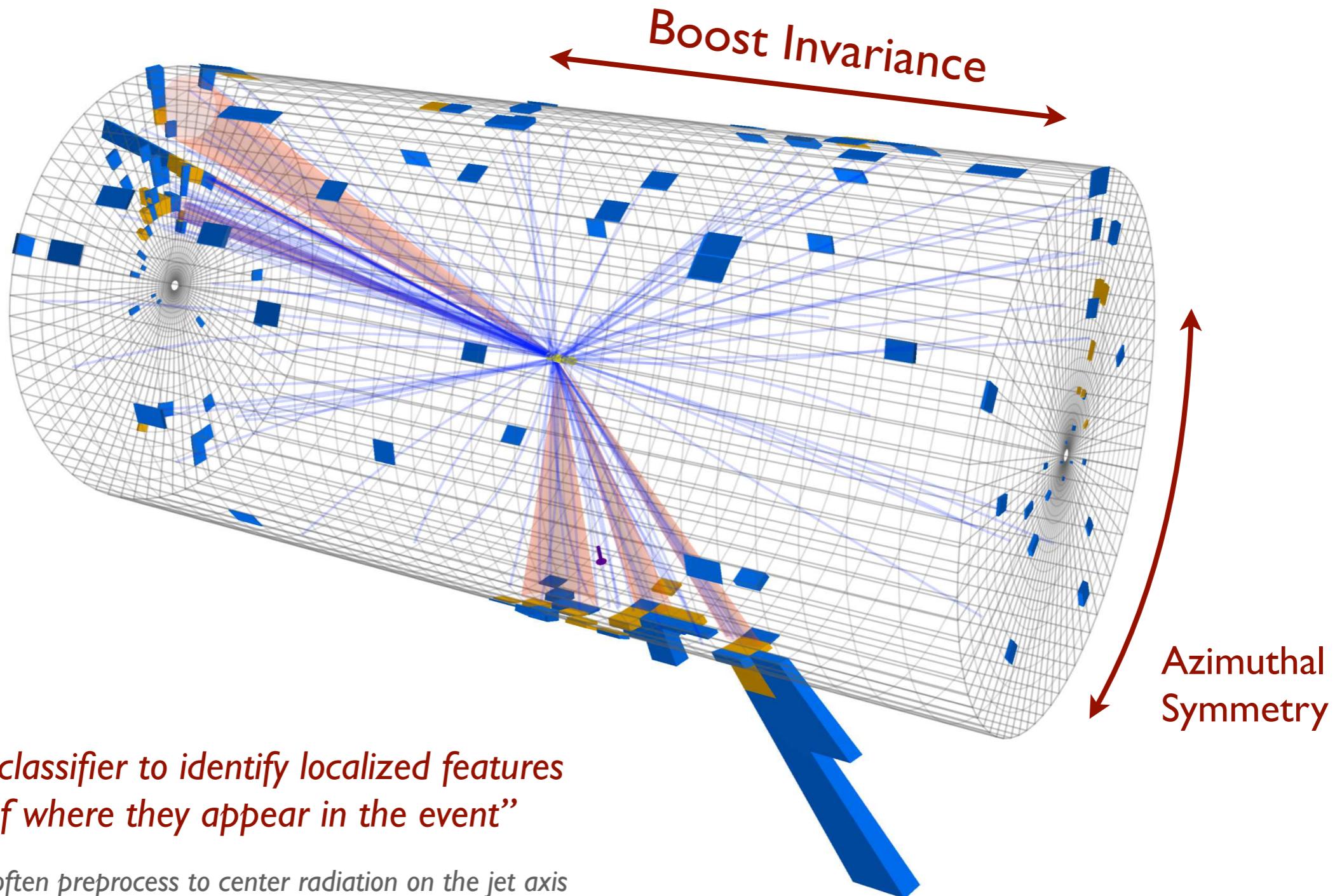
Symmetries of a CNN



"I want my classifier to identify localized features regardless of where they appear in the image"

[image from LeCun, Bottou, Bengio, Haffner, 1998]

Symmetries of Collision Events



[image from CMS, 2015]

The Physics-First Approach

Underlying Physics



“Deep Thinking”

Natural Data Representation



“Deep Learning”

Suitable Algorithm

The Buzzword-First Approach

Questionable Physics



“Wishful Thinking”

Unnatural Data Representation



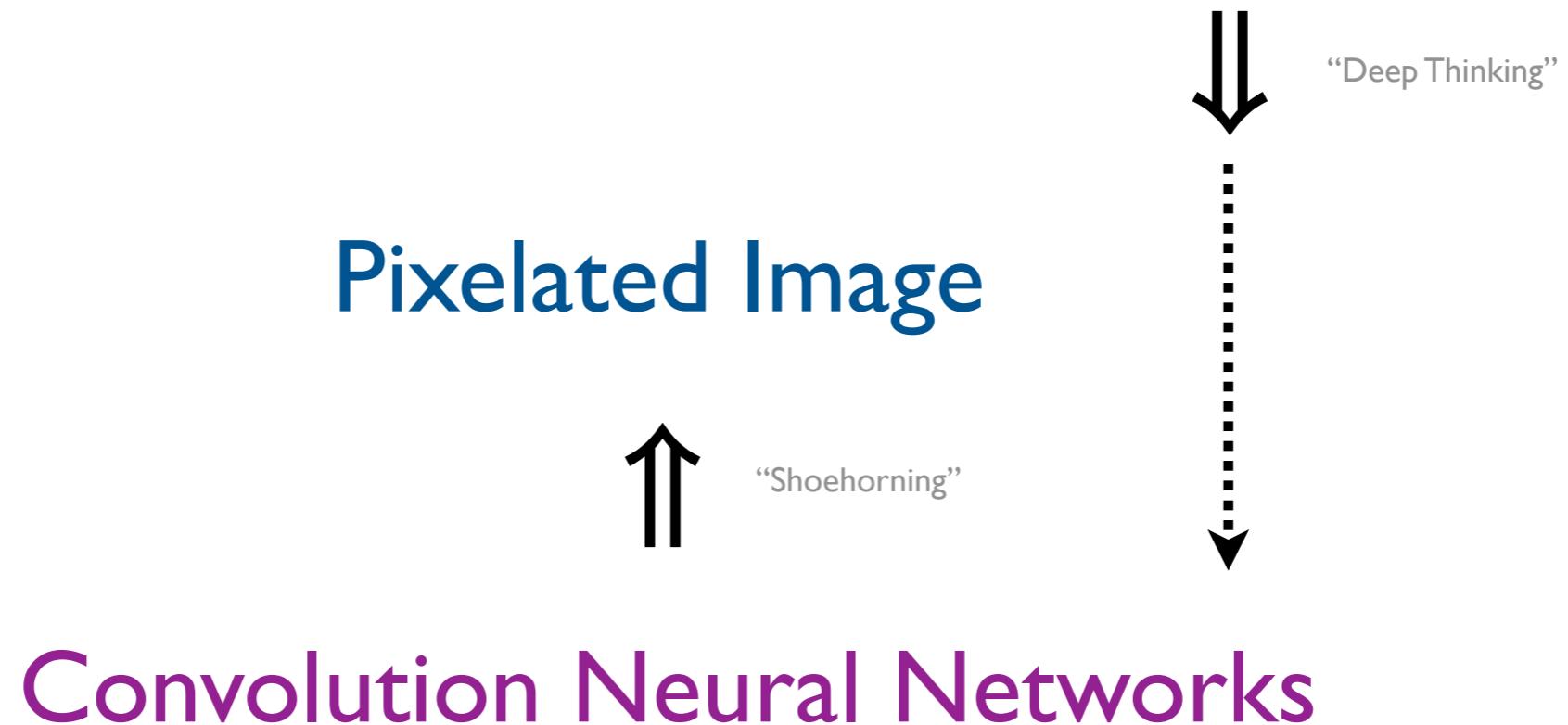
“Shoehorning”

Cool-Sounding Algorithm

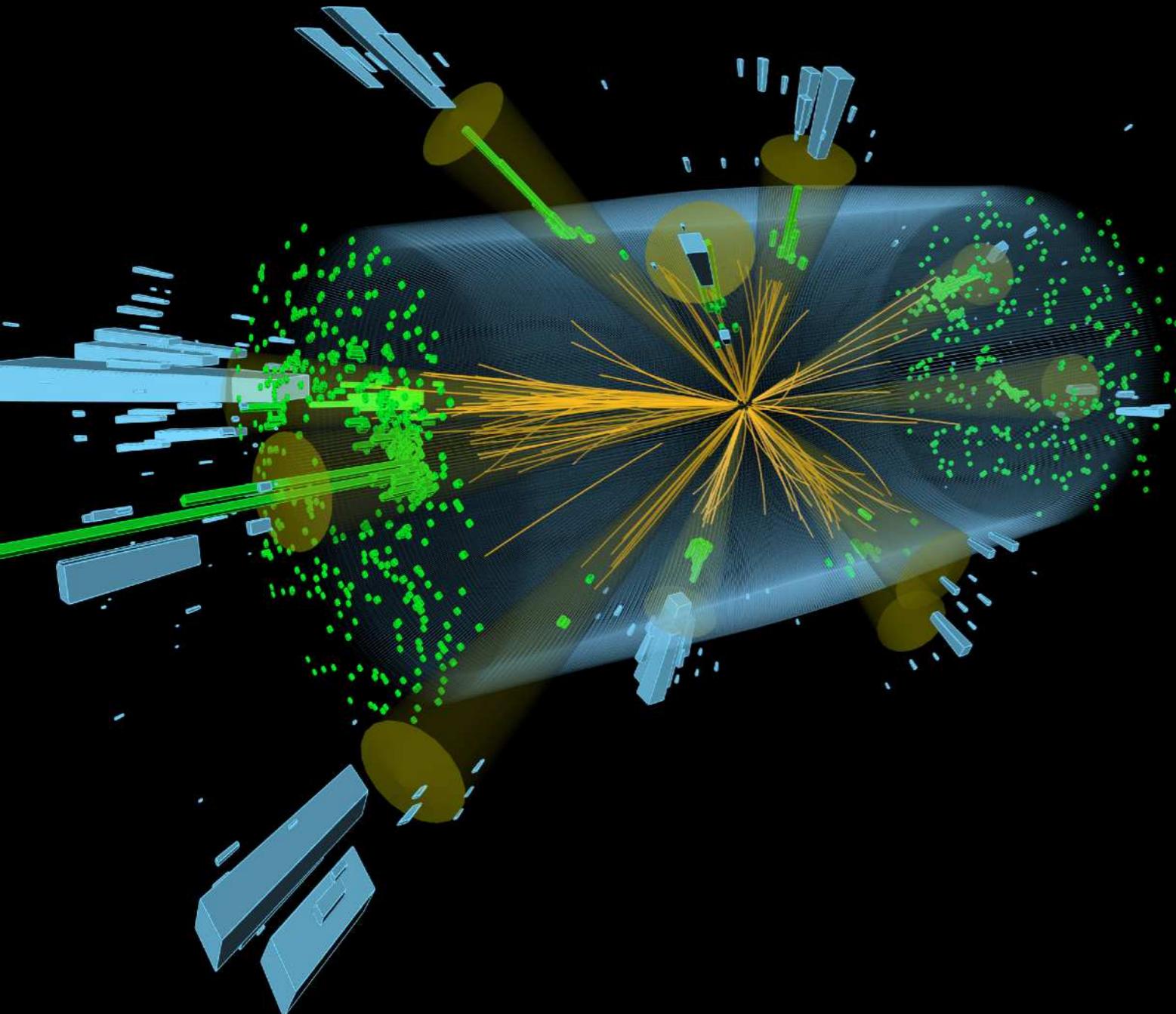
Why CNNs Aren't Ideal

Underlying Physics
Natural Data Representation
Suitable Algorithm

Translations/Boost Invariance



Debris Taxonomy



T E H M



γ

photon



e^+

electron



μ^+

muon



π^+

pion



K^+

kaon



K_L^0

K-long



p/\bar{p}

proton



n/\bar{n}

neutron

composite

elementary

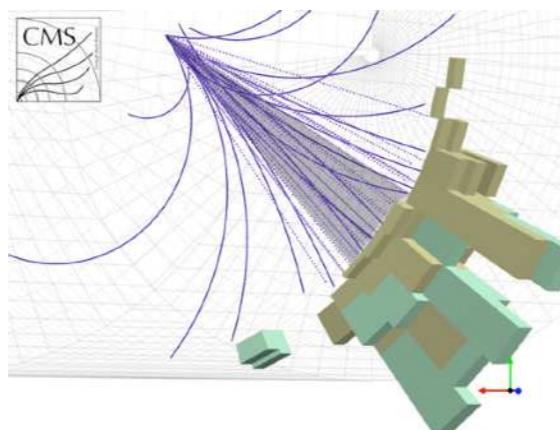
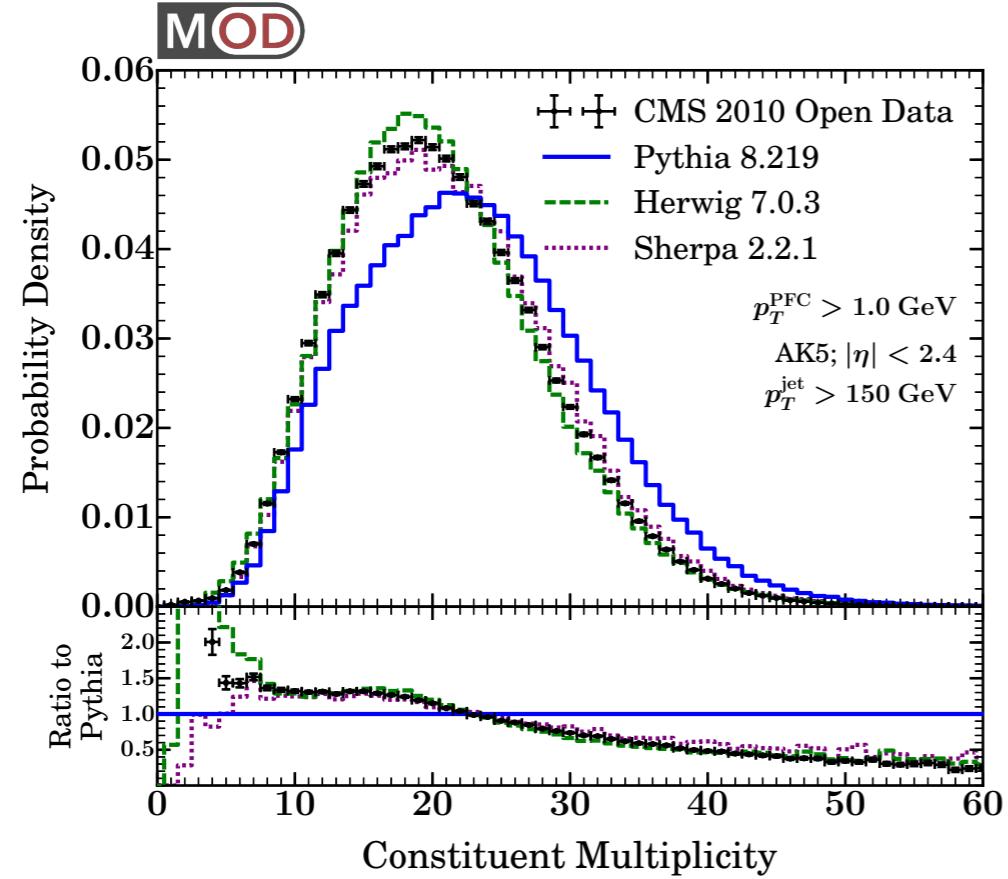
composite

Point Clouds



[Popular Science, 2013]

Key Fact #1



Jet constituents:
Particle-like objects
Variable-length
Unordered set

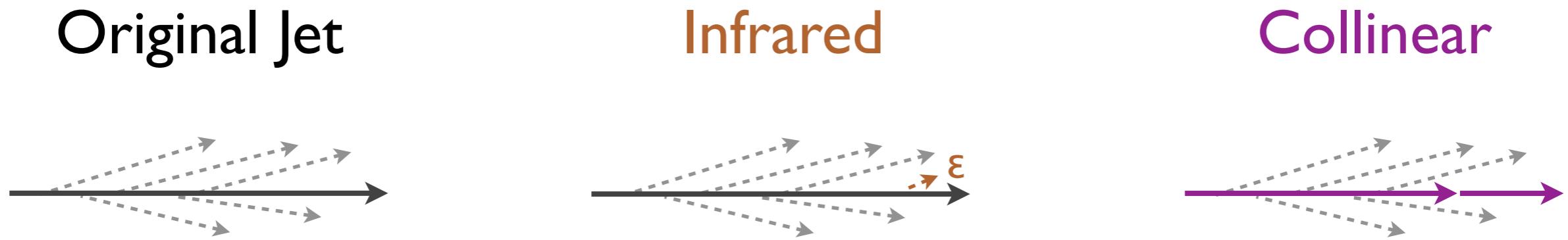
Per particle:
 $\{E, p_x, p_y, p_z\}$ or $\{p_T, \gamma, \Phi, m\}$
Flavor/charge labels
Vertex information
Quality criteria, etc.

[plot from Tripathee, Xue, Larkoski, Marzani, JDT, 1704.05842]

Key Fact #2

Wide range of interesting observables are “safe”

Interesting \approx Calculable in fixed-order perturbation theory



IRC Safe Observable: Insensitive to **IR** or **C** emissions

Enforces smooth interpolation between
variable-length inputs (i.e. $N \rightarrow N-1$)

In the Backup (2017)

Underlying Physics
Natural Data Representation
Suitable Algorithm

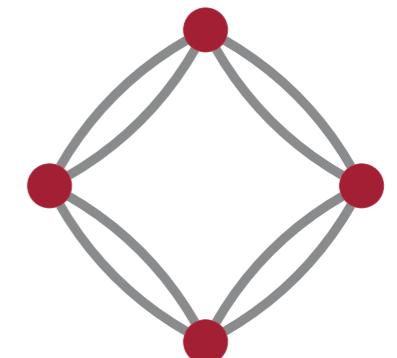
Infrared/Collinear Safety



Energy Flow Polynomials



Linear Regression



[Komiske, Metodiev, JDT, 1712.07124;
<https://energyflow.network>]

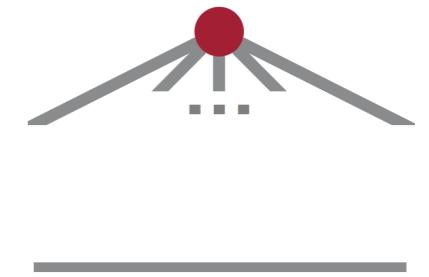
In the Backup (2019?)

Underlying Physics
Natural Data Representation
Suitable Algorithm

Infrared/Collinear Safety



Energy Flow Moments



Linear Regression + Linear Runtime

[Komiske, Metodiev, JDT, we've been promising this paper for 9 months]

Today's Talk (2018)

Underlying Physics
Natural Data Representation
Suitable Algorithm

Quantum-Mechanical Indistinguishability

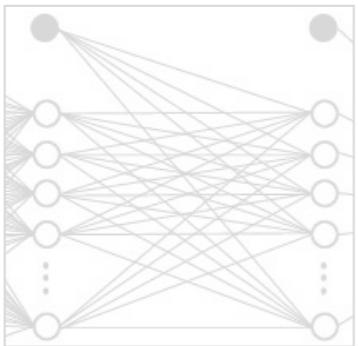


Variable-Length Unordered Sets of Particles

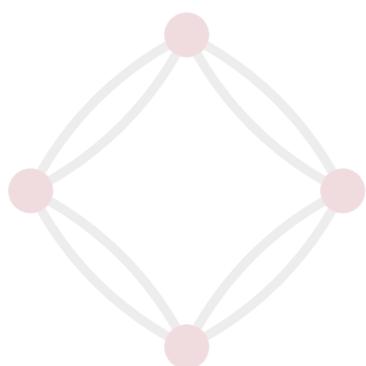


???

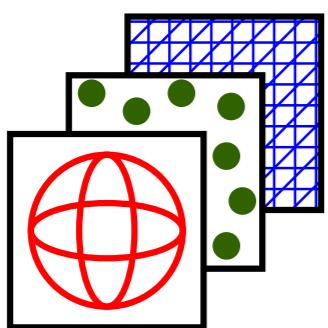
[Komiske, Metodiev, JDT, 1810.05165]



Into the Network



Symmetries & Safety



Deep Sets for Particle Jets

The Power of Addition

Additive Observable: $\mathcal{O} = \sum_{i \in \text{jet}} \Phi(E_i, \vec{p}_i, \dots)$

(often comes up in the context of
SCET factorization theorems)

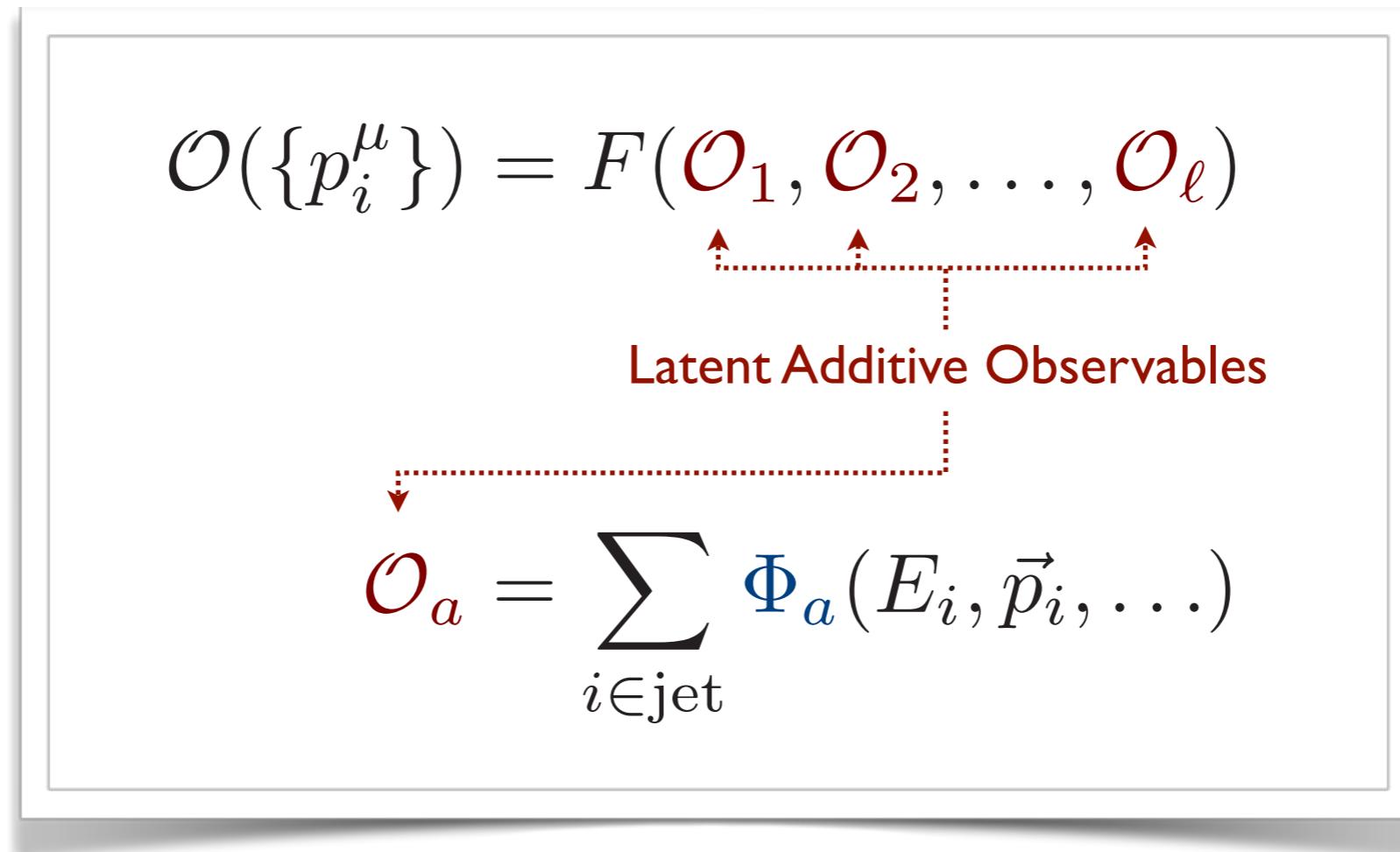
- Permutation invariant by construction
- Easily adapts to variable-length inputs
- Can approximate Φ with neural networks
- Can incorporate additional particle properties
- Linear runtime in number of particles

Additive Safe Observable: $\mathcal{O} = \sum_{i \in \text{jet}} E_i \Phi(\hat{p}_i) \quad \hat{p}_i = \frac{\vec{p}_i}{E_i}$

IRC safety guaranteed by energy weighting

Conjectured Generalization

Arbitrary permutation-symmetric observable?



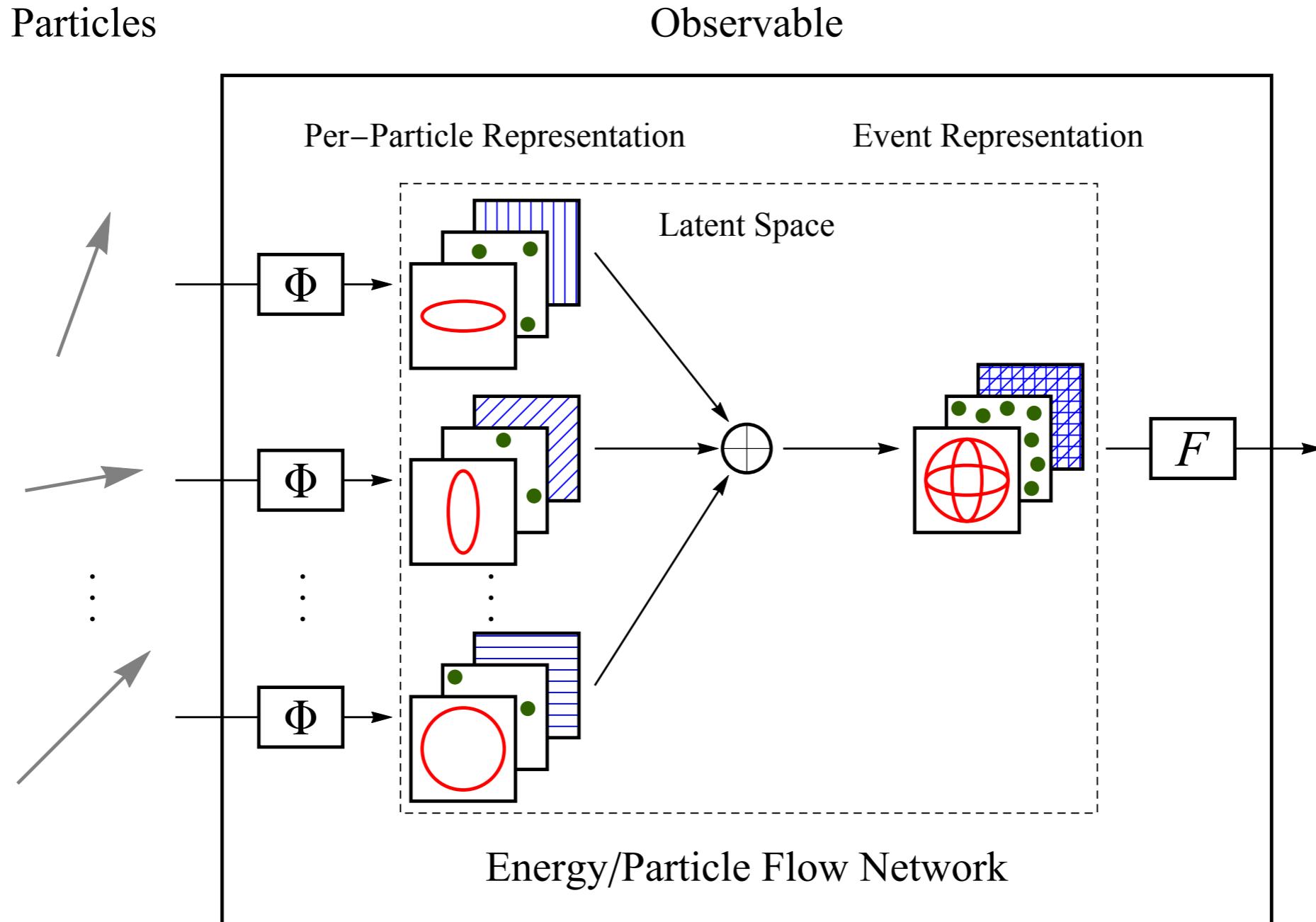
Energy / Particle Flow Networks

IRC-safe Φ

General Φ

[Komiske, Metodiev, JDT, 1810.05165]

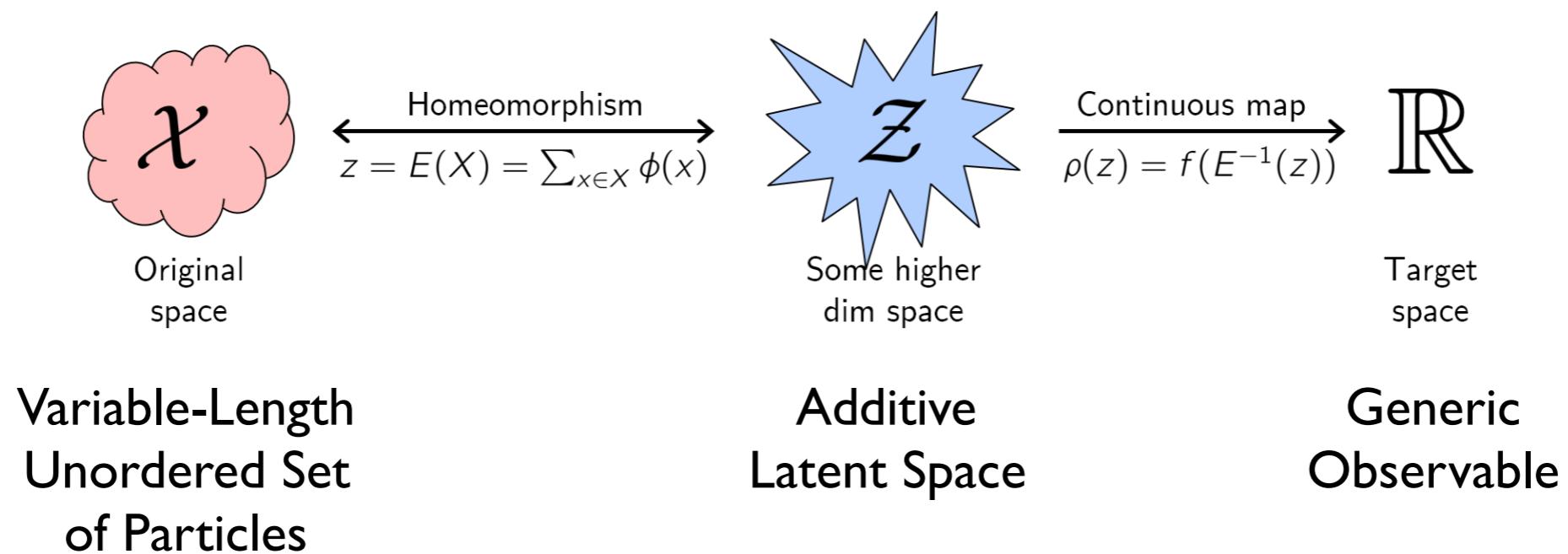
Conjectured Generalization



Meanwhile in ML-Land: Deep Sets

Theorem 2 A function $f(X)$ operating on a set X having elements from a countable universe, is a valid set function, i.e., **invariant** to the permutation of instances in X , iff it can be decomposed in the form $\rho \left(\sum_{x \in X} \phi(x) \right)$, for suitable transformations ϕ and ρ .

↑
(!)



[Zaheer, Kottur, Ravanbakhsh, Poczos, Salakhutdinov, Smola, 1703.06114]

Deep Sets for...

Celebrity Face Anomaly Detection



Point Cloud Classification



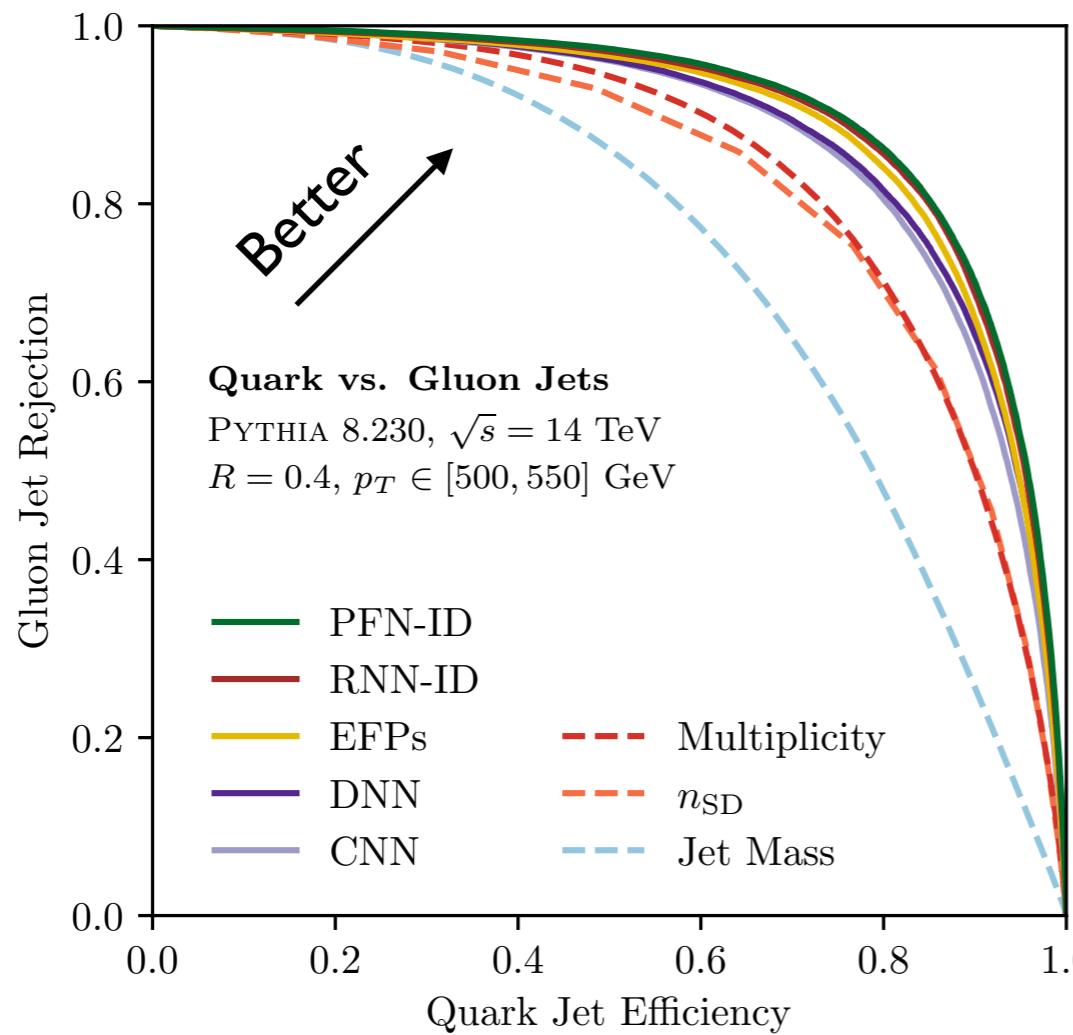
[Zaheer, Kottur, Ravanbakhsh, Poczos, Salakhutdinov, Smola, 1703.06114]

Deep Sets for Q vs. G

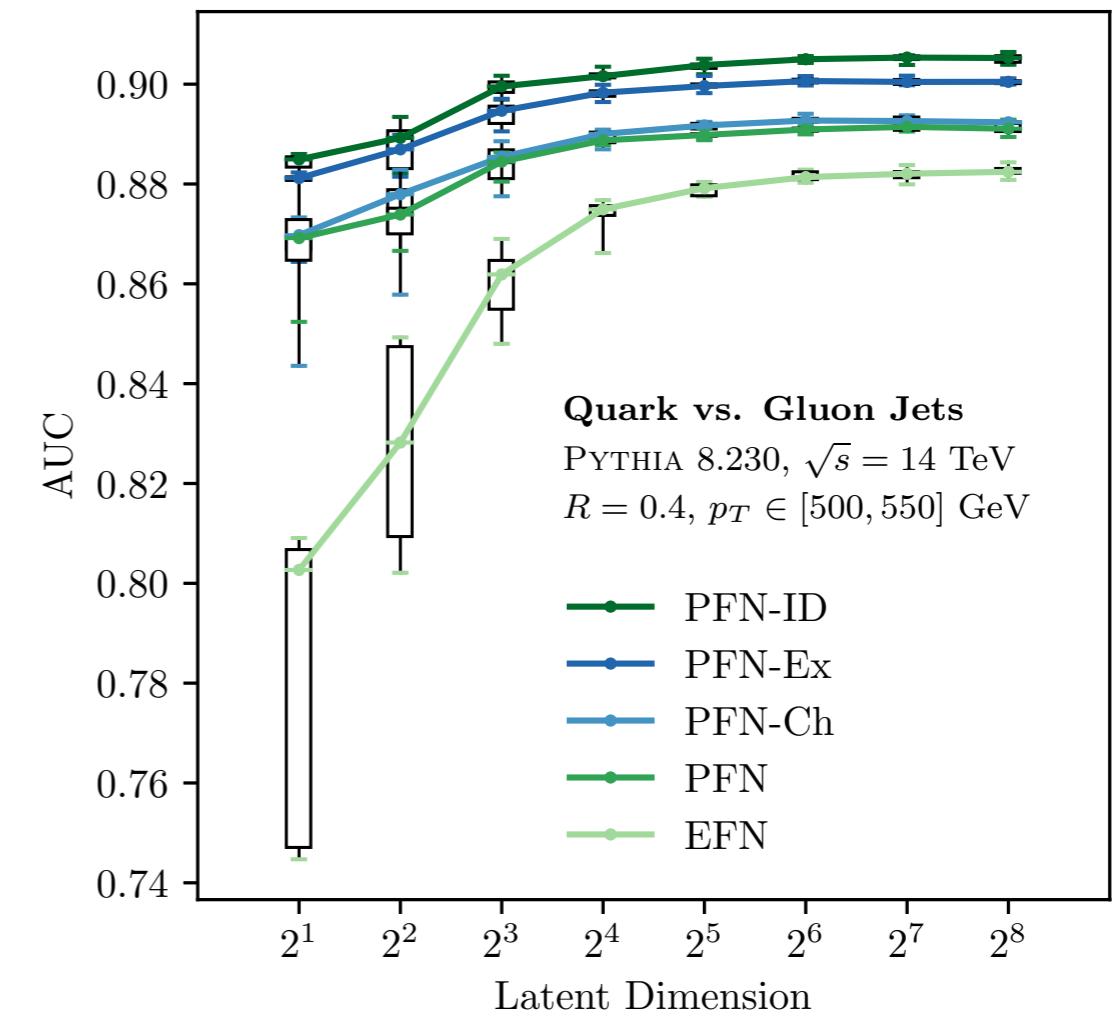
The “Hello, World!” of jet classification



Competitive with
previous methods



Performance improves
with more information

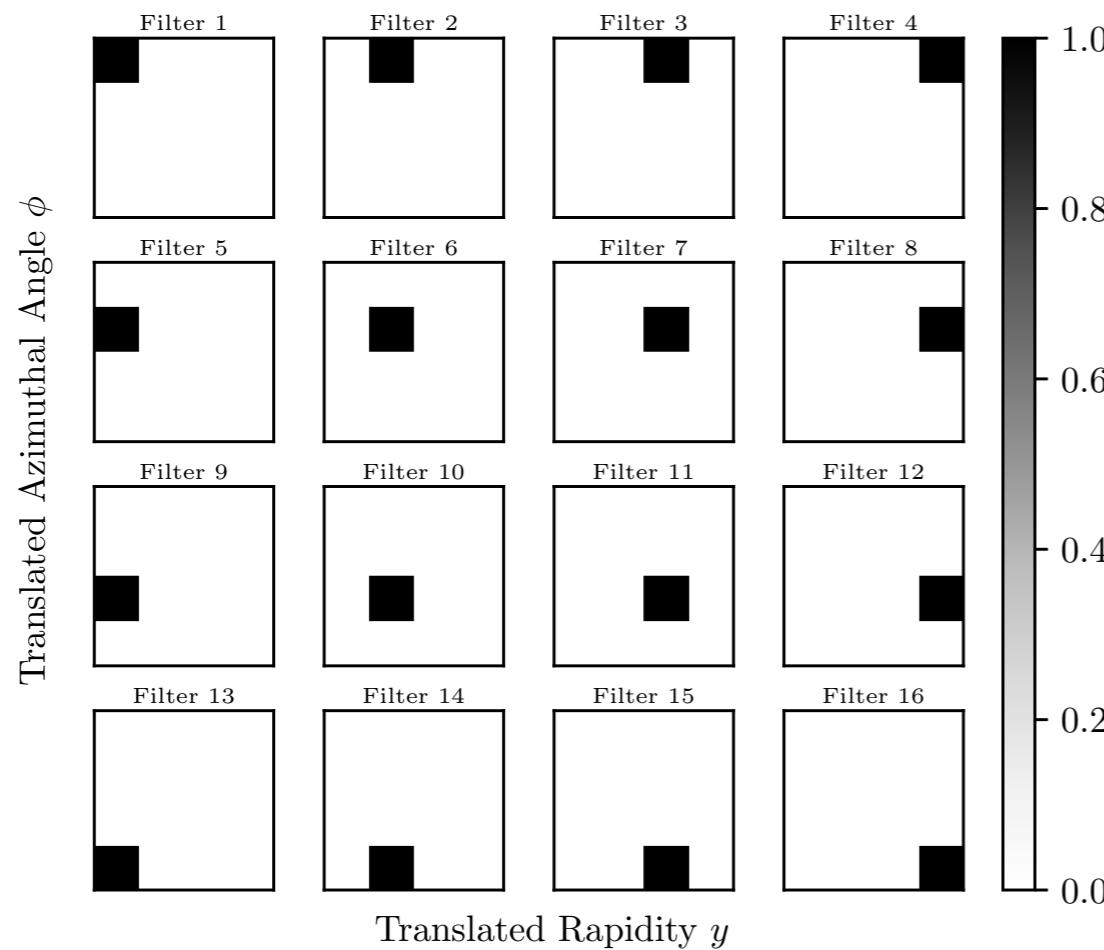


Latent Space Visualization

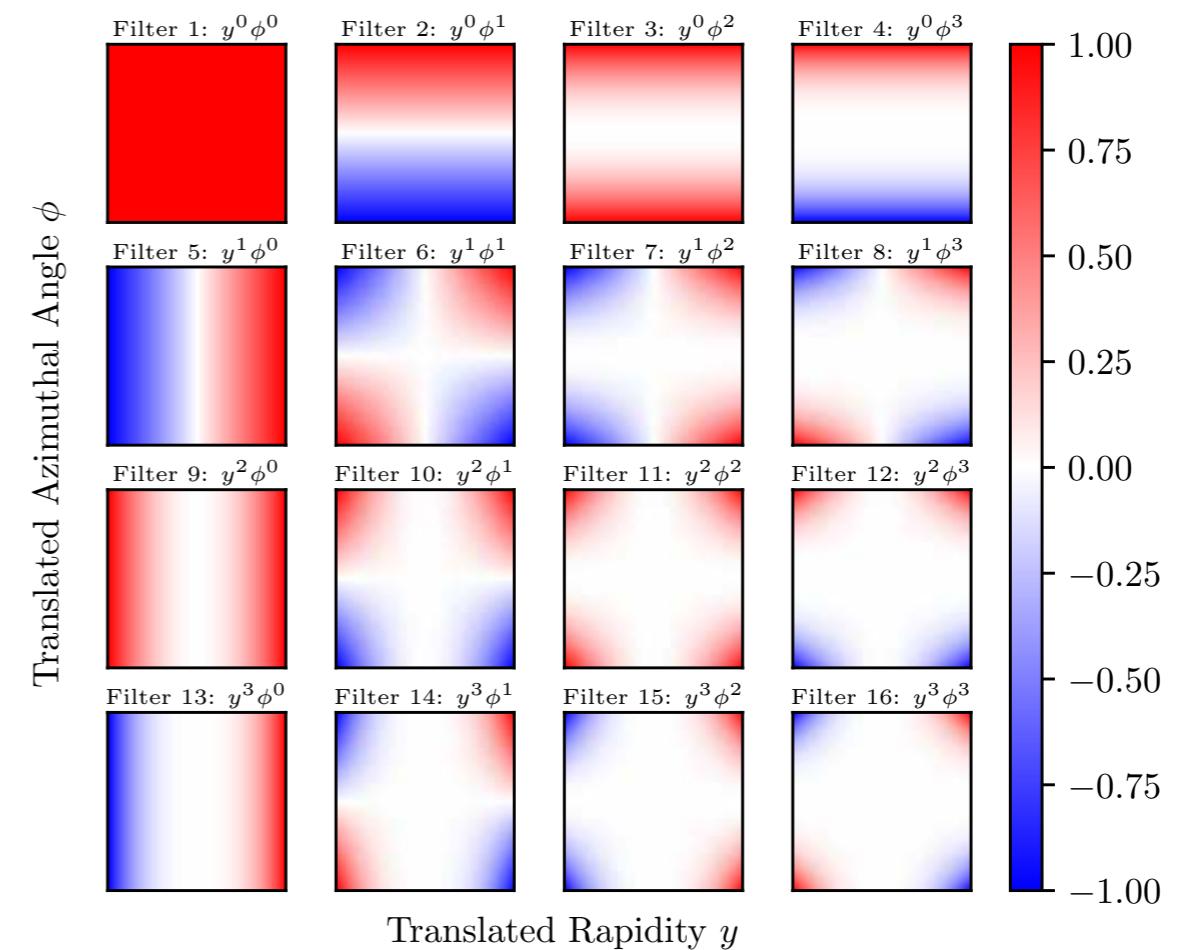
$$\mathcal{O}(\{p_i^\mu\}) = F(\mathcal{O}_1, \mathcal{O}_2, \dots, \mathcal{O}_\ell)$$

IRC-safe: $\mathcal{O}_a = \sum_{i \in \text{jet}} p_{Ti} \Phi_a(\phi_i, y_i)$

Calorimeter Pixels



Radiation Moments

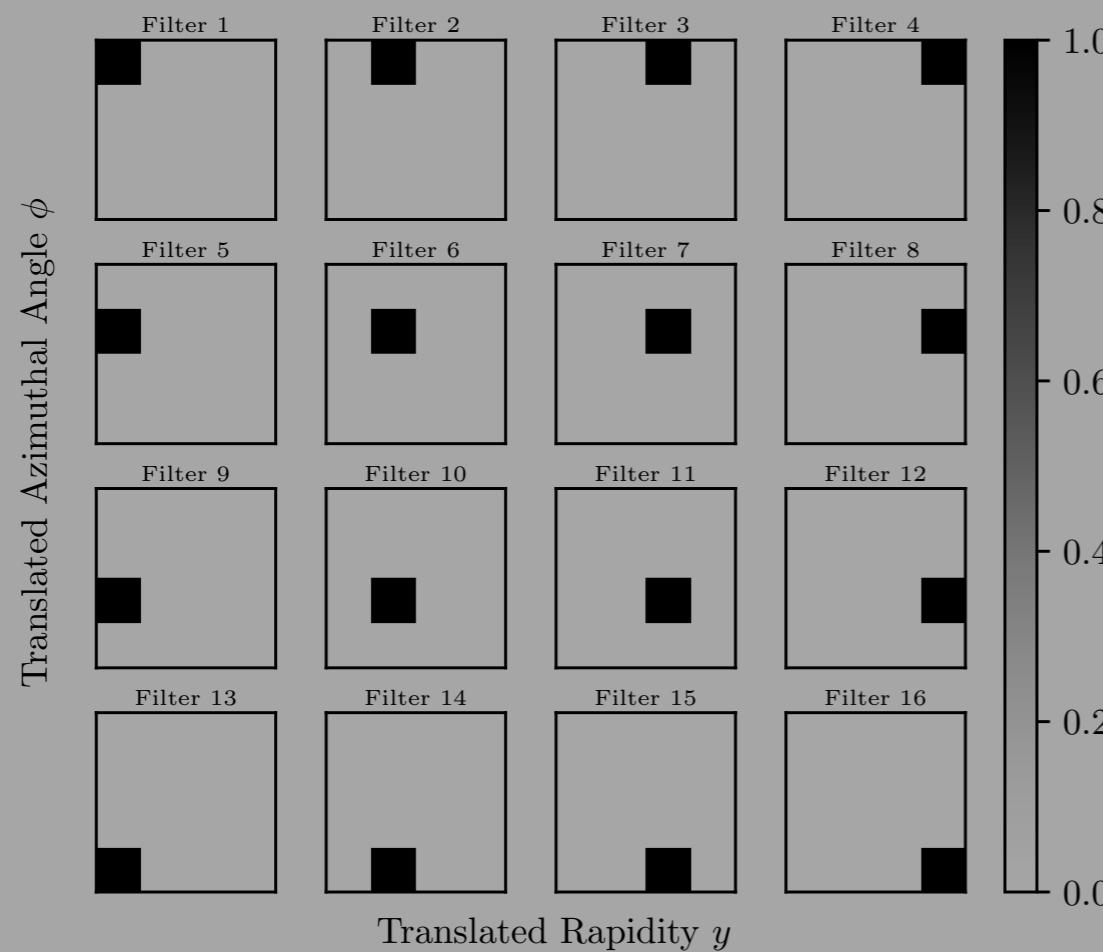


Latent Space Visualization

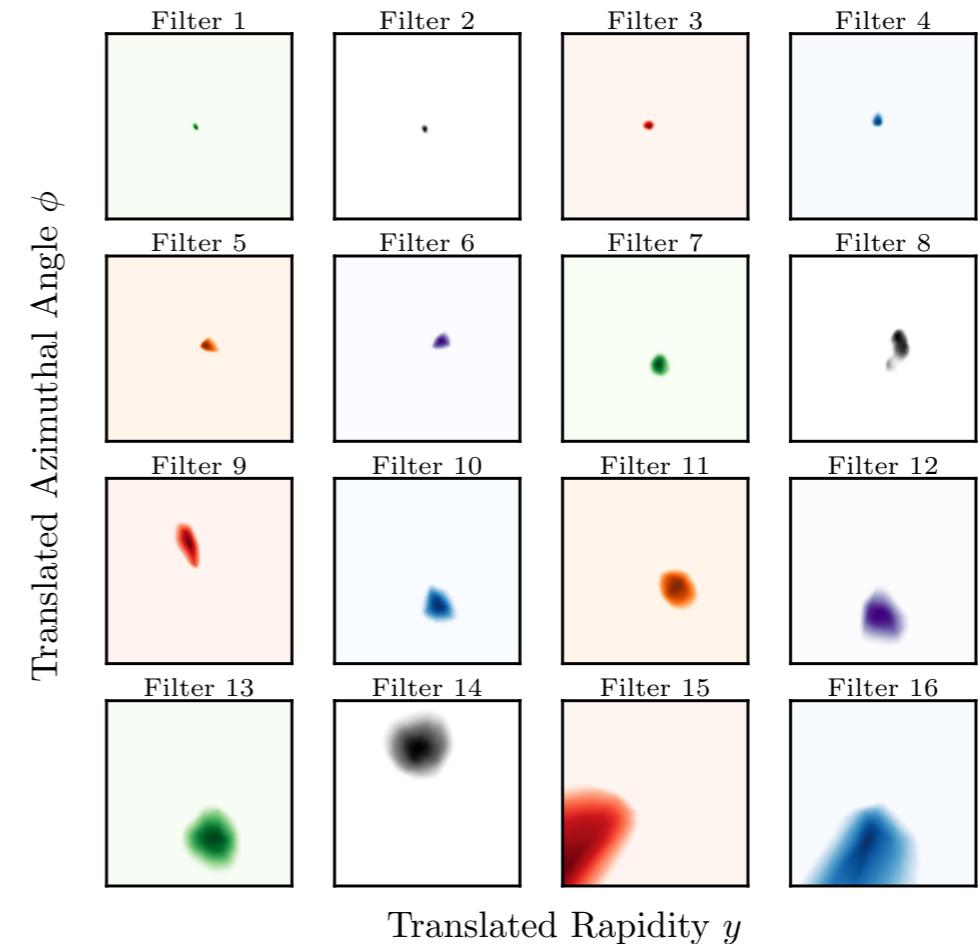
$$\mathcal{O}(\{p_i^\mu\}) = F(\mathcal{O}_1, \mathcal{O}_2, \dots, \mathcal{O}_\ell)$$

IRC-safe: $\mathcal{O}_a = \sum_{i \in \text{jet}} p_{Ti} \Phi_a(\phi_i, y_i)$

Calorimeter Pixels

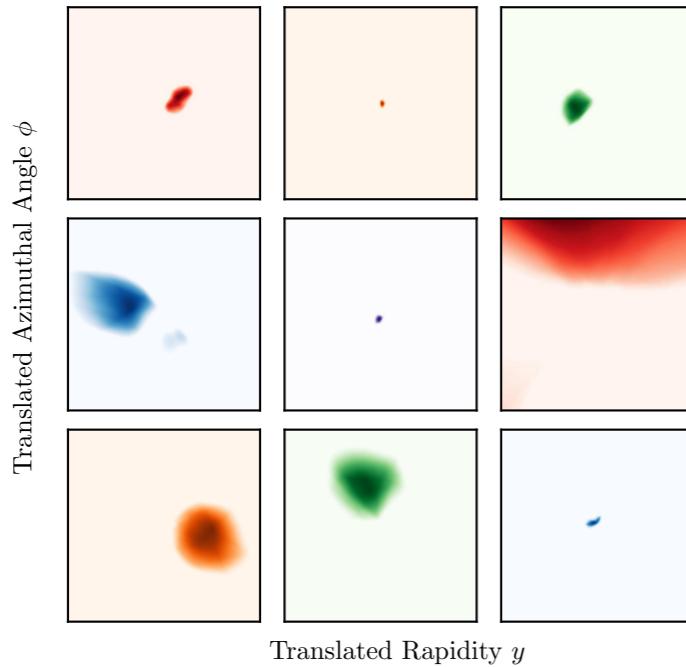


EFNs: Dynamic Pixilation

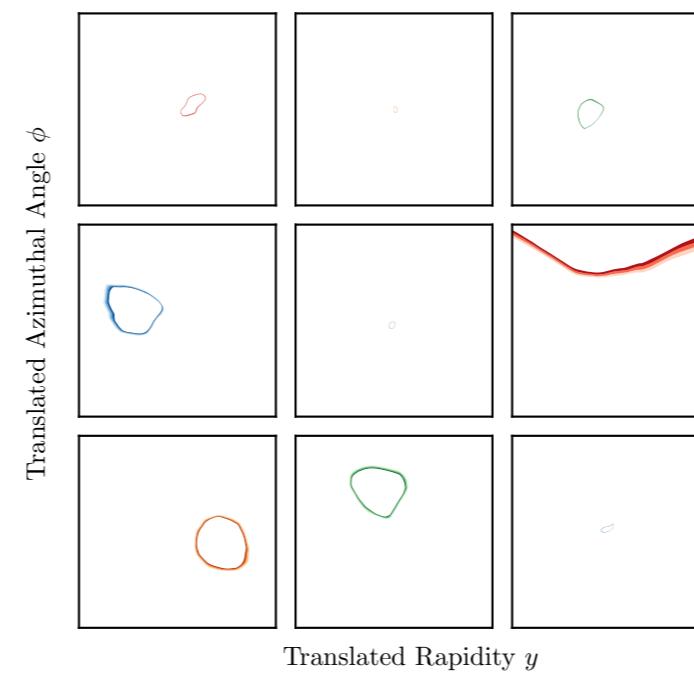


Psychedelic Visualization

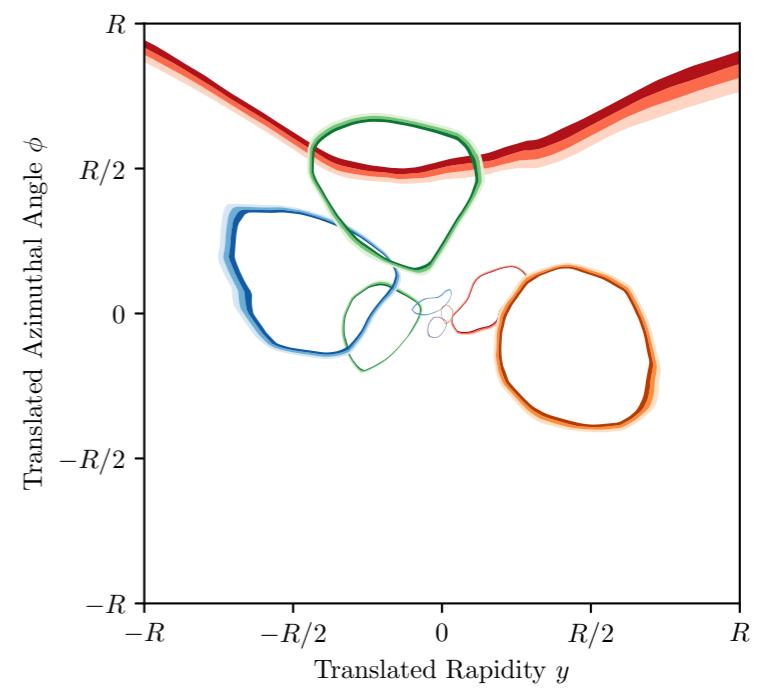
Latent Filters



50% Contours

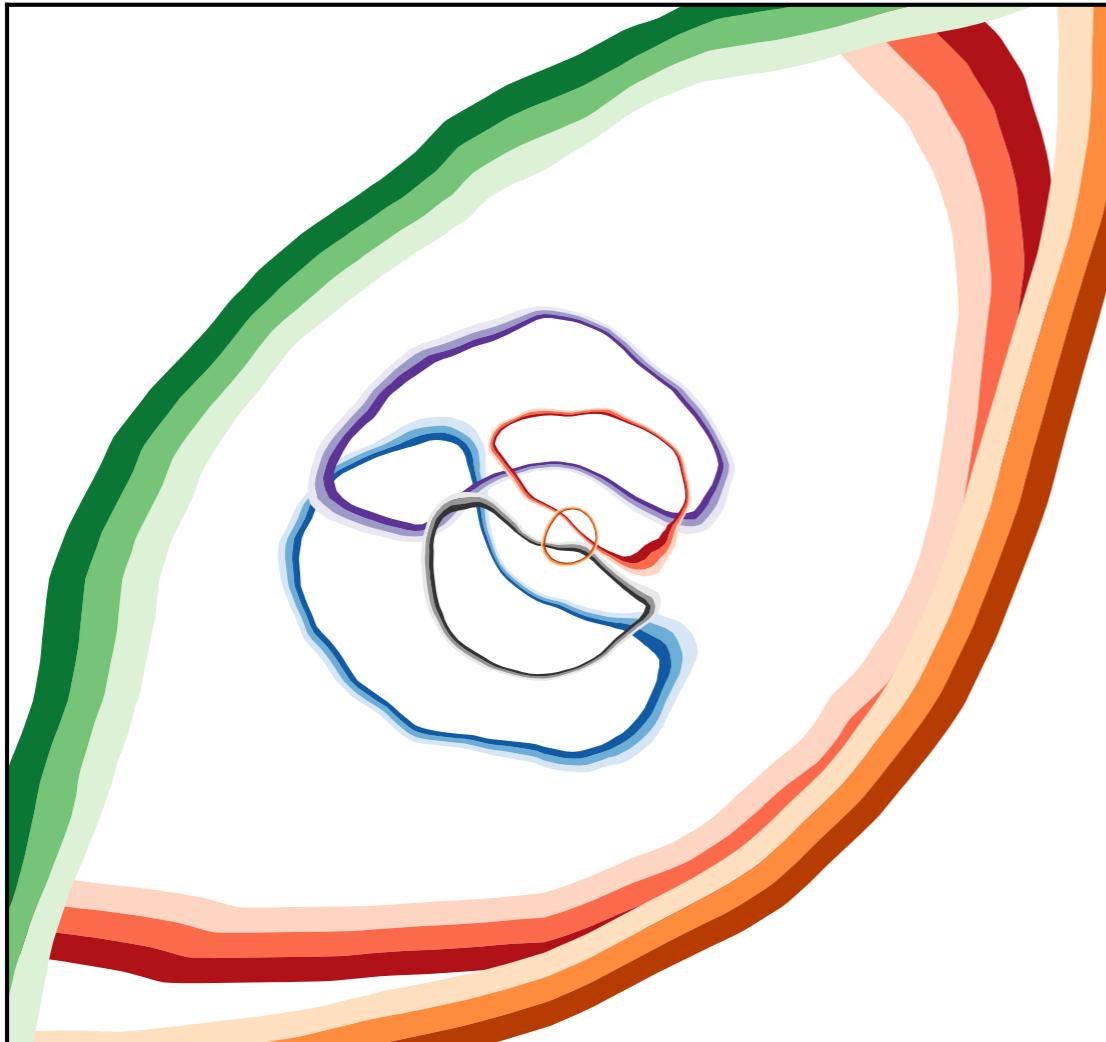


Overlay

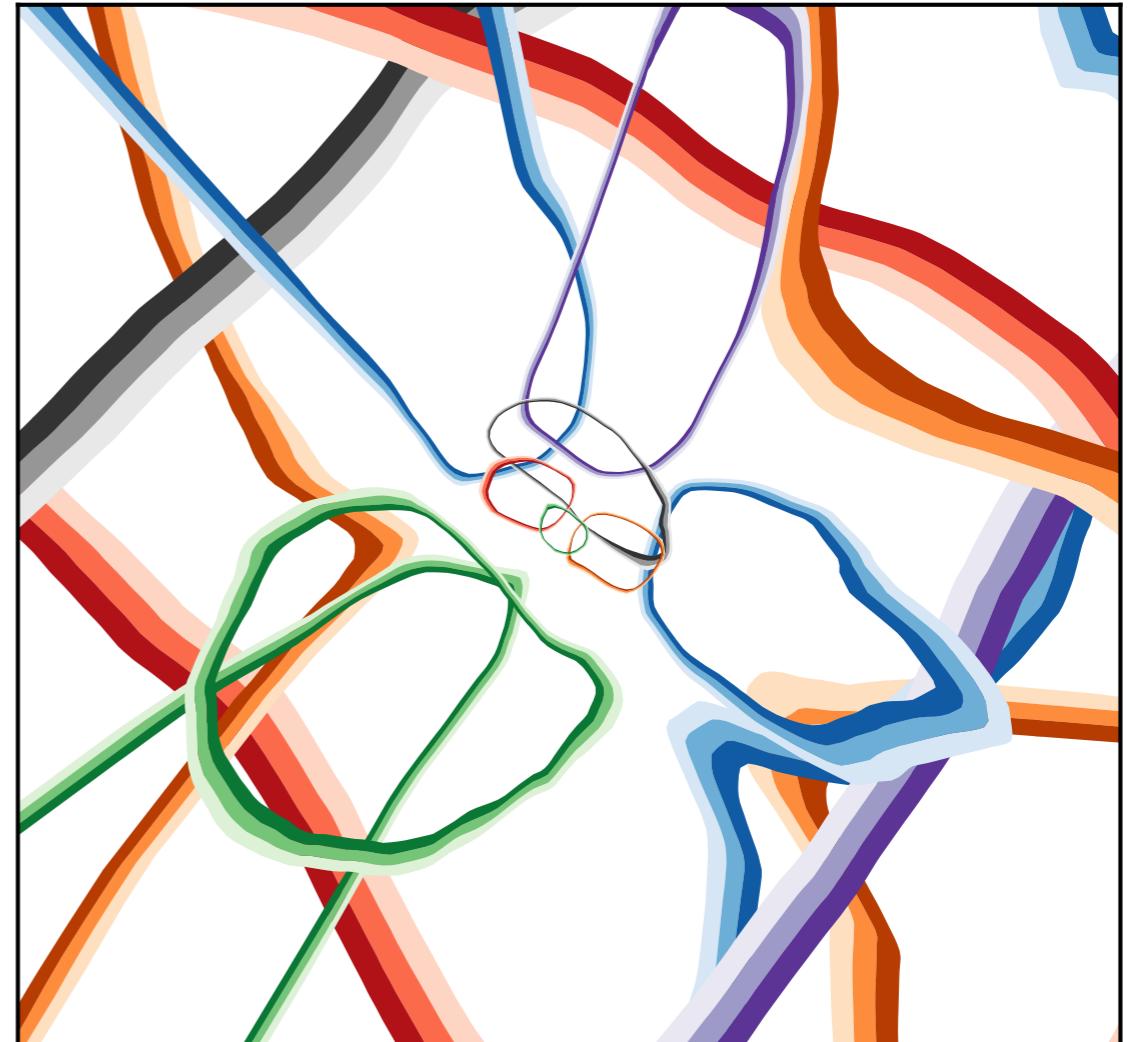


Psychedelic Visualization

Latent Dimension 8

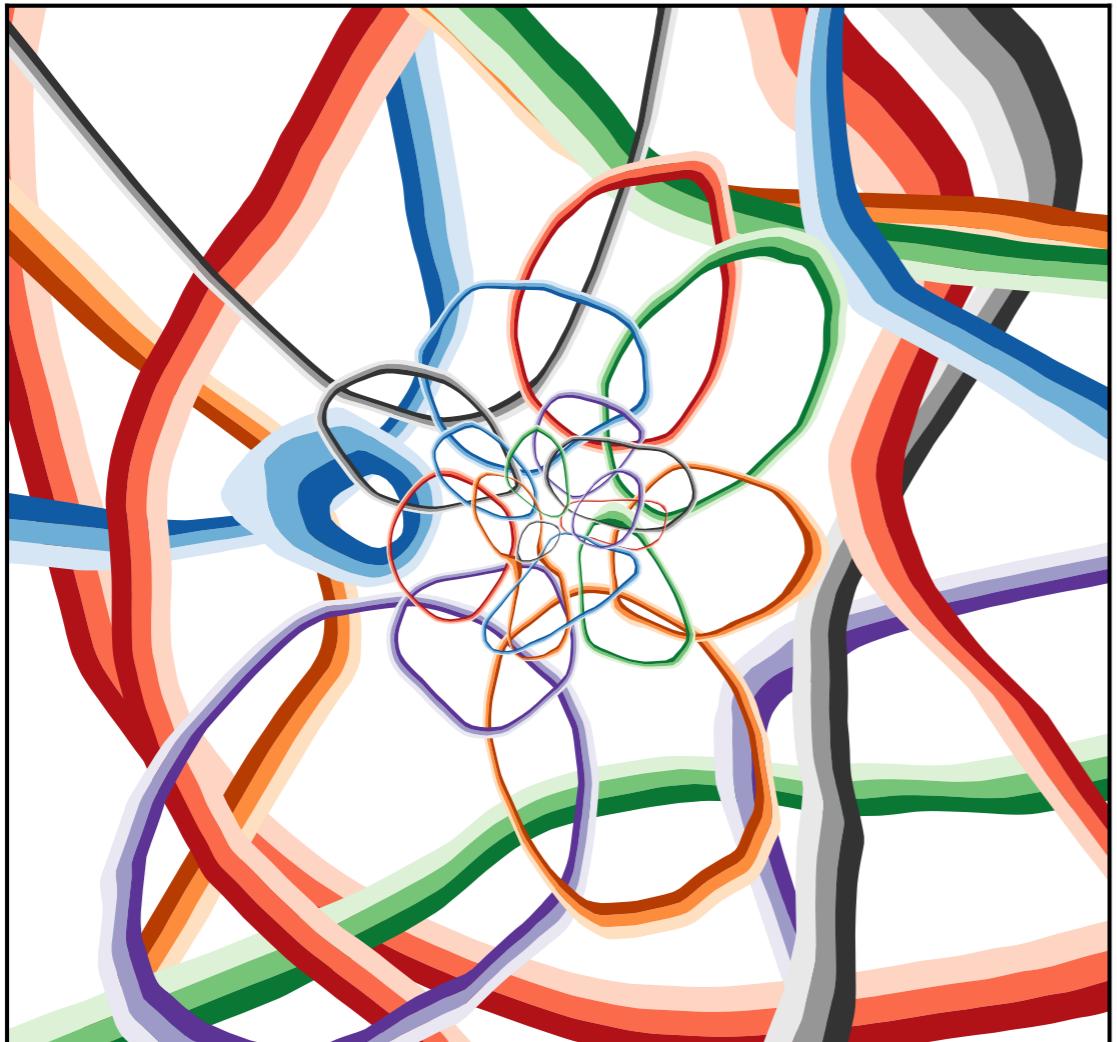


Latent Dimension 16

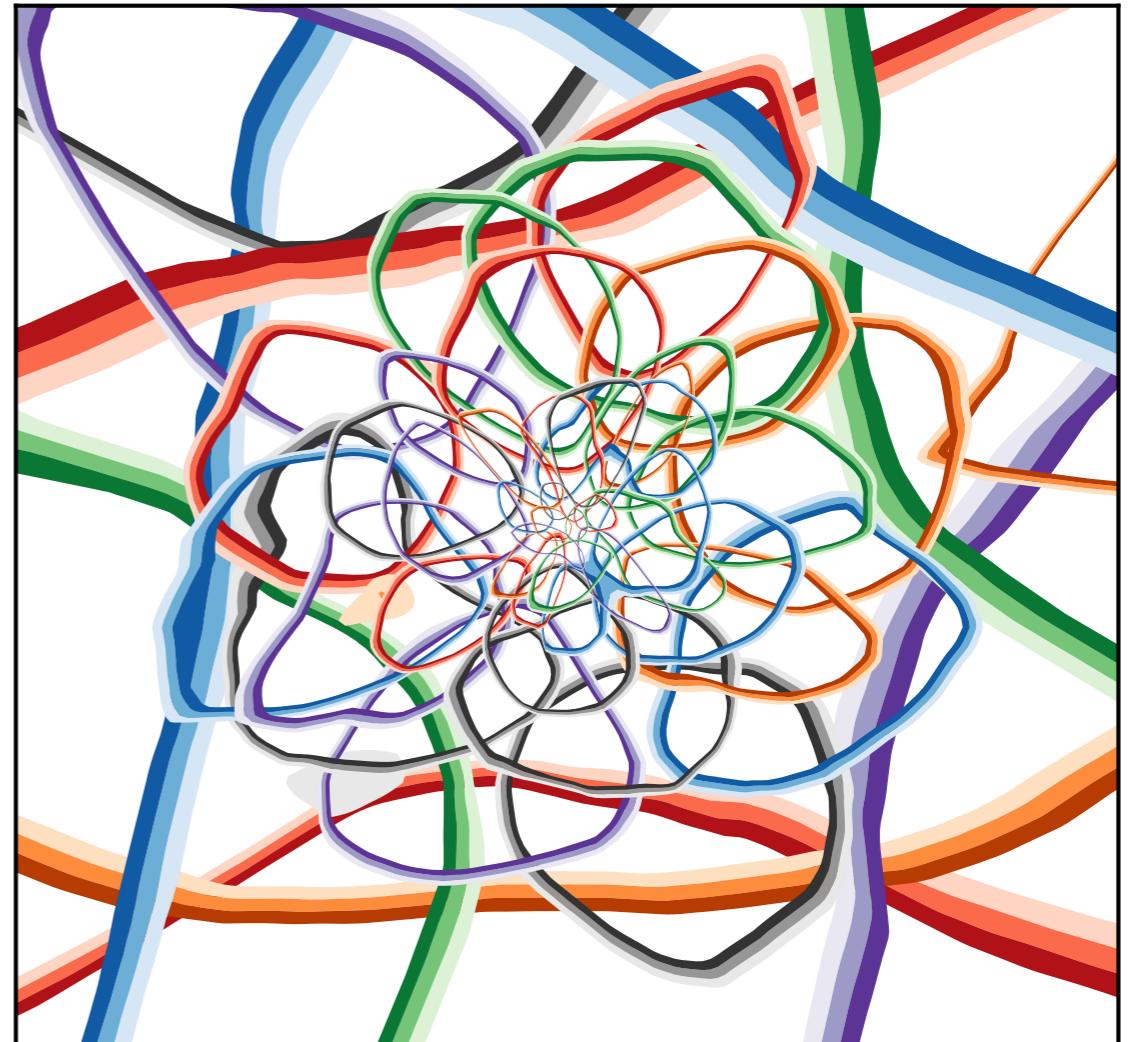


Psychedelic Visualization

Latent Dimension 32

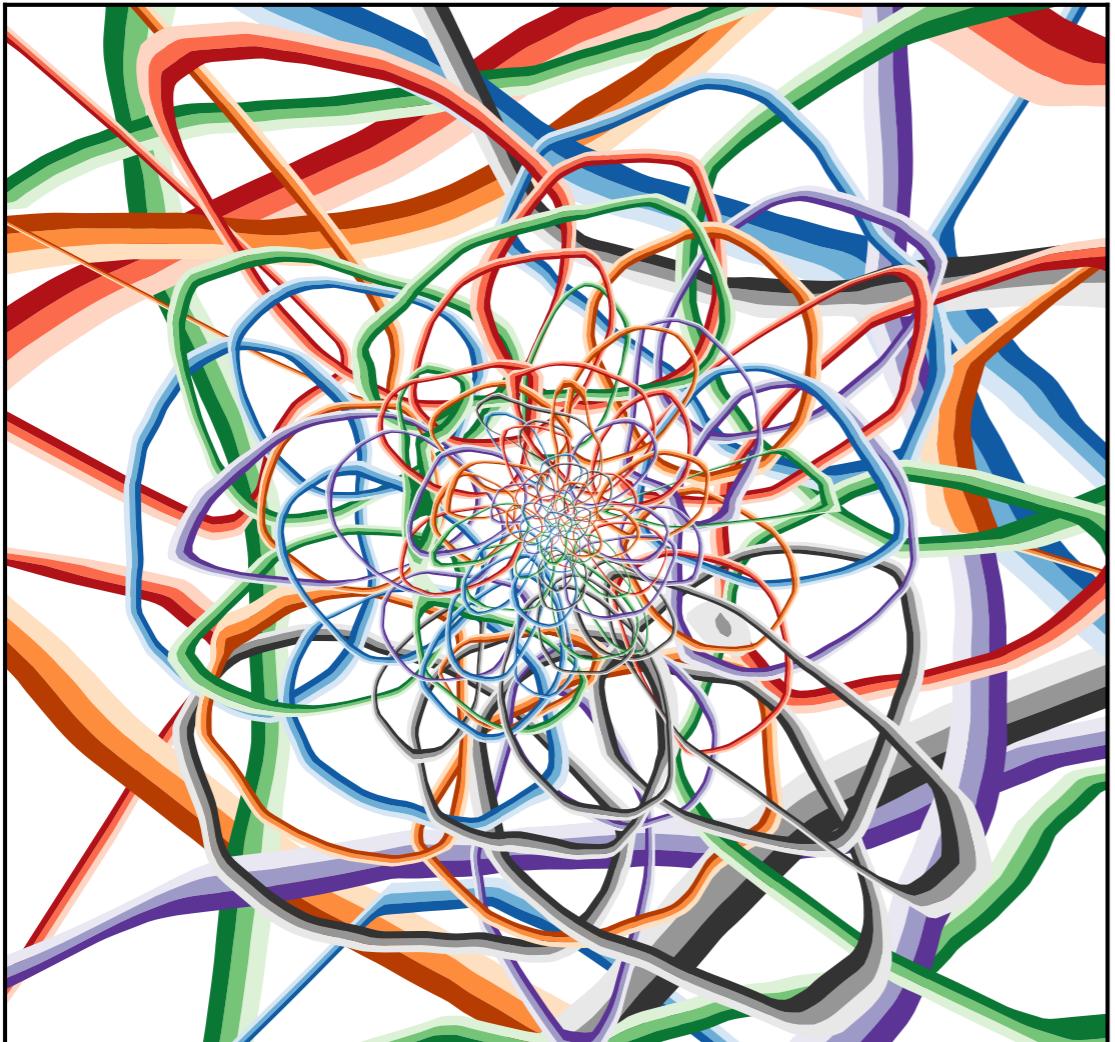


Latent Dimension 64

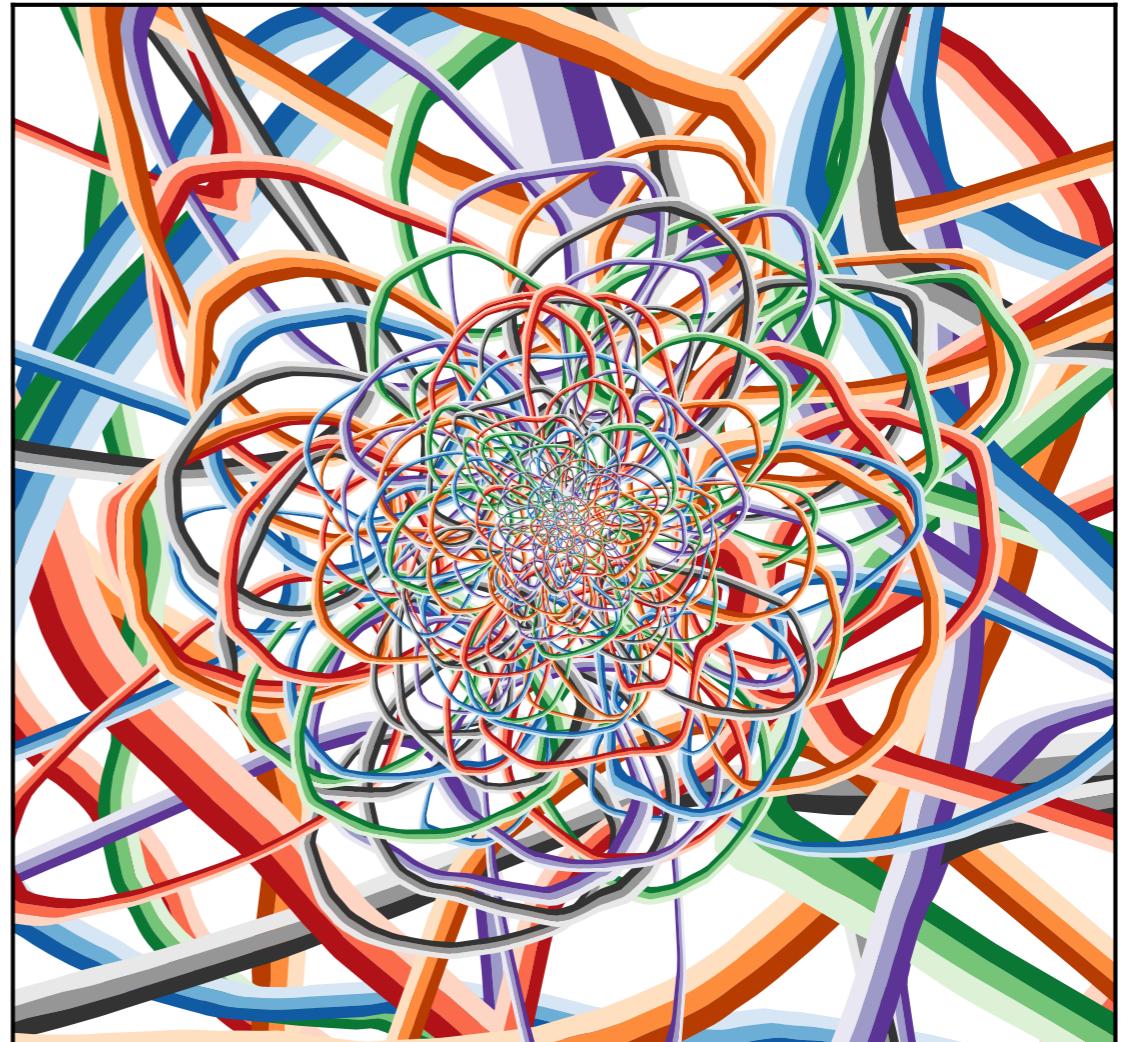


Psychedelic Visualization

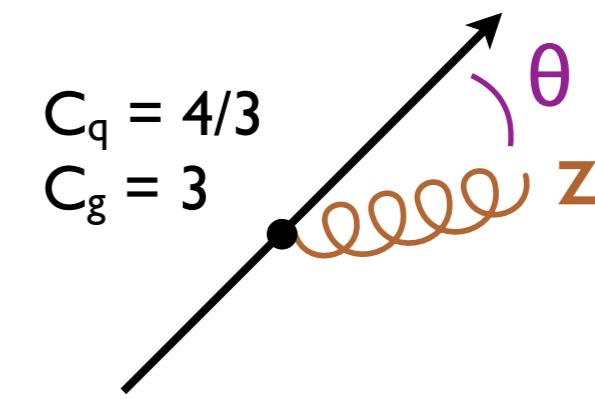
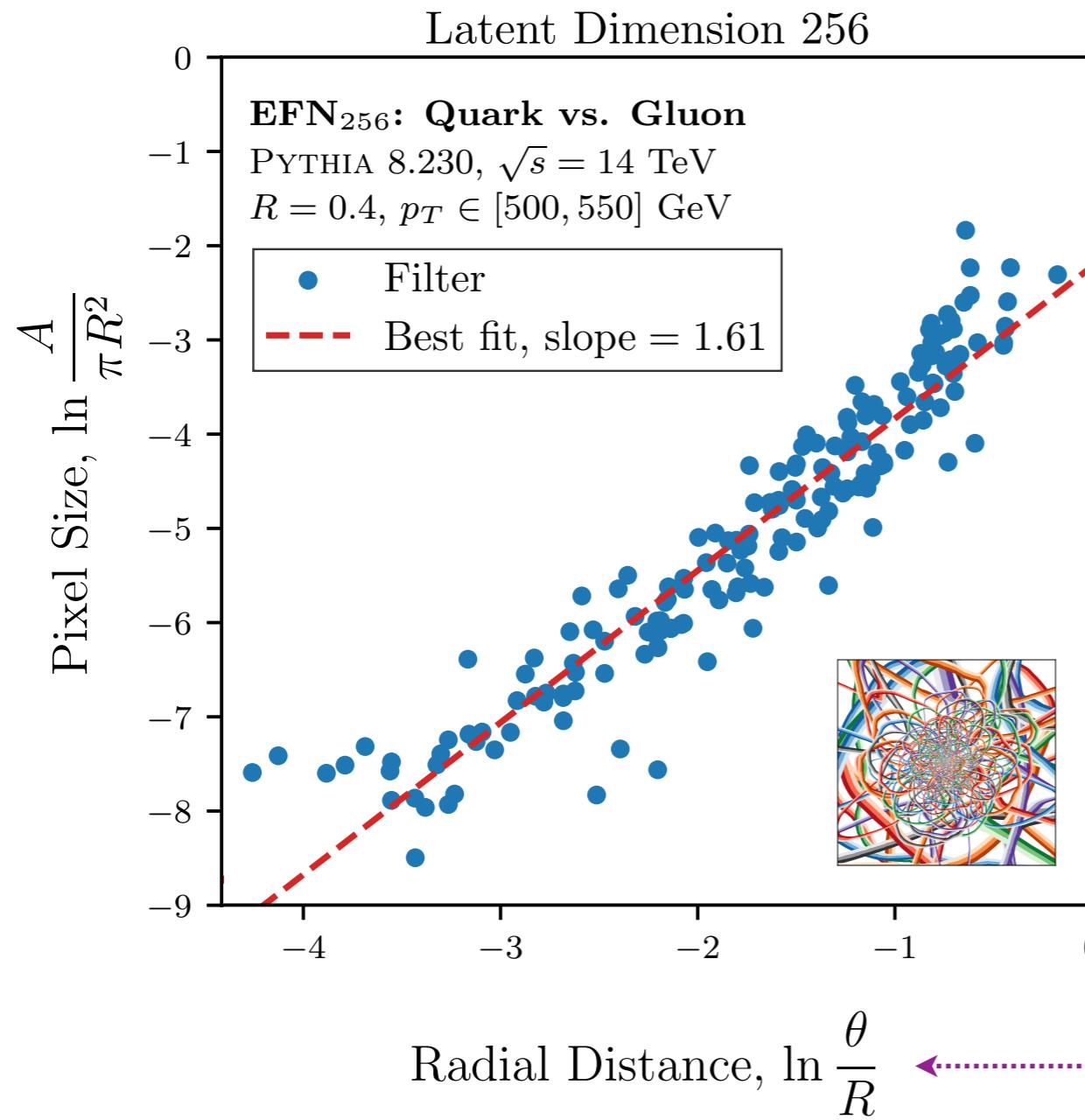
Latent Dimension 128



Latent Dimension 256



Learning the Singularity Structure of QCD

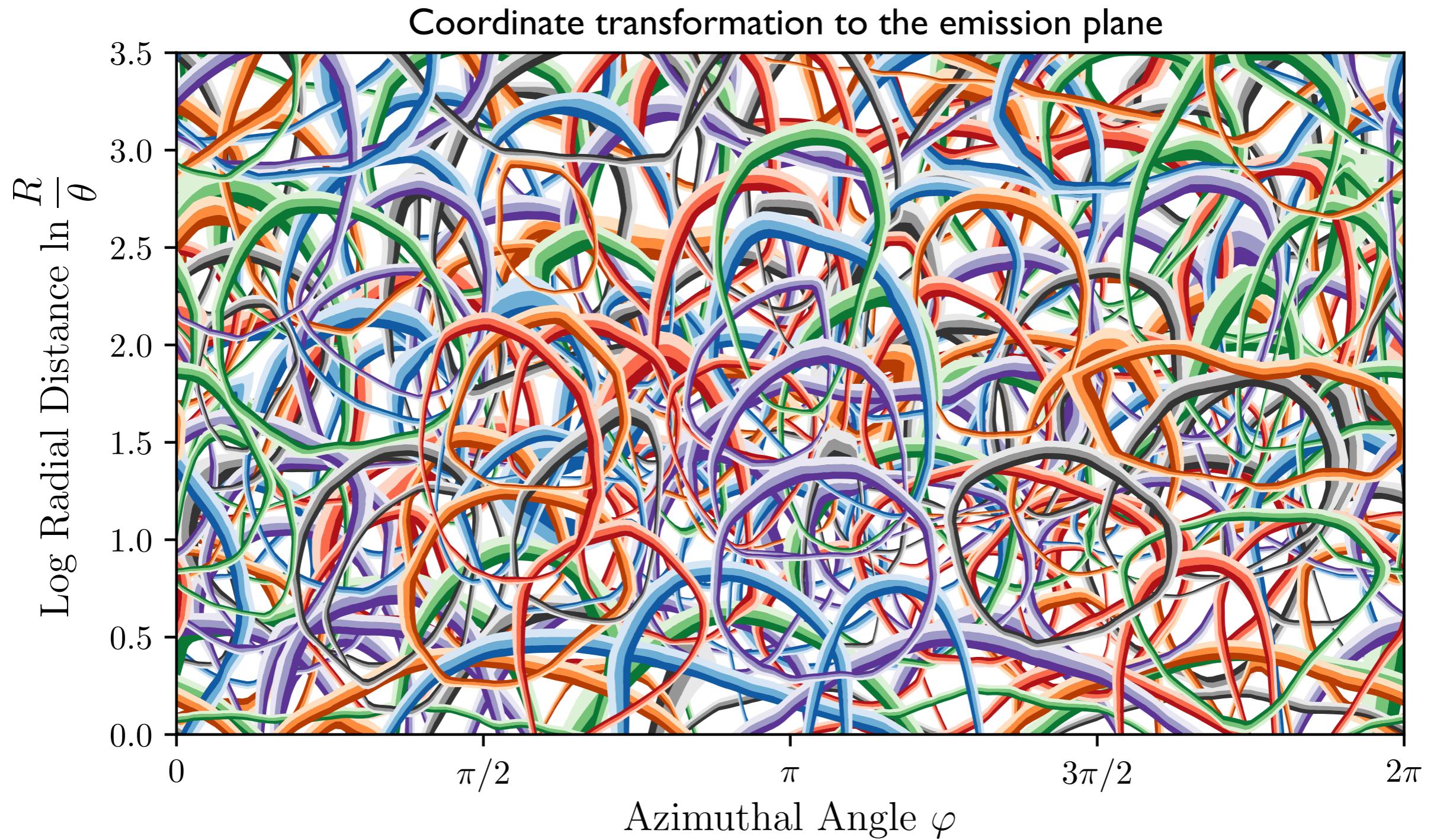


$$dP_{i \rightarrow ig} \sim \frac{2\alpha_s}{\pi} C_i \frac{d\theta}{\theta} \frac{dz}{z}$$

Collinear Soft

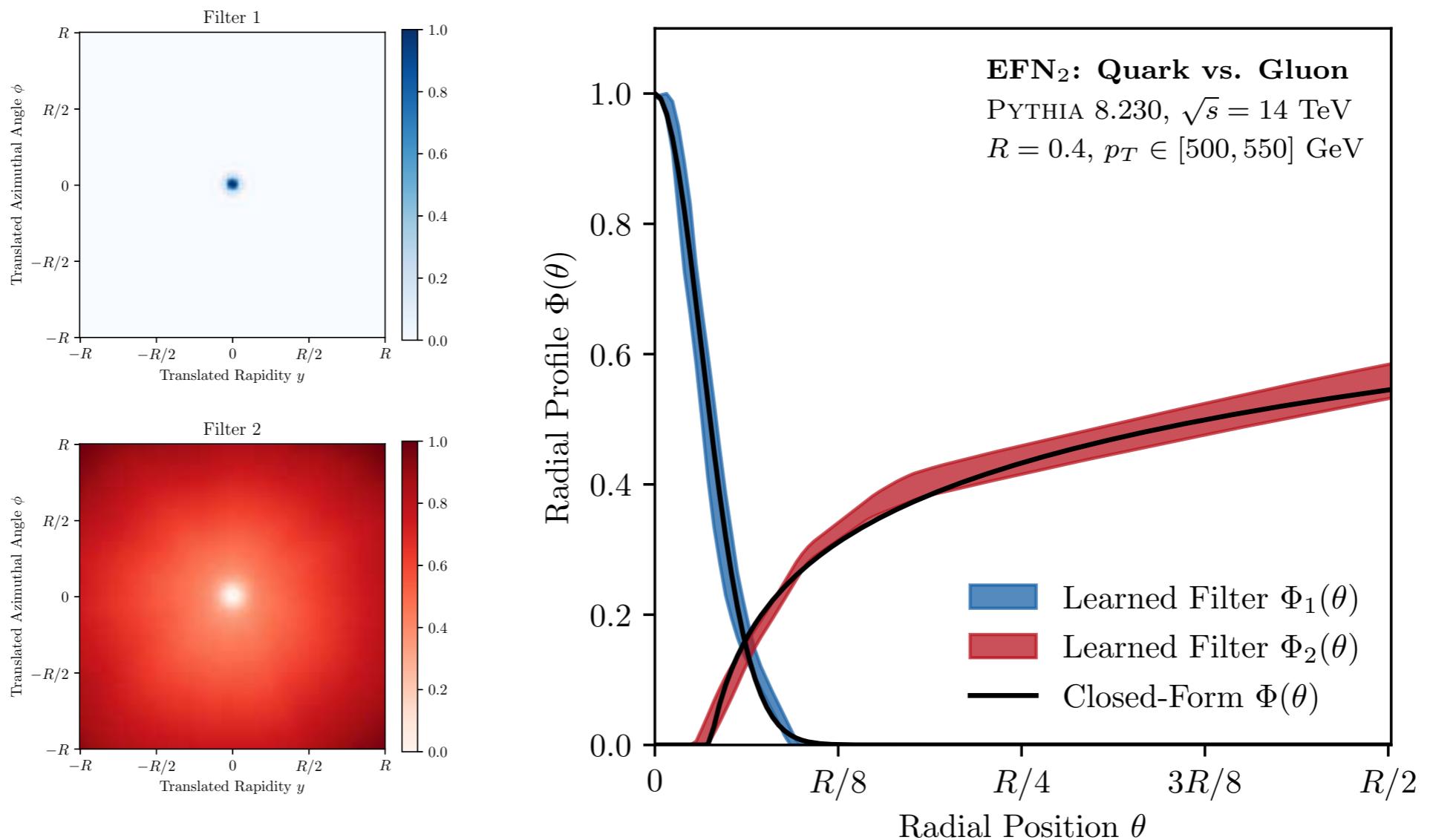
A dotted purple arrow points from the text "Collinear" to the purple bracket under "dθ/θ". Another dotted purple arrow points from the text "Soft" to the orange bracket under "dz/z".

Ready for the MoMA



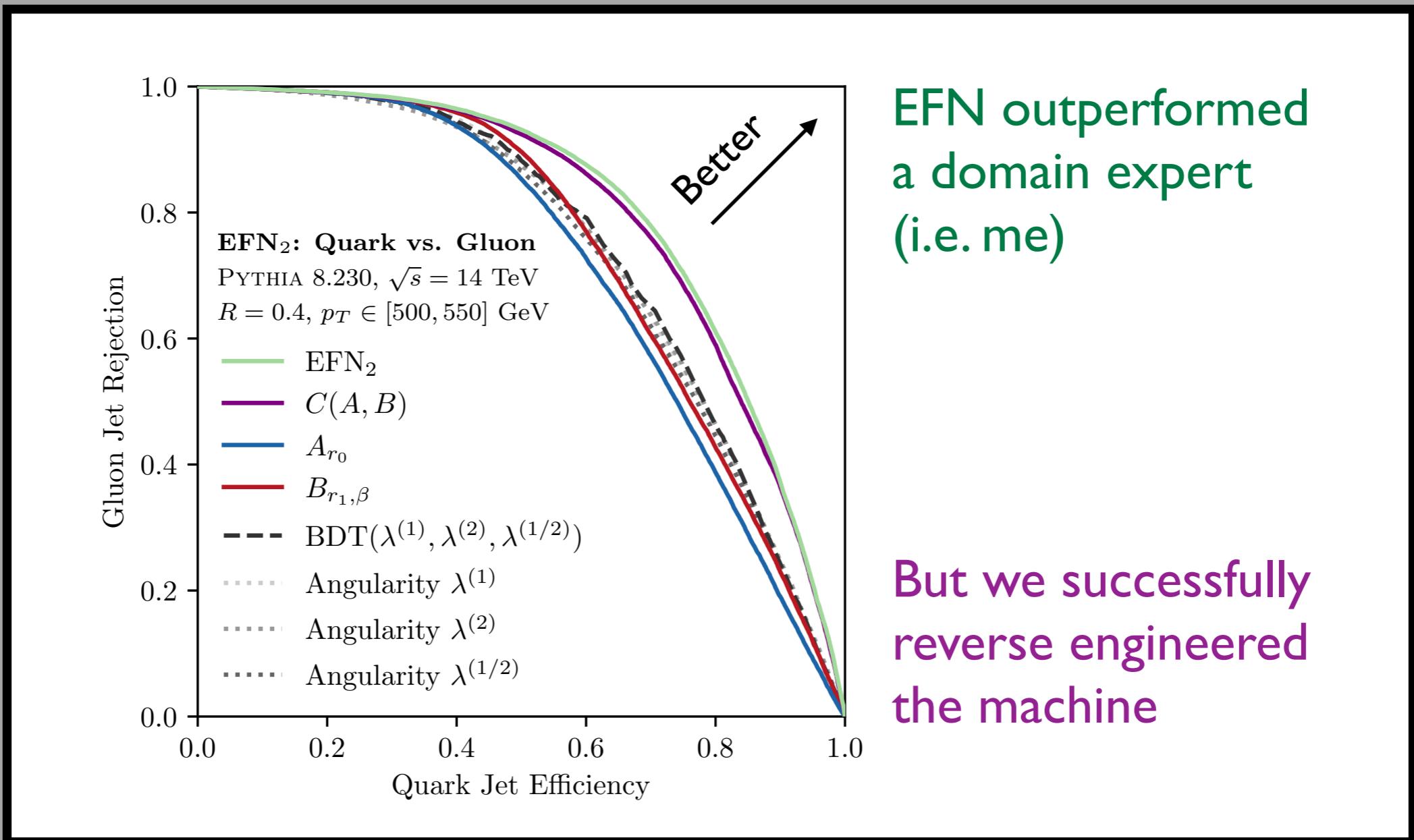
“What is the Machine Learning?”

For $\ell = 2$, radial moments: $\sum_{i \in \text{jet}} z_i f(\theta_i)$ cf. Angularities:
 $f(\theta) = \theta^\beta$



“What is the Machine Learning?”

For $\ell = 2$, radial moments: $\sum_{i \in \text{jet}} z_i f(\theta_i)$ cf. Angularities:
 $f(\theta) = \theta^\beta$



The Broader Lesson

“Deep Learning”

&

~~vs.~~

“Deep Thinking”

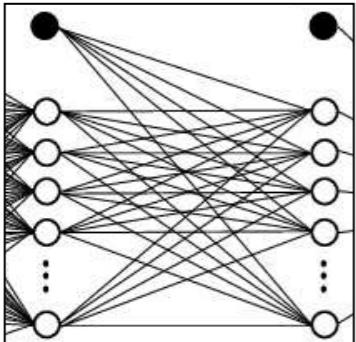
Advances in mathematics and computer science (Deep Sets)



Advances in collider physics (EFN/PFN)

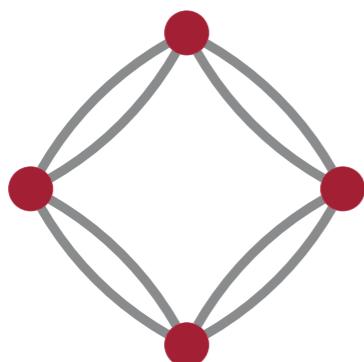
Weighted Point Sets \Leftrightarrow *IRC Safety*

Summary



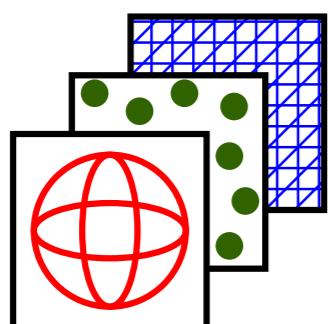
Into the Network

Embracing the rise of machine learning for collider physics



Symmetries & Safety

Importance of indistinguishability and energy weighting



Deep Sets for Particle Jets

EFN/PFNs as a step towards robustness, versatility, and transparency

energyflow.network

The screenshot shows a web browser displaying the [EnergyFlow](https://energyflow.network) documentation. The page has a dark theme with a red header. The header features the EnergyFlow logo (a diamond shape with internal lines) and the text "EnergyFlow". Below the logo is a search bar labeled "Search docs". The main content area has a white background. At the top of the content area, there is a breadcrumb navigation showing "Docs » Home". The main title is "Welcome to EnergyFlow" in a large, bold, red font. Below the title, there is a detailed description of what EnergyFlow is: "EnergyFlow is a Python package for computing Energy Flow Polynomials (EFPs), a collection of jet substructure observables which form a complete linear basis of IRC-safe observables, and for implementing Energy Flow Networks (EFNs) and Particle Flow Networks (PFNs). We also provide quick implementations of other architectures useful for particle physics, namely convolutional neural networks (CNNs) for jet images and dense neural networks (DNNs) for e.g. the N -subjettiness phase space basis." There is also a note about the current version being 0.10.3 and tests being available for version 0.7.0, with source code on GitHub. Below this, there are links to "installing EnergyFlow", "exploring the demo", and "running the examples". The sidebar on the left contains a navigation menu with sections like "Home", "Getting Started", "Documentation", "Measures", "Generation", "Energy Flow Polynomials", "Architectures", "Utils", and "Datasets". At the bottom of the sidebar are links to "GitHub" and "Next »".

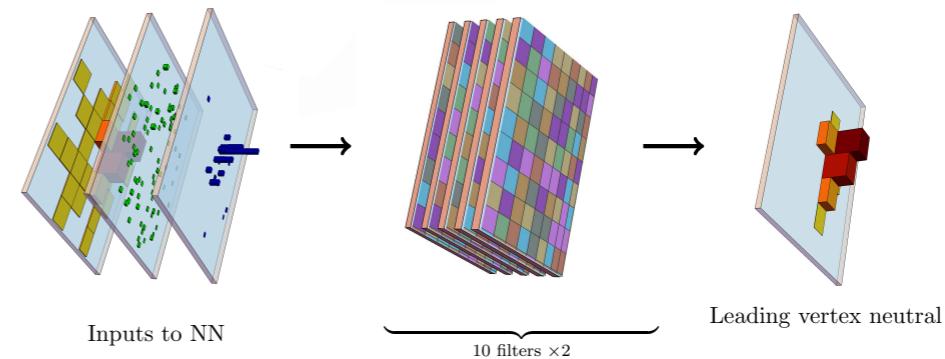
Backup Slides

Beyond Classification

PUMML

Pileup Mitigation

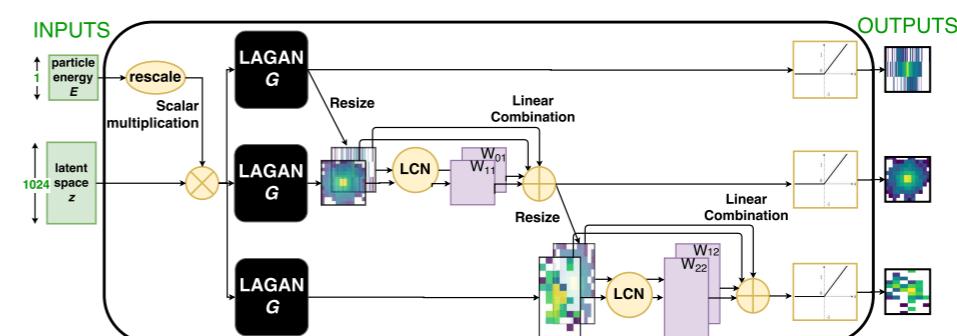
[Komiske, Metodiev, Nachman, Schwartz, 1707.08600]



CaloGAN

Fast Detector Simulation

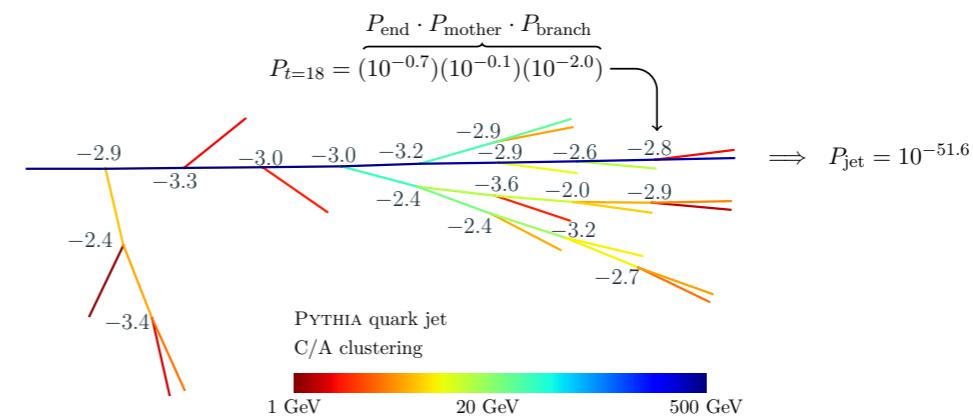
Paganini, de Oliveira, Nachman, 1705.02355, 1712.10321;
see also de Oliveira, Michela Paganini, Nachman, 1701.05927]



JUNIPR

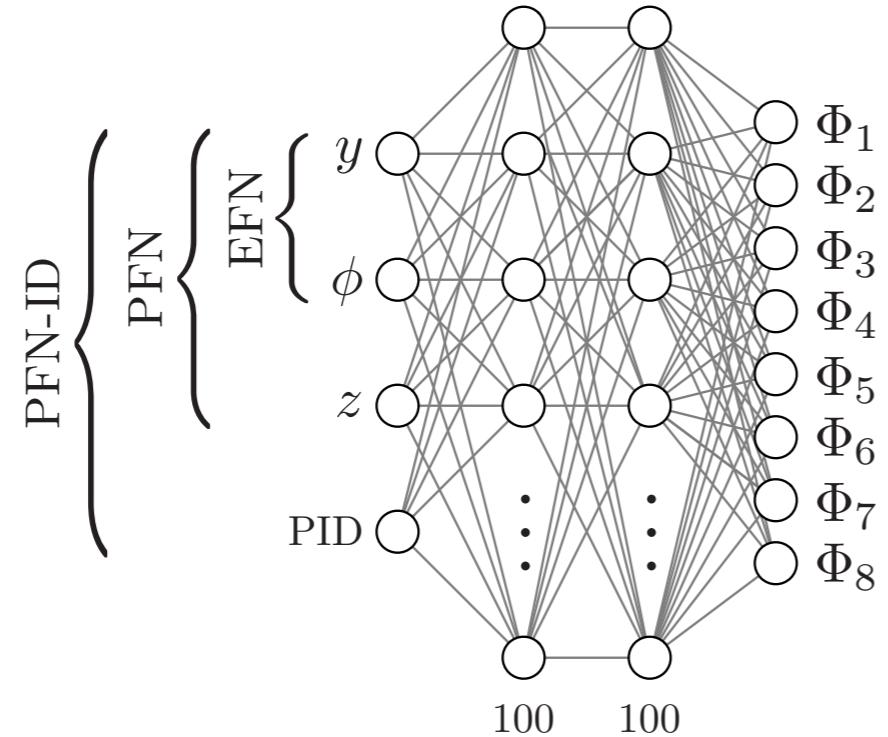
Probability Modeling

[Andreassen, Feige, Frye, Schwartz, 1804.09720;
see also Monk, 1807.03685]

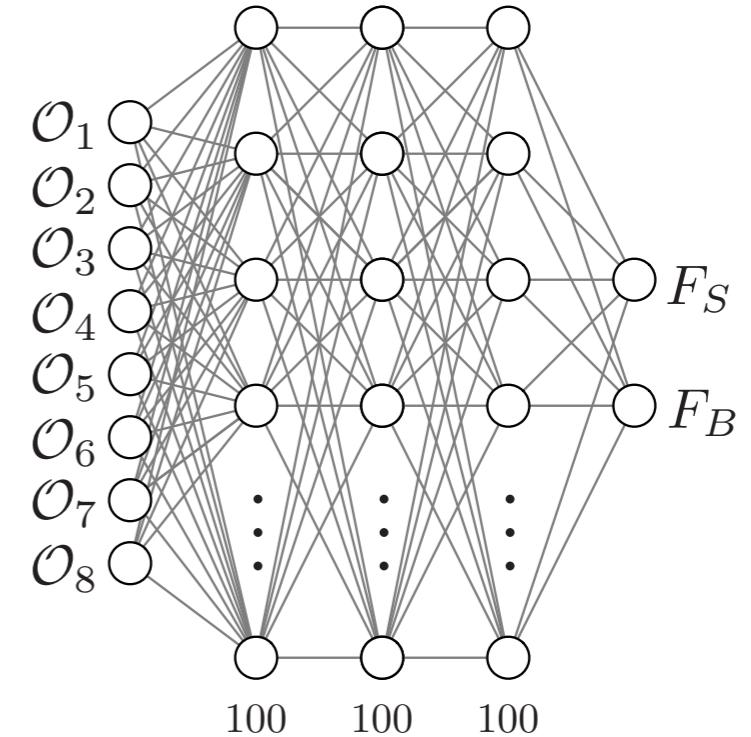


Architecture Details

Per-Particle:



Latent Combiner: F



Per-Jet (Latent Space):

$$\text{EFN: } \mathcal{O}_a = \sum_{i \in \text{jet}} z_i \Phi_a(y_i, \phi_i)$$

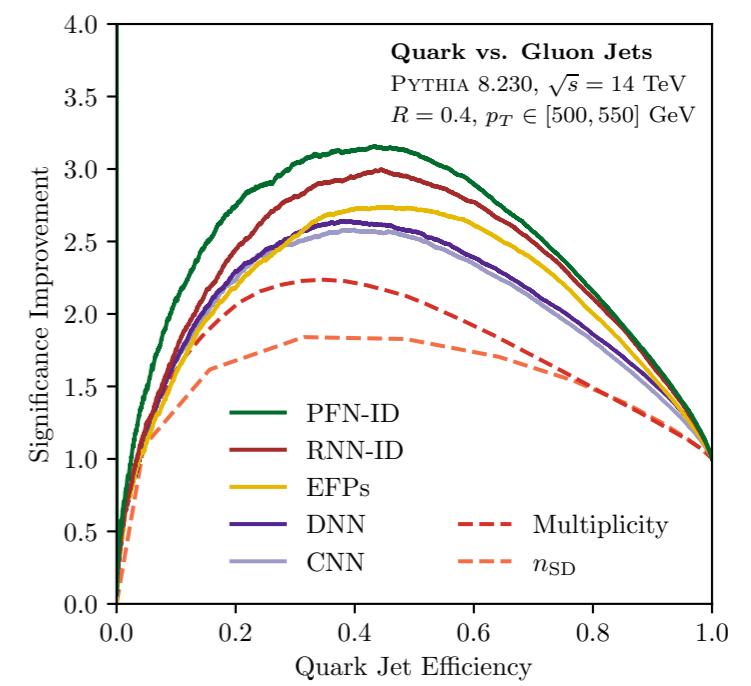
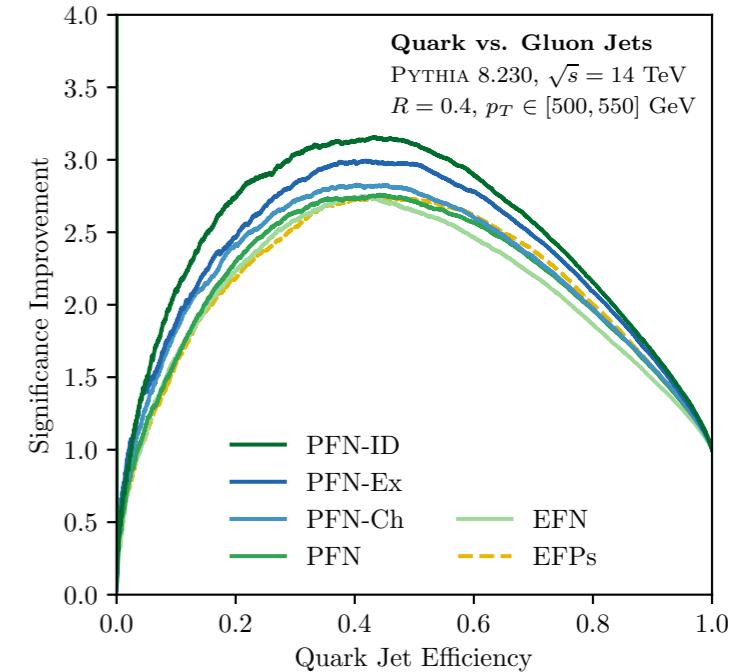
$$\text{PFN: } \mathcal{O}_a = \sum_{i \in \text{jet}} \Phi_a(y_i, \phi_i, z_i, \text{PID}_i)$$

Final Discriminant:

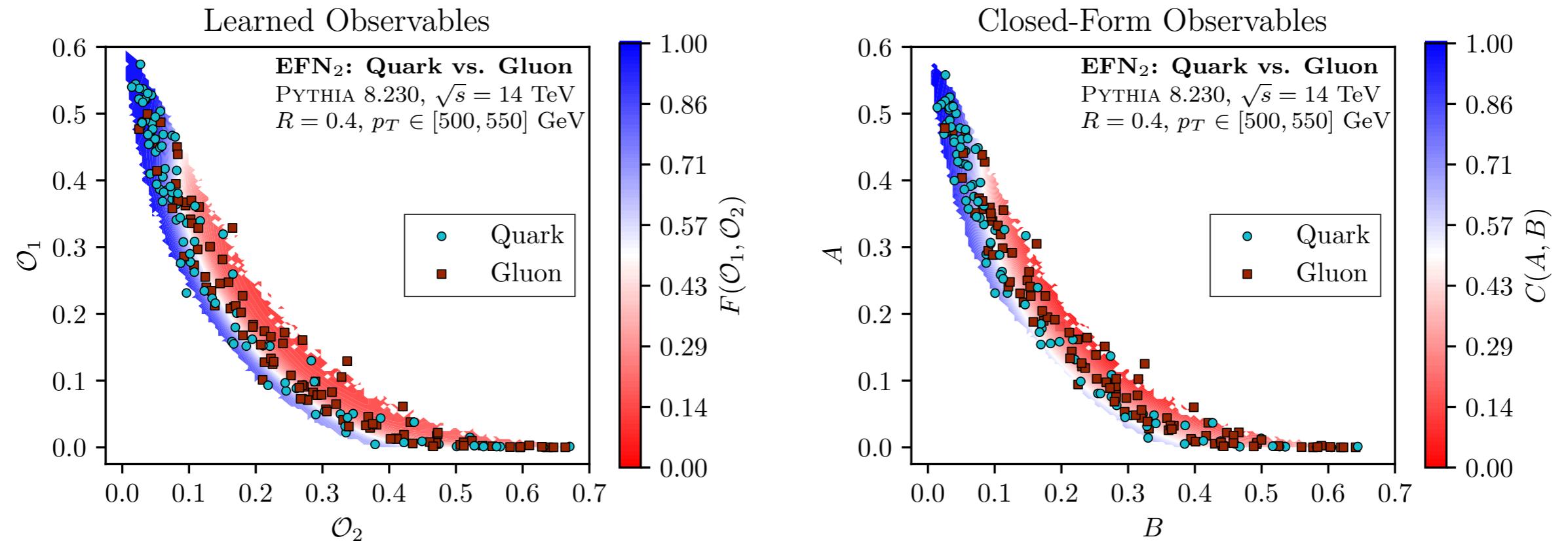
$$\text{softmax}(F_S, F_B)$$

More Quark/Gluon Performance

Model	AUC	$1/\varepsilon_g$ at $\varepsilon_q = 50\%$
PFN-ID	0.9052 ± 0.0007	37.4 ± 0.7
PFN-Ex	0.9005 ± 0.0003	34.7 ± 0.4
PFN-Ch	0.8924 ± 0.0001	31.2 ± 0.3
PFN	0.8911 ± 0.0008	30.8 ± 0.4
EFN	0.8824 ± 0.0005	28.6 ± 0.3
RNN-ID	0.9010	34.4
RNN	0.8899	30.5
EFP	0.8919	29.7
DNN	0.8849	26.4
CNN	0.8781	25.5
M	0.8401	19.0
n_{SD}	0.8297	14.2
m	0.7401	7.2



Reverse Engineering the Machine



Fascinating QCD question about why this is a better strategy than traditional angularities

Energy Flow Polynomials

- Underlying Physics
- Natural Data Representation
- Suitable Algorithm

What is the space of *all*
IRC-safe observables?

Examples from Jet Substructure

→ Underlying Physics
 Natural Data Representation
 Suitable Algorithm

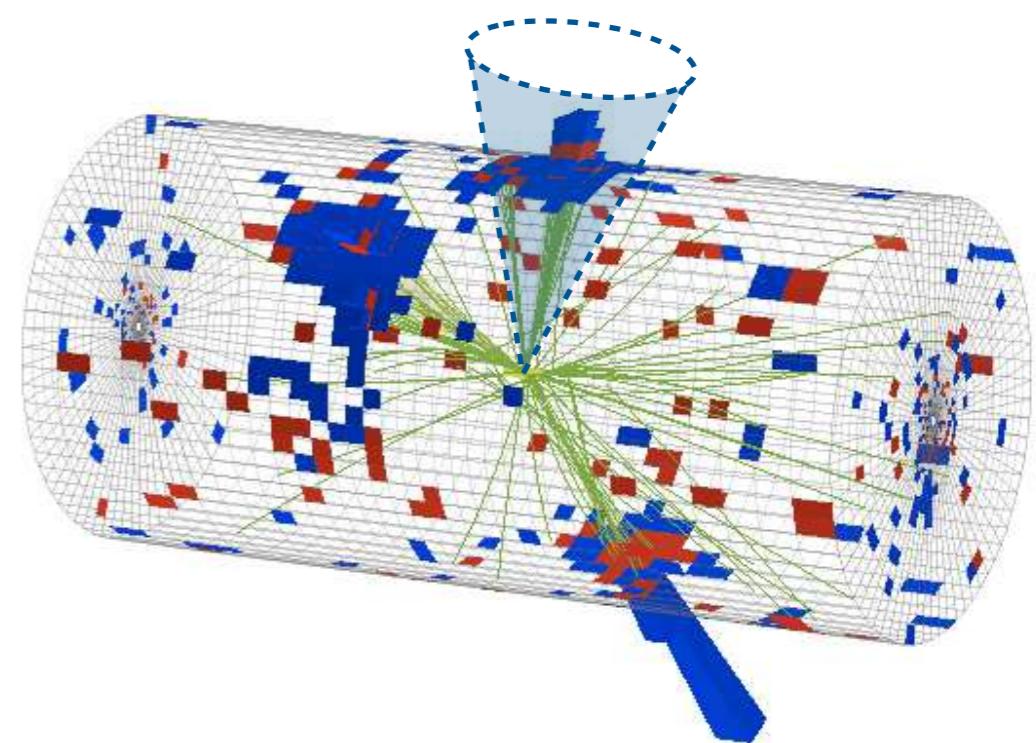
Jet pt: $\sum_{i \in \text{jet}} p_{T,i}$ **IRC Safe**

p_T^D :
[CMS HIG-11-027] $\sum_{i \in \text{jet}} \frac{p_{T,i}^2}{p_{T\text{jet}}^2}$ **IR Safe**
C Unsafe

Multiplicity: $\sum_{i \in \text{jet}} 1$ **IRC Unsafe**

Jet Mass: $\sum_{i,j \in \text{jet}} p_i \cdot p_j$ **IRC Safe**

N-subjettiness:
[JDT, Van Tilburg,
 1011.2268, 1108.2701] $\sum_{i \in \text{jet}} p_{T,i} \min \{ \Delta R_{i,1}, \Delta R_{i,2}, \dots, \Delta R_{i,N} \}^\beta$ **IRC Safe**
 But ratios are only “Sudakov safe”!



A Systematic Expansion

Underlying Physics
→ Natural Data Representation
Suitable Algorithm

Expand* any IRC safe observable in small energy limit

$$\begin{aligned} \mathcal{S} = & \sum_i E_i f_1^{\mathcal{S}}(\hat{n}_i) + \sum_{ij} E_i E_j f_2^{\mathcal{S}}(\hat{n}_i, \hat{n}_j) \\ & + \sum_{ijk} E_i E_j E_k f_3^{\mathcal{S}}(\hat{n}_i, \hat{n}_j, \hat{n}_k) + \dots \end{aligned}$$

Form enforced by:	Particle Relabeling	Infrared Safety	Collinear Safety
-------------------	---------------------	-----------------	------------------

Further expand* each angular function in pairwise angles

$$z_i = \frac{E_i}{E_{\text{jet}}} \quad \cos \theta_{ij} = \hat{n}_i \cdot \hat{n}_j$$

[Komiske, Metodiev, JDT, 1712.07124; see also Tkachov, hep-ph/9601308]

The Energy Flow Polynomials

Underlying Physics
→ Natural Data Representation
Suitable Algorithm

$$\text{EFP}_G = \sum_{i_1=1}^M \cdots \sum_{i_N=1}^M z_{i_1} \cdots z_{i_N} \prod_{(k,\ell) \in G} \theta_{i_k i_\ell}$$

Multigraph *Angular Scaling*

\downarrow $\downarrow \beta$

All N-tuples N Energy Fractions

Polynomial in Pairwise Angles

A Linear Basis for Jet Substructure (!)

[Komiske, Metodiev, JDT, 1712.07124]

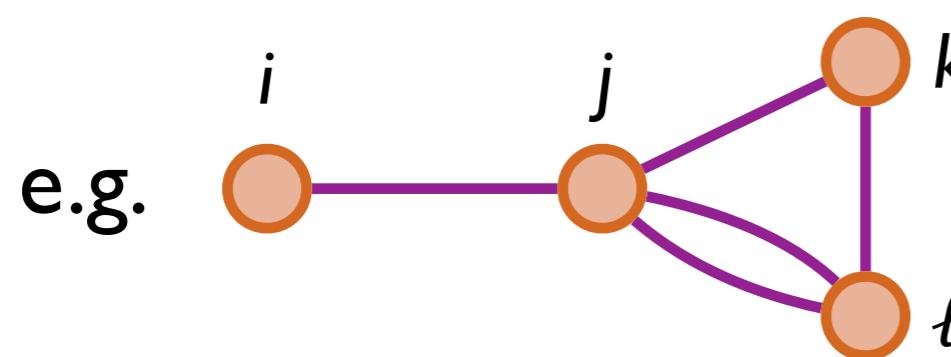
The Energy Flow Polynomials

Underlying Physics
 → Natural Data Representation
 Suitable Algorithm

$$\begin{array}{c}
 \text{Multigraph} \\
 \downarrow \\
 \text{EFP}_G = \sum_{i_1=1}^M \cdots \sum_{i_N=1}^M z_{i_1} \cdots z_{i_N} \prod_{(k,\ell) \in G} \theta_{i_k i_\ell}^\beta
 \end{array}$$

All N-tuples	$\text{N Energy Fractions}$	$\text{Polynomial in Pairwise Angles}$
-----------------------	-----------------------------	--

e.g.



$$= \sum_{ijkl} z_i z_j z_k z_l \theta_{ij} \theta_{jk} \theta_{jl}^2 \theta_{kl}$$

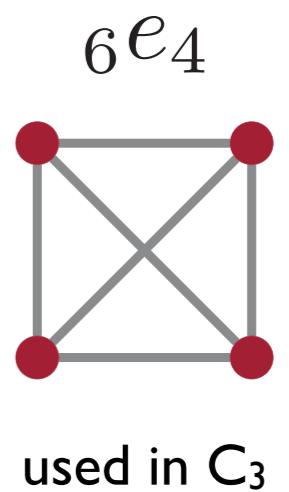
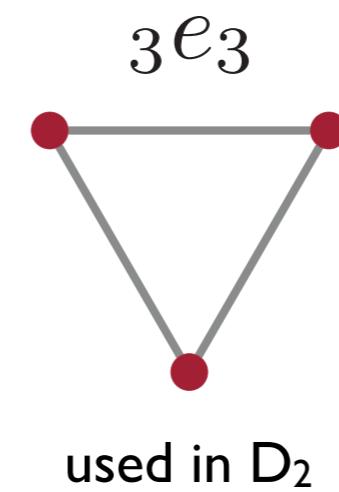
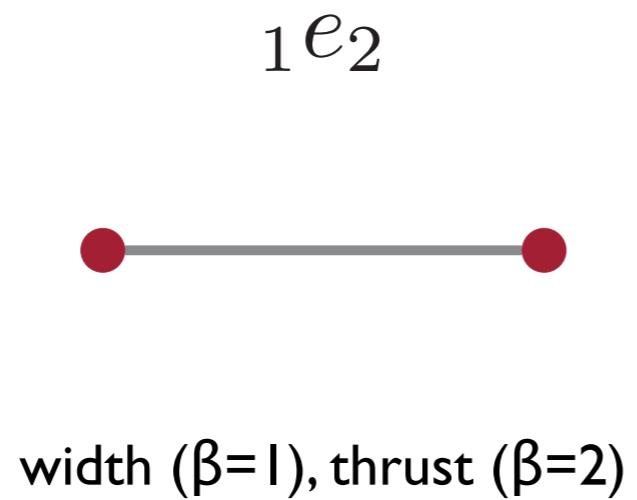
A Linear Basis for Jet Substructure (!)

[Komiske, Metodiev, JDT, 1712.07124]

Down the Rabbit Hole

Underlying Physics
→ Natural Data Representation
Suitable Algorithm

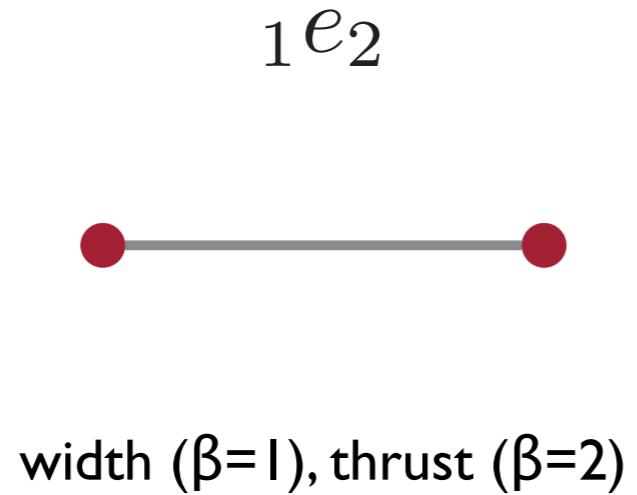
Known Structures:



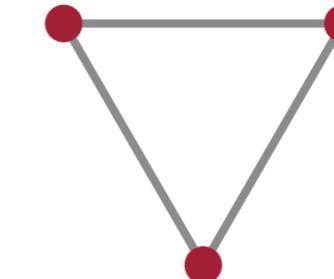
Down the Rabbit Hole

Underlying Physics
→ Natural Data Representation
Suitable Algorithm

Known Structures:

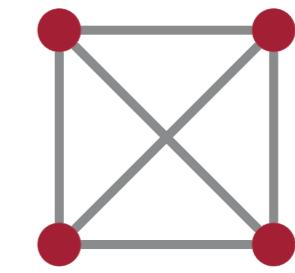


$3e_3$



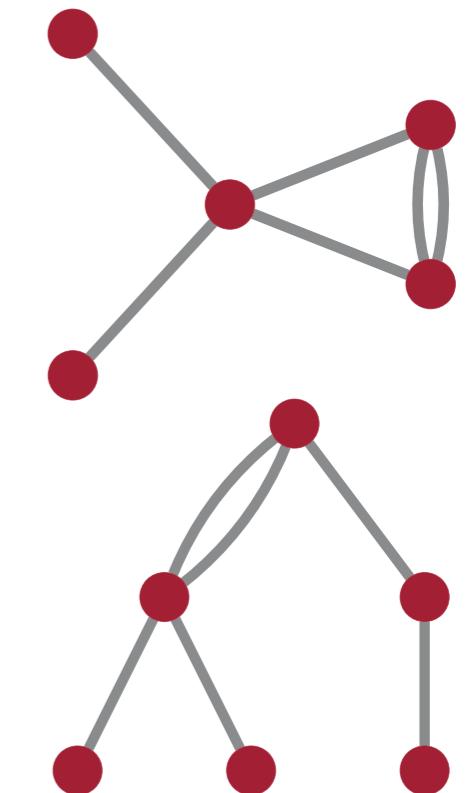
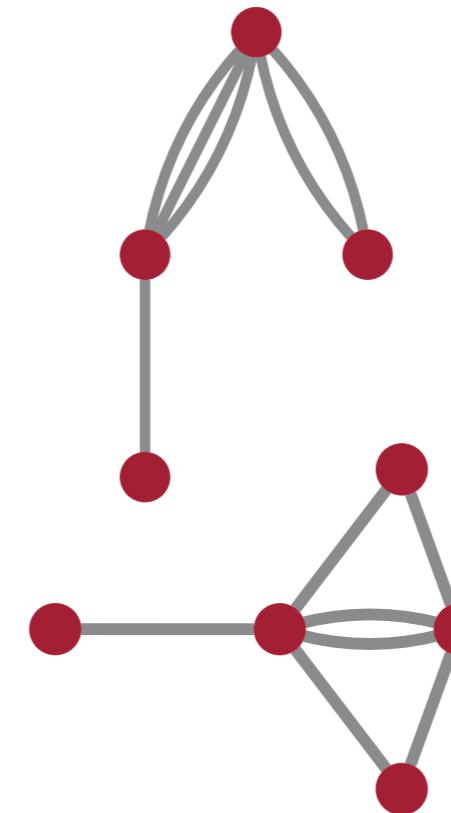
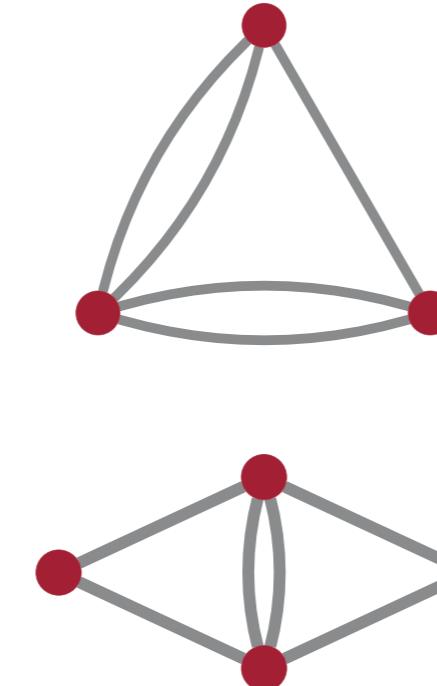
used in D_2

$6e_4$



used in C_3

No Idea:

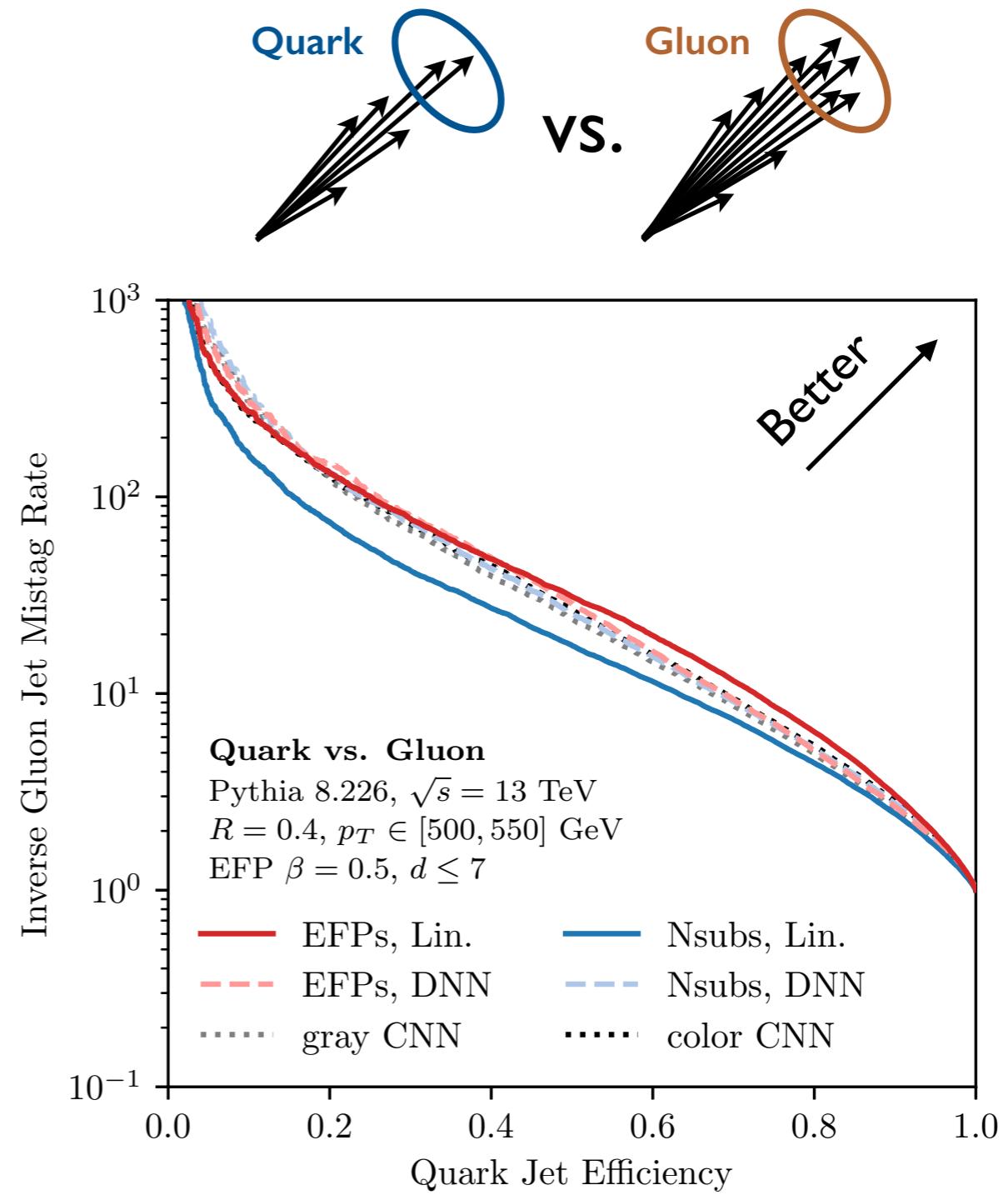
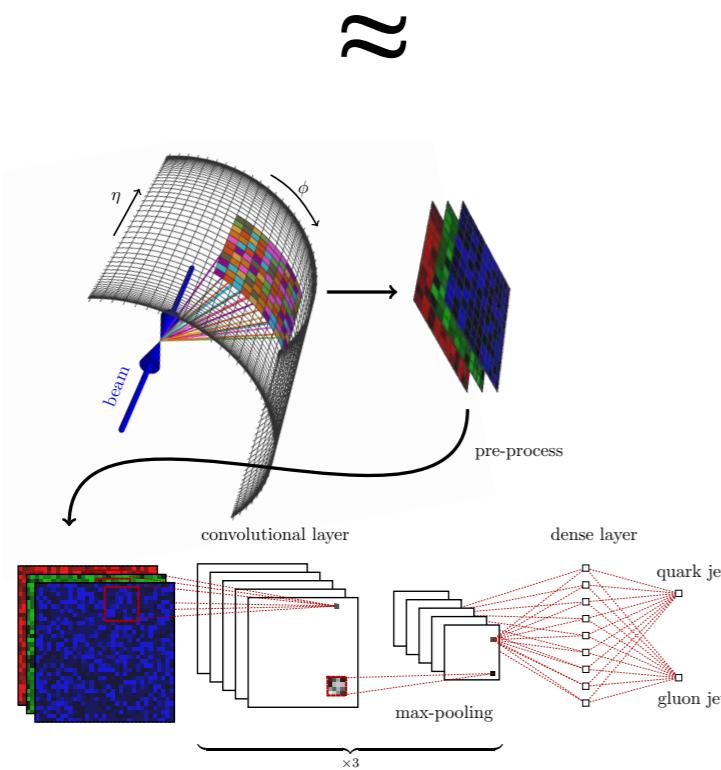


Linear Regression \approx CNN

Underlying Physics
 Natural Data Representation
 → Suitable Algorithm

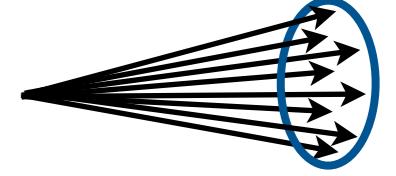
“...indistinguishable from magic”

$$\mathcal{S} = \sum_G s_G \text{EFP}_G$$



[Komiske, Metodiev, JDT, 1712.07124; Komiske, Metodiev, Schwartz, 1612.01551]

Comparing Data Representations



Original 4-Vectors: $\{p_1^\mu, p_2^\mu, \dots, p_N^\mu\}$

Variable-length, unordered set

EFP Basis: $\text{EFP}_G = \sum_{i_1=1}^M \cdots \sum_{i_N=1}^M z_{i_1} \cdots z_{i_N} \prod_{(k,\ell) \in G} \theta_{i_k i_\ell}$

*Automatically permutation invariant
Linear spanning basis (over-complete)
Computational nightmare of $O(M^N)$?*



Too naive, can use
variable elimination

The Physics-Meets-Computation Approach

Underlying Physics



Natural & Efficient Data Representation



Desired Computational Property

- Underlying Physics
- Natural Data Representation
- Suitable Algorithm

What is the space of *all*
linearly-computable
permutation-invariant
IRC-safe observables?

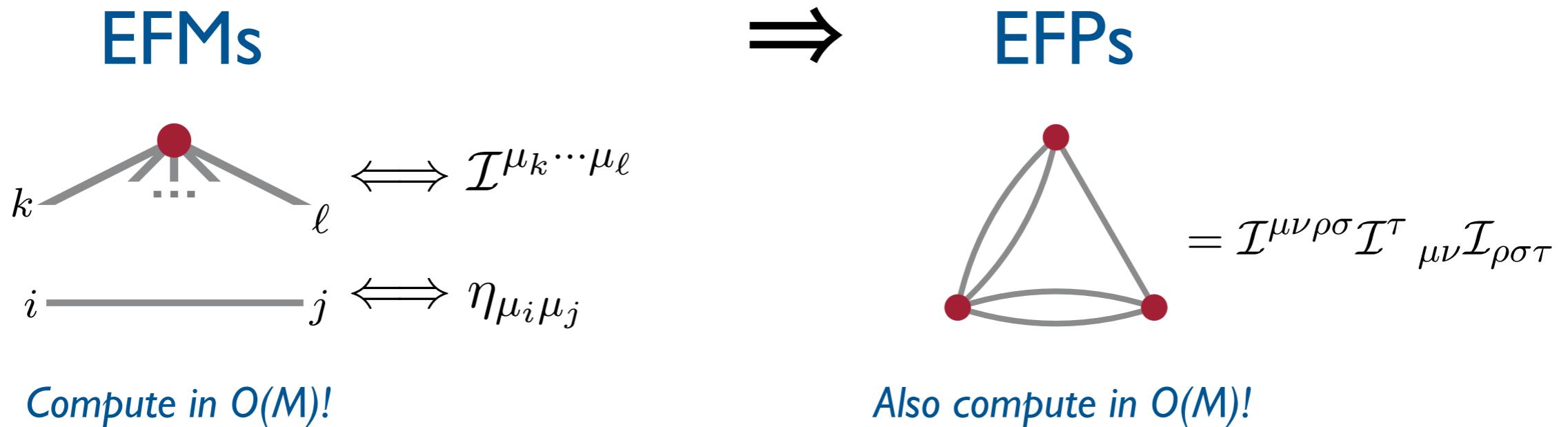
The Energy Flow Moments

Underlying Physics
 → Natural Data Representation
 Suitable Algorithm

$$\mathcal{I}^{\mu_1 \mu_2 \cdots \mu_v} = \sum_{i=1}^M E_i \hat{p}^{\mu_1} \hat{p}^{\mu_2} \cdots \hat{p}^{\mu_v}$$

Particle
Relabeling Infrared
Safety

Special Choice
of Angle $\theta_{ij} = 2 \eta_{\mu\nu} \hat{p}_i^\mu \hat{p}_j^\nu$



[Komiske, Metodiev, JDT, we've been promising this paper for 9 months]