Three Lectures on Collider Physics

Jesse Thaler

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1. Ingredients & Kinematics

2. From Diagrams to Cross Sections

3. An Introduction to Jets

Lecture 1: Ingredients & Kinematics.

Why colliders?

Smash particles at ever increasing energies to learn about fundamental physics?

Yes! It is our best (known) strategy to study

- heavy objects
- short lifetimes
- rarely produced.

E.g. Higgs boson $m_h \approx 125$ GeV

$c\tau_h \approx 5 \times 10^{-11}$ mm

$\sigma_h^{LHC} \approx 50$ pb

Have to simultaneously produce <u>and</u> study.

If you come up with a better method, you will revolutionize particle physics.

These lectures will focus on hadron colliders, in particular the LHC. (Though some calculations for e^+e^-)

Parameters of the LHC: $p \longrightarrow \quad \longleftarrow p$

proton - proton $\qquad E_{cm} = 7, 8, 13$, eventually 14 TeV

Collision rate: $40\ \text{MHz} = \frac{1}{25 \text{ nano seconds}}$

protons per bunch $\approx 10^{11}$ (similar to number of stars in the milky way)

Instantaneous Luminosity $= \dfrac{N_1 N_2 f}{\underbrace{A_{eff}}_{\approx (.1\text{mm})^2}} \approx \dfrac{2 \times 10^{34}}{\text{cm}^2 \text{ sec}}$

Integrated Luminosity $= \int \underset{\substack{\uparrow \\ \text{actual uptime} \\ (2 \text{ months})}}{dt}\, \mathcal{L} \approx 100\ \text{fb}^{-1} = \ ??$

what does this mean?

Proton radius $\approx$ fm $= 10^{-15}$ m $\approx \frac{1}{200 \text{ MeV}}$

Barn $= 100\ \text{fm}^2 = 10^{-28}\ \text{m}^2$

$\sigma_{pp} \approx 0.1$ b (total cross section)

$\Longrightarrow 100\ \text{fb}^{-1} \cdot 0.1\ \text{b} = 10^{16}$ proton-proton collisions.

Some helpful numbers to keep in mind....

How many proton-proton collisions per beam crossing?

$$N_{xing} = \underset{\uparrow}{\mathcal{L}} \cdot \underset{\uparrow}{T} \cdot \sigma \approx 50 \quad (!)$$

$\mathcal{L}$: $\dfrac{2\times10^{34}}{cm^2\,sec}$ $\quad$ T: 25 ns $\quad$ σ: 0.1 b

but most of these are uninteresting "Pileup"

How many collisions can you write to tape?

1 collision ≈ 1 cute picture of my kid ≈ 1 MB (with compression)

Total data rate $\quad 40\ \text{MHz} \cdot 1\,\text{MB} = 40000\ \dfrac{\text{GB}}{\text{sec}}$

Rough factor by which you have to reduce the data volume. "Triggering"

An array of fast hard drives.

How many Higgs bosons have been produced in diphoton channel?

$$N_{H\to\gamma\gamma} = \underbrace{\mathcal{L}\cdot T}_{\substack{100\ fb^{-1} \\ \text{(end of run 2)}}} \cdot \underbrace{\sigma^{LHC}_{Higgs}}_{50\,pb} \cdot \underbrace{Br(H\to\gamma\gamma)}_{2\cdot10^{-3}}$$

≈ 10000

A big number! But have to wrestle with $pp\to\gamma\gamma$ background.

The Master Formula.

$$AB \to 1,\ 2,\ 3,\ \cdots\ n$$

$$\sigma_M = \frac{1}{2E_{cm}^2} \sum_{n=2}^{\infty} \int d\Phi_n \left| \mathcal{M}_{AB \to 123\cdots n} \right|^2 f_M(\Phi_n)$$

- σ_M: a measurement ("observable")
- $\frac{1}{2E_{cm}^2}$: a geometric factor (assuming massless beams)
- $\sum_{n=2}^{\infty}$: all possible final states
- $d\Phi_n$: Lorentz invariant phase space.
- $\mathcal{M}_{AB \to 123\cdots n}$: Scattering amplitude
- f_M: A measurement function.

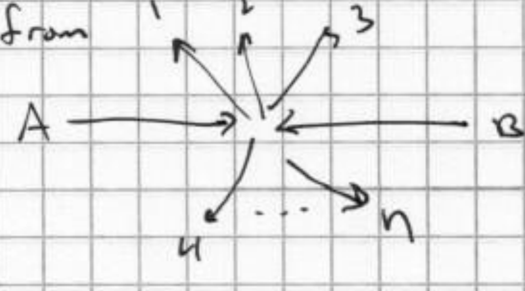
Key: The cross section you measure depends on what you measure. ("observables")

Have to think carefully about what choice of f_M to use for a given physical process, then have to sum over <u>all</u> final states that contribute to f_M.

This will be very important when we talk about jets.

Note: Cross section has units of area, saturated by $\frac{1}{2E_{cm}^2}$

Goal: Go from



to σ.

A Tour of the PDG*, organized by lifetime.

Absolutely Stable

p	proton	$m_p = 938$ MeV	We know how to make beams of these.
e^-	electron	$m_e = 511$ keV	
γ	photon	$m_\gamma = 0$	
$\nu_1\ \nu_2\ \nu_3$	neutrinos	$m_\nu \simeq 0$	No idea how to make beams of these.
G	graviton	$m_G = 0$	

Workhorse colliders: e^+e^-, pp, $p\bar{p}$, e^-p

(For nuclear physics, also heavy ion colliders.)

* Particle Data Group. Order your 2018 edition today!

Collider Stable ($c\tau > 1$ m)

n neutron $m_n \approx 940$ MeV $c\tau \approx 2.7 \times 10^{14}$ mm

(can make beams of, but hard to control)

μ^- muon $m_\mu \approx 106$ MeV $c\tau \approx 6.6 \times 10^5$ mm

(might eventually make beams of, but definitely design detectors around measuring them)

K_L kaon-long $m_{K_L} \approx 498$ MeV $c\tau \approx 1.5 \times 10^4$ mm

(need hadronic calorimeters to catch these)

$\pi^\pm$ charged pion $m_{\pi^\pm} \approx 140$ MeV $c\tau \approx 7.8 \times 10^3$ mm

$K^\pm$ charged kaon $m_{K^\pm} \approx 494$ MeV $c\tau \approx 3.7 \times 10^3$ mm

(mostly stable, detectable as charged tracks, their decays can give ~~your~~ punchthrough)

Sort of Stable ($c\tau > 10$ mm)

Ξ^0 Λ^0

Ξ^- Σ^- — strangeness carrying baryons — $m \simeq 1.2 - 1.7$ GeV $\quad c\tau \approx 24 - 87$ mm

Ω^- Σ^+

K_s kaon-short $\quad m_{K_s} \approx 498$ MeV $\quad c\tau \approx 27$ mm

(decays to $\pi^0\pi^0$ or $\pi^+\pi^-$, intermediate lifetime makes this kind of a pain)

(Remember, by time dilation, lifetime in lab frame scaled by Lorentz γ factor.)

Displaced Vertex ($c\tau > .01$ mm)

B^+ B^0 B_s^0	bottom mesons	$m_B \simeq 5.3$ GeV	$c\tau \simeq .44 - .49$ mm
Λ_b^0 Ξ_b^- etc.	bottom baryons		
Ξ_{bc}^+, etc.	bottom-charm baryons	$m \simeq 5.6 - 15.1$ GeV	$c\tau \simeq .36$ mm
Ω_{bb}^- etc	bottom-bottom (-bottom) baryons		
D^+ D_s^+ (B_c^+) D_0	Charm mesons	$m \simeq 1.9$ GeV (6.2 GeV)	$c\tau \simeq .12 - .15$ mm
Ξ_c^+, Λ_c^+	Charm baryon	$m \simeq 2.5 - 2.7$ GeV	$c\tau \simeq .13$ mm $- .02$ mm
Ξ_{cc}^+ etc.	charm-charm baryon	$m \simeq 3.6 - 3.8$ GeV	$c\tau \simeq .1$ mm
Ω_{ccc}^{*++}	c-c-c baryon	$m \simeq 4.9$ GeV	$c\tau \simeq .1$ mm
τ^-	tau lepton	$m_\tau \simeq 1.78$ GeV	$c\tau \simeq .087$ mm

(has diverse decays that must be reconstructed.)

Everything else can not be resolved directly at a collider! (And these displaced vertex decays are hard!)

These are only ingredients you need for $AB \to 123 \cdots n$

Wait! You mentioned $\pi^\pm$. Where is the neutral pion?

π^0 neutral pion $\quad m_{\pi^0} \simeq 135$ MeV $\quad c\tau \simeq 2.6 \times 10^{-5}$ mm

Unstable on collider time scales, seen as $\pi^0 \to \gamma\gamma$.
(might mistake for a single photon)

[What about the rest of the standard model?
More on that later...]

At ATLAS and CMS, not all of these particles are distinguishable. Generic categories are:

- "$e^\pm$" electron | positron candidates
- "$\mu^\pm$" ~~m~~ muon | anti-muon candidate
- "$\pi^\pm$" charged hadron (includes $K^\pm$, p, etc.)
- "K^0_L" neutral hadron (includes n, etc.)
- "γ" photon (may or may not include $\pi^0 \to \gamma\gamma$)

$\not{E}_T$ missing transverse momentum (includes neutrinos)

The Rest of the Standard Model

So far we've mentioned:

γ

$e \quad \mu \quad \tau$

$\nu_1 \quad \nu_2 \quad \nu_3$

Every other particle was a composite hadron.

How do we "see" the other particles in the standard model?

Jets

Jets : Collimated sprays of particles, focus of lecture 3

g gluon

d down

u up

s strange

Very challenging to distinguish. Spray of $\mathcal{O}(10)$ hadrons initiated by parton.

$m_{parton} \approx 0$

$m_{jet} \approx 10\% \cdot E_{jet}$ (not massless!)

c charm

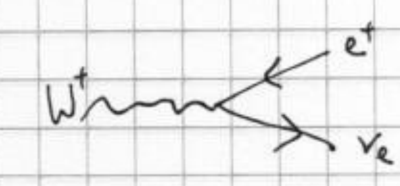
b bottom

Hadronize to a heavy-flavor hadron to yield displaced vertex signature.

Heavy Resonances : Decay to lighter states

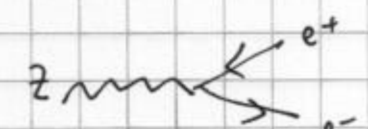
W^+ W boson $m_W \approx 80.4$ GeV

$11\% \rightarrow e^+ \nu_e$
$11\% \rightarrow \mu^+ \nu_\mu$
$11\% \rightarrow \tau^+ \nu_\tau$
$67\% \rightarrow$ hadrons (two jets)



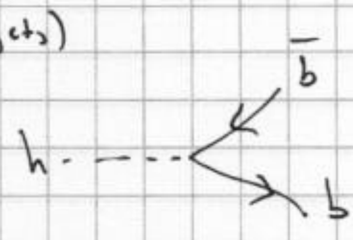
Z^0 Z boson $m_Z \approx 91.2$ GeV

$3.4\% \rightarrow e^+ e^-$
$3.4\% \rightarrow \mu^+ \mu^-$
$3.4\% \rightarrow \tau^+ \tau^-$
$20.5\% \rightarrow$ neutrinos
$70\% \rightarrow$ hadrons (two jets)



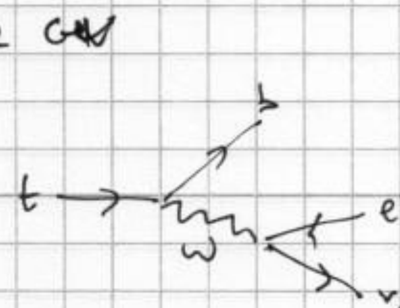
h^0 Higgs boson $m_h \approx 125$ GeV

$60\% \rightarrow b\bar{b}$ (two b-tagged jets)
$21\% \rightarrow WW^*$ (see above)
$9\% \rightarrow gg$
$5\% \rightarrow \tau^+ \tau^-$
$2.5\% \rightarrow c\bar{c}$
$2.5\% \rightarrow ZZ^*$ (see above)
$0.2\% \rightarrow \gamma\gamma$
$0.15\% \rightarrow Z\gamma$



t top quark $m_t \approx 172$ GeV

$99\% \rightarrow bW$ (see above)



To set up the next lecture, we need to talk about ...

Kinematics

$$p^\mu = (E, p_x, p_y, p_z) = (E, \vec{p})$$

a 4-vector

$$p^\mu p_\mu \equiv p^2 = E^2 - p_x^2 - p_y^2 - p_z^2$$

(collider physics uses mostly minus metric.)

Lorentz-invariant phase space for single particle

$$\underbrace{\int \frac{d^4 p}{(2\pi)^4}}_{\text{manifestly Lorentz invariant}} \underbrace{(2\pi)\,\delta(p^2 - m^2)}_{\text{on-shell}} \underbrace{\Theta(E)}_{\text{positive energy}}$$

$$= \int \frac{d^3\vec{p}}{(2\pi)^3} \frac{1}{2E} \quad \text{where } E = \sqrt{m^2 + \vec{p}^{\,2}}$$

not so manifestly Lorentz invariant.

Total energy-momentum conservation

$$(2\pi)^4 \delta^{(4)}\Big(p_{CM} - \underbrace{p_1 - p_2 - \cdots - p_n}_{n\text{-particle final state.}}\Big)$$

For example, total decay width for particle of mass M.

$$\Gamma_A = \frac{1}{2M} \sum_{n=2}^{\infty} \int d\Phi_n \, \left|\mathcal{M}_{A\to 12\ldots n}\right|^2$$

(has dimension of mass)

Where

$$\int d\Phi_n = \int \prod_{i=1}^{n} \frac{d^4p_i}{(2\pi)^4} (2\pi)\, \delta(p_i^2 - m_i^2)\, \Theta(E_i)$$

$$\times (2\pi)^4 \delta^{(4)}\left(p_{CM} - p_1 - p_2 - \cdots - p_n\right)$$

$\Rightarrow$ $4n$ degrees of freedom (off-shell)

$-\ n$ on-shell conditions

$-\ 4$ energy momentum constraints.

$= 3n-4$ physical degrees of freedom.

n	2	3	4	5	6
d.o.f	2	5	8	11	14

etc.

+3 +3 +3 +3

So just by dimension counting:

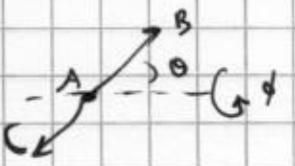
$$\Gamma = \frac{1}{2M} \sum_{n=2}^{\infty} \int d\Phi_n \, |\mathcal{M}_{A \to 12 \ldots n}|^2$$

mass dimension: -1 $\quad 2n-4 \quad$ $2 \cdot (4-n-1) \Rightarrow +1$

why not 3? b/c of $\delta(p^2 - m^2)$

Let's do a simple case of a $1 \to 2$ decay

$$A \to B\, C$$



$$p_A = (m_A, 0, 0, 0)$$

$$p_B = \left(\sqrt{m_B^2 + k^2},\ k\sin\theta\cos\phi,\ k\sin\theta\sin\phi,\ k\cos\theta\right)$$

$$p_C = \left(\sqrt{m_C^2 + k^2},\ \underbrace{-k\sin\theta\cos\phi,\ -k\sin\theta\sin\phi,\ -k\cos\theta}_{\text{automatically conserves 3-momentum}}\right)$$

Subject to constraint

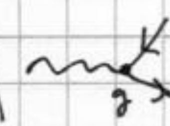
$$M_A = \sqrt{m_B^2 + k^2} + \sqrt{m_C^2 + k^2}$$

A good exercise to show:

$$k = \frac{m_A}{2}\sqrt{1-\frac{(m_B+m_C)^2}{m_A^2}}\sqrt{1-\frac{(m_B-m_C)^2}{m_A^2}}$$

$$\int d\Phi_2 = \int \frac{d\Omega}{(2\pi)^2}\cdot\frac{k}{4M_A} = \int \frac{d\Omega}{32\pi^2}\,\frac{2k}{m_A}$$

Note that $d\Phi_2 \to 0$ as $k \to 0$ ("no phase space for decay")

For a typical amplitude $|M|^2 = \Big|$  $\Big|^2$

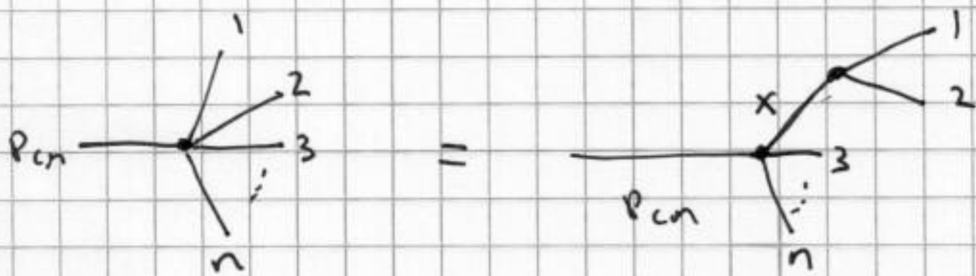
$$\simeq |g\cdot m_A|^2 \qquad (m_B \sim m_C \sim 0) \quad \left(\frac{2k}{m_A}\to 1\right)$$

$$\Gamma \sim \frac{g^2}{64\pi^2}\, m_A \underbrace{\int d\Omega}_{4\pi} \sim \frac{g^2}{16\pi}\, m_A$$

a typical (partial) width, have to add up multiple decay channels.

Finally, to close out this lecture, a very useful decomposition of n-body phase space.

$$d\Phi_n\left(p_{cm} \to p_1\, p_2 \cdots p_n\right) = d\Phi_{n-1}\left(p_{cm} \to p_x\; p_3 \cdots p_n\right) \times \frac{dm_x^2}{2\pi} \cdot d\Phi_2\left(p_x \to p_1\, p_2\right)$$



where m_x ranges over all allowed values.

This can be straightforwardly derived by inserting a dummy 4-vector with dummy mass

$$\underbrace{\frac{d^4 p_x}{(2\pi)^4}\,(2\pi)^4\,\delta^{(4)}\left(p_x - p_1 - p_2\right)}_{\text{Fancy way to write } 1.}\;\underbrace{\frac{dm_x^2}{(2\pi)}\,\underbrace{(2\pi)\,\delta\left(p_x^2 - m_x^2\right)\theta(E_x)}_{\text{putting dummy on-shell}}}_{\text{Fancy way to write } 1.}$$

This will be important to go from Feynman diagrams to cross sections.