

# The Energy Flow Basis

Jesse Thaler

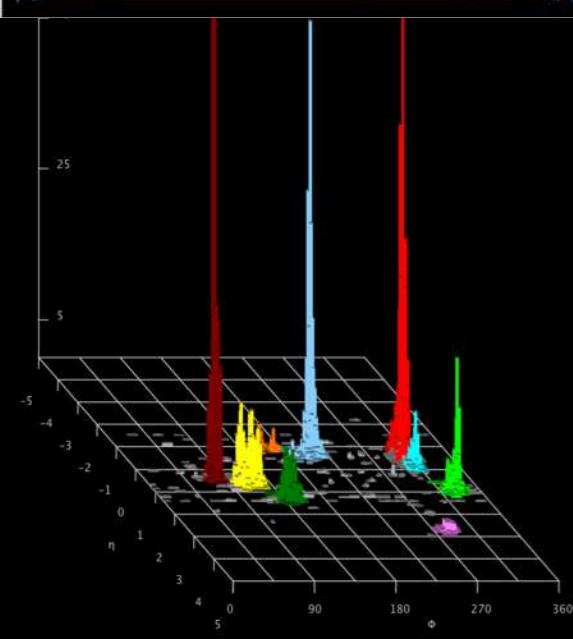
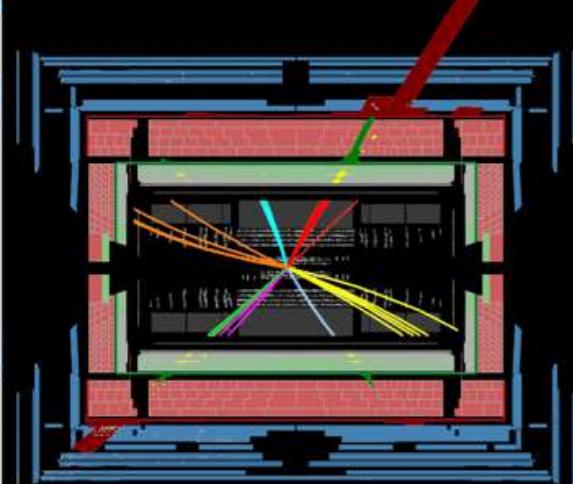


*with Patrick Komiske & Eric Metodiev (1712.07124, ...)*

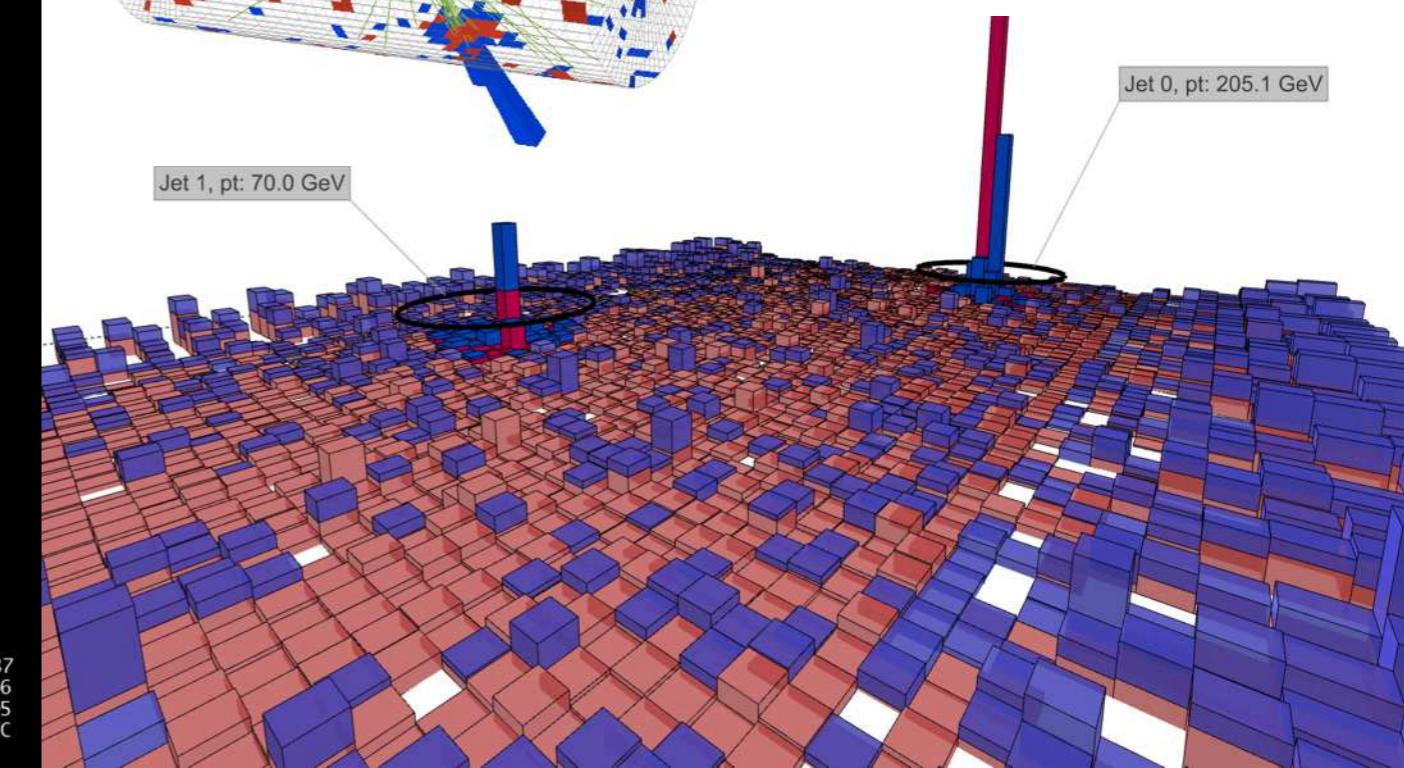
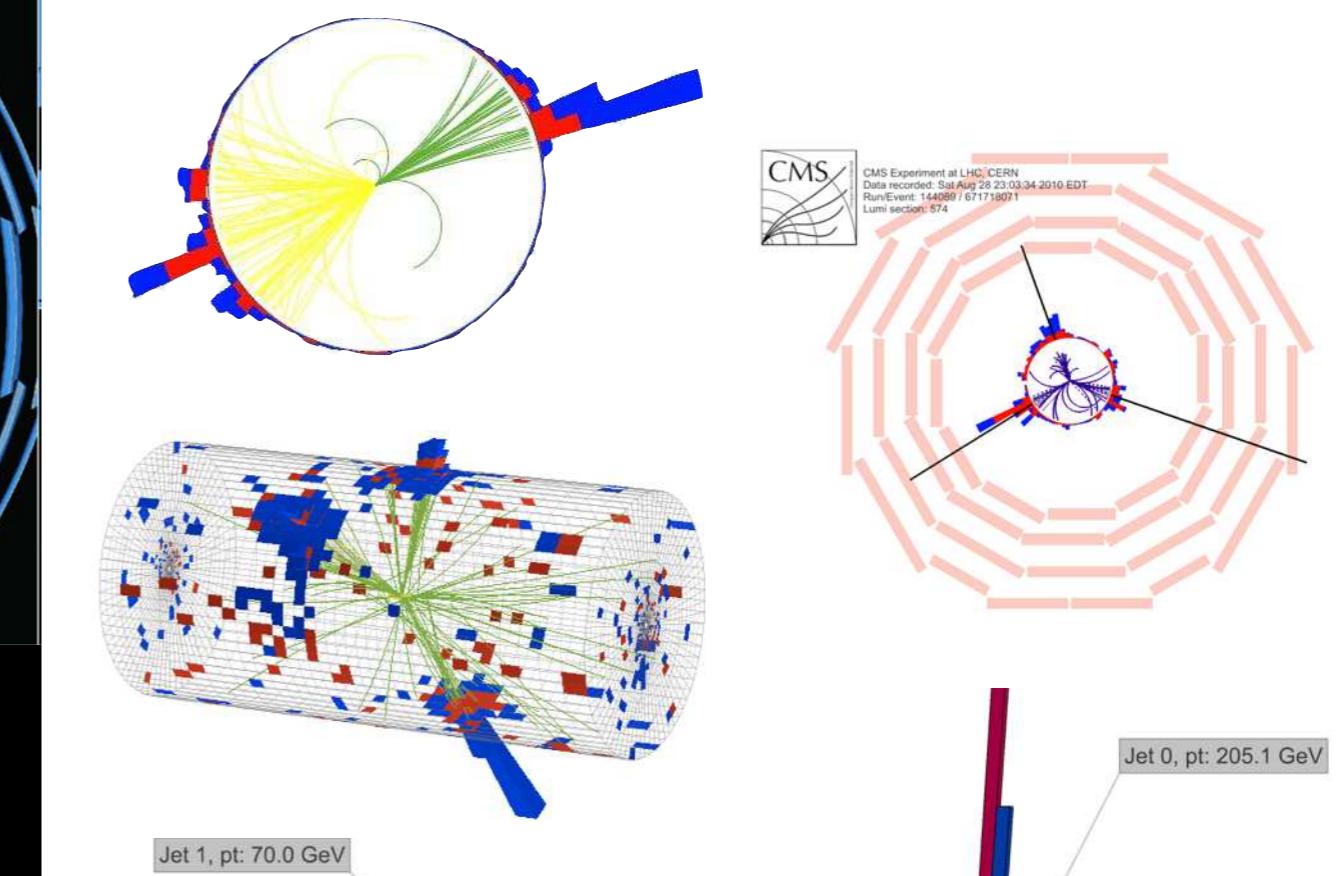
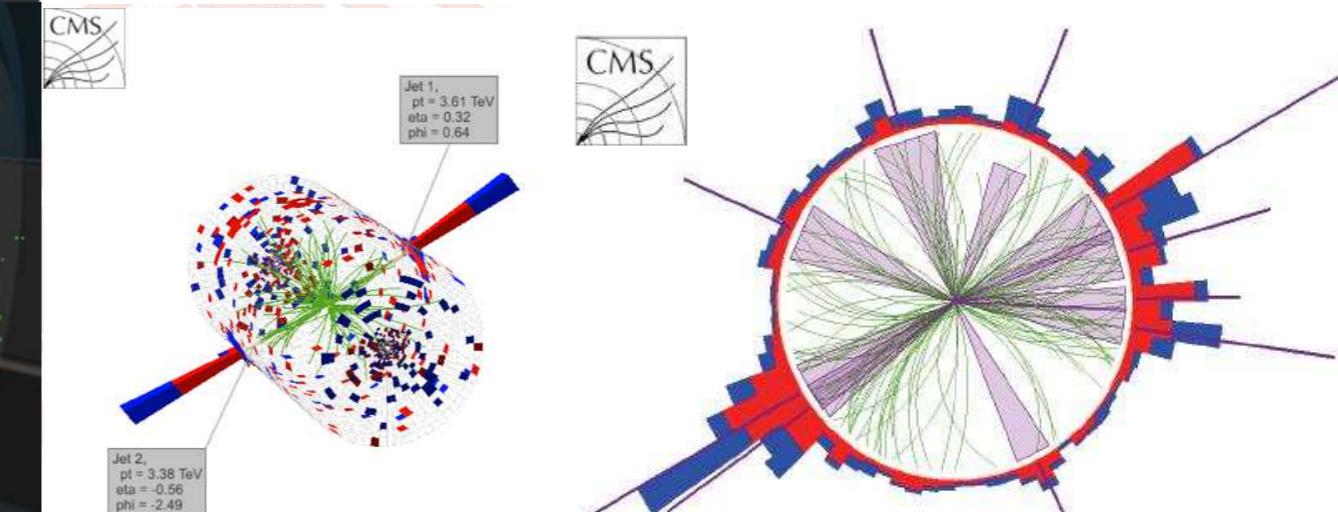
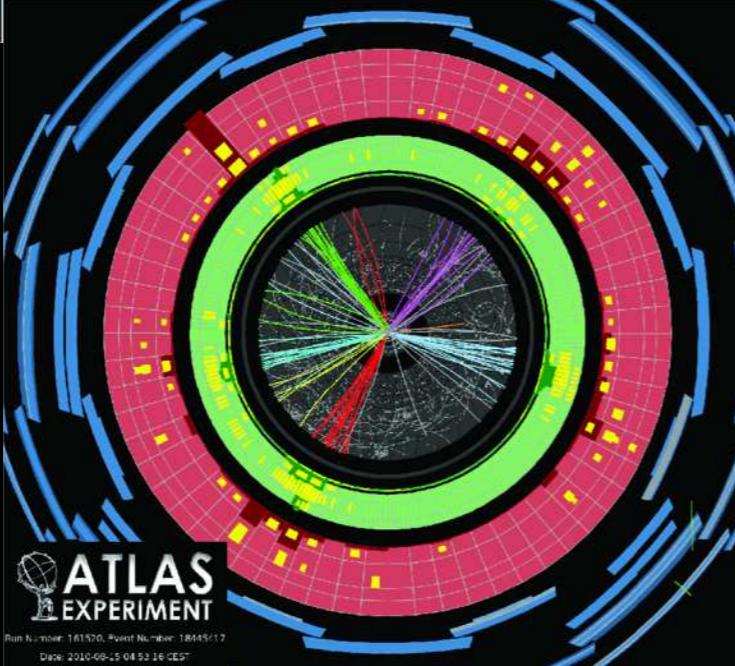
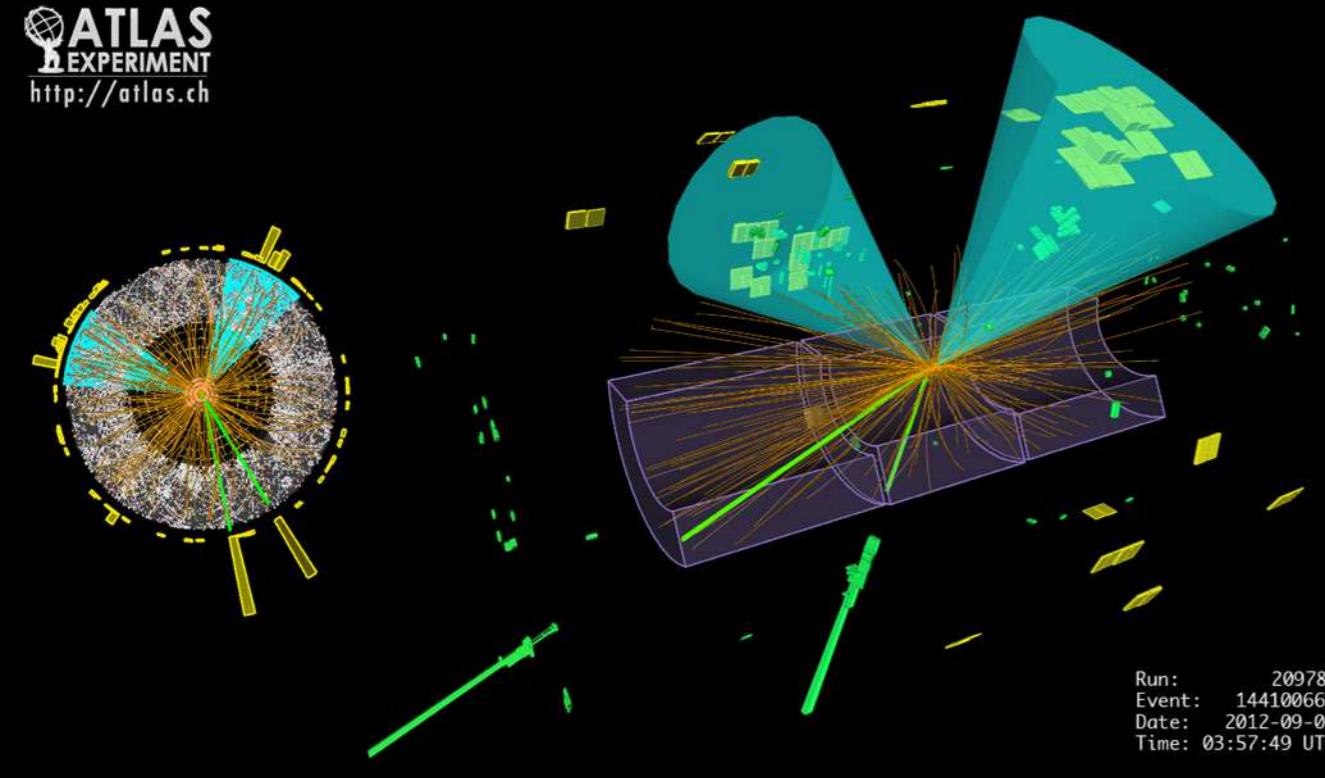
Reconstruction, Trigger, and Machine Learning for the HL-LHC, MIT — April 27, 2018

Run Number: 159224, Event Number: 3533152

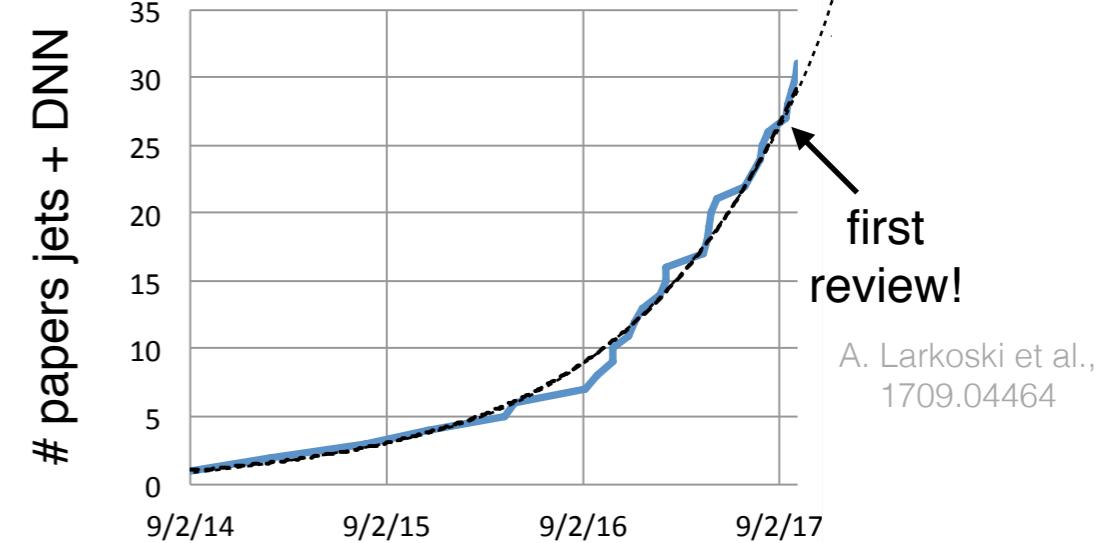
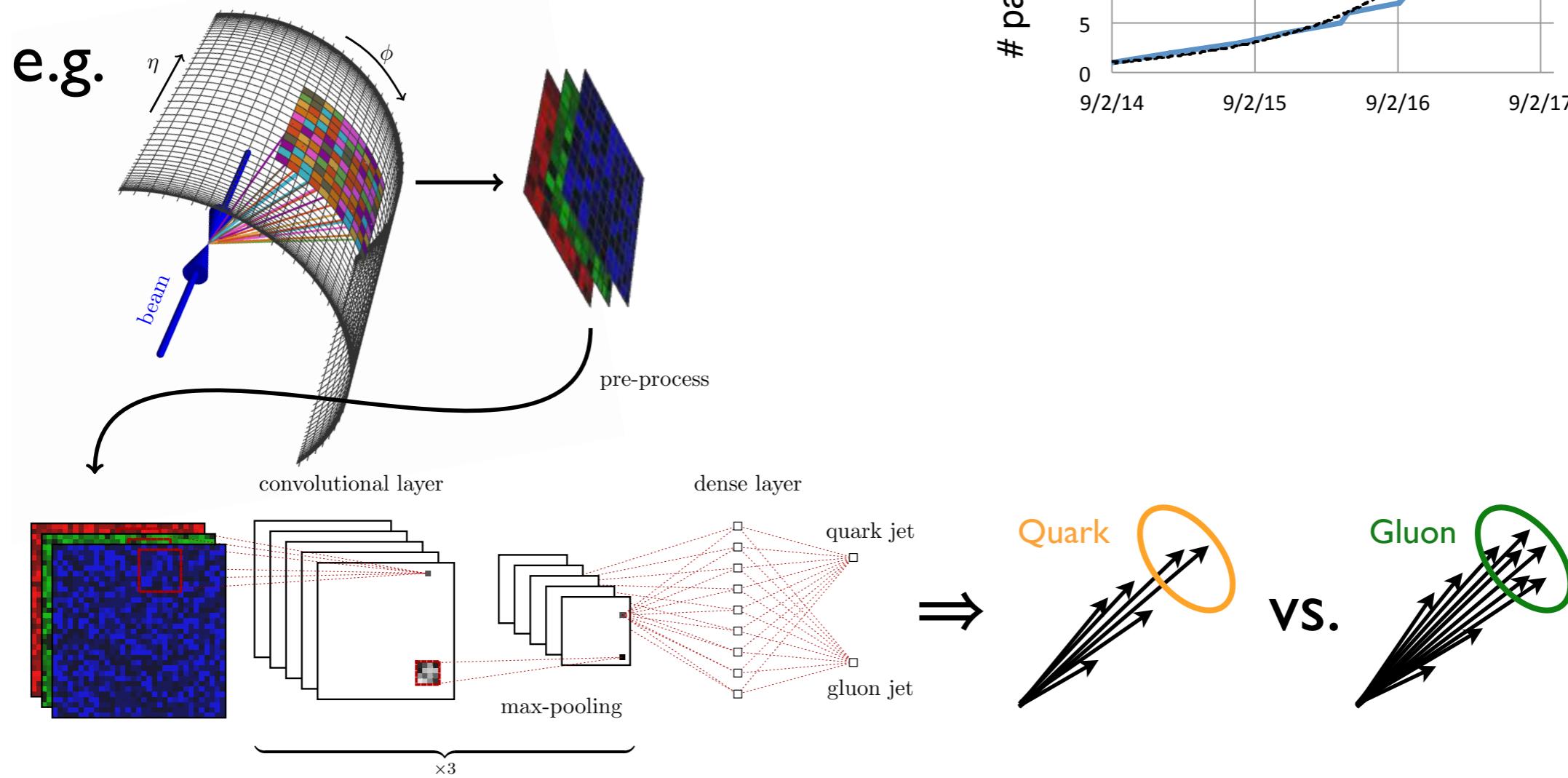
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**ATLAS**  
EXPERIMENT  
<http://atlas.ch>



# The Rise of Machine Learning for Jets



A. Larkoski et al.,  
1709.04464

[e.g. Komiske, Metodiev, Schwartz, 2016; Nachman, Machine Learning for Jets Workshop, 2017]

# My Perspective c. 2016

“Deep Learning”      vs.      “Deep Thinking”

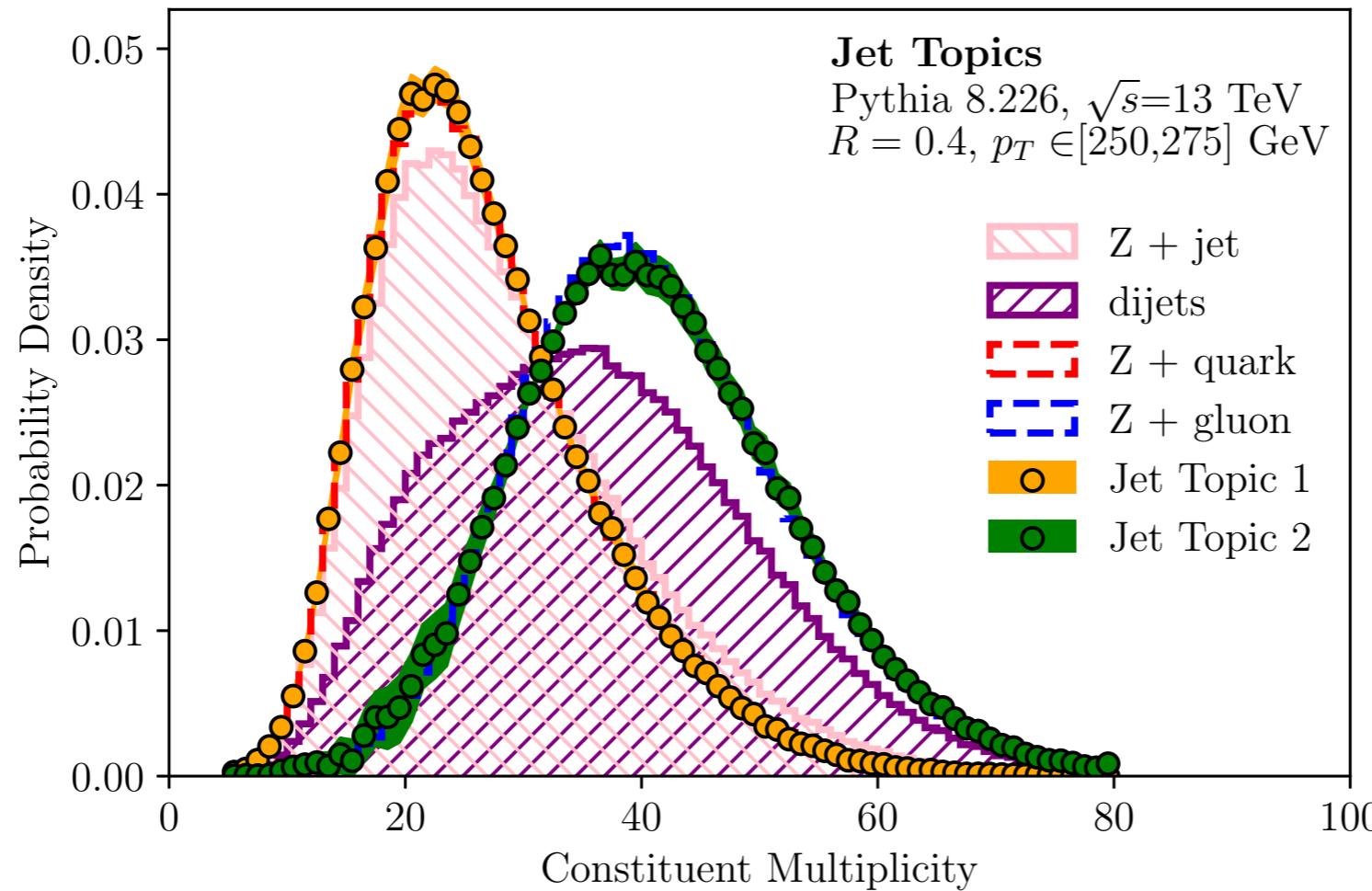
# My Perspective c. 2018

“Deep Learning”      &      ~~vs.~~      “Deep Thinking”

New first-principles studies of QCD  
facilitated by advances in  
statistics, mathematics, and computer science

# E.g. Jet Topics for Quark/Gluon Jets

Extract jet categories from data...



...solely\* from the assumption they exist

*Sample Independence, Different Fractions, Mutual Irreducibility*

[Metodiev, JDT, 1802.00008]

# Extrapolate Lessons to HL-LHC?

The goal of this workshop is to develop cross-experiment collaborations to work on common problems that will be faced by HL-LHC experiments. Beyond this, we hope to build collaborations between the HEP and the CS communities. **The irregular 3-D geometries of HEP detectors, their heterogeneity, and the extremely small latencies provide unique and interesting data science challenges.** Conversely, we have excellent models of how particles behave in our detectors, and excellent simulations of these detectors, which makes obtaining vast training samples possible. By building HEP-CS collaborations, we hope that HEP data sets can be used for cutting-edge ML research, providing benefits to both communities.

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Jets in Theory-land:  
**Regular geometry, homogeneous inputs, offline analysis**

# The Theory-Land Approach

Underlying Physics



“Deep Thinking”

Natural Data Representation



“Deep Learning”

Suitable Algorithm

# The Buzzword Approach

Questionable Physics



“Wishful Thinking”

Unnatural Data Representation



“Shoehorning”

Cool-Sounding Algorithm

# Logic for the Energy Flow Basis

Underlying Physics  
Natural Data Representation  
Suitable Algorithm

Infrared and Collinear Safety



Energy Flow Polynomials

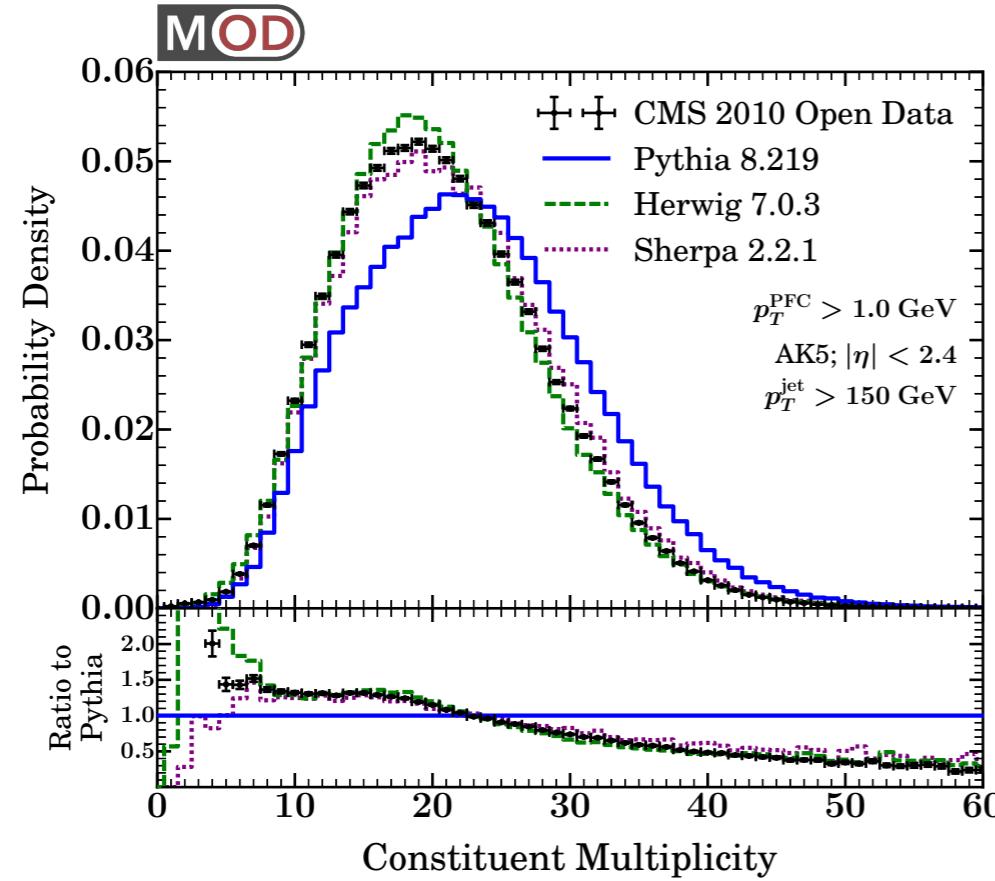


Linear Regression

[Komiske, Metodiev, JDT, 1712.07124]

# Key Fact #1

→ Underlying Physics  
Natural Data Representation  
Suitable Algorithm



Jet constituents:  
*Particle-like objects*  
*Variable-length*  
*Unordered set*

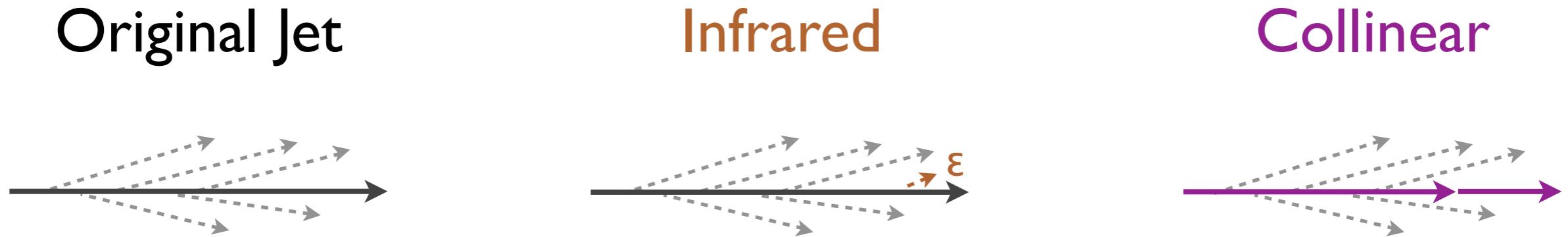
Per particle:  
 $\{E, p_x, p_y, p_z\}$  or  $\{p_T, \gamma, \Phi, m\}$   
*Flavor/charge labels*  
*Vertex information*  
*Quality criteria, etc.*

[plot from Tripathee, Xue, Larkoski, Marzani, JDT, 1704.05842]

# Key Fact #2

- Underlying Physics
- Natural Data Representation
- Suitable Algorithm

Wide range of interesting observables are “safe”



**IRC Safe Observable:** Insensitive to IR or C emissions

Enforces smooth interpolation between  
variable-length inputs (i.e.  $N \rightarrow N-1$ )

# Examples from Jet Substructure

→ Underlying Physics  
 Natural Data Representation  
 Suitable Algorithm

Jet pt:  $\sum_{i \in \text{jet}} p_{T,i}$  **IRC Safe**

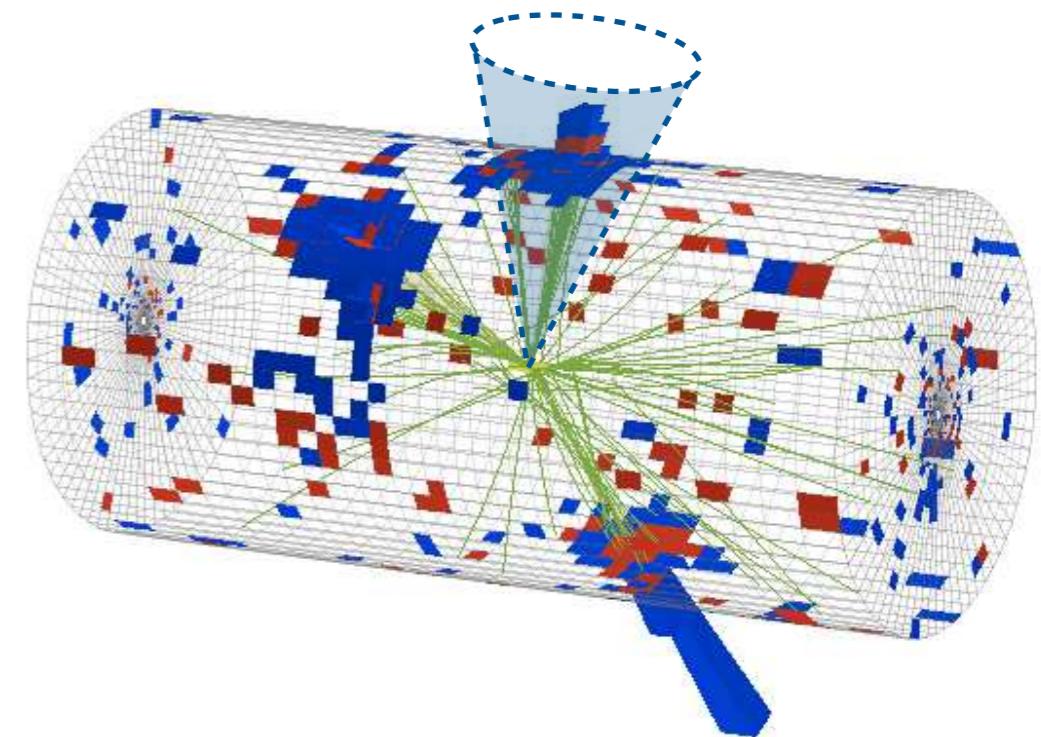
$p_T^D$ :  
[CMS HIG-11-027]  $\sum_{i \in \text{jet}} \frac{p_{T,i}^2}{p_{T\text{jet}}^2}$  **IR Safe**  
**C Unsafe**

Multiplicity:  $\sum_{i \in \text{jet}} 1$  **IRC Unsafe**

Jet Mass:  $\sum_{i,j \in \text{jet}} p_i \cdot p_j$  **IRC Safe**

N-subjettiness:  $\sum_{i \in \text{jet}} p_{T,i} \min \{ \Delta R_{i,1}, \Delta R_{i,2}, \dots, \Delta R_{i,N} \}^\beta$  **IRC Safe**

[JDT, Van Tilburg, 1011.2268, 1108.2701]



- Underlying Physics
- Natural Data Representation
- Suitable Algorithm

What is the space of *all*  
IRC-safe observables?

# A Systematic Expansion

Underlying Physics  
→ Natural Data Representation  
Suitable Algorithm

Expand\* any IRC safe observable in small energy limit

$$\begin{aligned} \mathcal{S} = & \sum_i E_i f_1^{\mathcal{S}}(\hat{n}_i) + \sum_{ij} E_i E_j f_2^{\mathcal{S}}(\hat{n}_i, \hat{n}_j) \\ & + \sum_{ijk} E_i E_j E_k f_3^{\mathcal{S}}(\hat{n}_i, \hat{n}_j, \hat{n}_k) + \dots \end{aligned}$$

Form enforced by:	Particle Relabeling	Infrared Safety	Collinear Safety
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Further expand\* each angular function in pairwise angles

$$z_i = \frac{E_i}{E_{\text{jet}}} \quad \cos \theta_{ij} = \hat{n}_i \cdot \hat{n}_j$$

[Komiske, Metodiev, JDT, 1712.07124; see also Tkachov, hep-ph/9601308]

# The Energy Flow Polynomials

- Underlying Physics
- Natural Data Representation
- Suitable Algorithm

$$\text{EFP}_G = \sum_{i_1=1}^M \cdots \sum_{i_N=1}^M z_{i_1} \cdots z_{i_N} \prod_{(k,\ell) \in G} \theta_{i_k i_\ell}^\beta$$

↓ Multigraph  
N Energy Fractions  
Polynomial in Pairwise Angles

# A Linear Basis for Jet Substructure (!)

# The Energy Flow Polynomials

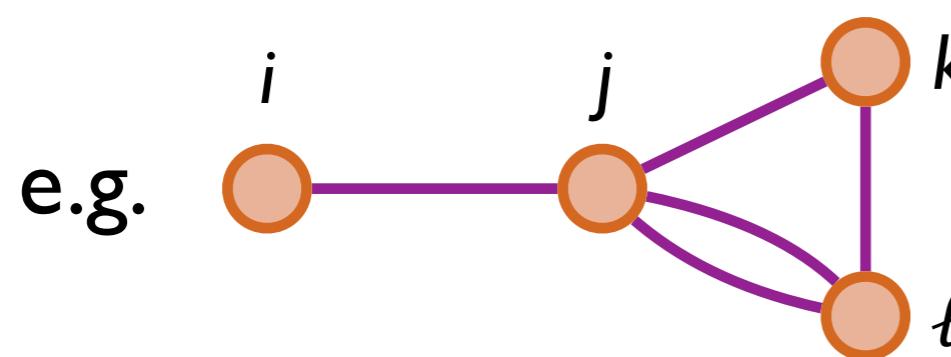
Underlying Physics  
 → Natural Data Representation  
 Suitable Algorithm

$$\text{EFP}_G = \sum_{i_1=1}^M \cdots \sum_{i_N=1}^M z_{i_1} \cdots z_{i_N} \prod_{(k,\ell) \in G} \theta_{i_k i_\ell}$$

*Multigraph*
*Angular Scaling*

All N-tuples
N Energy Fractions
Polynomial in Pairwise Angles

e.g.



$$= \sum_{ijkl} z_i z_j z_k z_l \theta_{ij} \theta_{jk} \theta_{jl}^2 \theta_{kl}$$

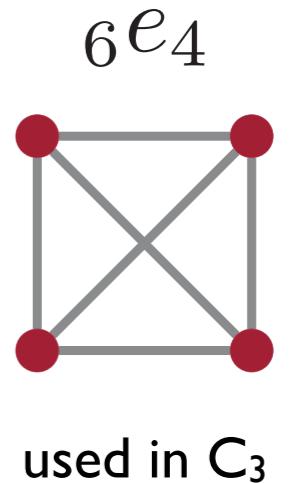
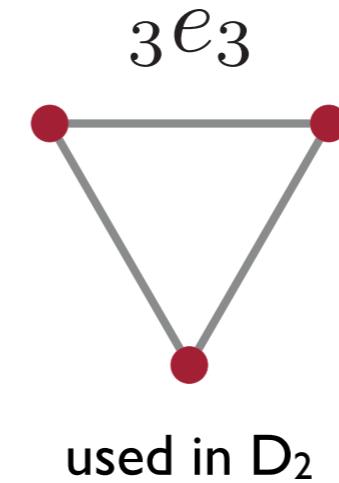
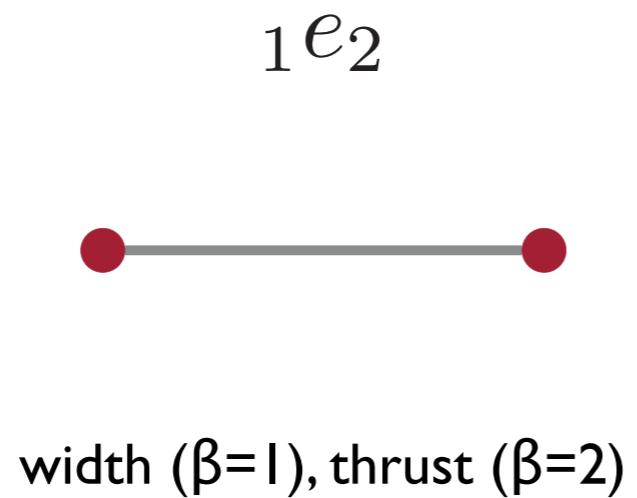
A Linear Basis for Jet Substructure (!)

[Komiske, Metodiev, JDT, 1712.07124]

# Down the Rabbit Hole

Underlying Physics  
→ Natural Data Representation  
Suitable Algorithm

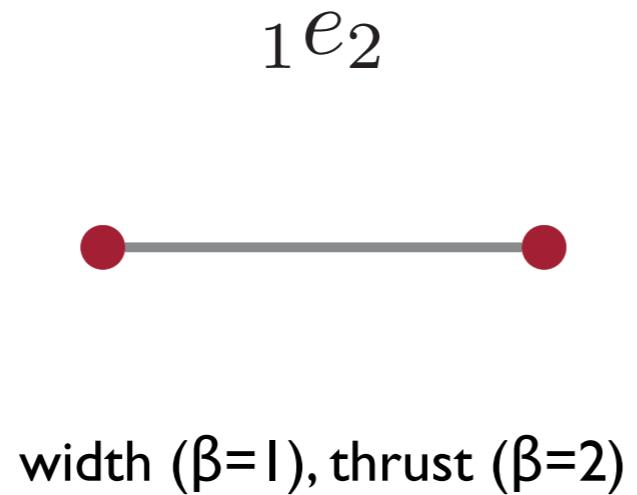
Known Structures:



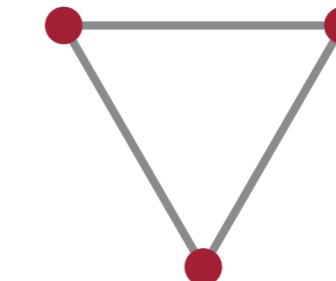
# Down the Rabbit Hole

Underlying Physics  
→ Natural Data Representation  
Suitable Algorithm

Known Structures:

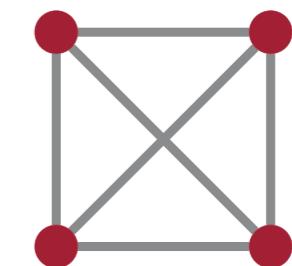


$3e_3$



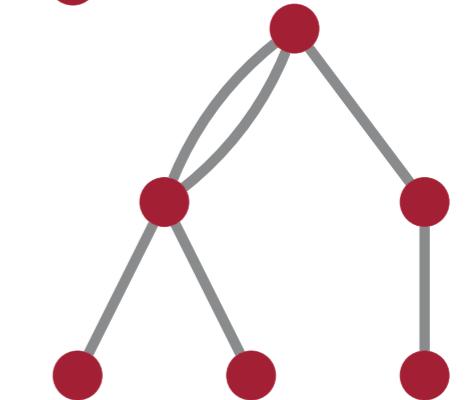
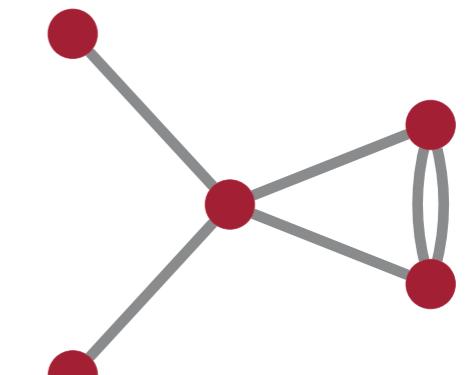
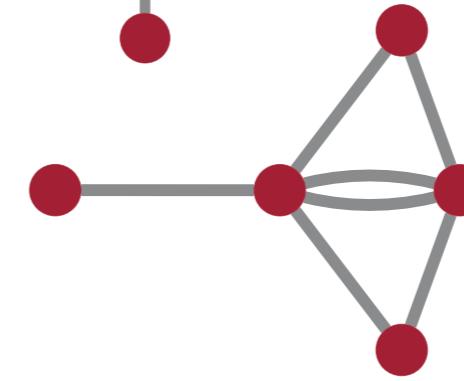
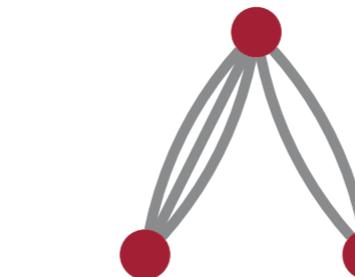
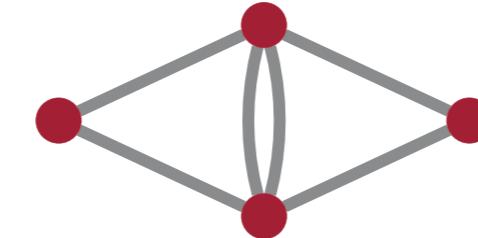
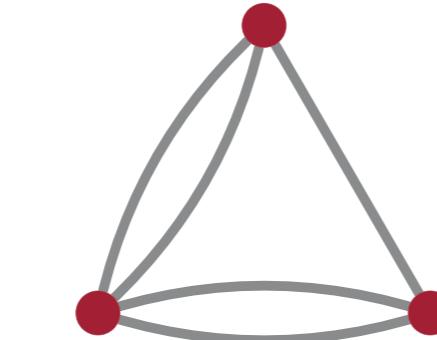
used in  $D_2$

$6e_4$



used in  $C_3$

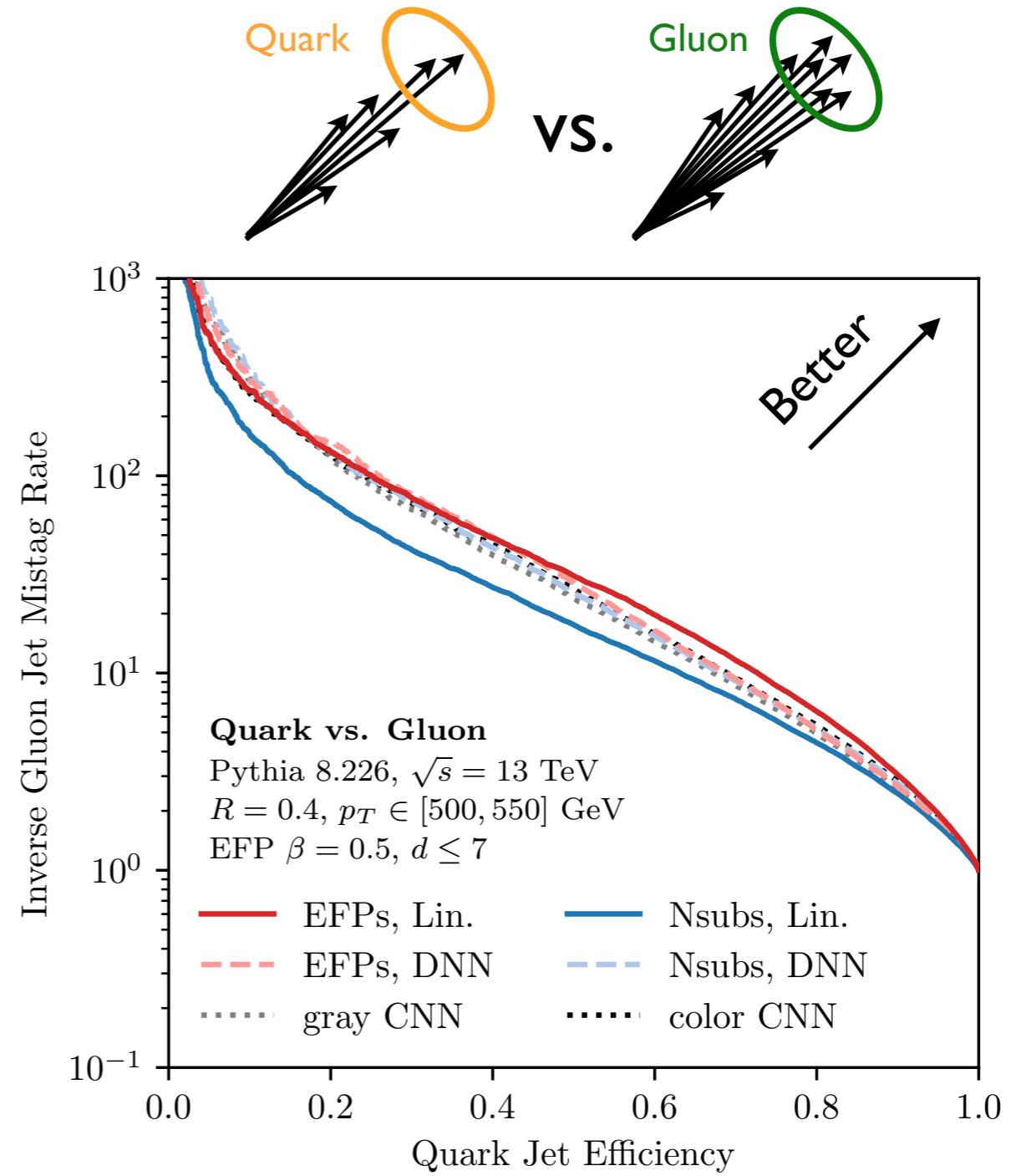
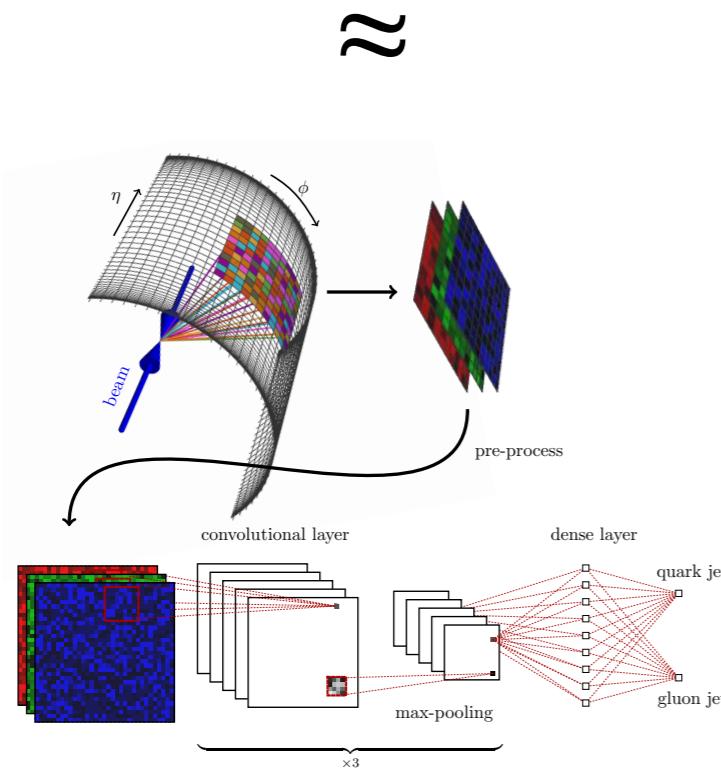
No Idea:



# Linear Regression vs. CNN

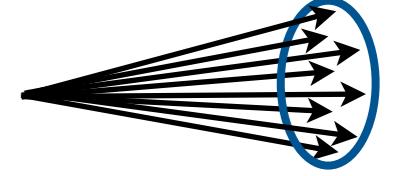
Underlying Physics  
 Natural Data Representation  
 → Suitable Algorithm

$$\mathcal{S} = \sum_G s_G \text{EFP}_G$$



[Komiske, Metodiev, JDT, 1712.07124; Komiske, Metodiev, Schwartz, 1612.01551]

# Comparing Data Representations



Original 4-Vectors:  $\{p_1^\mu, p_2^\mu, \dots, p_N^\mu\}$

*Variable-length, unordered set*

EFP Basis:  $\text{EFP}_G = \sum_{i_1=1}^M \cdots \sum_{i_N=1}^M z_{i_1} \cdots z_{i_N} \prod_{(k,\ell) \in G} \theta_{i_k i_\ell}$

*Automatically permutation invariant  
Linear spanning basis (over-complete)  
Computational nightmare of  $O(M^N)$ ?*



Too naive, can use  
variable elimination

# The Theory-Meets-Computation Approach

Underlying Physics



Natural & Efficient Data Representation



Desired Computational Property

- Underlying Physics
- Natural Data Representation
- Suitable Algorithm

What is the set of *all*  
linearly-computable  
permutation-invariant  
IRC-safe structures?

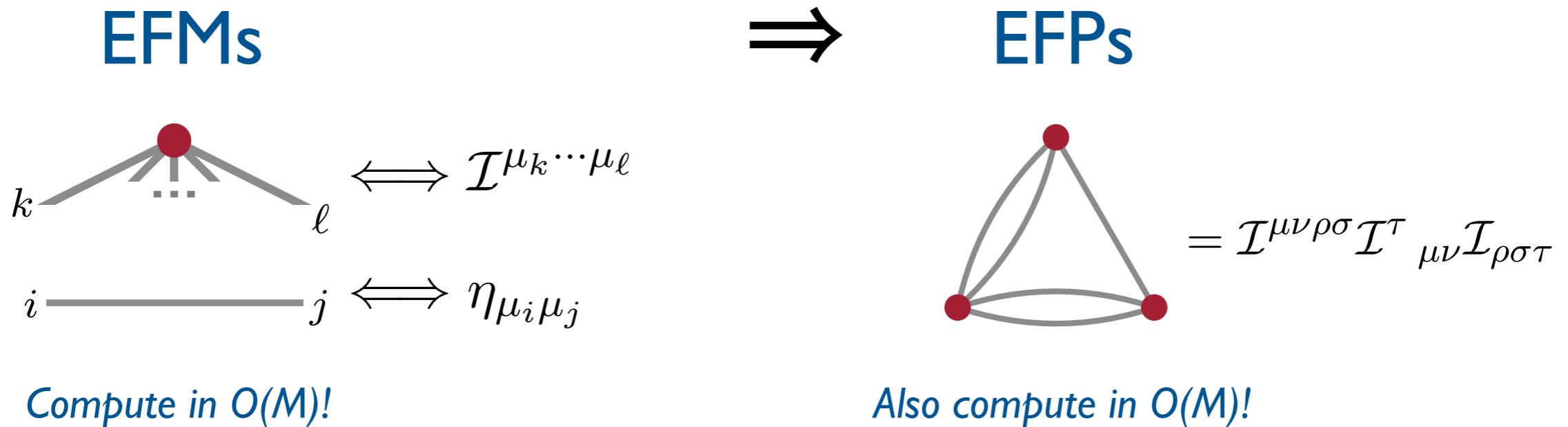
# The Energy Flow Moments

Underlying Physics  
 → Natural Data Representation  
 Suitable Algorithm

$$\mathcal{I}^{\mu_1 \mu_2 \cdots \mu_v} = \sum_{i=1}^M E_i \hat{p}^{\mu_1} \hat{p}^{\mu_2} \cdots \hat{p}^{\mu_v}$$

Particle  
Relabeling      Infrared  
Safety

Special Choice  
of Angle       $\theta_{ij} = 2 \eta_{\mu\nu} \hat{p}_i^\mu \hat{p}_j^\nu$



[Komiske, Metodiev, JDT, to appear]

The screenshot shows a web browser window with the title "Home - EnergyFlow". The URL in the address bar is <https://pkomiske.github.io/EnergyFlow/>. The page content is the "Welcome to EnergyFlow" documentation.

The left sidebar contains a navigation menu with the following items:

- Home
- Welcome to EnergyFlow
- References
- Getting Started
- Installation
- Tutorial
- FAQ
- Documentation
- Measures
- Generation
- Energy Flow Polynomials
- Energy Flow Moments
- Utils

At the bottom of the sidebar are links for "GitHub" and "Next »".

The main content area has a red header with the EnergyFlow logo (a diamond shape with internal lines) and the text "EnergyFlow". Below the header is a search bar labeled "Search docs".

The breadcrumb navigation shows "Docs » Home".

## Welcome to EnergyFlow

EnergyFlow is a Python package for computing Energy Flow Polynomials (EFPs), a collection of jet substructure observables which form a complete, linear basis of IRC-safe observables. The source code can be found on [GitHub](#).

**Note:** As of version [0.7.0](#), all EFP code has been thoroughly tested. EFM code is still under development.

## References

[1] P.T. Komiske, E.M. Metodiev, and J. Thaler, "Energy Flow Polynomials: A complete linear basis for jet substructure." [\[1712.07124\]](#).

[Next ➔](#)

Built with [MkDocs](#) using a theme provided by [Read the Docs](#).

# Punchlines

Underlying Physics  
Natural Data Representation  
Suitable Algorithm

## Jet Constituents:

*Variable-length, unordered set of particle-like objects*

## Energy Flow Polynomials:

*Linear spanning basis for IRC-safe jet observables*

## Energy Flow Moments:

*Linearly computable building blocks for EFPs*

## Linear Regression:

*If it works, use it*

## Deep Learning or Deep Thinking ?

*Both*