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Casimir Meets Poisson (Sorry Sudakov)

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UT Austin

Chris Frye, Andrew Larkoski,
JDT, Kevin Zhou

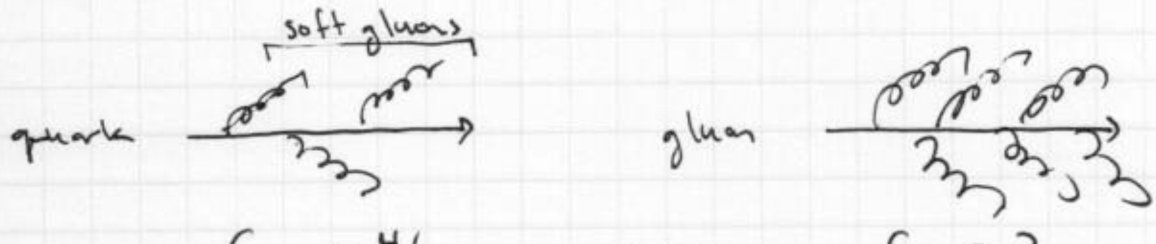
17xx.xxxxx

My colloquium this week is about jets. Incredible progress using jets for new physics searches and for pushing frontiers of (perturbative) QCD 🎉

Today, I want to take you behind the scenes of a rather technical issue, but hopefully convince you that analytic methods can yield important insights into jet physics.

Question: How to optimally distinguish quark-initiated from gluon-initiated jets?

At lowest order:



$$C_F = 4/3 \quad \text{vs.} \quad C_A = 3$$

(Casimir factors)

$$3/\tfrac{4}{3} = 9/4 \simeq 2$$

0^{th}-order question?

Are quark & gluon jets well-defined?

Yes! In the same sense as jets themselves are well-defined once you choose a jet algorithm.

More accurately: quark-enriched vs. gluon-enriched.

e.g. $pp \to Z + \text{jet}$ ← quark-enriched.

$pp \to \text{dijet}$ ← gluon enriched

1^{st}-order question?

Is distinguishing quarks vs. gluons useful?

Absolutely! e.g. SUSY cascade decays give you mainly quark jets.

$$\tilde{g} \to q \tilde{q} \hookrightarrow q \tilde{\chi}$$

Backgrounds are more gluon-enriched

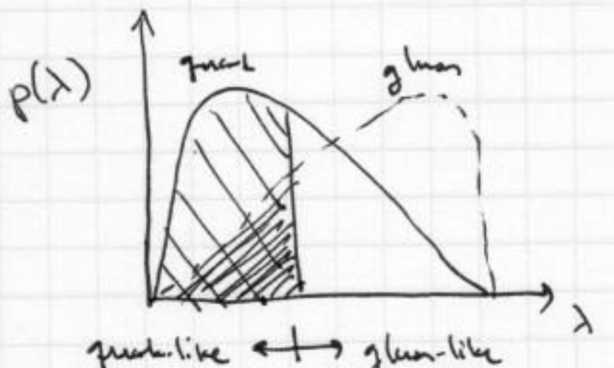
2^{nd}-order question?

Can you get any information beyond C_F vs C_A?

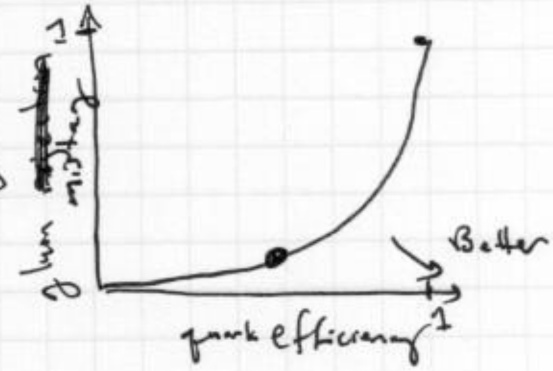
Sure! Subleading terms in splitting function, spin correlations, matrix element effects.

Today's question: If you *only* have C_F vs. C_A, what is the optimal separation power?

Measure property of jet : λ "ROC curve"



$\Longrightarrow$



Fact 1: almost all IRC safe discriminants have

$$\underset{20\%}{\text{gluon mis.}} = \underset{50\%}{(\text{quark eff.})}^{C_A/C_F}$$

(I'll prove later)

$$\frac{Q}{\sqrt{G}} \Longrightarrow \frac{Q}{\sqrt{G}}\frac{\varepsilon_q}{\sqrt{\varepsilon_g}} = \frac{Q}{\sqrt{G}}\,\varepsilon_q^{\left(1-\frac{1}{2}\frac{C_A}{C_F}\right)}$$

almost zero!

Need to have multijet final state for this to be worthwhile.

Fact 2: Hadron multiplicity (IRC unsafe) is close to a factor of 2 better!

50% quark eff ⇒ 10% gluon rej.

Just counting hadrons? Is this better because it is IRC unsafe, or is there other physics at play?

Answer: Key is Poisson distribution (multiplicity)
versus Sudakov distribution (e.g. mass)

We have an IRC safe version of multiplicity.

Matches performance of hadron multiplicity with calculability of jet mass.

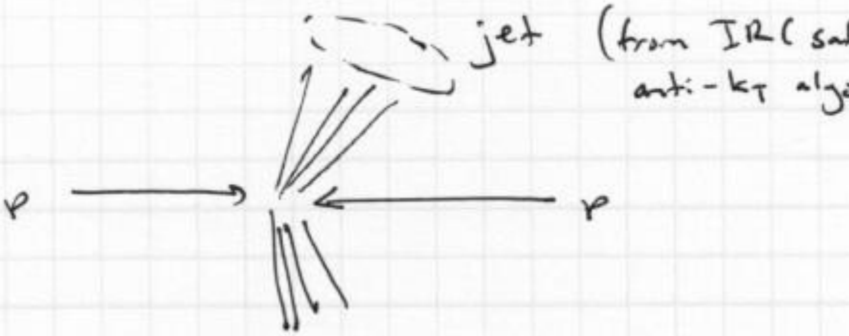
Infrared / Collinear Safety:

Condition to be calculable order-by-order in perturbation theory.

(In colloquium, I'll show an example where you can go beyond IRC safety but still be calculable.)

Key: real emissions $\Longleftrightarrow$ virtual emissions.

Examples:



jet p_T: $\approx \sum_i p_{Ti}$ IRC safe

jet p_T^D: $= \dfrac{\sqrt{\sum_i p_{Ti}^2}}{\sum_i p_{Ti}}$ IR safe, C <u>unsafe</u>

jet mass: $\sim \sum_{ij} 2 p_i \cdot p_j$ IRC safe

hadron multiplicity in jet: $= \sum_i 1$ IRC <u>unsafe</u>

Standard Lore:

IRC safe $\Rightarrow$ Calculable in pQCD

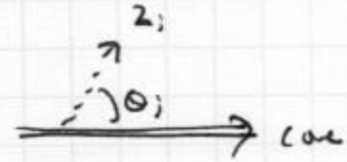
C unsafe but IR safe $\Rightarrow$ Calculable with fragmentation functions.

IRC unsafe $\Rightarrow$ No luck.

Always a good idea to challenge the standard lore, but we'll stick to the standard picture here.

Typical quark/gluon discriminant.

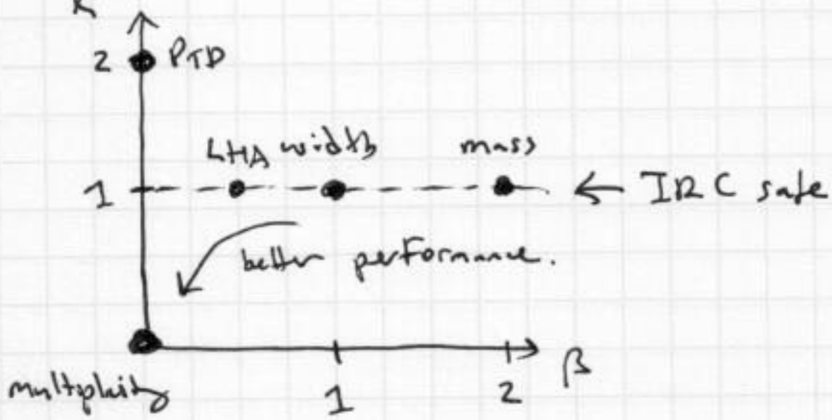
$$\lambda^{\kappa}_{\beta} = \sum_{i \in \text{jet}} z_i^{\kappa}\, \theta_i^{\beta}$$



$$z_i = \frac{p_{Ti}}{p_{T\,\text{jet}}}$$

$$\theta_i \approx R_i / R$$

jet radius.



Let's do a quick and dirty QCD calculation!

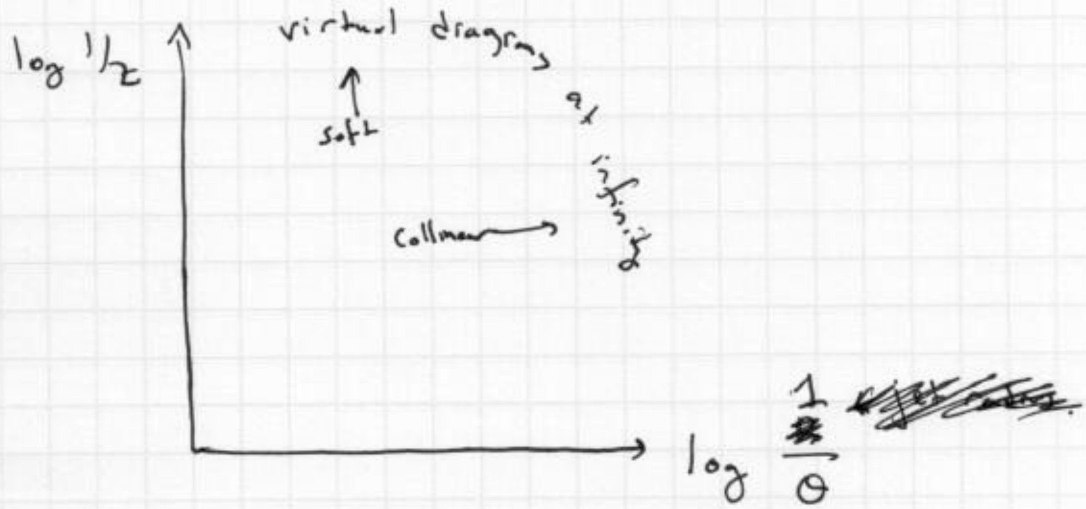
$$dP_{i \to ig} = \frac{2\alpha_s}{\pi}\, C_i\, \frac{d\theta}{\theta}\, \frac{dz}{z}$$

Casimir factor
$C_F = 4/3$
$C_A = 3$

Collinear singularity

Soft singularity

Uniform emissions in $\log\theta$, $\log z$ plane.

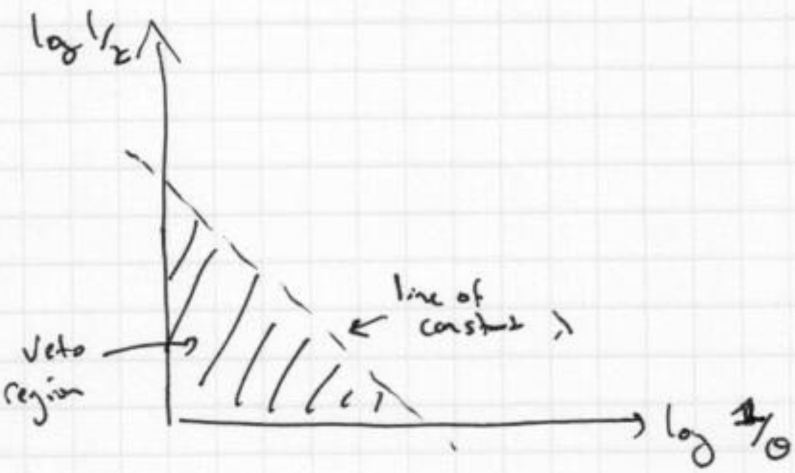


$\kappa = 1$ IRC safe

This argument works for any $\beta > 0$

What is cumulative probability to have $\lambda < \lambda_{max}$?

Can't have any emissions that contribute more than λ_{max} to the observable.



$$\log 1/\lambda \simeq \cancel{\kappa}^{\,1} \log 1/z + \beta \log 1/\theta$$

$$\Sigma(\lambda_{max}) = e^{-\frac{2\alpha_s}{\pi} C_i \, \lambda_{max}}$$

Sudakov exponent.

$$\Sigma_{gluon}(\lambda_{max}) = \left(\Sigma_{quark}(\lambda_{max})\right)^{C_A/C_F}$$

True for any Sudakov-vetoed observable at LL order.

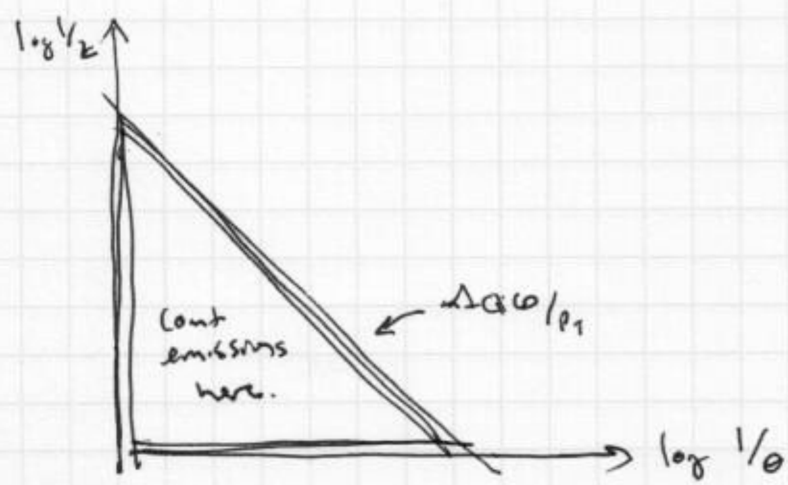
This is why quark/gluon separation (using IRC observables) is so hard.

How can you break Casimir scaling?

Still has to be function of C_A & C_F, but need to get rid of veto.

Here's the idea: Count the number of emissions in some perturbative region of phase space.

$$\text{perturbative} \Rightarrow \quad z\theta > \frac{\Lambda_{QCD}}{p_T R}$$



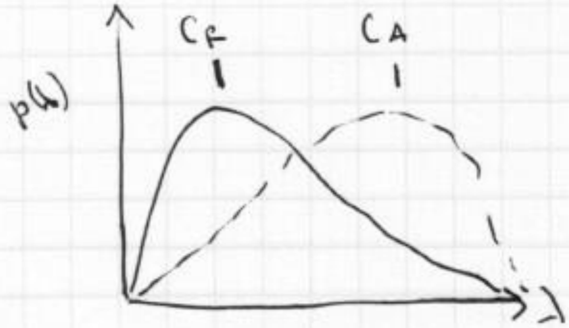
$$P(n_{emit}) = \frac{\lambda_{ave}^{n_{emit}}\, e^{-\lambda_{ave}}}{n_{emit}!} \quad \text{(Poisson process)}$$

$$\lambda_{ave} = \frac{2\alpha_s C_i}{\pi}$$

$$\frac{1}{2}\log^2 \frac{2\Lambda_{QCD}}{p_T R}$$

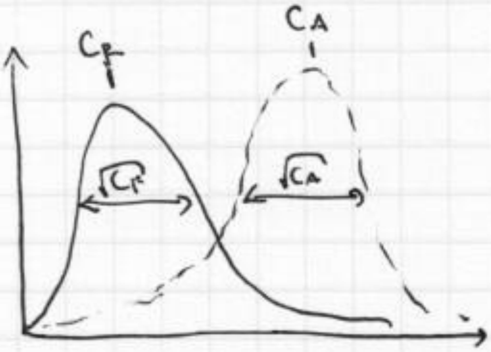
Still a function of C_i, but as $p_T R \to \infty$, becomes more like Gaussian, overlap decreases.

Sudakov distributions



widths don't change with p_T very much.

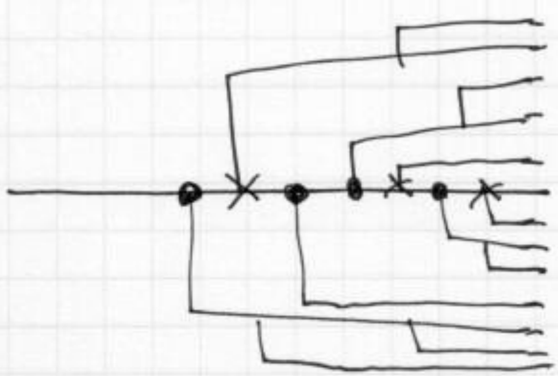
Poisson distributions



As you go to higher p_T, widths get smaller and smaller, overlap decreases.

Is there an observable that actually lets you count in an IRC safe way?

yes! "Soft drop multiplicity"



Recurse through trunk of tree, and count number of emissions where

$$z\theta > \frac{\Lambda_{QCD}}{p_T R}.$$

Because built on tree, ~~[illegible]~~ because of $z\theta$ cut, this is IRC safe.

Lesson: whether or not you care about quark/gluon separation, you should care about having analytic control over jet observables.

Push towards highly automated machine learning for jet physics. This is a good thing, because it highlights places where we can exploit new information

But "Deep Learning" needs to feedback into "Deep Thinking". Determine which information is being used, and then go back and calculate from first principles.

For my colloquium, I will show an observable using soft drop that is <u>independent</u> of C_i, which is very surprising from splitting function perspective.