

Collision Course

Particle Physics meets Machine Learning

Jesse Thaler



Harvard Physics Colloquium — November 2, 2020

The NSF AI Institute for Artificial Intelligence and Fundamental Interactions (IAIFI)



[<http://iaifi.org>, MIT News Announcement]

The NSF AI Institute for Artificial Intelligence and Fundamental Interactions (IAIFI)

“eye-phi”



*Advance physics knowledge — from the smallest building blocks of nature
to the largest structures in the universe — and galvanize AI research innovation*



Pulkit Agrawal
Lisa Barsotti
Isaac Chuang
William Detmold
Bill Freeman
Philip Harris
Kerstin Perez
Alexander Rakhlin

Phiala Shanahan
Tracy Slatyer
Marin Soljacic
Justin Solomon
Washington Taylor
Max Tegmark
Jesse Thaler
Mike Williams



Demba Ba
Edo Berger
Cora Dvorkin
Daniel Eisenstein
Doug Finkbeiner
Matthew Schwartz
Yaron Singer
Todd Zickler



James Halverson
Brent Nelson



Taritree Wongjirad

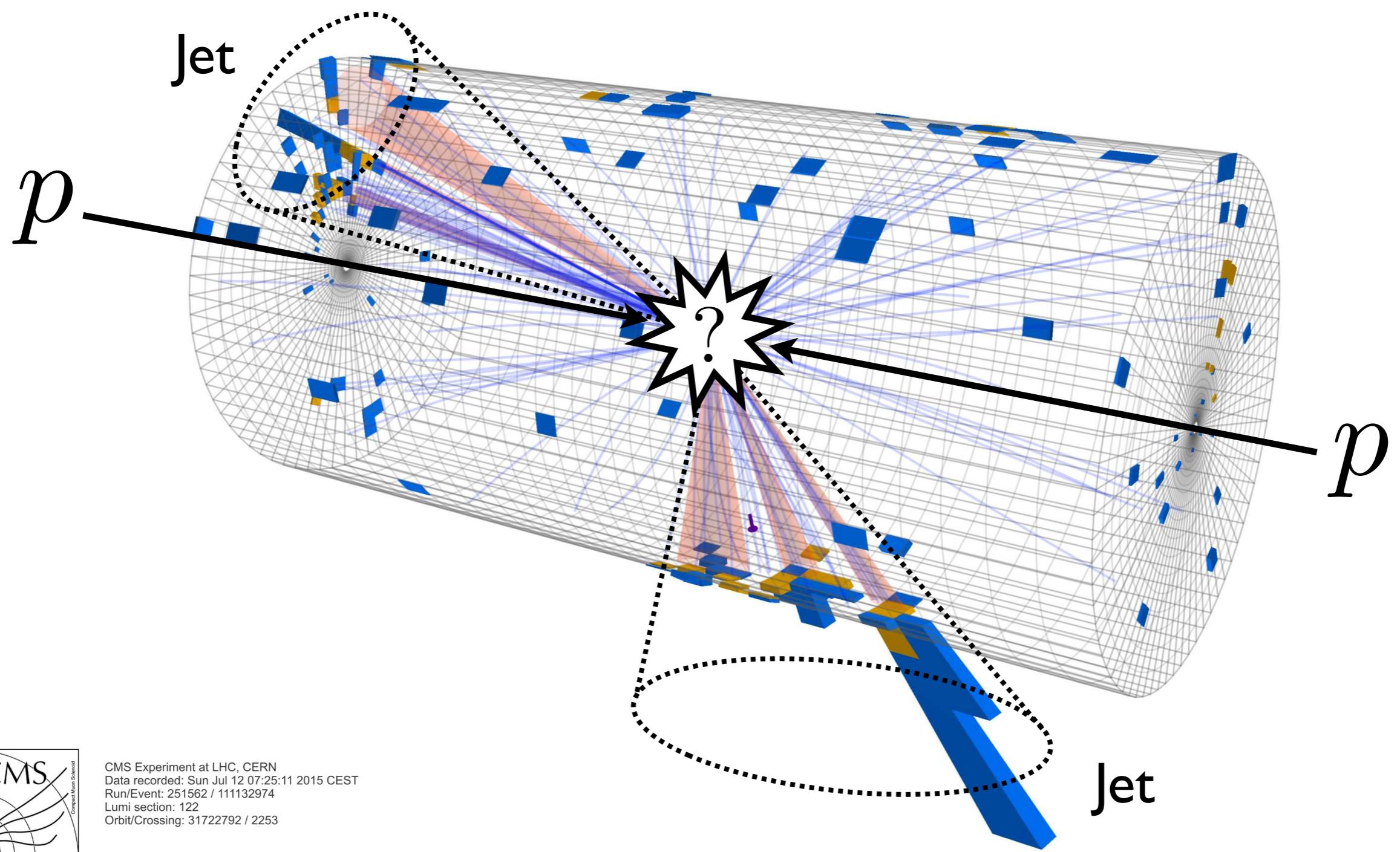


“Subway Sabbatical”

Thanks, SF !

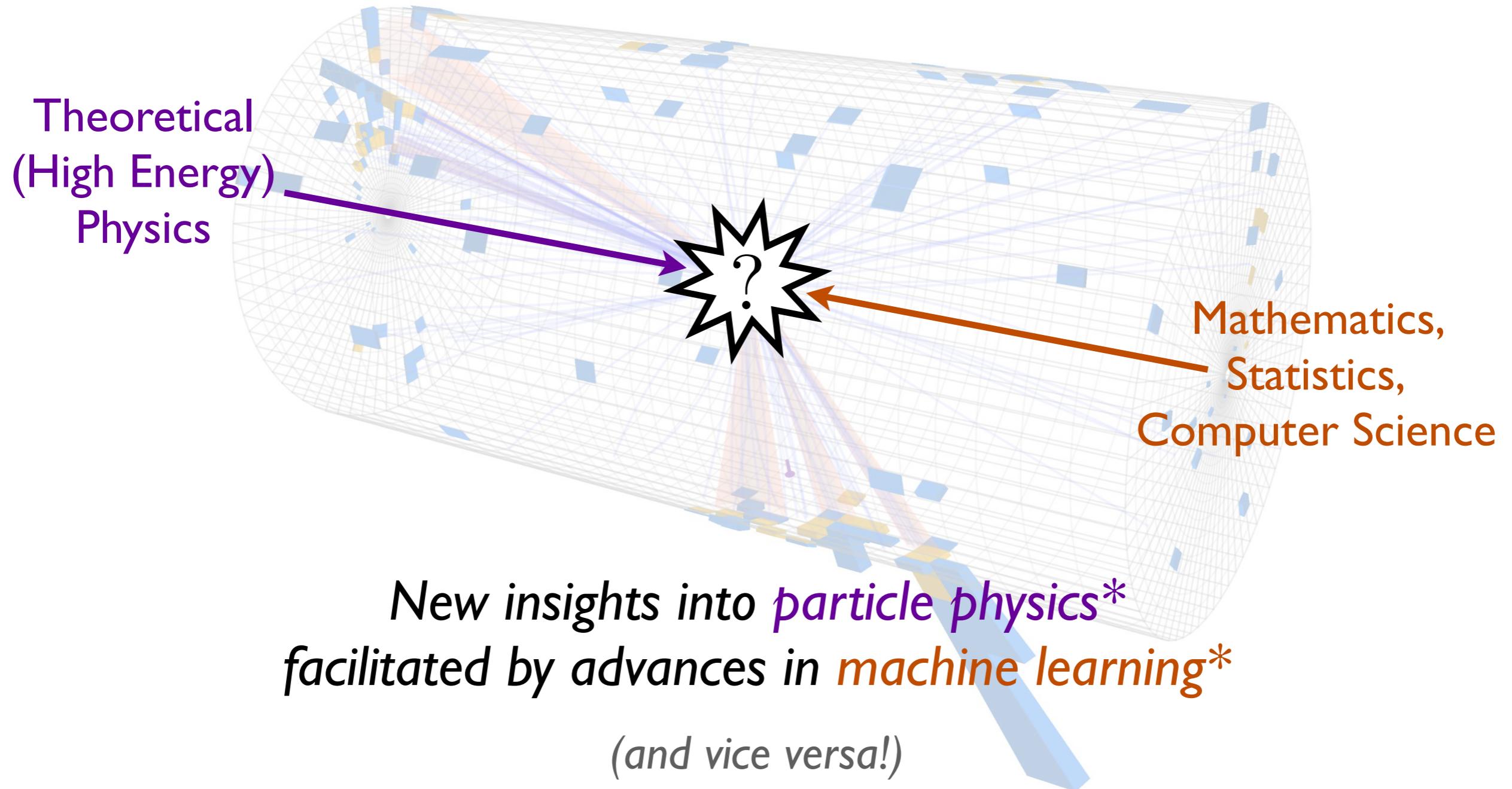


“Collision Course”



“Collision Course”

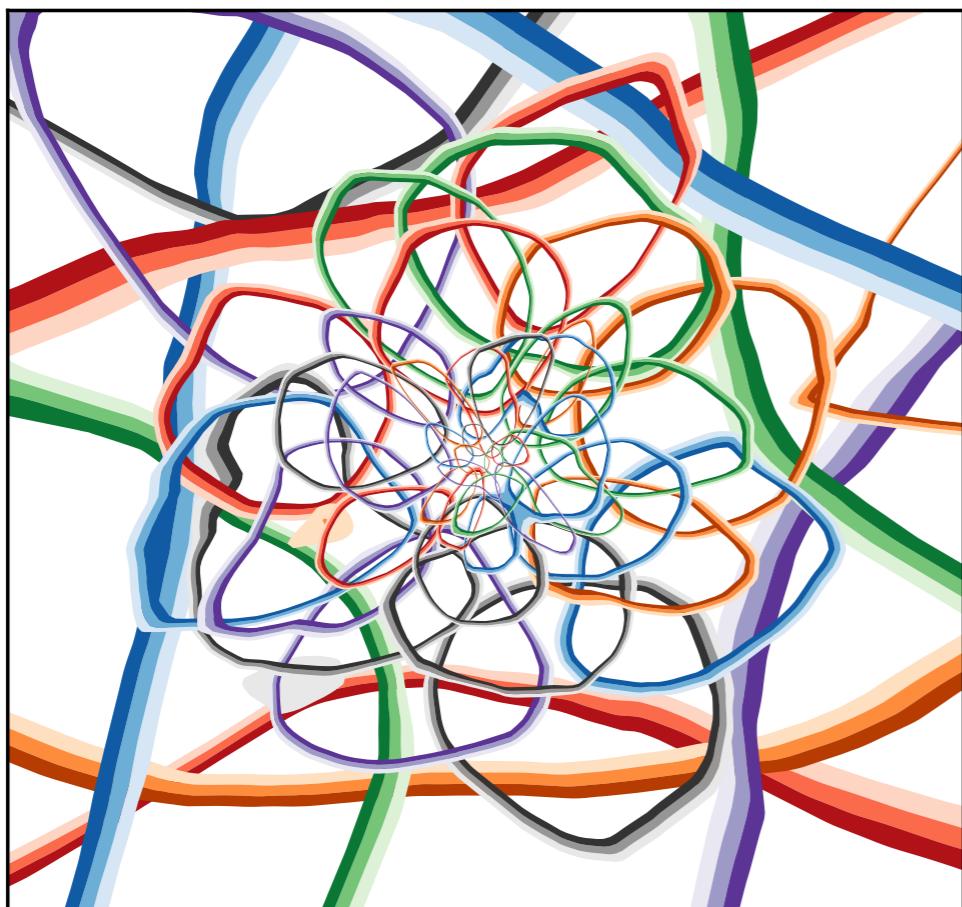
“Theoretical Physics for Machine Learning”
Aspen Center for Physics, January 2019



Two Anecdotes

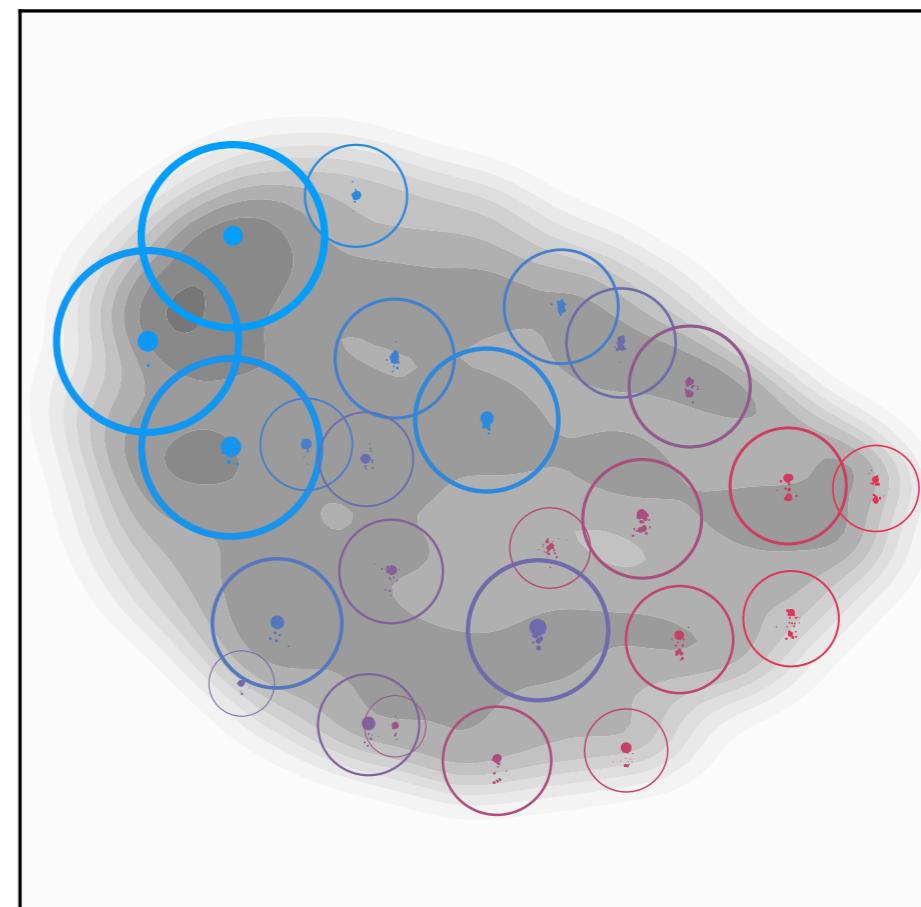


Teaching a Machine to “Think Like a Physicist”



[Komiske, Metodiev, JDT, [JHEP 2019](#)]

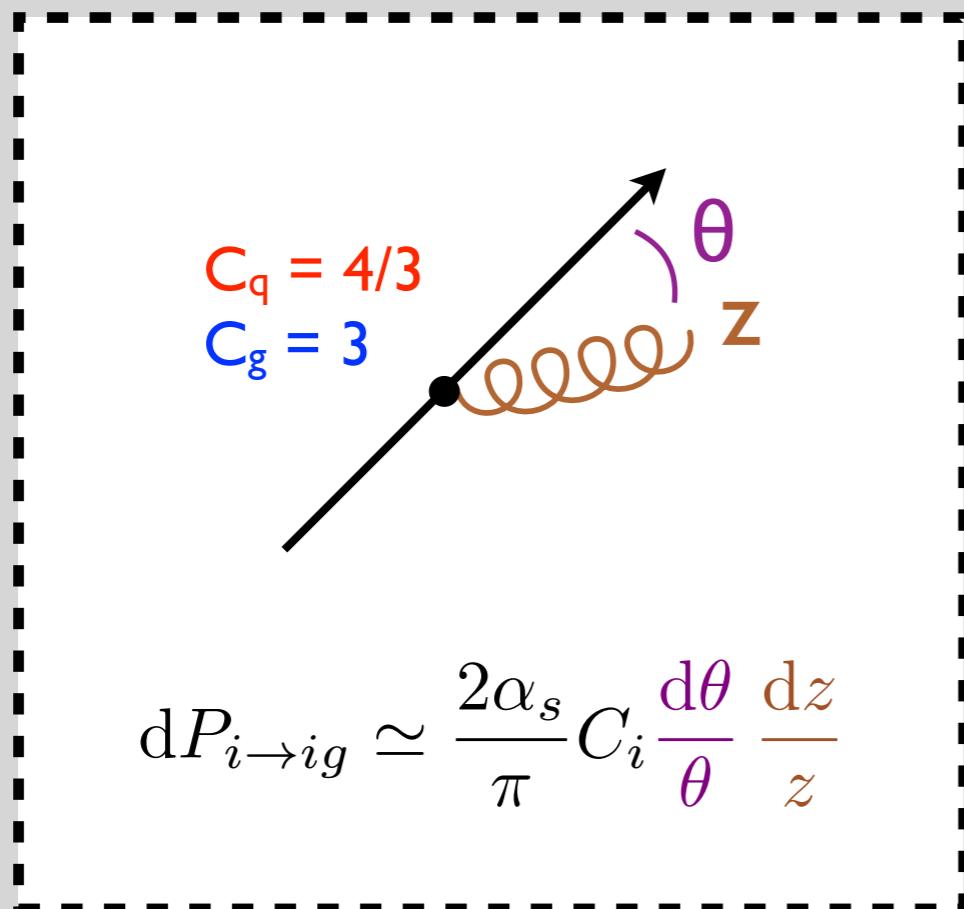
Letting Collider Data Speak for Itself



[Komiske, Mastandrea, Metodiev, Naik, JDT, [PRD 2020](#);
based on Komiske, Metodiev, JDT, [PRL 2019](#)]

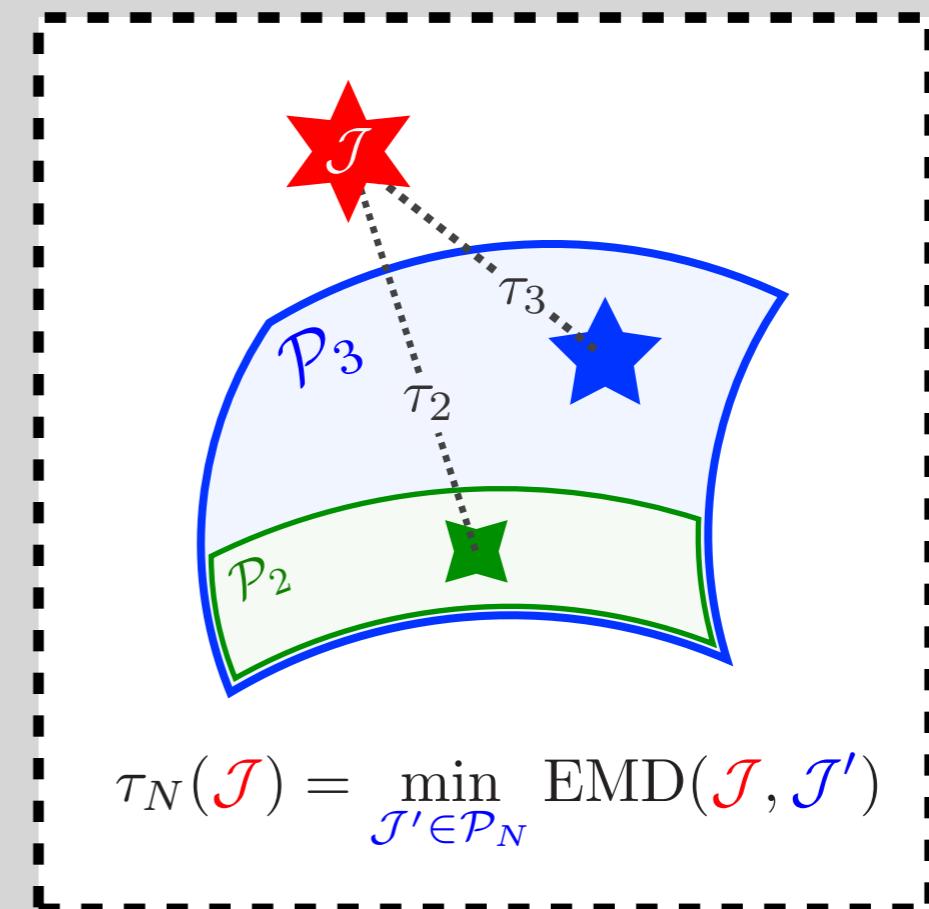
*Data analysis strategies motivated by the
symmetries and structures of particle physics*

Exploiting a Core Prediction of QCD



[Altarelli, Parisi, [NPB 1977](#)]

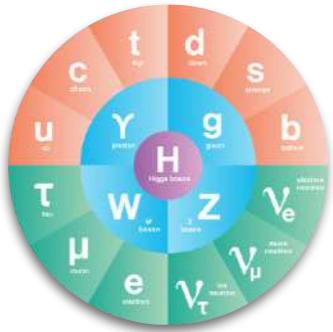
Nested Singularities of Gauge Theories



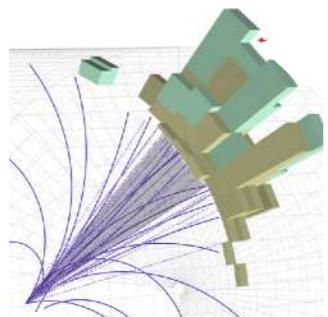
[Stewart, Tackmann, Waalewijn, [PRL 2010](#);
JDT, Van Tilburg, [JHEP 2011](#), [JHEP 2012](#);
rephrased via Komiske, Metodiev, JDT, [JHEP 2020](#)]

New perspectives on key theoretical concepts

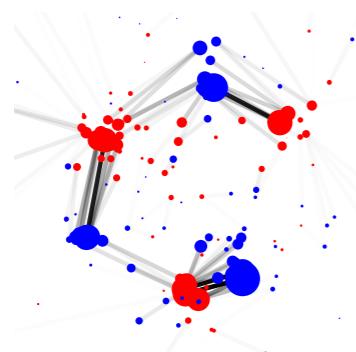
Outline



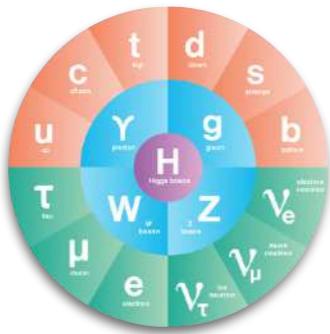
Particle Physics 101



What is a Collider Event?



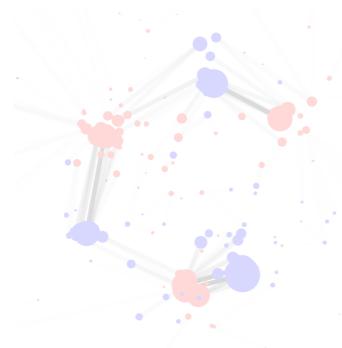
When are Collider Events Similar?



Particle Physics 101

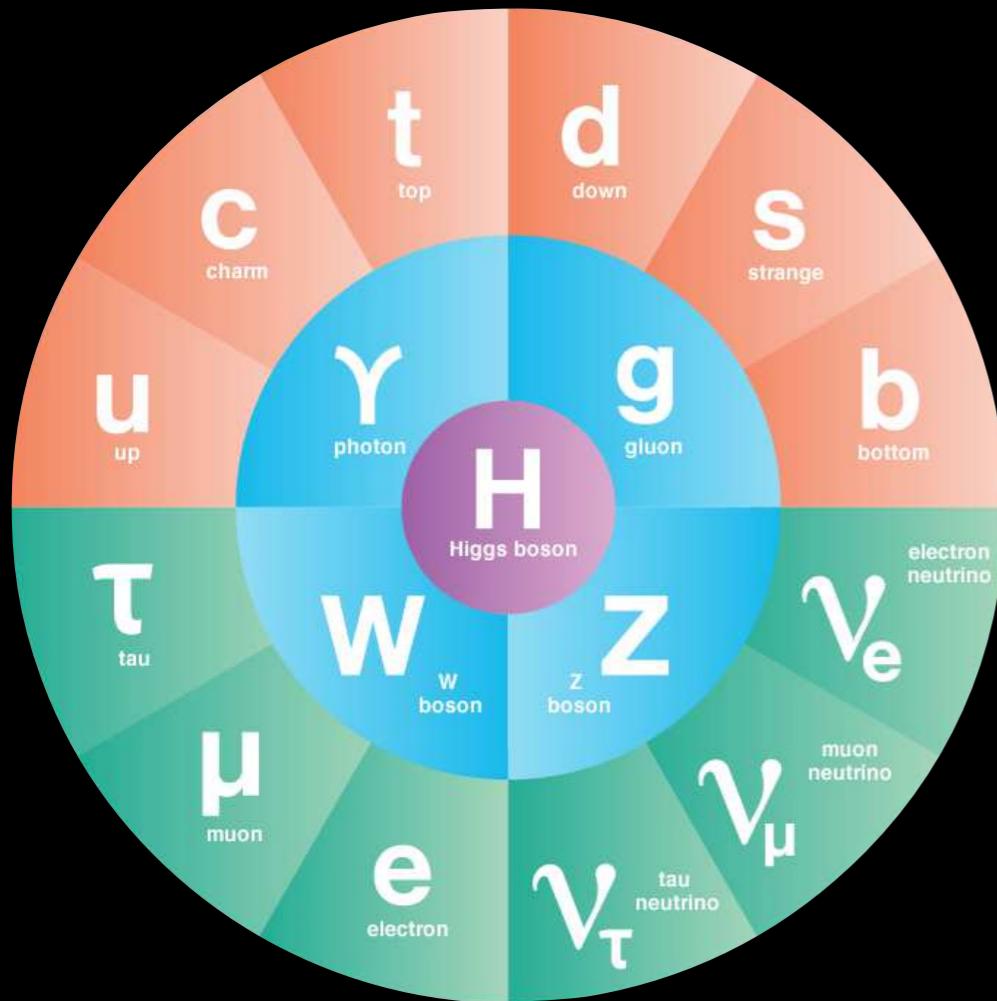


What is a Collider Event?

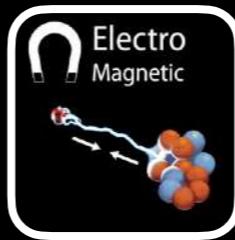
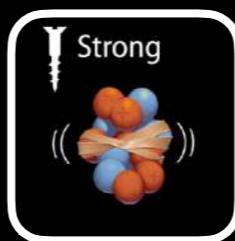


When are Collider Events Similar?

Particle Physics 101



Quarks & Gluons



γ	photon
e^\pm	electron
μ^\pm	muon
π^\pm	pion
K^\pm	kaon
K_L^0	K-long
p/\bar{p}	proton
n/\bar{n}	neutron

T E H M



γ

photon



e^+

electron



μ^+

muon



π^+

pion



K^+

kaon



K_L^0

K-long



p/\bar{p}

proton



n/\bar{n}

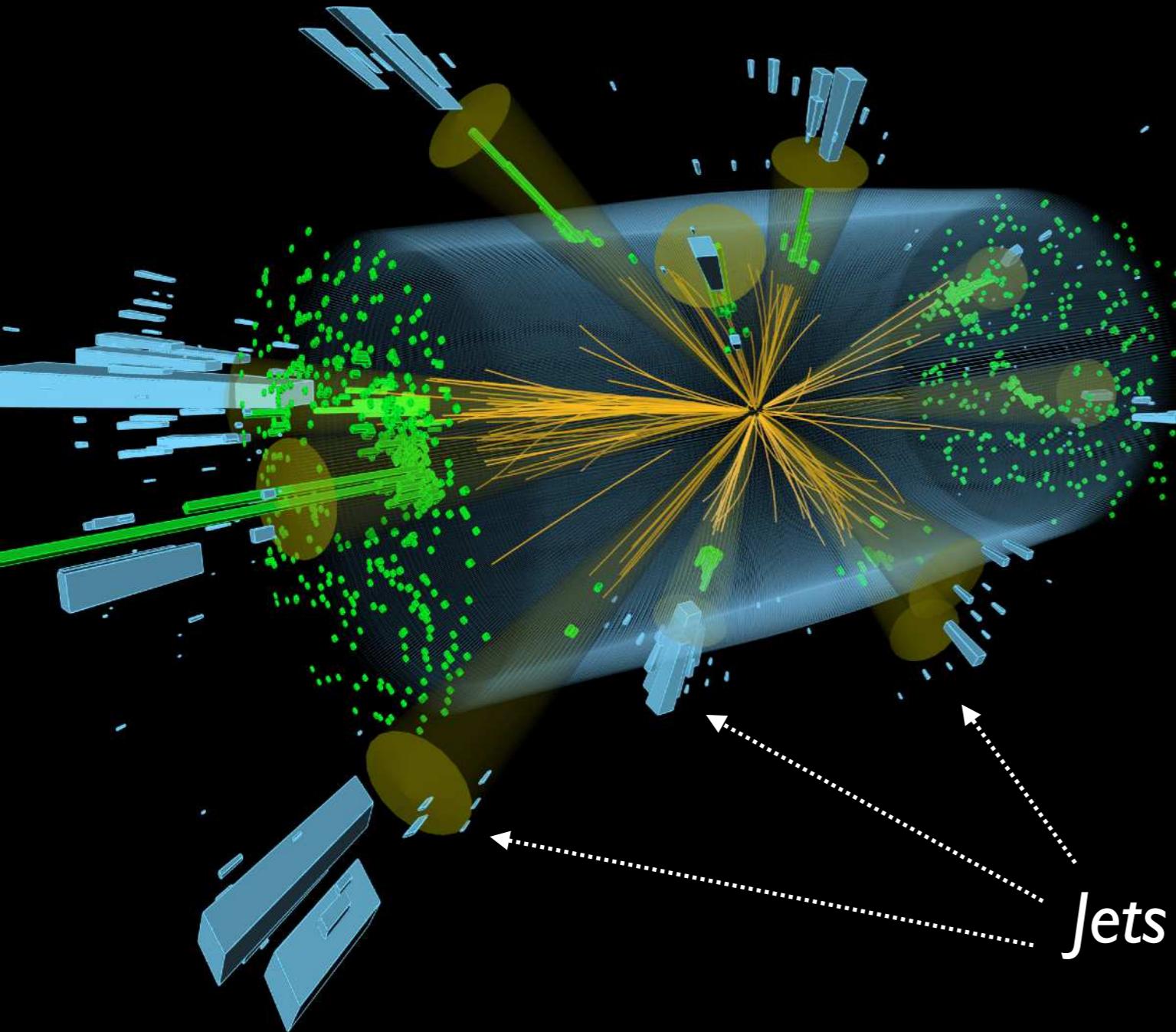
neutron

elementary

composite

Collider Event

Every 25 nanoseconds at the LHC



T E H M



γ

photon



e^+

electron



μ^+

muon



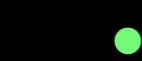
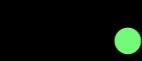
π^+

pion



K^+

kaon



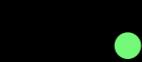
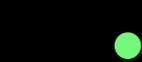
K_L^0

K-long



p/\bar{p}

proton

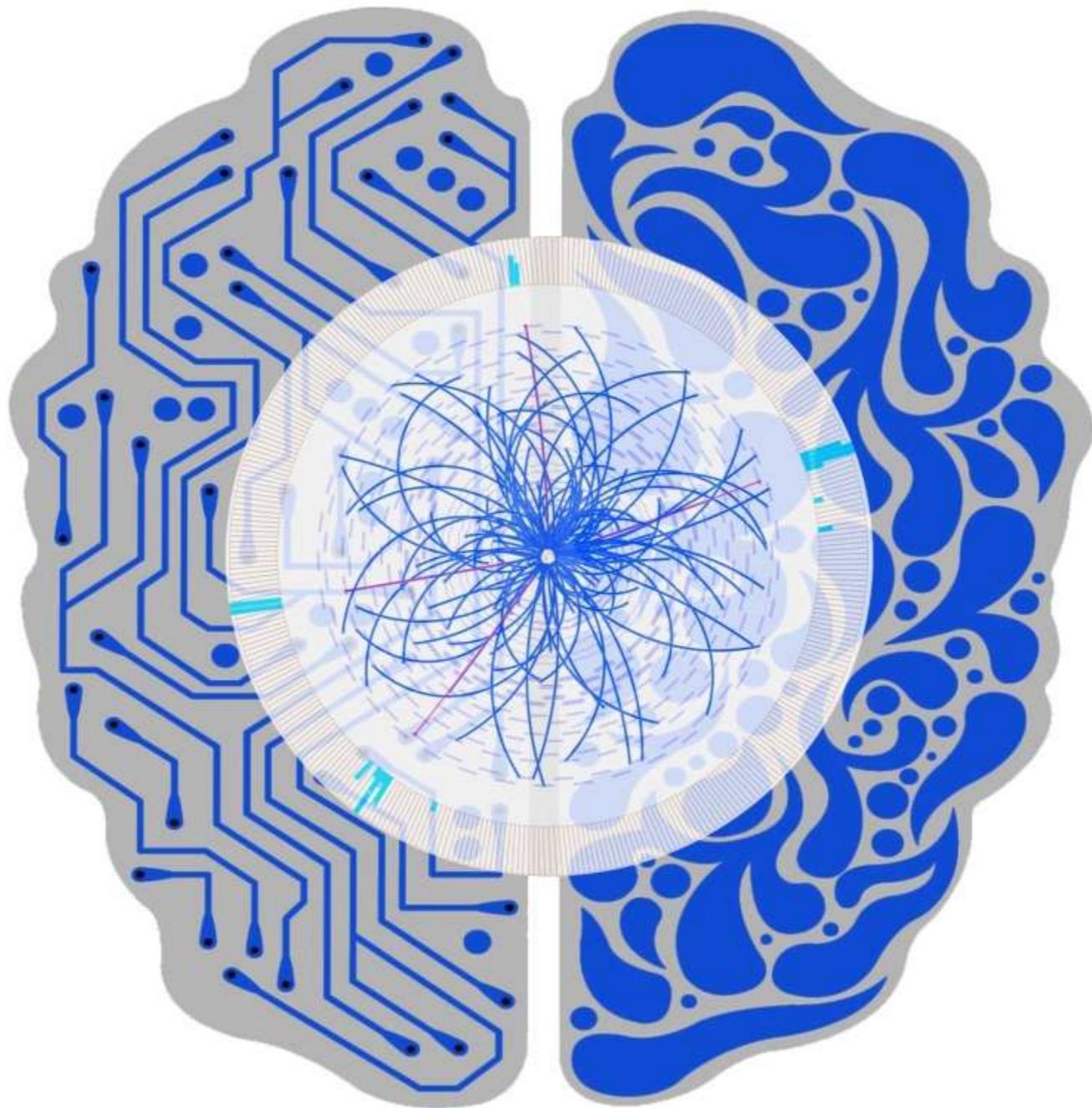


n/\bar{n}

neutron

elementary

composite



The Rise of Machine Learning

From Curmudgeon...

Jet classification via image recognition



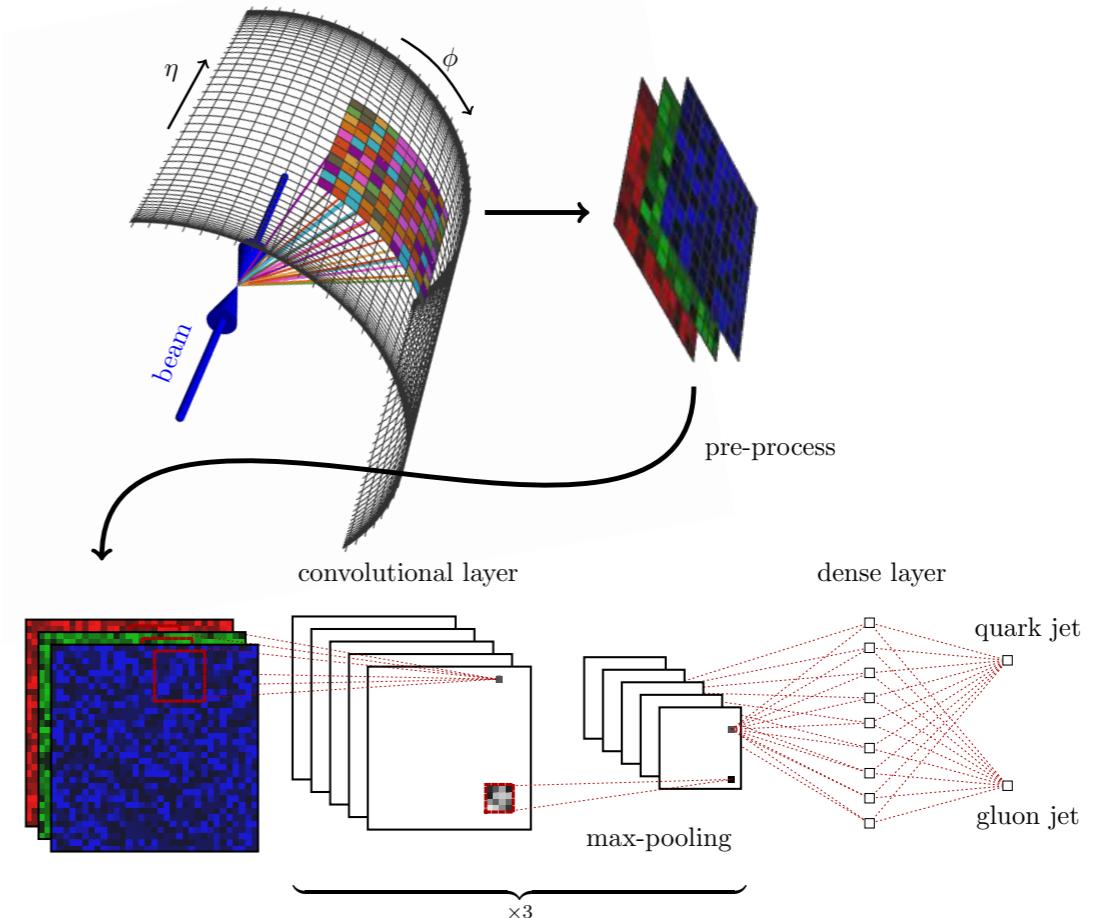
Quark

VS.



Gluon

Multi-channel convolutional neural networks



[e.g. Komiske, Metodiev, Schwartz, [JHEP 2017](#);
cf. Gras, Höche, Kar, Larkoski, Lönnblad, Plätzer, Siódak, Skands, Soyez, [JHEP 2017](#)]



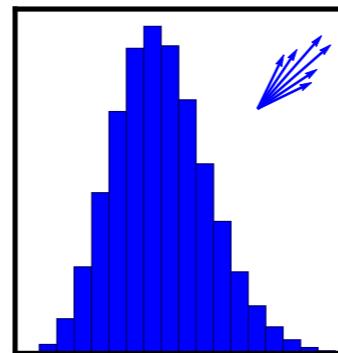
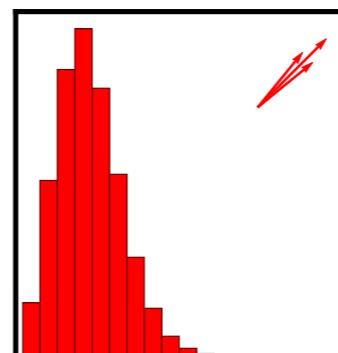
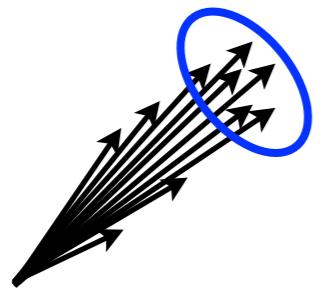
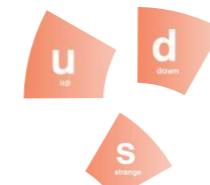
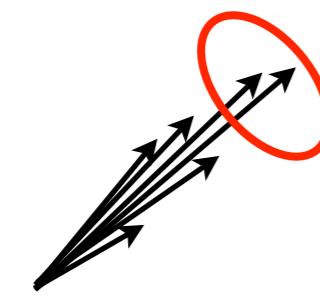
...to Evangelist

Jet flavor definitions via natural language processing

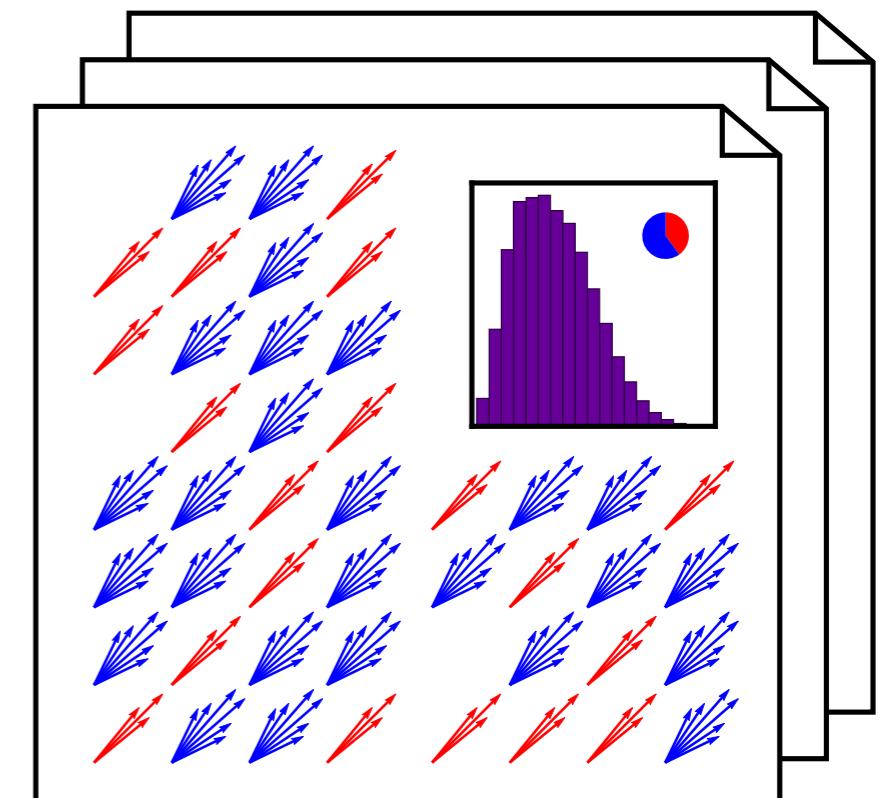
Quark

vs.

Gluon



Topic Modeling / Blind Source Separation

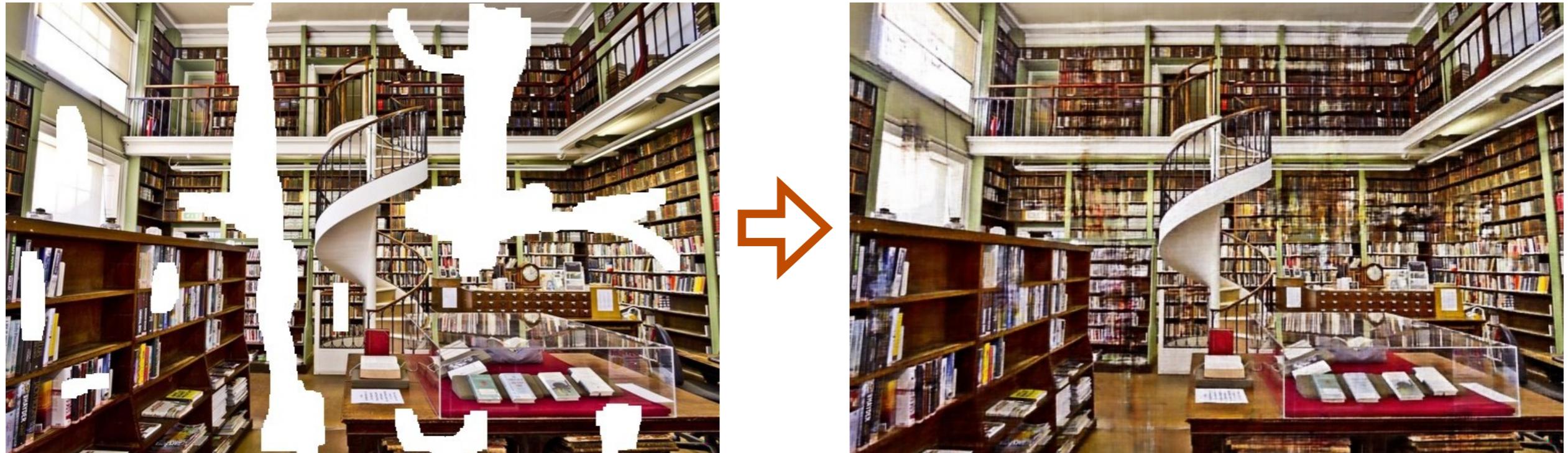


[Komiske, Metodiev, JDT, [JHEP 2018](#);
using Metodiev, Nachman, JDT, [JHEP 2017](#); Metodiev, JDT, [PRL 2018](#); Komiske, Metodiev, JDT, [JHEP 2019](#)]



Deep Learning

E.g. Inpainting

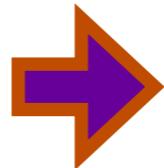


increased computational power and large data sets

[Ulyanov, Vedaldi, Lempitsky, CVPR 2018]

Deep Learning meets Deep Thinking

E.g. *Inpainting*



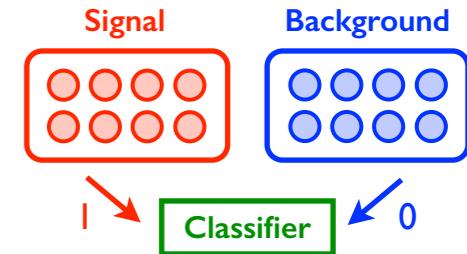
Using randomly initialized neural network (!)

Progress made by understanding the structure of problems
(not just increased computational power and large data sets)

[Ulyanov, Vedaldi, Lempitsky, CVPR 2018]

Quark/Gluon Classification

“Hello, World!” of Jet Physics



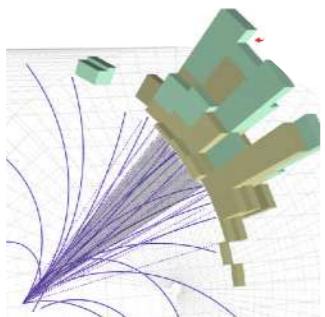
Find $h\left(\begin{array}{c} \text{jet} \\ \text{image} \end{array}\right)$ such that

$$h(\text{Quark}) = 1$$
$$h(\text{Gluon}) = 0$$

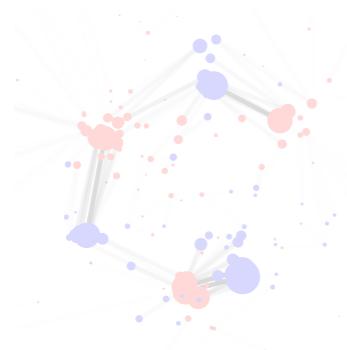
Best you can do: $h(\mathcal{J}) = \frac{p(\mathcal{J}|Q)}{p(\mathcal{J}|Q) + p(\mathcal{J}|G)}$
(Neyman-Pearson lemma)



Particle Physics 101



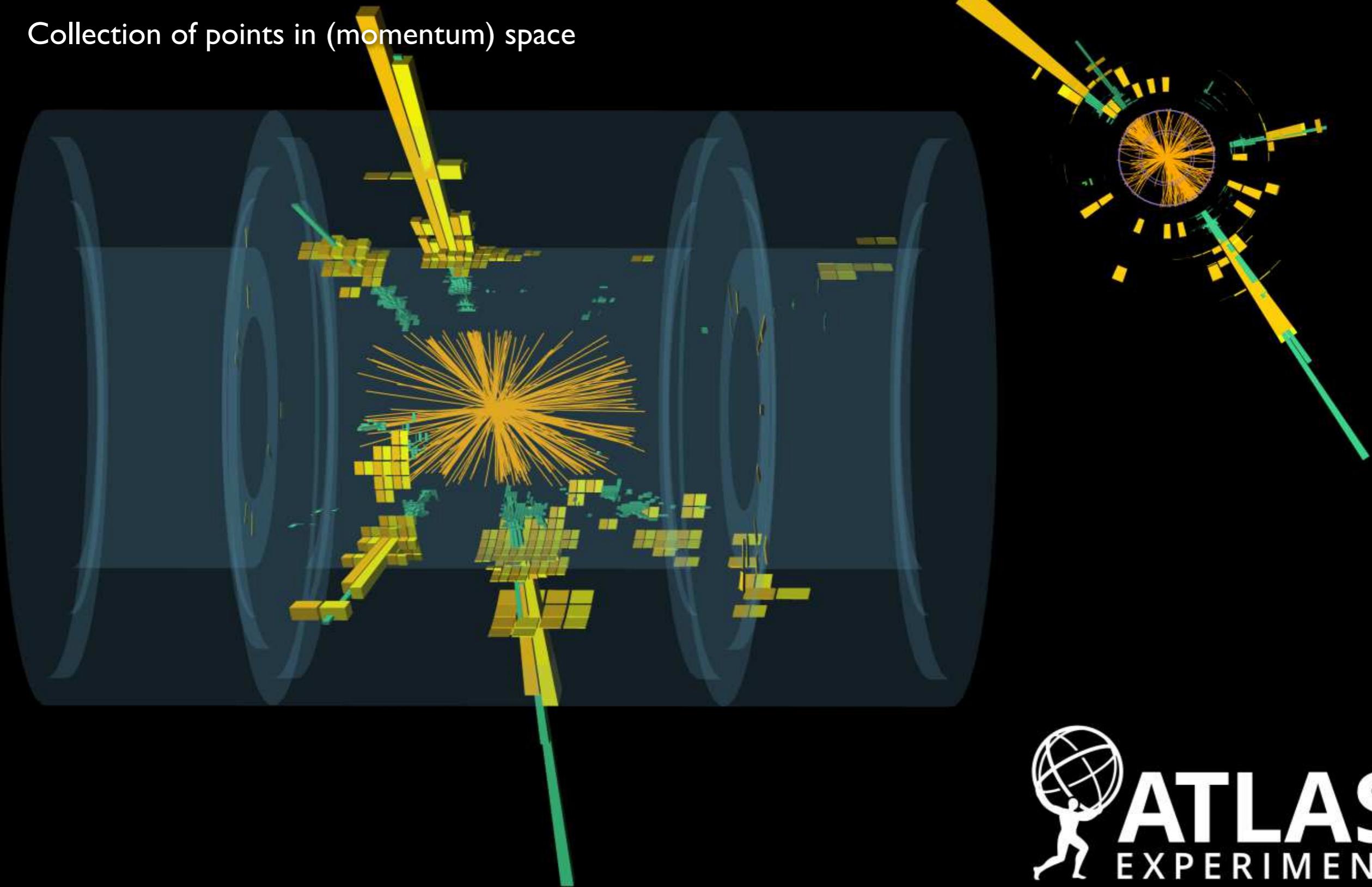
What is a Collider Event?



When are Collider Events Similar?

Collider Event

Collection of points in (momentum) space



Point Cloud

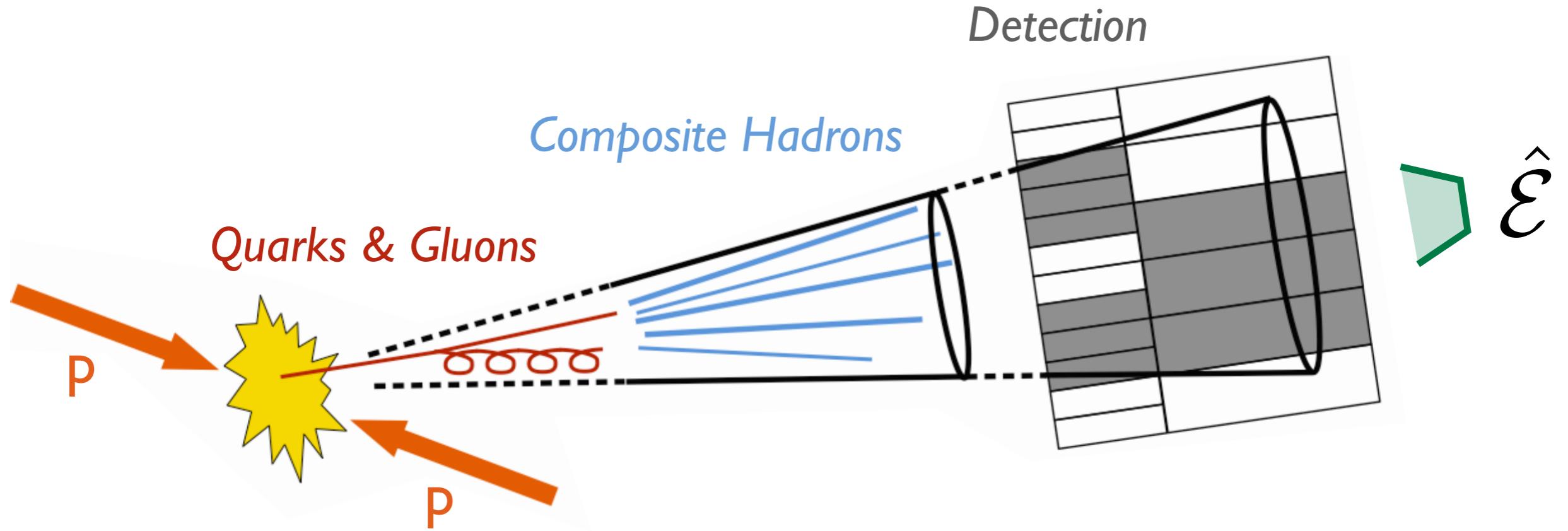
Collection of points in (position) space



[Popular Science, 2013]

Jet Formation from QCD

Theory

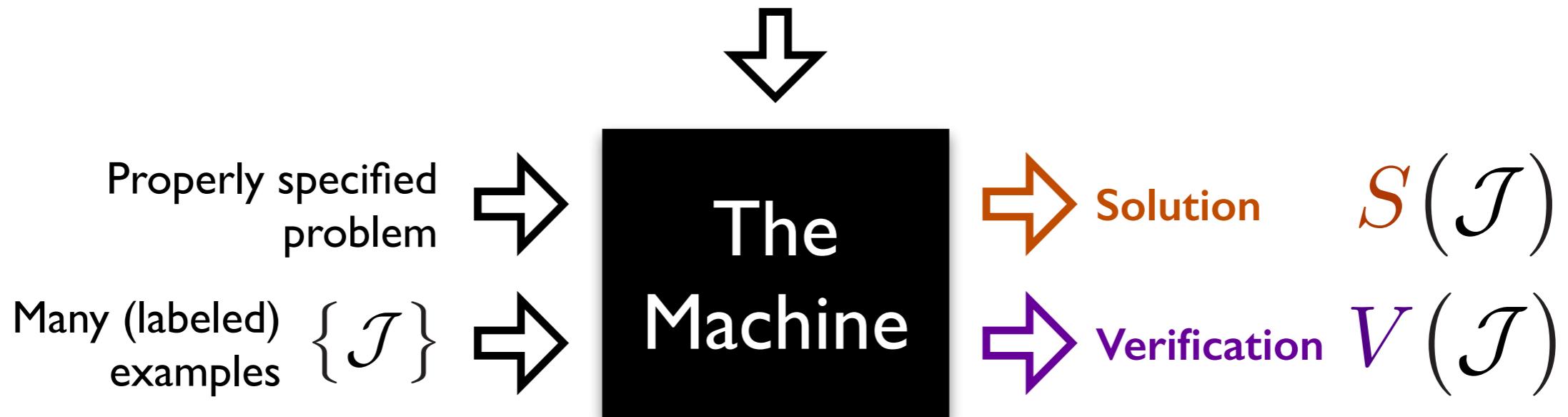
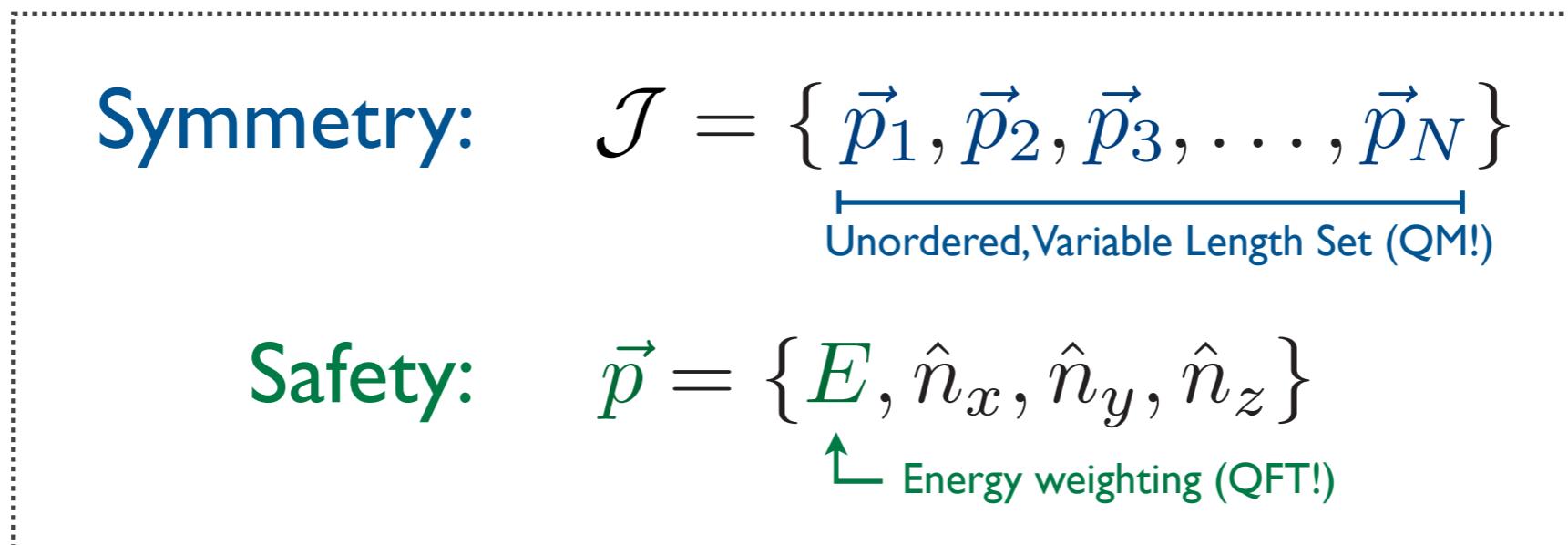


Energy Flow:
Robust to hadronization and detector effects

$$\hat{\mathcal{E}} \simeq \lim_{t \rightarrow \infty} \hat{n}_i T^{0i}(t, vt\hat{n})$$

[see e.g. Sveshnikov, Tkachov, [PLB 1996](#); Hofman, Maldacena, [JHEP 2008](#); Mateu, Stewart, [JDT, PRD 2013](#); Belitsky, Hohenegger, Korchemsky, Sokatchev, Zhiboedov, [PRL 2014](#); Chen, Moult, Zhang, Zhu, [PRD 2020](#)]

“Thinking” Like a Physicist



*Check that answer
is physically sensible*

Energy Flow Networks

Architecture designed around **symmetries** and **interpretability**



$$S(\mathcal{J}) = F(V_1, V_2, \dots, V_\ell)$$
$$V_a(\mathcal{J}) = \sum_{i \in \mathcal{J}} E_i \Phi_a(\hat{n}_i)$$

Permutation invariant \downarrow Linear weights (i.e. safe) \downarrow

Parametrized with Neural Networks

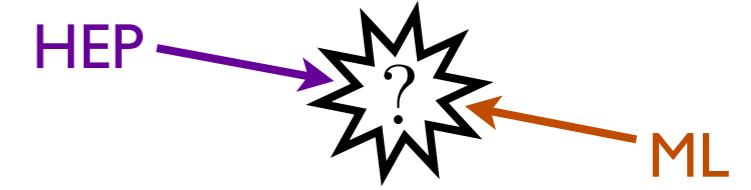
Provably describes any **safe** observable (!)
Excellent jet classification performance*

[Komiske, Metodiev, JDT, [JHEP 2019](#); see also Komiske, Metodiev, JDT, [JHEP 2018](#); code at [energyflow.network](#); special case of Zaheer, Kottur, Ravanbakhsh, Poczos, Salakhutdinov, Smola, [NIPS 2017](#)]



Energy Flow Networks

Architecture designed around symmetries and *interpretability*



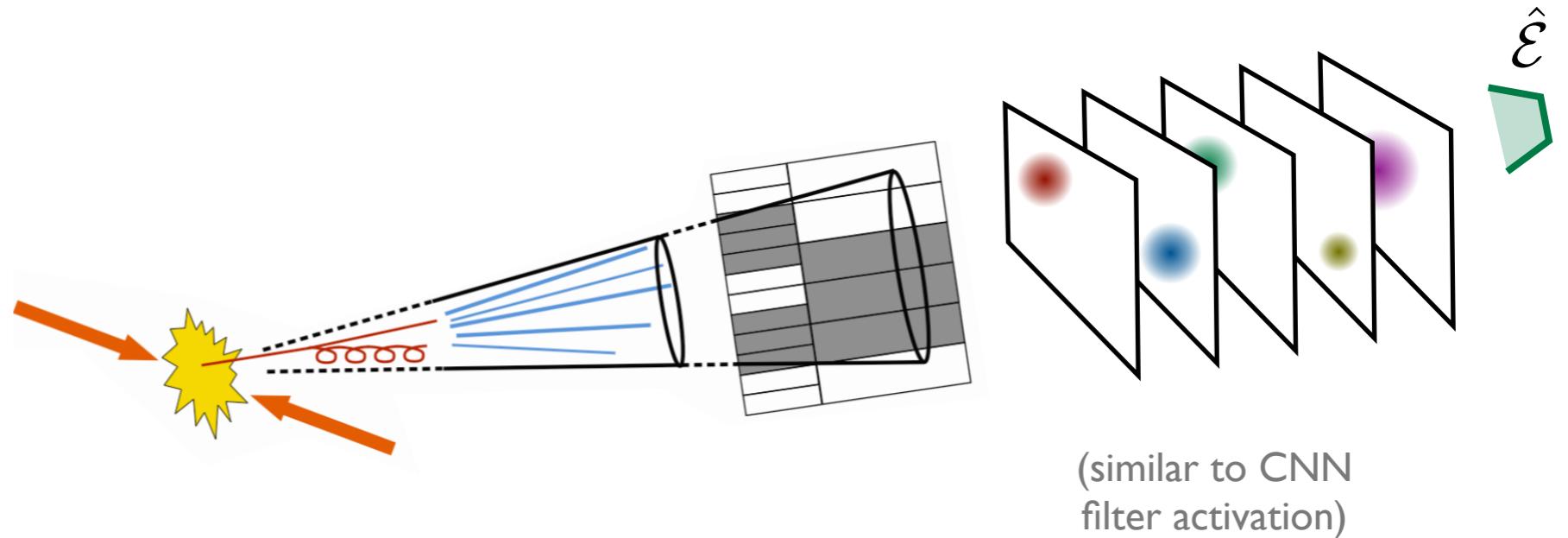
$$S(\mathcal{J}) = F(V_1, V_2, \dots, V_\ell)$$

Latent space of dim ℓ

$V_a(\mathcal{J}) = \sum_{i \in \mathcal{J}} E_i \Phi_a(\hat{n}_i)$

Can visualize if ℓ is small

Easy to plot!

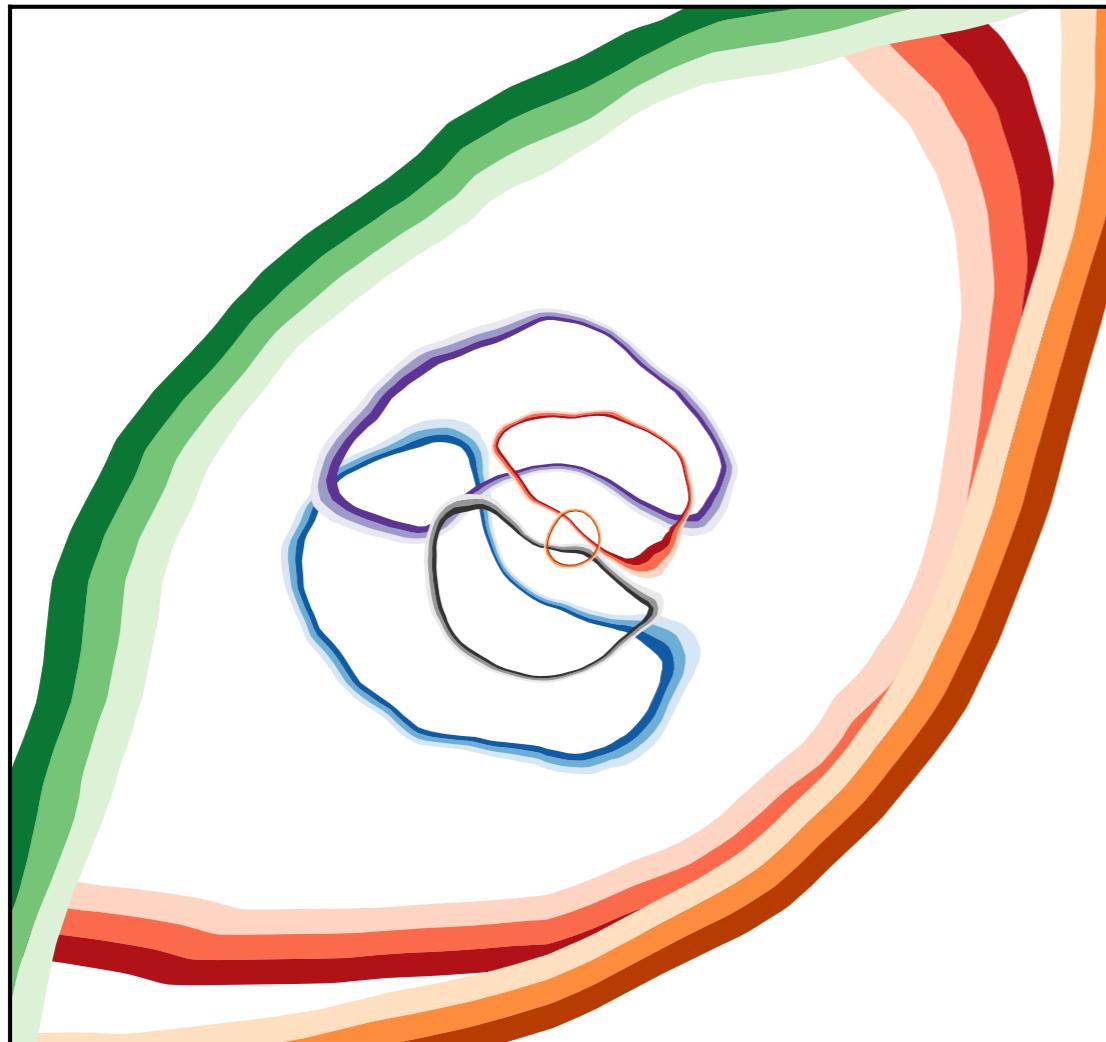


[Komiske, Metodiev, JDT, [JHEP 2019](#); see also Komiske, Metodiev, JDT, [JHEP 2018](#); code at [energyflow.network](#); special case of Zaheer, Kottur, Ravanbakhsh, Poczos, Salakhutdinov, Smola, [NIPS 2017](#)]

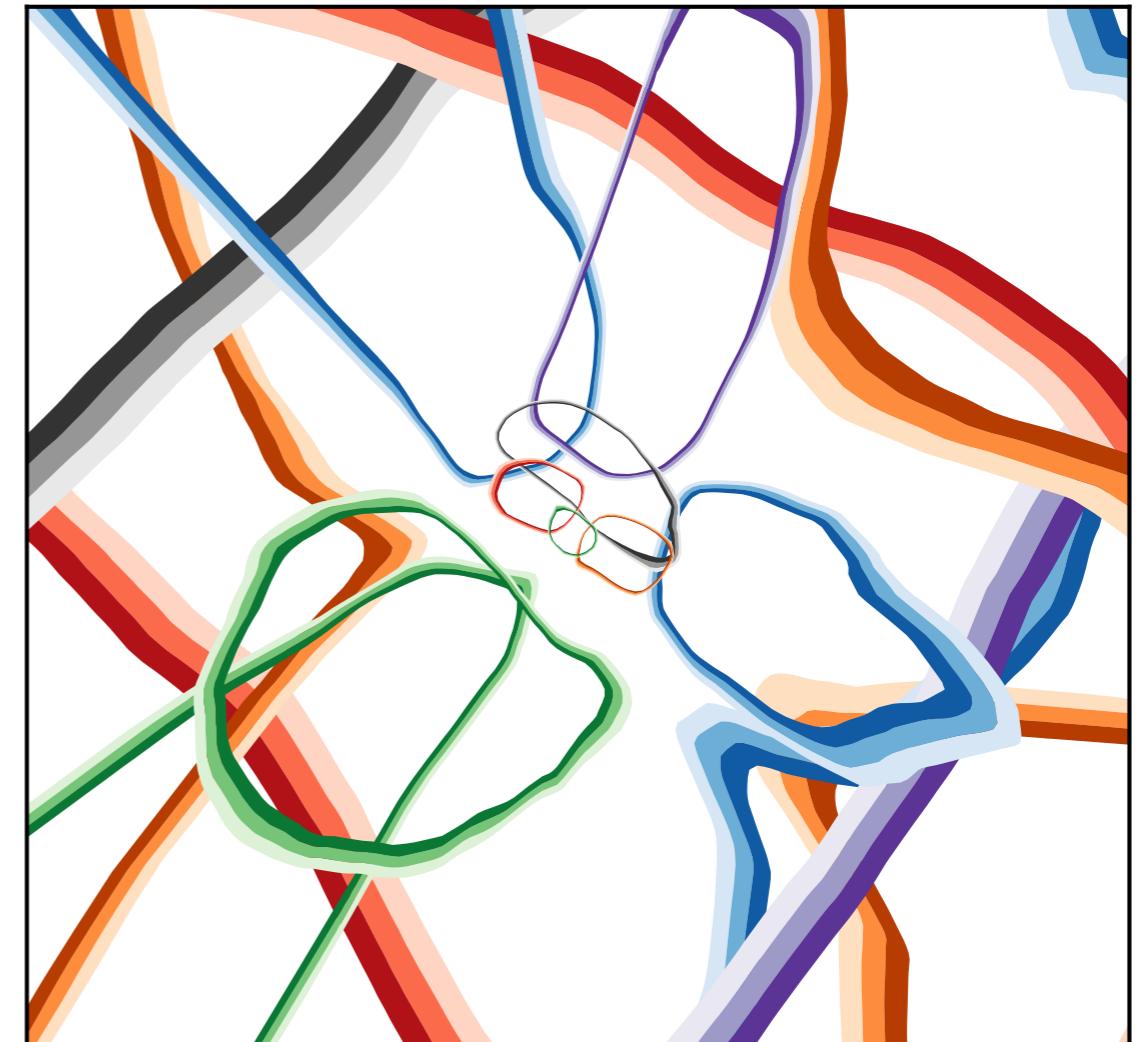


Psychedelic Network Visualization

Latent Dimension 8



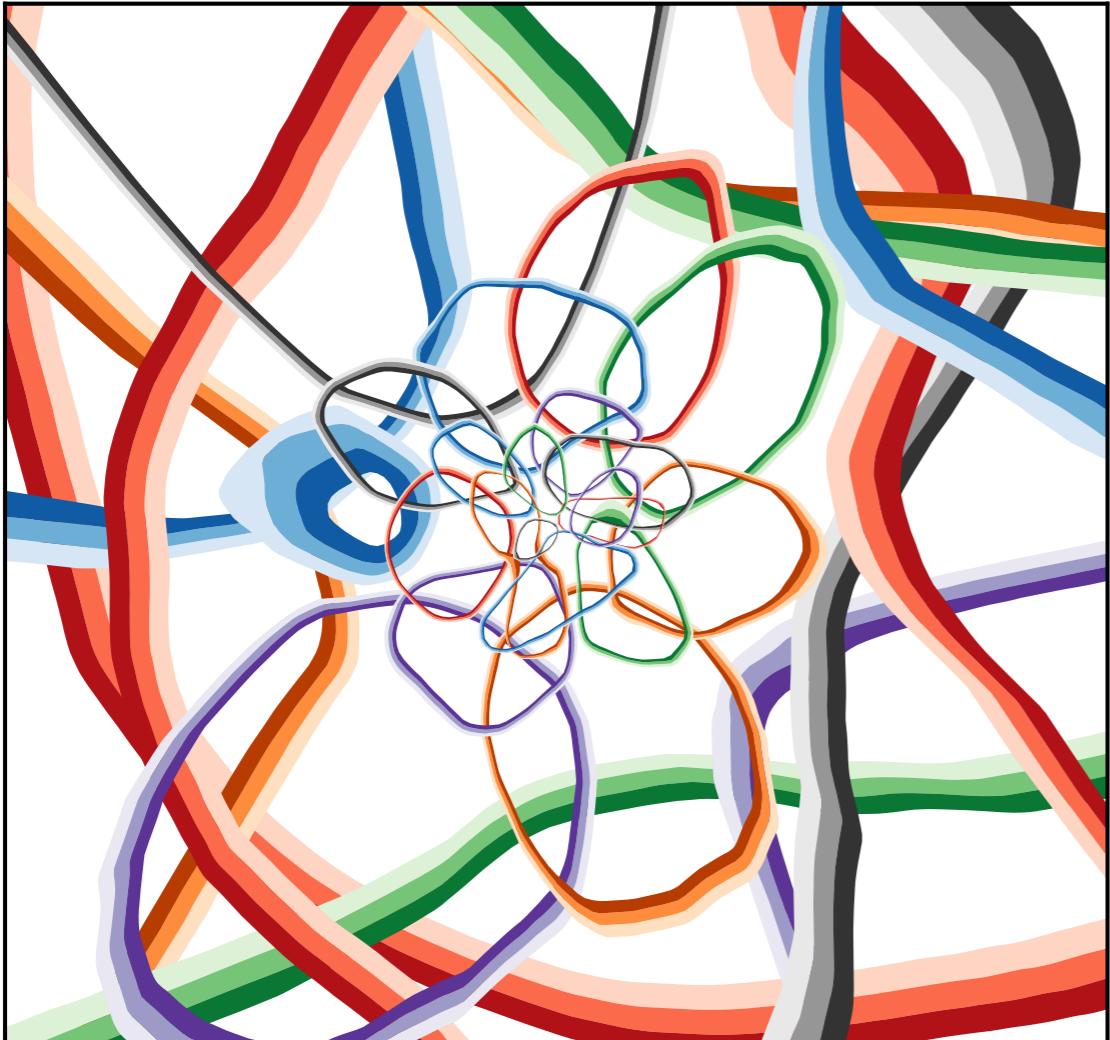
Latent Dimension 16



For the case of **quark** vs. **gluon** classification

Psychedelic Network Visualization

Latent Dimension 32

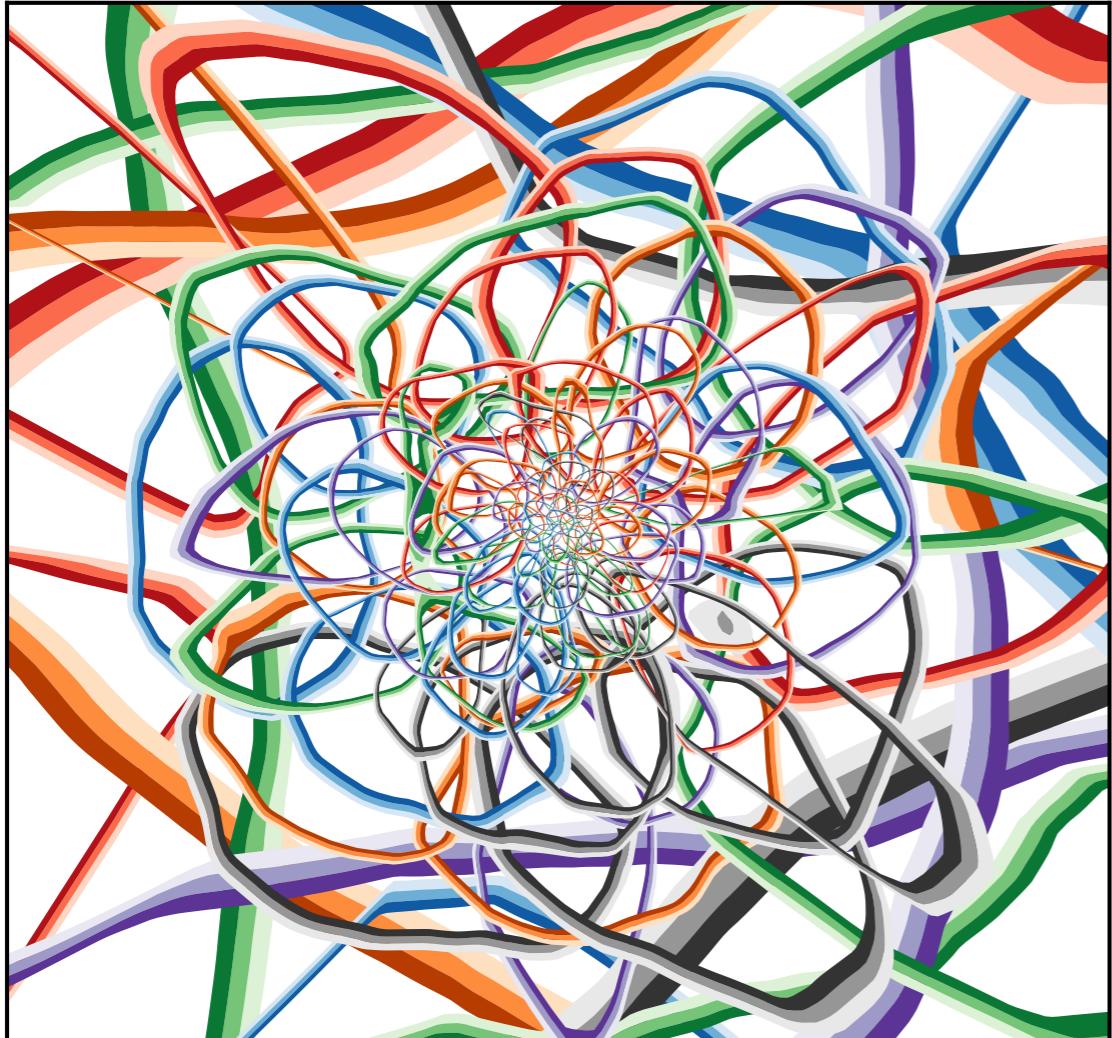


Latent Dimension 64

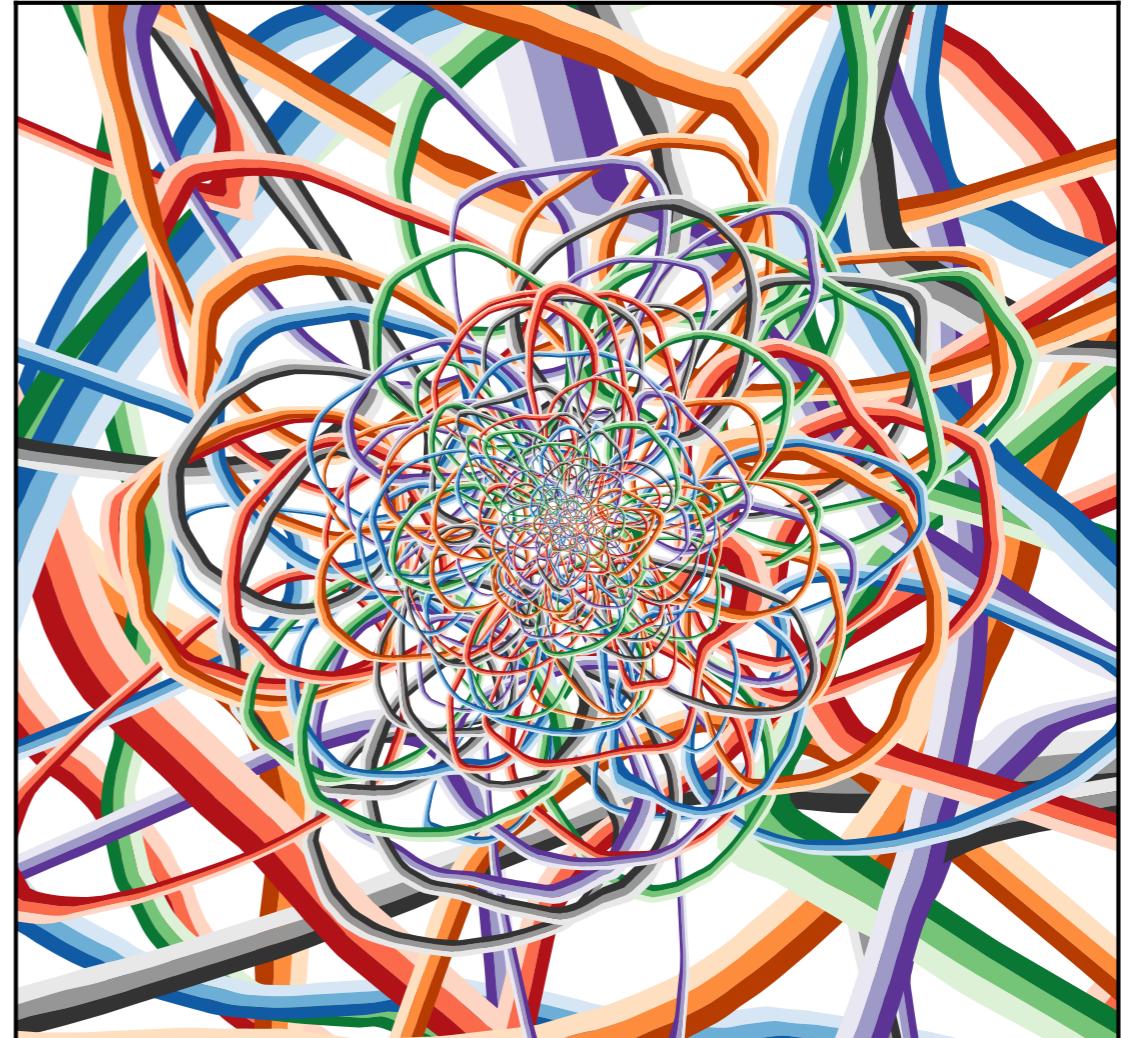


Psychedelic Network Visualization

Latent Dimension 128

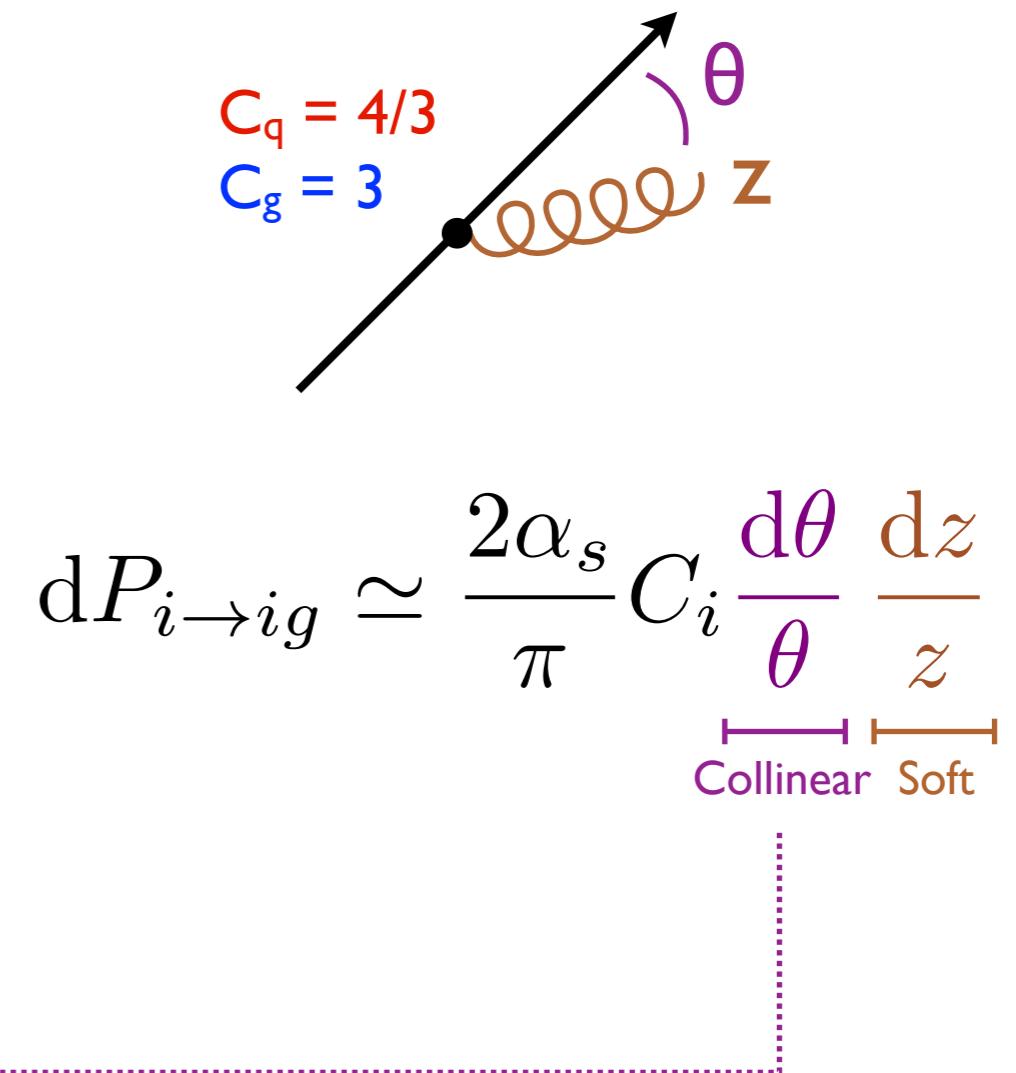
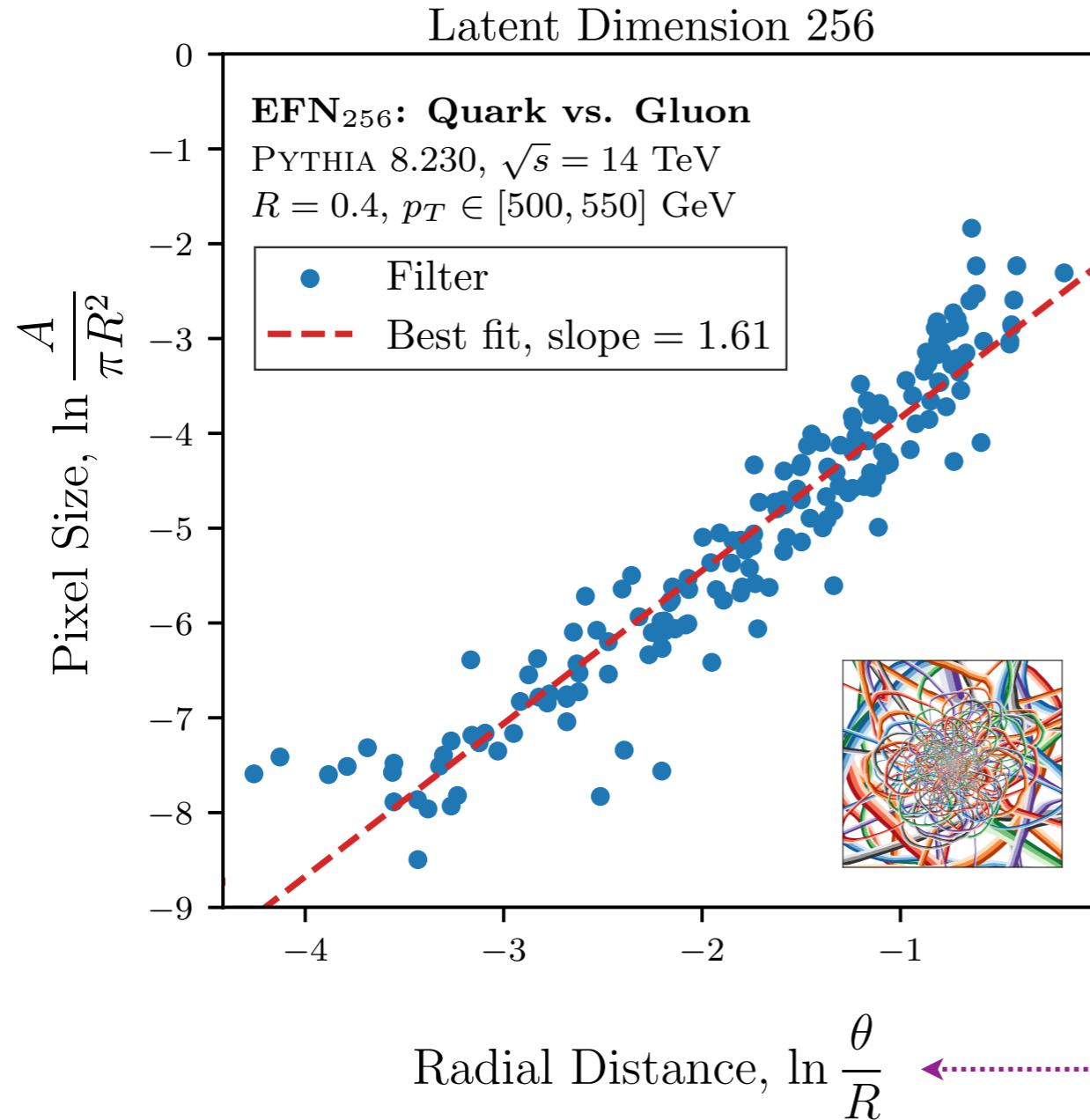


Latent Dimension 256



Singularity structure of QCD!

Machine Learning Collinear QCD

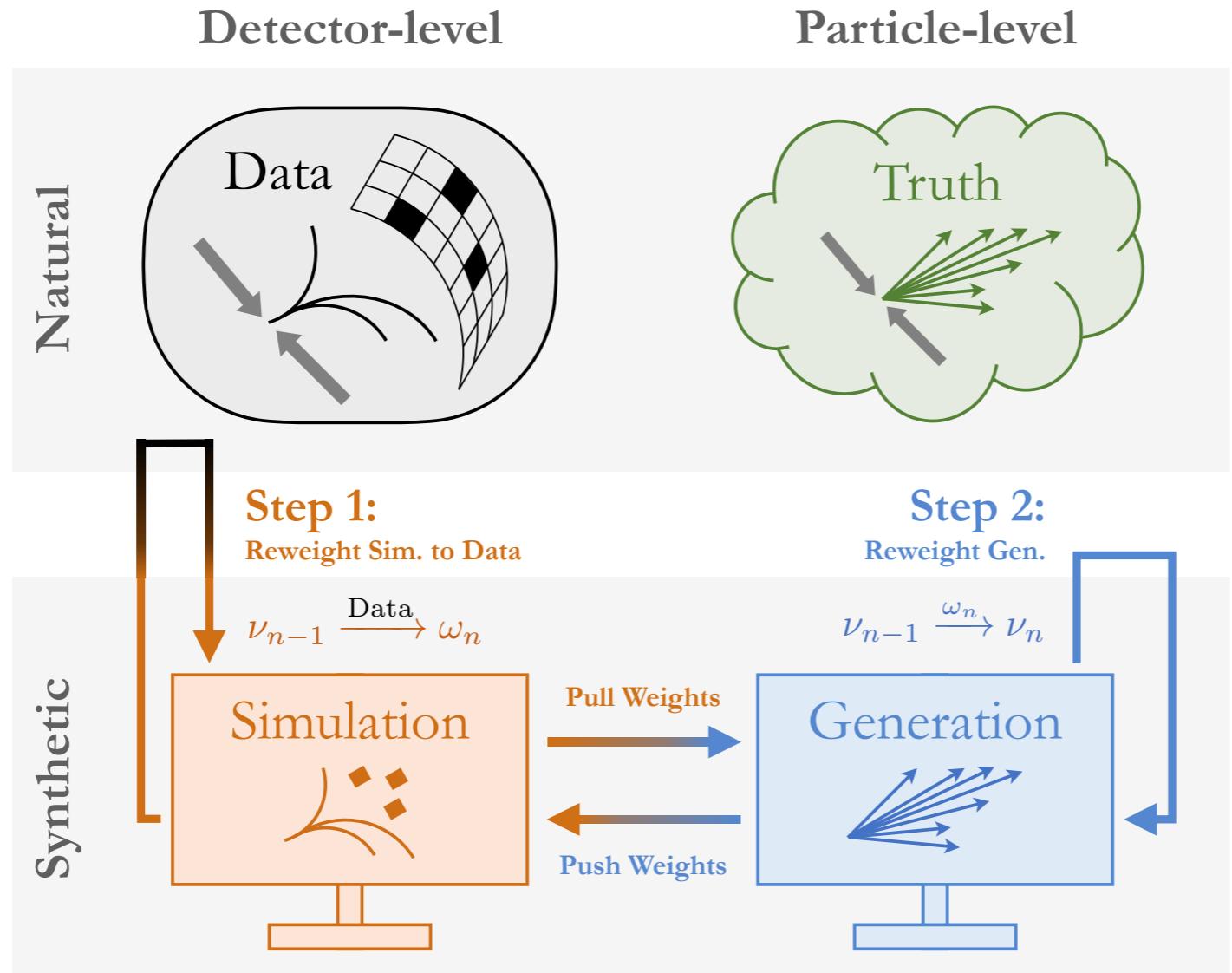
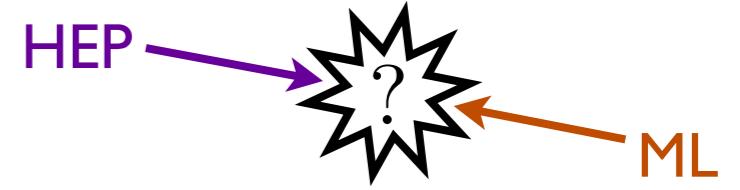


[Komiske, Metodiev, JDT, JHEP 2019]

*“Okay, but can you really do
something you couldn’t do before?”*

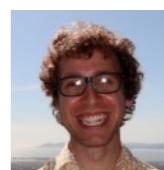
[see Harvard colloquium by Kyle Cranmer, [Sept 10, 2018](#)]

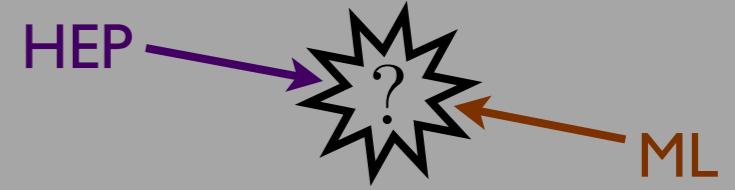
OmniFold



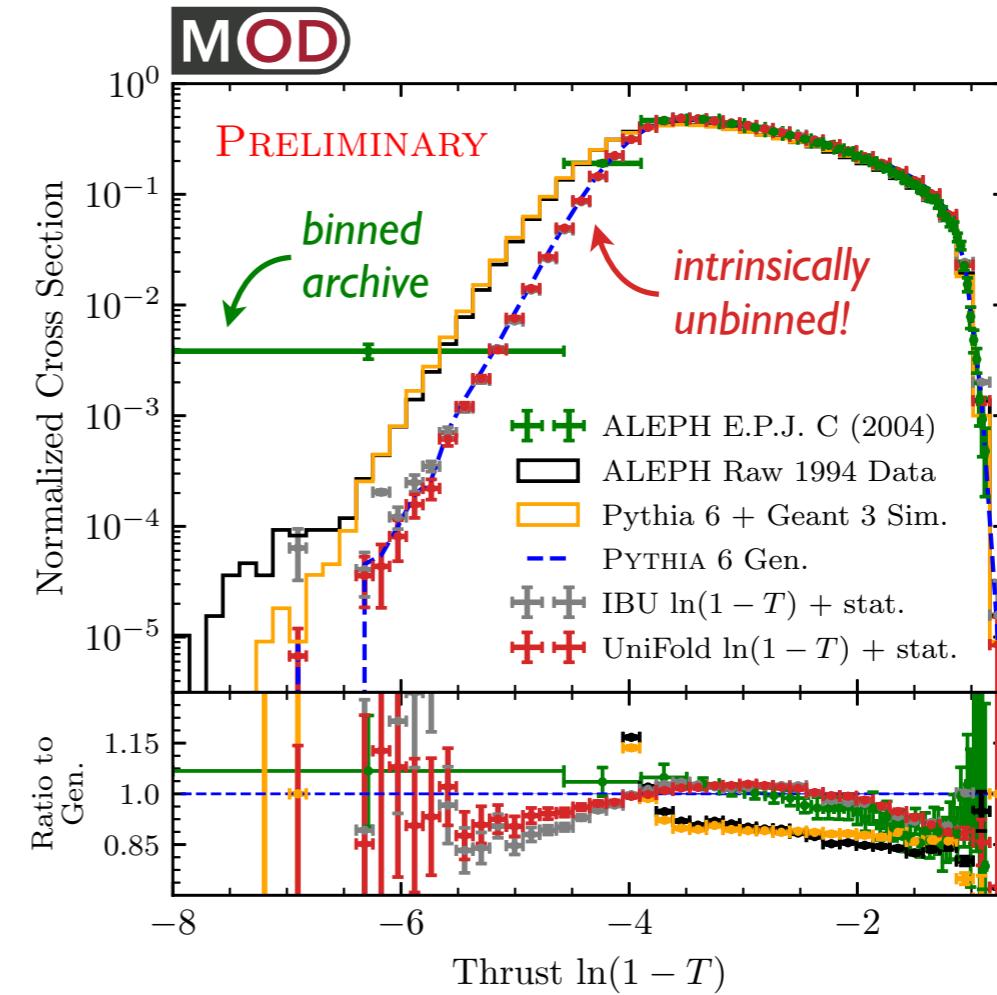
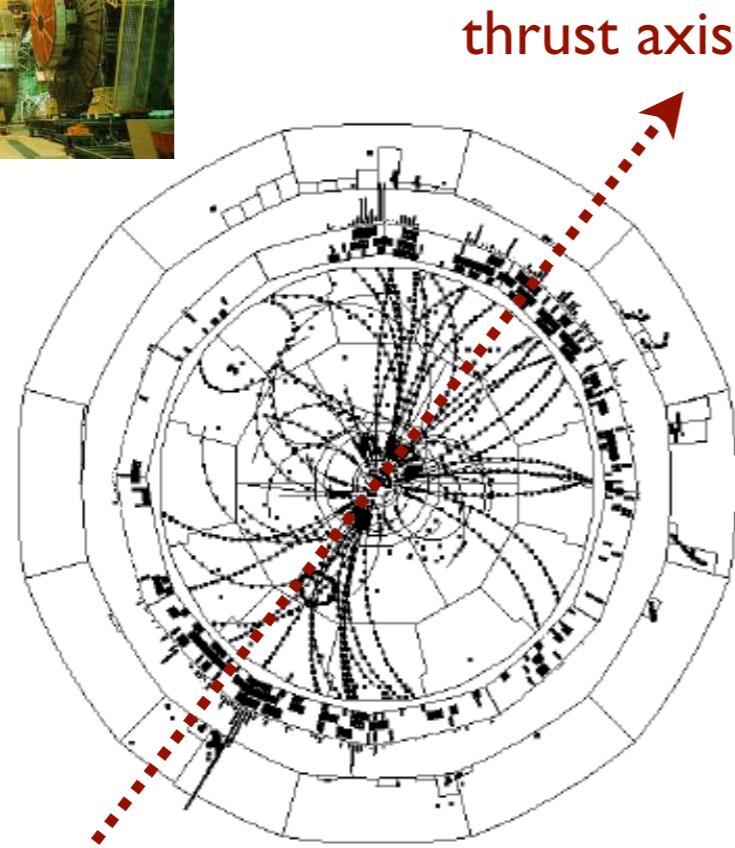
Multi-dimensional unbinned detector corrections via iterated binary classification

[Andreassen, Komiske, Metodiev, Nachman, JDT, [PRL 2020](#)]

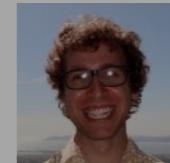
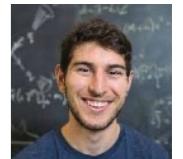




Back to the Future with ALEPH Archival Data



[talk by Badea, [ICHEP 2020](#); cf. ALEPH, [EPJC 2004](#)
 [see also Badea, Baty, Chang, Innocenti, Maggi, McGinn, Peters, Sheng, JDT, Lee, [PRL 2019](#)]

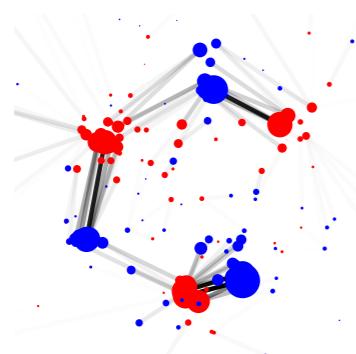




Particle Physics 101



What is a Collider Event?

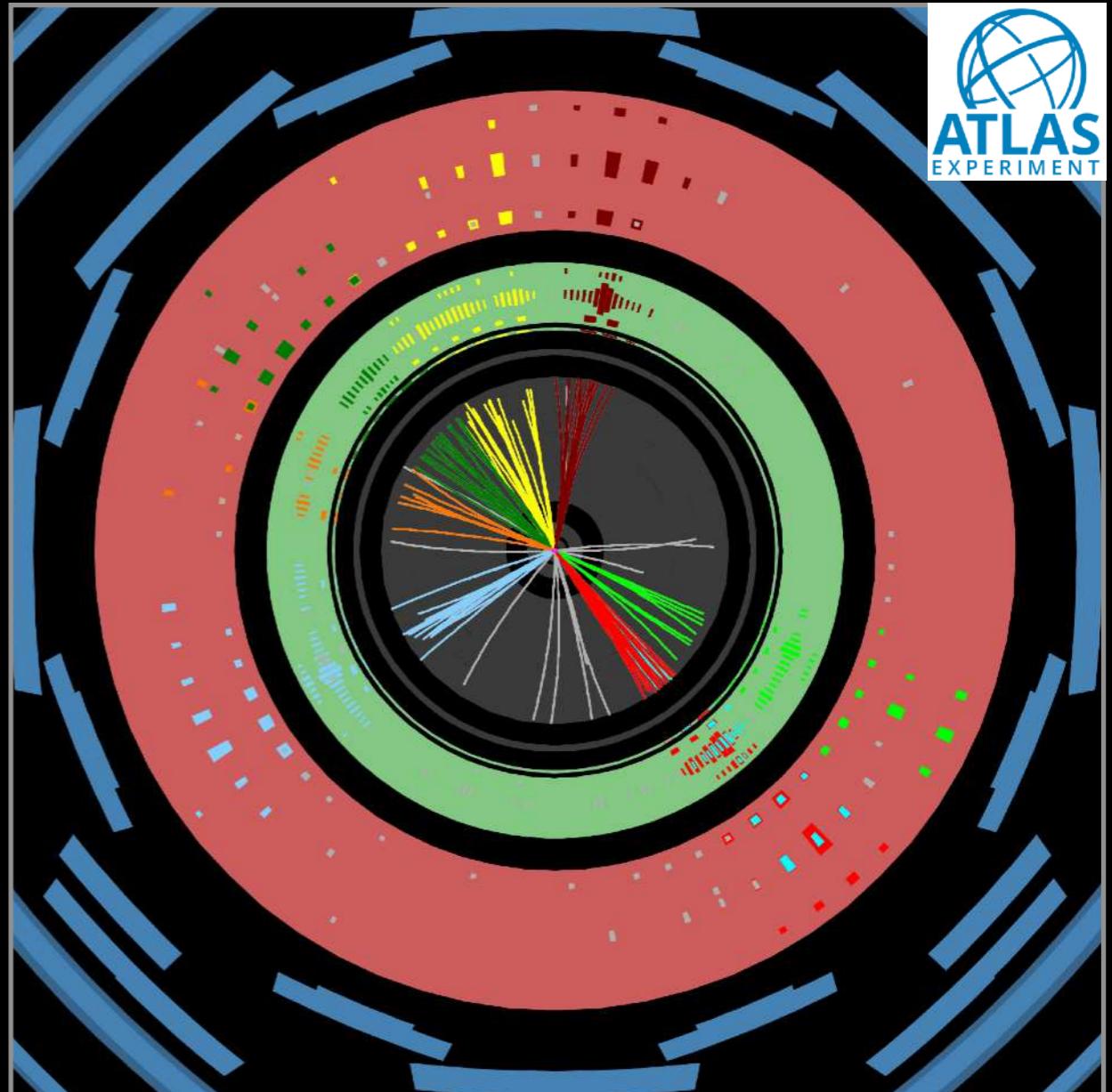
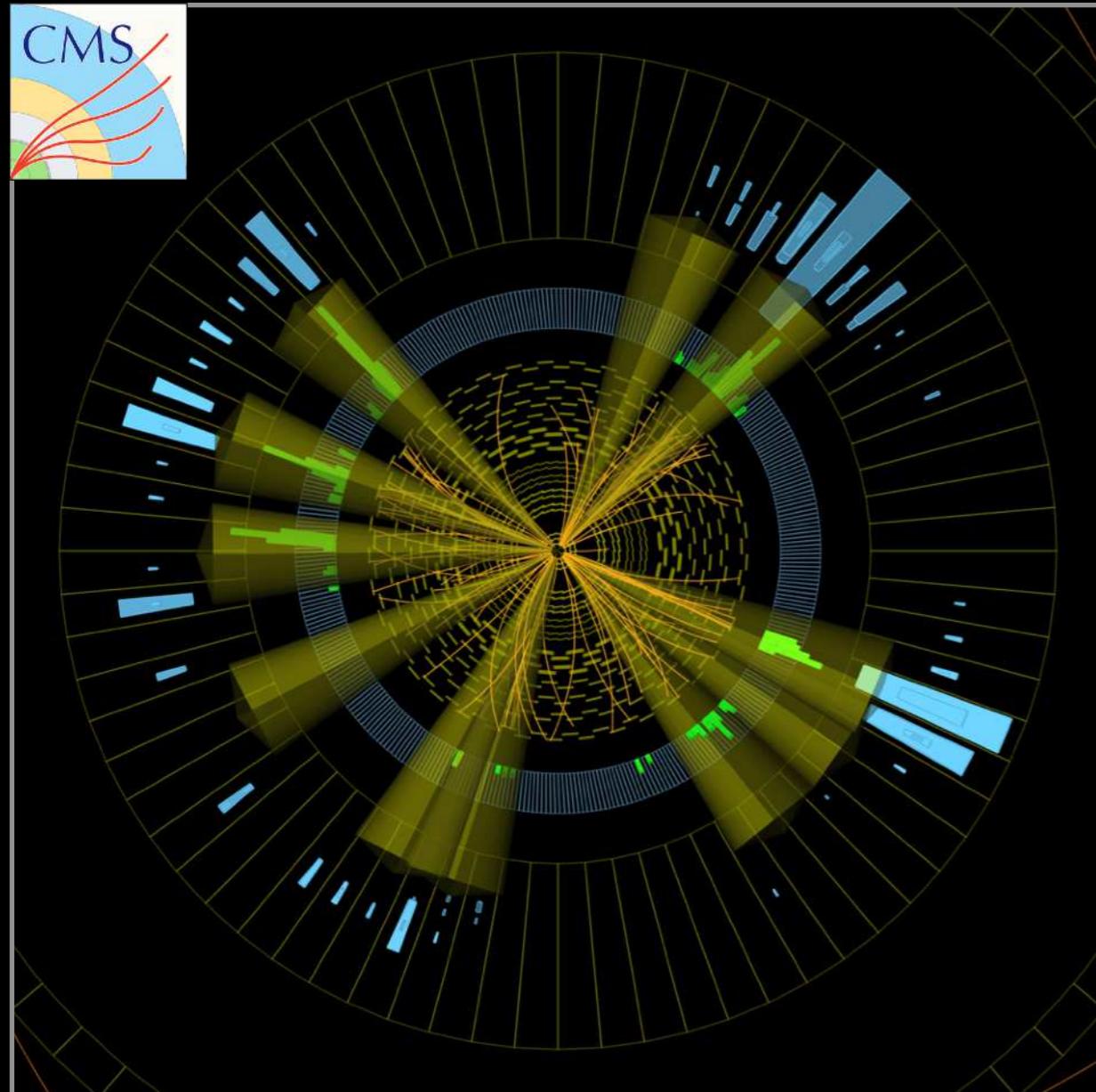


When are Collider Events Similar?

Two Collider Events

Two collections of points in (momentum) space

How “close” are these?

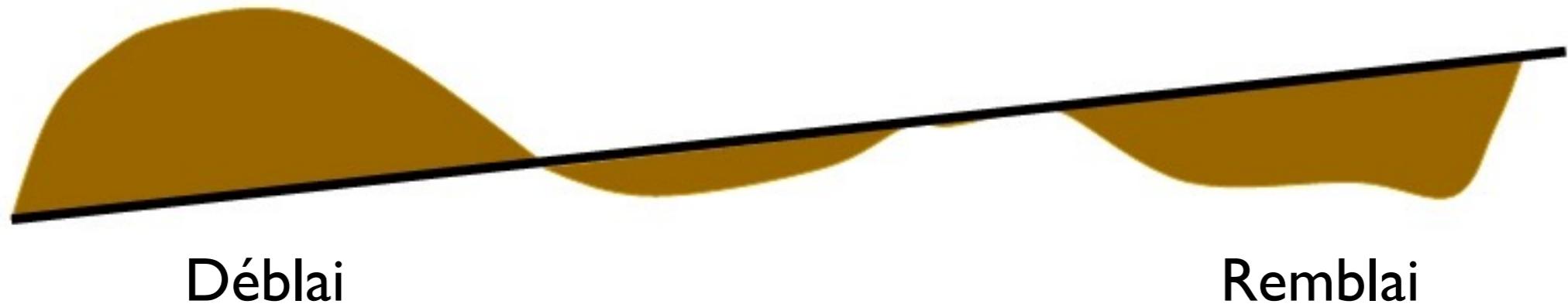


The Earth Mover's Distance

Optimal Transport:

[Peleg, Werman, Rom, [IEEE 1989](#);
Rubner, Tomasi, Guibas, [ICCV 1998](#), [ICCV 2000](#);
Pele, Werman, [ECCV 2008](#); Pele Taskar, [GSI 2013](#)]

Minimum “work” (stuff \times distance) to make one distribution look like another distribution



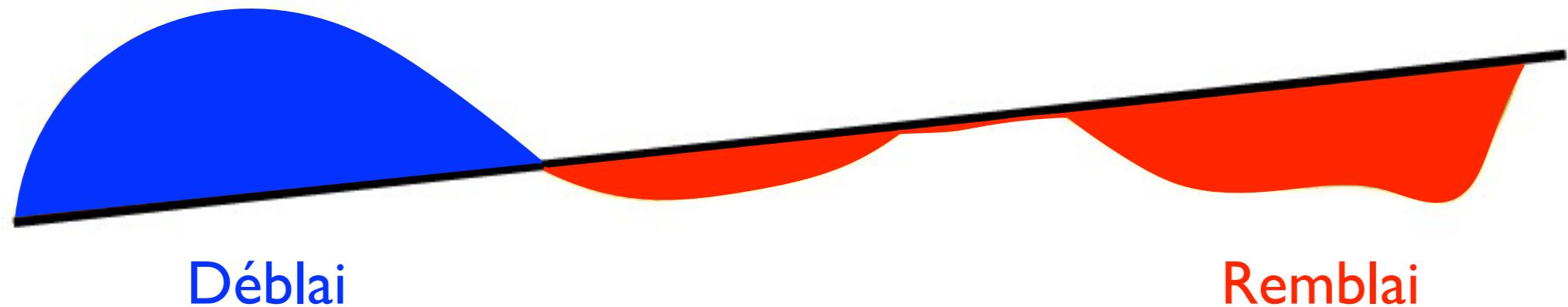
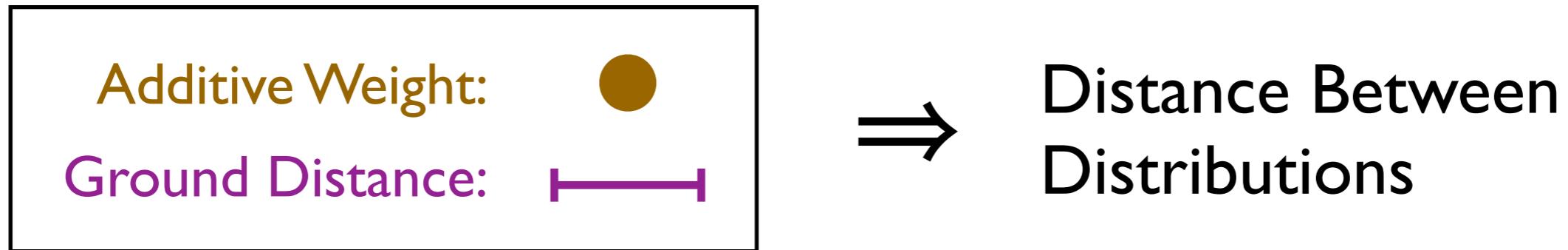
[h/t Niles-Weed, [ML4Jets 2020](#); Monge, 1781; Vaserštejn, 1969; [Wikipedia](#)]

The Earth Mover's Distance

Optimal Transport:

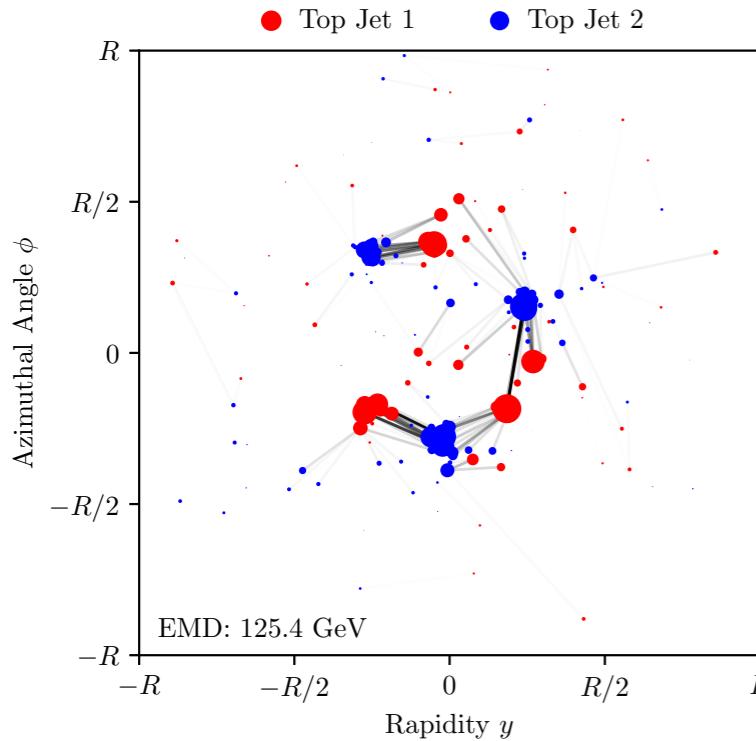
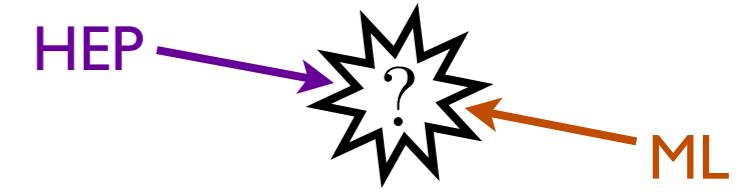
[Peleg, Werman, Rom, [IEEE 1989](#);
Rubner, Tomasi, Guibas, [ICCV 1998](#), [ICCV 2000](#);
Pele, Werman, [ECCV 2008](#); Pele Taskar, [GSI 2013](#)]

Minimum “work” (**stuff** × **distance**) to make
one distribution look like **another distribution**



[h/t Niles-Weed, [ML4Jets 2020](#); Monge, 1781; Vaserštejn, 1969; [Wikipedia](#)]

The Energy Mover's Distance

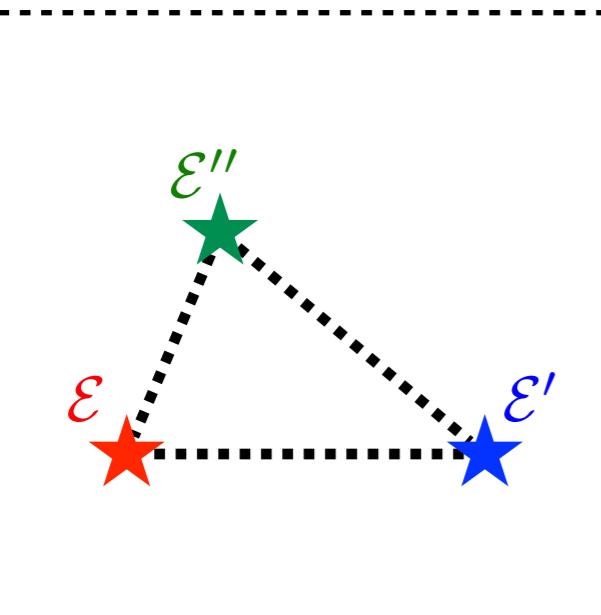


Optimal transport of energy flows...

$$\text{EMD}(\mathcal{E}, \mathcal{E}') = \min_{\{f\}} \sum_i \sum_j f_{ij} \frac{\theta_{ij}}{R} + \left| \sum_i E_i - \sum_j E'_j \right|$$

↑
in GeV

Cost to move energy Cost to create energy



...defines a metric on the space of events

$$0 \leq \text{EMD}(\mathcal{E}, \mathcal{E}') \leq \text{EMD}(\mathcal{E}, \mathcal{E}'') + \text{EMD}(\mathcal{E}', \mathcal{E}'')$$

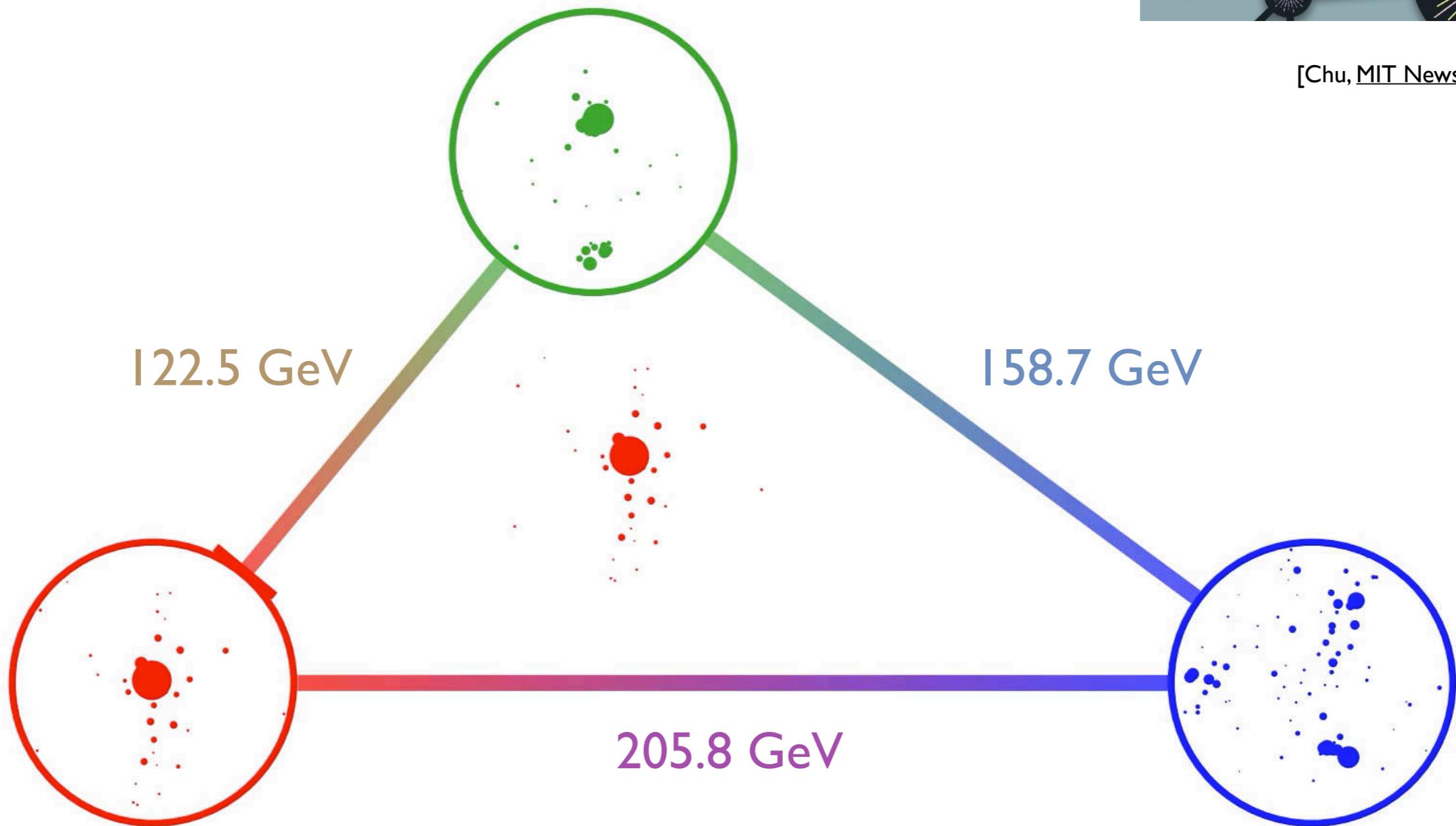
(assuming $R \geq \theta_{\max}/2$, i.e. $R \geq$ jet radius for conical jets)

[Komiske, Metodiev, JDT, [PRL 2019](#);
see also Pele, Werman, [ECCV 2008](#); Pele, Taskar, [GSI 2013](#);
see flavored variant in Crispim Romão, Castro, Milhano, Pedro, Vale, [arXiv 2020](#)]

Triangulating the Space of Jets



[Chu, MIT News July 2019]



[Komiske, Metodiev, JDT, PRL 2019; code at Komiske, Metodiev, JDT, [energyflow.network](#)]

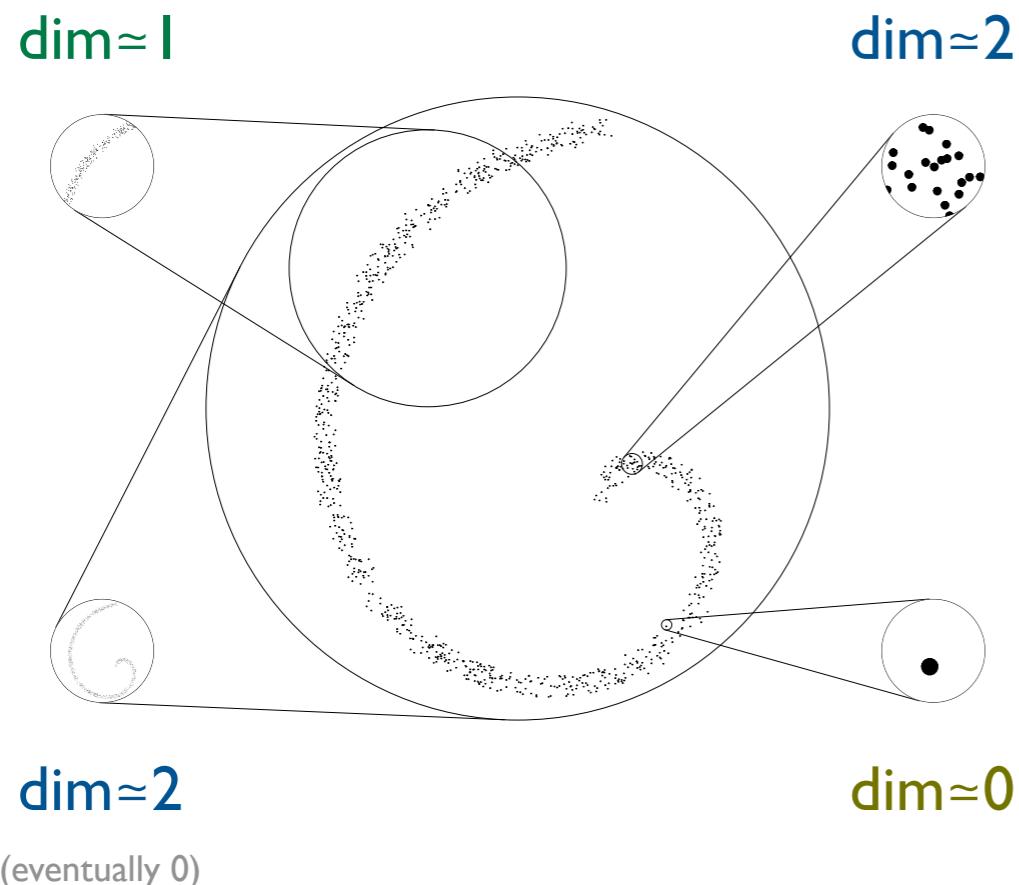
*“Ok, kind of cool, but what can
you actually do with a metric?”*

Dimensionality of Space of Jets

$$N_{\text{neighbors}}(r) \sim r^{\dim}$$

$$\Rightarrow \dim(r) \sim r \frac{\partial}{\partial r} \ln N_{\text{neighbors}}(r)$$

[Grassberger, Procaccia, [PRL 1983](#); Kégl, [NIPS 2002](#)]



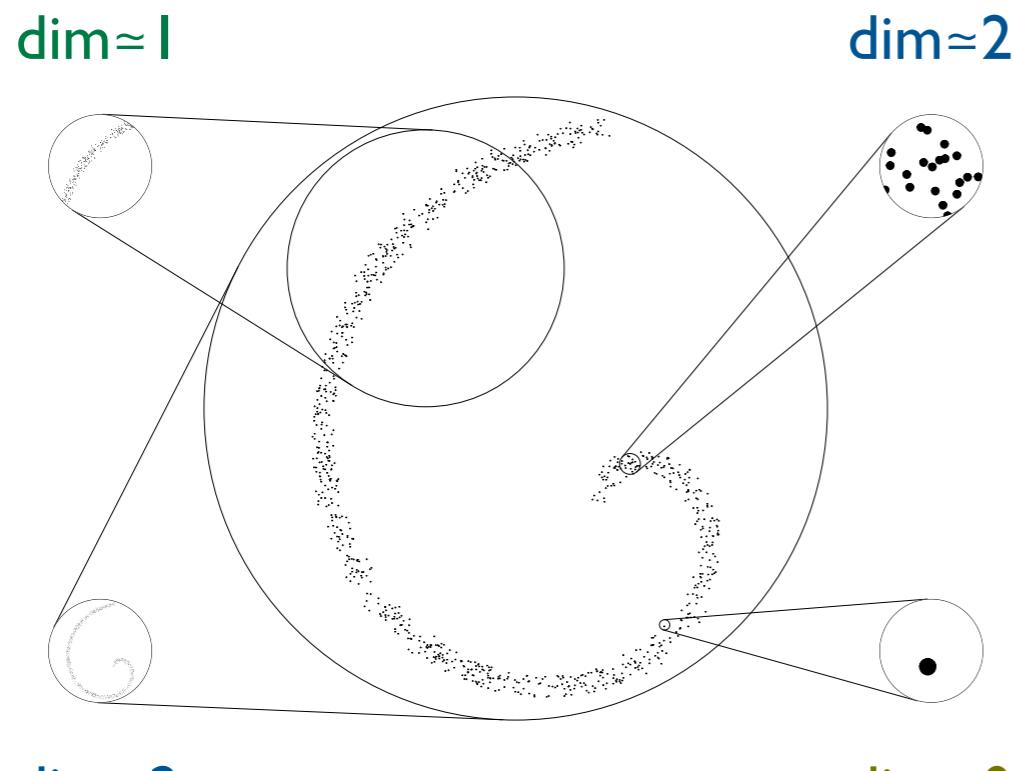
Dimensionality of Space of Jets



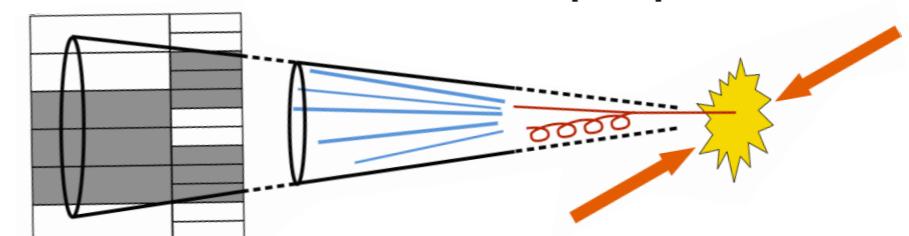
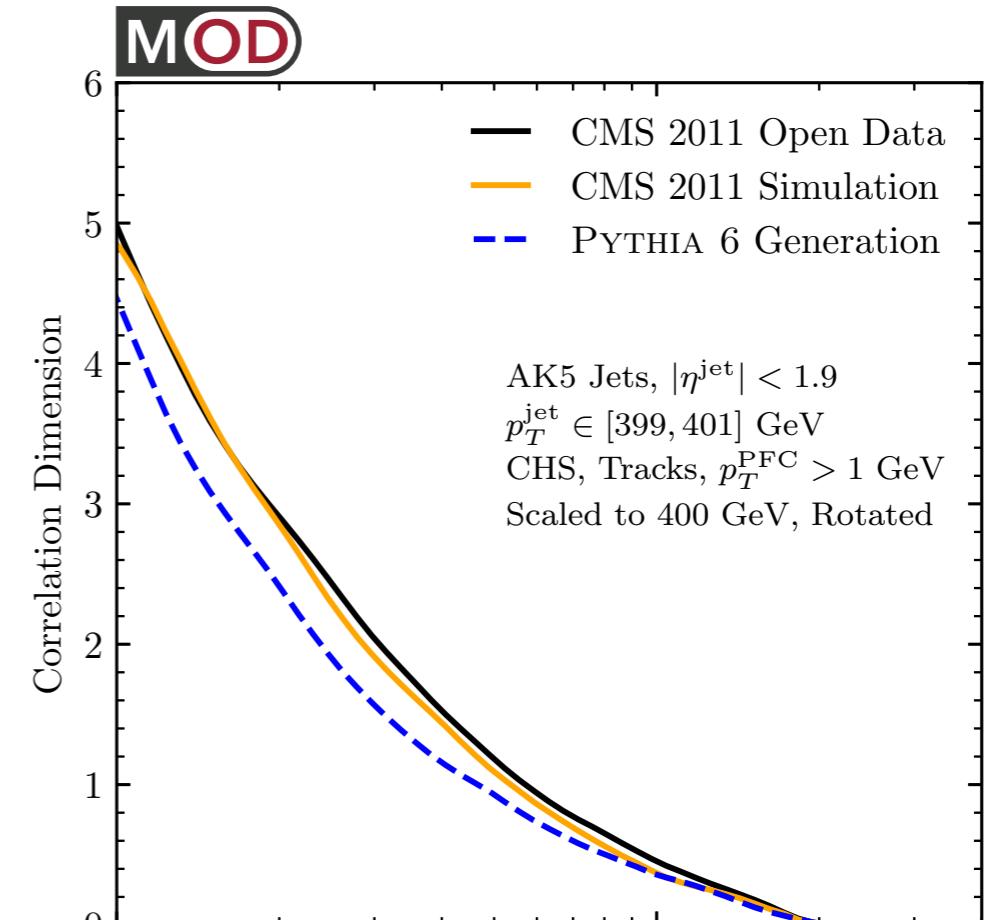
$$N_{\text{neighbors}}(r) \sim r^{\dim}$$

$$\Rightarrow \dim(r) \sim r \frac{\partial}{\partial r} \ln N_{\text{neighbors}}(r)$$

[Grassberger, Procaccia, [PRL 1983](#); Kégl, [NIPS 2002](#)]



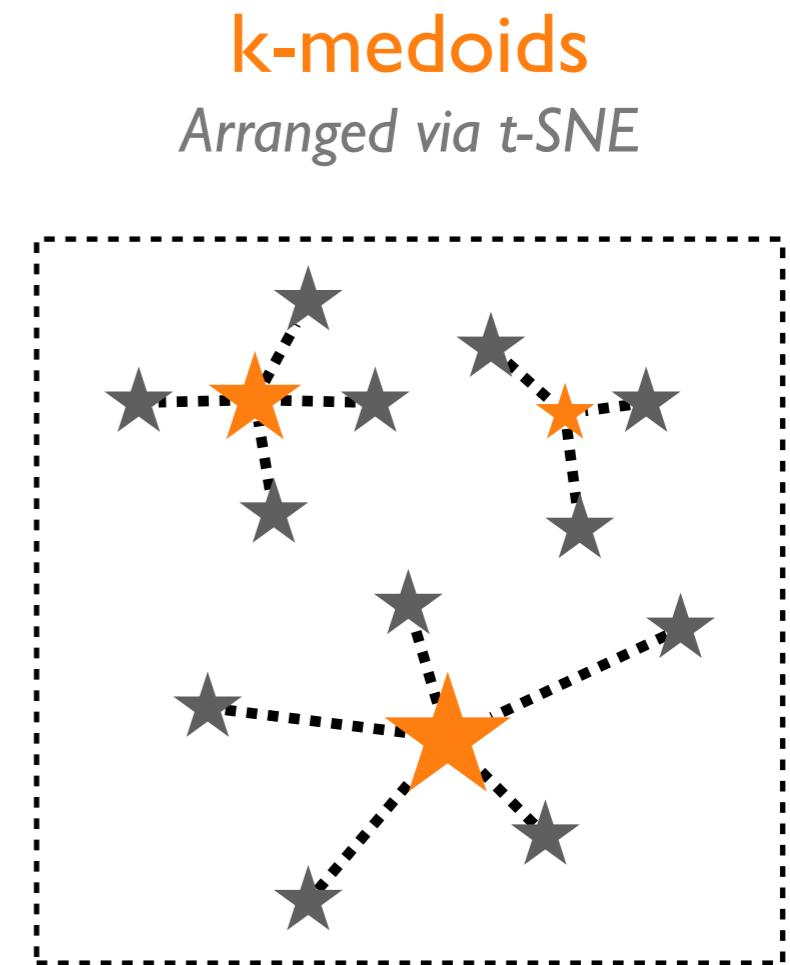
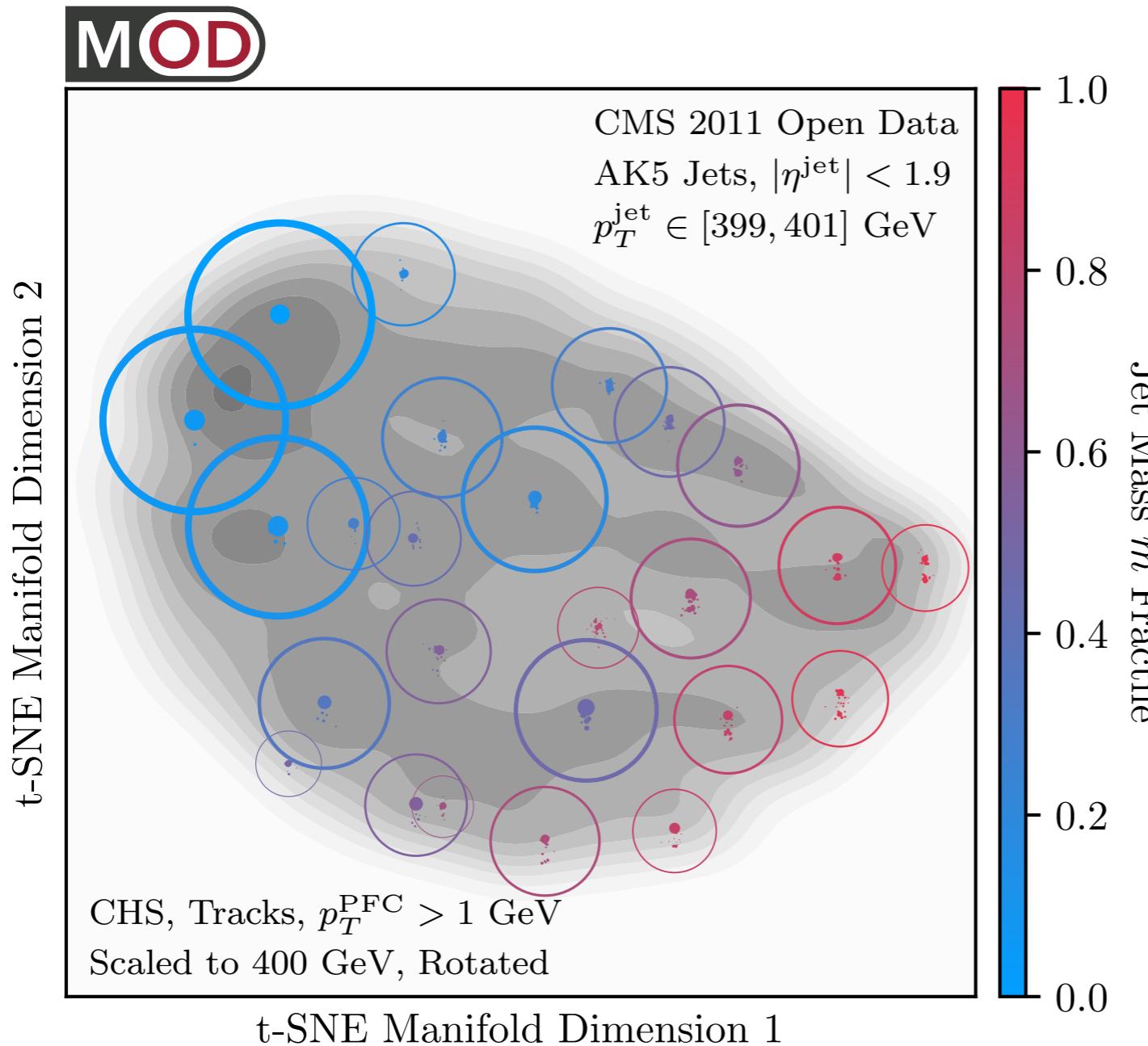
(eventually 0)



[Komiske, Mastandrea, Metodiev, Naik, [JDT, PRD 2020](#);
using [CMS Open Data](#)]



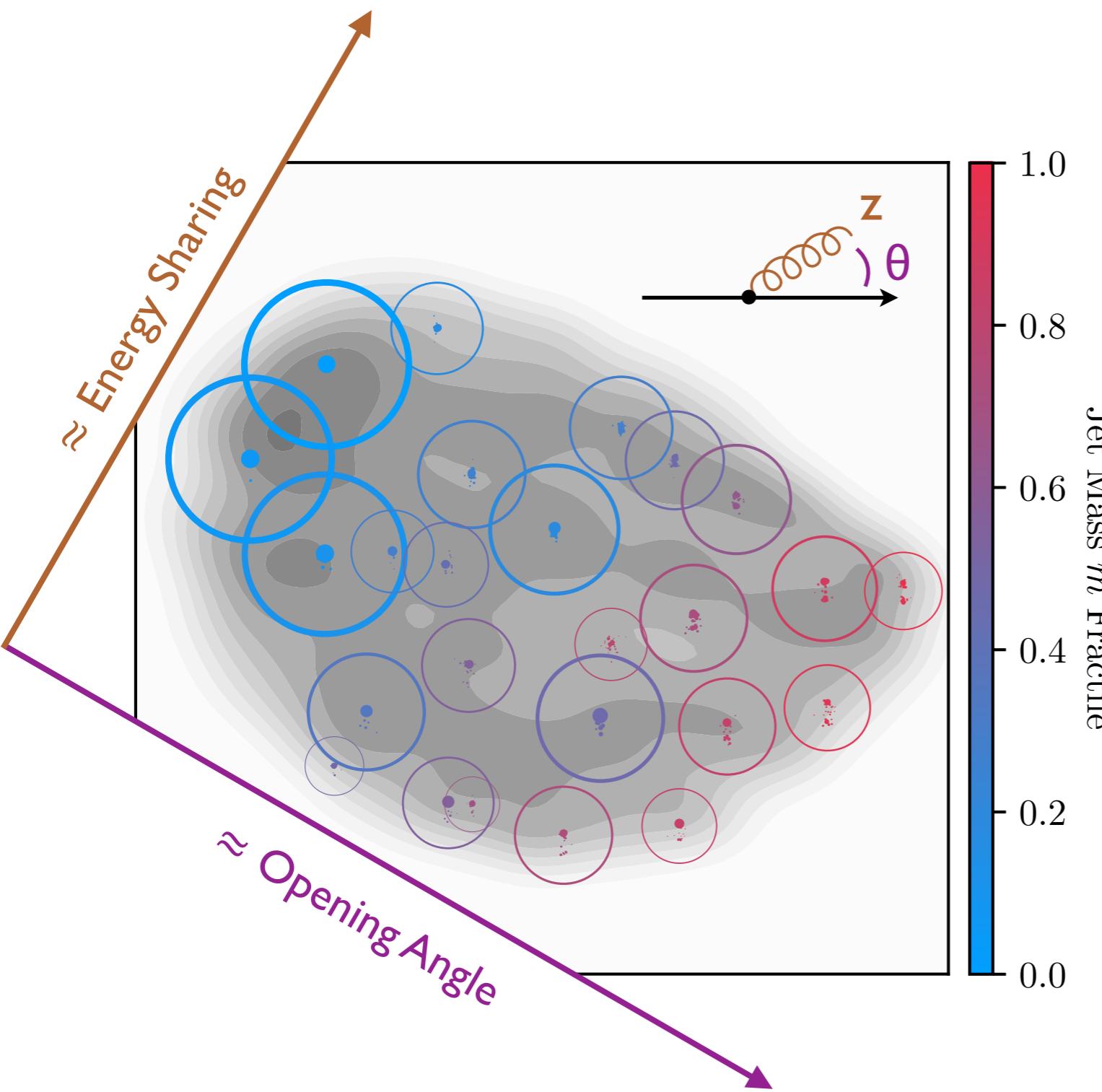
Most Representative Jets



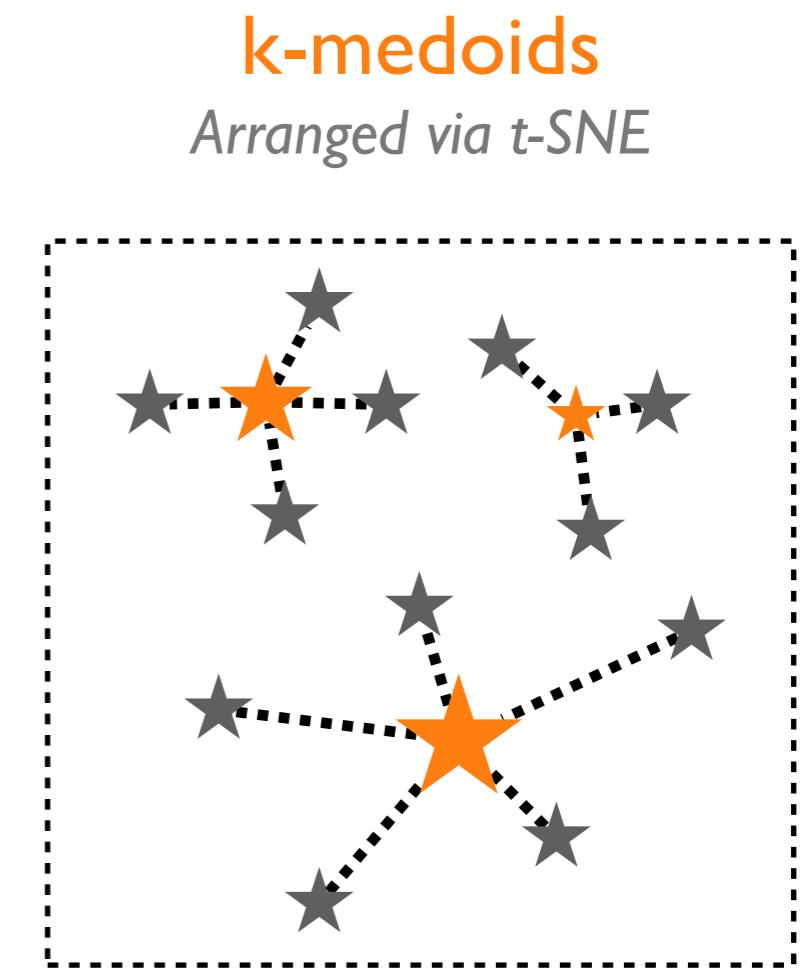
[Komiske, Mastandrea, Metodiev, Naik, JDT, [PRD 2020](#);
using van der Maaten, Hinton, [JMLR 2008](#)]



Most Representative Jets

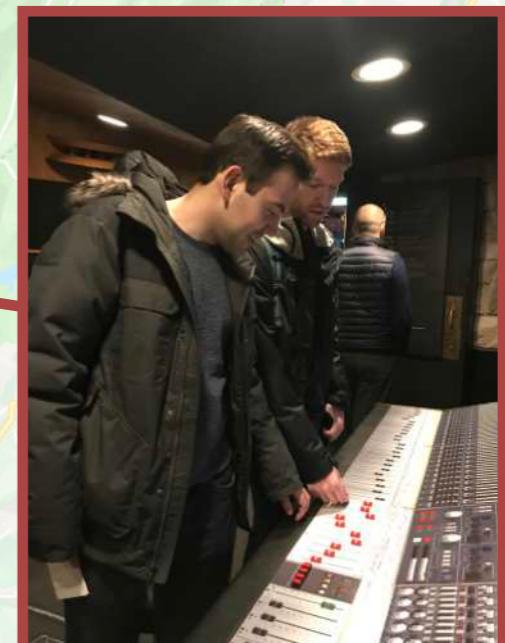


[Komiske, Mastandrea, Metodiev, Naik, JDT, [PRD 2020](#);
using van der Maaten, Hinton, [JMLR 2008](#)]



*“Okay, but are you sure this
hasn’t all been done before?”*

Scenes from My Sabbatical



Eric Metodiev

Patrick Komiske

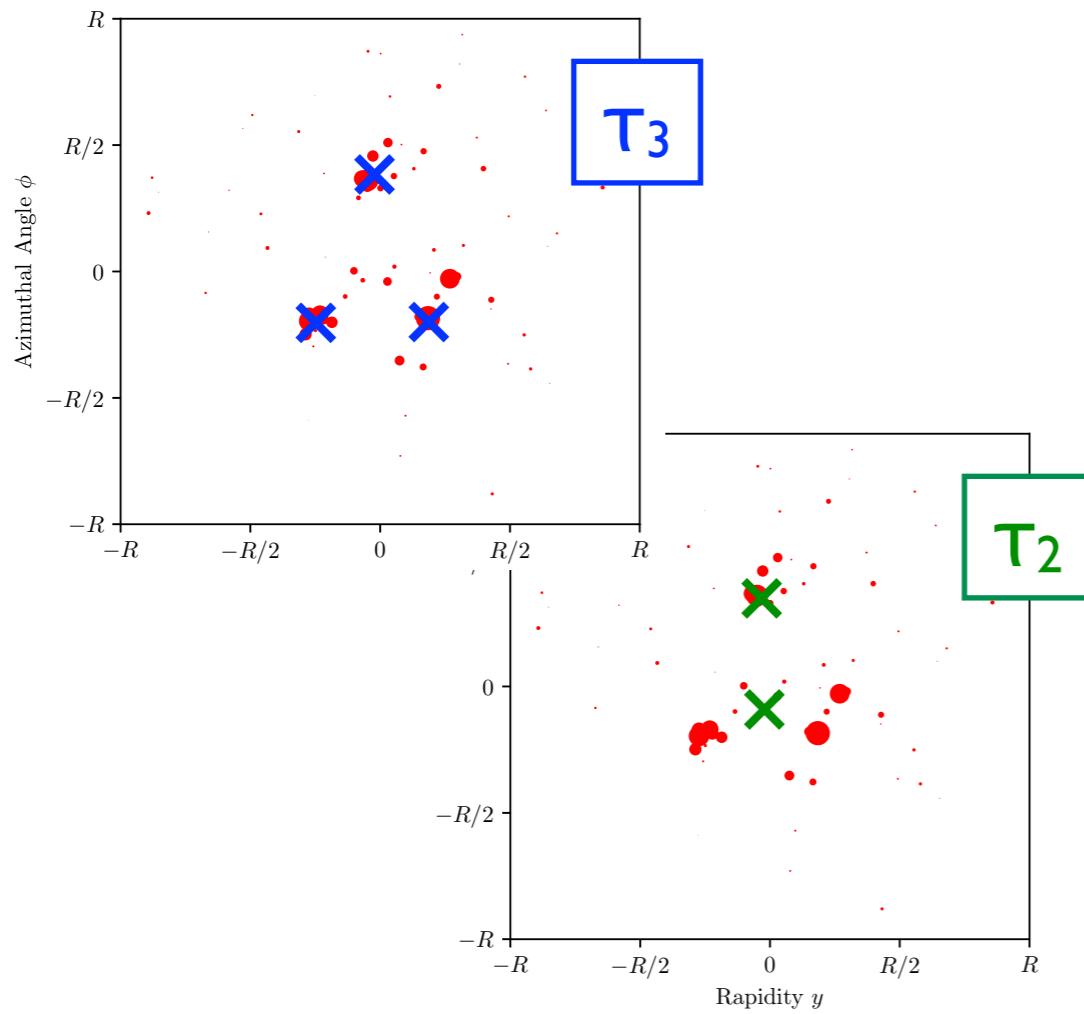
[February 2019;
Simons Sabbatical Fellowship]

SF

N-subjettiness

Ubiquitous jet substructure observable used for almost a decade...

$$\tau_N(\mathcal{J}) = \min_{N \text{ axes}} \sum_i E_i \min \{\theta_{1,i}, \theta_{2,i}, \dots, \theta_{N,i}\}$$

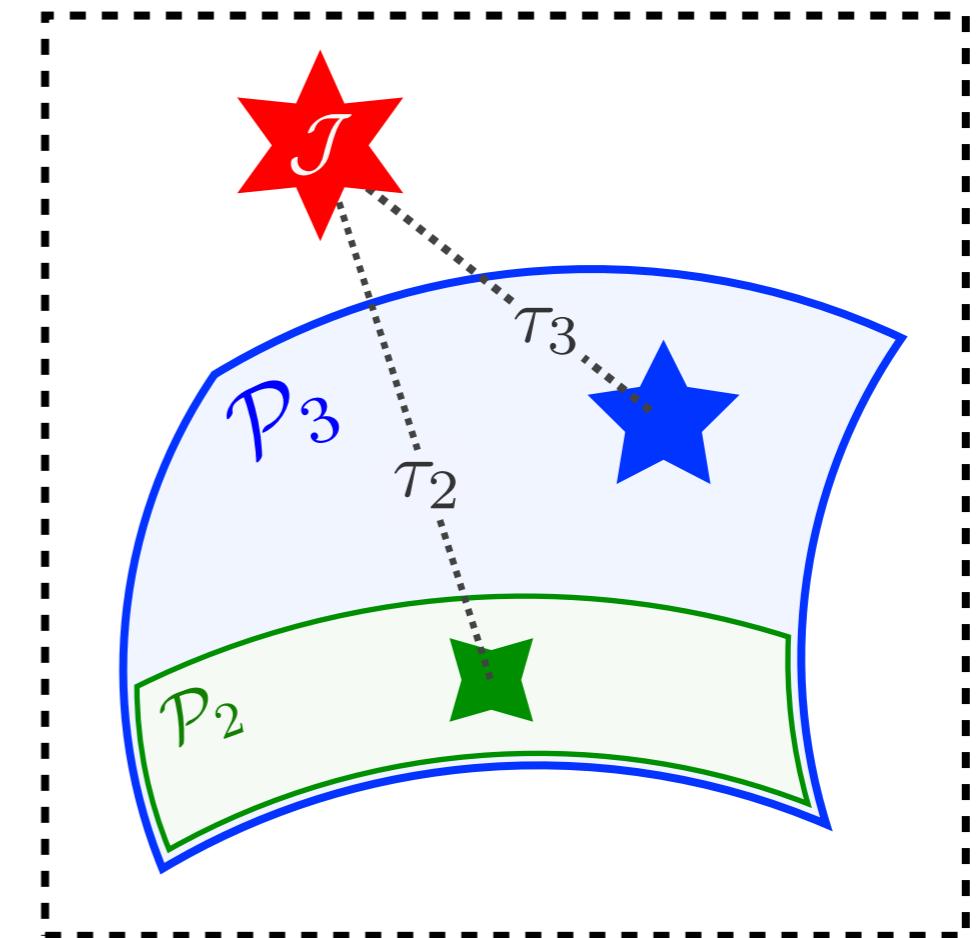
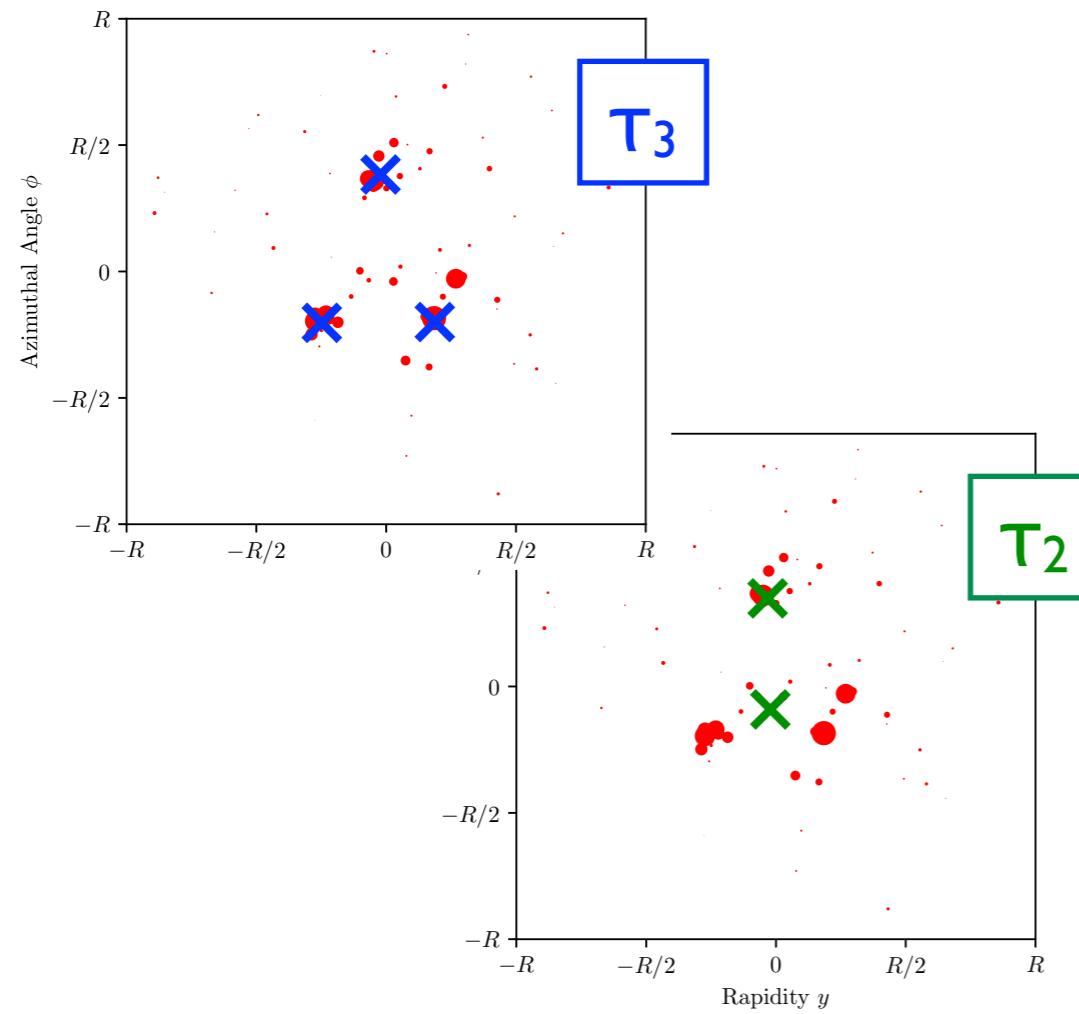


[JDT, Van Tilburg, [JHEP 2011](#), [JHEP 2012](#);
based on Brandt, Dahmen, [ZPC 1979](#); Stewart, Tackmann, Waalewijn, [PRL 2010](#)]

N-subjettiness = Point to Manifold EMD

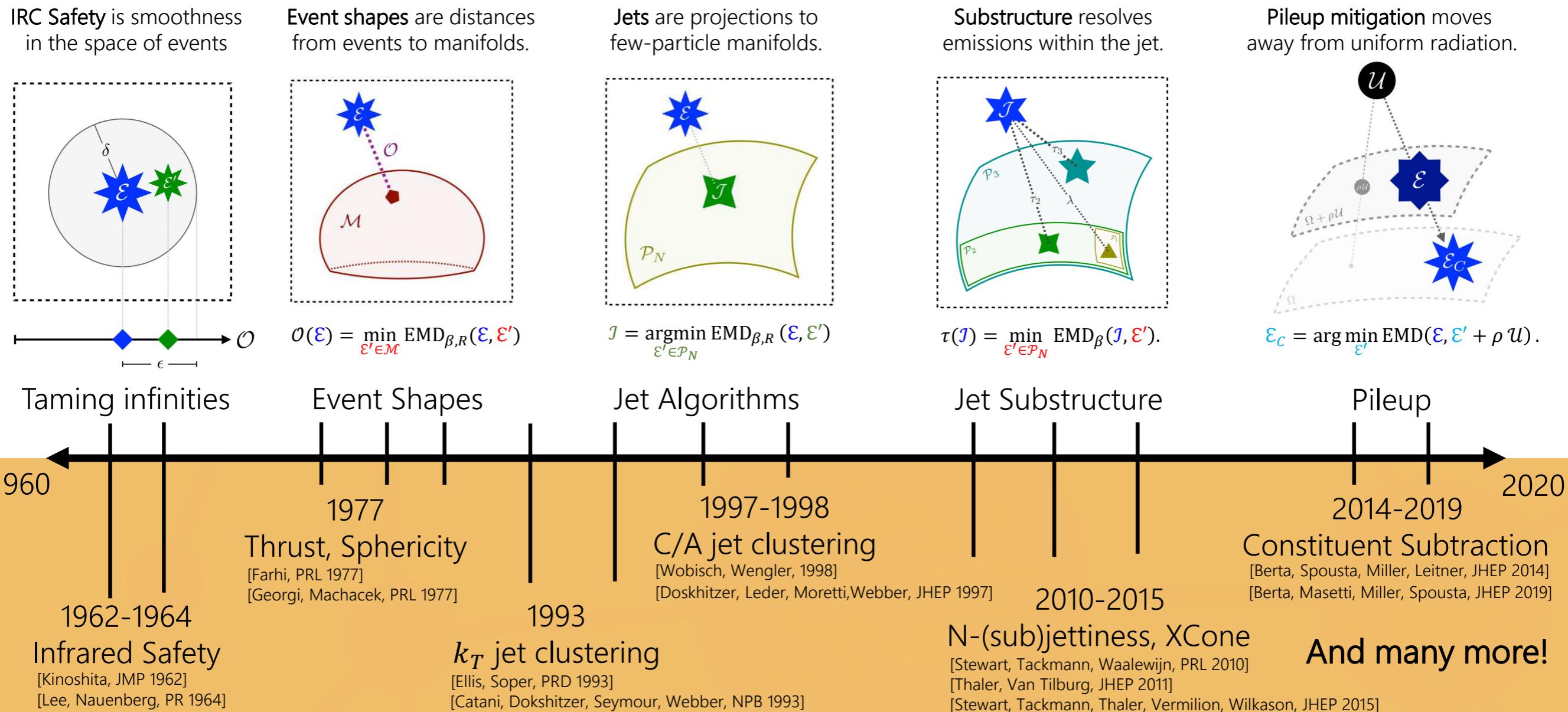
...is secretly an optimal transport problem

$$\tau_N(\mathcal{J}) = \min_{\mathcal{J}' \in \mathcal{P}_N} \text{EMD}(\mathcal{J}, \mathcal{J}')$$



[JDT, Van Tilburg, [JHEP 2011](#), [JHEP 2012](#);
rephrased via Komiske, Metodiev, JDT, [JHEP 2020](#); see opposite limit in Cesarotti, JDT, [JHEP 2020](#)]

Six Decades of Collider Physics Translated into a New Geometric Language!

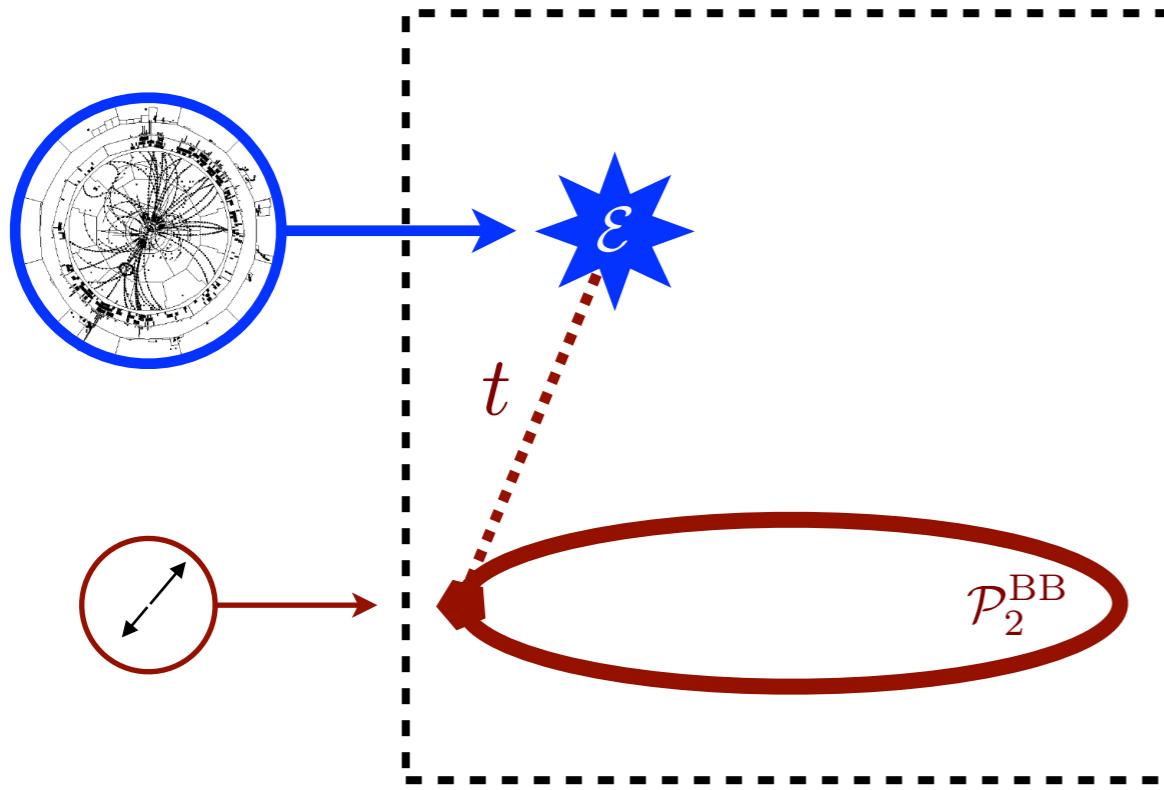


[Komiske, Metodiev, JDT, JHEP 2020; timeline by Metodiev]

Back to the Future with Thrust

How dijet-like is an event?

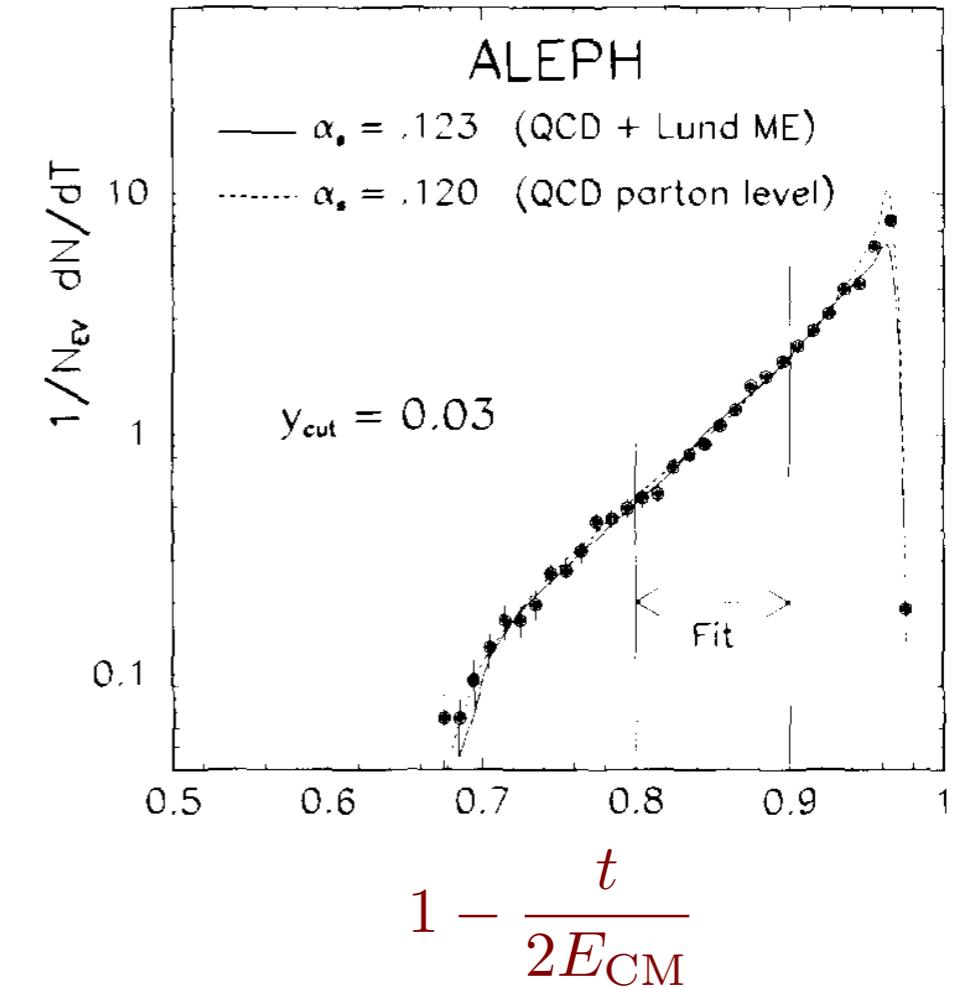
$$t(\mathcal{E}) = \min_{\mathcal{E}' \in \mathcal{P}_2^{\text{BB}}} \text{EMD}_2(\mathcal{E}, \mathcal{E}')$$



All Back-to-Back Two Particle Configurations

$$\mathcal{P}_2^{\text{BB}} = \left\{ \begin{array}{c} \text{red circles with arrows} \\ \dots \end{array} \right\}$$

(using $\beta=2$ EMD variant)

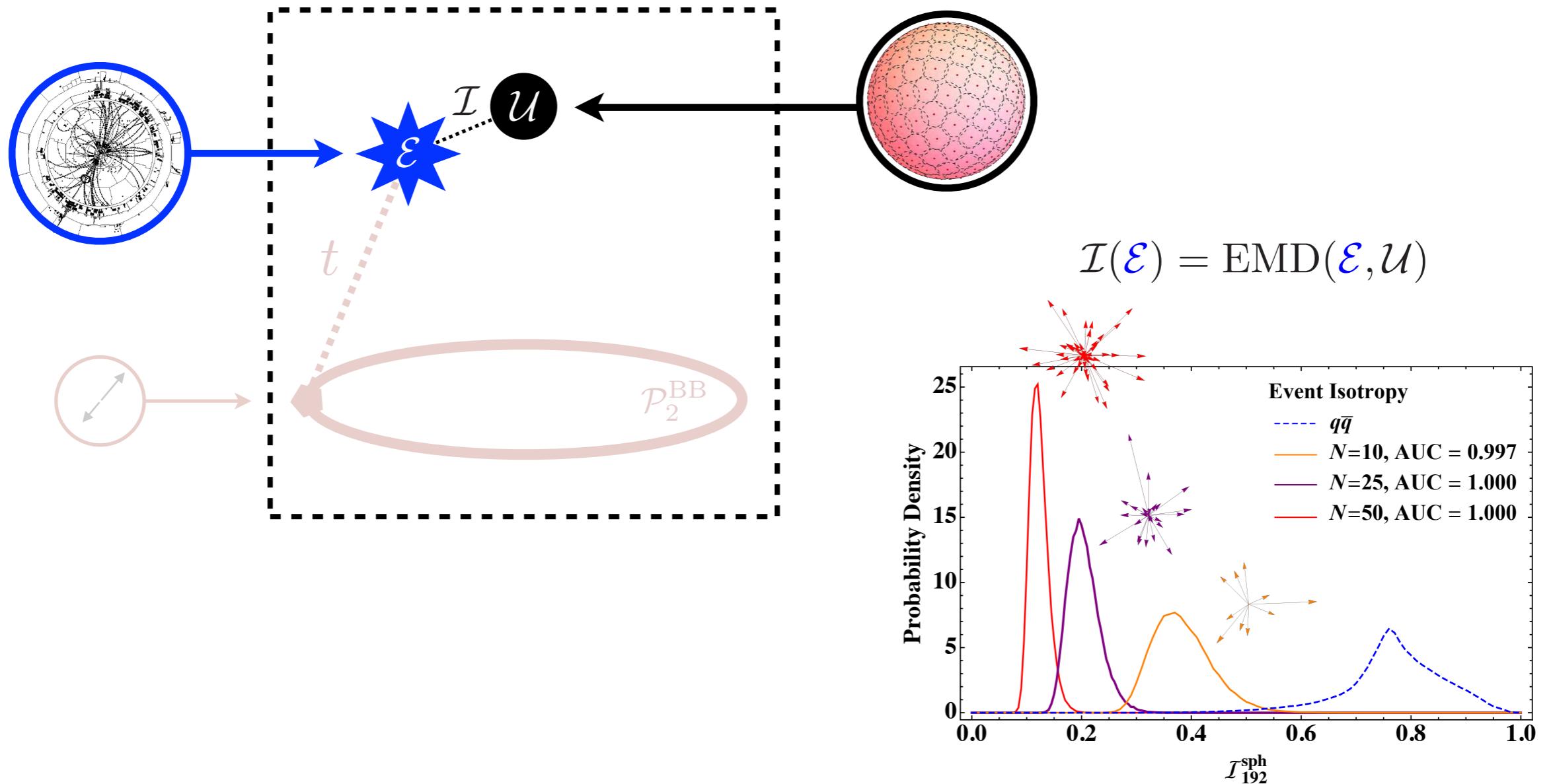


(flipped, linear version of
ALEPH thrust plot from before)

[Komiske, Metodiev, JDT, JHEP 2020]
[Brandt, Peyrou, Sosnowski, Wroblewski, PL 1964; Farhi, PRL 1977; ALEPH, PLB 1991]

Event Isotropy from Collider Geometry

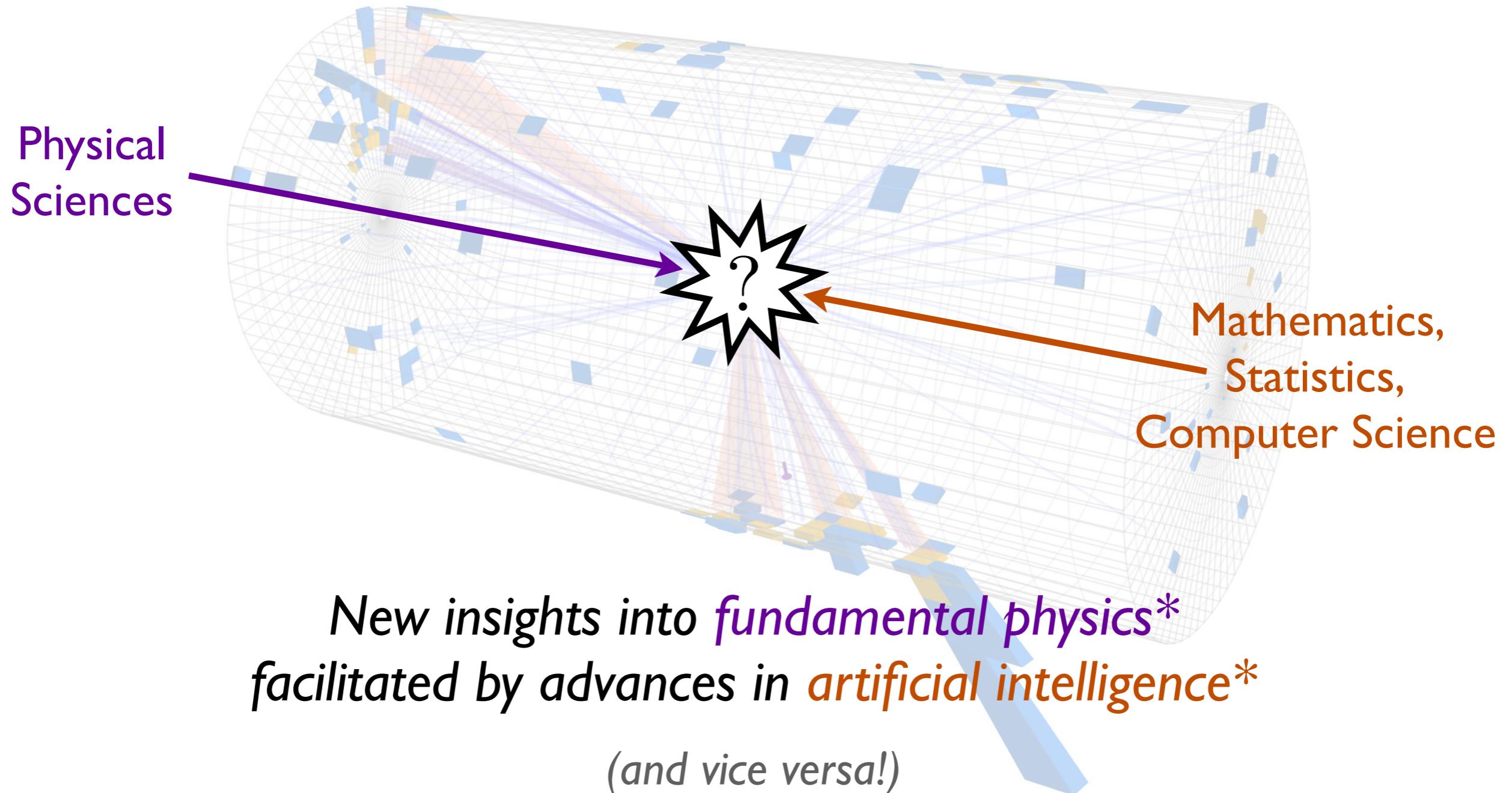
How isotropic is an event?



[Cesarotti, JDT, [JHEP 2020](#);
see also Cesarotti, Reece, Strassler, [arXiv 2020](#)]



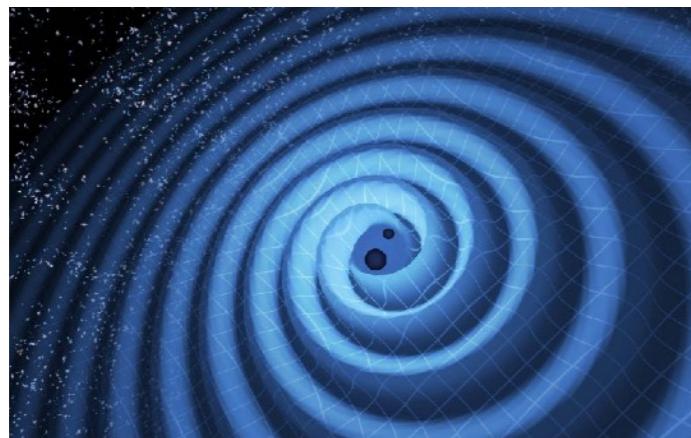
“Collision Course”



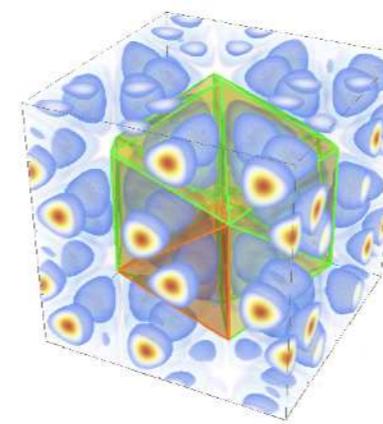
Artificial Intelligence \leftrightarrow Fundamental Interactions



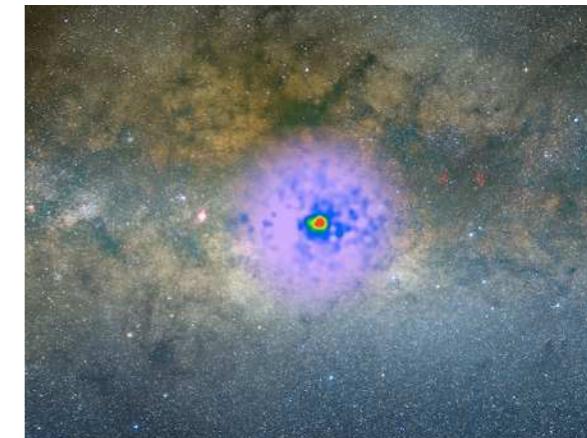
Gravitational Waves



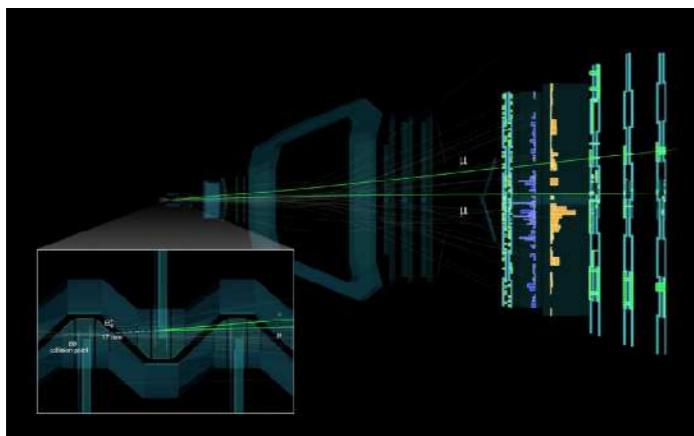
Nuclear Physics



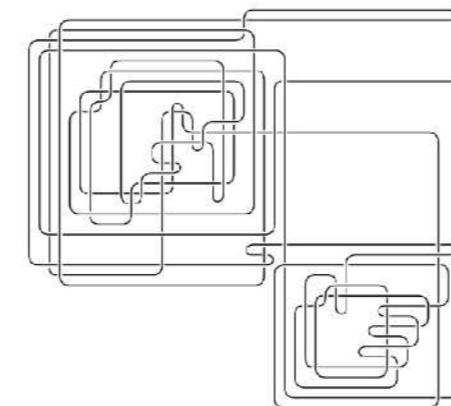
Astrophysics



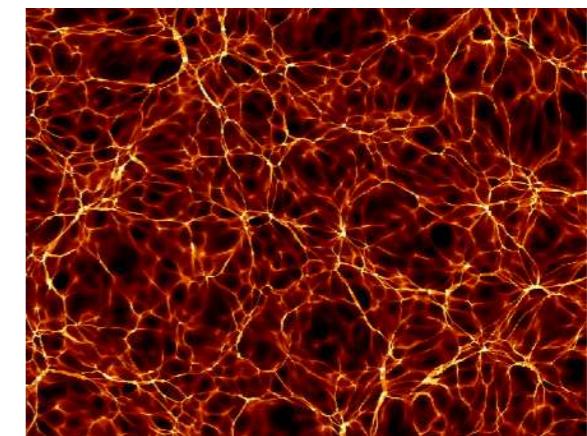
Particle Colliders



Mathematical Physics



Dark Matter

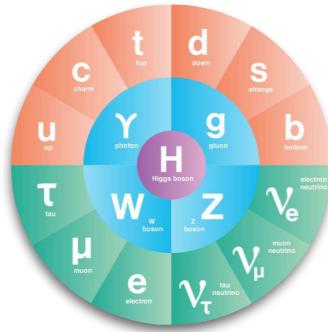
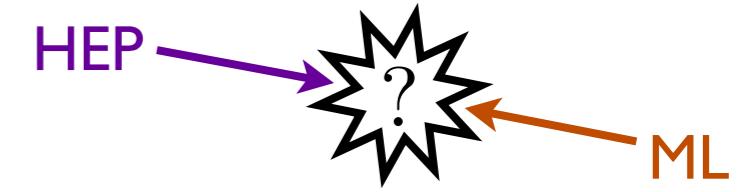


...

Machine learning that incorporates first principles, best practices, and domain knowledge from fundamental physics

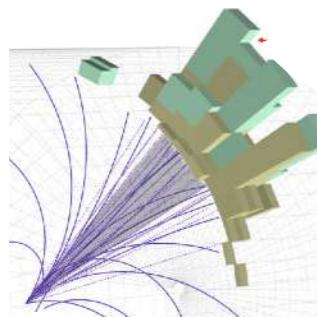
[<http://iaifi.org>]

Summary



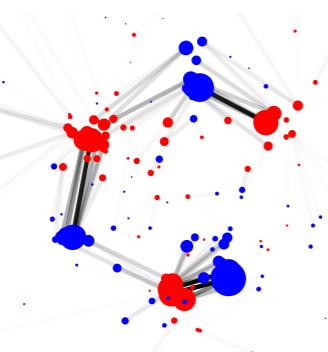
Particle Physics 101

*High-energy collisions can yield insights into fundamental physics
Machine learning offers powerful tools to analyze collision debris*



What is a Collider Event?

*Unordered set of particles describing energy flow of jets
Inspires network architectures designed for symmetry and safety*

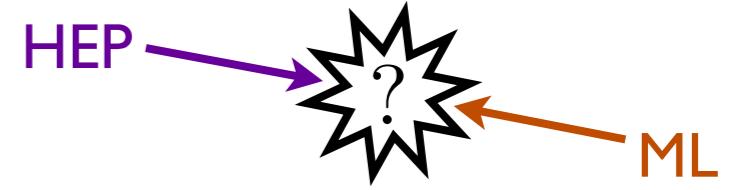


When are Collider Events Similar?

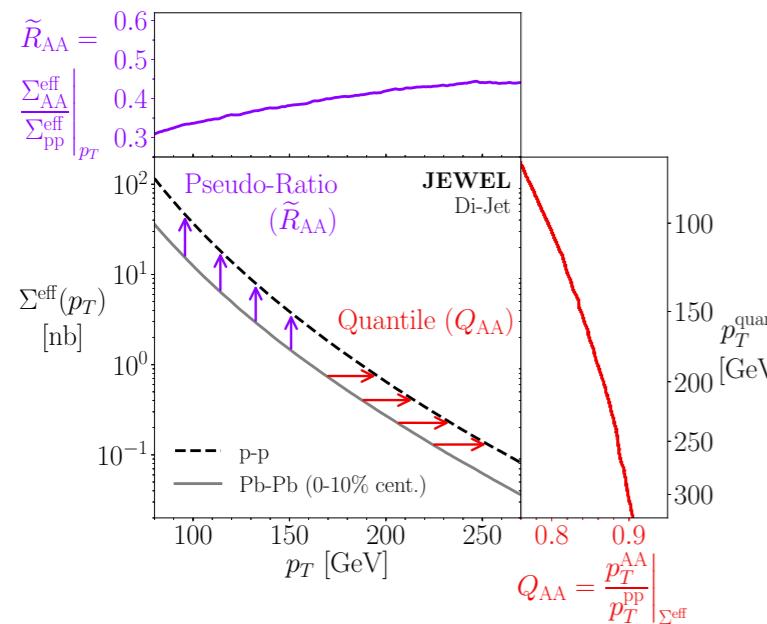
*When their energy flows are similar
Inspires unsupervised learning strategies based on event geometry*

Backup Slides

More Collisions

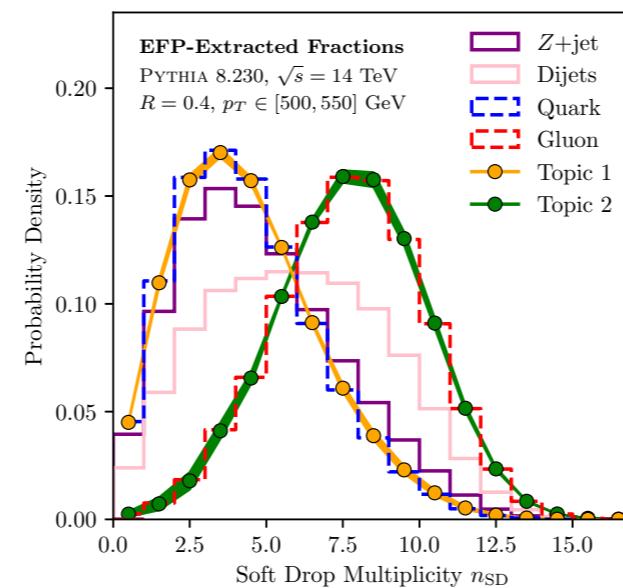


Jet Quenching via Optimal Transport



[Brewer, Milhano, JDT, PRL 2019]

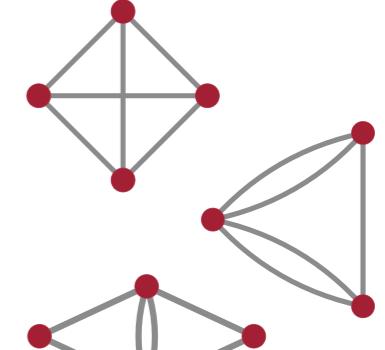
Quark/Gluon Definitions via Blind Source Separation



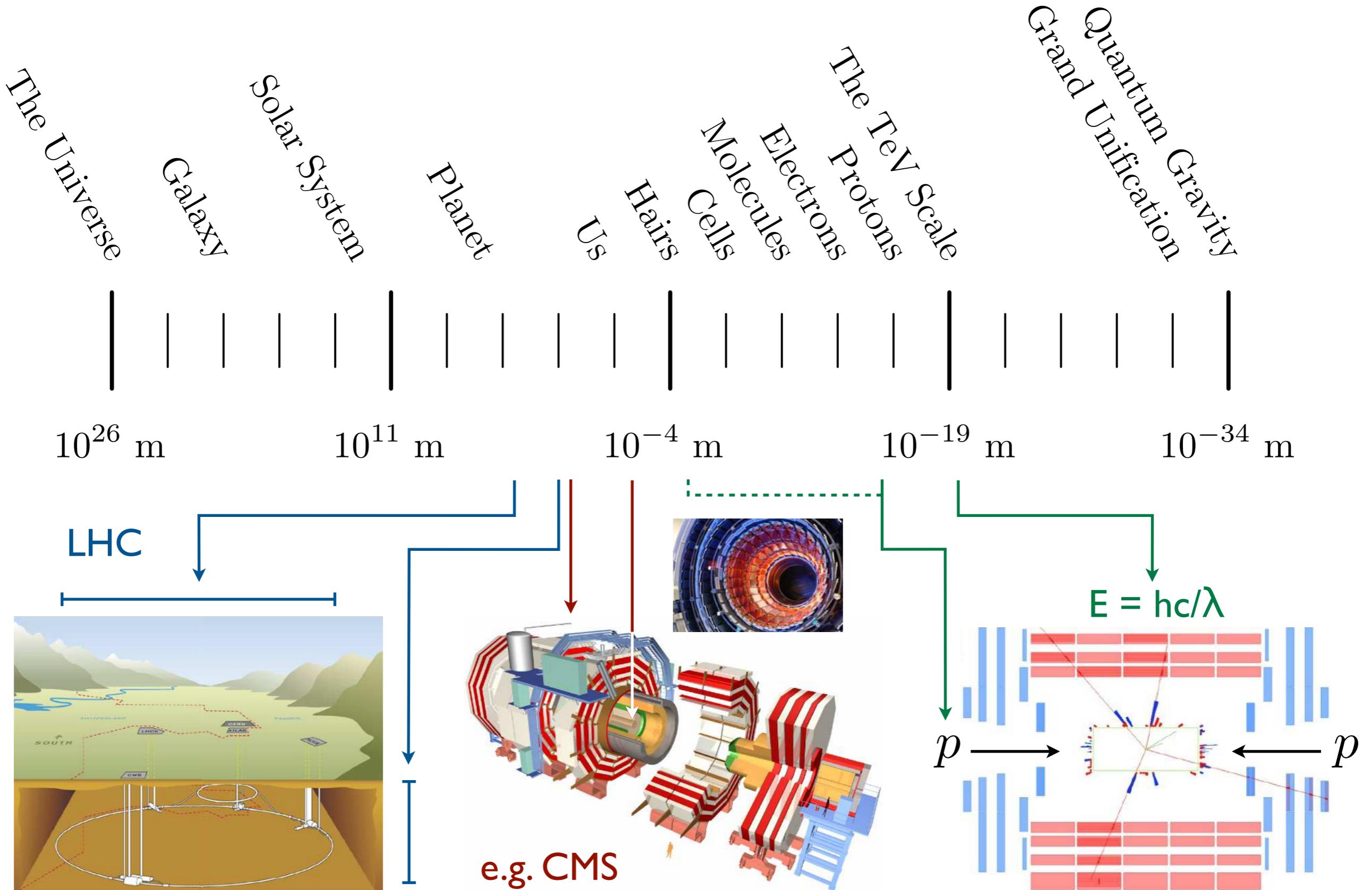
[Komiske, Metodiev, JDT, JHEP 2018;
Brewer, JDT, Turner; arXiv 2020]

Kinematic Decomposition via Graph Theory

Edges d	Leafless Multigraphs		
	Connected	All	A307316
1	0	0	0
2	1	1	1
3	2	2	2
4	4	5	5
5	9	11	11
6	26	34	34
7	68	87	87
8	217	279	279
9	718	897	897
10	2 553	3 129	3 129
11	9 574	11 458	11 458
12	38 005	44 576	44 576
13	157 306	181 071	181 071
14	679 682	770 237	770 237
15	3 047 699	3 407 332	3 407 332
16	14 150 278	15 641 159	15 641 159



[Komiske, Metodiev, JDT,
JHEP 2018, PRD 2020]



Mesons

$\pi^\pm \pi^0 \eta K^\pm K^0 \eta' D^\pm D^0 D_s^\pm \eta_c B^\pm B^0 B_s^0 \eta_b \dots$
 $\rho^\pm \rho^0 \omega K^{*\pm} K^{*0} \phi D^{*\pm} D^{*0} D_s^{*\pm} J/\psi B^{*\pm} B^{*0} B_s^{*0} \Upsilon \dots$

Baryons

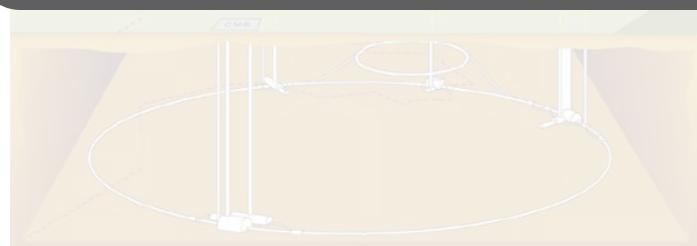
$p n \Lambda^0 \Sigma^+ \Sigma^0 \Sigma^- \Xi^0 \Xi^- \dots$
 $\Delta^{++} \Delta^+ \Delta^0 \Delta^- \Sigma^{*+} \Sigma^{*0} \Sigma^{*-} \Xi^{*0} \Xi^{*-} \Omega^- \dots$

Tetraquarks

$X(3872) Y(4260) Z(4430) \dots$

Pentaquarks

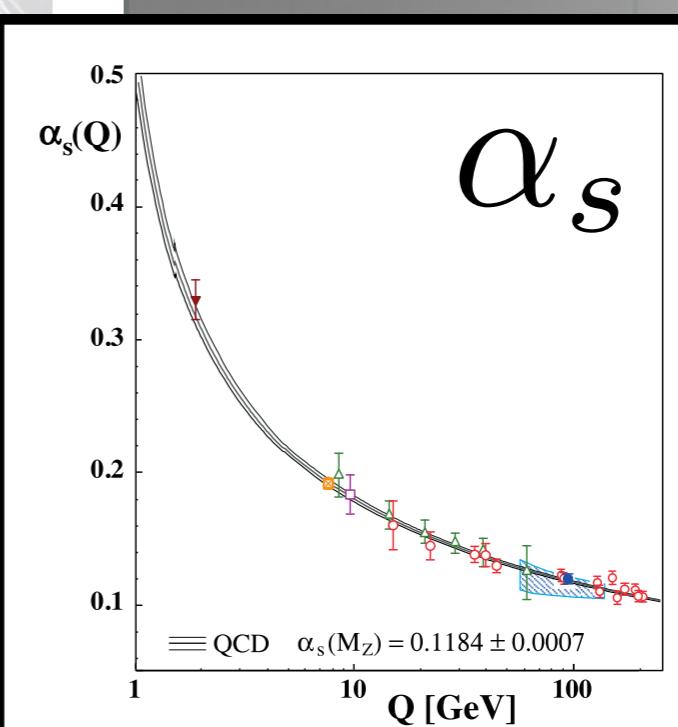
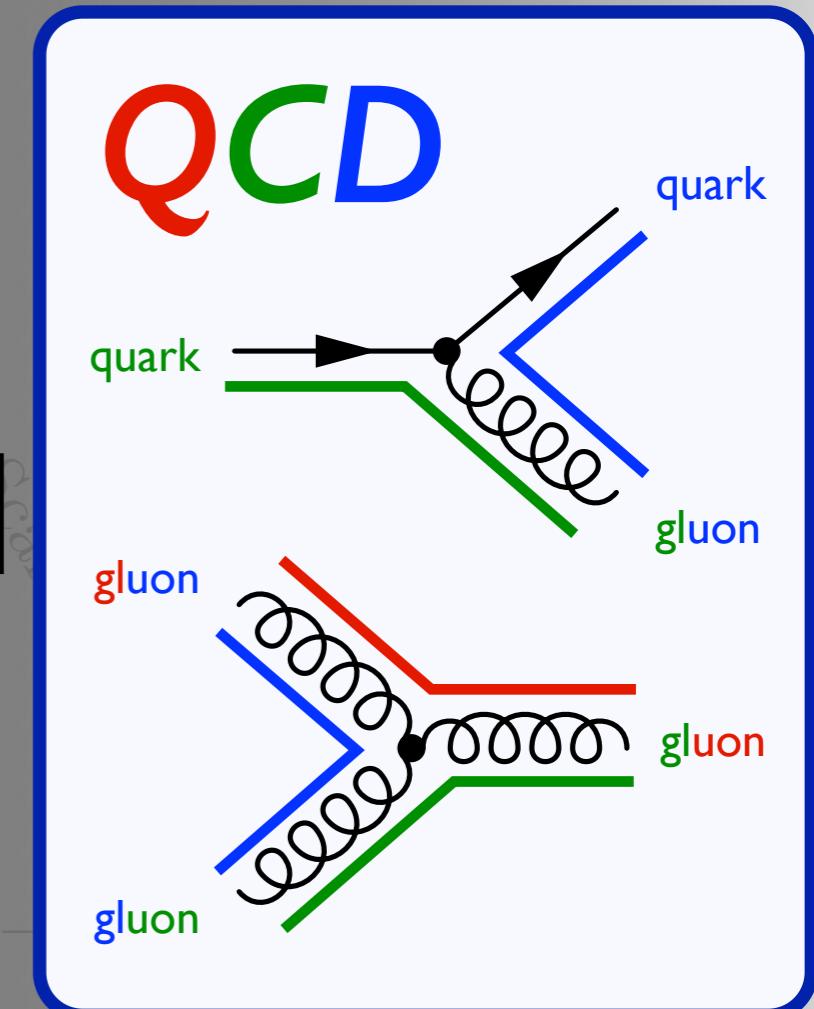
$P_c^+(4450) \dots$



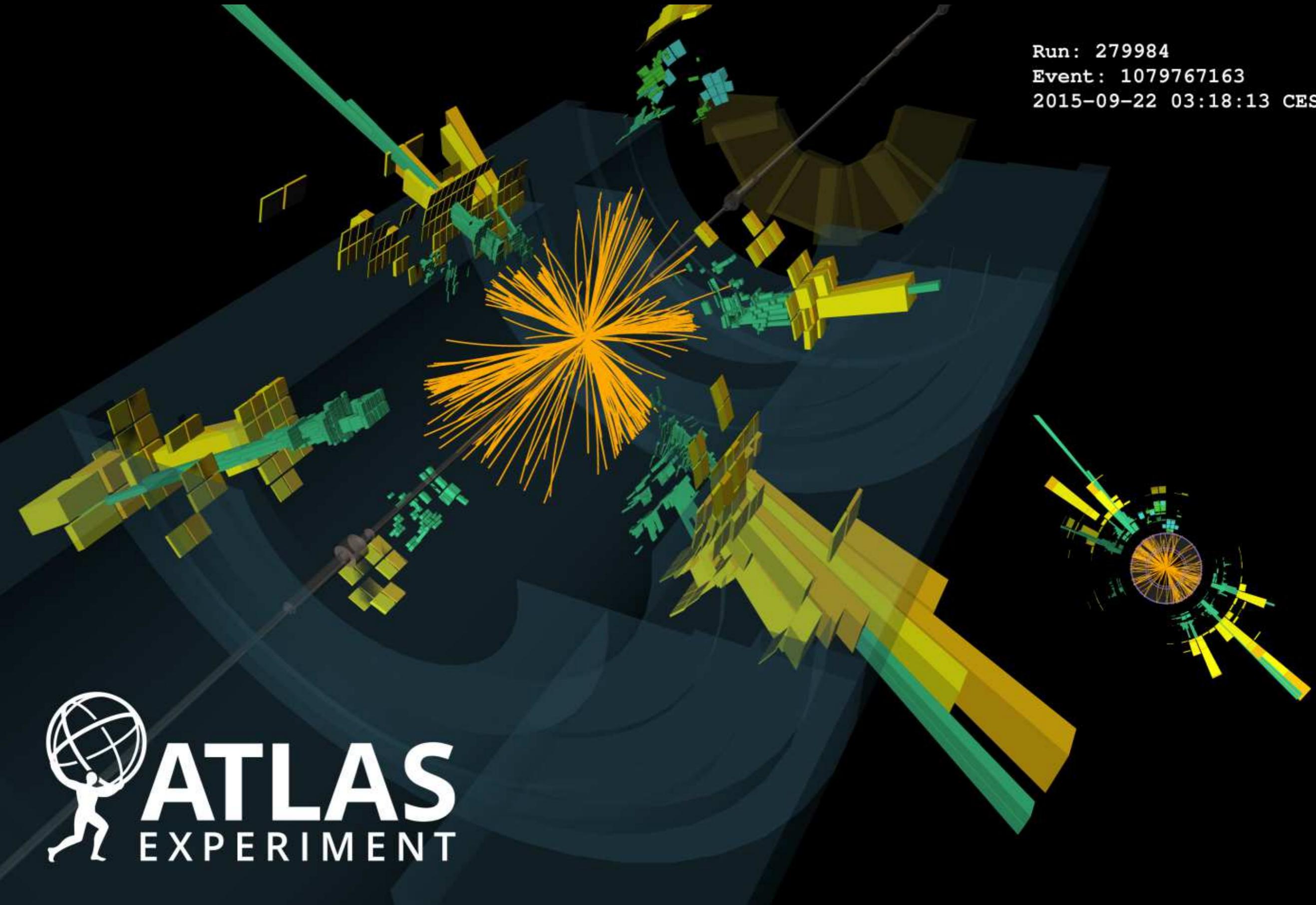
|

e.g. CMS

Jets

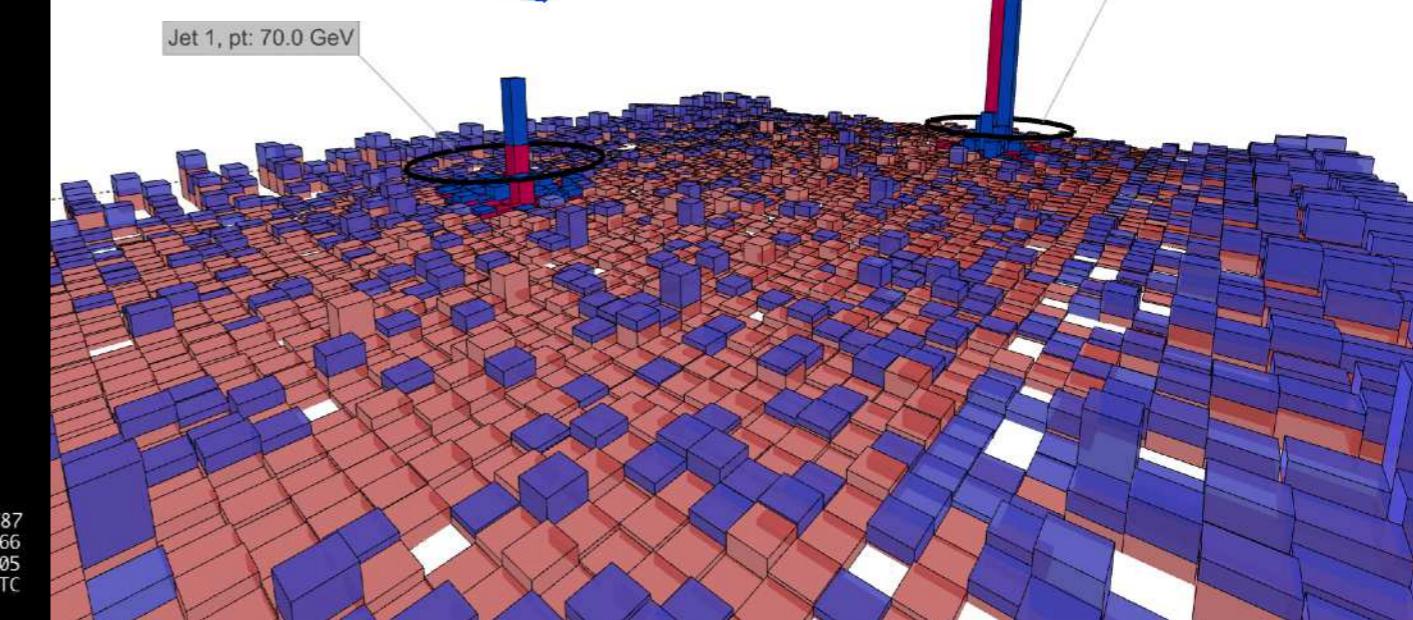
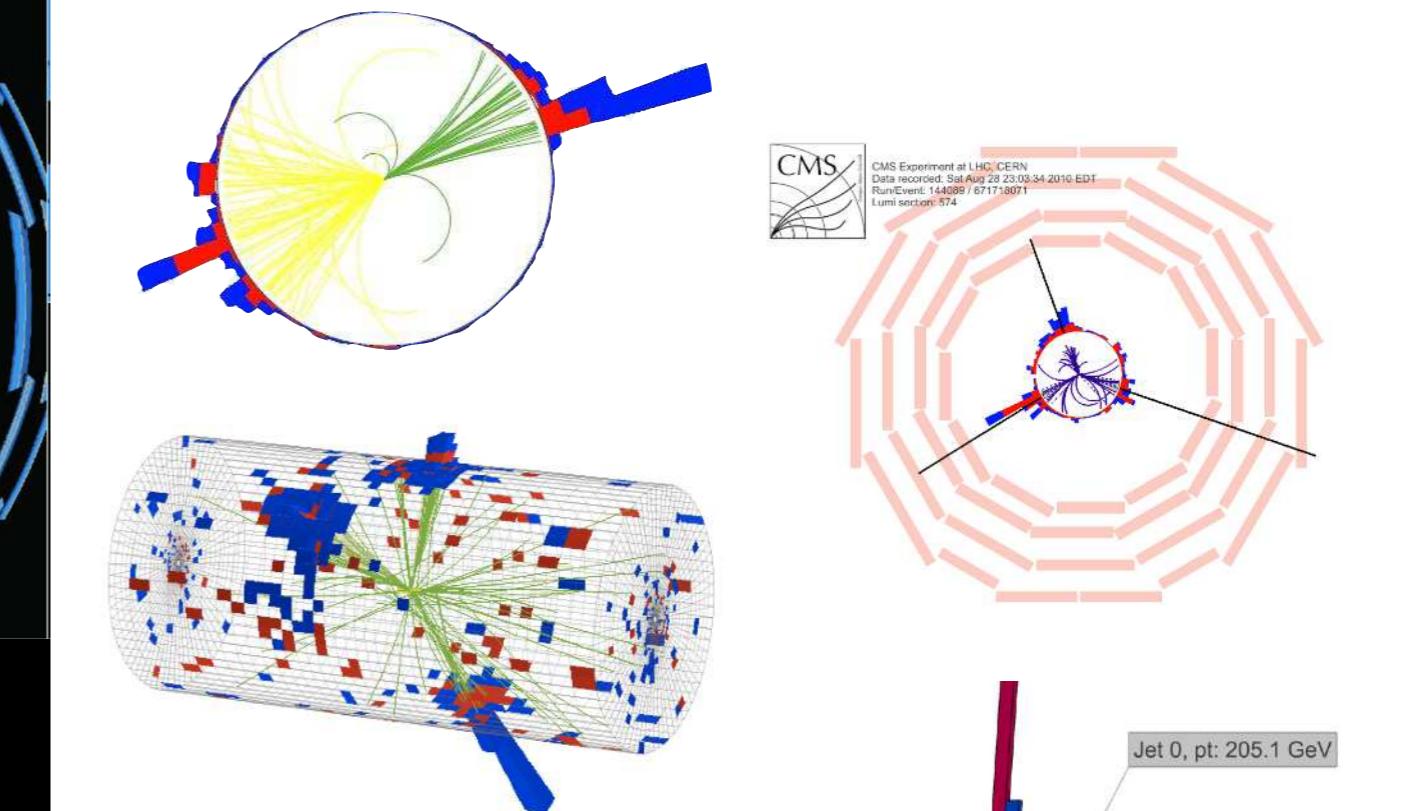
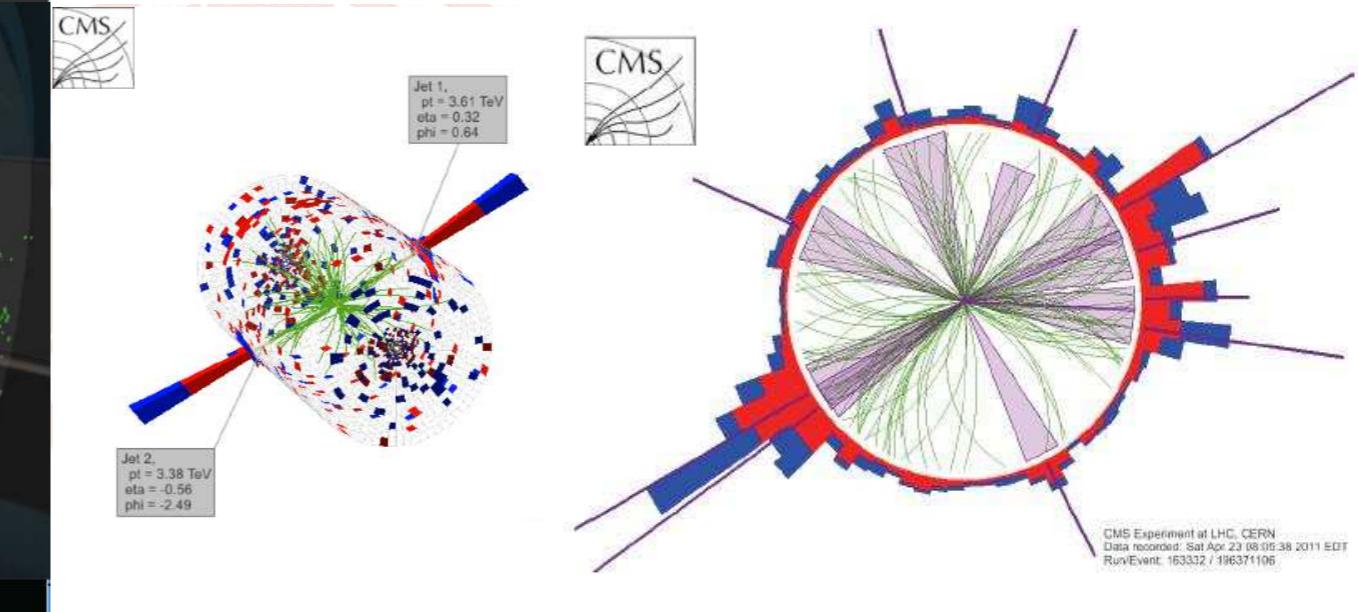
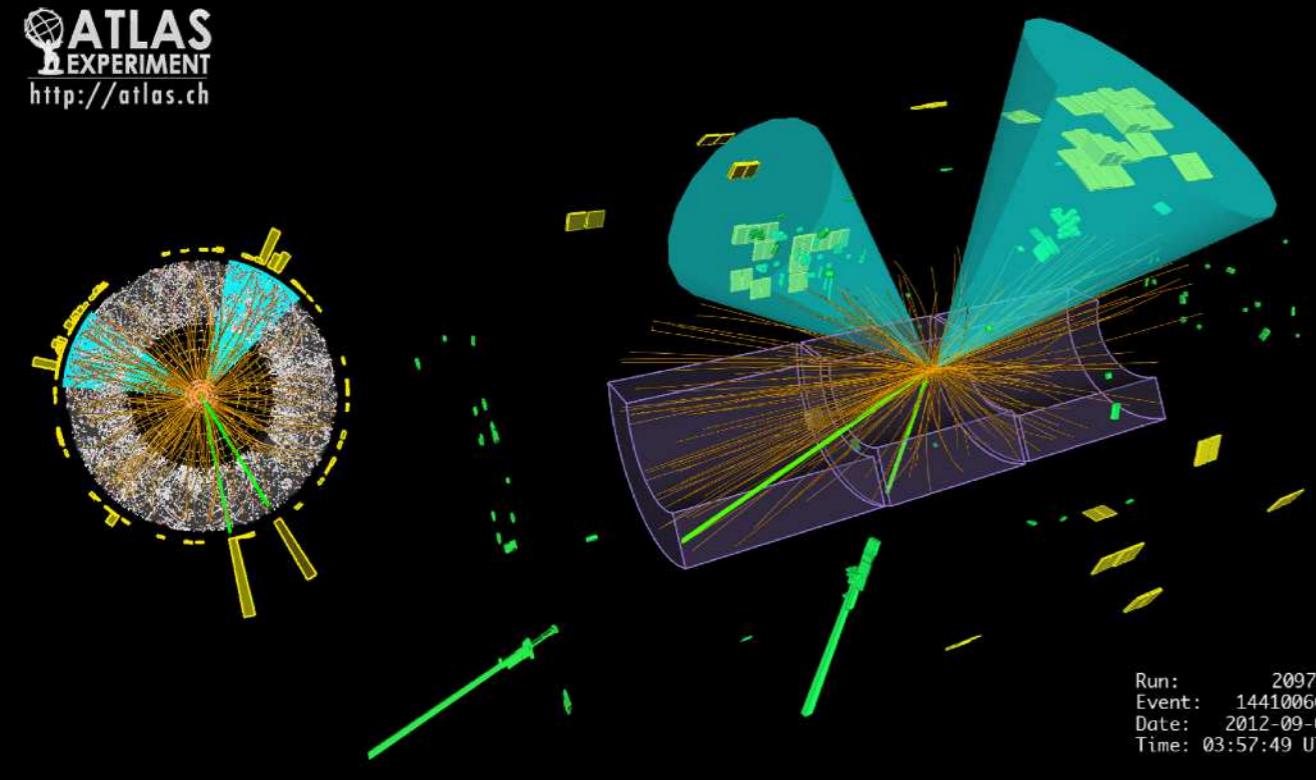
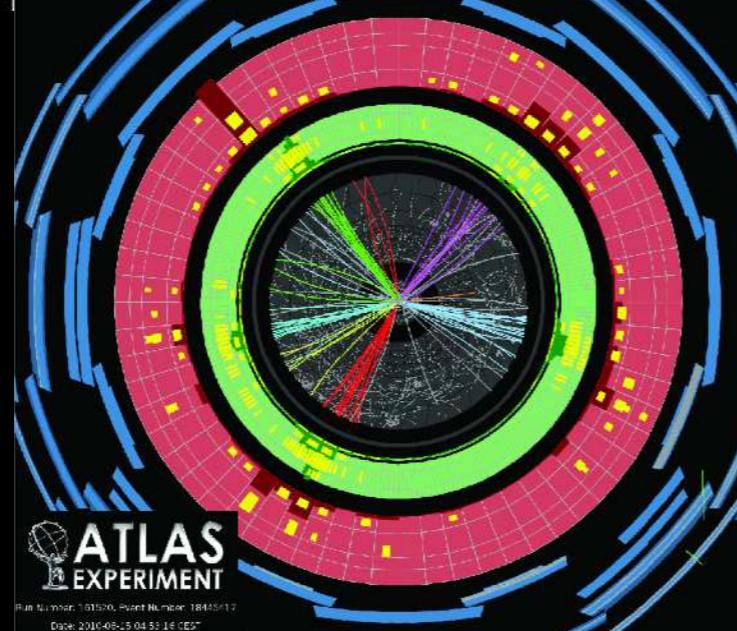
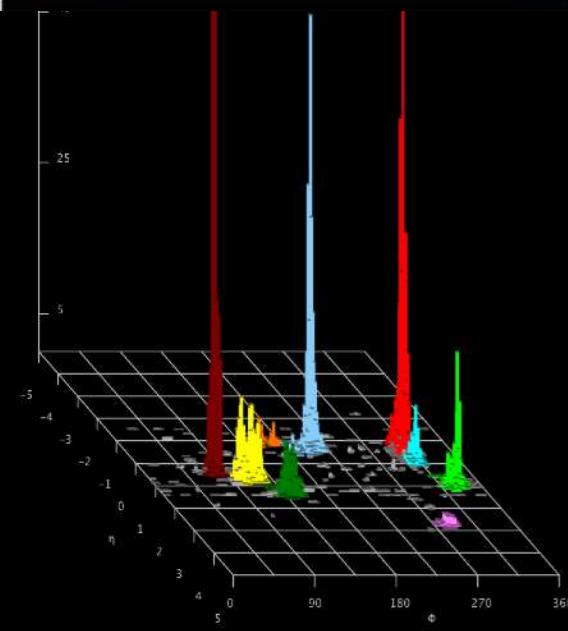
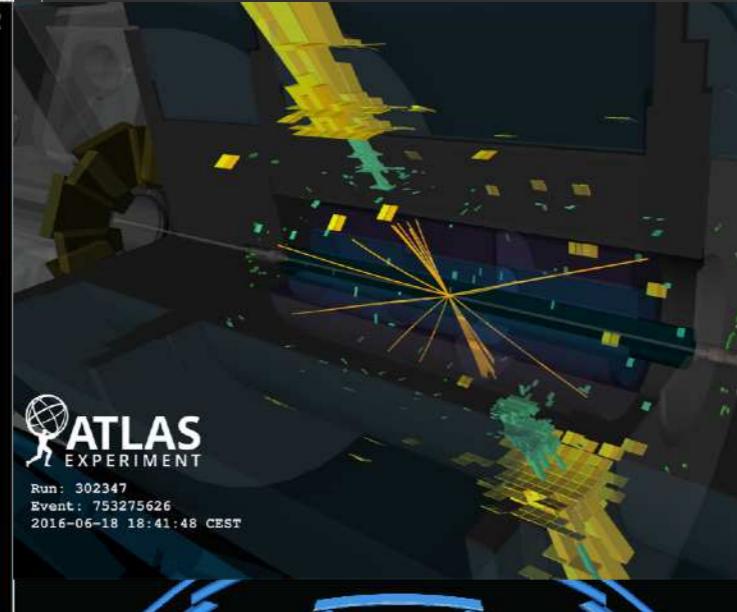
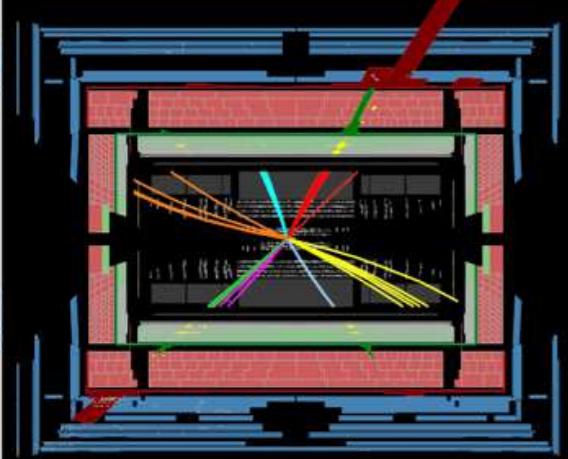


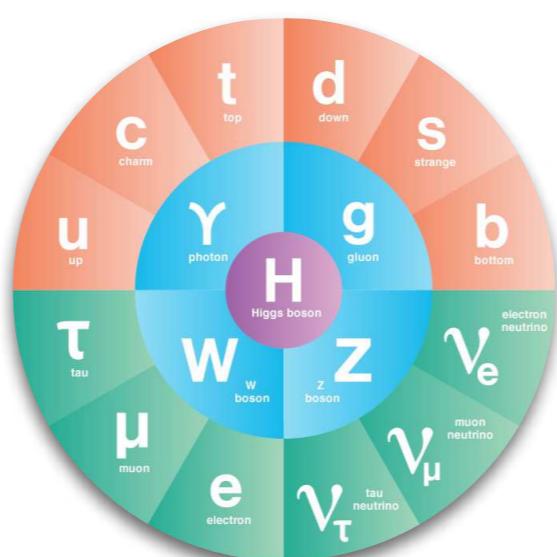
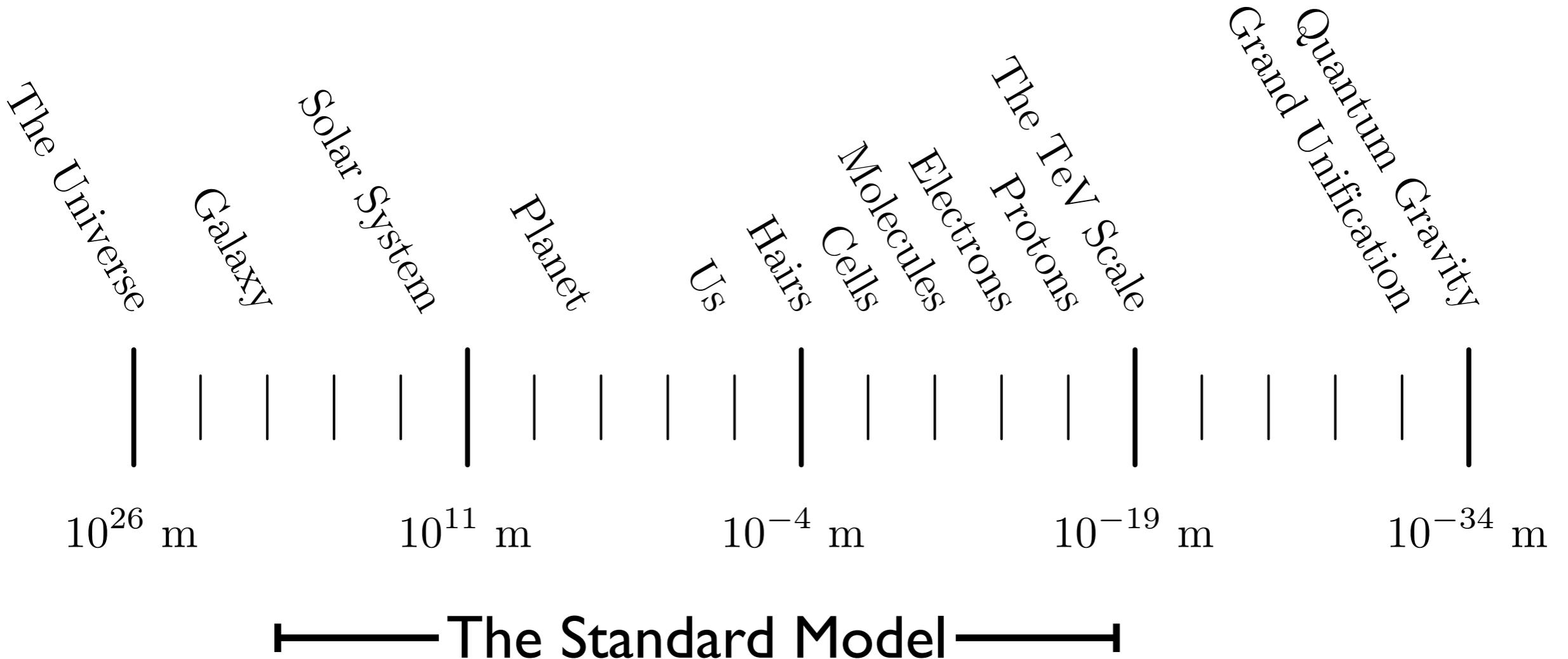
Run: 279984
Event: 1079767163
2015-09-22 03:18:13 CEST



Run Number: 159224, Event Number: 3533152

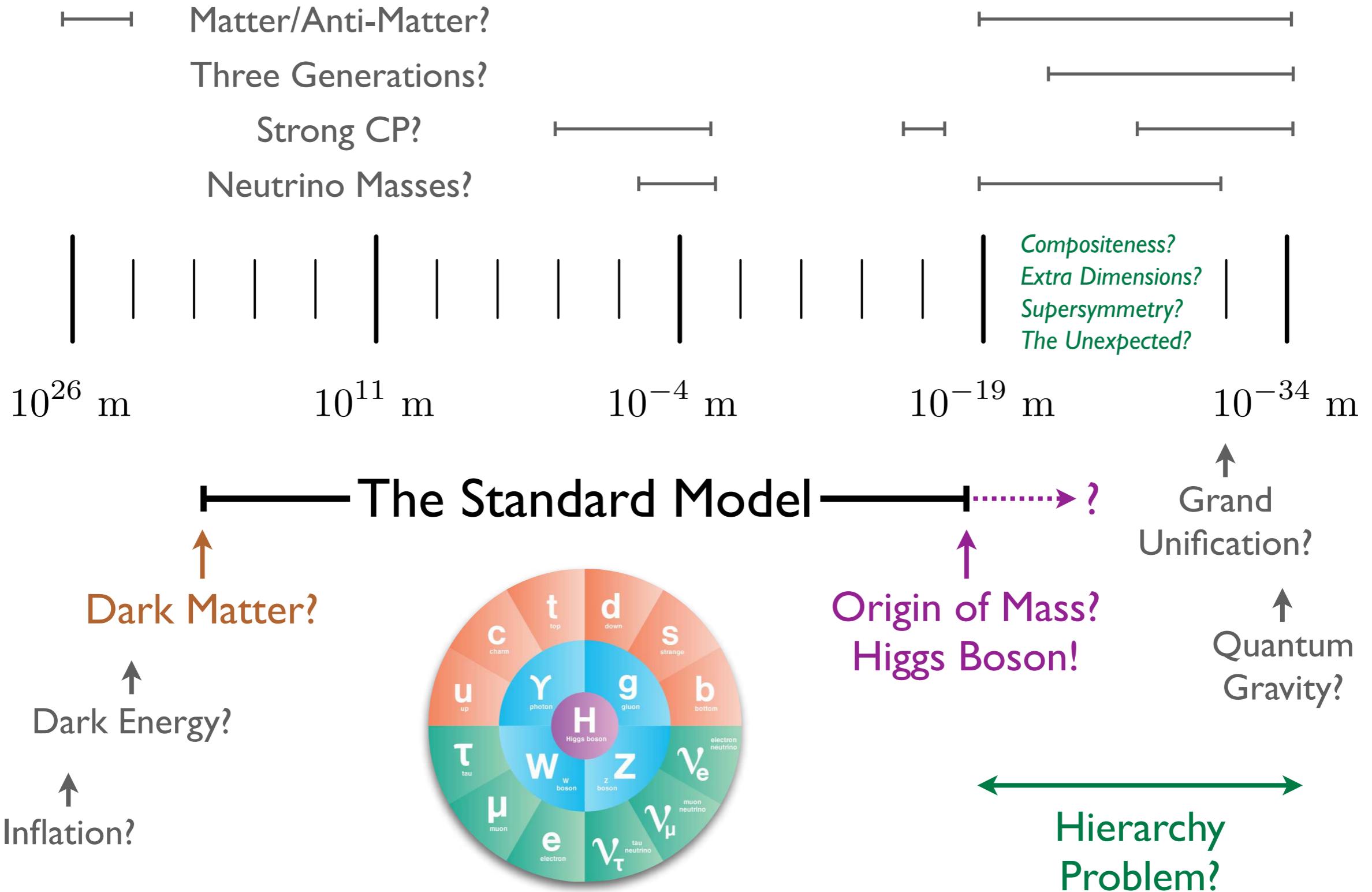
Date: 2010-07-18 11:05:54 CEST

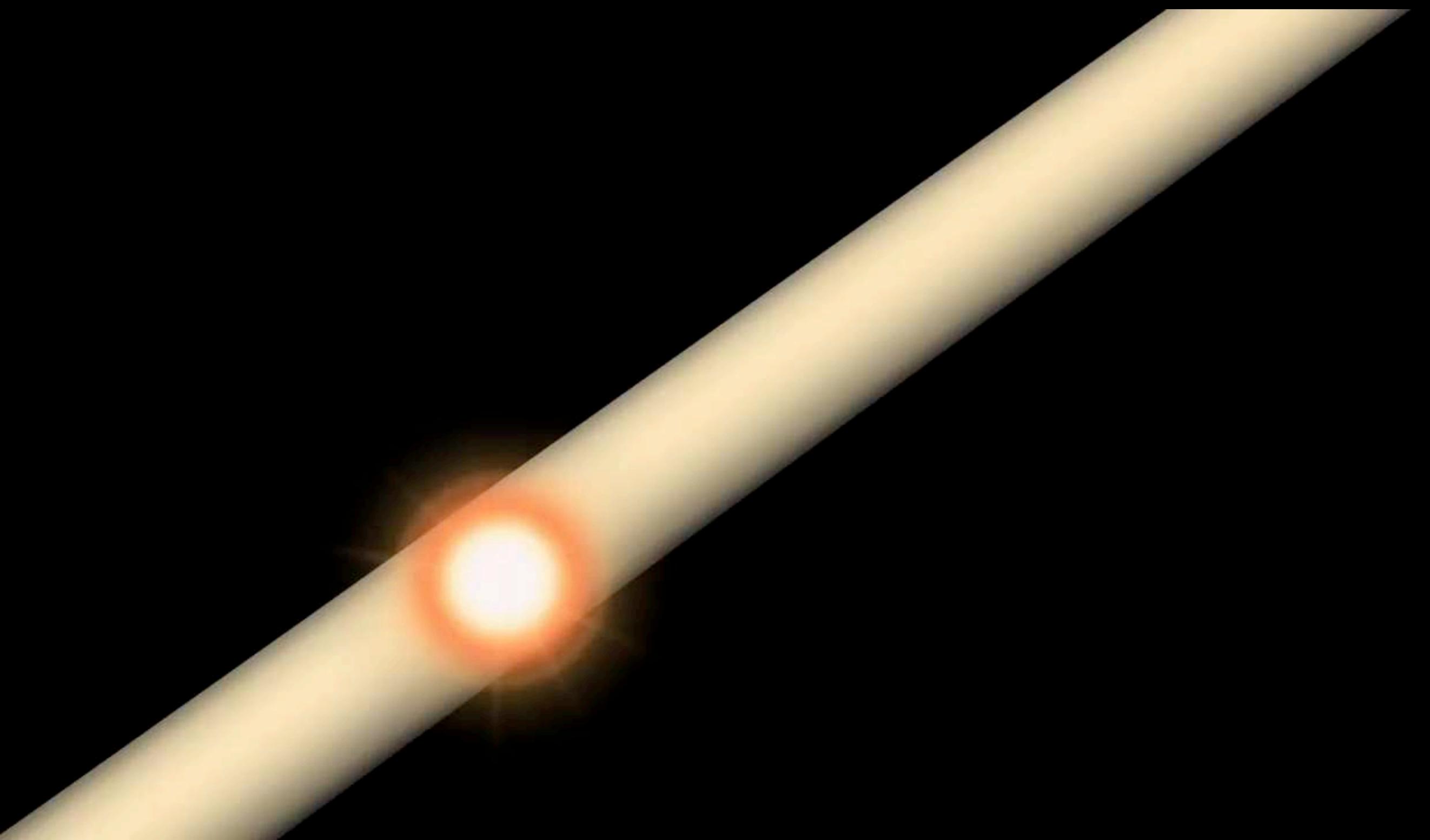




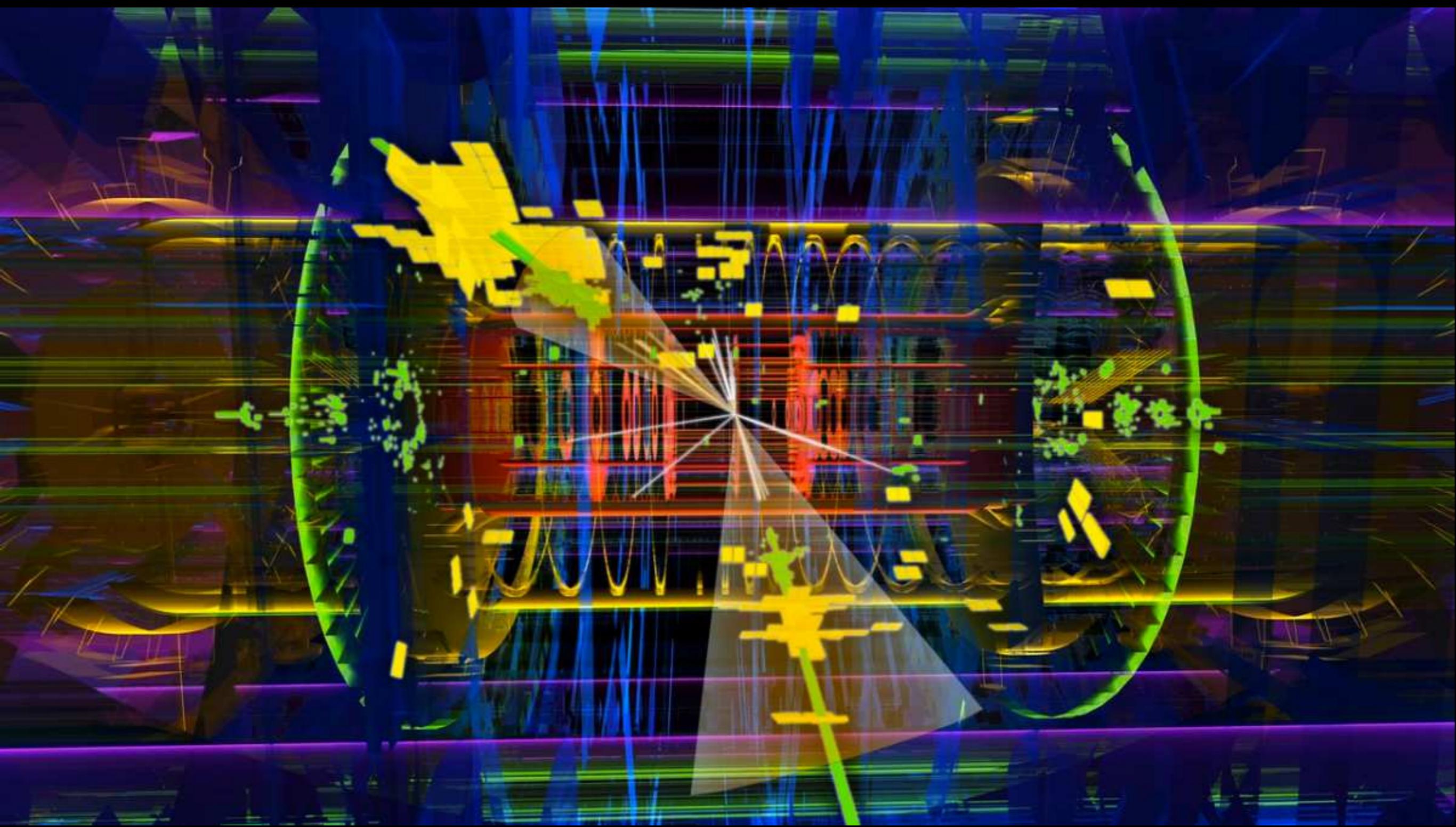
Puzzles in Particle Physics

SnowMass2021





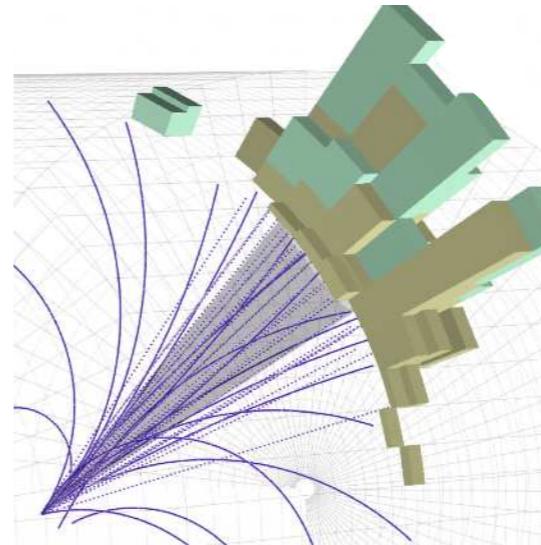




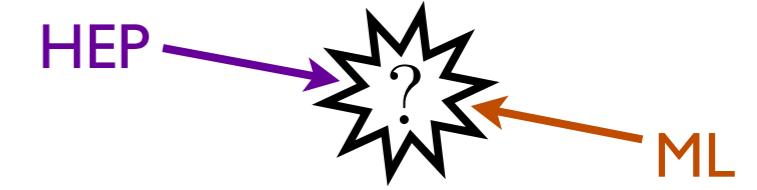
Cartoon of Machine Learning

“ML4Jets”
NYU, January 2020

For this talk: \mathcal{J} = jet



E.g. Search for Supersymmetry

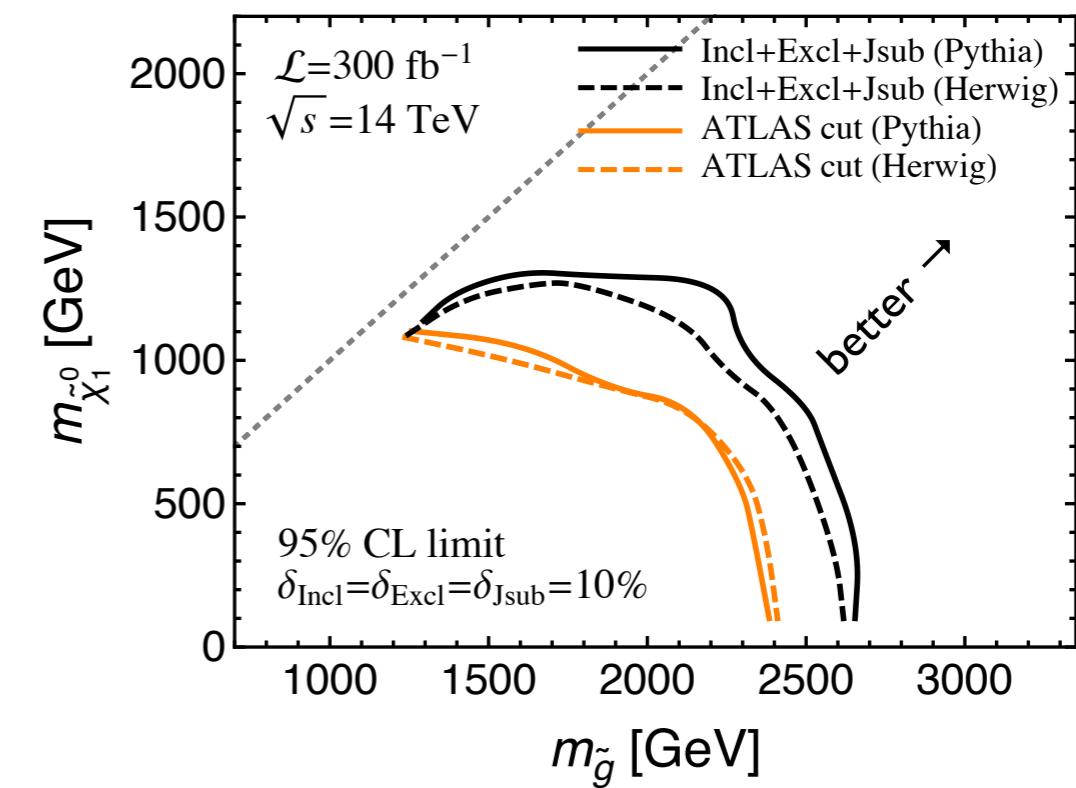
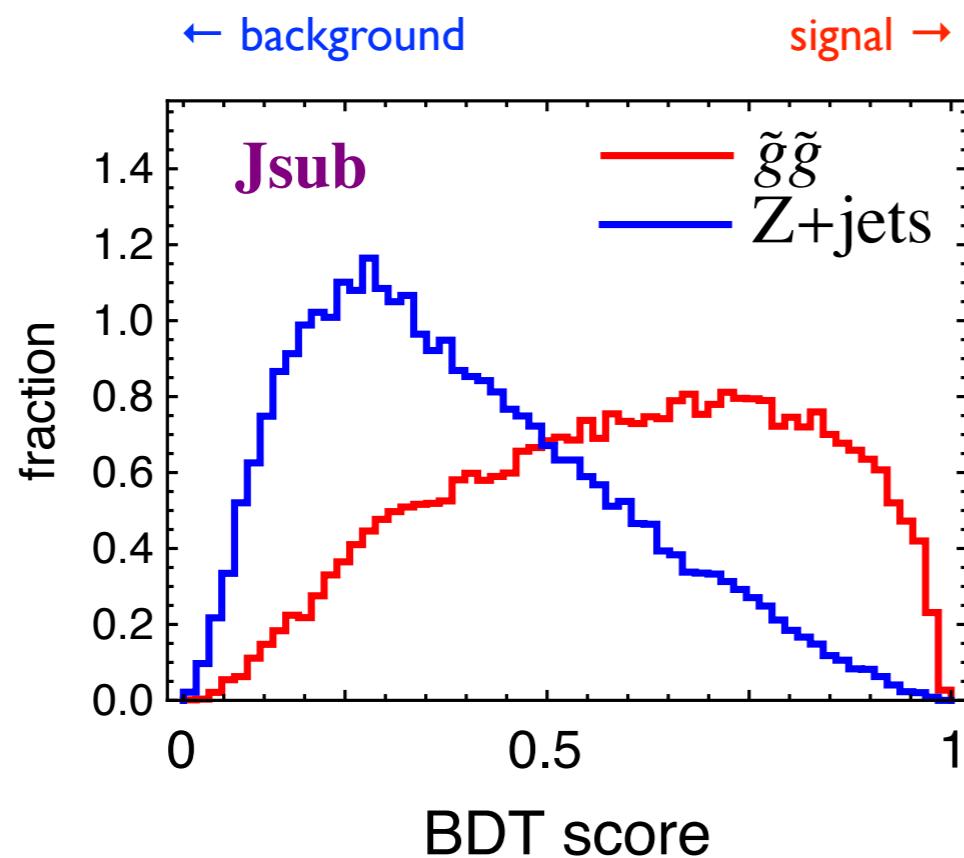
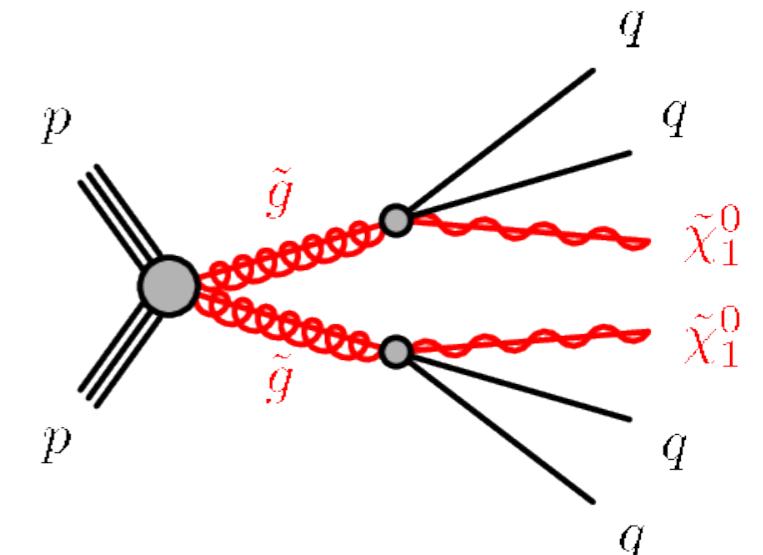


Classifier: Boosted decision tree (for each of 4 jets)

Inputs: Jet mass, width, track multiplicity

Signal: Quark enriched

Background: Gluon enriched



[Bhattacherjee, Mukhopadhyay, Nojiri, Sakakie, Webber, [JHEP 2017](#)]

Principles of Fundamental Physics

Robustness of Energy Flow

[Komiske, Metodiev, JDT, JHEP 2018]



Patrick Komiske



Eric Metodiev



S F



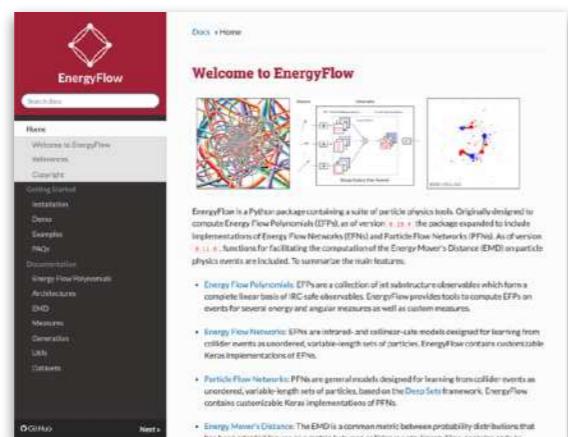
Power of Artificial Intelligence

Point Cloud Learning

Zaheer, Kottur, Ravanbakhsh, Poczos, Salakhutdinov, Smola, NIPS 2017

Energy Flow Networks

<https://energyflow.network/>
[Komiske, Metodiev, JDT, JHEP 2019]



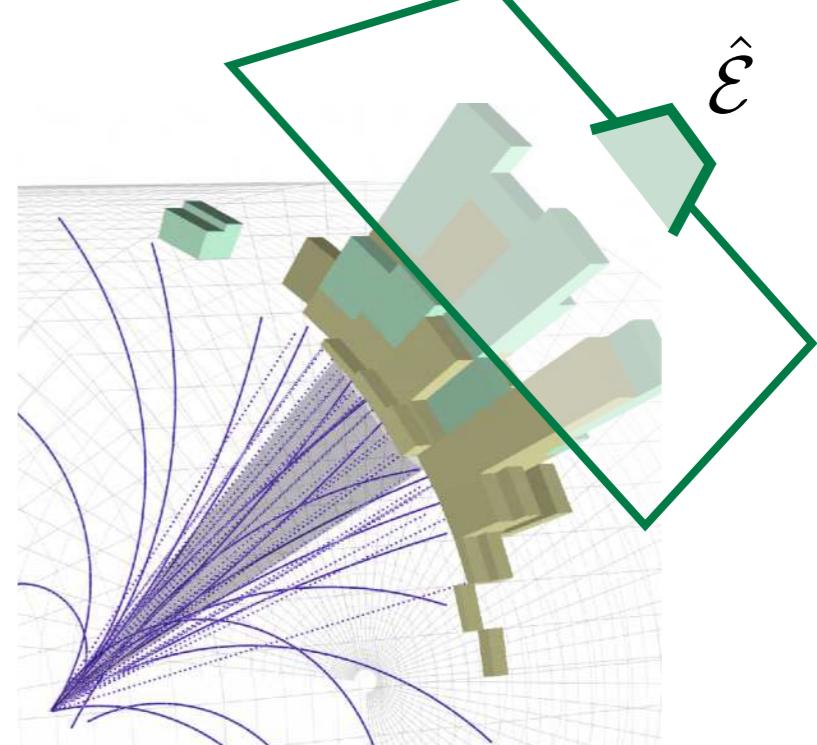
Jets as Weighted Point Clouds

- Energy-Weighted Directions

$$\vec{p} = \{E, \hat{n}_x, \hat{n}_y, \hat{n}_z\}$$

↑ |
Energy Direction

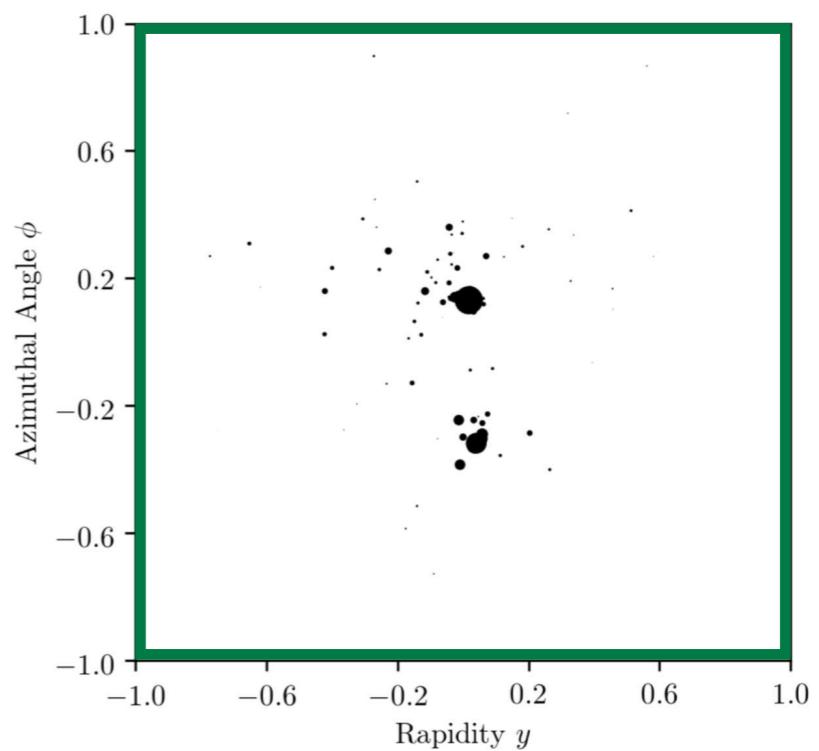
(suppressing “unsafe” charge/flavor information)



- Equivalently: Energy Density

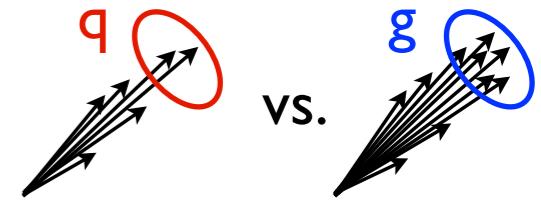
$$\rho(\hat{n}) = \sum_{i \in \mathcal{J}} E_i \delta^{(2)}(\hat{n} - \hat{n}_i)$$

↑ ↑
Energy Direction

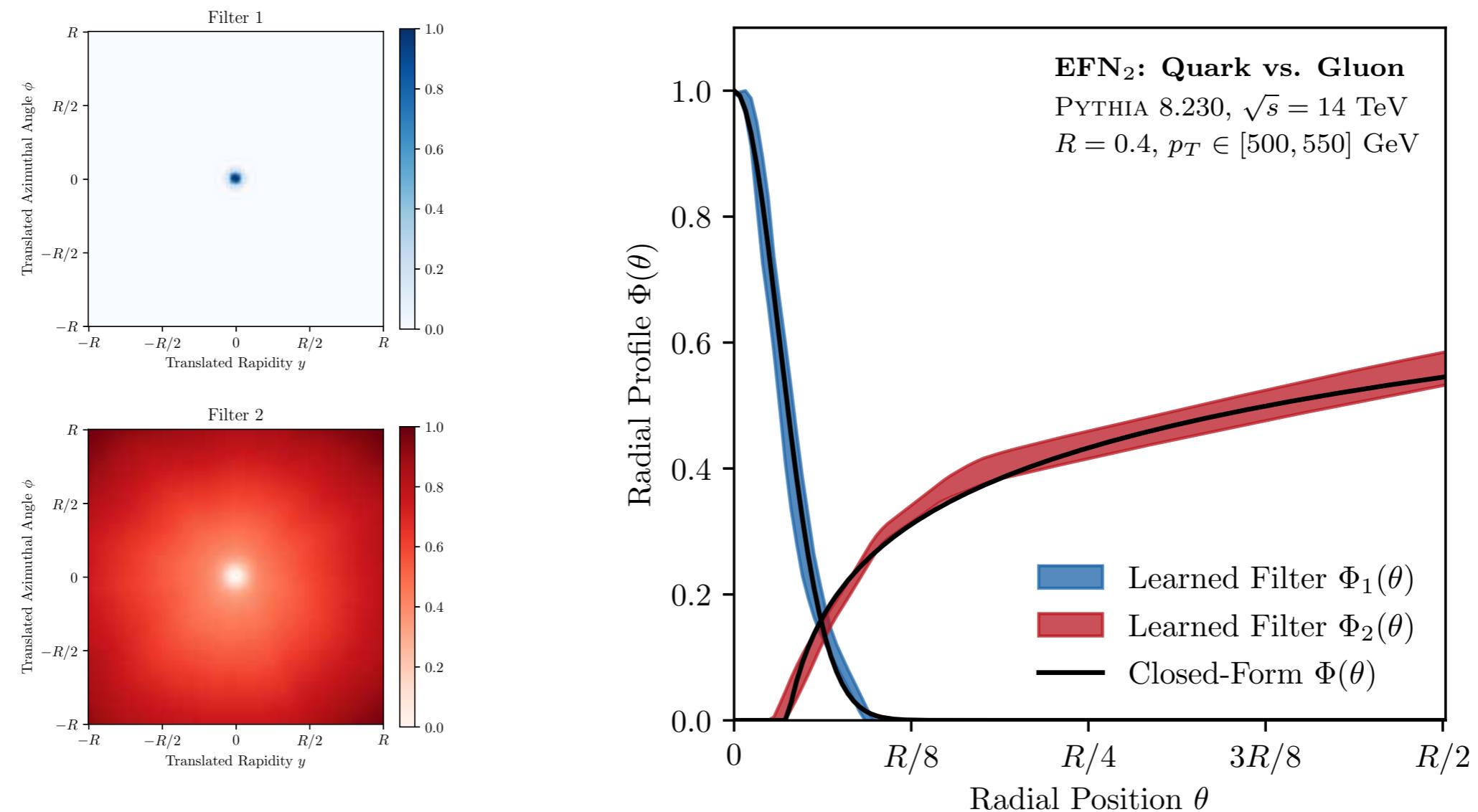


*“Ok, but did you really learn something
you didn’t already know?”*

Learning from the Machine



For $\ell = 2$ EFN, radial moments: $\sum_{i \in \text{jet}} z_i f(\theta_i)$ cf. Angularities:
 $f(\theta) = \theta^\beta$



[Komiske, Metodiev, JDT, JHEP 2019;
cf. Larkoski, JDT, Waalewijn, JHEP 2014; using Berger, Kucs, Sterman, PRD 2003; Ellis, Vermilion, Walsh, Hornig, Lee, JHEP 2010]

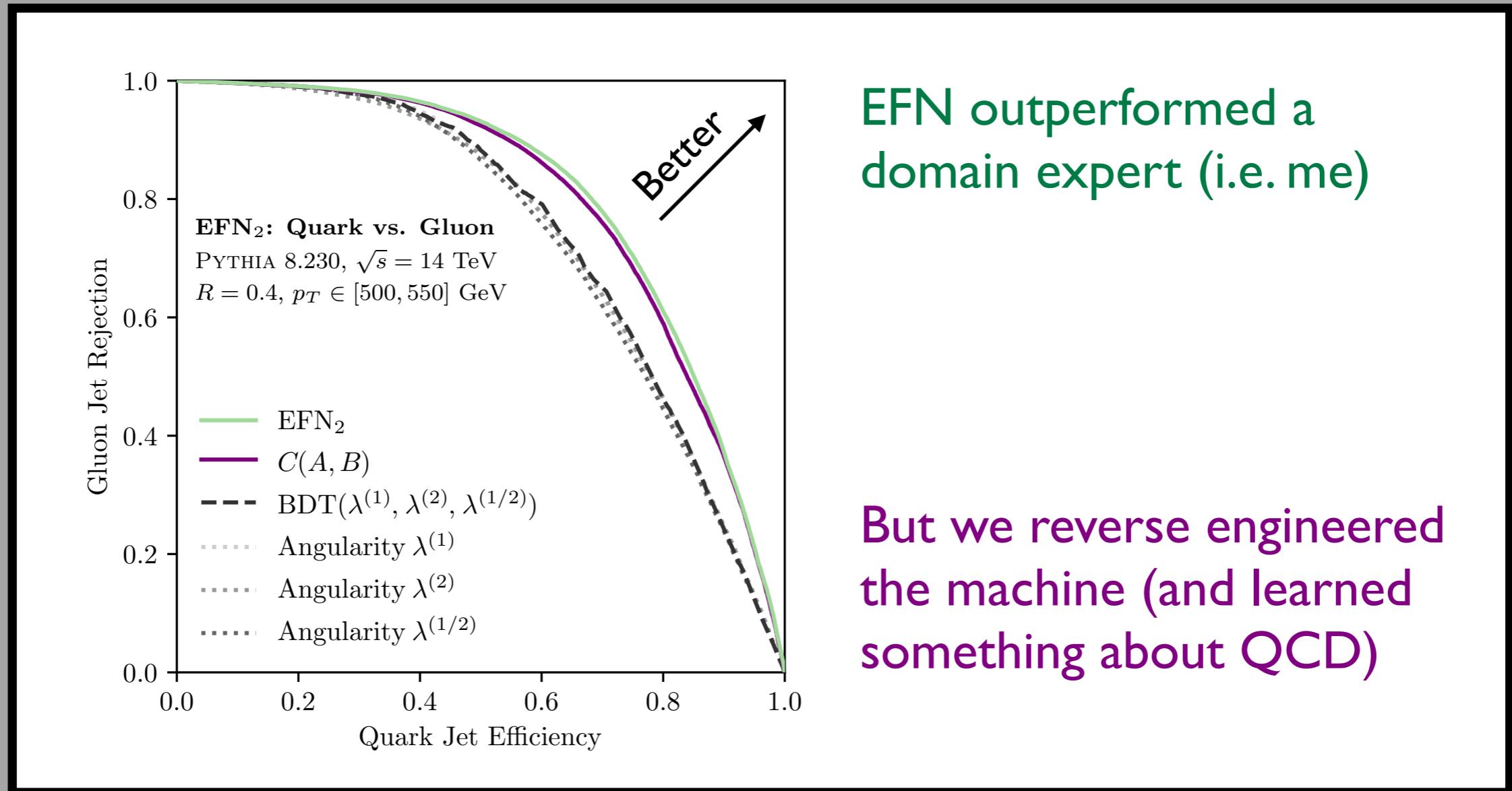
Learning from the Machine



For $\ell = 2$ EFN, radial moments:

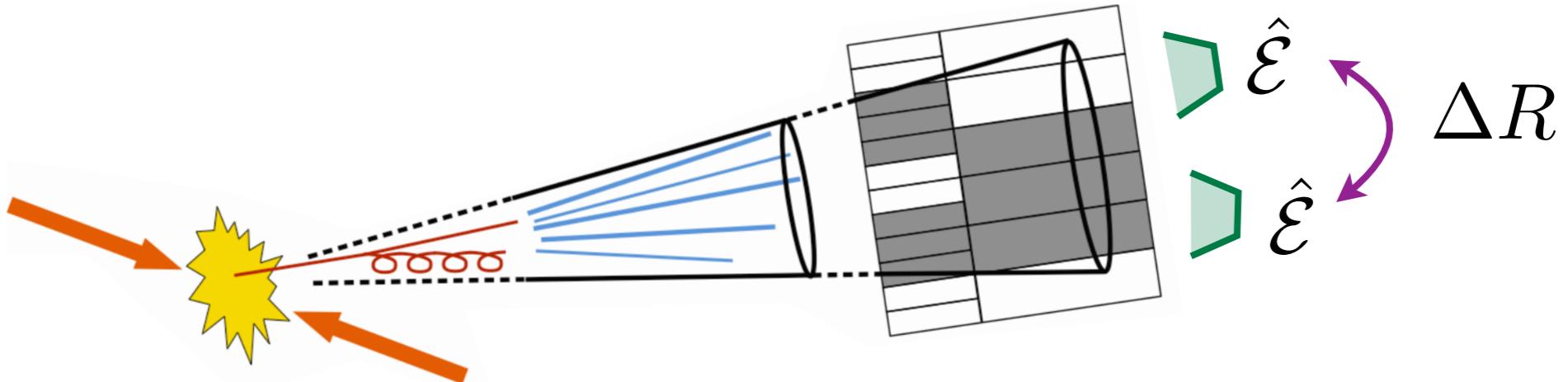
$$\sum_{i \in \text{jet}} z_i f(\theta_i)$$

cf. Angularities:
 $f(\theta) = \theta^\beta$

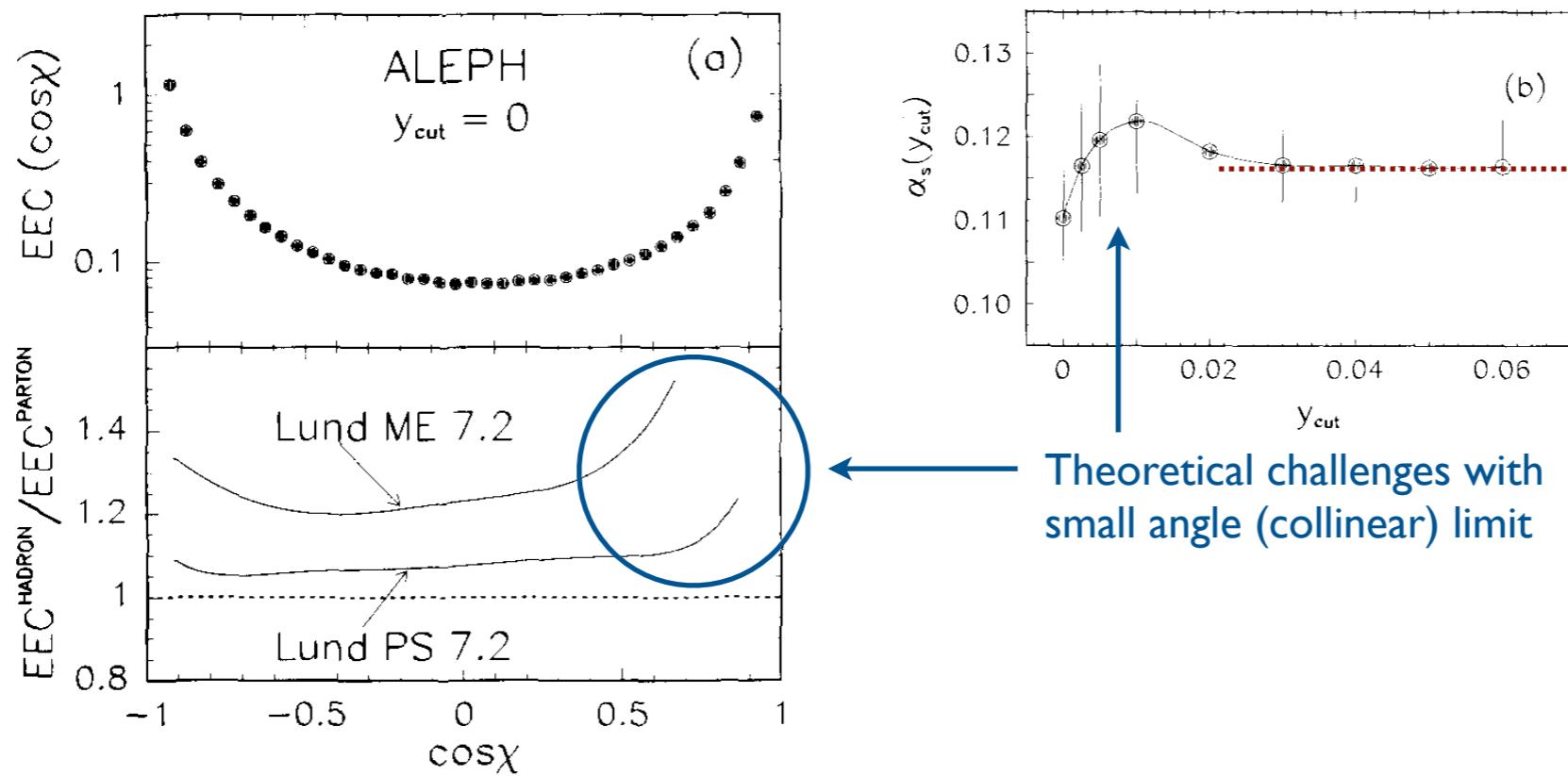


[Komiske, Metodiev, JDT, [JHEP 2019](#);
cf. Larkoski, JDT, Waalewijn, [JHEP 2014](#); using Berger, Kucs, Sterman, [PRD 2003](#); Ellis, Vermilion, Walsh, Hornig, Lee, [JHEP 2010](#)]

Energy-Energy Correlators

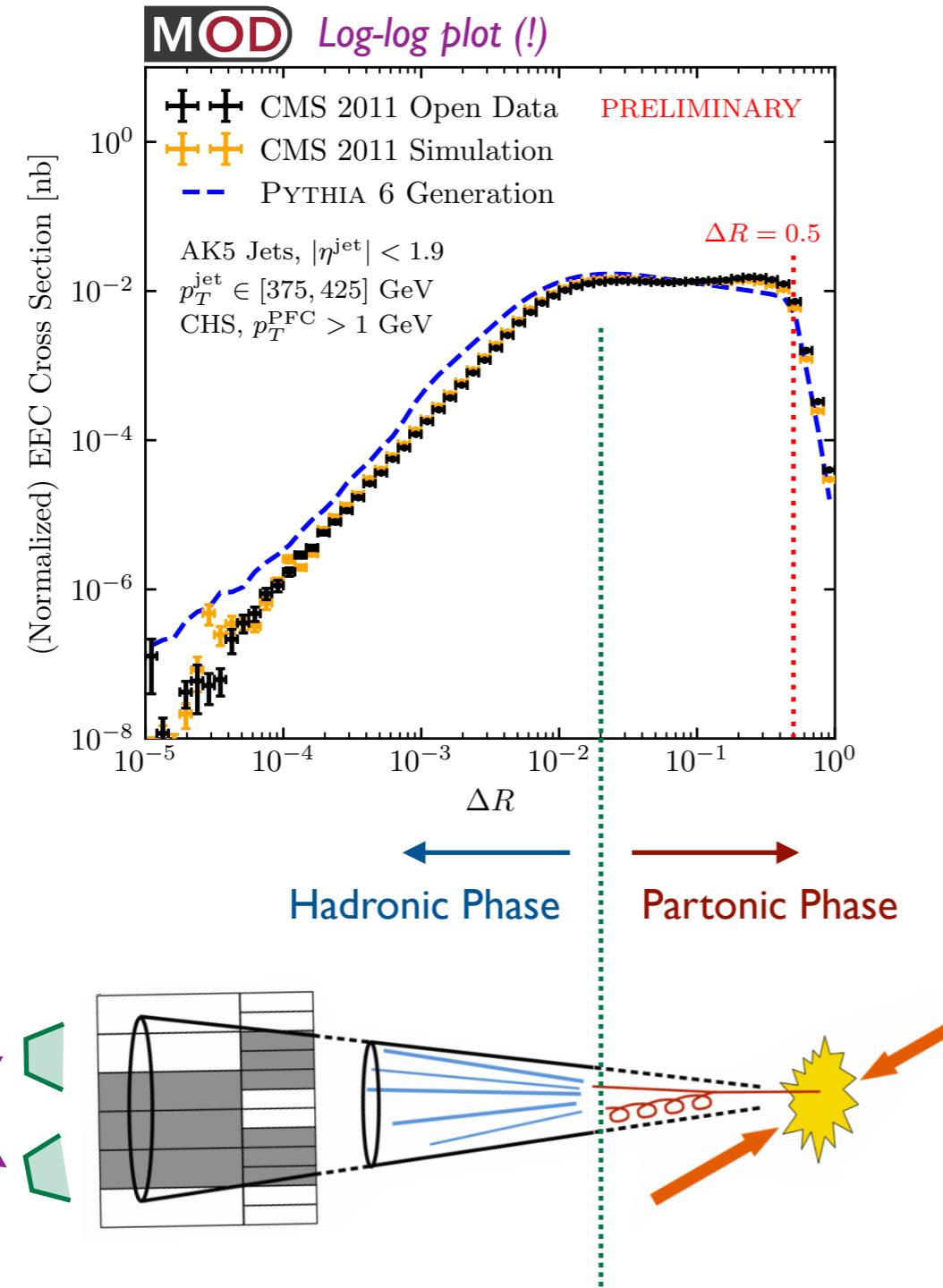


A long history in probing collinear dynamics of QCD



[Basham, Brown, Ellis, Love, [PRL 1978](#); ALEPH, [PLB 1991](#);
see Chen, Moult, Zhang, Zhu, [PRD 2020](#)]

QCD Phase Transition in Jets?



Behind the scenes at **BOSTON 2019**

Jesse Thaler

EE(E)C?

To: Ian Moult

July 24, 2019 at 2:01 PM

JT

Hi Ian,

I appreciate the provocative lunch! At some point, we should talk about what you might want to see measured on the CMS Open Data with respect to EE(E)C.

Cheers,
Jesse

Jesse Thaler

Preliminary EEC plot

To: Ian Moult, Cc: Patrick Komiske

July 24, 2019 at 10:16 PM

JT

Hi Ian,

Below is a preliminary EEC plot on the CMS Open Data, made by Patrick. (Strictly speaking, this is the pT-pT-Correlator...)

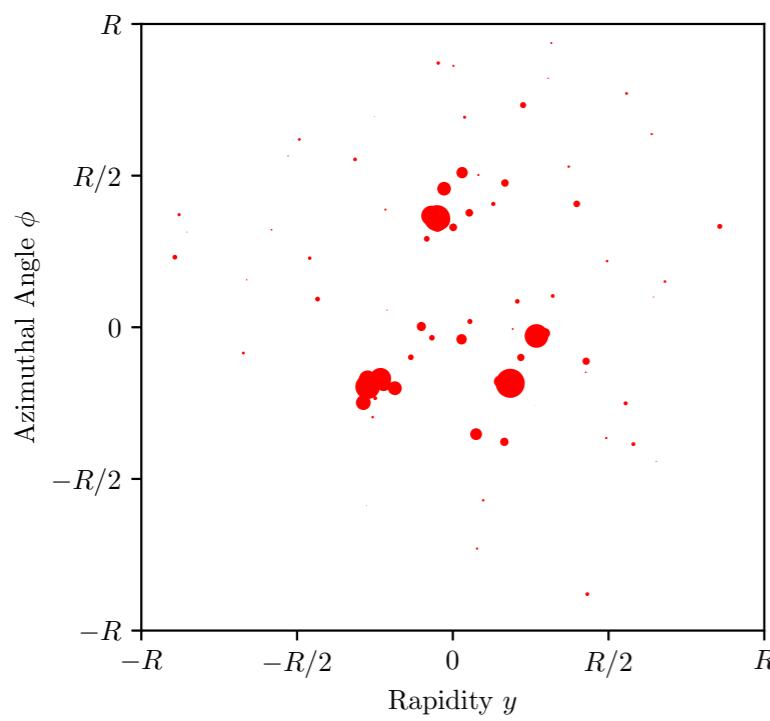
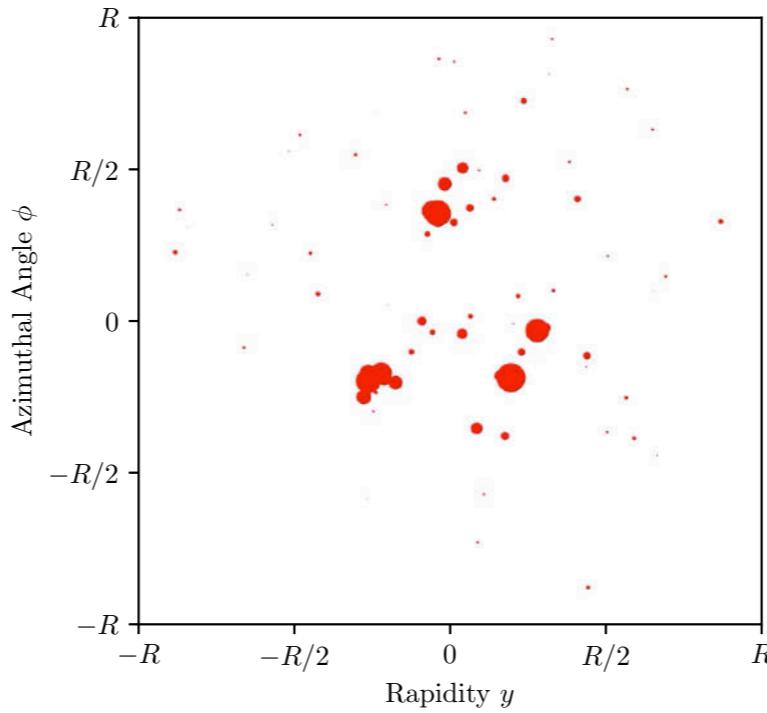
We do this within single jets in the pT range specified in the plot, and the dashed vertical red line is the jet radius (beyond which things don't really make sense).

[Komiske, Moult, JDT, et al., in progress; see talks by Moult, [BOOST 2019](#), [BOOST 2020](#)]

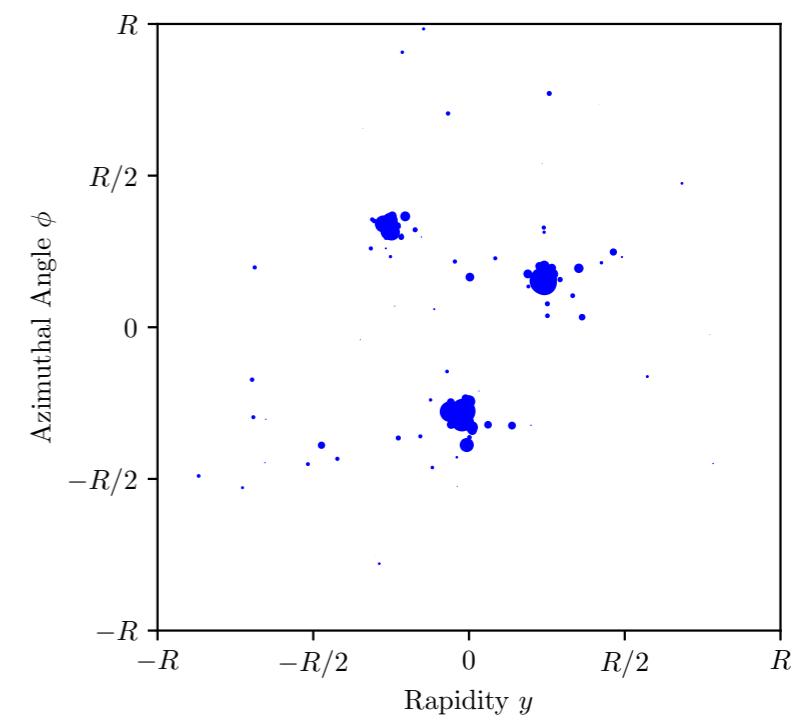


Similarity of Two Energy Flows?

$$\mathcal{E}(\hat{n}) = \sum_i E_i \delta(\hat{n} - \hat{n}_i)$$



Optimal Transport:
Earth Mover's Distance
a.k.a. *1-Wasserstein metric*



[Komiske, Metodiev, JDT, PRL 2019; code at Komiske, Metodiev, JDT, [energyflow.network](#)]

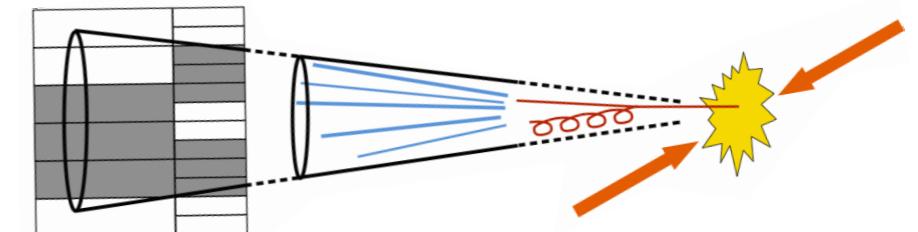
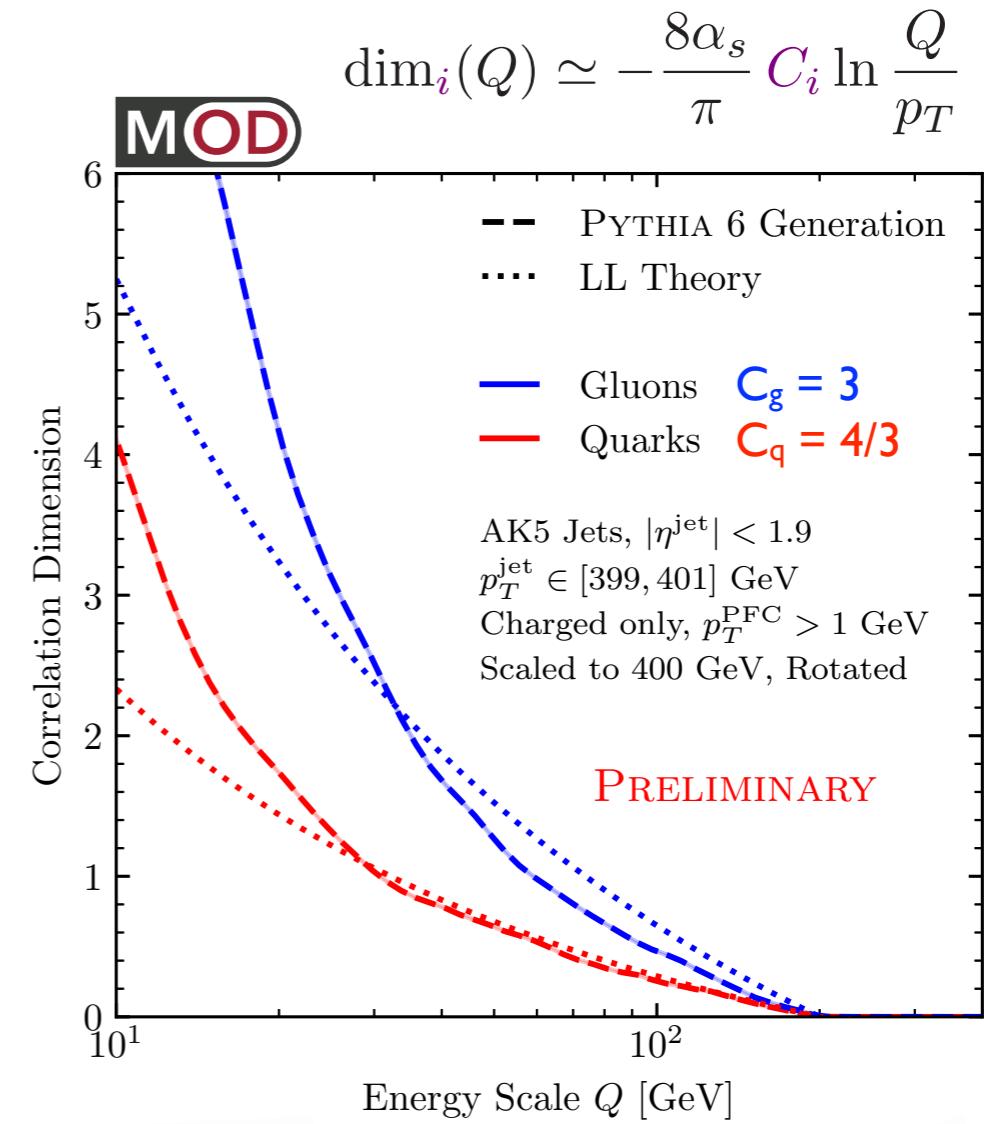
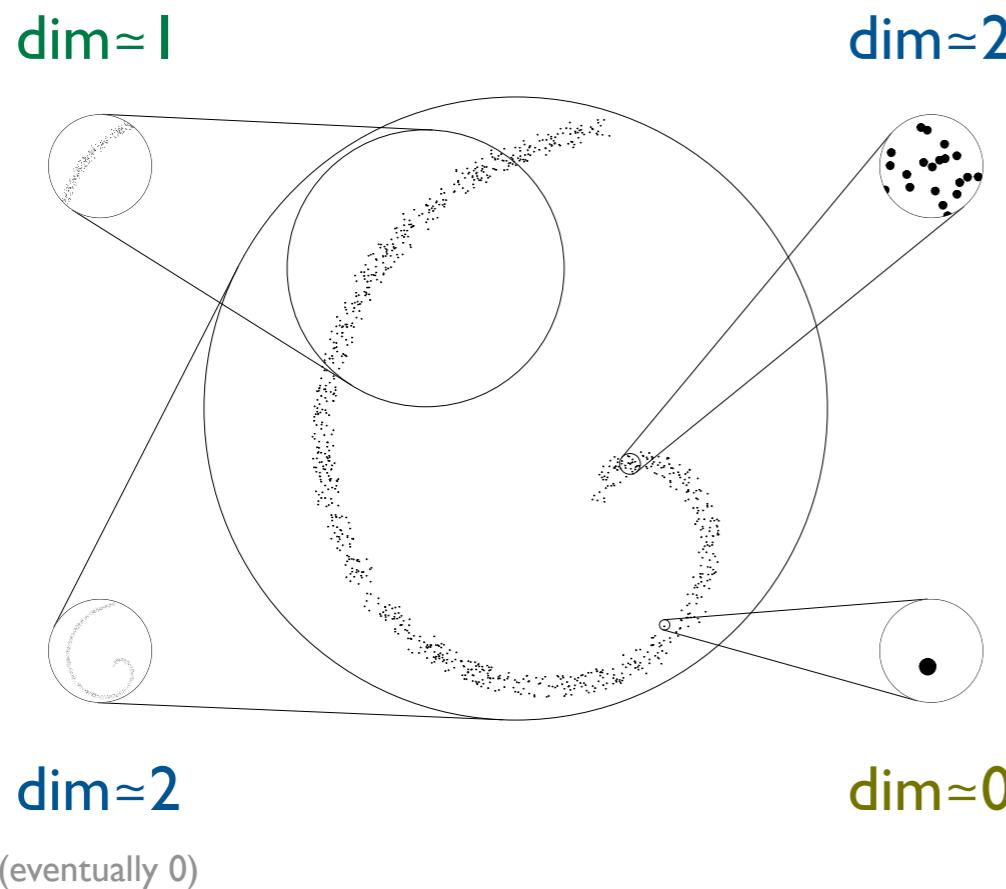
Dimensionality of Space of Jets



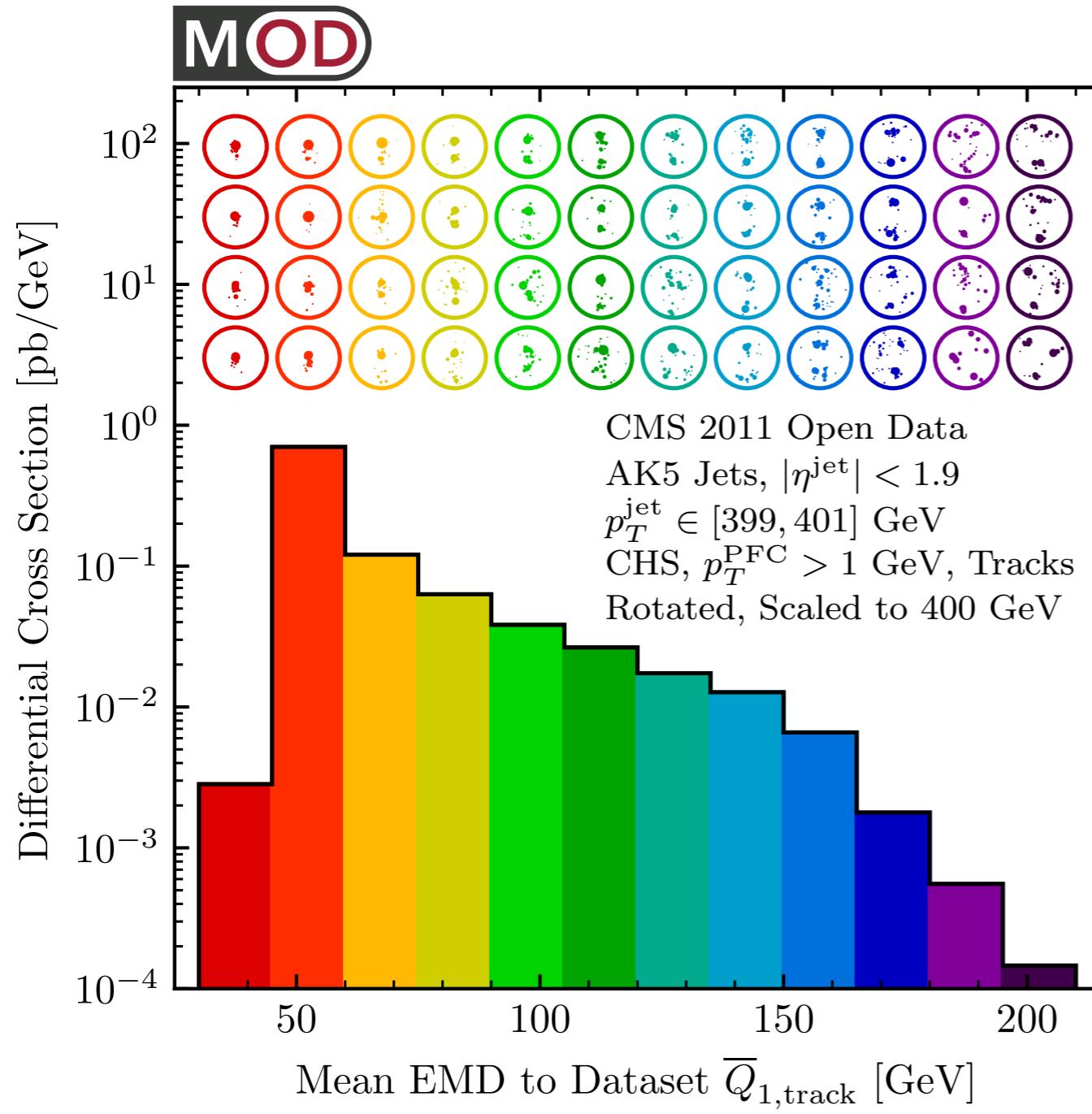
$$N_{\text{neighbors}}(r) \sim r^{\dim}$$

$$\Rightarrow \dim(r) \sim r \frac{\partial}{\partial r} \ln N_{\text{neighbors}}(r)$$

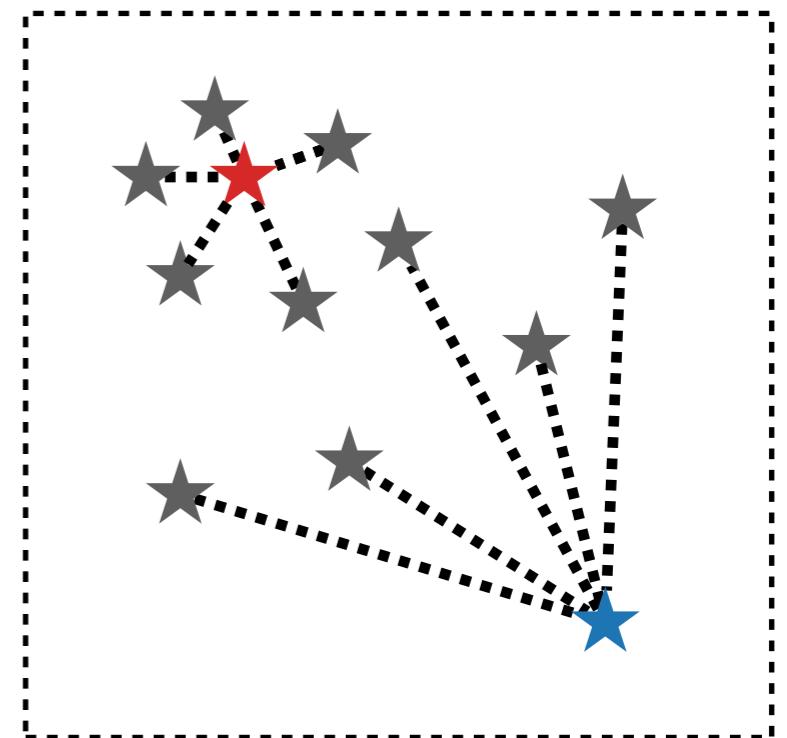
[Grassberger, Procaccia, [PRL 1983](#); Kégl, [NIPS 2002](#)]



Least Representative Jets



New Physics?
Or tails of QCD?



[Komiske, Mastandrea, Metodiev, Naik, JDT, PRD 2020]

