The Geometry of Particle Collisions: Hidden in Plain Sight

Jesse Thaler

Physics Colloquium, Brandeis University — February 8, 2022

The Geometry of Particle Collisions: Hidden in “Plane” Sight?

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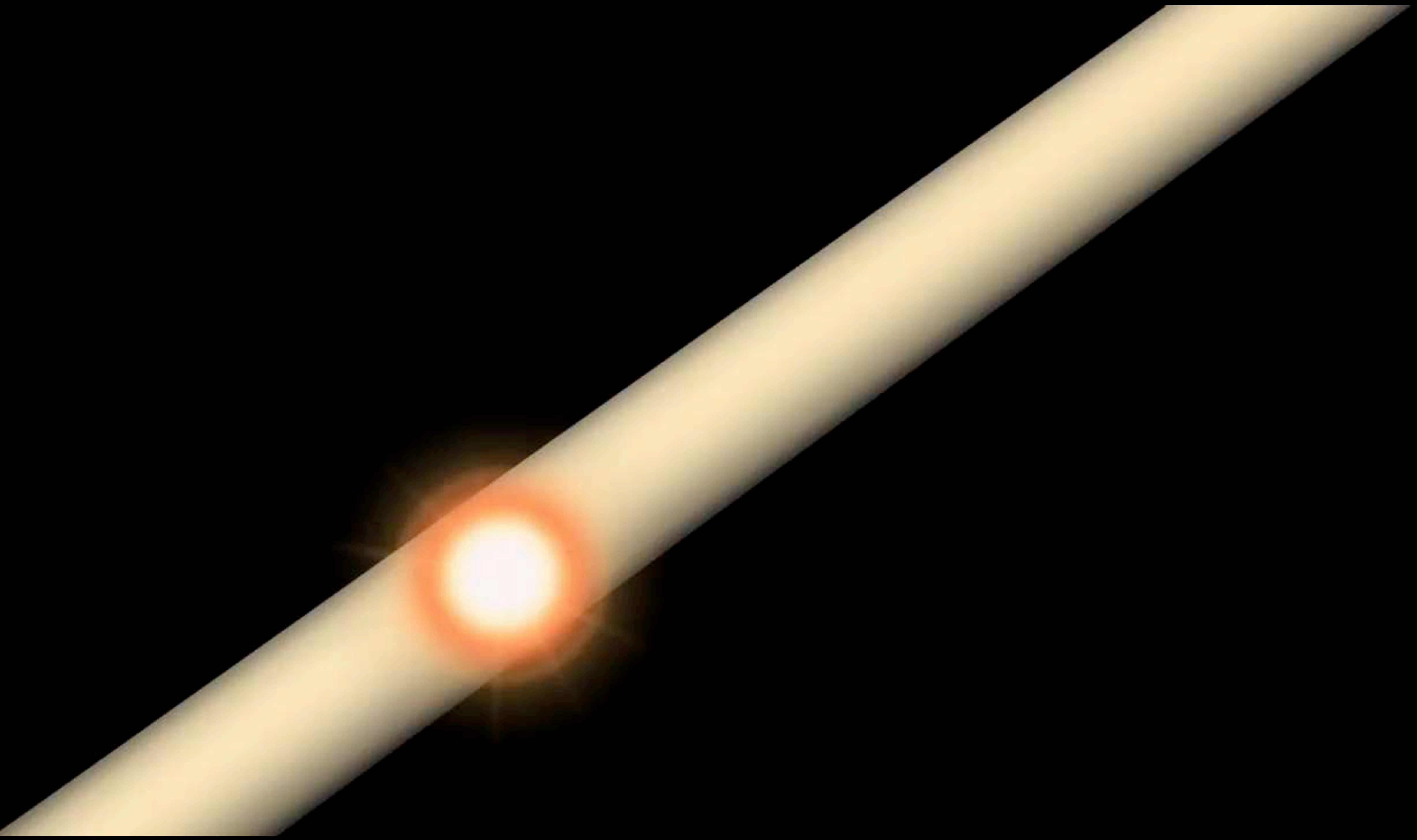
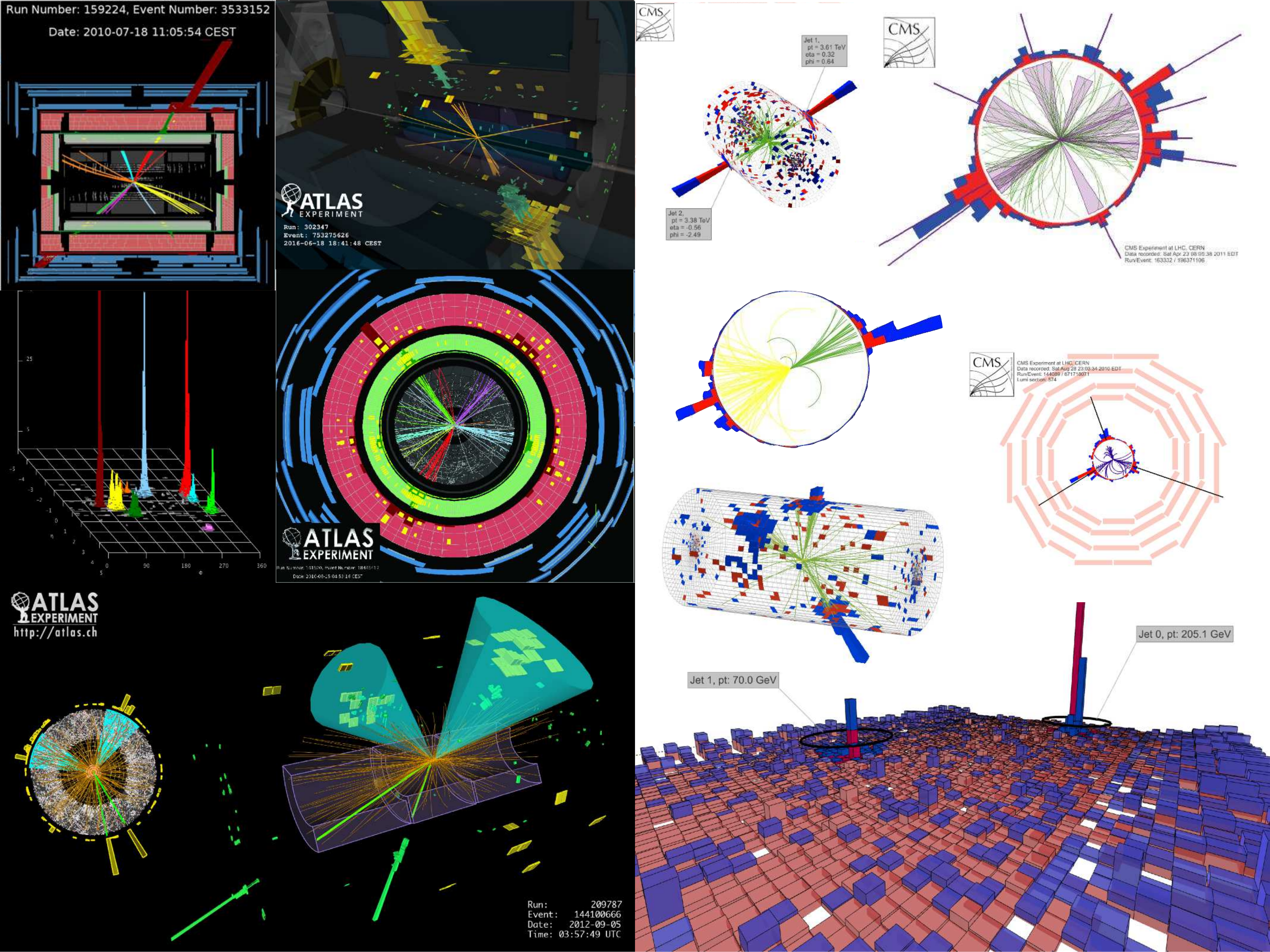



"Jets" initiated by
quarks & gluons in QCD



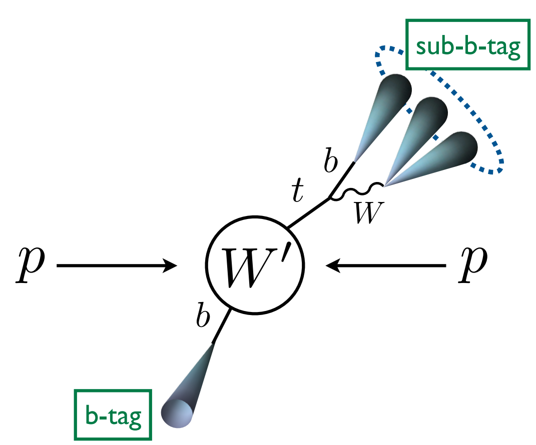
Run Number: 159224, Event Number: 3533152
Date: 2010-07-18 11:05:54 CEST
ATLAS
EXPERIMENT
Run: 302347
Event: 753275626
2016-06-18 18:41:48 CEST
CMS
Jet 1,
pt = 3.61 TeV
eta = 0.32
phi = 0.64
Jet 2,
pt = 3.38 TeV
eta = -0.56
phi = -2.49
CMS
ATLAS
EXPERIMENT
CMS
ATLAS
EXPERIMENT
http://atlas.ch
Run: 209787
Event: 144100666
Date: 2012-09-05
Time: 03:57:49 UTC
Jet 0, pt: 205.1 GeV
Jet 1, pt: 70.0 GeV

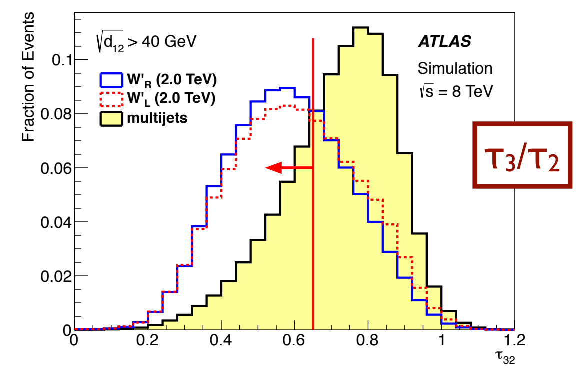
2015 Brandeis Colloquium

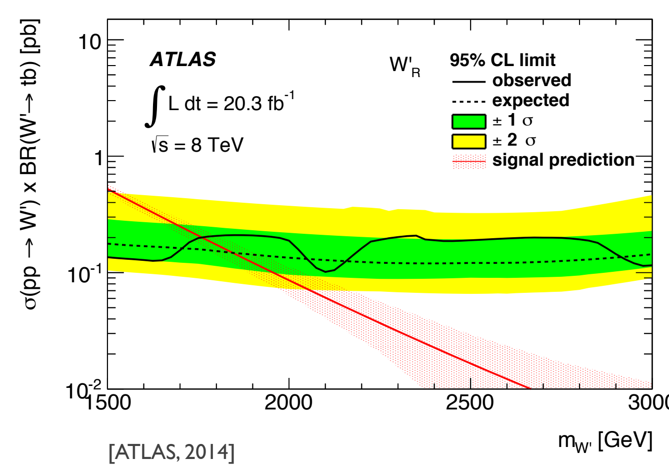
The Rise of Jet Substructure in Searching for New Phenomena at Colliders

ATLAS Search for Heavy W Bosons

Trimming + *B-tagging*
+ *N-subjettiness*







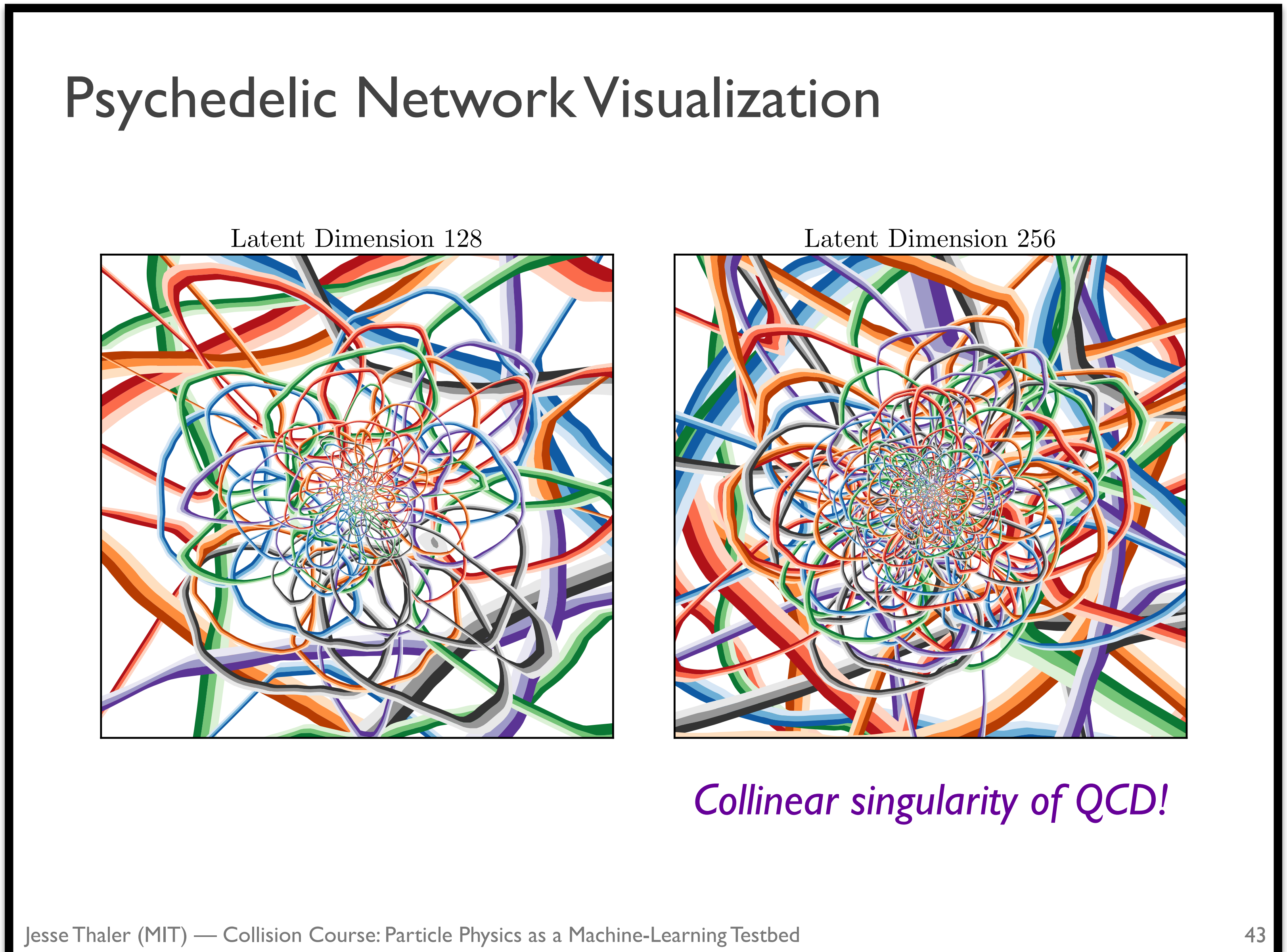
[ATLAS, 2014]

Jesse Thaler — The Rise of Jet Substructure 29

“Why is N-subjettiness the preferred analysis strategy?”

2019 Brandeis Seminar

The Rise of Machine Learning for Deciphering Collider Data



"Can we ever hope to understand what the machine is learning?"

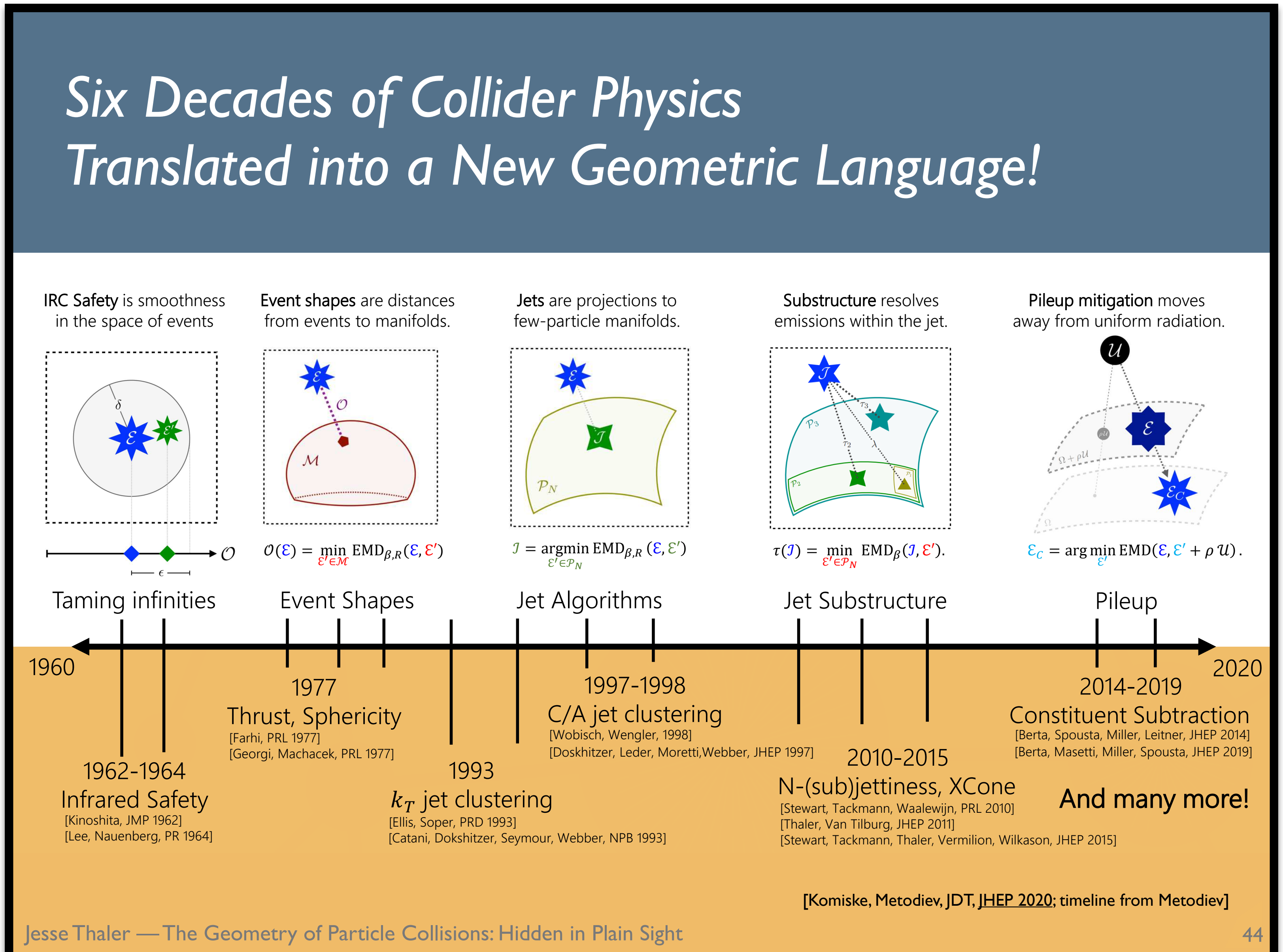
The NSF AI Institute for Artificial Intelligence and Fundamental Interactions (IAIFI) *"eye-phi"*

Advance physics knowledge — from the smallest building blocks of nature to the largest structures in the universe — and galvanize AI research innovation

[http://iaifi.org, MIT News Announcement]

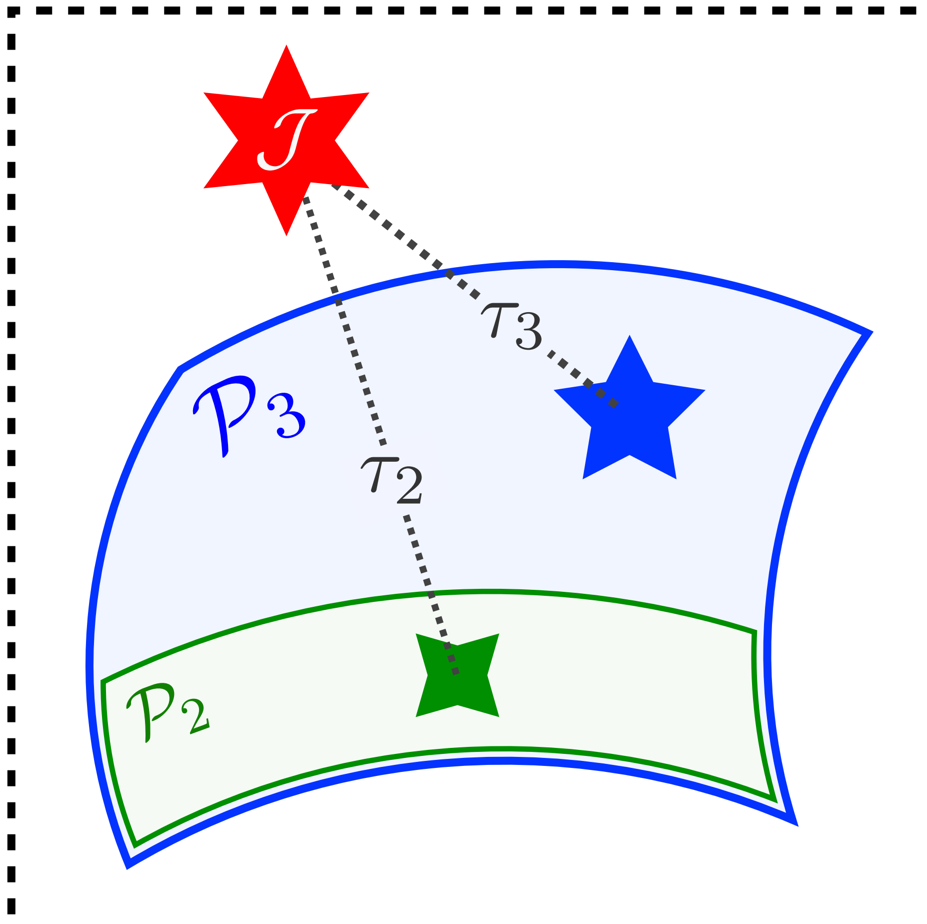
2022 Brandeis Colloquium

The more things change, the more they stay the same…



Physics intelligence translated into language of artificial intelligence

Today's Punchline



N-subjettiness is a natural "geometric" object defined on the "space" of collider events

Suggests exciting applications of optimal transport theory for the LHC and beyond

Optimal transport for new sampling algorithms and efficient generative models

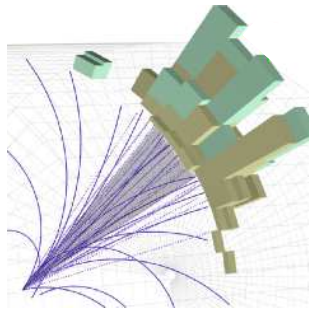
January 25, 2022

Tyler Maunu, Brandeis University

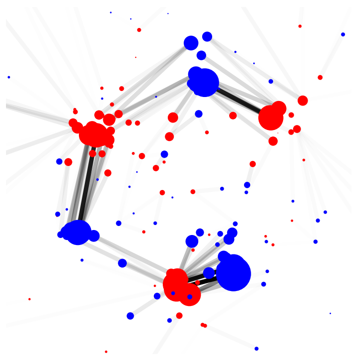
Host: Aram Apyan

Coincidence?
I think not.

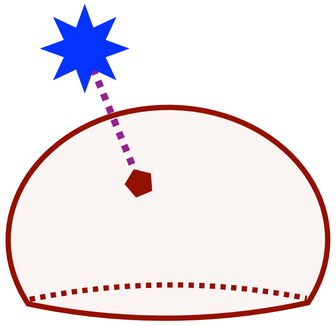
Outline



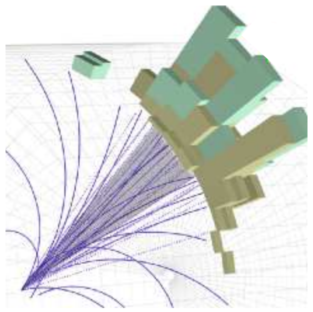
Going with the (Energy) Flow



The Energy Mover's Distance



Revealing a Hidden Geometry



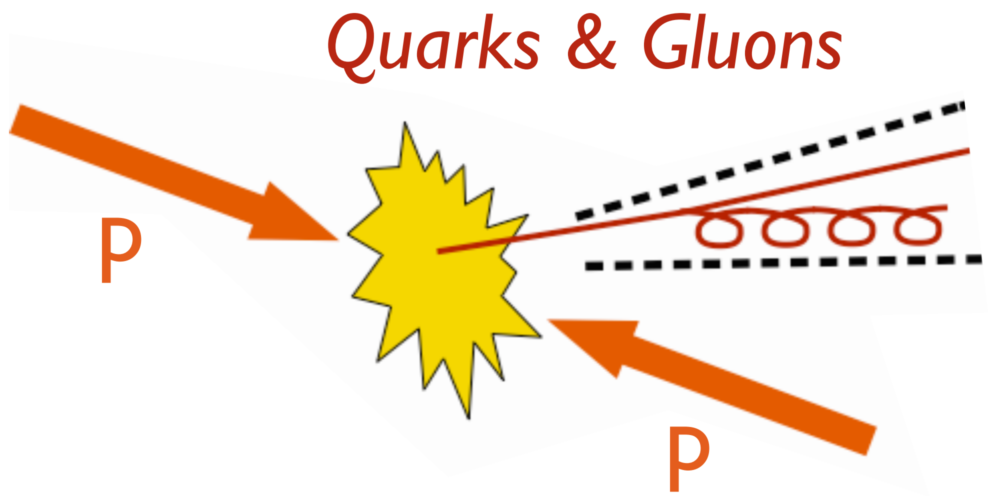
Going with the (Energy) Flow

The Energy Mover's Distance

Revealing a Hidden Geometry

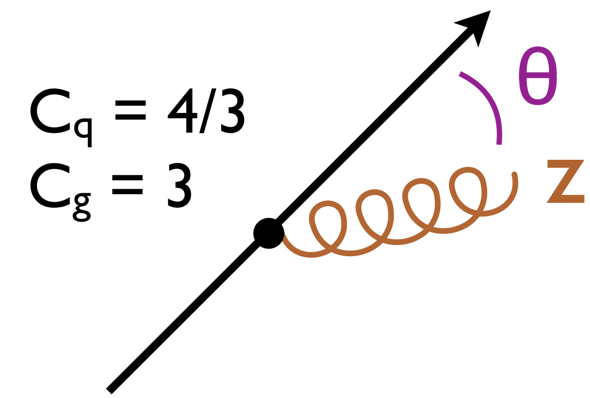
Introducing Energy Flow

Respects infrared and collinear safety



Altarelli-Parisi Splitting

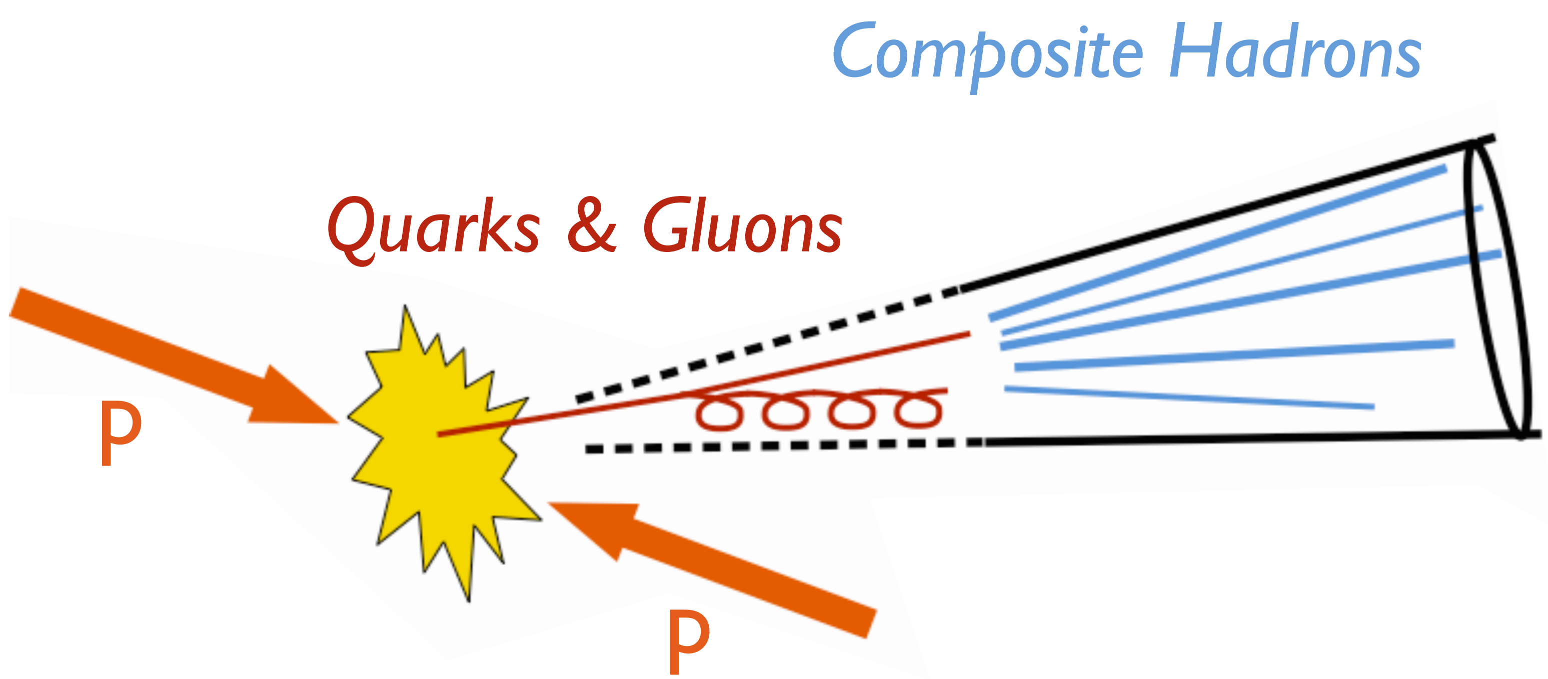
Core prediction of QCD



$$\mathrm{d}P_{i\to ig} \simeq \frac{2\alpha_s}{\pi} C_i \underbrace{\frac{\mathrm{d}\theta}{\theta}}_{\text{Collinear}} \underbrace{\frac{\mathrm{d}z}{z}}_{\text{Soft}}$$

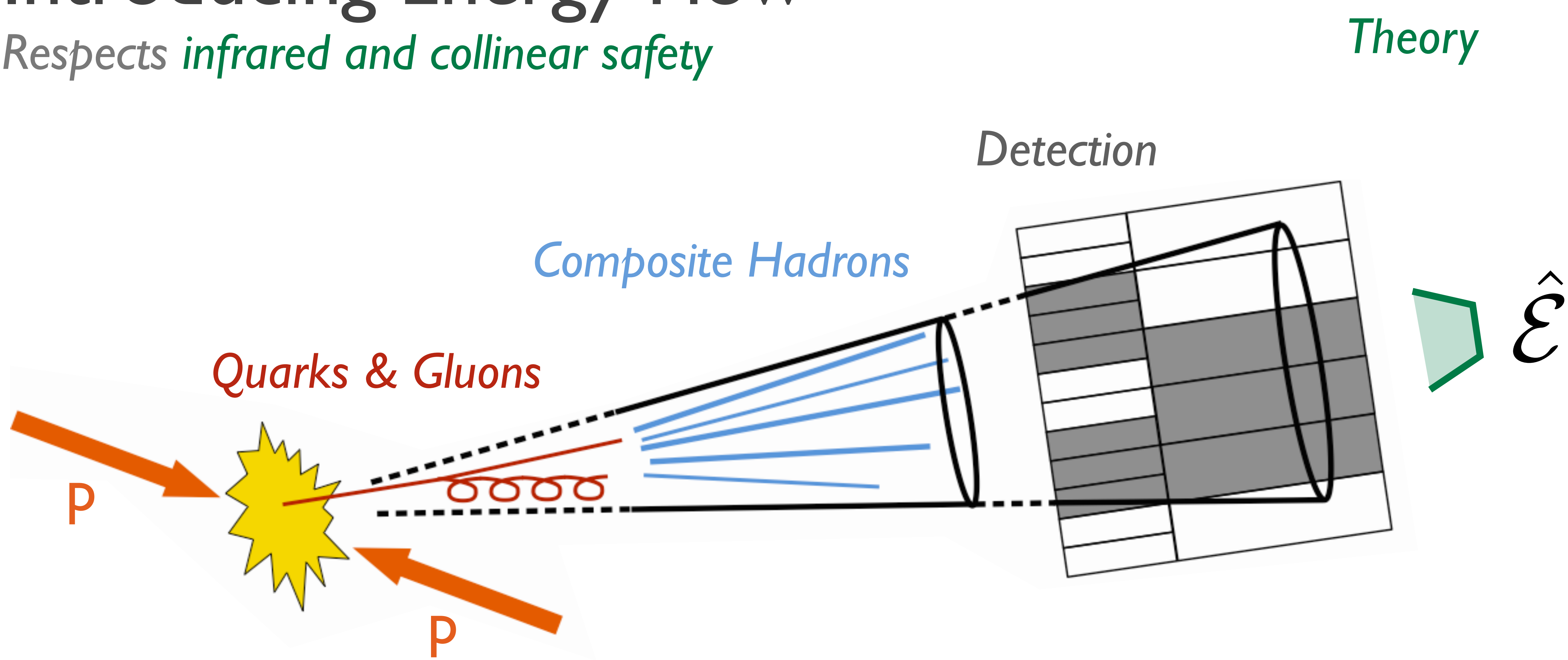
Introducing Energy Flow

Respects ***infrared and collinear safety***



Introducing Energy Flow

Respects infrared and collinear safety



Energy Flow:

Robust to hadronization and detector effects

$$\boxed{\hat{\mathcal{E}} \simeq \lim_{t\to\infty} \hat{n}_i T^{0i}(t, vt\hat{n})}$$

[see e.g. Sveshnikov, Tkachov, PLB 1996; Hofman, Maldacena, JHEP 2008; Mateu, Stewart, JDT, PRD 2013; Belitsky, Hohenegger, Korchemsky, Sokatchev, Zhiboedov, PRL 2014; Chen, Moult, Zhang, Zhu, PRD 2020]

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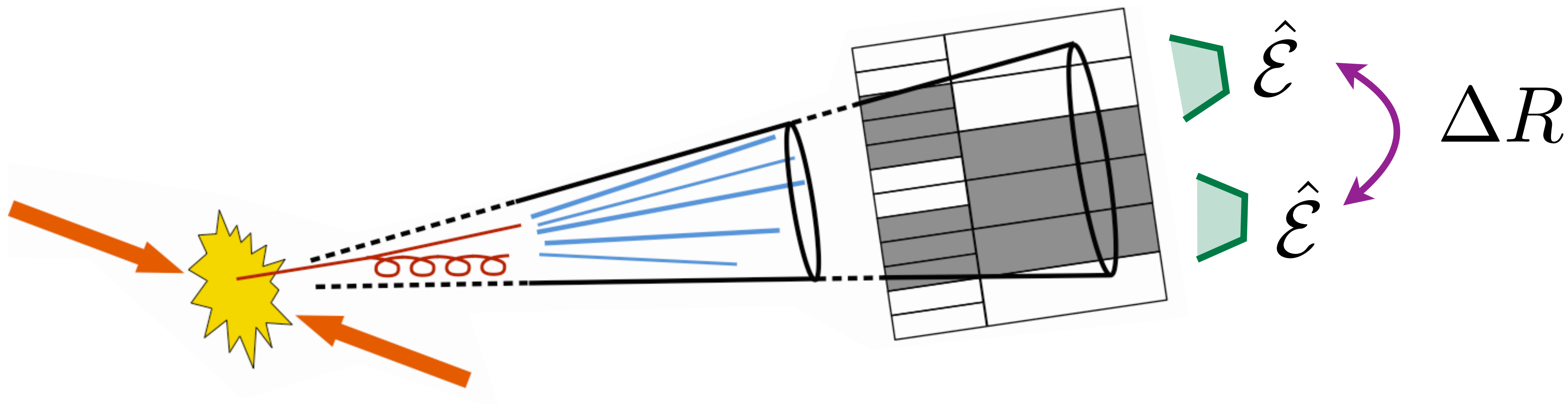
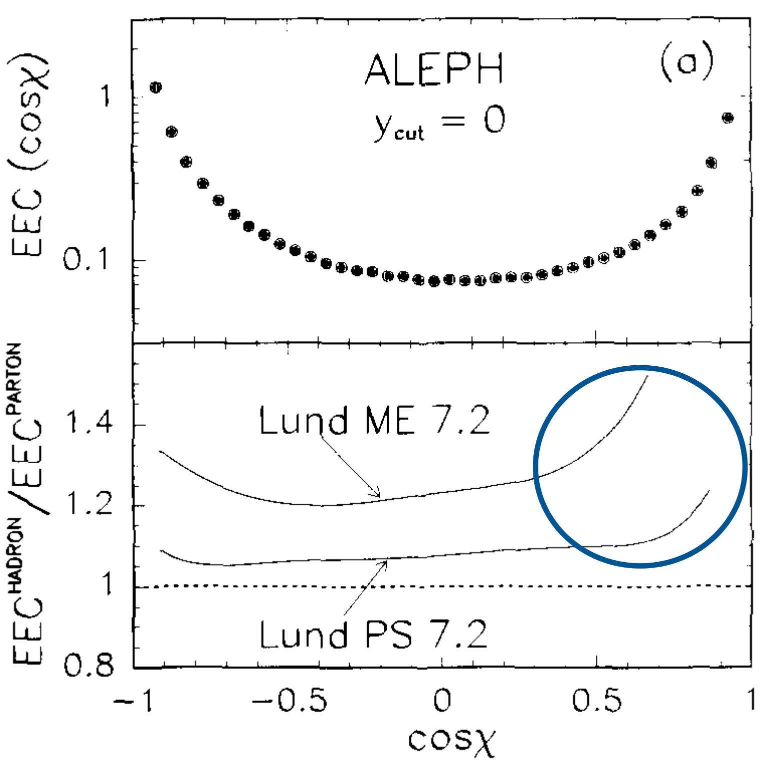
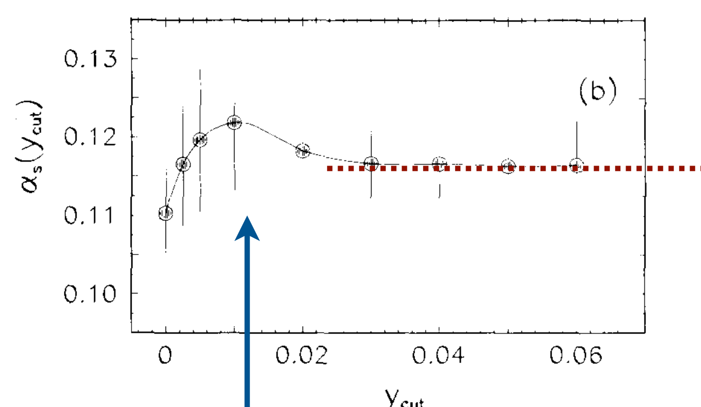
Energy-Energy Correlators



A long history in probing collinear dynamics of QCD





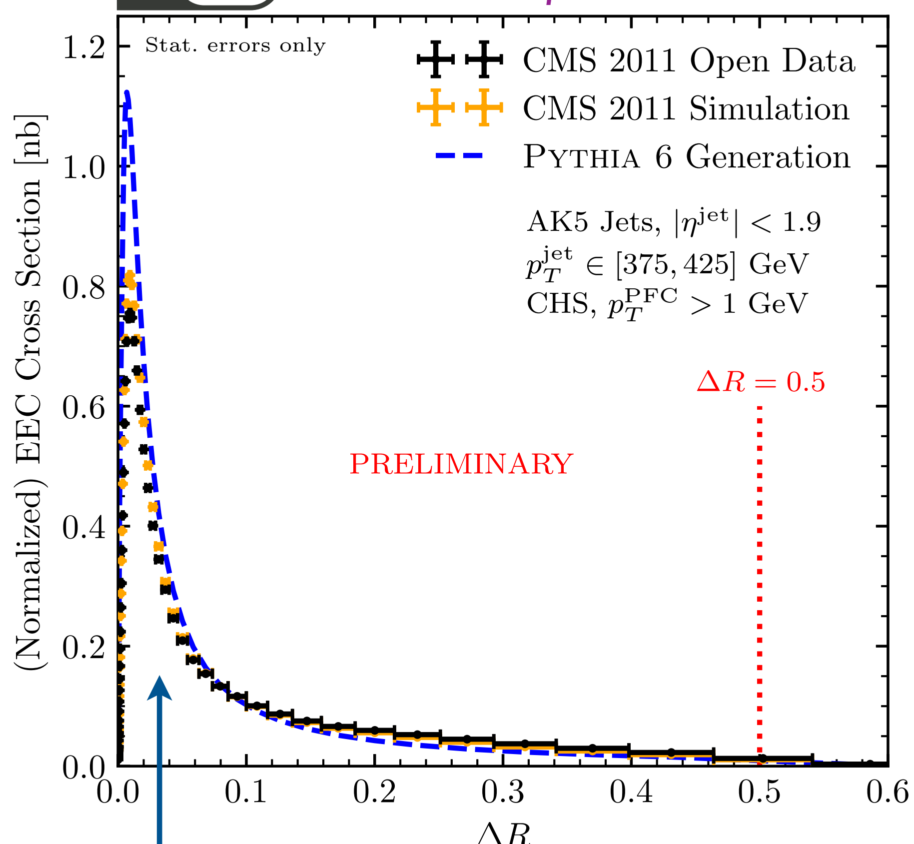
Extracting the strong coupling constant

Theoretical challenges with small angle (collinear) limit

[Basham, Brown, Ellis, Love, PRL 1978; ALEPH, PLB 1991; see Chen, Moult, Zhang, Zhu, PRD 2020]

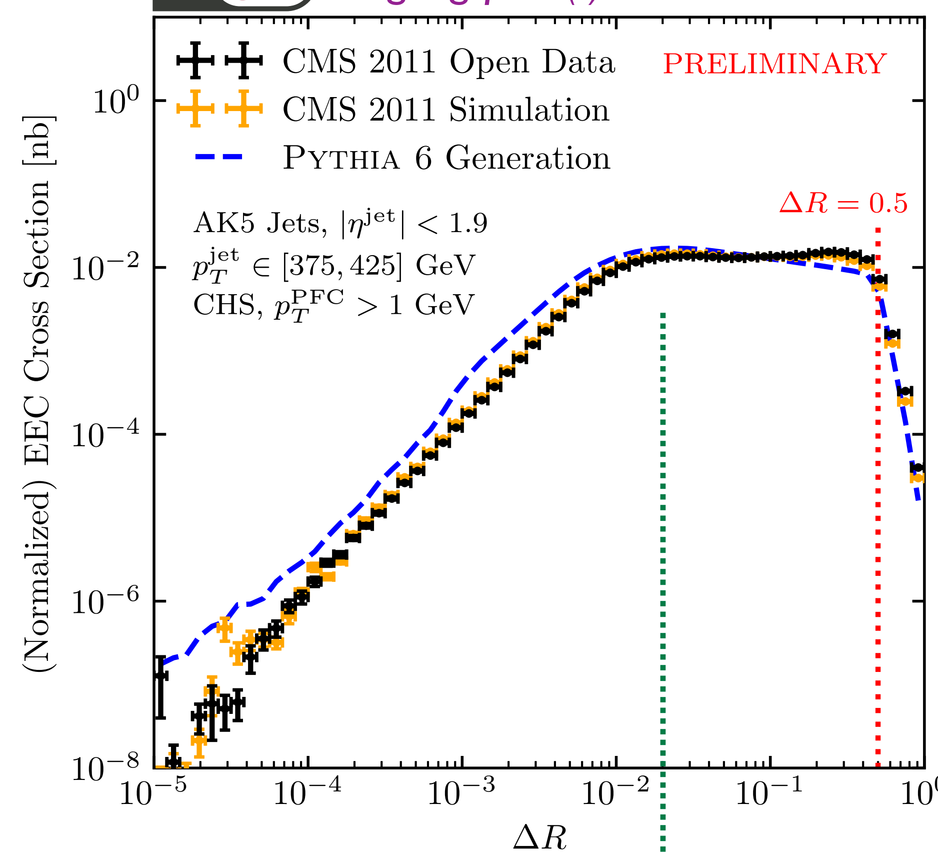
QCD Phase Transition in Jets?

MOD *Linear-linear plot*



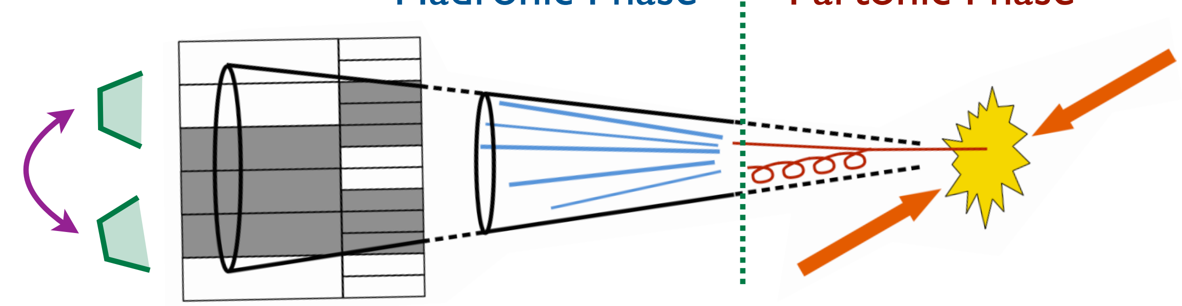
Are we learning something about small angle limit of QCD?

MOD *Log-log plot (!)*



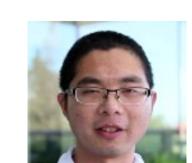
Hadronic Phase

Partonic Phase



First Jet EEC Plot from the LHC (!)





[Komiske, Moult, JDT, Zhu, arXiv 2022; see talks by Moult, BOOST 2019, BOOST 2020]

This music video images the
real-time confinement transition in QCD
at the shortest time scale
in real LHC data
from free hadrons at low energies (small angles)
to interacting quarks and gluons with non-trivial correlations
at high energies (large angles).

[Zhu on YouTube]

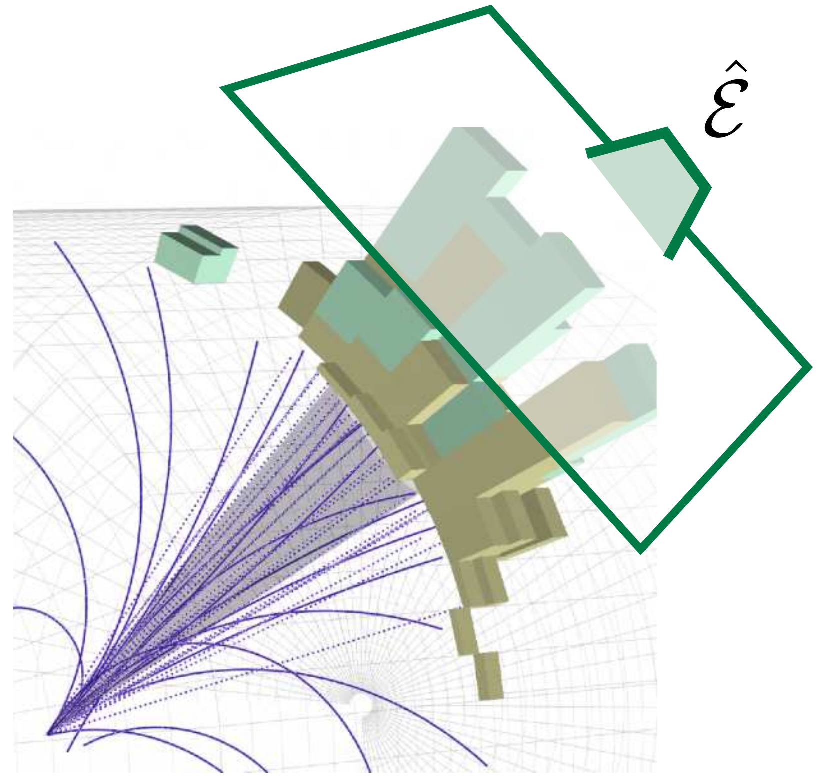
)

Jets as Weighted Point Clouds

- Energy-Weighted Directions

$$\vec{p} = \{\underbrace{E}_{\text{Energy}}, \underbrace{\hat{n}_x, \hat{n}_y, \hat{n}_z}_{\text{Direction}}\}$$

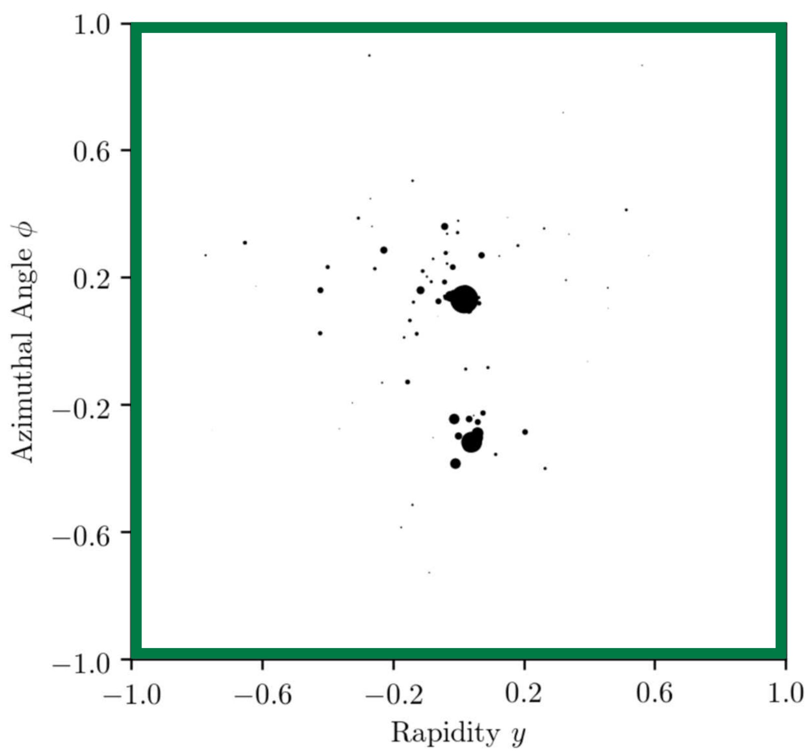
(suppressing "unsafe" charge/flavor information)

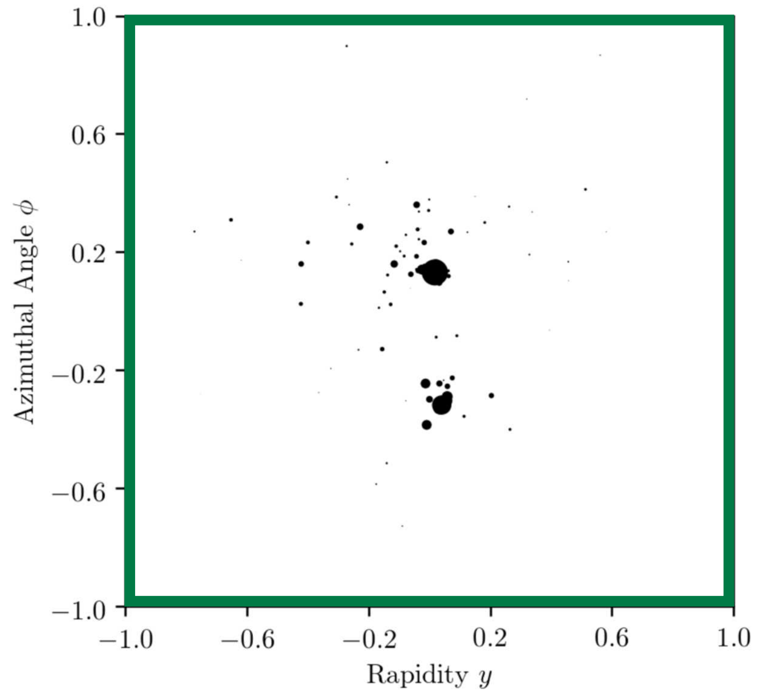


- Equivalently: Energy Density

$$\rho(\hat{n}) = \sum_{i\in\mathcal{J}} E_i \, \delta^{(2)}(\hat{n} - \hat{n}_i)$$

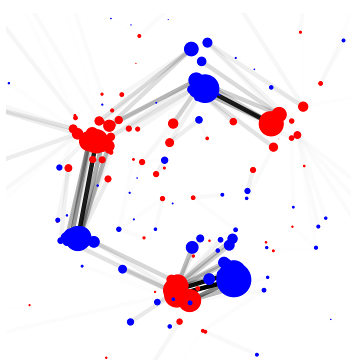
Energy   Direction





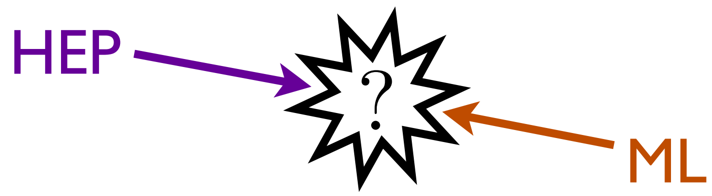
*When restricted to IRC safe information,
jets/events are naturally represented
as energy densities*

Going with the (Energy) Flow



The Energy Mover’s Distance

Revealing a Hidden Geometry



If you ask your local computational geometry expert how to process densities…

The Earth Mover’s Distance

Optimal Transport:

[Peleg, Werman, Rom, IEEE 1989; Rubner, Tomasi, Guibas, ICCV 1998, ICJV 2000; Pele, Werman, ECCV 2008; Pele Taskar, GSI 2013]

Minimum “work” (stuff x distance) to make one distribution look like another distribution

Déblai

Remblai

[h/t Niles-Weed, ML4Jets 2020; Monge, 1781; Kantorovich, 1939; Vaseršteĭn, 1969; Wikipedia]

The Earth Mover's Distance

Optimal Transport: Minimum "work" (stuff x distance) to make one distribution look like another distribution

[Peleg, Werman, Rom, IEEE 1989; Rubner, Tomasi, Guibas, ICCV 1998, ICJV 2000; Pele, Werman, ECCV 2008; Pele Taskar, GSI 2013]

Additive Weight:
Ground Distance:

$\Rightarrow$ Distance Between Distributions

Déblai Remblai

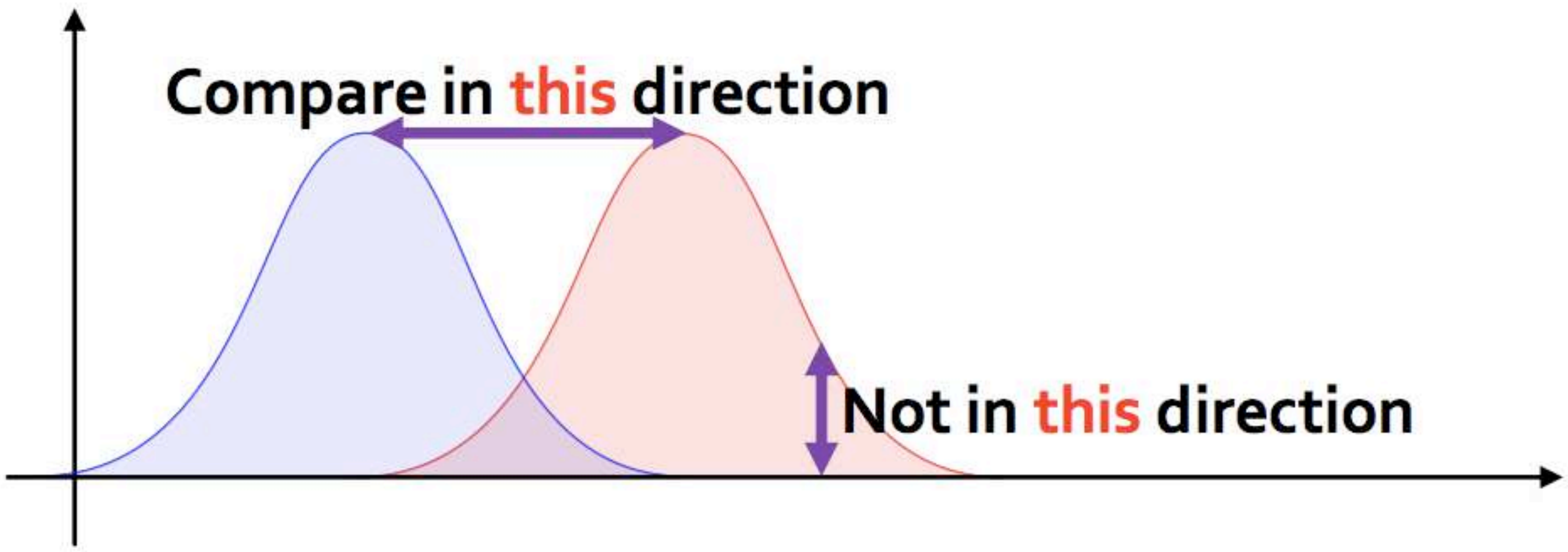
[h/t Niles-Weed, ML4Jets 2020; Monge, 1781; Kantorovich, 1939; Vaseršteĭn, 1969; Wikipedia]

The Earth Mover's Distance

Optimal Transport: Minimum "work" (stuff x distance) to make one distribution look like another distribution

[Peleg, Werman, Rom, IEEE 1989; Rubner, Tomasi, Guibas, ICCV 1998, ICJV 2000; Pele, Werman, ECCV 2008; Pele Taskar, GSI 2013]

"Horizontal" comparison (EMD) yields better dynamic range than "vertical" comparison (e.g. KL)

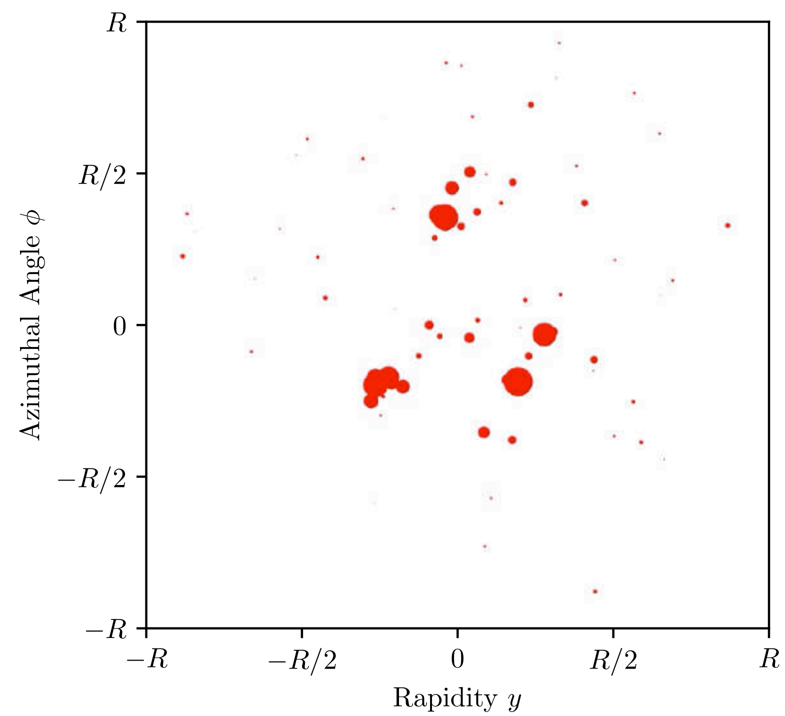


[figure from Kun, Math ∩ Programming]

[h/t Niles-Weed, ML4Jets 2020; Monge, 1781; Kantorovich, 1939; Vaseršteĭn, 1969; Wikipedia]

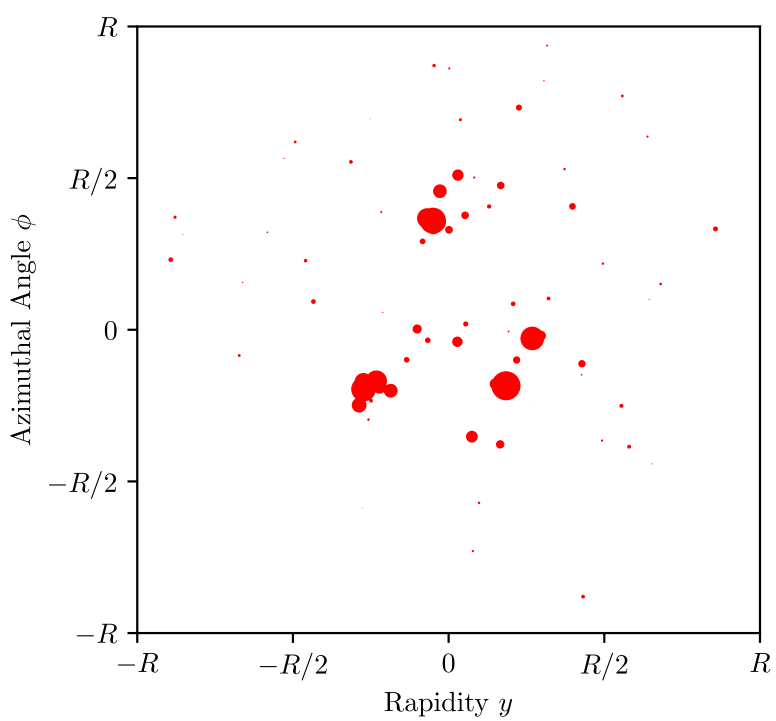
Similarity of Two Energy Flows

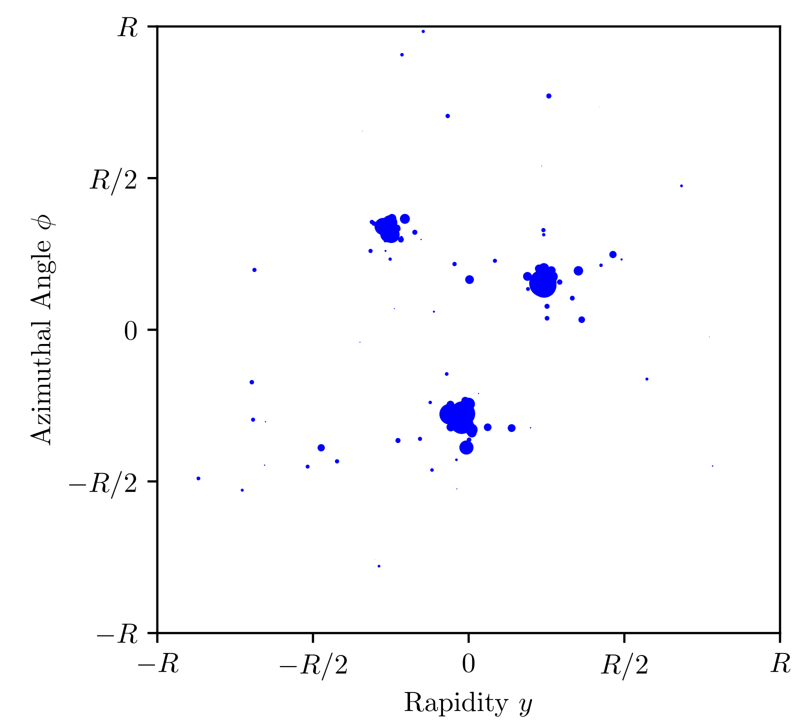
$$\mathcal{E}(\hat{n}) = \sum_i E_i \, \delta(\hat{n} - \hat{n}_i)$$



Optimal Transport:

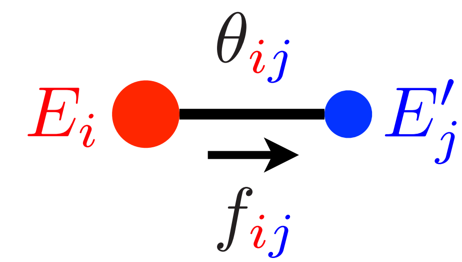
Earth Mover's Distance
a.k.a. 1-Wasserstein metric

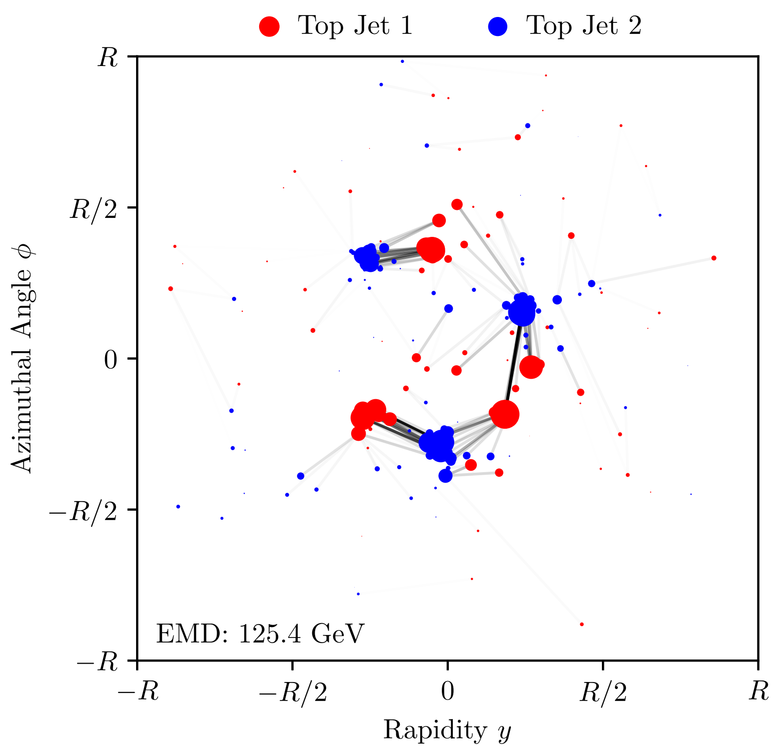




[Komiske, Metodiev, JDT, PRL 2019; code at Komiske, Metodiev, JDT, energyflow.network]

The Energy Mover's Distance

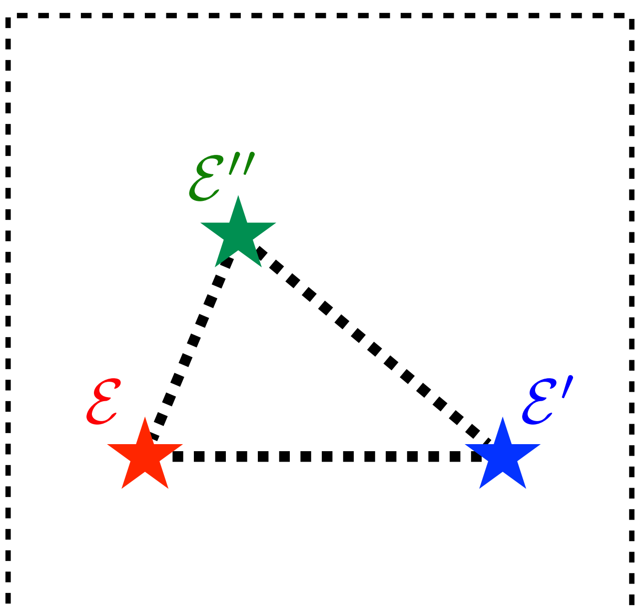




Optimal transport between energy flows…

$$\mathrm{EMD}(\mathcal{E}, \mathcal{E}') = \underbrace{\min_{\{f\}} \sum_i \sum_j f_{ij} \frac{\theta_{ij}}{R}}_{\text{Cost to move energy}} + \underbrace{\left| \sum_i E_i - \sum_j E'_j \right|}_{\text{Cost to create energy}}$$

in GeV



…defines a metric on the space of events

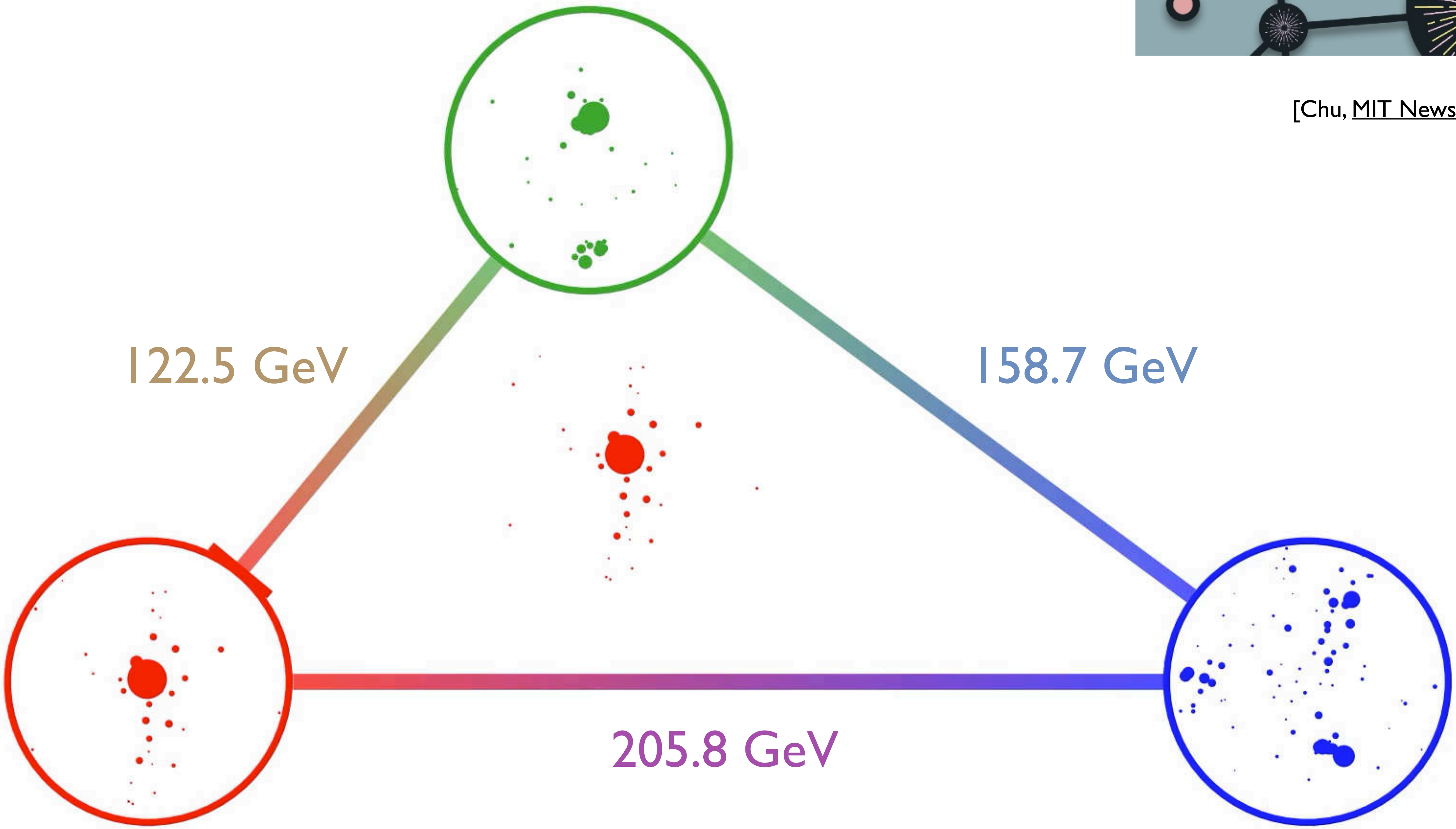
$$0 \le \mathrm{EMD}(\mathcal{E}, \mathcal{E}') \le \mathrm{EMD}(\mathcal{E}, \mathcal{E}'') + \mathrm{EMD}(\mathcal{E}', \mathcal{E}'')$$

(assuming R ≥ $\theta_{max}/2$, i.e. R ≥ jet radius for conical jets)

[Komiske, Metodiev, JDT, PRL 2019;
see also Pele, Werman, ECCV 2008; Pele, Taskar, GSI 2013;
[see flavored variant in Crispim Romão, Castro, Milhano, Pedro, Vale, EPJC 2021]
[see computational speed up in Cai, Cheng, Craig, Craig, PRD 2020]

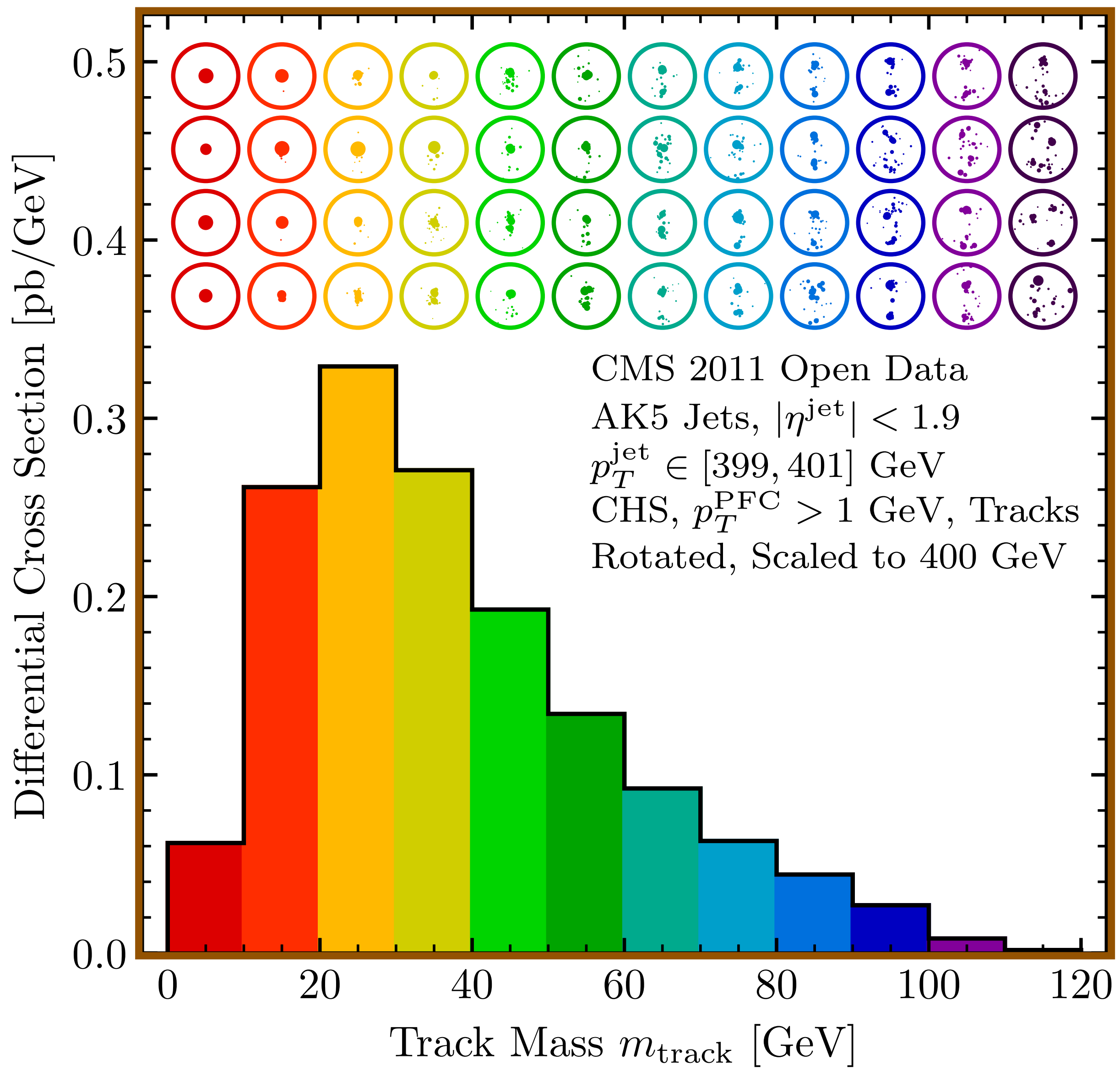
Similarity of Three Energy Flows

[Chu, MIT News July 2019]

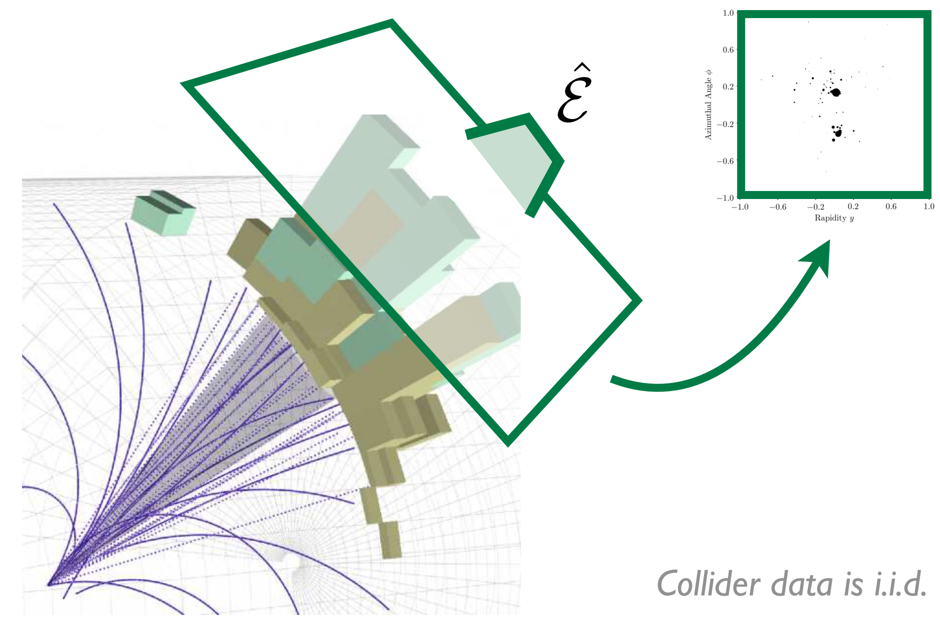


[Komiske, Metodiev, JDT, PRL 2019; code at Komiske, Metodiev, JDT, energyflow.network;
see alternative graph network approach in Mullin, Pacey, Parker, White, Williams, JHEP 2021]

The Forest and the Trees



A Histogram of Observables

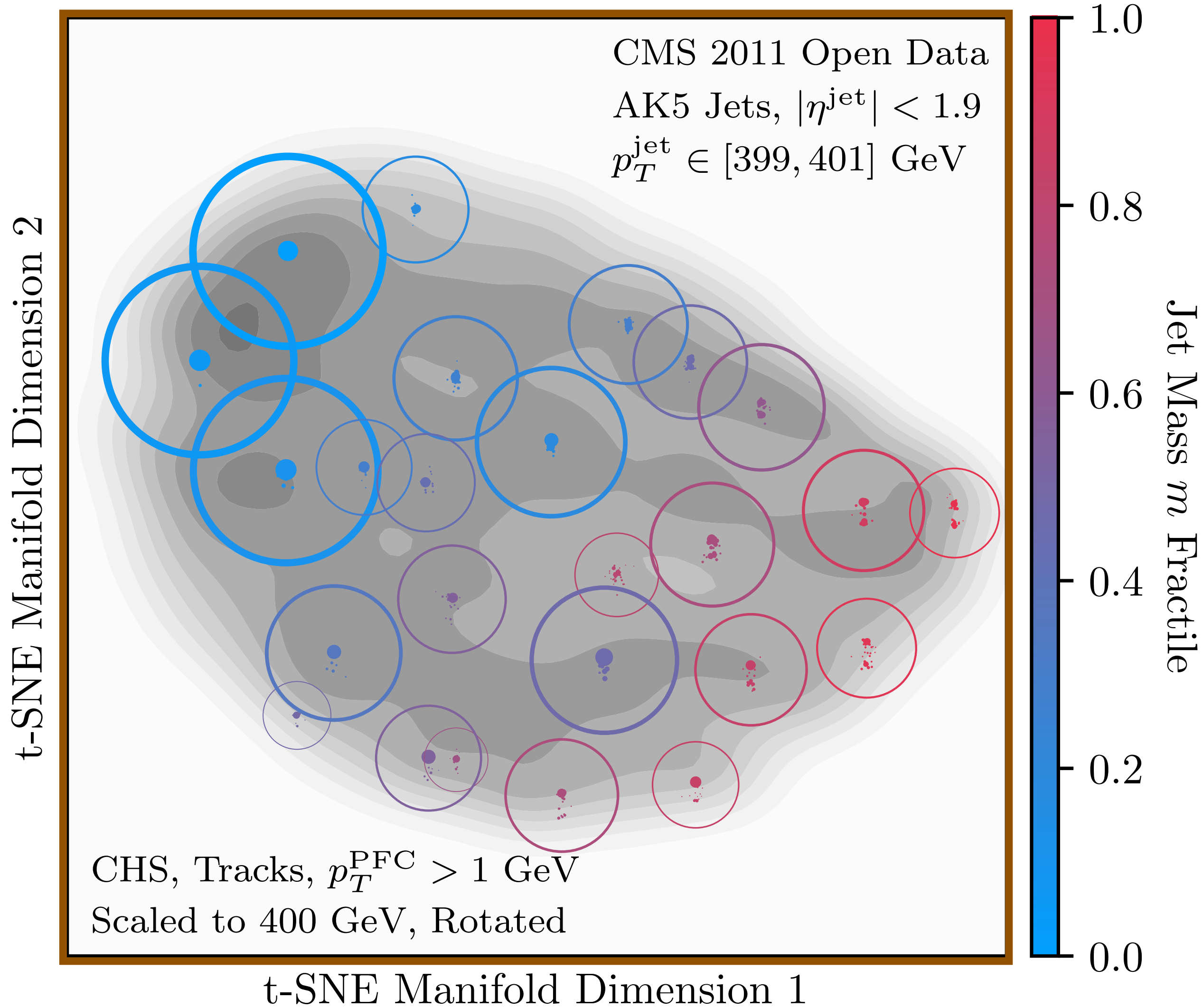


Collider data is i.i.d.

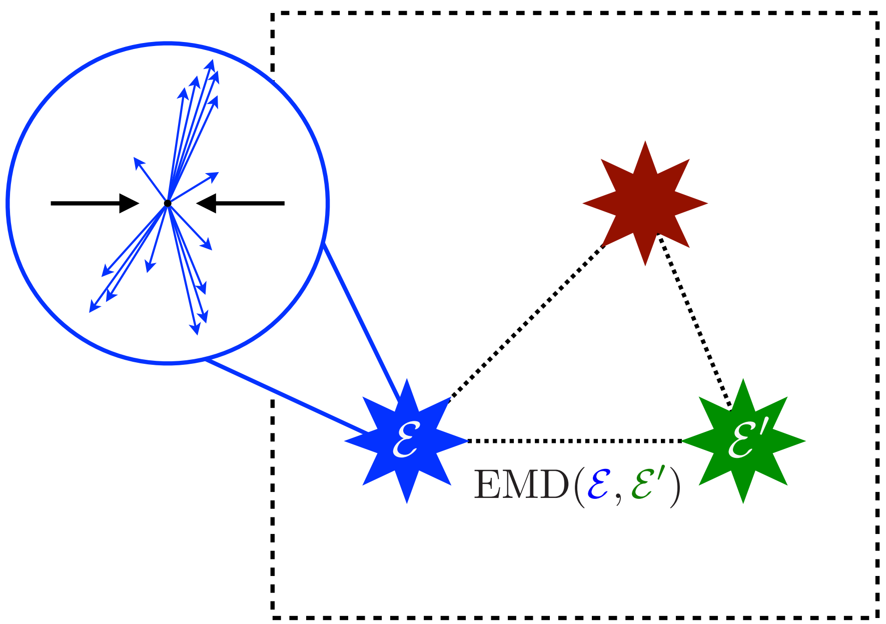
[Komiske, Mastandrea, Metodiev, Naik, JDT, PRD 2020; using CMS Open Data]




Building the Forest from the Trees

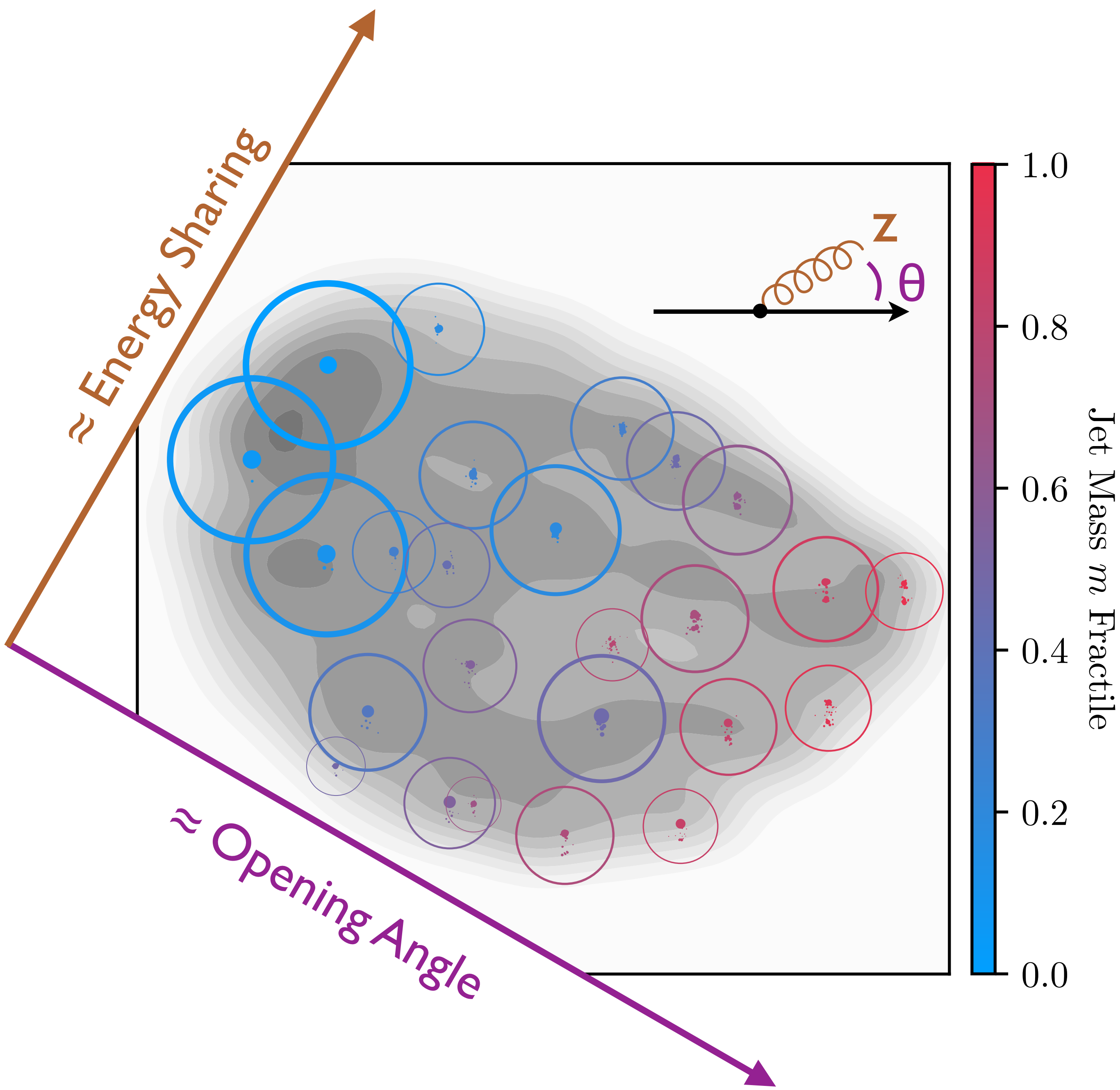


The Space of Energy Flows

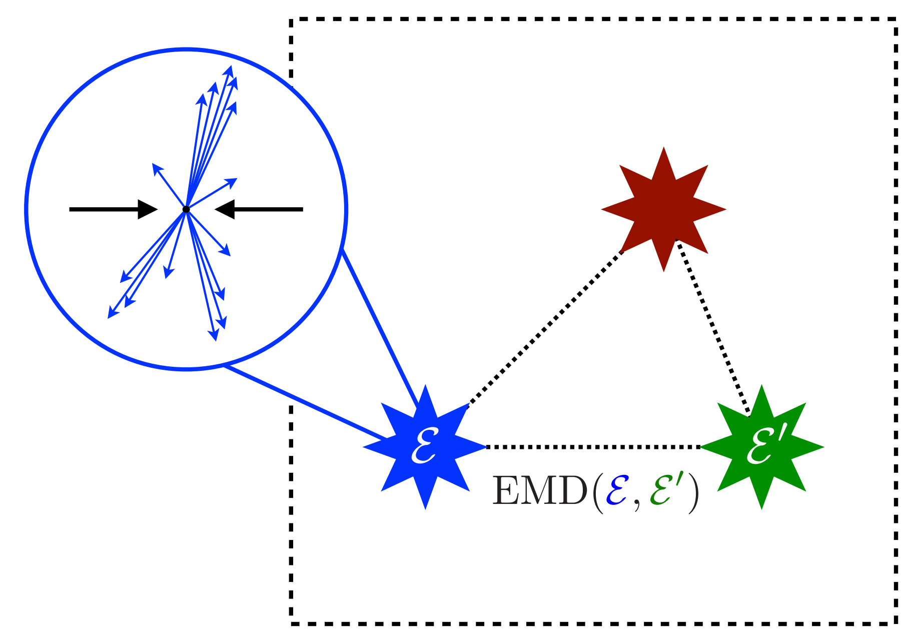


[Komiske, Mastandrea, Metodiev, Naik, JDT, PRD 2020;
using van der Maaten, Hinton, JMLR 2008; using CMS Open Data]

Building the Forest from the Trees



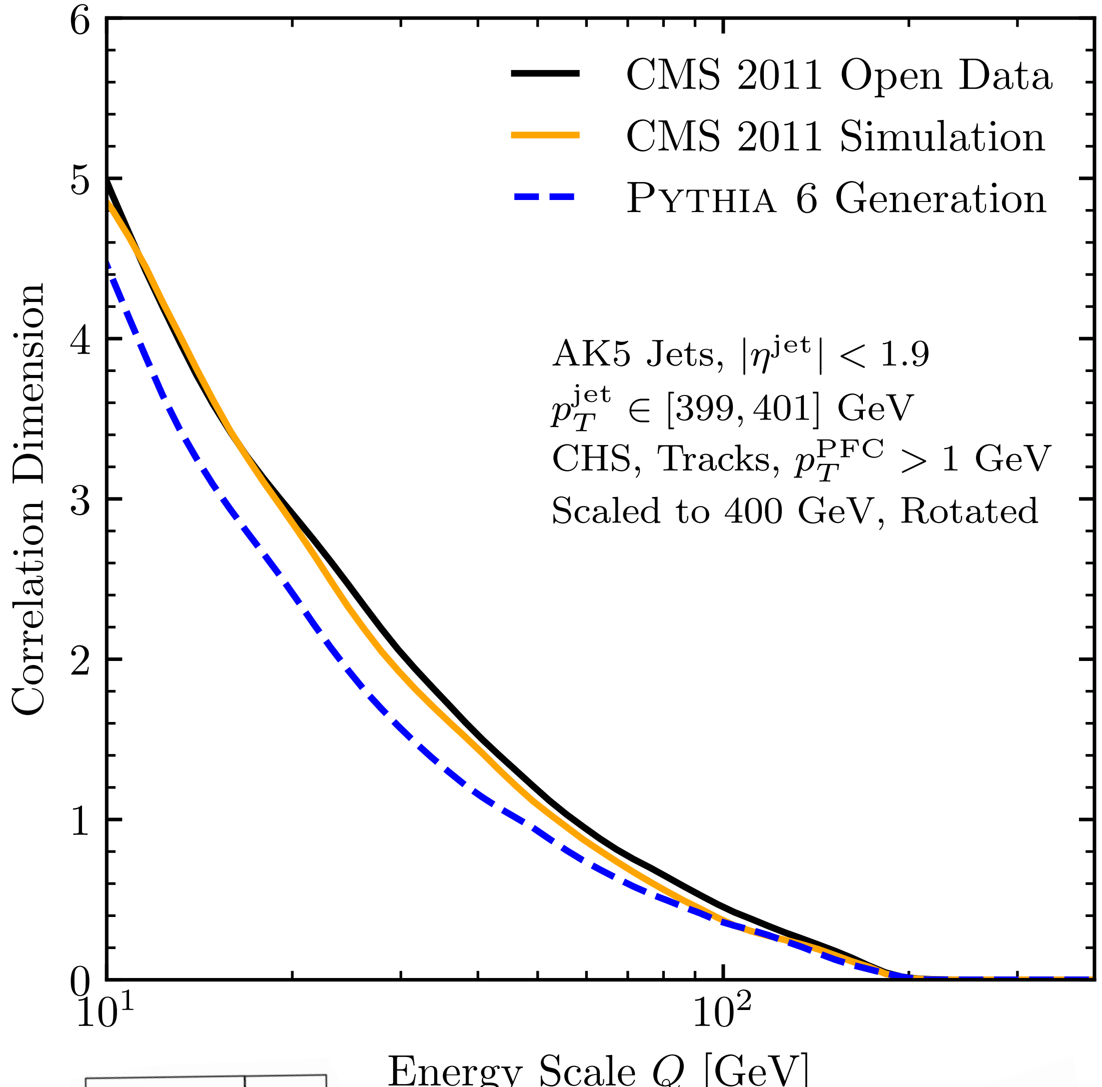
The Space of Energy Flows

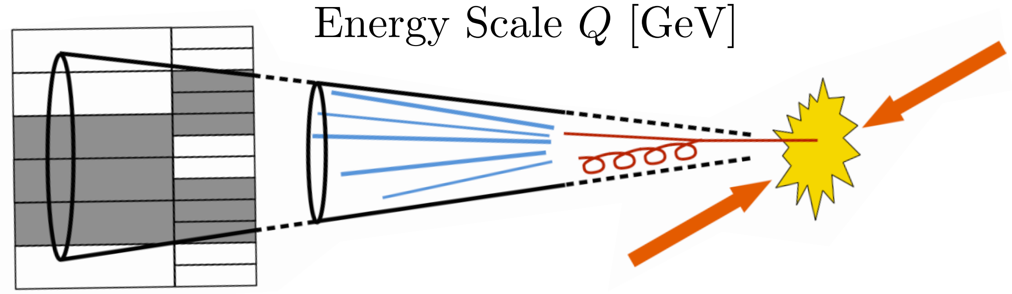




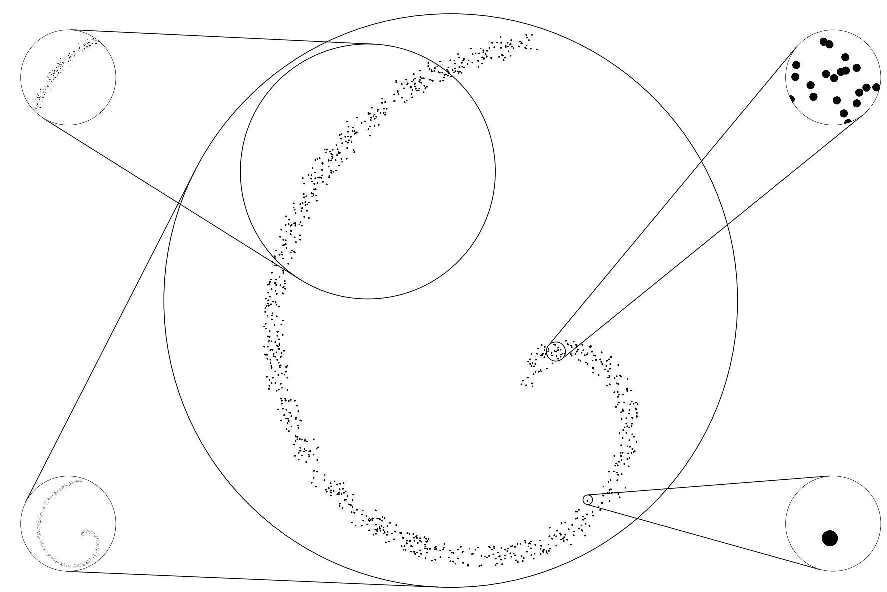

[Komiske, Mastandrea, Metodiev, Naik, JDT, PRD 2020;
using van der Maaten, Hinton, JMLR 2008; using CMS Open Data]

A Super-Fractal Forest made from Trees

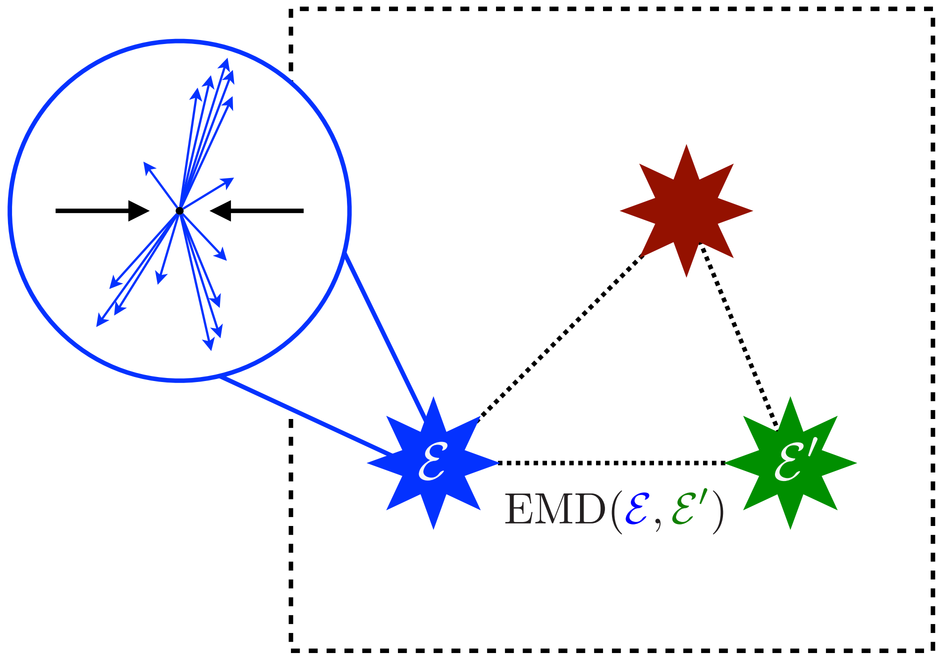




Dimension of Space of Energy Flows



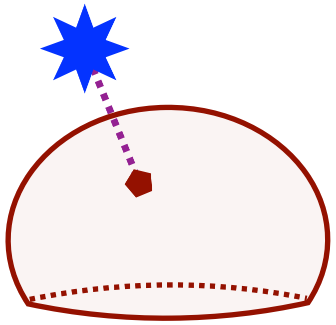
[Komiske, Mastandrea, Metodiev, Naik, JDT, PRD 2020;
using Grassberger, Procaccia, PRL 1983; Kégl, NIPS 2002; using CMS Open Data]



Viewed through the data science lens,
the EMD unlocks a suite of
geometric analysis strategies

Going with the (Energy) Flow

The Energy Mover's Distance

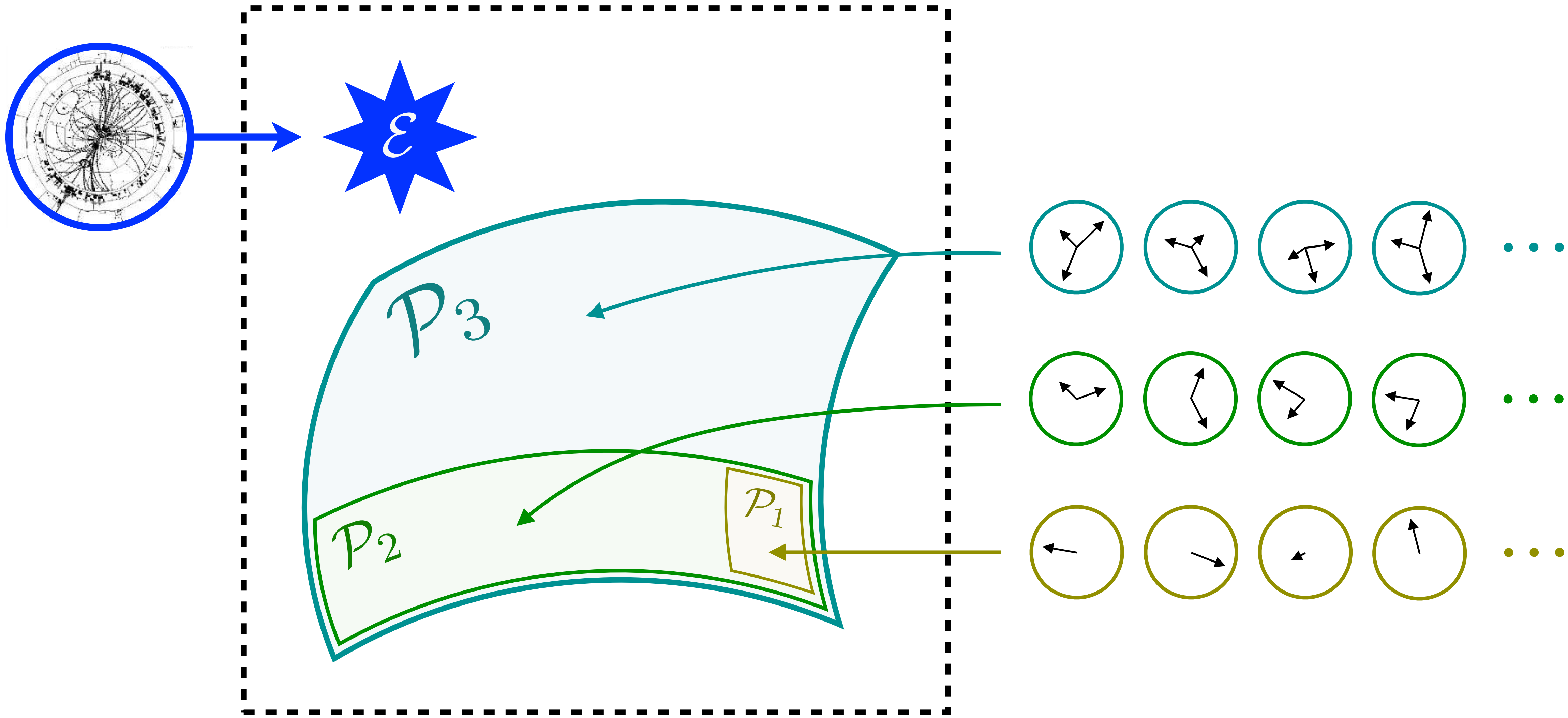


Revealing a Hidden Geometry

Given a metric space, the first geometric object you might think to construct is…

Introducing N-particle Manifolds

$\mathcal{P}_N$ = set of all N-particle configurations

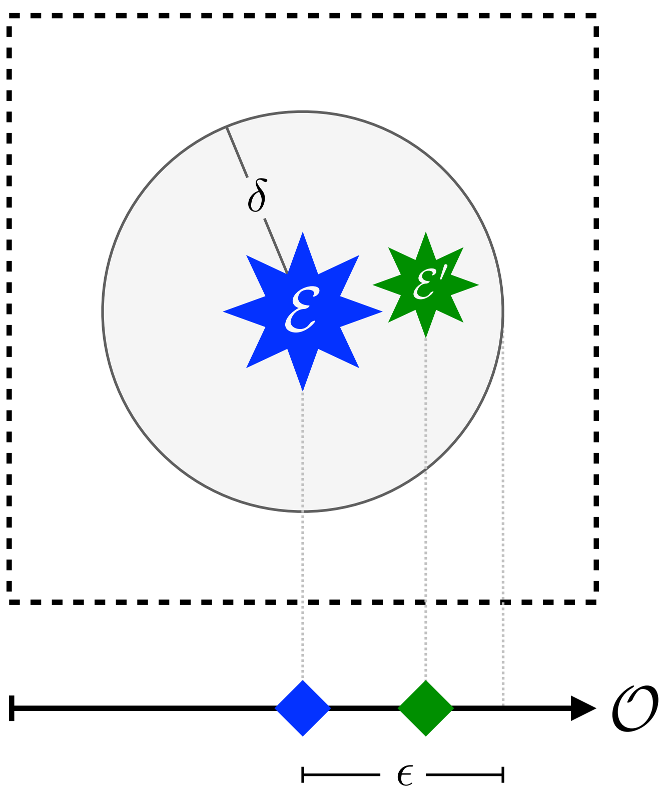


$\mathcal{P}_N \supset \mathcal{P}_{N-1} \supset \ldots \supset \mathcal{P}_2 \supset \mathcal{P}_1$ by soft/collinear limits

[see related discussion in Larkoski, Melia, PRD 2020]

Introducing N-particle Manifolds

$\mathcal{P}_N$ = set of all N-particle configurations



Infrared & Collinear Safety

≈ calculable in perturbative quantum field theory

*is**

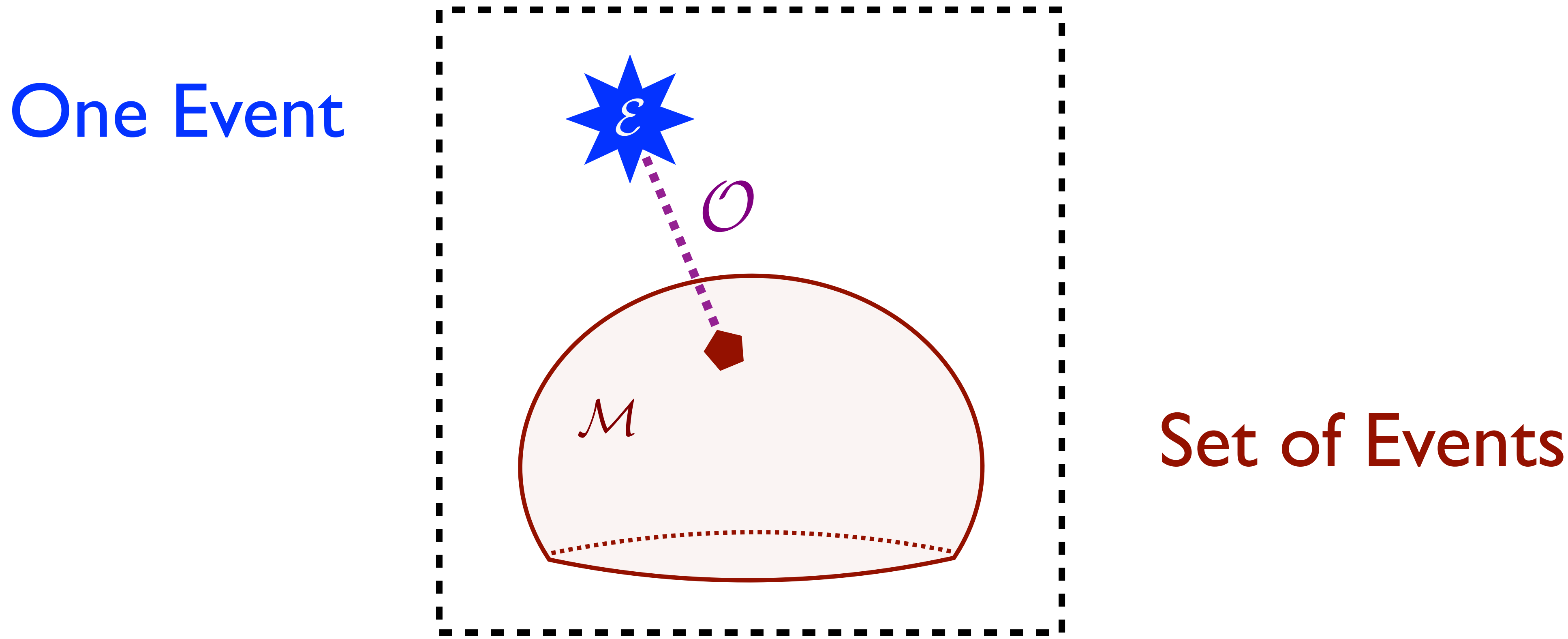
Continuity in EMD Space

[Komiske, Metodiev, JDT, JHEP 2020]
[Sterman, Weinberg, PRL 1977; Sterman, PRD 1979]
[see also Banfi, Salam, Zanderighi, JHEP 2005; Larkoski, Marzani, JDT, PRD 2015]

$\mathcal{P}_N \supset \mathcal{P}_{N-1} \supset \ldots \supset \mathcal{P}_2 \supset \mathcal{P}_1$ by soft/collinear limits

[see related discussion in Larkoski, Melia, PRD 2020]

Manifolds for Observables



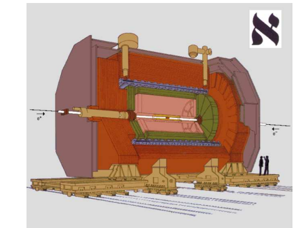
Distance of Closest Approach ⇒ Observable

$$\mathcal{O}(\mathcal{E}) = \min_{\mathcal{E}' \in \mathcal{M}} \mathrm{EMD}(\mathcal{E}, \mathcal{E}')$$

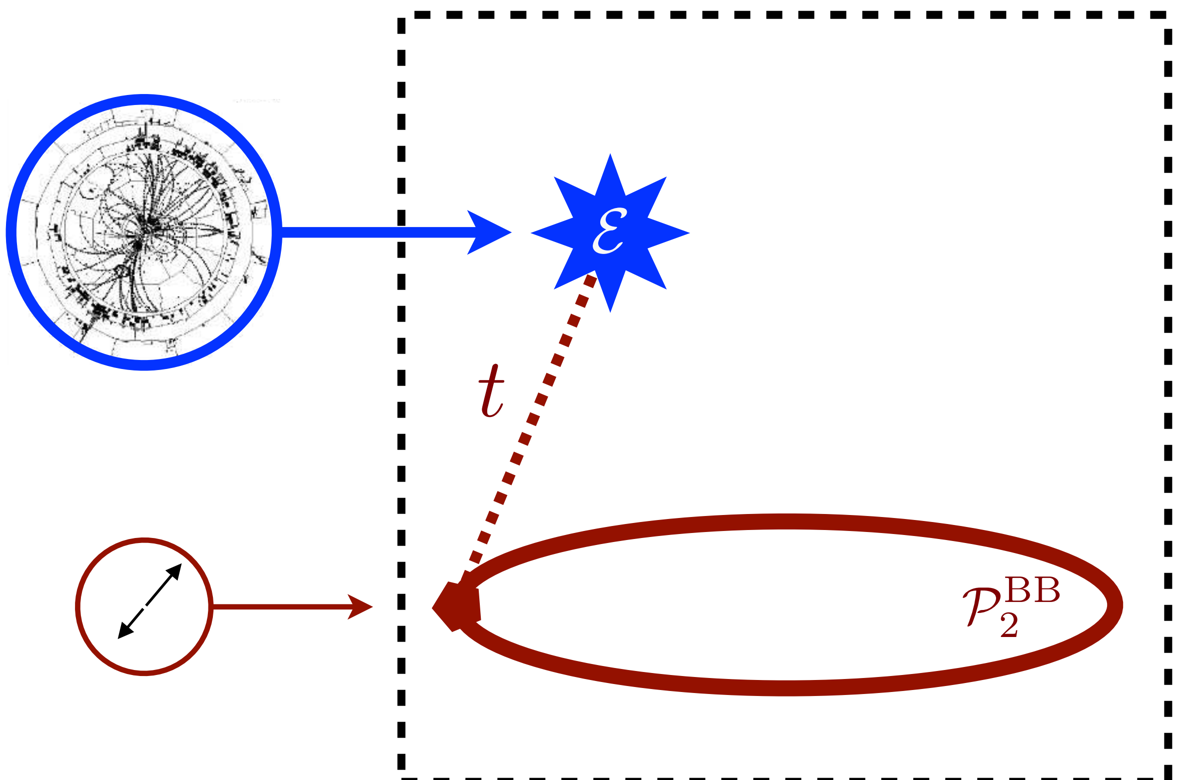
[Komiske, Metodiev, JDT, JHEP 2020]

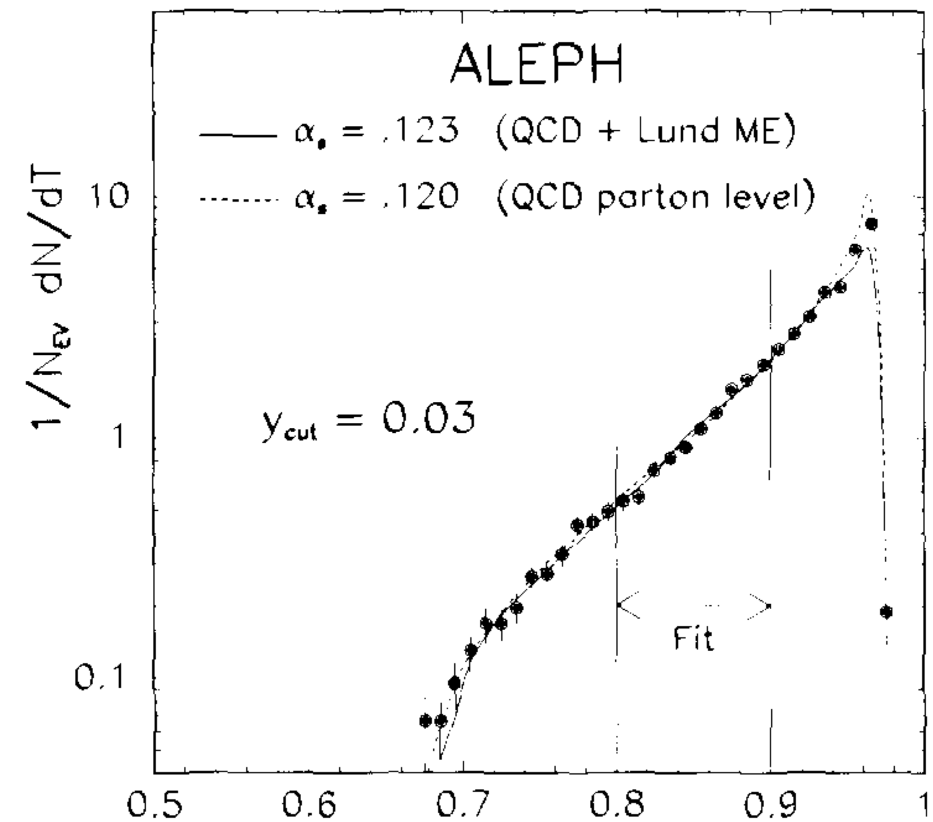
E.g. Thrust

How dijet-like is an event?



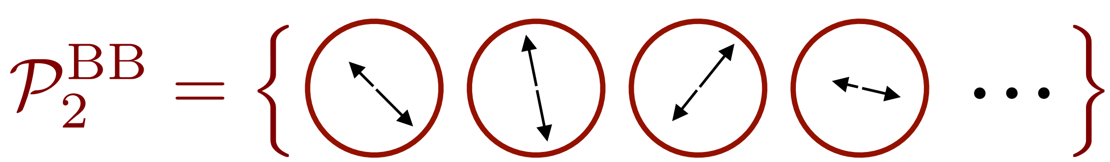
$$t(\mathcal{E}) = \min_{\mathcal{E}' \in \mathcal{P}_2^{\rm BB}} \mathrm{EMD}_2(\mathcal{E}, \mathcal{E}')$$





$$1 - \frac{t}{2E_{\rm CM}}$$

All Back-to-Back Two Particle Configurations



$$\mathcal{P}_2^{\rm BB} = \{ \cdots \}$$

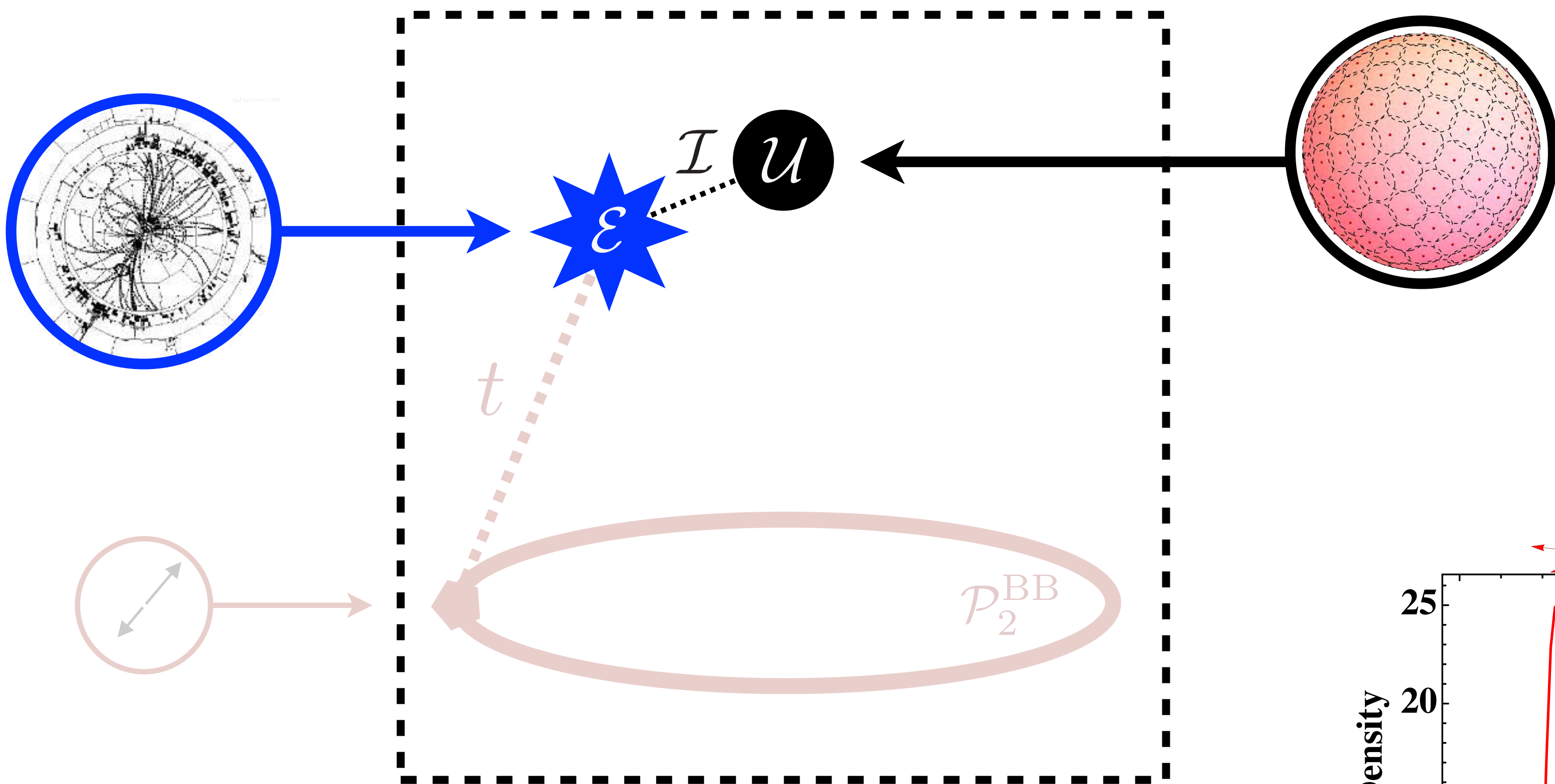
(using β=2 EMD variant)

cf. $T(\mathcal{E}) = \max_{\hat{n}} \frac{\sum_i |\vec{p}_i \cdot \hat{n}|}{\sum_j |\vec{p}_j|}$

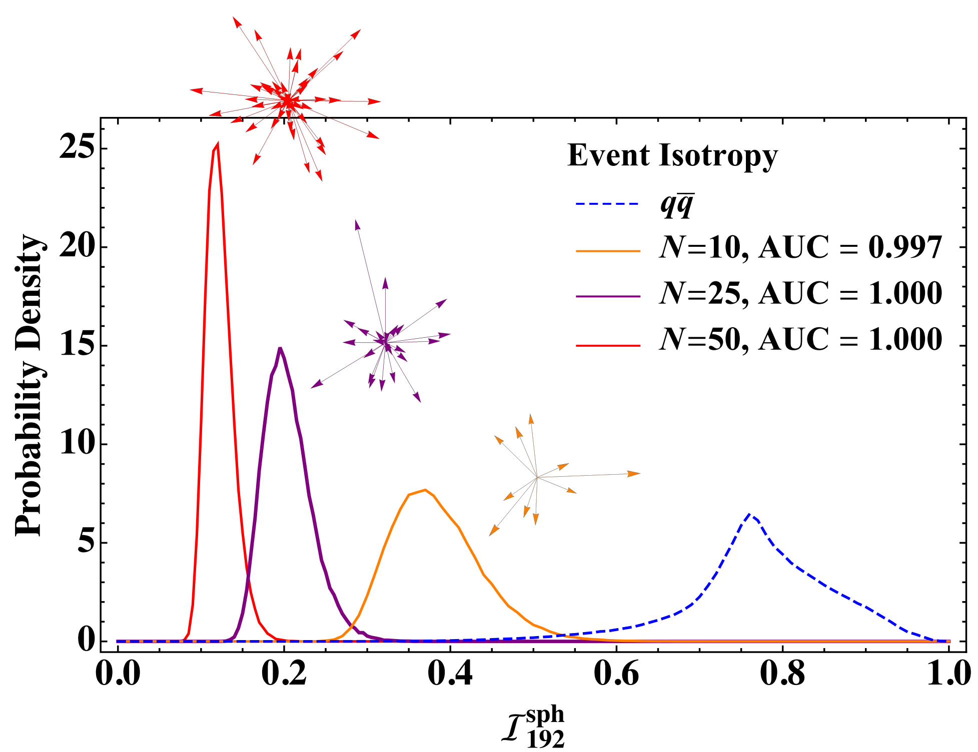
[Komiske, Metodiev, JDT, JHEP 2020]
[Brandt, Peyrou, Sosnowski, Wroblewski, PL 1964; Farhi, PRL 1977; ALEPH, PLB 1991]

New! Event Isotropy

How isotropic is an event?



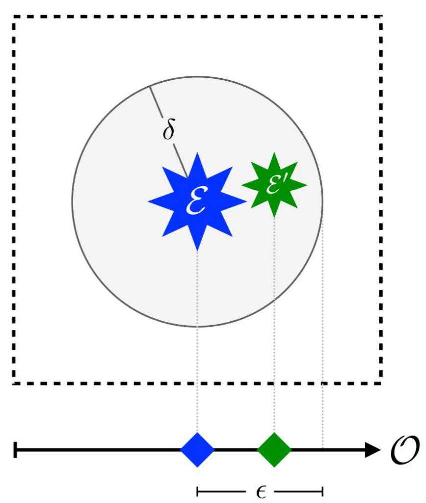
$$\mathcal{I}(\mathcal{E}) = \mathrm{EMD}(\mathcal{E}, \mathcal{U})$$



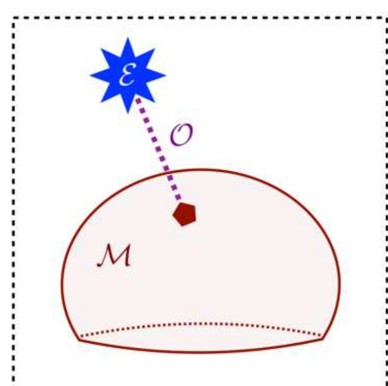
[Cesarotti, JDT, JHEP 2020;
see also Cesarotti, Reece, Strassler, JHEP 2021]

Six Decades of Collider Physics Translated into a New Geometric Language!

IRC Safety is smoothness in the space of events

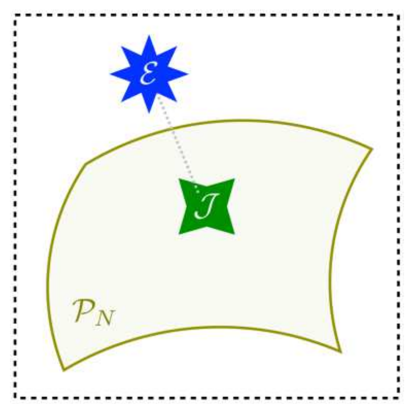


Event shapes are distances from events to manifolds.



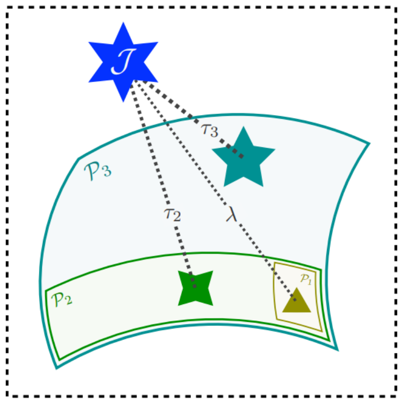
$\mathcal{O}(\mathcal{E}) = \min_{\mathcal{E}' \in \mathcal{M}} \mathrm{EMD}_{\beta,R}(\mathcal{E}, \mathcal{E}')$

Jets are projections to few-particle manifolds.



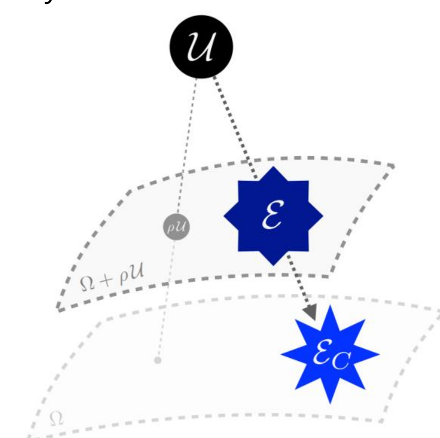
$\mathcal{J} = \operatorname{argmin}_{\mathcal{E}' \in \mathcal{P}_N} \mathrm{EMD}_{\beta,R}(\mathcal{E}, \mathcal{E}')$

Substructure resolves emissions within the jet.



$\tau(\mathcal{J}) = \min_{\mathcal{E}' \in \mathcal{P}_N} \mathrm{EMD}_{\beta}(\mathcal{J}, \mathcal{E}').$

Pileup mitigation moves away from uniform radiation.



$\mathcal{E}_C = \arg\min_{\mathcal{E}'} \mathrm{EMD}(\mathcal{E}, \mathcal{E}' + \rho\,\mathcal{U})\,.$

Taming infinities | Event Shapes | Jet Algorithms | Jet Substructure | Pileup

1960 — 2020

1962-1964
Infrared Safety
[Kinoshita, JMP 1962]
[Lee, Nauenberg, PR 1964]

1977
Thrust, Sphericity
[Farhi, PRL 1977]
[Georgi, Machacek, PRL 1977]

1993
k_T jet clustering
[Ellis, Soper, PRD 1993]
[Catani, Dokshitzer, Seymour, Webber, NPB 1993]

1997-1998
C/A jet clustering
[Wobisch, Wengler, 1998]
[Doskhitzer, Leder, Moretti,Webber, JHEP 1997]

2010-2015
N-(sub)jettiness, XCone
[Stewart, Tackmann, Waalewijn, PRL 2010]
[Thaler, Van Tilburg, JHEP 2011]
[Stewart, Tackmann, Thaler, Vermilion, Wilkason, JHEP 2015]

2014-2019
Constituent Subtraction
[Berta, Spousta, Miller, Leitner, JHEP 2014]
[Berta, Masetti, Miller, Spousta, JHEP 2019]

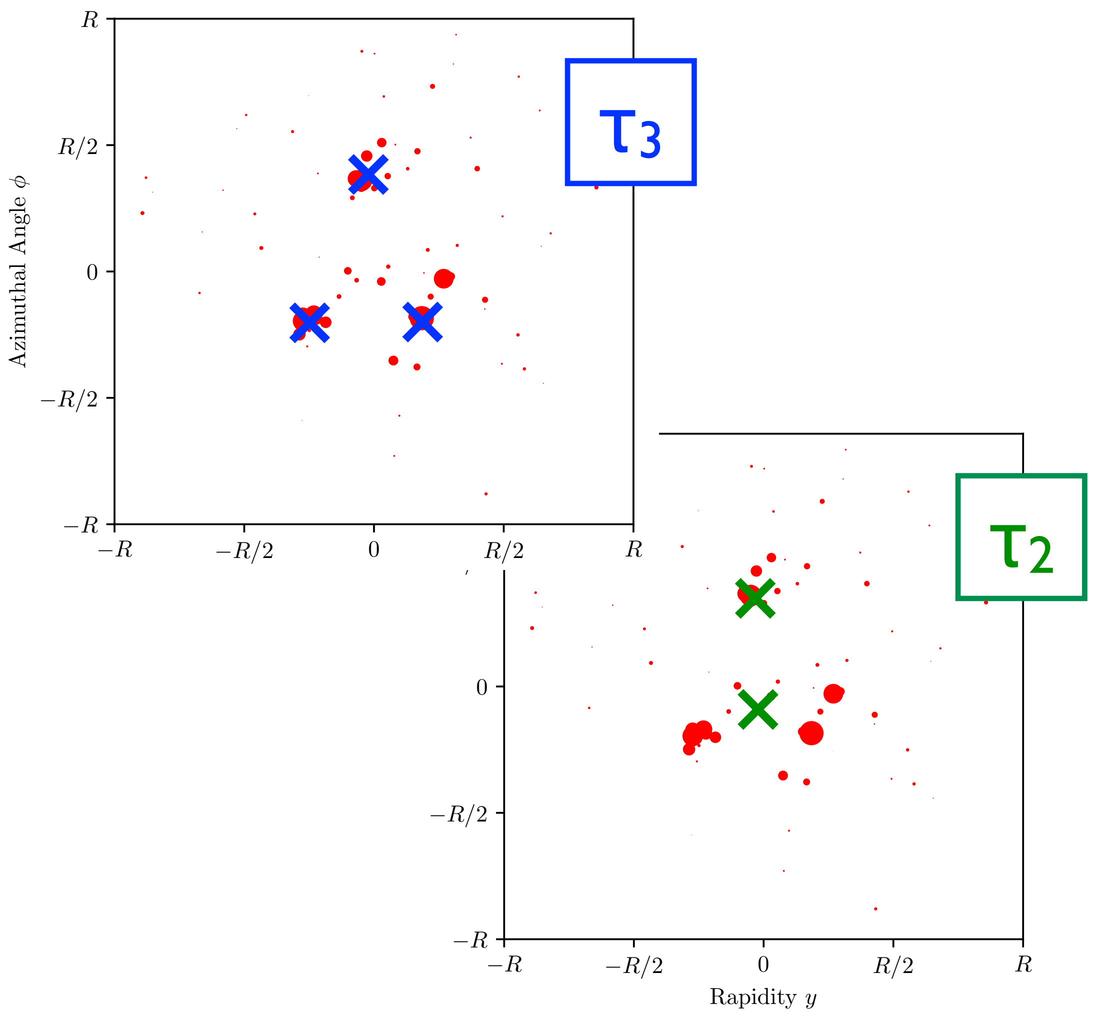
And many more!

[Komiske, Metodiev, JDT, JHEP 2020; timeline from Metodiev]

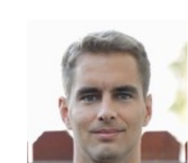
N-subjettiness

Ubiquitous jet substructure observable used for almost a decade…

$$\tau_N(\mathcal{J}) = \min_{N\ \text{axes}} \sum_i E_i \min\left\{\theta_{1,i}, \theta_{2,i}, \ldots, \theta_{N,i}\right\}$$



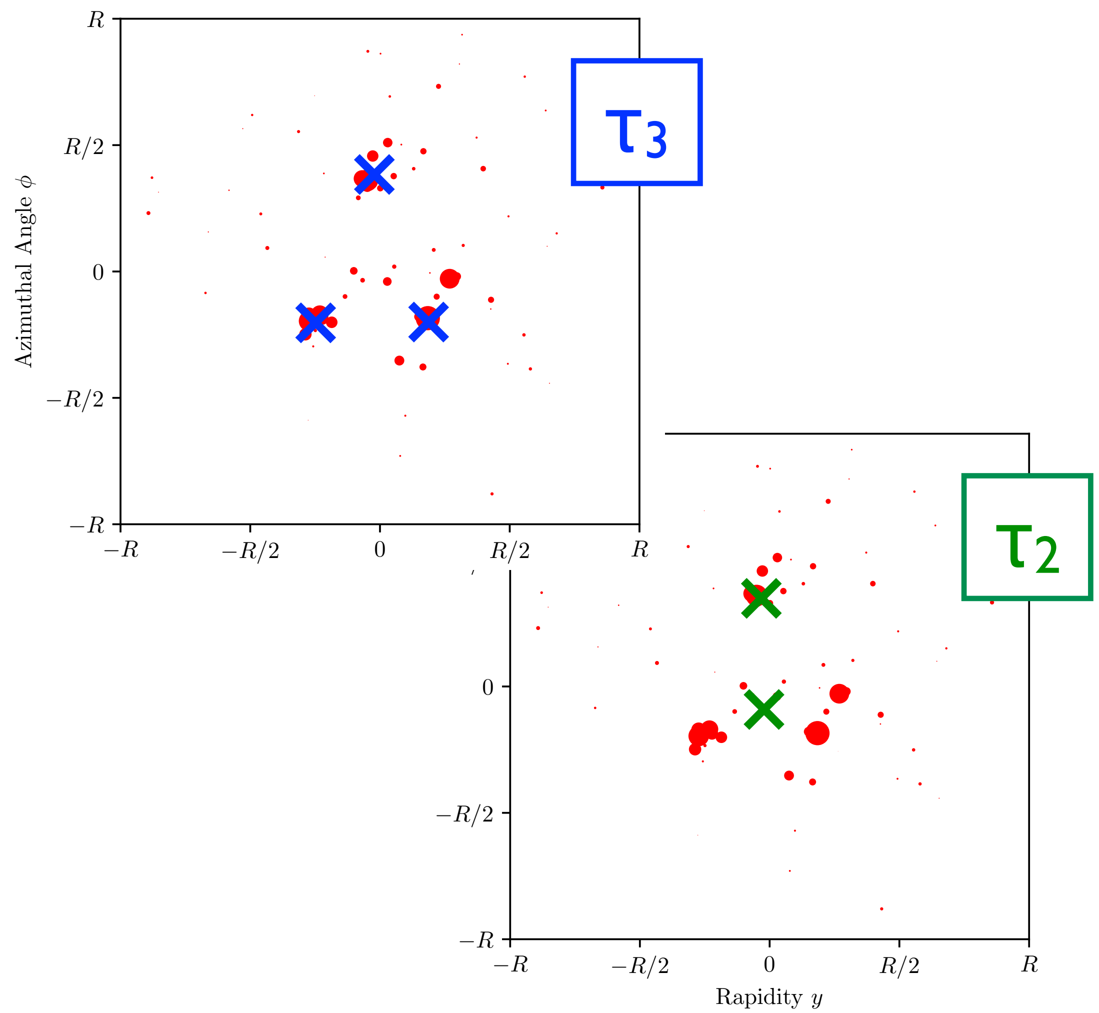
[JDT, Van Tilburg, JHEP 2011, JHEP 2012;
based on Brandt, Dahmen, ZPC 1979; Stewart, Tackmann, Waalewijn, PRL 2010]

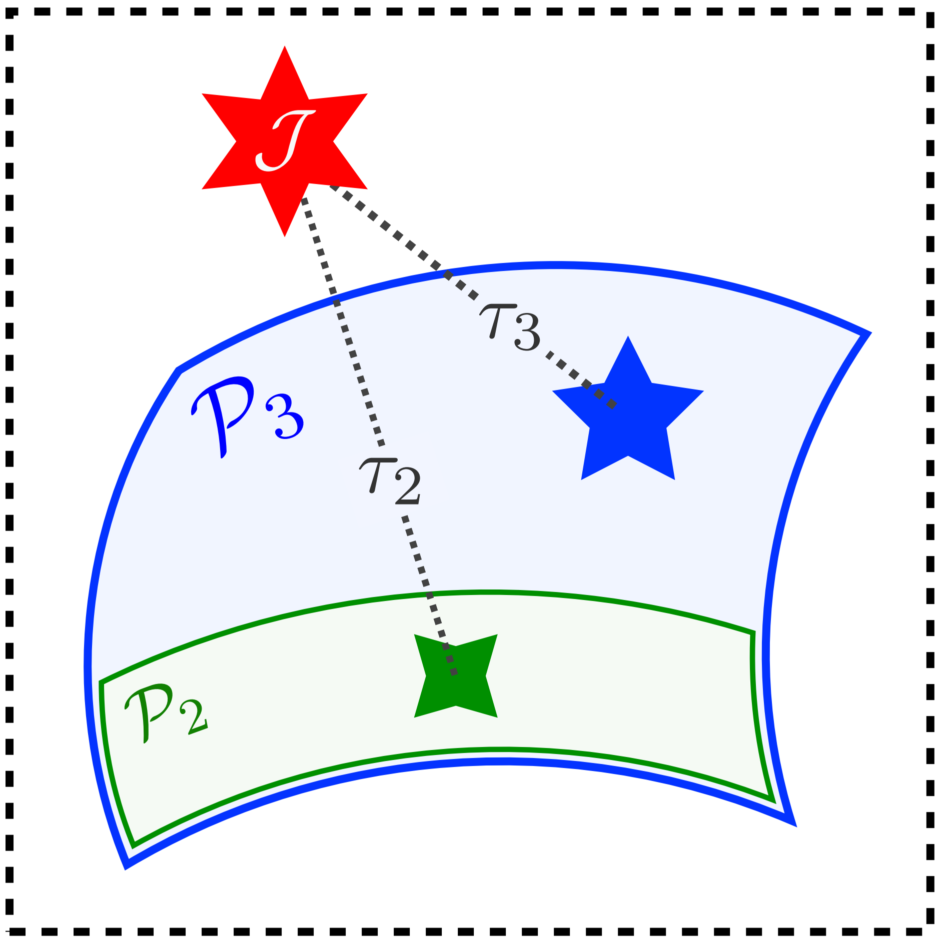


N-subjettiness = Point to Manifold EMD

…is secretly an optimal transport problem

$$\tau_N(\mathcal{J}) = \min_{\mathcal{J}' \in \mathcal{P}_N} \mathrm{EMD}(\mathcal{J}, \mathcal{J}')$$





[JDT, Van Tilburg, JHEP 2011, JHEP 2012;
rephrased in the language of Komiske, Metodiev, JDT, PRL 2019]

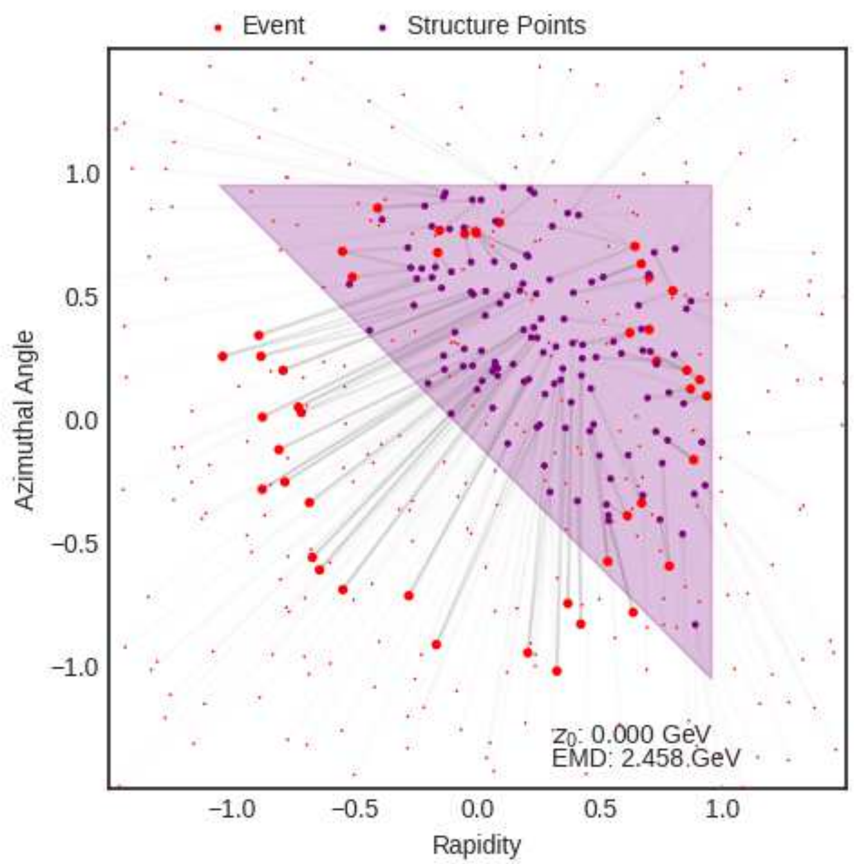


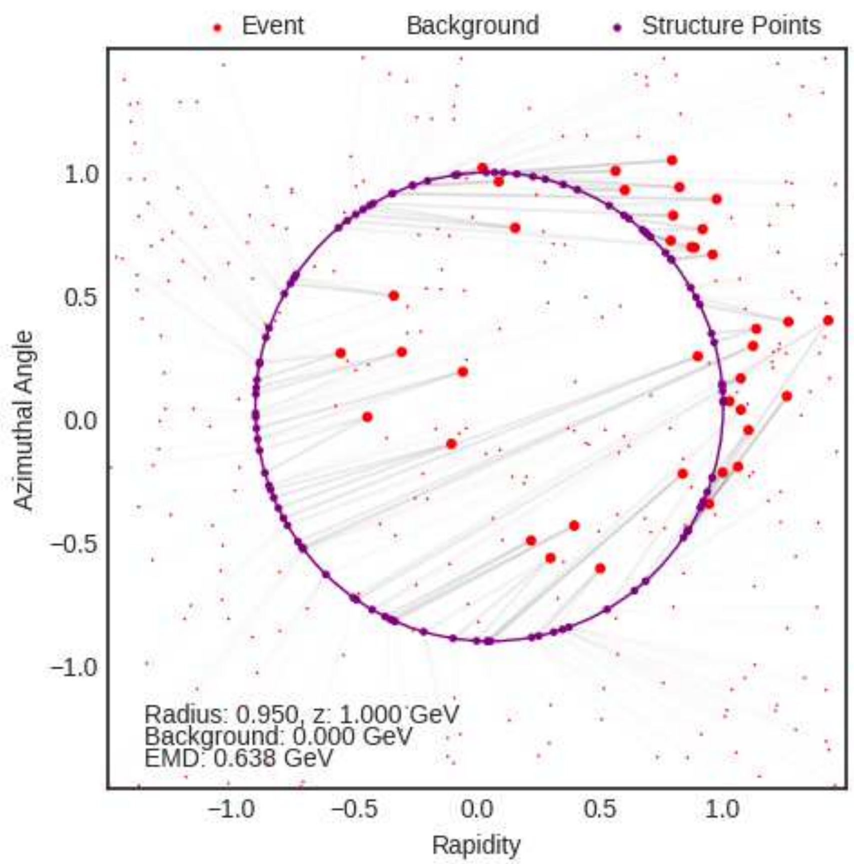
Deep Manifold Learning

Optimal transport meets gradient descent

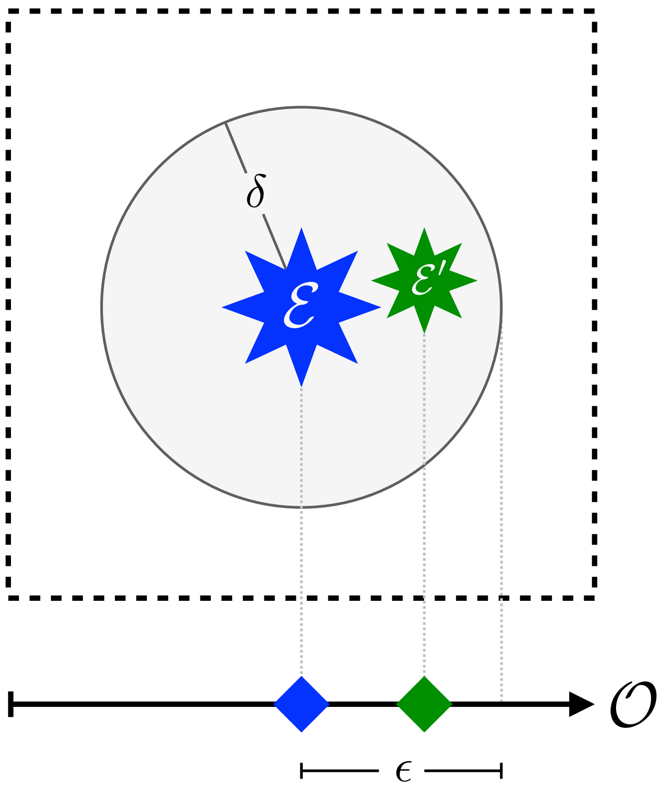
$$\mathcal{O}(\mathcal{E}) = \min_{\mathcal{E}' \in \mathcal{M}} \text{EMD}(\mathcal{E}, \mathcal{E}')$$

How triangle-like / ring-like is this jet?



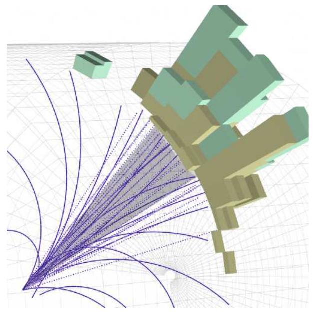


[Ba, Dogra, Gambhir, JDT, in progress;
inspired by Tankala, Tasissa, Murphy, Ba, arXiv 2020]



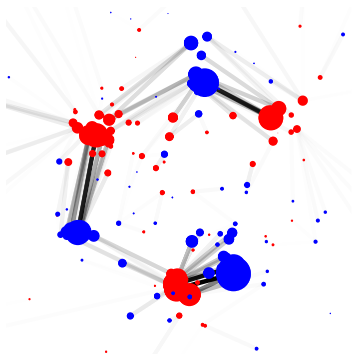
We are just beginning to leverage the conceptual richness of optimal transport for high-energy physics application

Summary



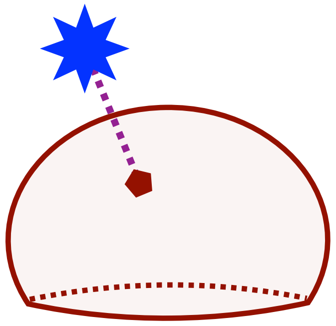
Going with the (Energy) Flow

Restricting our attention to IRC safe information is a theoretically motivated data analysis strategy



The Energy Mover's Distance

Optimal transport allows us to triangulate the space of collider events and define an emergent geometry

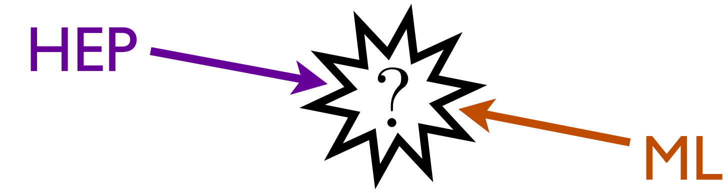


Revealing a Hidden Geometry

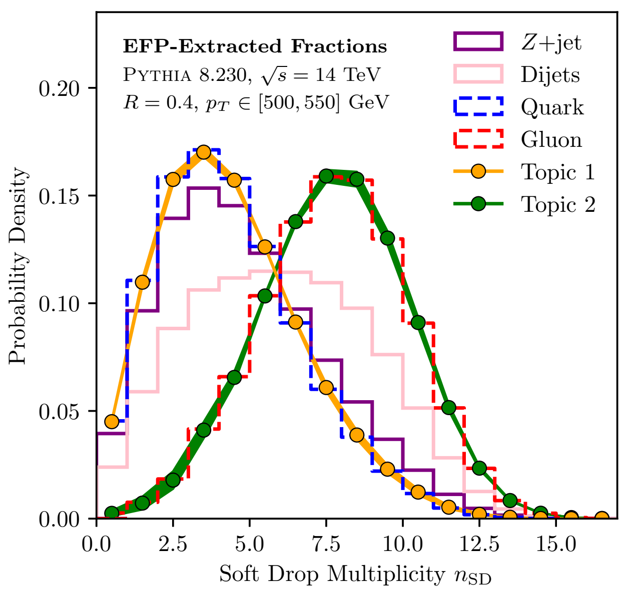
We can gain new perspectives on concepts/techniques in QFT and collider physics from the last half century

Backup Slides

Recent Collisions

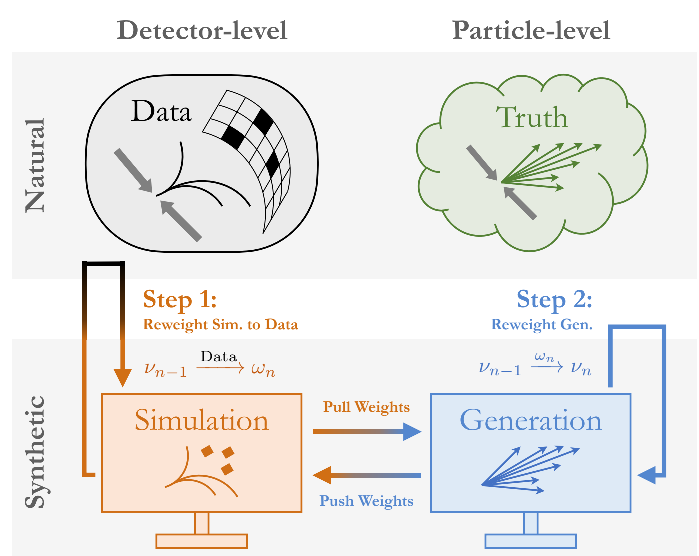


Quark/Gluon Definitions via Blind Source Separation



[Komiske, Metodiev, JDT, JHEP 2018; Brewer, JDT, Turner; PRD 2021]

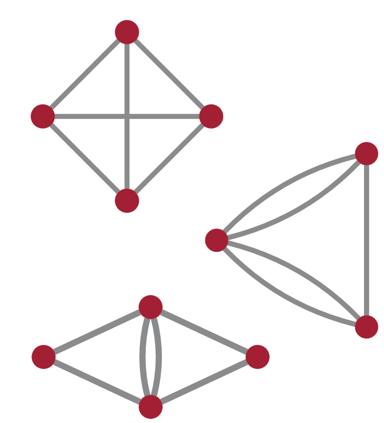
Detector Deconvolution via Binary Classification



[Andreassen, Komiske, Metodiev, Nachman, JDT, PRL 2020]

Kinematic Decomposition via Graph Theory

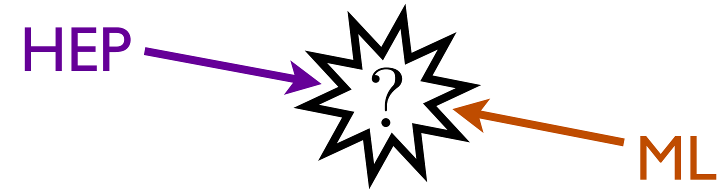
	Leafless Multigraphs	
	Connected	All
Edges d	A307317	A307316
1	0	0
2	1	1
3	2	2
4	4	5
5	9	11
6	26	34
7	68	87
8	217	279
9	**718**	**897**
10	**2 553**	**3 129**
11	**9 574**	**11 458**
12	**38 005**	**44 576**
13	**157 306**	**181 071**
14	**679 682**	**770 237**
15	**3 047 699**	**3 407 332**
16	**14 150 278**	**15 641 159**



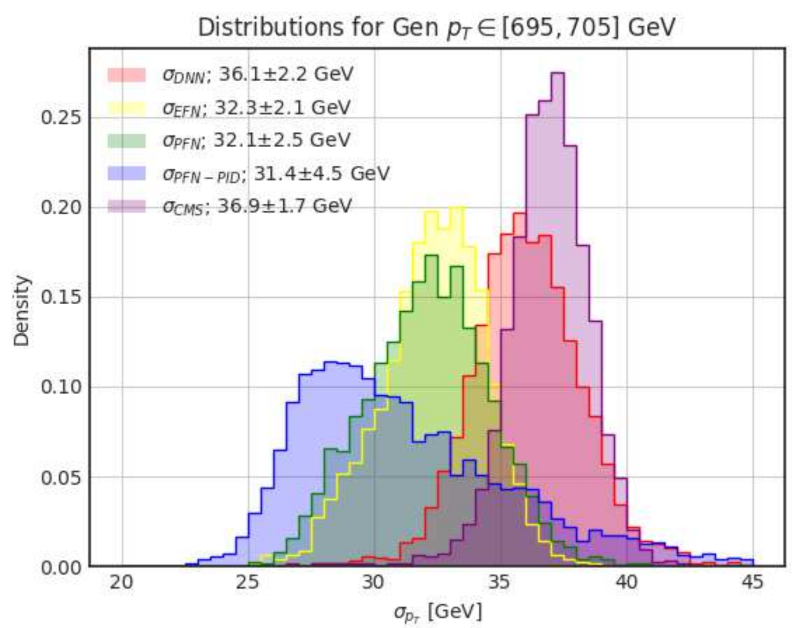
[Komiske, Metodiev, JDT, JHEP 2018, PRD 2020]

High Energy Physics ⇔ Mathematics, Statistics & Computer Science

Ongoing Collisions

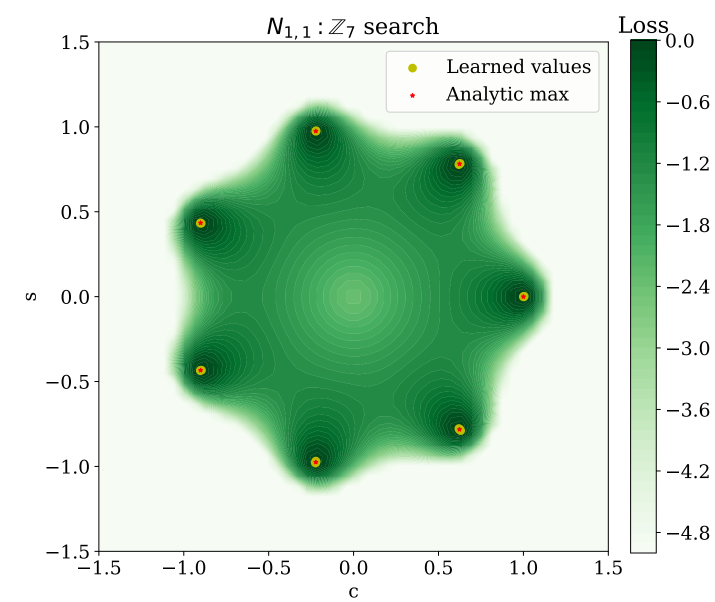


Frequentist Jet Calibration
via Gaussian Ansatz



[Nachman, Gambhir, JDT, in progress]

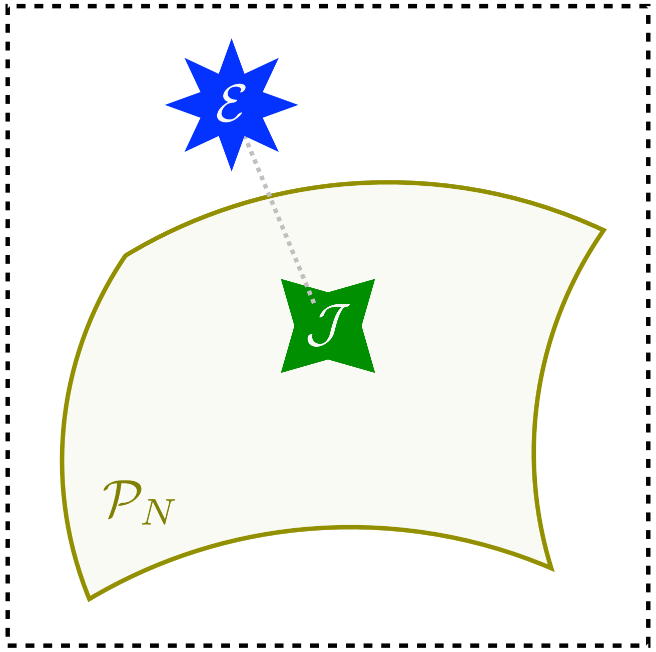
Symmetry Discovery
via Adversarial Learning



[Desai, Nachman, JDT, NeurIPS 2021 ML4PS]

High Energy Physics ⇔ Mathematics, Statistics & Computer Science

More Fun with N-particle Manifolds



N-jettiness

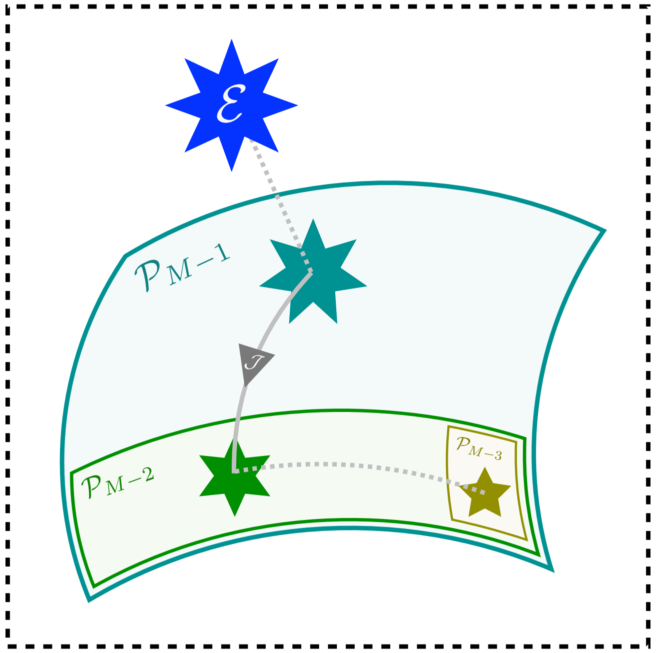
Distance of closest approach to N-particle manifold

[Brandt, Dahmen, ZPC 1979; Stewart, Tackmann, Waalewijn, PRL 2010]

Exclusive Cone Jet Finding

Point of closest approach on N-particle manifold

[Stewart, Tackmann, JDT, Vermilion, Wilkason, JHEP 2015]



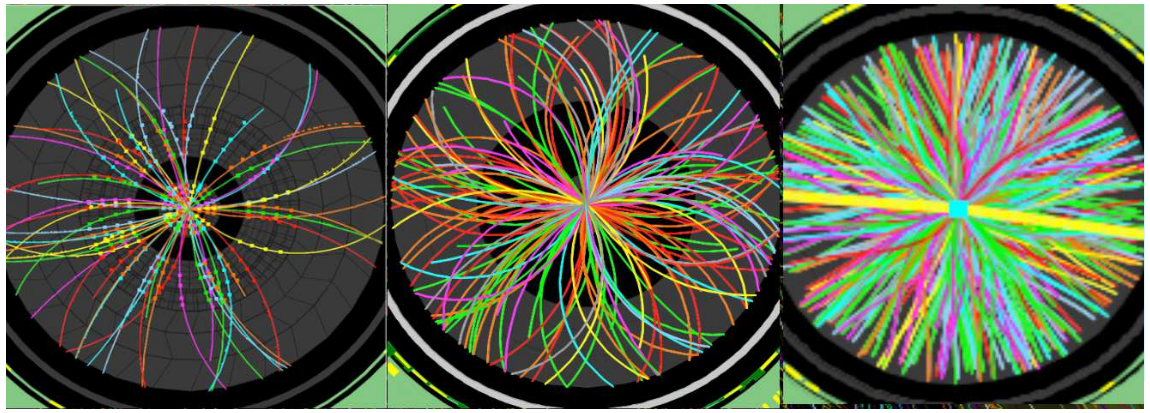
Sequential Jet Recombination

Iteratively stepping between various N-particle manifolds

[Catani, Dokshitzer, Seymour, Webber, NPB 1993; Ellis, Soper, PRD 1993]
[Dokshitzer, Leder, Moretti, Webber, JHEP 1997; Wobisch, Wengler, arXiv 1999]
[Butterworth, Couchman, Cox, Waugh, CPC 2003; Larkoski, Neill, JDT, JHEP 2014]

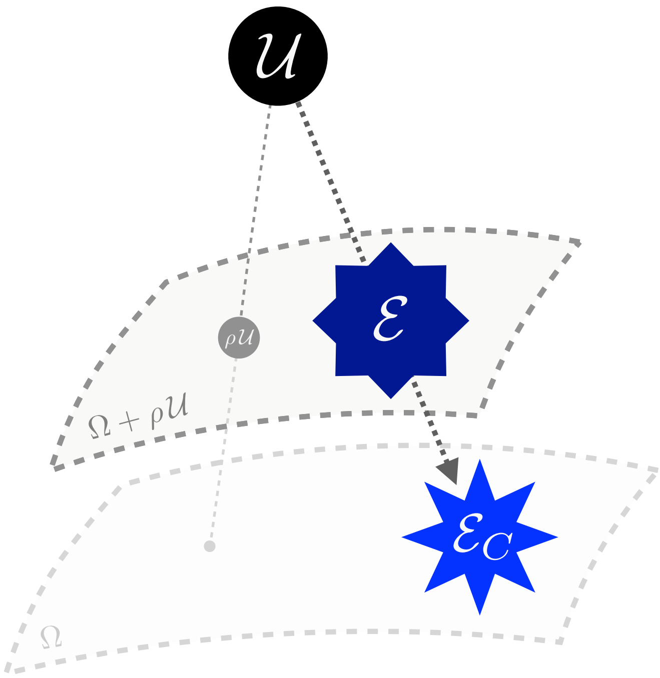
[Komiske, Metodiev, JDT, JHEP 2020]

Pileup Mitigation



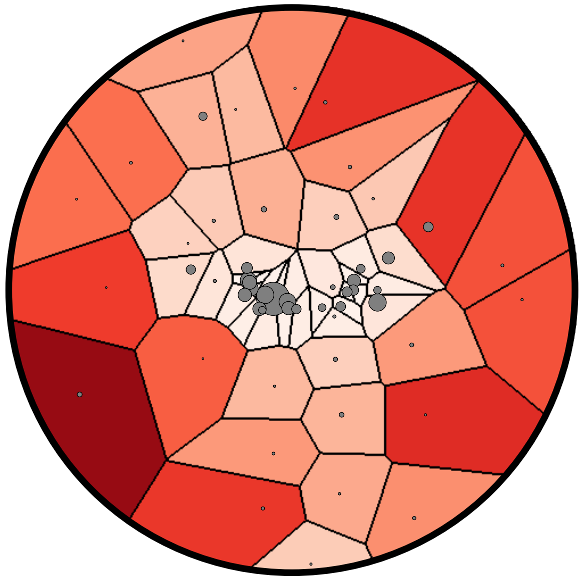
[see review in Soyez, PR 2019]

Uniform event contamination from overlapping proton-proton collisions



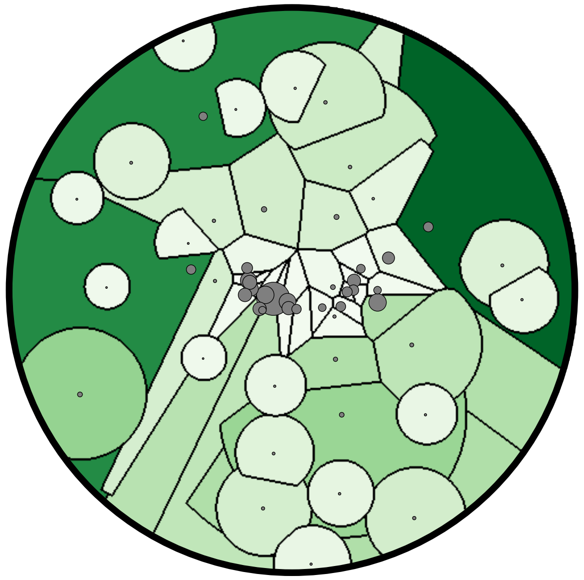
Pileup Mitigation:
"Move away" from uniform event

Voronoi



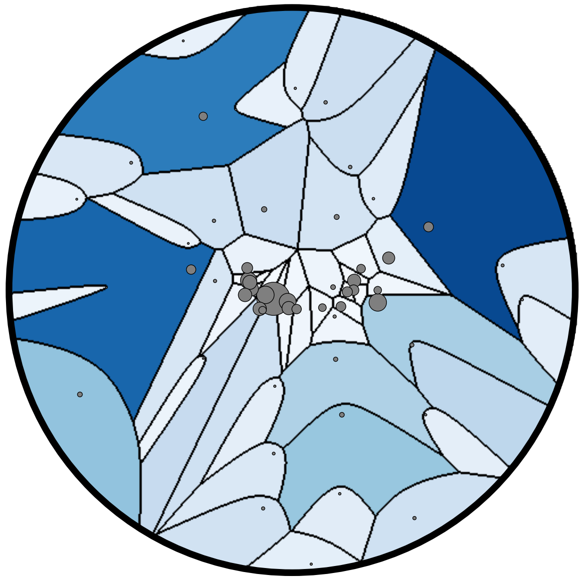
[Cacciari, Salam, Soyez, JHEP 2008]

Constituent Subtraction



[Berta, Spousta, Miller, Leitner, JHEP 2014]

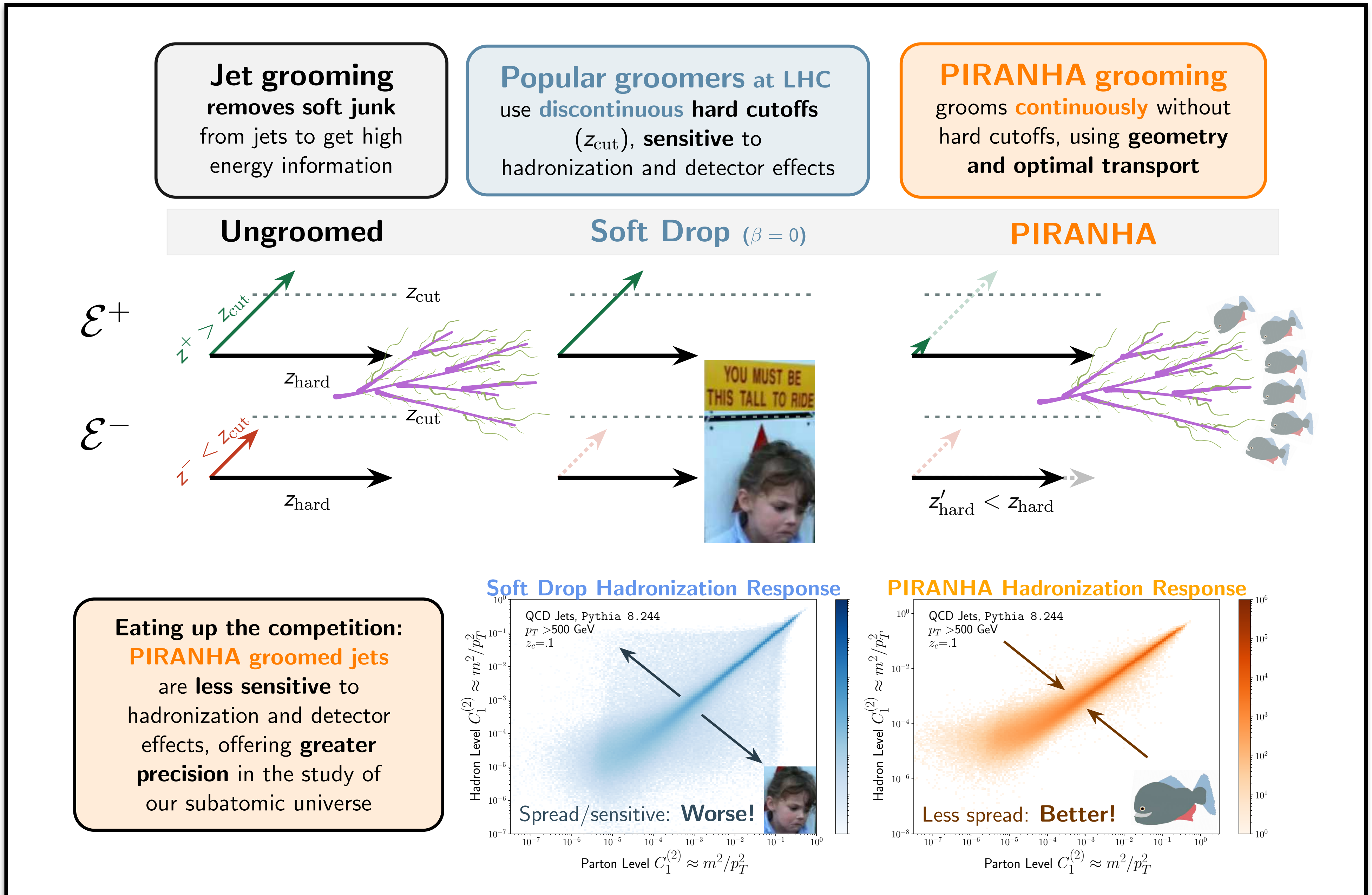
Apollonius



[Komiske, Metodiev, JDT, JHEP 2020]

Pileup and Infrared Radiation AnNiHilAtion

Recursive Safe Subtraction: tree-based approx. to optimal transport grooming



[Slides from Sam Alipour-fard]
[Alipour-fard, Komiske, Metodiev, JDT, in progress]

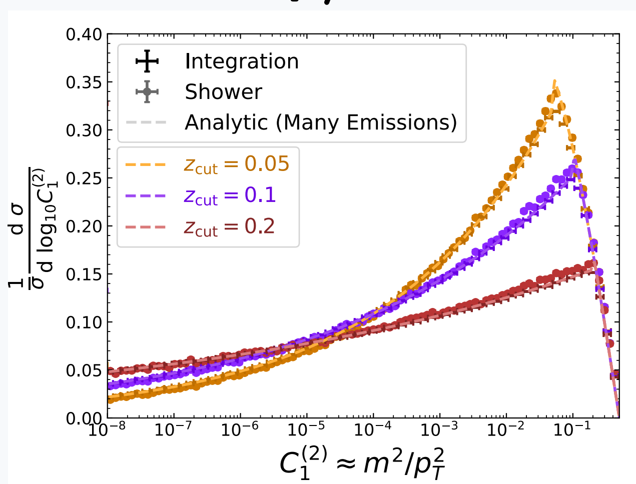


Pileup and Infrared Radiation AnNiHilAtion

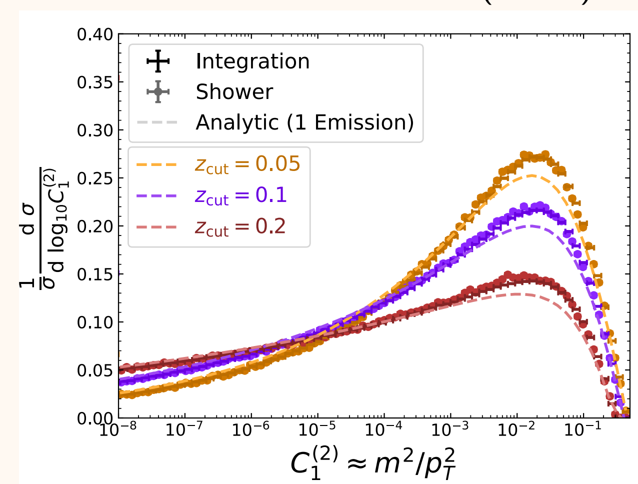
Recursive Safe Subtraction: tree-based approx. to optimal transport grooming

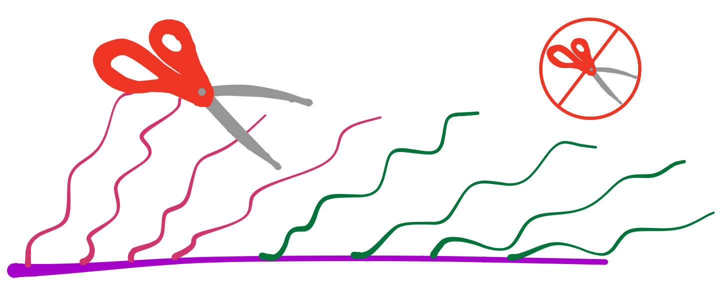
Fixed coupling, **multiple emission** calculations:

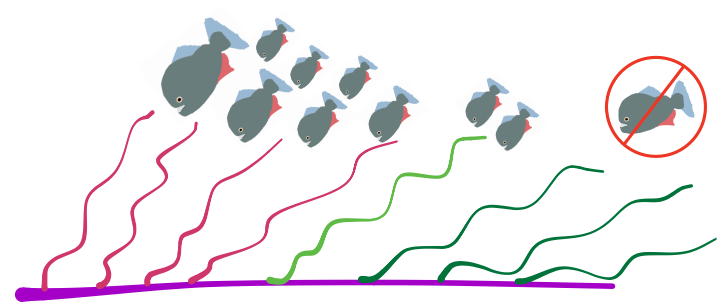
Soft Drop/mMDT



PIRANHA-RSS $(f = 1)$







Sharp cutoff $\rightarrow$ kink

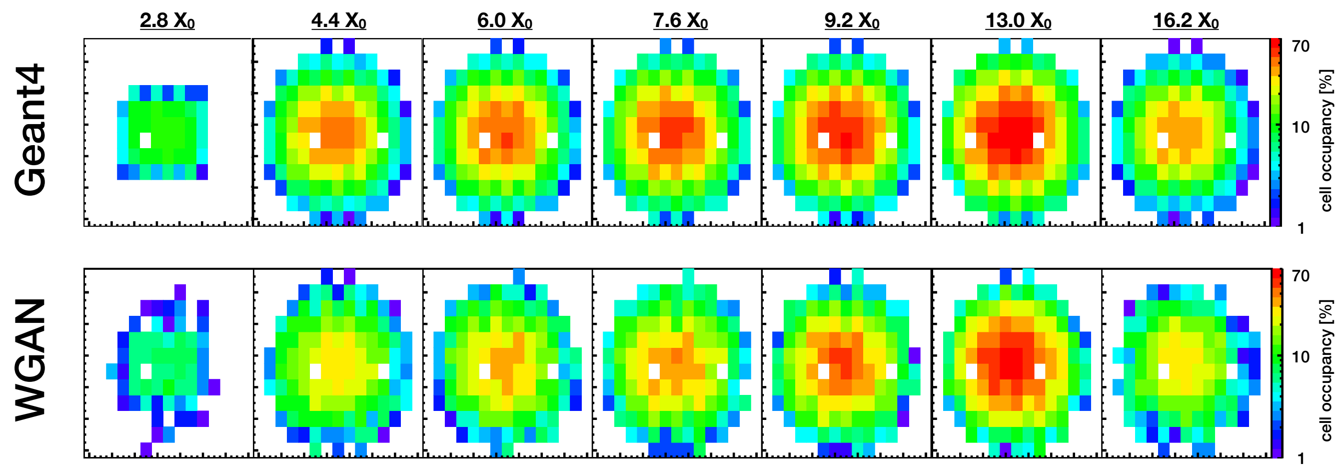
No sharp cutoff $\rightarrow$ smooth

[Slides from Sam Alipour-fard]
[Alipour-fard, Komiske, Metodiev, JDT, in progress]



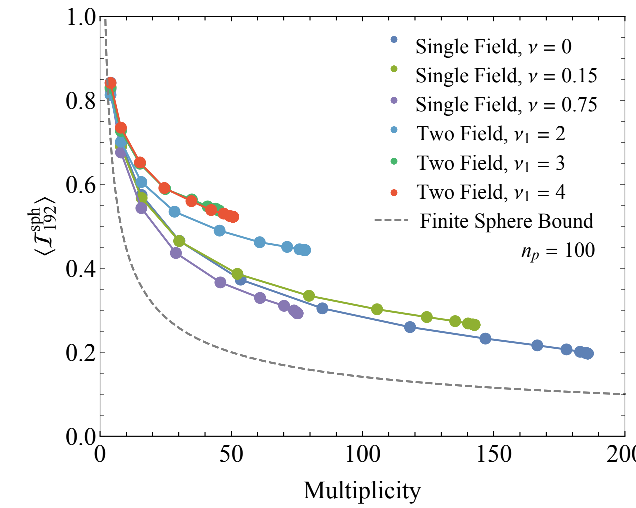
Wasserstein in HEP

Generative Modeling



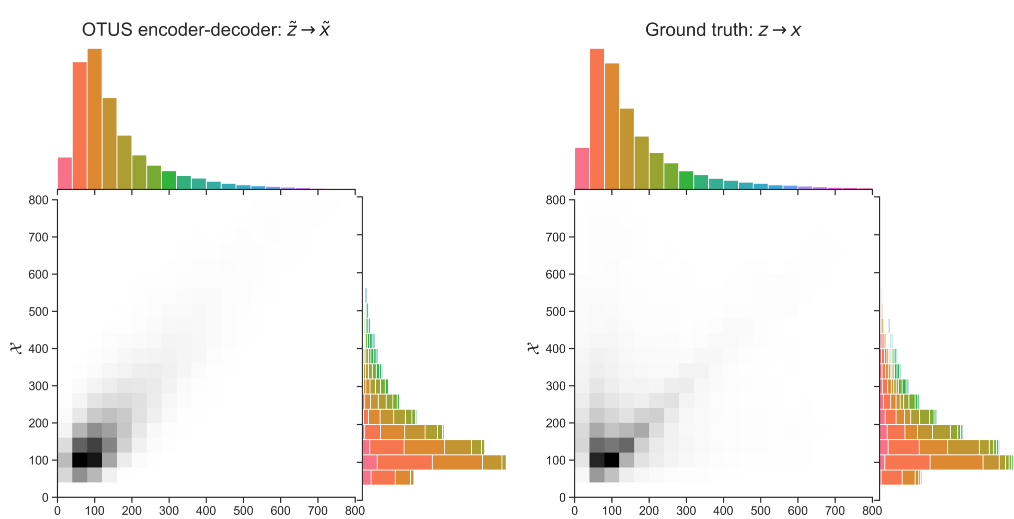
[Erdmann, Geiger, Glombitza, Schmidt, CSBS 2018; Erdmann, Glombitza, Quast, CSBS 2019; Chekalina, Orlova, Ratnikov, Ulyanov, Ustyuzhanin, Zakharov, CHEP 2018]

BSM Characterization



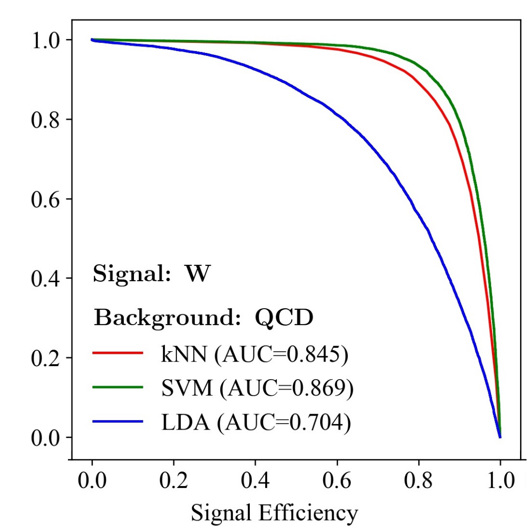
[Cesarotti, Reece, Strassler, JHEP 2021, arXiv 2020]

Estimated Simulation/Unfolding



[Howard, Mandt, Whiteson, Yang, arXiv 2021]

Jet Classification



[Cai, Cheng, Craig, Craig, PRD 2020]

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