The Hidden Geometry of Particle Collisions

Jesse Thaler

Heidelberg Teilchentee / MITP Virtual Workshop Seminar — June 24, 2021

The NSF AI Institute for Artificial Intelligence and Fundamental Interactions (IAIFI) *"eye-phi"*

Advance physics knowledge — from the smallest building blocks of nature to the largest structures in the universe — and galvanize AI research innovation

[http://iaifi.org, MIT News Announcement]

“Collision Course”

Robustness of Energy Flow

[Komiske, Metodiev, JDT, JHEP 2018]

HEP → ? ← ML

Power of Point Cloud Learning

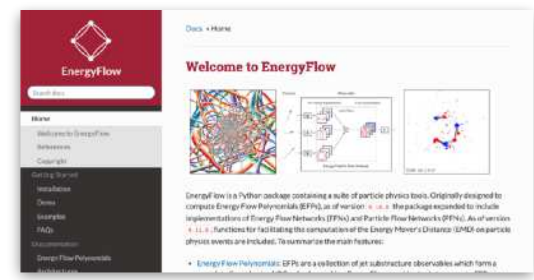
[Zaheer, Kottur, Ravanbakhsh, Poczos, Salakhutdinov, Smola, NIPS 2017]

Patrick Komiske

Eric Metodiev

Energy Flow Networks

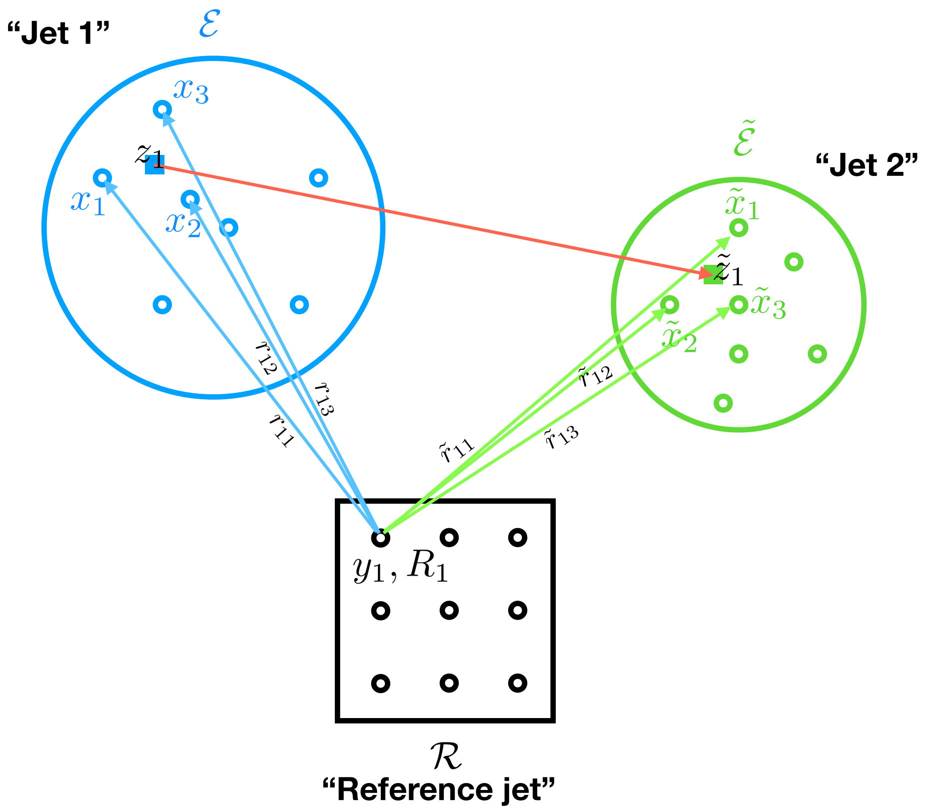
https://energyflow.network/
[Komiske, Metodiev, JDT, JHEP 2019]



Optimal Transport for HEP

See talks by Nathaniel and Jessica on Monday!

Linearized OT for Jet Classification

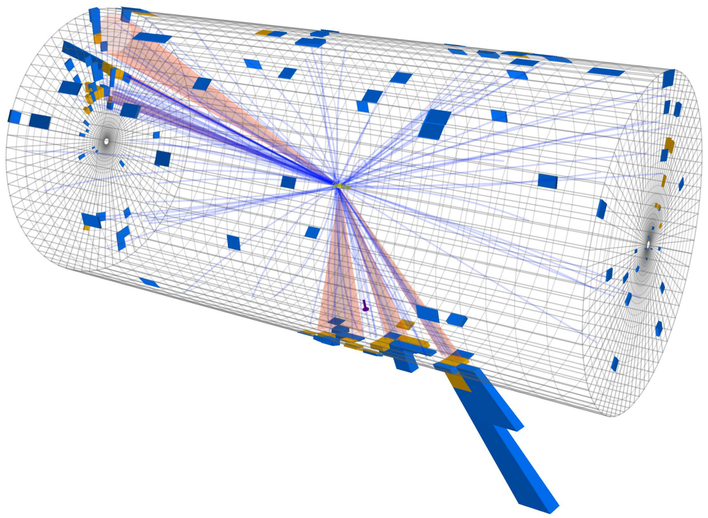


[Cai, Cheng, Craig, Craig, PRD 2020]

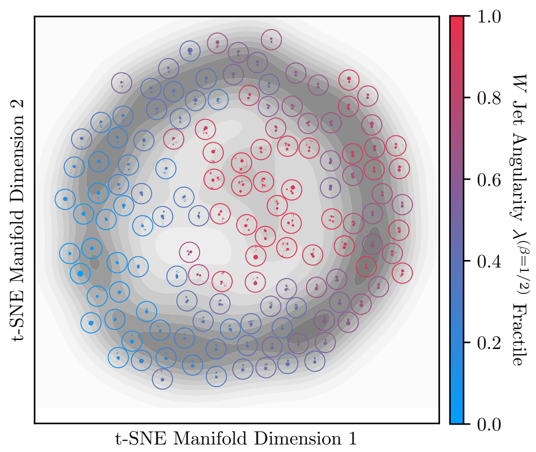
Sliced Wasserstein for Simulation/Unfolding

[Howard, Mandt, Whiteson, Yang, arXiv 2021]

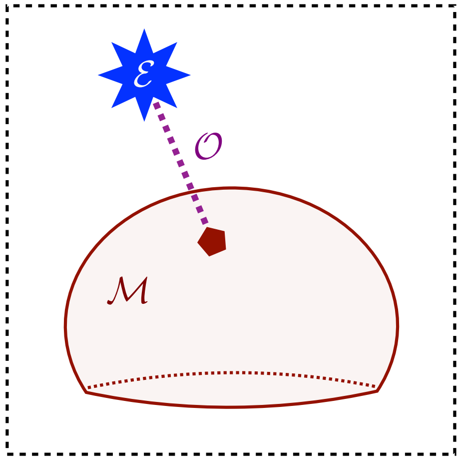
What does moving "earth" have to do with collider physics?



From Manifest Geometry…



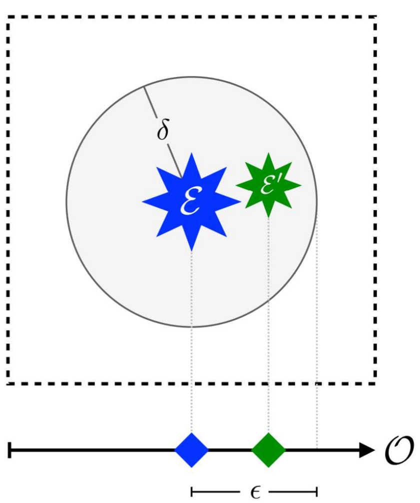
…to Emergent Geometry…



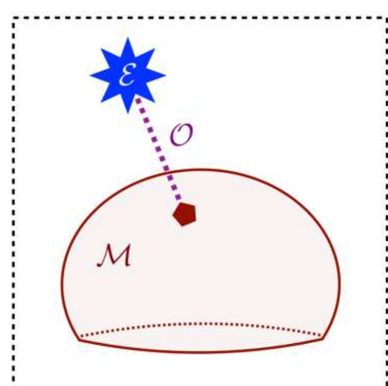
…to Hidden Geometry!

Six Decades of Collider Physics Translated into a New Geometric Language!

IRC Safety is smoothness in the space of events

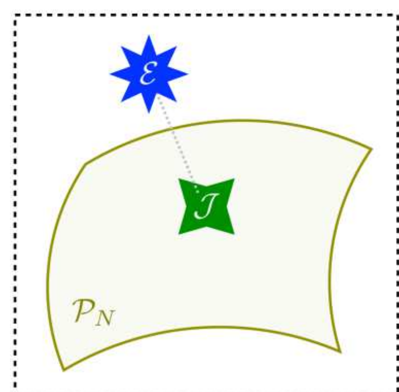


Event shapes are distances from events to manifolds.



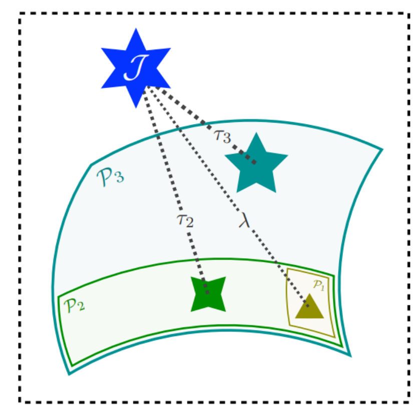
$\mathcal{O}(\mathcal{E}) = \min_{\mathcal{E}'\in\mathcal{M}} \mathrm{EMD}_{\beta,R}(\mathcal{E},\mathcal{E}')$

Jets are projections to few-particle manifolds.



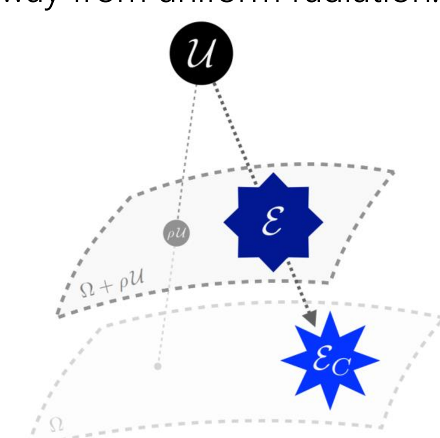
$\mathcal{J} = \operatorname{argmin}_{\mathcal{E}'\in\mathcal{P}_N} \mathrm{EMD}_{\beta,R}(\mathcal{E},\mathcal{E}')$

Substructure resolves emissions within the jet.



$\tau(\mathcal{J}) = \min_{\mathcal{E}'\in\mathcal{P}_N} \mathrm{EMD}_{\beta}(\mathcal{J},\mathcal{E}')$.

Pileup mitigation moves away from uniform radiation.



$\mathcal{E}_C = \arg\min_{\mathcal{E}'} \mathrm{EMD}(\mathcal{E},\mathcal{E}'+\rho\,\mathcal{U})$.

Taming infinities | Event Shapes | Jet Algorithms | Jet Substructure | Pileup

1960 — 2020

1962-1964
Infrared Safety
[Kinoshita, JMP 1962]
[Lee, Nauenberg, PR 1964]

1977
Thrust, Sphericity
[Farhi, PRL 1977]
[Georgi, Machacek, PRL 1977]

1993
k_T jet clustering
[Ellis, Soper, PRD 1993]
[Catani, Dokshitzer, Seymour, Webber, NPB 1993]

1997-1998
C/A jet clustering
[Wobisch, Wengler, 1998]
[Doskhitzer, Leder, Moretti,Webber, JHEP 1997]

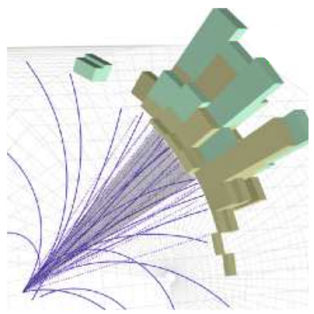
2010-2015
N-(sub)jettiness, XCone
[Stewart, Tackmann, Waalewijn, PRL 2010]
[Thaler, Van Tilburg, JHEP 2011]
[Stewart, Tackmann, Thaler, Vermilion, Wilkason, JHEP 2015]

2014-2019
Constituent Subtraction
[Berta, Spousta, Miller, Leitner, JHEP 2014]
[Berta, Masetti, Miller, Spousta, JHEP 2019]

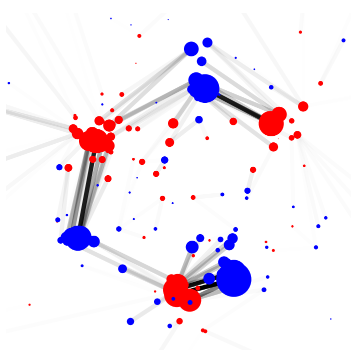
And many more!

[timeline from Eric Metodiev]

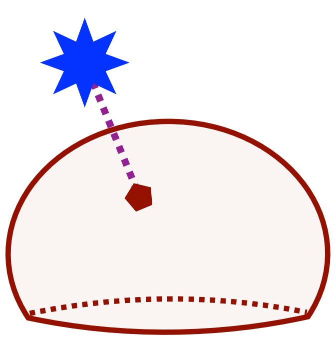
Outline



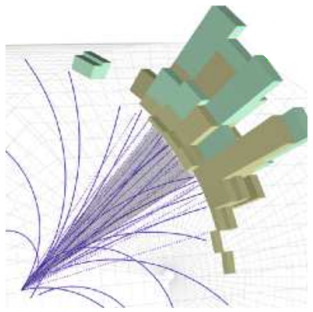
Going with the (Energy) Flow



The Energy Mover's Distance



Revealing a Hidden Geometry



Going with the (Energy) Flow

The Energy Mover’s Distance

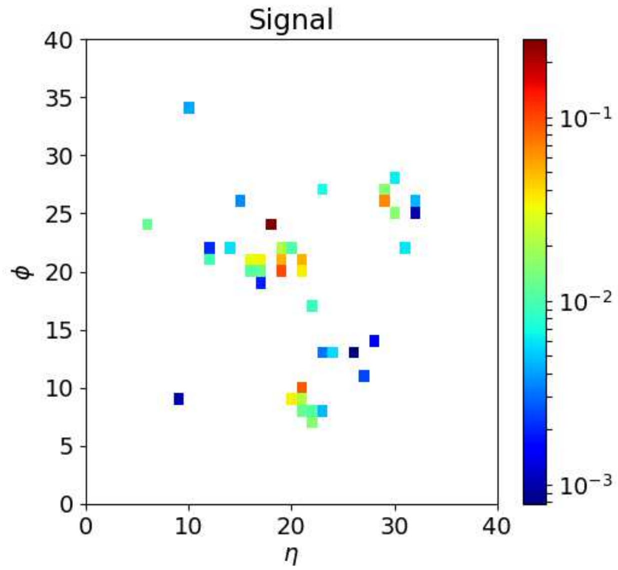
Revealing a Hidden Geometry

Taking a step back to supervised machine learning…

Jet Representations

Pixelized Image

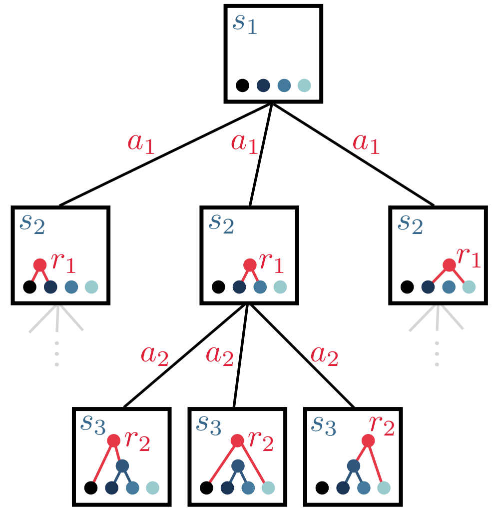
Calorimetry



[review in Kagan, arXiv 2020]

Hierarchical Tree

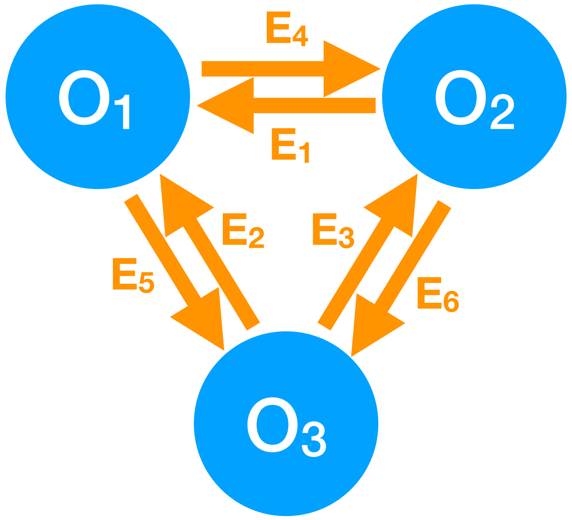
Binary Splittings



[e.g. Brehmer, Macaluso, Pappadopulo, Cranmer, NeurIPS 2020]

Graphs

Pairwise Interactions

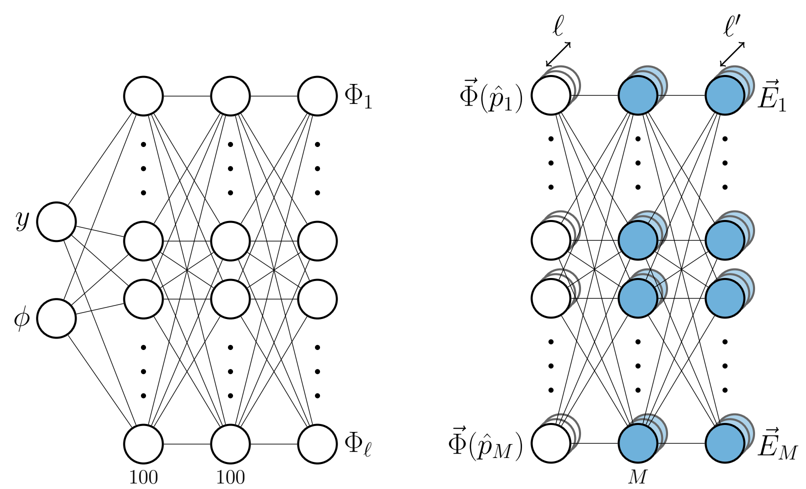


[e.g. Moreno, Cerri, Duarte, Newman, Nguyen, Periwal, Pierini, Serikova, Spiropulu, Vlimant, EPJC 2020]

Imposes implicit theoretical prior (typically a good thing!)
Influences choice of network architecture

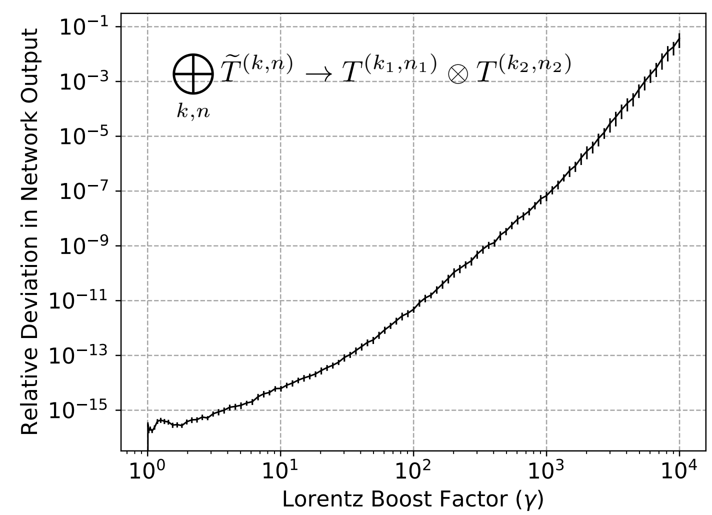
From Principles to Network Architectures

Permutation Equivariance



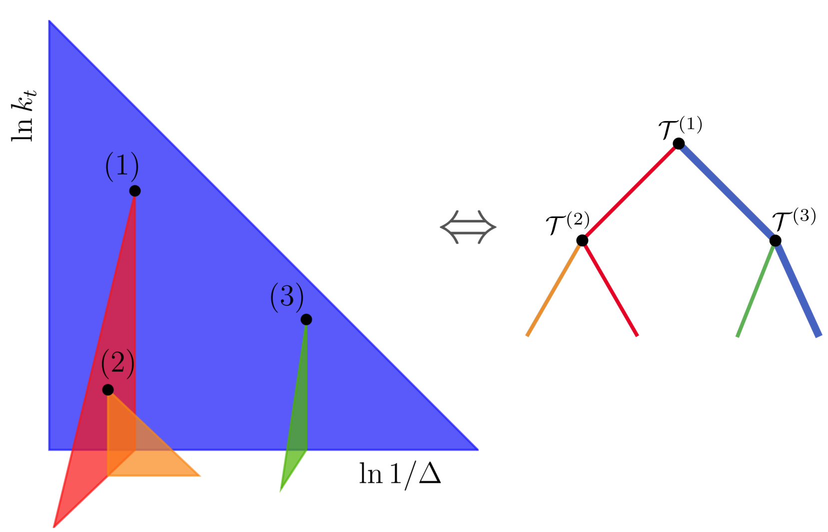
[Dolan, Ore, PRD 2021]

Lorentz Equivariance



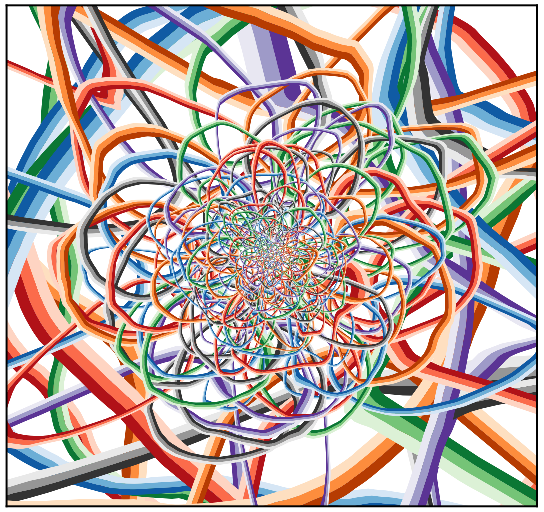
[Bogatskiy, Anderson, Offermann, Roussi, Miller, Kondor, arXiv 2020]

Lund Plane Emissions



[Dreyer, Qu, JHEP 2021]

Infrared and Collinear Safety

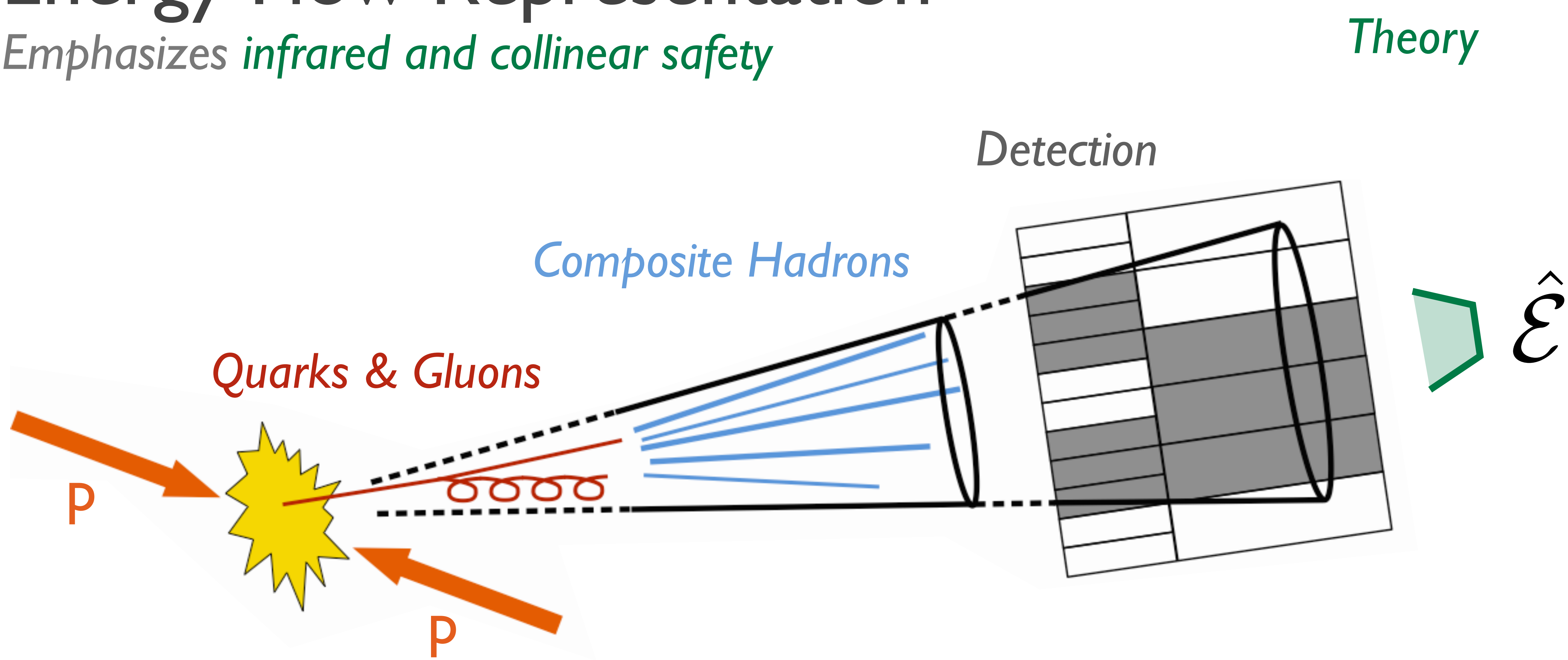


[Komiske, Metodiev, JDT, JHEP 2019]

. . .

Energy Flow Representation

Emphasizes infrared and collinear safety



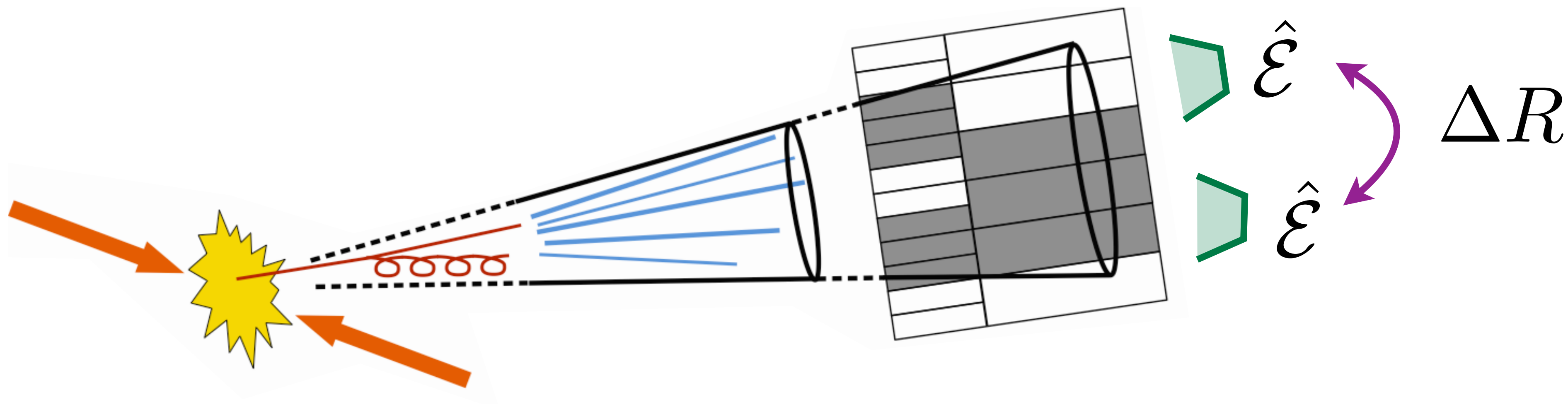
Energy Flow:

Robust to hadronization and detector effects
Well-defined for massless gauge theories

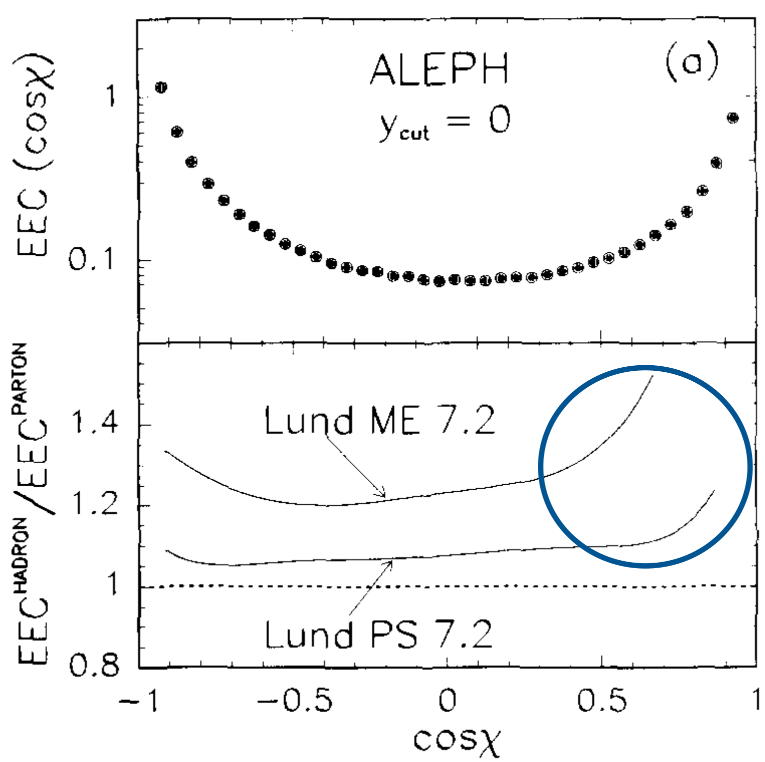
$$\boxed{\hat{\mathcal{E}} \simeq \lim_{t\to\infty} \hat{n}_i T^{0i}(t, vt\hat{n})}$$

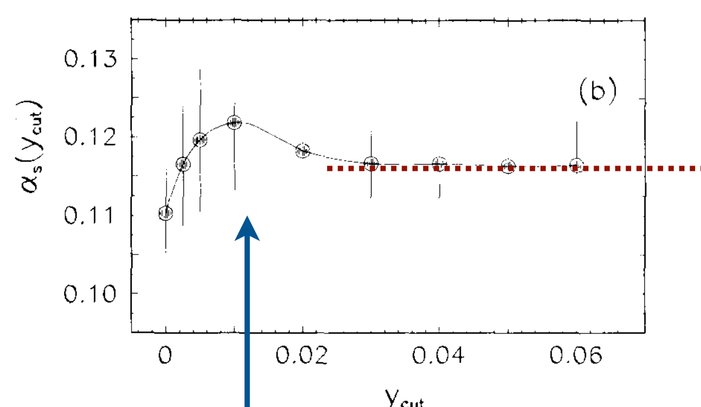
[see e.g. Sveshnikov, Tkachov, PLB 1996; Hofman, Maldacena, JHEP 2008; Mateu, Stewart, JDT, PRD 2013; Belitsky, Hohenegger, Korchemsky, Sokatchev, Zhiboedov, PRL 2014; Chen, Moult, Zhang, Zhu, PRD 2020]

Energy-Energy Correlators



A long history in probing collinear dynamics of QCD





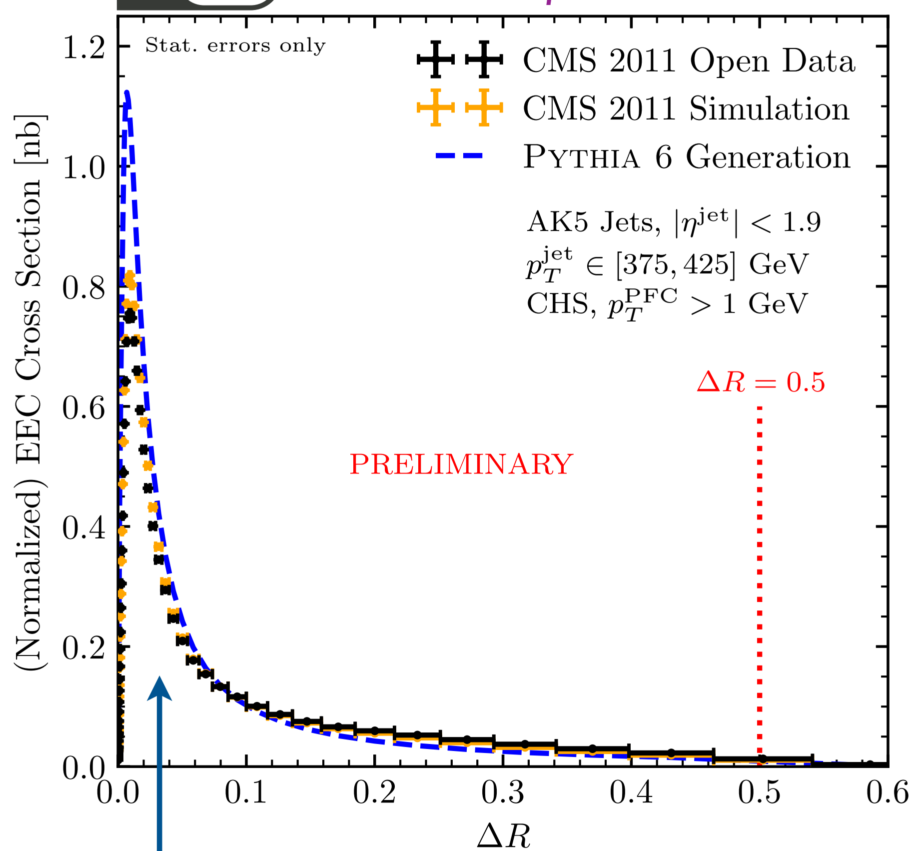
Extracting the strong coupling constant

Theoretical challenges with small angle (collinear) limit

[Basham, Brown, Ellis, Love, PRL 1978; ALEPH, PLB 1991; see Chen, Moult, Zhang, Zhu, PRD 2020]

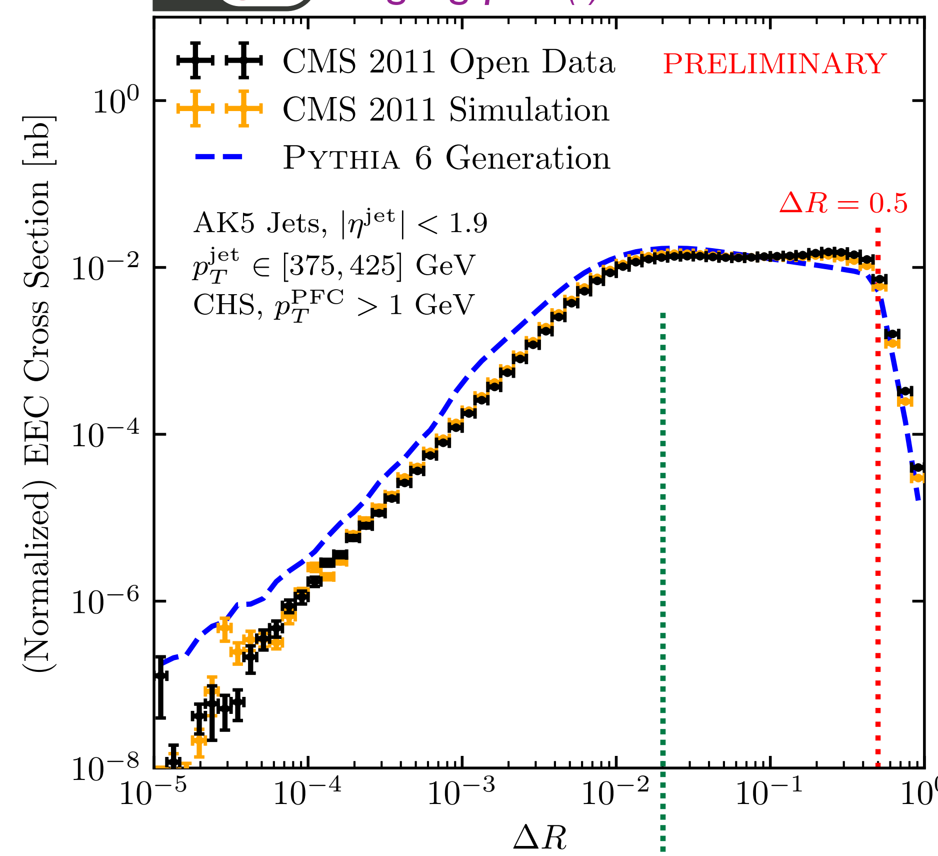
QCD Phase Transition in Jets?

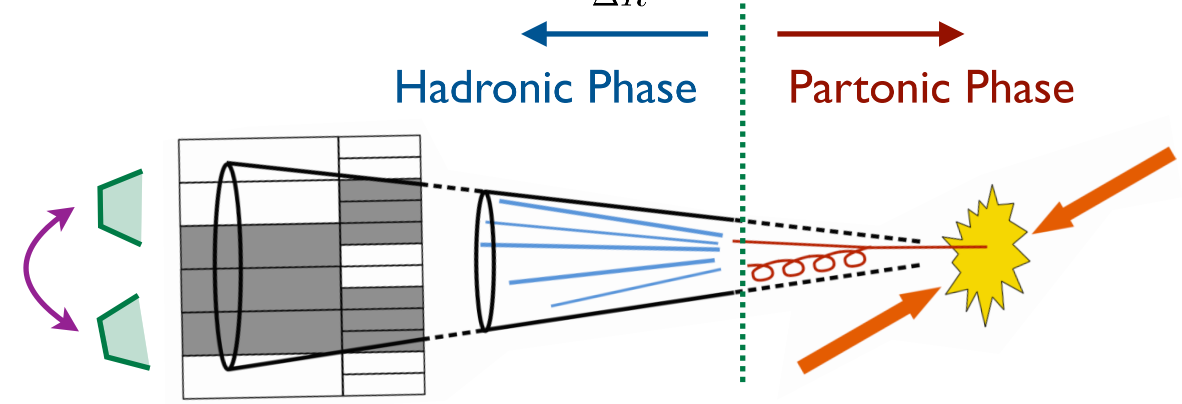
Linear-linear plot



Are we learning something about small angle limit of QCD?

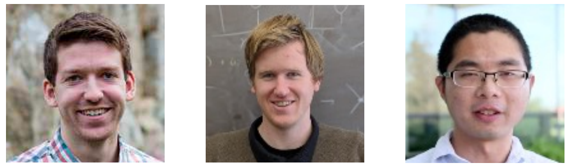
Log-log plot (!)





First Jet EEC Plot from the LHC (!)

[Komiske, Moult, JDT, Zhu, in progress; see talks by Moult, BOOST 2019, BOOST 2020]



Jets as Weighted Point Clouds

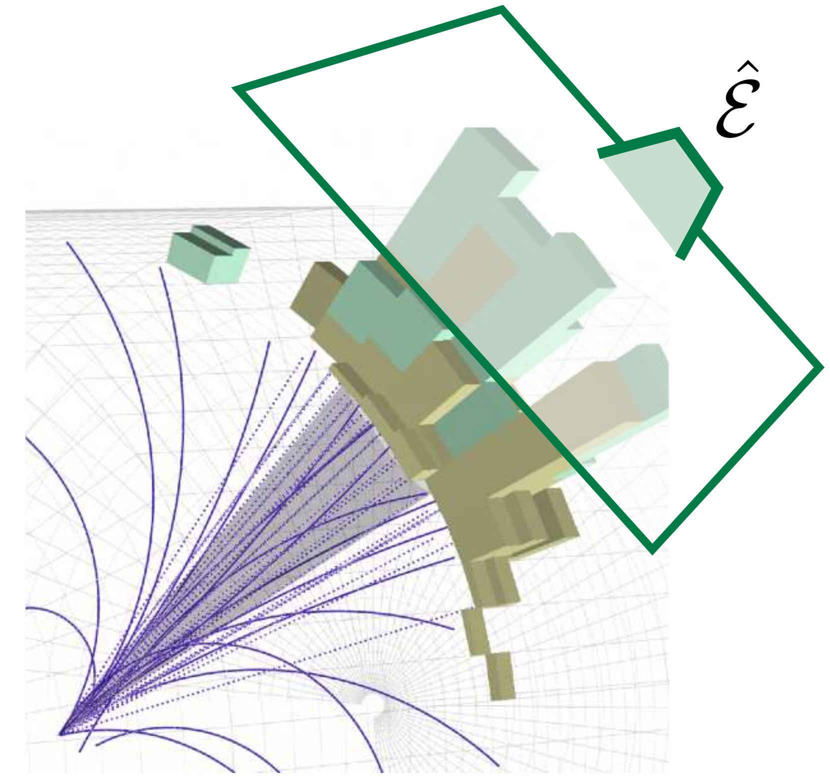
- Energy-Weighted Directions

$$\vec{p} = \{E, \underbrace{\hat{n}_x, \hat{n}_y, \hat{n}_z}_{\text{Direction}}\}$$

Energy

Direction

(suppressing "unsafe" charge/flavor information)

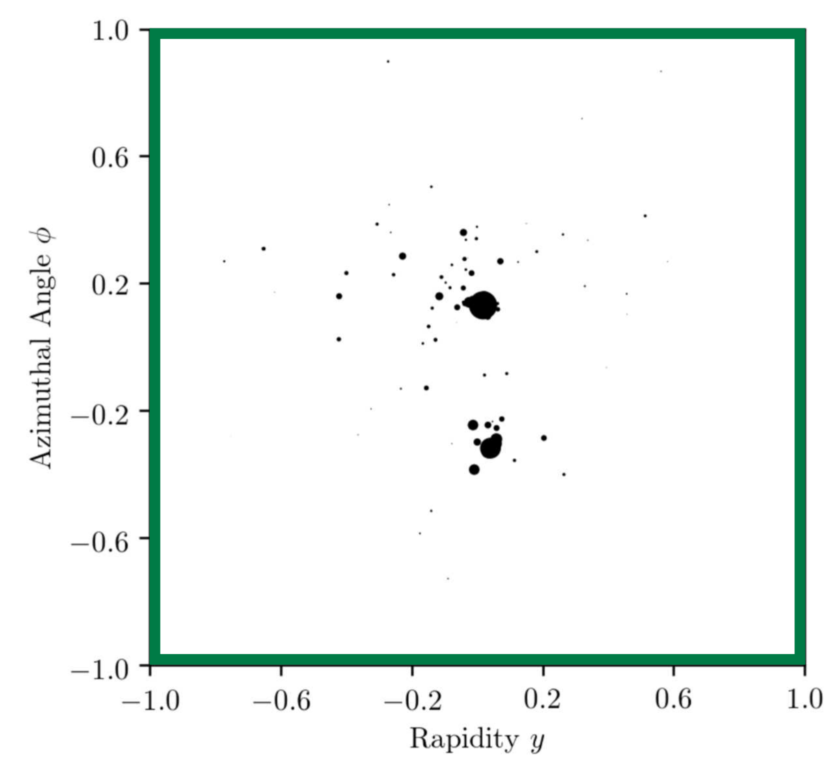


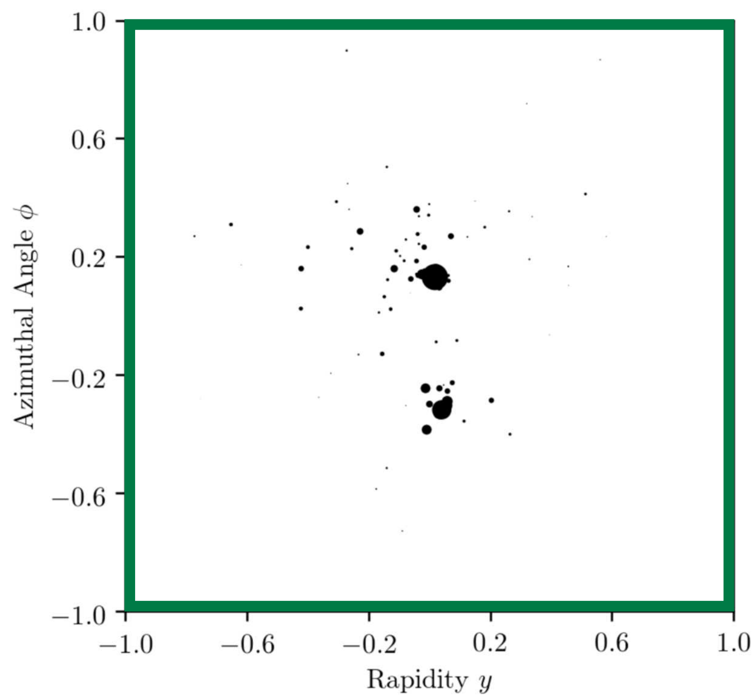
- Equivalently: Energy Density

$$\rho(\hat{n}) = \sum_{i \in \mathcal{J}} E_i \, \delta^{(2)}(\hat{n} - \hat{n}_i)$$

Energy

Direction

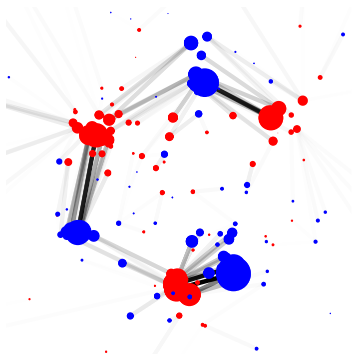




When restricted to IRC safe information, jets/events are naturally represented as energy densities

There is no obvious way to include unsafe information in this picture, since flow of charge/flavor is theoretically delicate

Going with the (Energy) Flow



The Energy Mover’s Distance

Revealing a Hidden Geometry

If you ask your local computational geometry expert how to process densities…

The Earth Mover's Distance

See Nathaniel's talk for a more extensive history

Optimal Transport: Minimum "work" (stuff x distance) to make one distribution look like another distribution

[Peleg, Werman, Rom, IEEE 1989; Rubner, Tomasi, Guibas, ICCV 1998, ICJV 2000; Pele, Werman, ECCV 2008; Pele Taskar, GSI 2013]

[h/t Niles-Weed, ML4Jets 2020; Monge, 1781; Kantorovich, 1939; Vaseršteĭn, 1969; Wikipedia]

The Earth Mover's Distance

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[Peleg, Werman, Rom, IEEE 1989;
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Pele, Werman, ECCV 2008; Pele Taskar, GSI 2013]

Additive Weight:

Ground Distance:

$\Rightarrow$ Distance Between Distributions

Déblai Remblai

[h/t Niles-Weed, ML4Jets 2020; Monge, 1781; Kantorovich, 1939; Vaseršteĭn, 1969; Wikipedia]

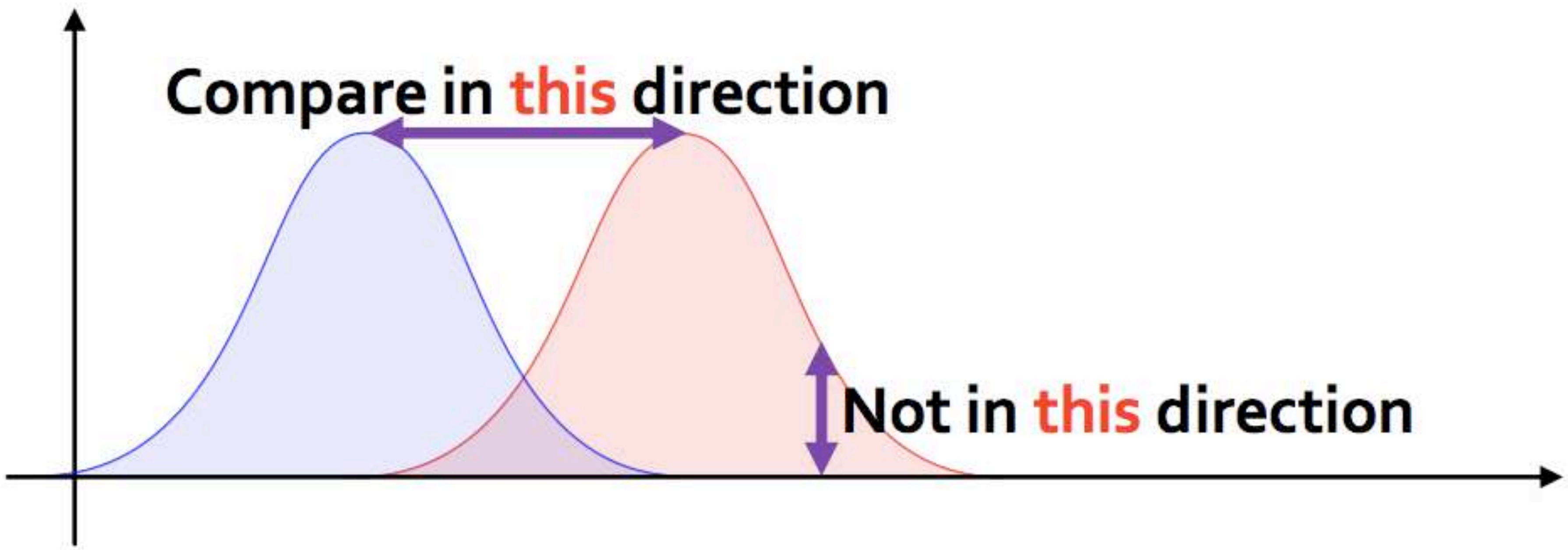
The Earth Mover's Distance

Optimal Transport:

[Peleg, Werman, Rom, IEEE 1989; Rubner, Tomasi, Guibas, ICCV 1998, ICJV 2000; Pele, Werman, ECCV 2008; Pele Taskar, GSI 2013]

Minimum "work" (stuff x distance) to make one distribution look like another distribution

"Horizontal" comparison (EMD) yields better dynamic range than "vertical" comparison (e.g. KL)

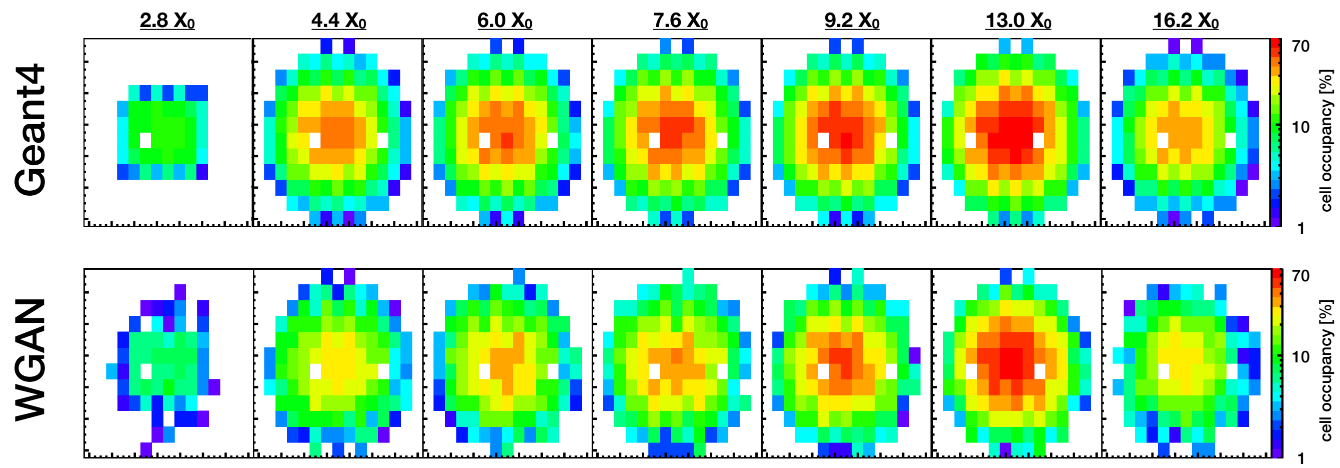


[figure from Kun, Math ∩ Programming]

[h/t Niles-Weed, ML4Jets 2020; Monge, 1781; Kantorovich, 1939; Vaseršteĭn, 1969; Wikipedia]

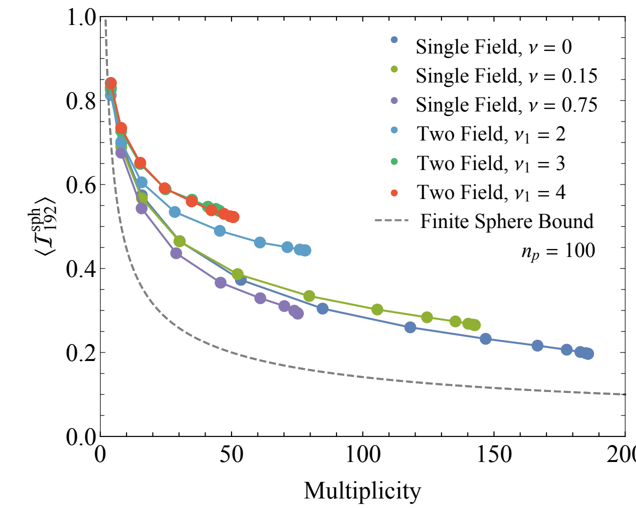
Wasserstein in HEP

Generative Modeling



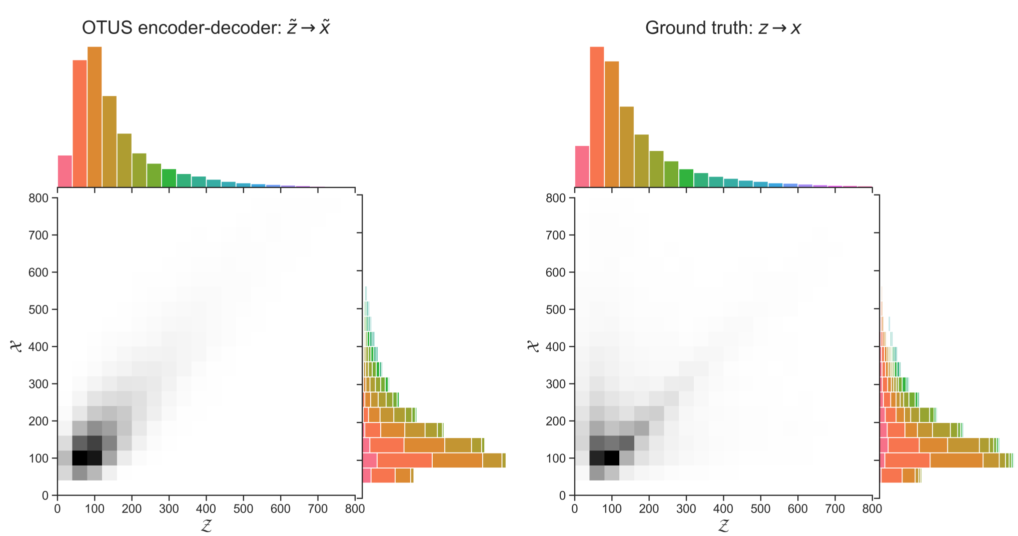
[Erdmann, Geiger, Glombitza, Schmidt, CSBS 2018; Erdmann, Glombitza, Quast, CSBS 2019; Chekalina, Orlova, Ratnikov, Ulyanov, Ustyuzhanin, Zakharov, CHEP 2018]

BSM Characterization



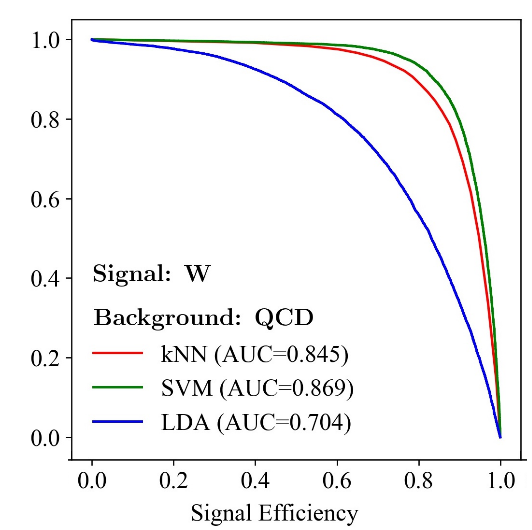
[Cesarotti, Reece, Strassler, JHEP 2021, arXiv 2020]

Estimated Simulation/Unfolding



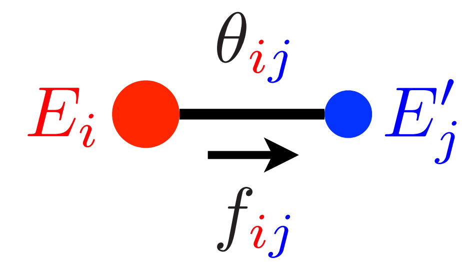
[Howard, Mandt, Whiteson, Yang, arXiv 2021]

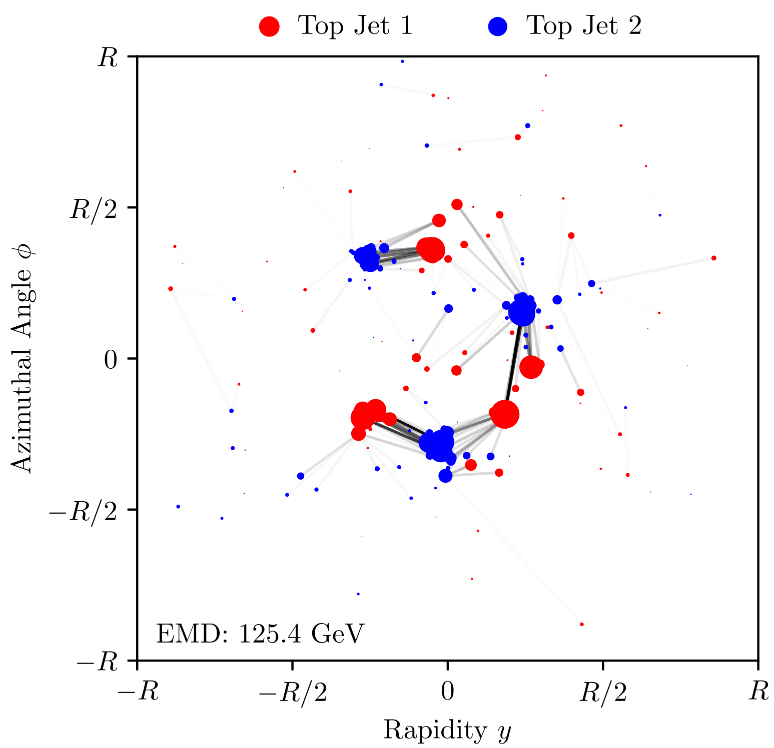
Jet Classification



[Cai, Cheng, Craig, Craig, PRD 2020]

The Energy Mover's Distance

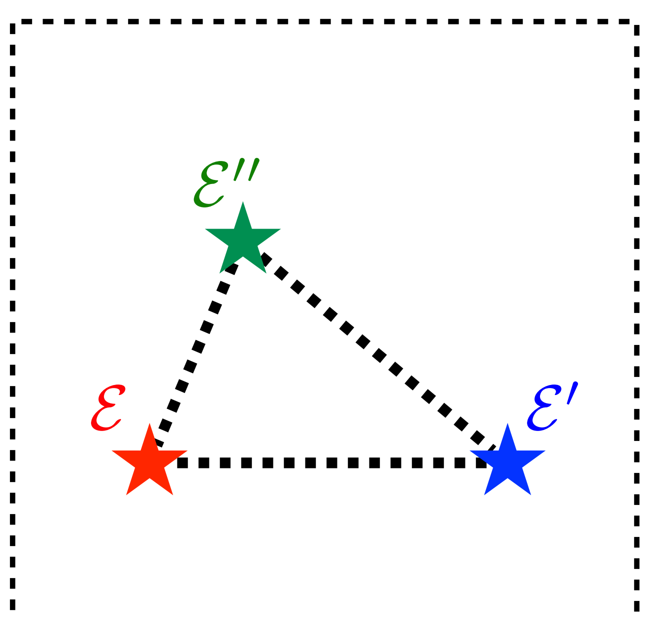




Optimal transport between energy flows…

$$\text{EMD}(\mathcal{E}, \mathcal{E}') = \underbrace{\min_{\{f\}} \sum_i \sum_j f_{ij} \frac{\theta_{ij}}{R}}_{\text{Cost to move energy}} + \underbrace{\left| \sum_i E_i - \sum_j E'_j \right|}_{\text{Cost to create energy}}$$

in GeV



…defines a metric on the space of events

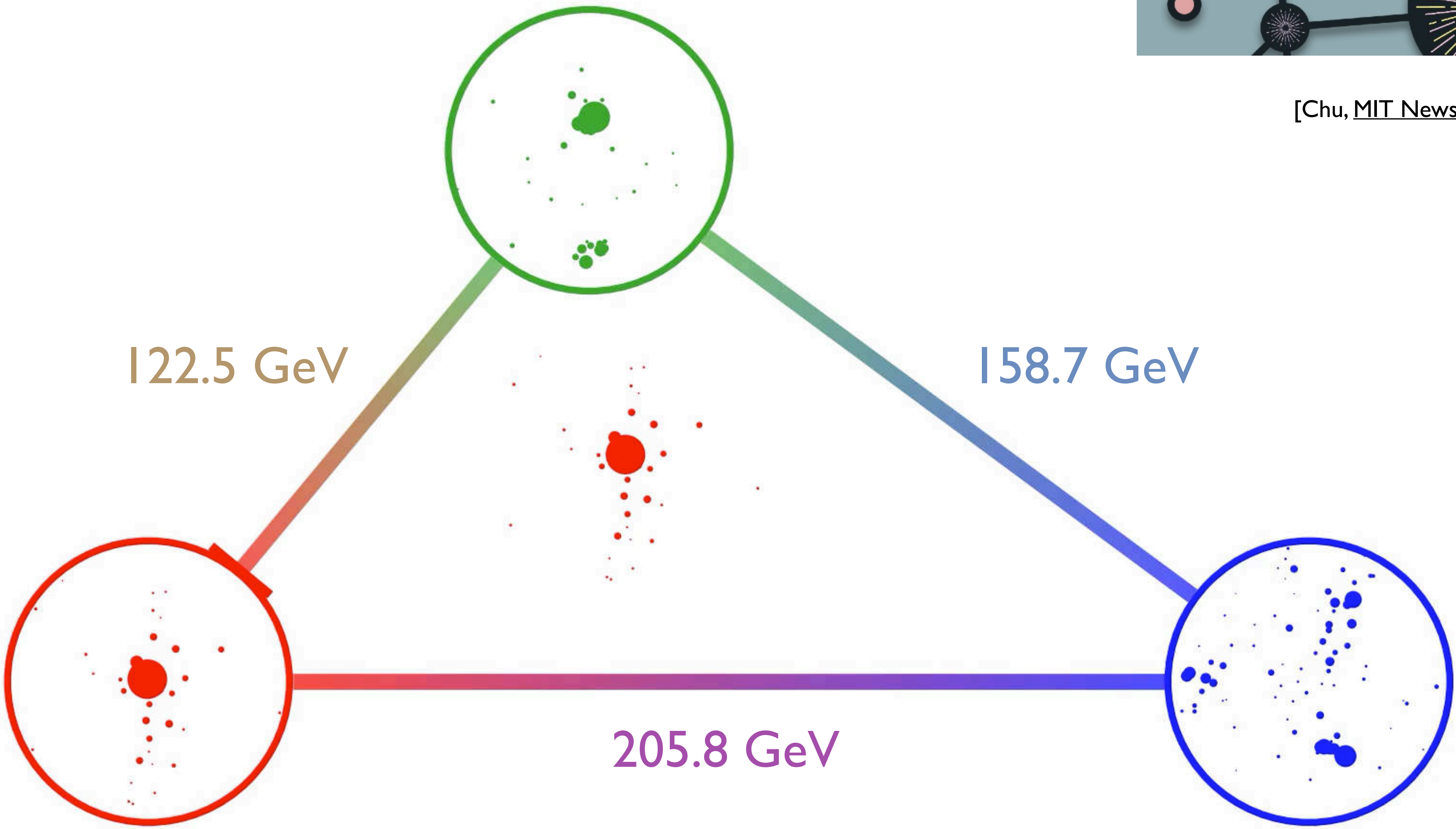
$$0 \leq \text{EMD}(\mathcal{E}, \mathcal{E}') \leq \text{EMD}(\mathcal{E}, \mathcal{E}'') + \text{EMD}(\mathcal{E}', \mathcal{E}'')$$

(assuming R ≥ θ_{max}/2, i.e. R ≥ jet radius for conical jets)

[Komiske, Metodiev, JDT, PRL 2019;
see also Pele, Werman, ECCV 2008; Pele, Taskar, GSI 2013;
[see flavored variant in Crispim Romão, Castro, Milhano, Pedro, Vale, EPJC 2021]
[see computational speed up in Cai, Cheng, Craig, Craig, PRD 2020]

Similarity of Three Energy Flows?

[Chu, MIT News July 2019]



[Komiske, Metodiev, JDT, PRL 2019; code at Komiske, Metodiev, JDT, energyflow.network; see alternative graph network approach in Mullin, Pacey, Parker, White, Williams, JHEP 2021]

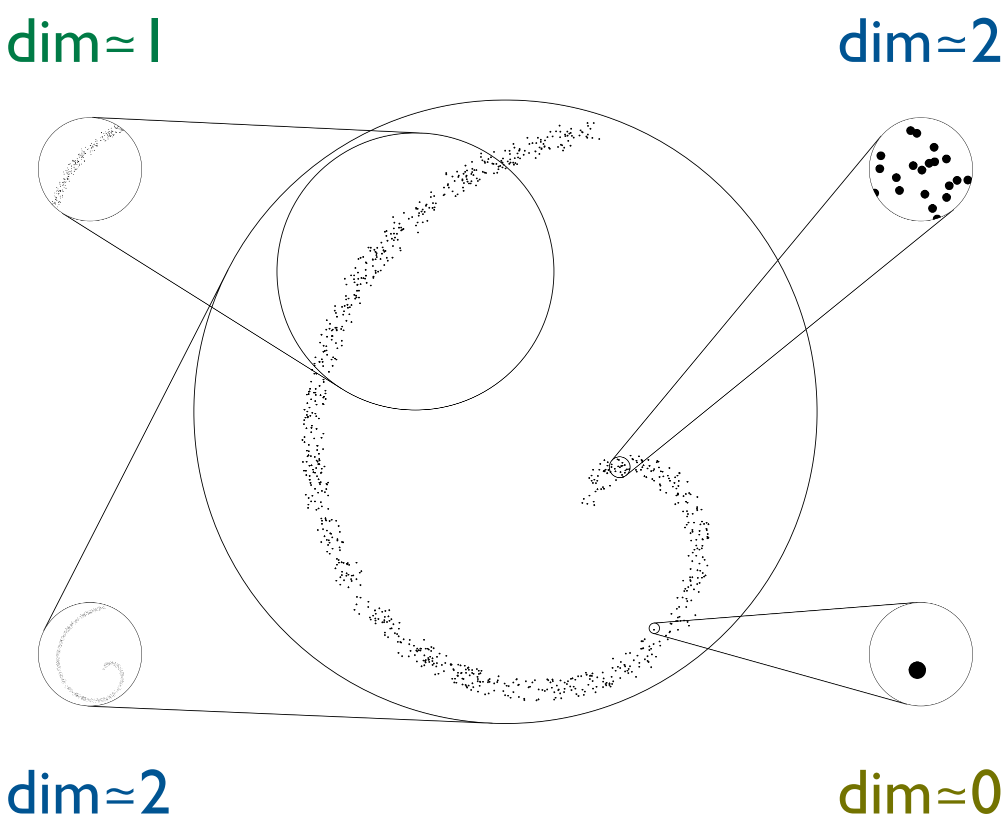
Dimensionality of Space of Jets

QCD

$$N_{\text{neighbors}}(r) \sim r^{\text{dim}}$$

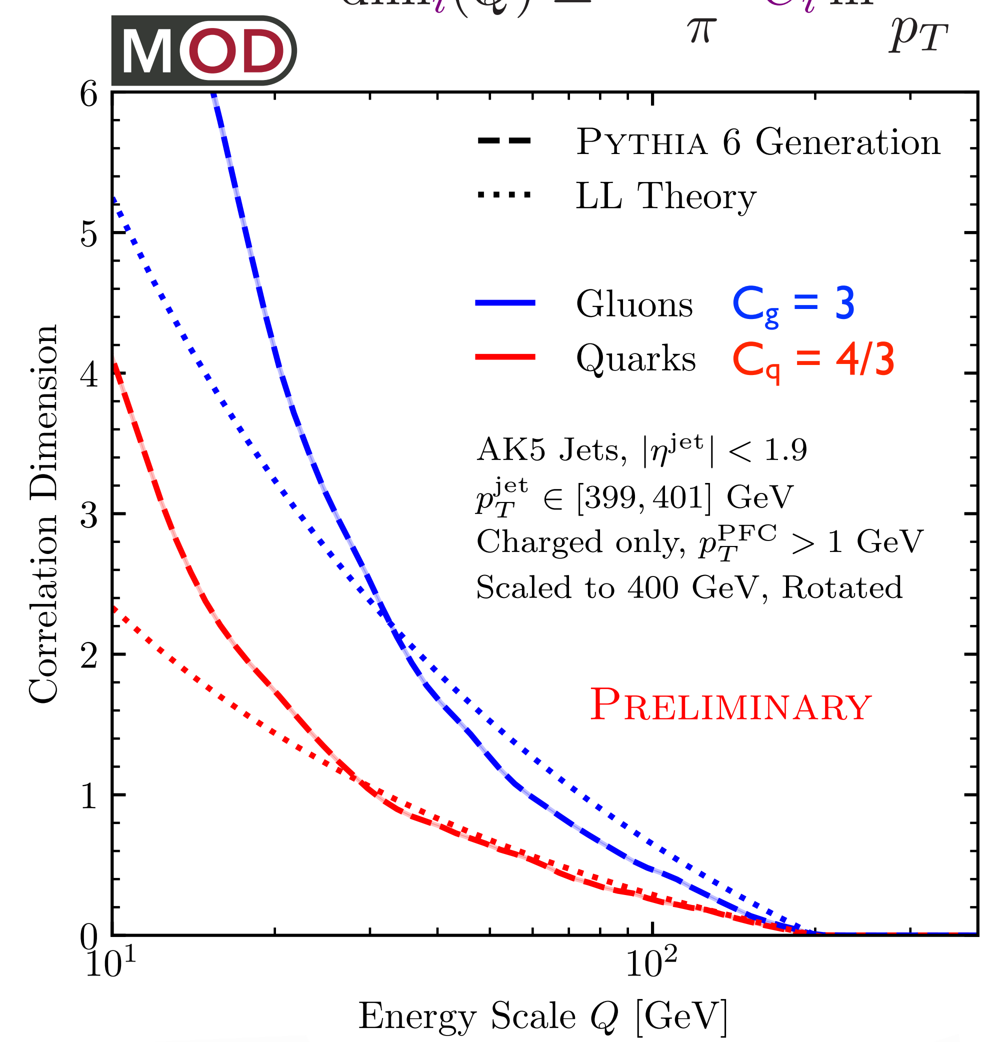
$$\Rightarrow \quad \text{dim}(r) \sim r \frac{\partial}{\partial r} \ln N_{\text{neighbors}}(r)$$

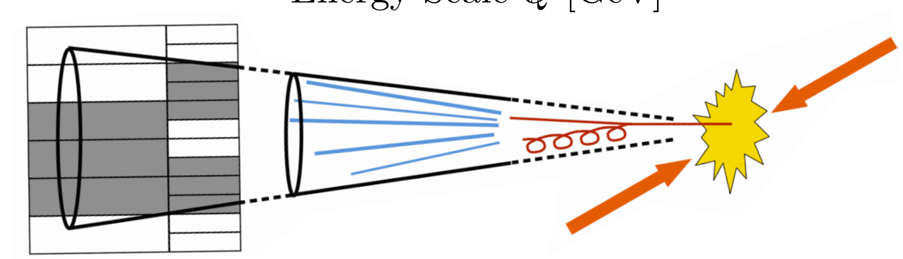
[Grassberger, Procaccia, PRL 1983; Kégl, NIPS 2002]



(eventually 0)

$$\text{dim}_i(Q) \simeq -\frac{8\alpha_s}{\pi} C_i \ln \frac{Q}{p_T}$$





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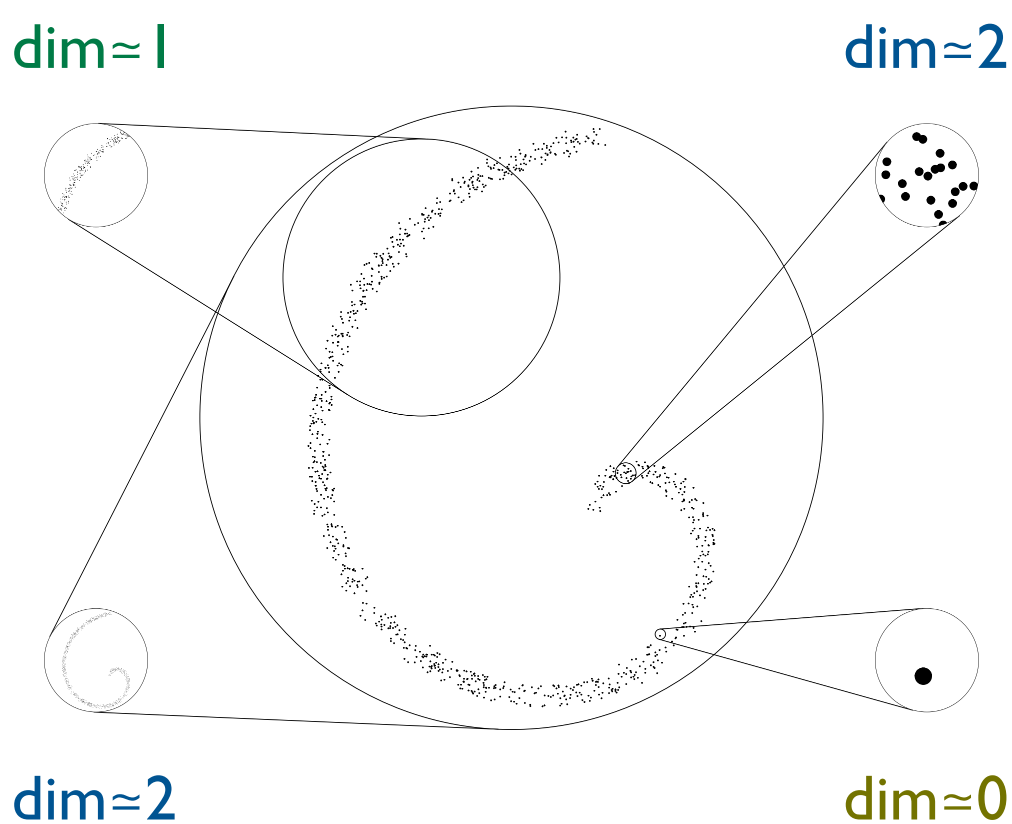
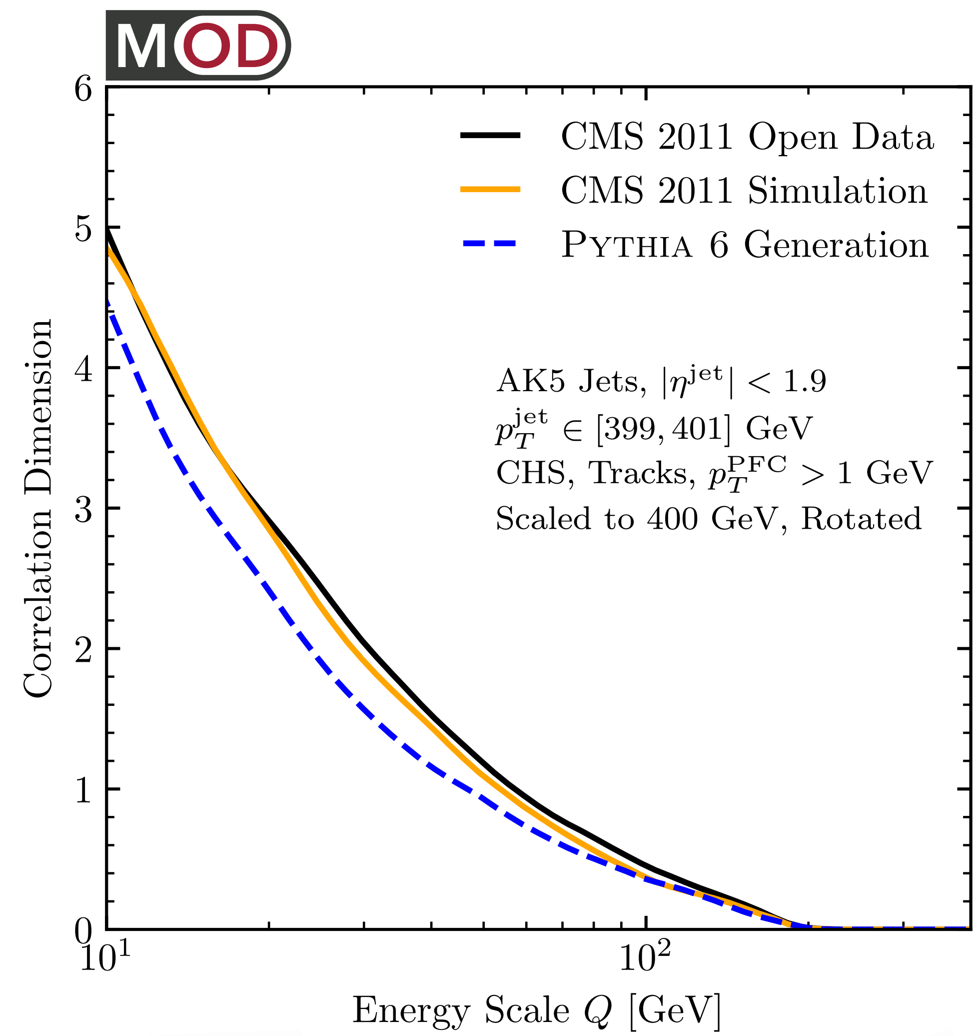
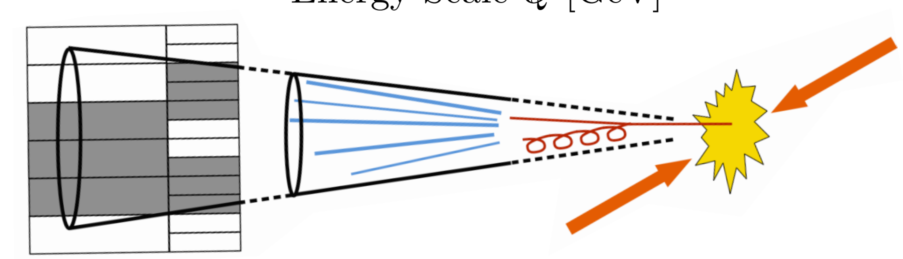


(eventually 0)

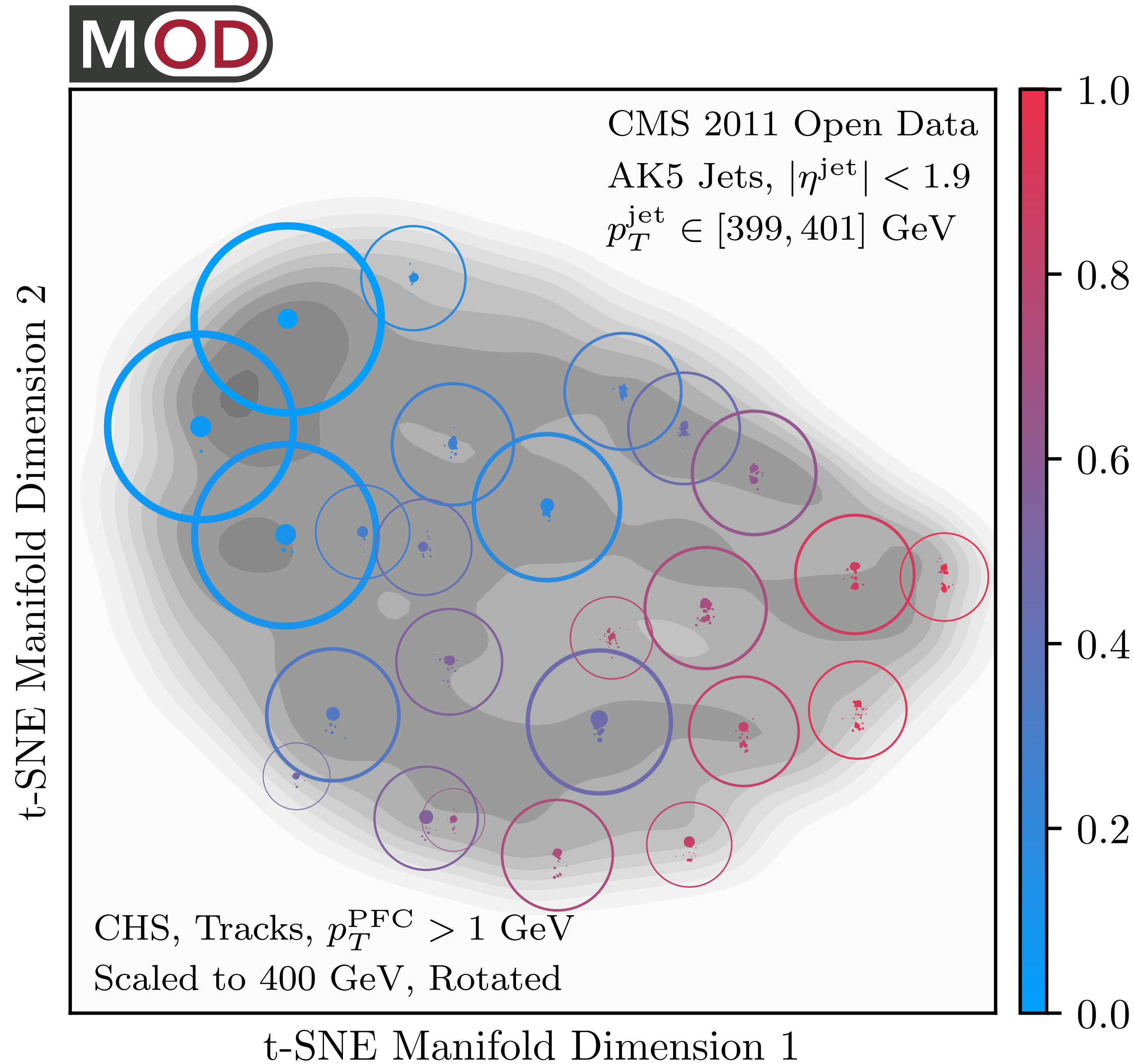




[Komiske, Mastandrea, Metodiev, Naik, JDT, PRD 2020; using CMS Open Data]

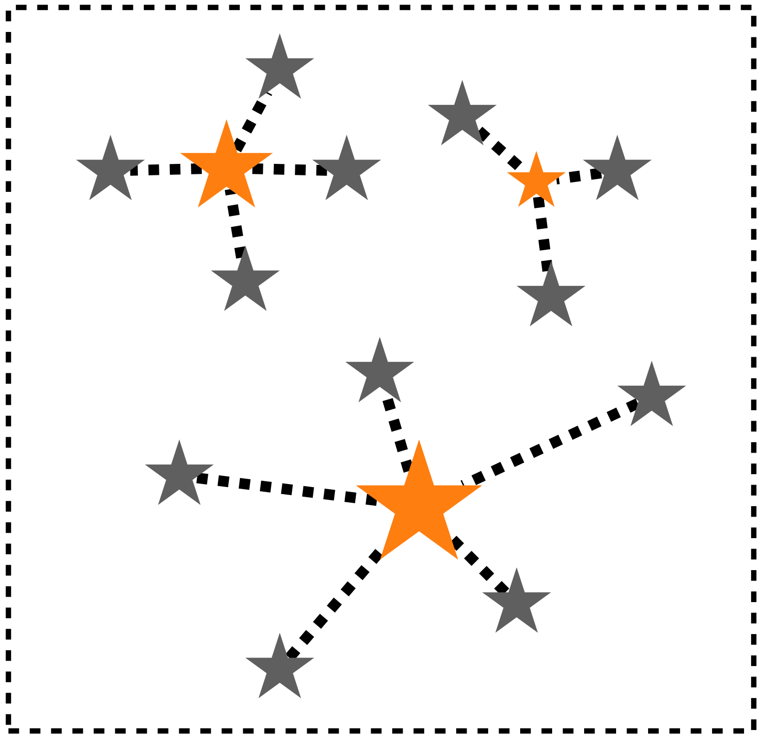
Most Representative Jets

[http://opendata.cern.ch/]



k-medoids

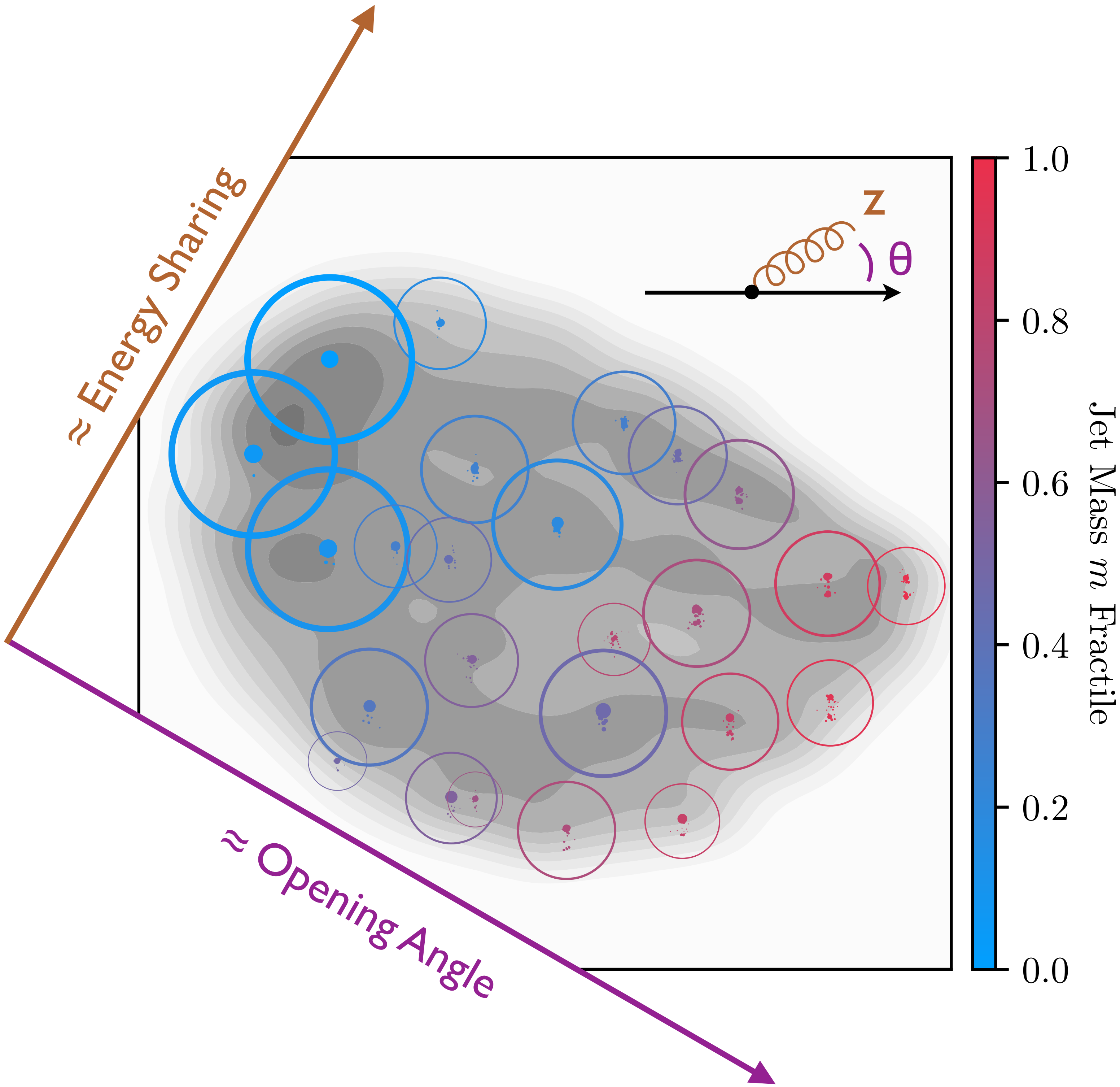
Arranged via t-SNE



[Komiske, Mastandrea, Metodiev, Naik, JDT, PRD 2020; using van der Maaten, Hinton, JMLR 2008]

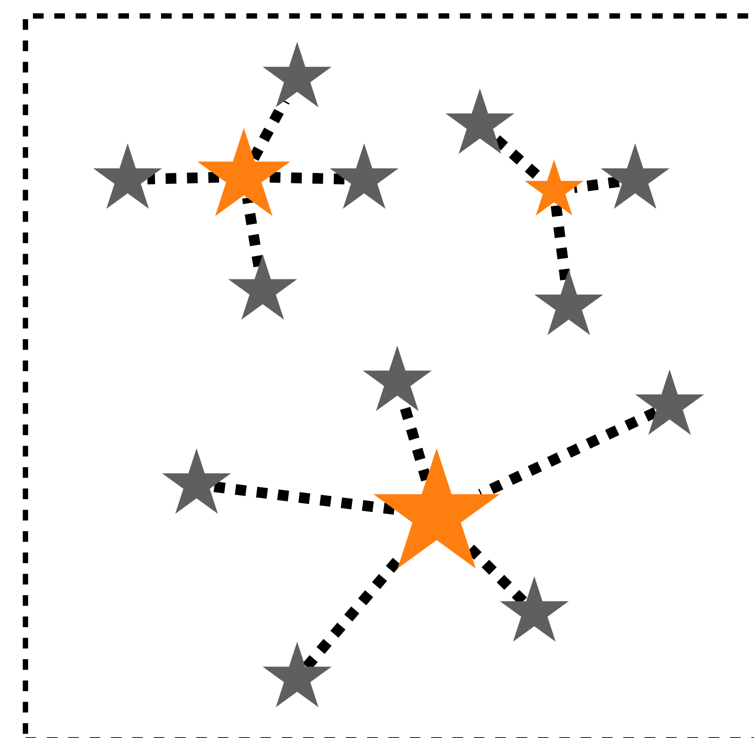
Most Representative Jets

[http://opendata.cern.ch/]



k-medoids

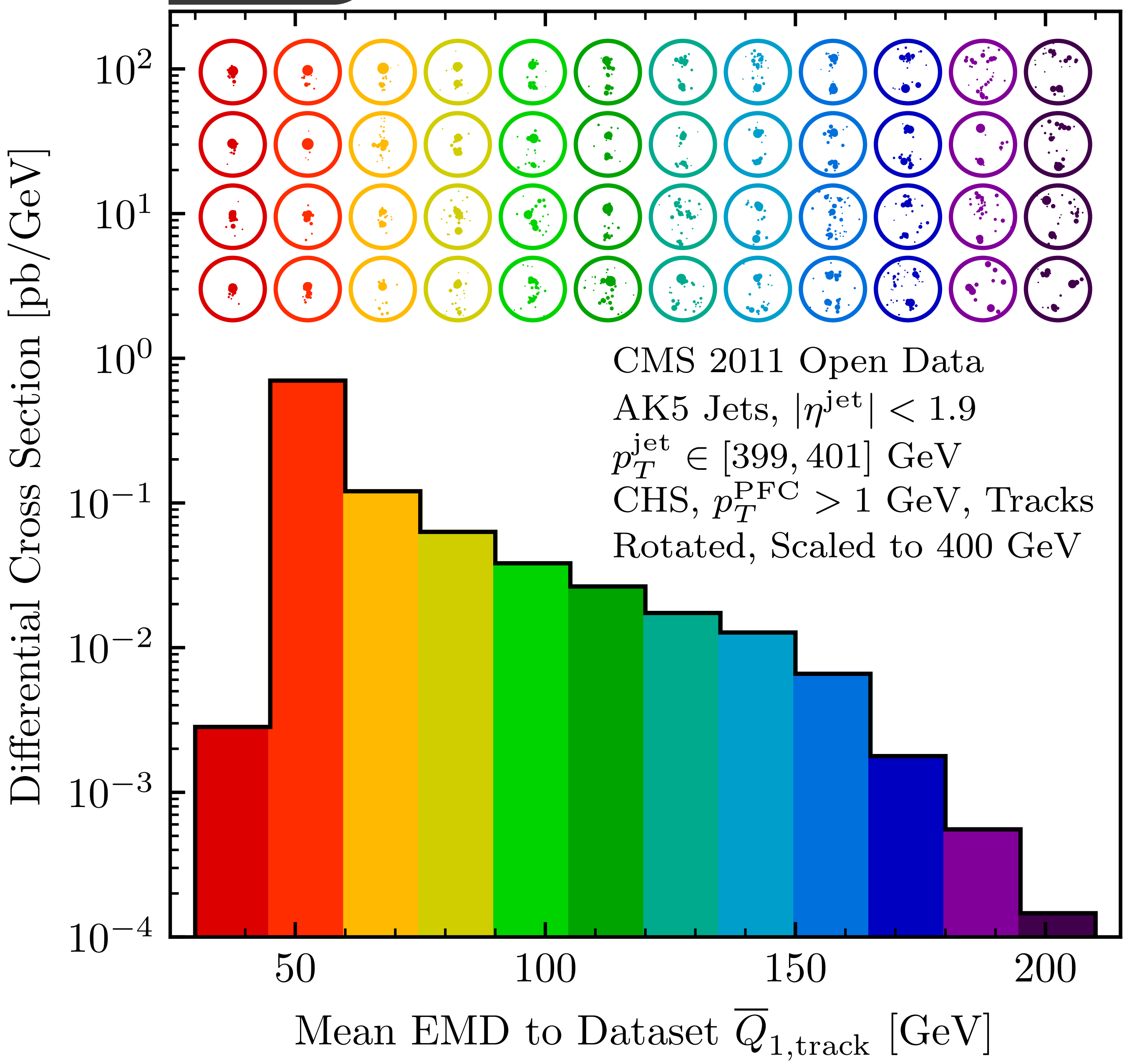
Arranged via t-SNE



[Komiske, Mastandrea, Metodiev, Naik, JDT, PRD 2020; using van der Maaten, Hinton, JMLR 2008]

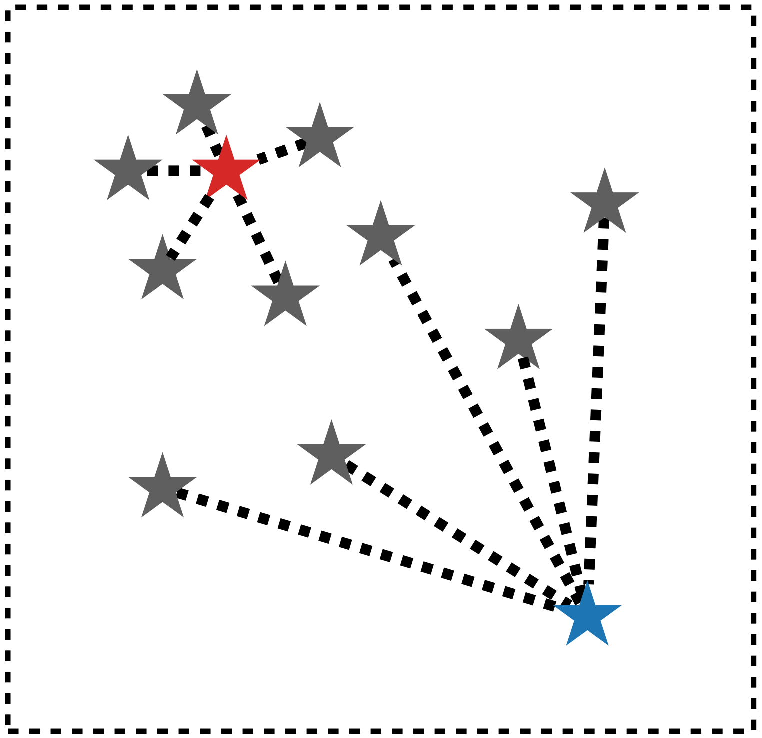
Least Representative Jets

[http://opendata.cern.ch/]

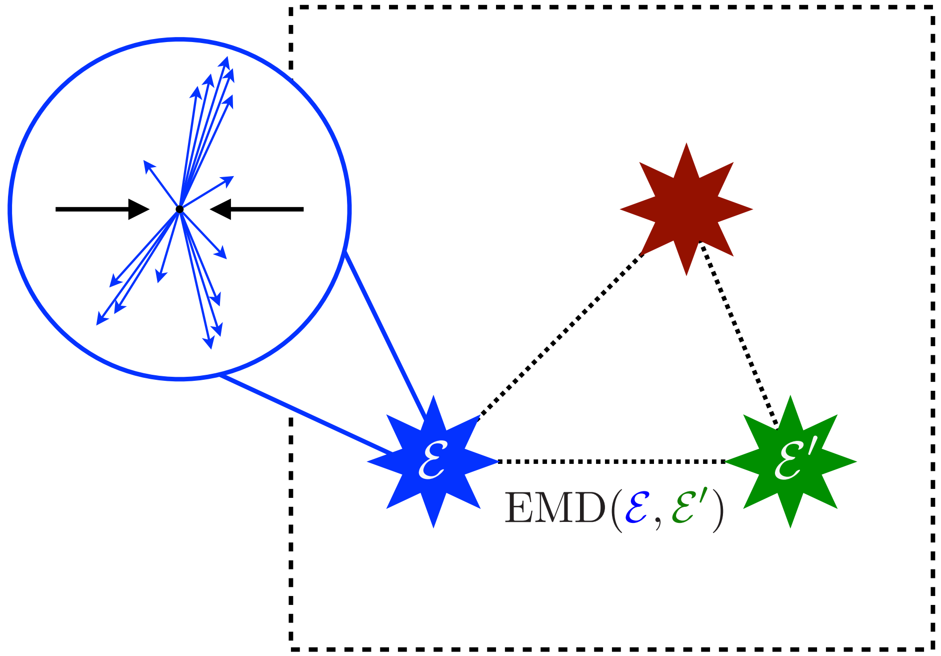


New Physics?

Or tails of QCD?



[Komiske, Mastandrea, Metodiev, Naik, JDT, PRD 2020]

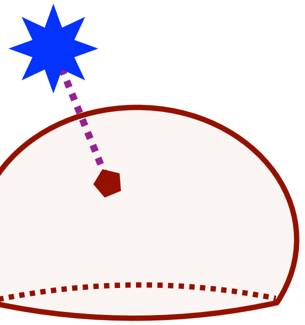


Viewed through the data science lens, the EMD unlocks a suite of geometric analysis strategies

At this point, it should not be obvious why optimal transport distances should be particularly well-suited to collider applications

Going with the (Energy) Flow

The Energy Mover's Distance

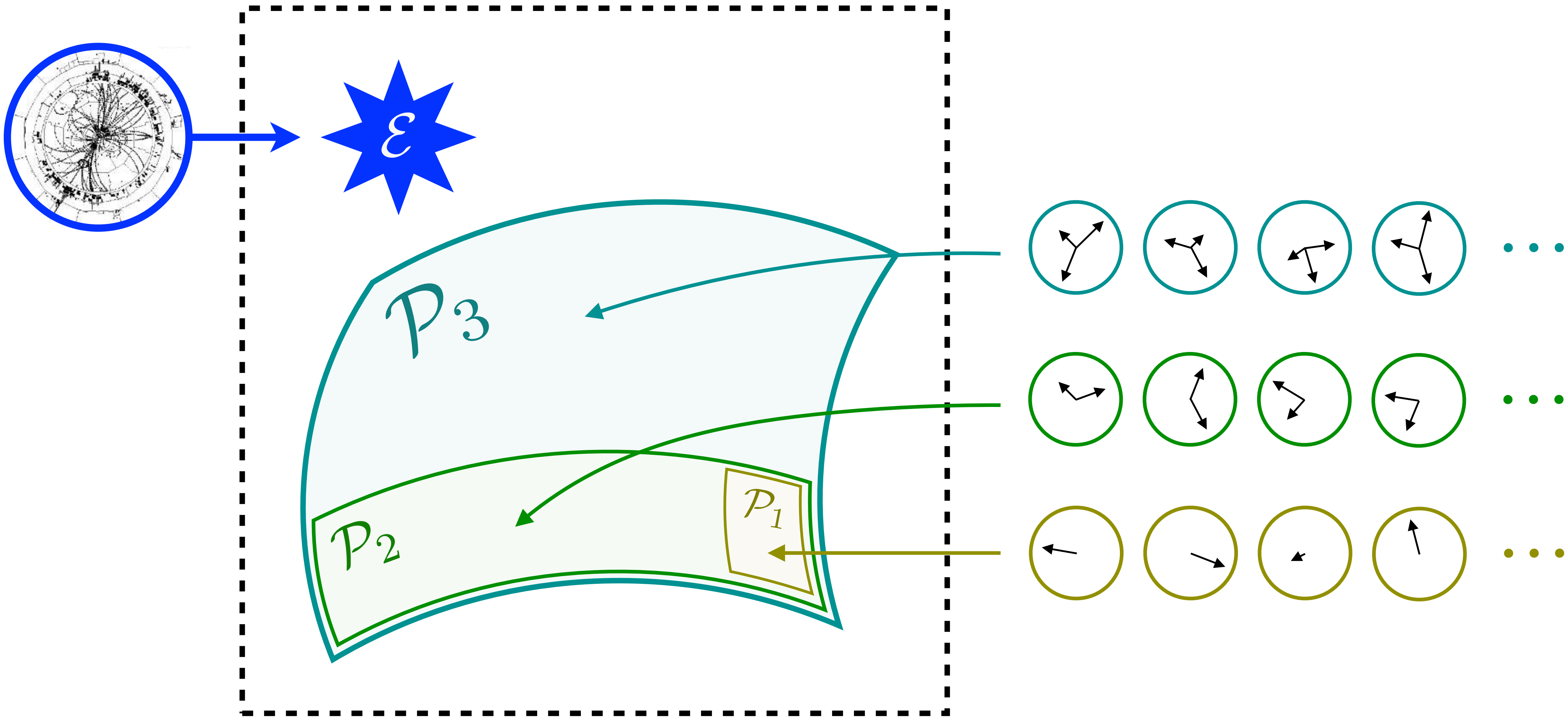


Revealing a Hidden Geometry

Given a metric space, the first geometric object you might think to construct is…

Introducing N-particle Manifolds

$\mathcal{P}_N$ = set of all N-particle configurations



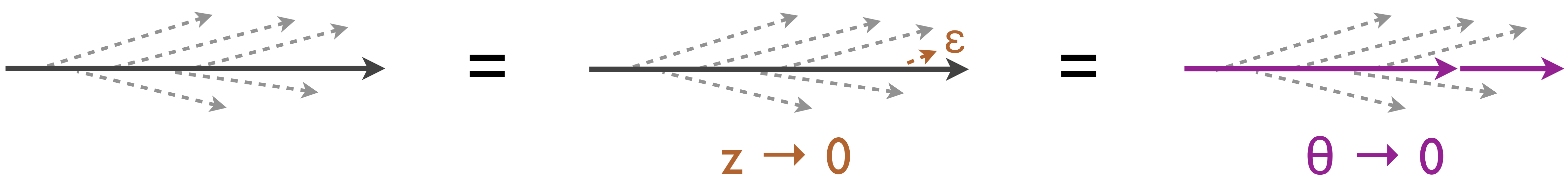
$\mathcal{P}_N \supset \mathcal{P}_{N-1} \supset \ldots \supset \mathcal{P}_2 \supset \mathcal{P}_1$ by soft/collinear limits

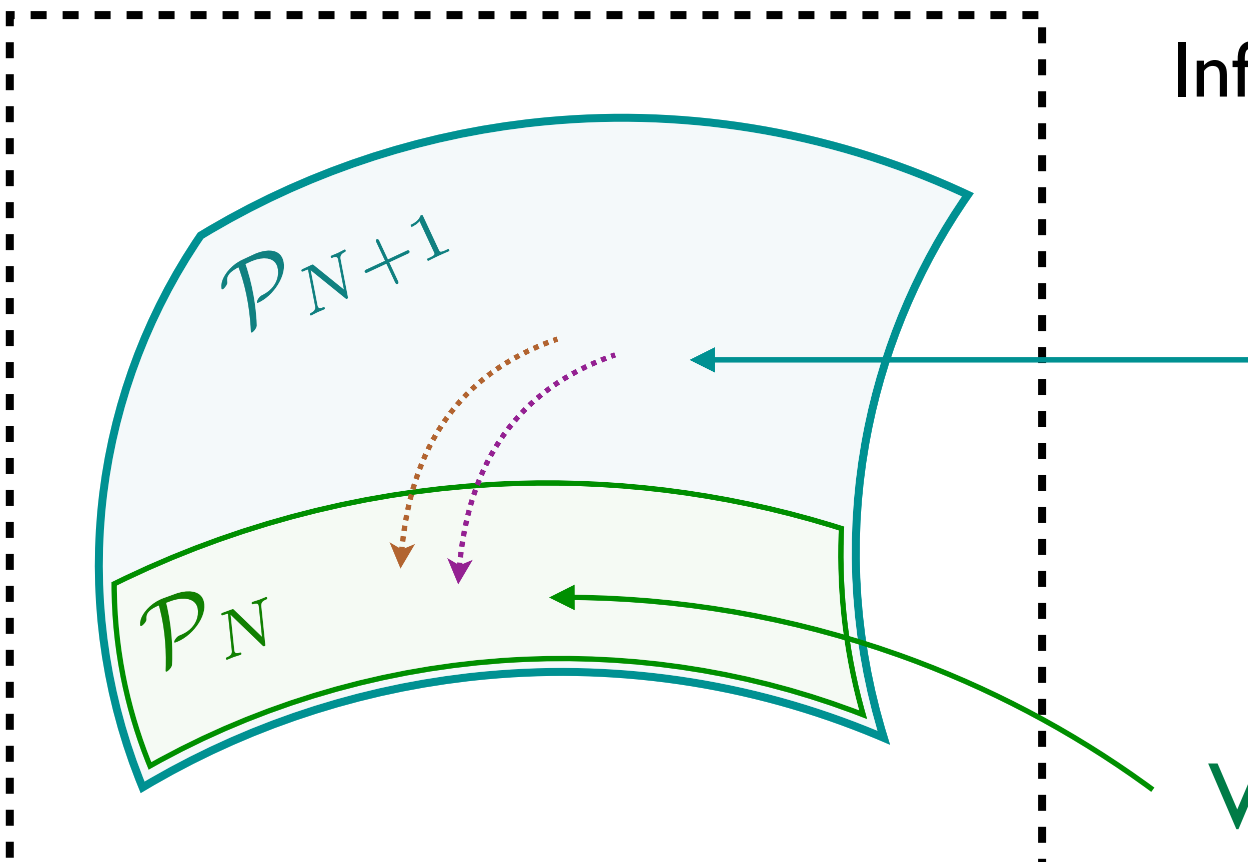
[see related discussion in Larkoski, Melia, PRD 2020]

When are Two Events the Same?

$$\mathcal{E}(\hat{n}) = \sum_i E_i\, \delta(\hat{n} - \hat{n}_i)$$

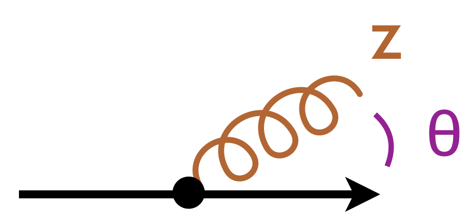
Energy Flow unchanged by infinitesimal soft/collinear emissions





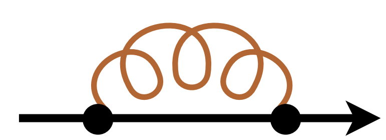
Infrared divergences "live" together!

Real:



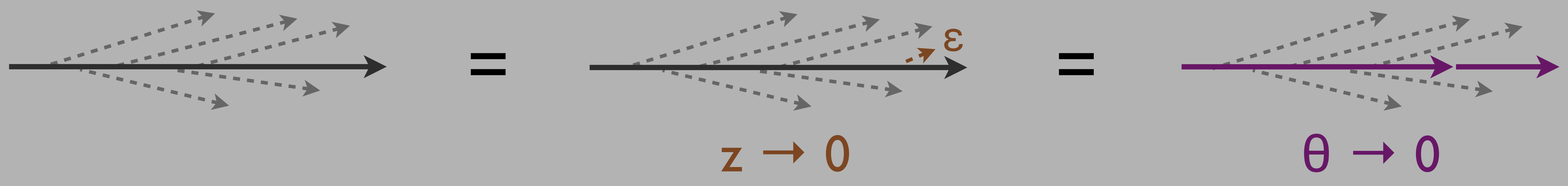
$$\mathrm{d}P_{i\to ig} \simeq \frac{2\alpha_s}{\pi} C_i \frac{\mathrm{d}z}{z} \frac{\mathrm{d}\theta}{\theta}$$

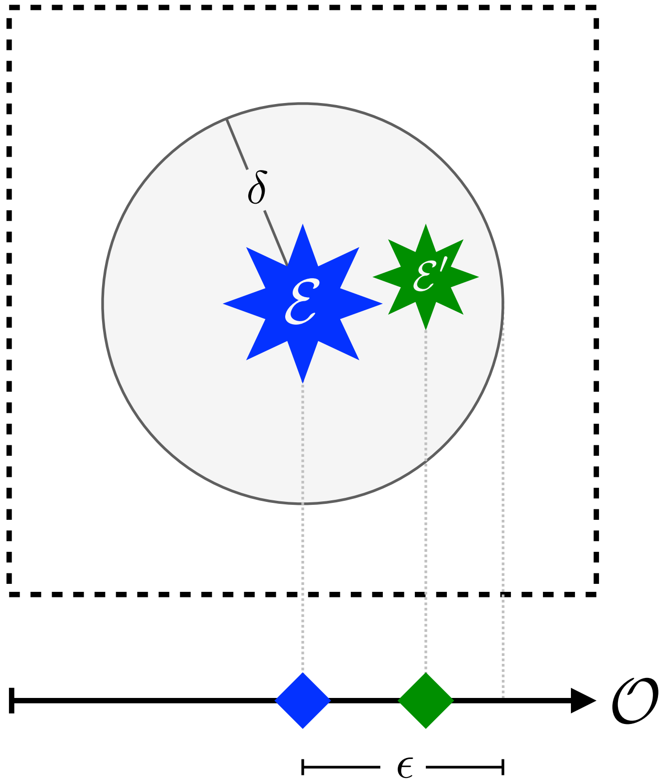
Virtual:



When are Two Events the Same?

Energy Flow unchanged by infinitesimal soft/collinear emissions





Infrared & Collinear Safety

≈ calculable in perturbative quantum field theory

*is** ⟵ (see backup for subtleties)

Continuity in EMD Space

[Komiske, Metodiev, JDT, JHEP 2020]
[Sterman, Weinberg, PRL 1977; Sterman, PRD 1979]
[see also Banfi, Salam, Zanderighi, JHEP 2005; Larkoski, Marzani, JDT, PRD 2015]

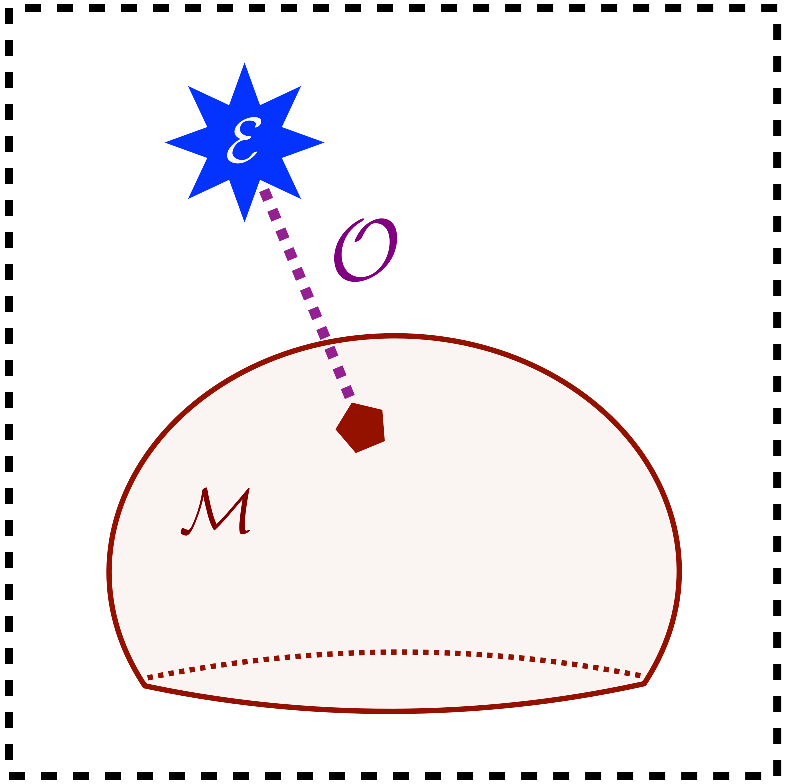
The EMD seems to define the "natural" geometry for massless gauge theories

Open question: Can you define $\left|\mathcal{M}_{AB\to 12\ldots n}\right|^2$ directly in this space?

What does it mean to "integrate" in this space?

Manifolds for Observables

One Event



Set of Events

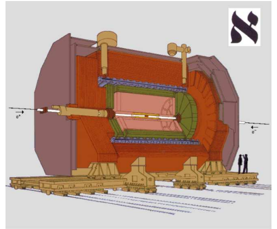
Distance of Closest Approach ⇒ Observable

$$\mathcal{O}(\mathcal{E}) = \min_{\mathcal{E}' \in \mathcal{M}} \mathrm{EMD}(\mathcal{E}, \mathcal{E}')$$

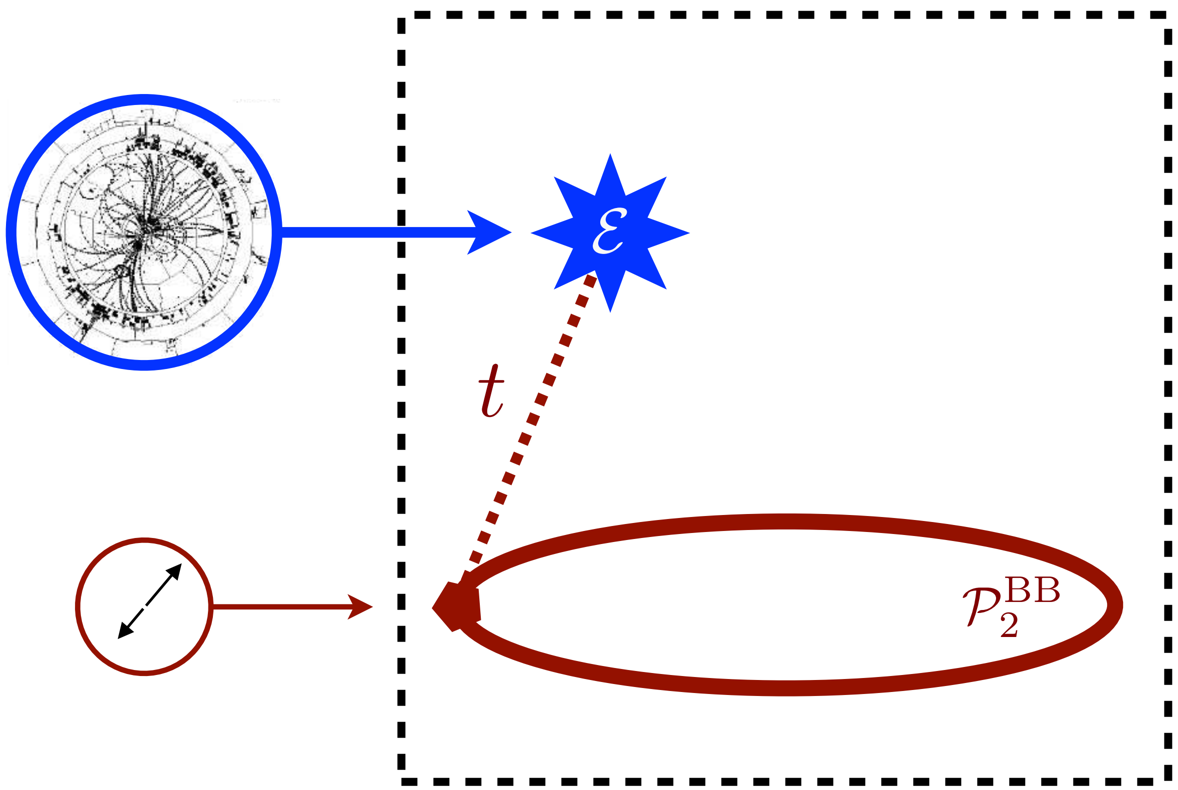
[Komiske, Metodiev, JDT, JHEP 2020]

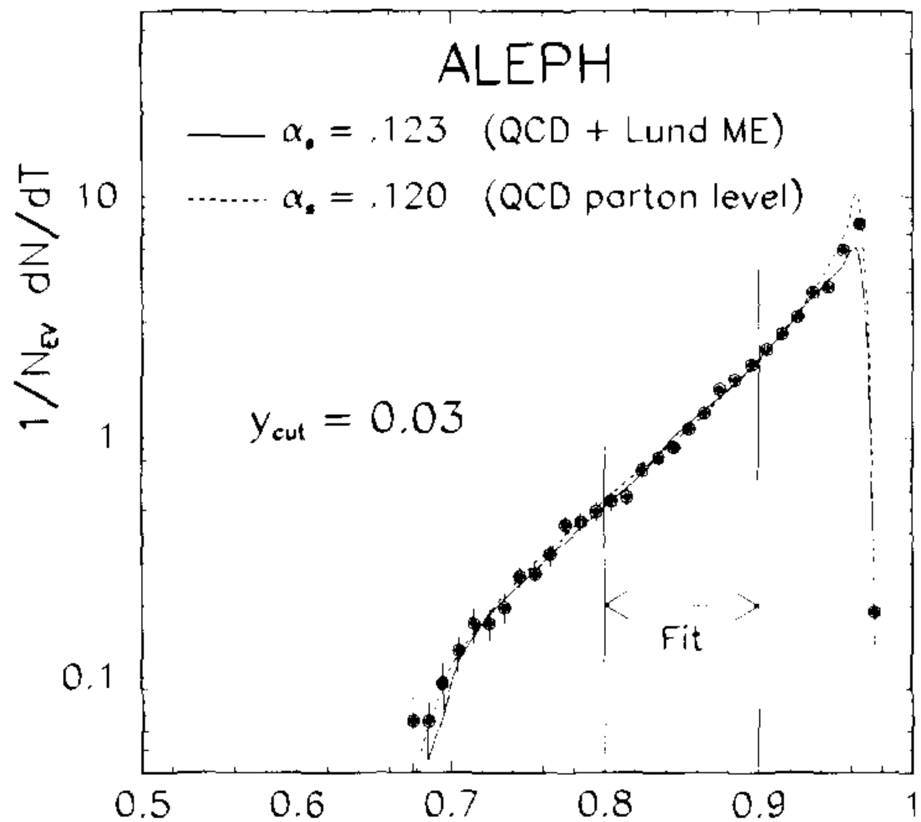
E.g. Thrust

How dijet-like is an event?



$$t(\mathcal{E}) = \min_{\mathcal{E}' \in \mathcal{P}_2^{\mathrm{BB}}} \mathrm{EMD}_2(\mathcal{E}, \mathcal{E}')$$

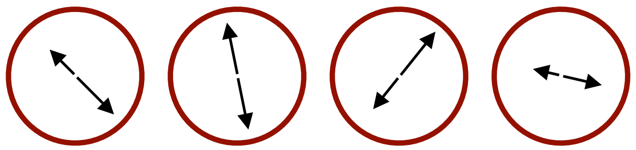




$$1 - \frac{t}{2E_{\mathrm{CM}}}$$

All Back-to-Back Two Particle Configurations

$$\mathcal{P}_2^{\mathrm{BB}} = \{ \quad \cdots \}$$



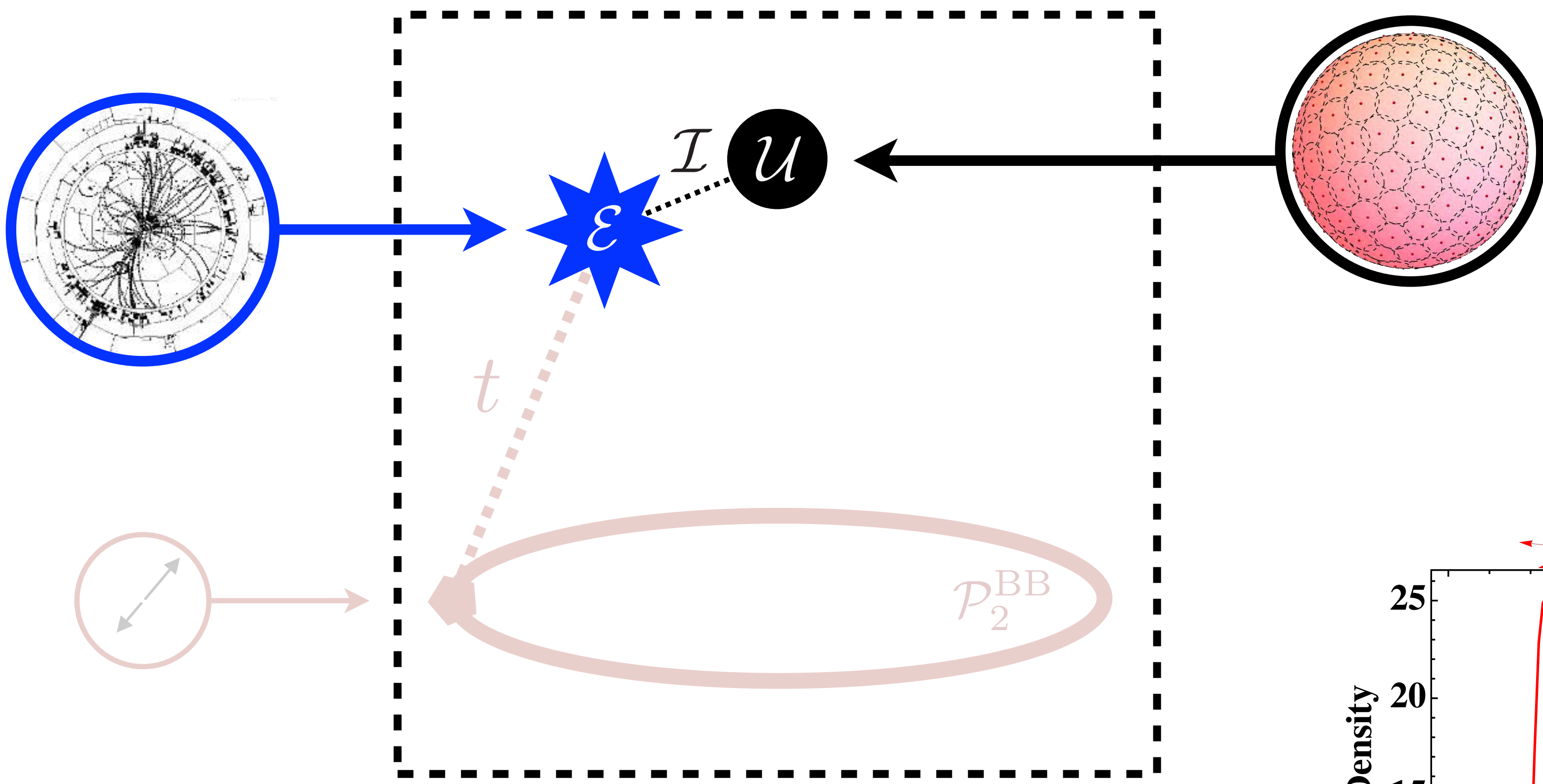
(using β=2 EMD variant)

cf. $T(\mathcal{E}) = \max_{\hat{n}} \frac{\sum_i |\vec{p}_i \cdot \hat{n}|}{\sum_j |\vec{p}_j|}$

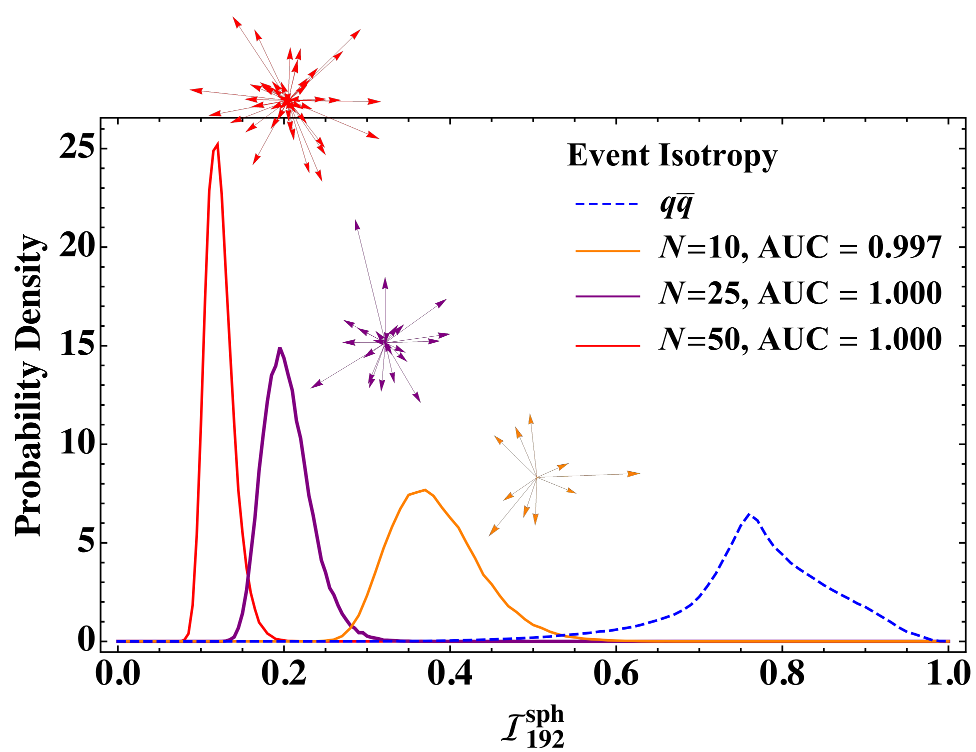
[Komiske, Metodiev, JDT, JHEP 2020]
[Brandt, Peyrou, Sosnowski, Wroblewski, PL 1964; Farhi, PRL 1977; ALEPH, PLB 1991]

New! Event Isotropy

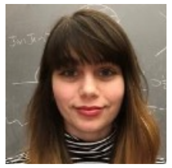
How isotropic is an event?



$$\mathcal{I}(\mathcal{E}) = \text{EMD}(\mathcal{E}, \mathcal{U})$$

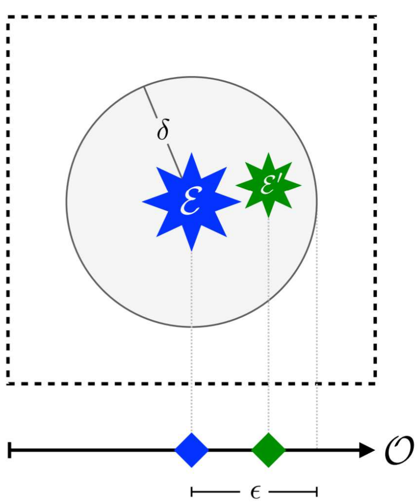


[Cesarotti, JDT, JHEP 2020;
see also Cesarotti, Reece, Strassler, arXiv 2020]

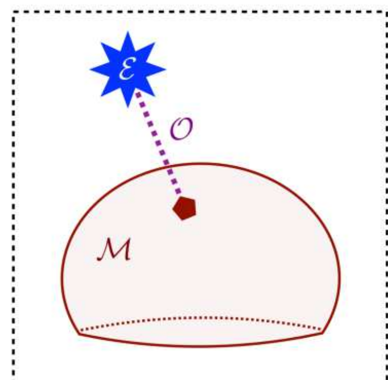


Six Decades of Collider Physics
Translated into a New Geometric Language!

IRC Safety is smoothness in the space of events

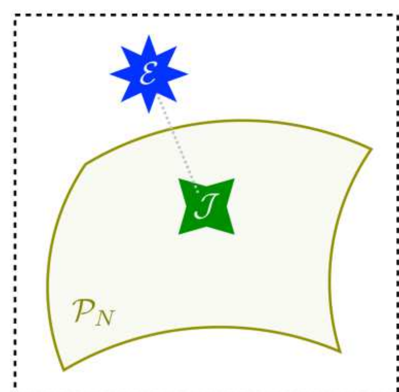


Event shapes are distances from events to manifolds.



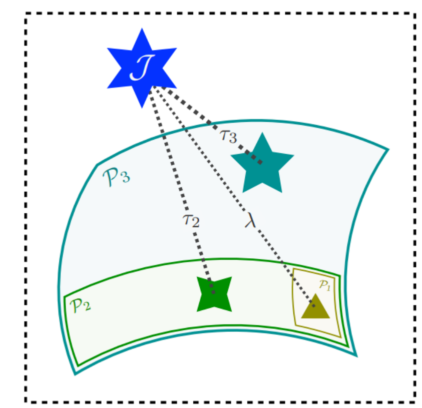
$\mathcal{O}(\mathcal{E}) = \min_{\mathcal{E}'\in\mathcal{M}} \mathrm{EMD}_{\beta,R}(\mathcal{E},\mathcal{E}')$

Jets are projections to few-particle manifolds.



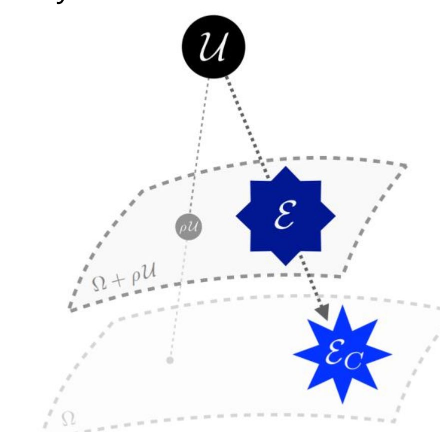
$\mathcal{J} = \mathrm{argmin}_{\mathcal{E}'\in\mathcal{P}_N} \mathrm{EMD}_{\beta,R}(\mathcal{E},\mathcal{E}')$

Substructure resolves emissions within the jet.



$\tau(\mathcal{J}) = \min_{\mathcal{E}'\in\mathcal{P}_N} \mathrm{EMD}_{\beta}(\mathcal{J},\mathcal{E}')$.

Pileup mitigation moves away from uniform radiation.



$\mathcal{E}_C = \arg\min_{\mathcal{E}'} \mathrm{EMD}(\mathcal{E},\mathcal{E}' + \rho\,\mathcal{U})$.

Taming infinities — Event Shapes — Jet Algorithms — Jet Substructure — Pileup

1960 — 2020

1962-1964 Infrared Safety
[Kinoshita, JMP 1962]
[Lee, Nauenberg, PR 1964]

1977 Thrust, Sphericity
[Farhi, PRL 1977]
[Georgi, Machacek, PRL 1977]

1993 k_T jet clustering
[Ellis, Soper, PRD 1993]
[Catani, Dokshitzer, Seymour, Webber, NPB 1993]

1997-1998 C/A jet clustering
[Wobisch, Wengler, 1998]
[Doskhitzer, Leder, Moretti,Webber, JHEP 1997]

2010-2015 N-(sub)jettiness, XCone
[Stewart, Tackmann, Waalewijn, PRL 2010]
[Thaler, Van Tilburg, JHEP 2011]
[Stewart, Tackmann, Thaler, Vermilion, Wilkason, JHEP 2015]

2014-2019 Constituent Subtraction
[Berta, Spousta, Miller, Leitner, JHEP 2014]
[Berta, Masetti, Miller, Spousta, JHEP 2019]

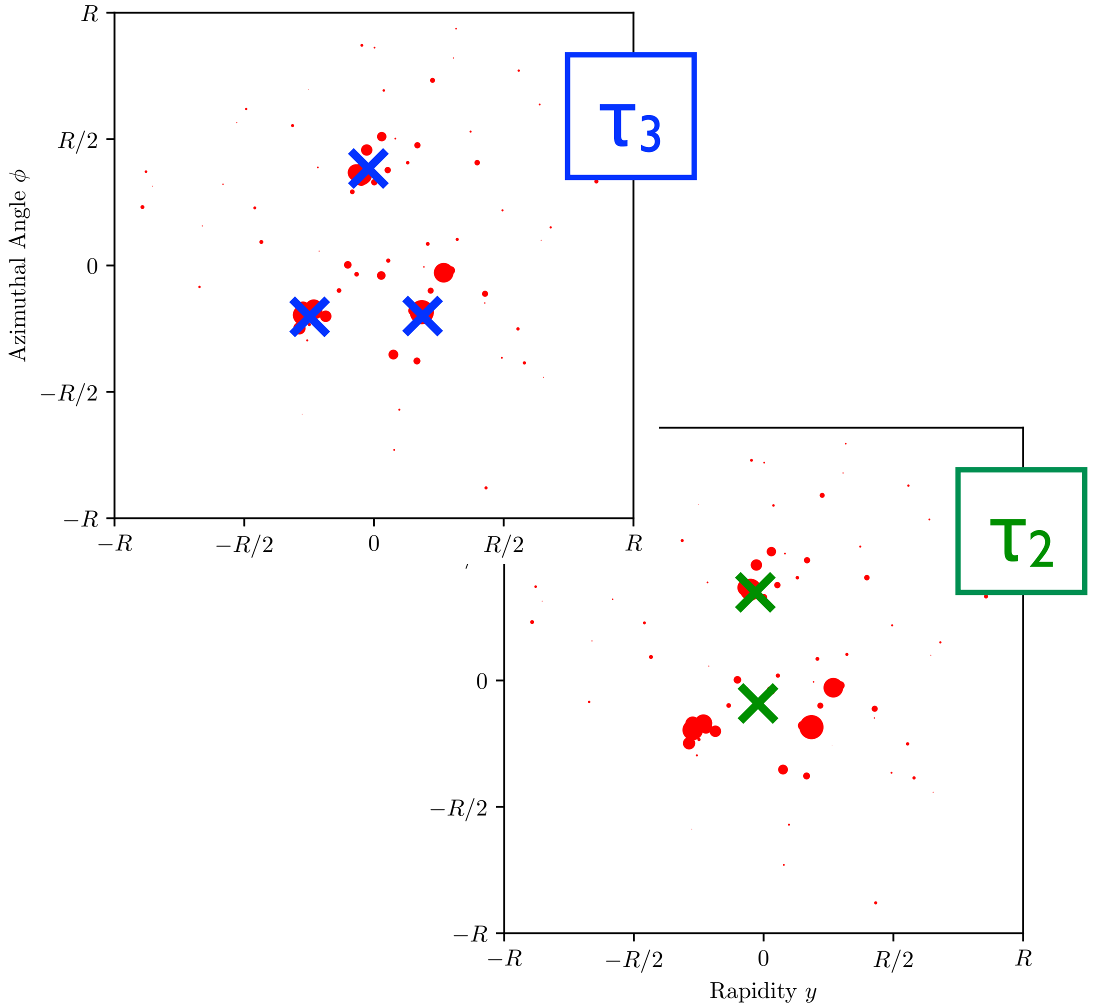
And many more!

[timeline from Eric Metodiev]

N-subjettiness

Ubiquitous jet substructure observable used for almost a decade…

$$\tau_N(\mathcal{J}) = \min_{N \text{ axes}} \sum_i E_i \min\{\theta_{1,i}, \theta_{2,i}, \ldots, \theta_{N,i}\}$$

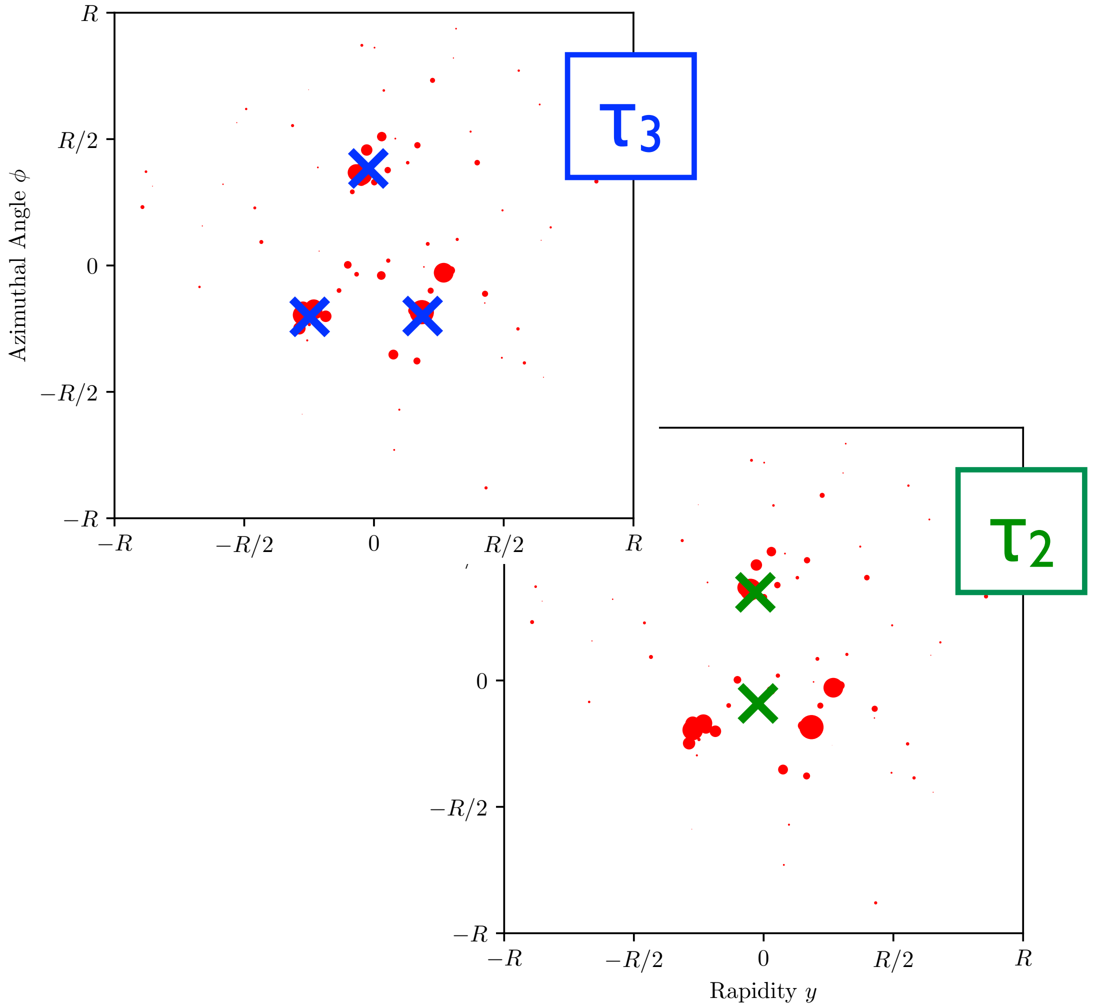


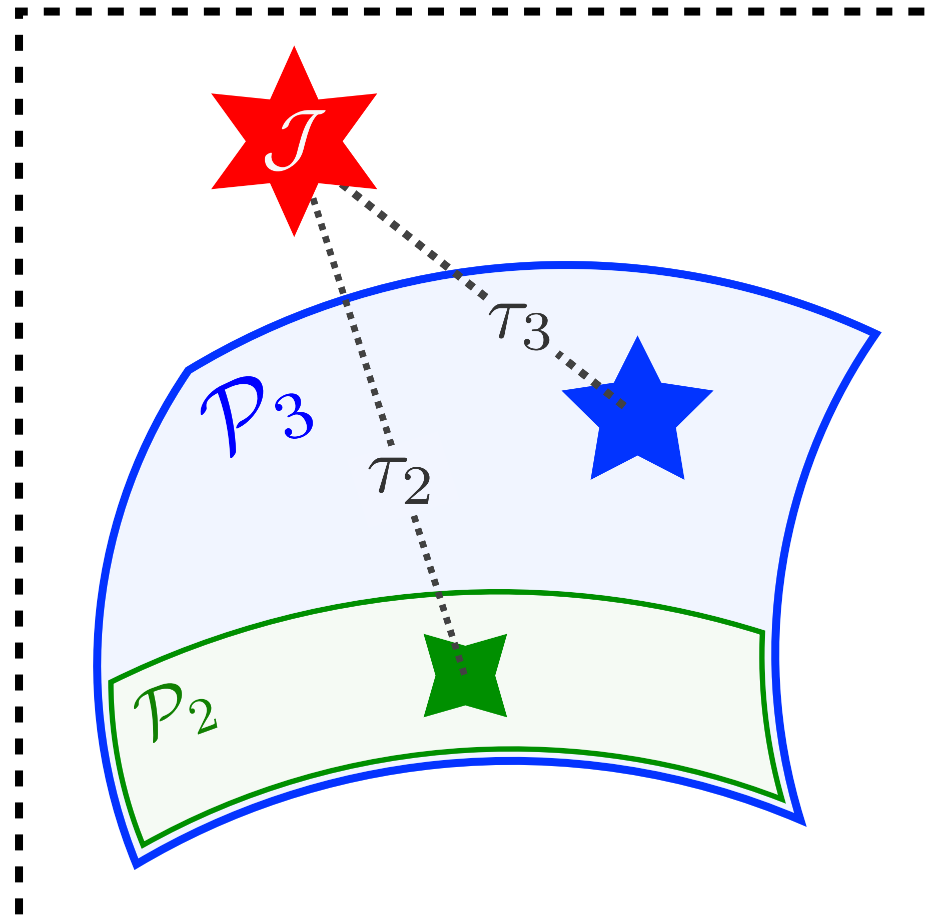
[JDT, Van Tilburg, JHEP 2011, JHEP 2012;
based on Brandt, Dahmen, ZPC 1979; Stewart, Tackmann, Waalewijn, PRL 2010]

N-subjettiness = Point to Manifold EMD

…is secretly an optimal transport problem

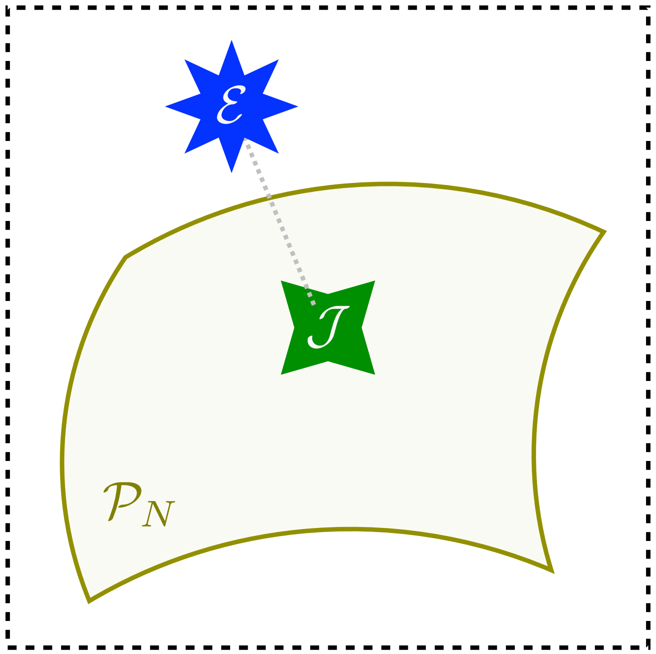
$$\tau_N(\mathcal{J}) = \min_{\mathcal{J}' \in \mathcal{P}_N} \mathrm{EMD}(\mathcal{J}, \mathcal{J}')$$





[JDT, Van Tilburg, JHEP 2011, JHEP 2012;
rephrased in the language of Komiske, Metodiev, JDT, PRL 2019]

More Fun with N-particle Manifolds



N-jettiness

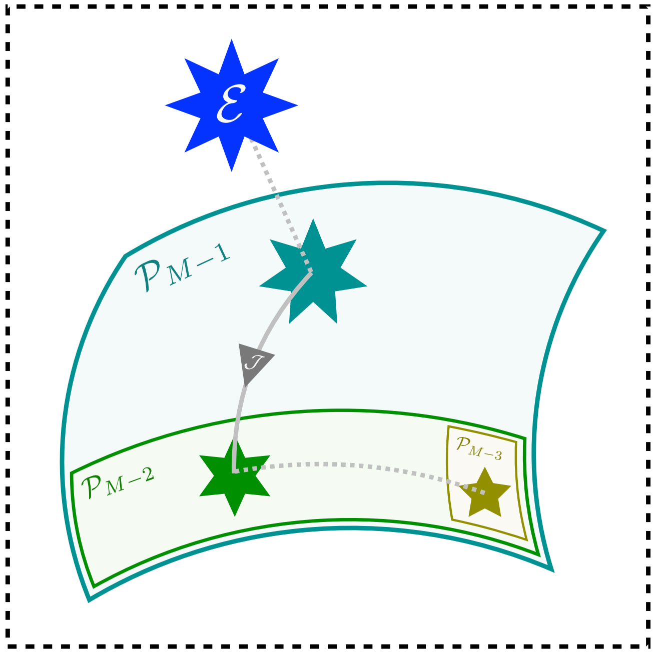
Distance of closest approach to N-particle manifold

[Brandt, Dahmen, ZPC 1979; Stewart, Tackmann, Waalewijn, PRL 2010]

Exclusive Cone Jet Finding

Point of closest approach on N-particle manifold

[Stewart, Tackmann, JDT, Vermilion, Wilkason, JHEP 2015]



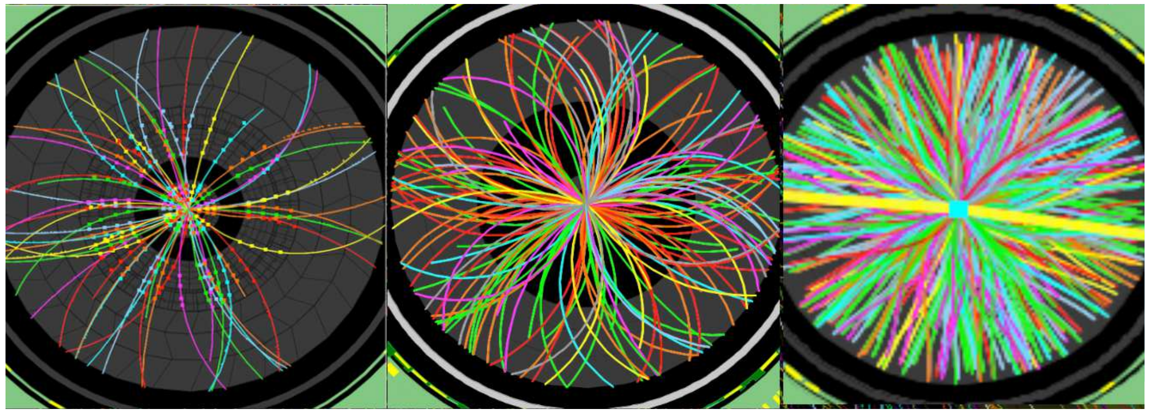
Sequential Jet Recombination

Iteratively stepping between various N-particle manifolds

[Catani, Dokshitzer, Seymour, Webber, NPB 1993; Ellis, Soper, PRD 1993]
[Dokshitzer, Leder, Moretti, Webber, JHEP 1997; Wobisch, Wengler, arXiv 1999]
[Butterworth, Couchman, Cox, Waugh, CPC 2003; Larkoski, Neill, JDT, JHEP 2014]

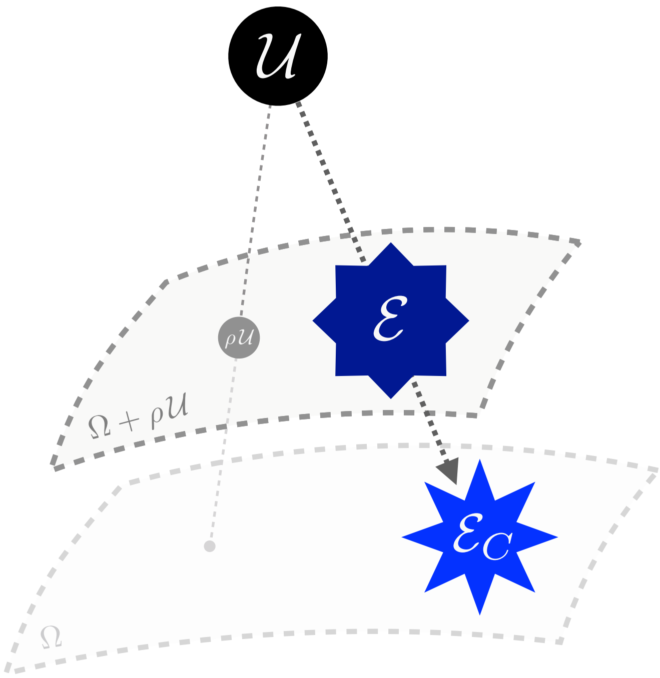
[Komiske, Metodiev, JDT, JHEP 2020]

Pileup Mitigation



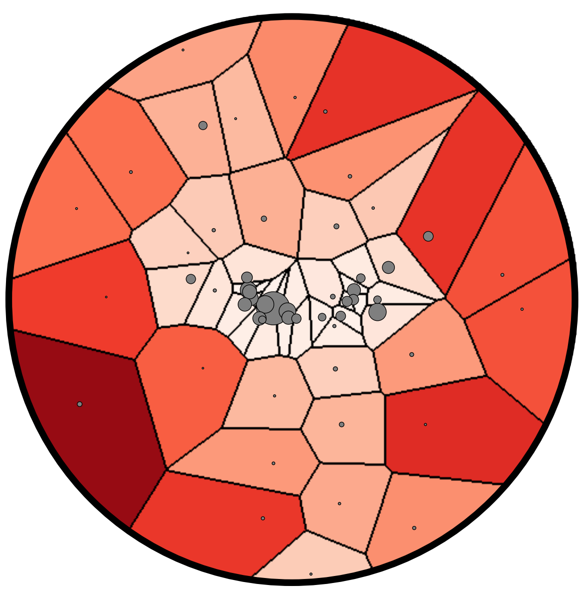
[see review in Soyez, PR 2019]

Uniform event contamination from overlapping proton-proton collisions



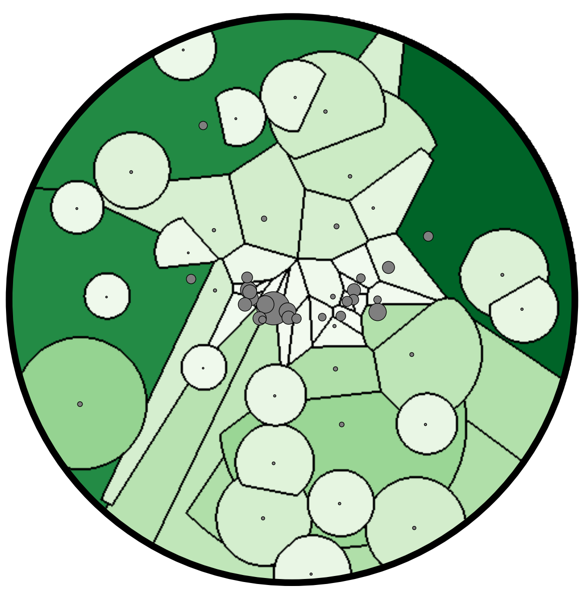
Pileup Mitigation:
"Move away" from uniform event

Voronoi



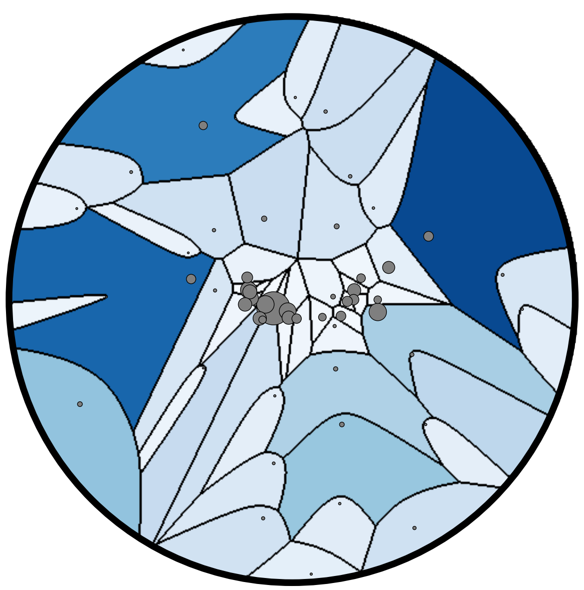
[Cacciari, Salam, Soyez, JHEP 2008]

Constituent Subtraction

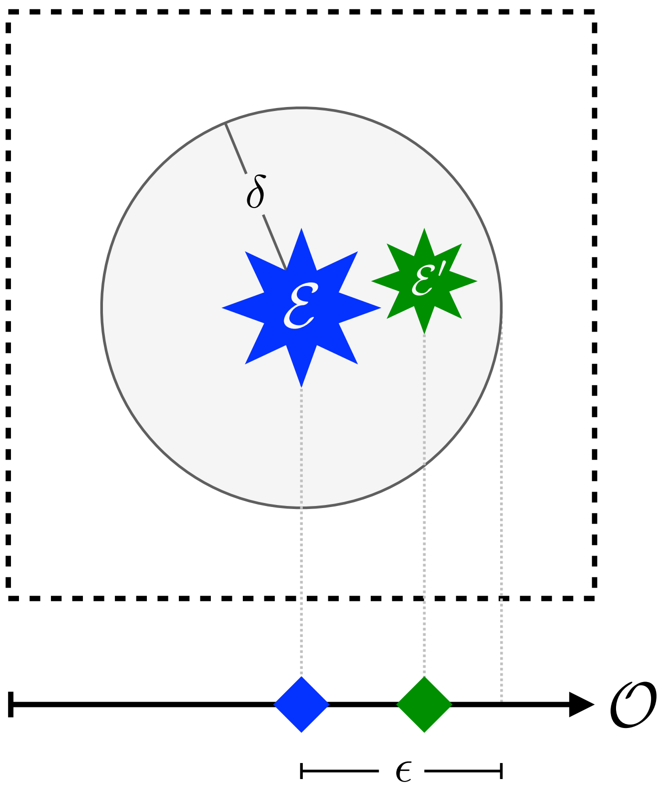


[Berta, Spousta, Miller, Leitner, JHEP 2014]

Apollonius



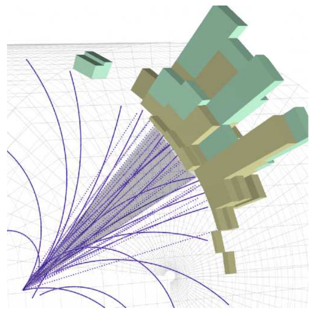
[Komiske, Metodiev, JDT, JHEP 2020]



We are just beginning to leverage the conceptual richness of optimal transport for high-energy physics application

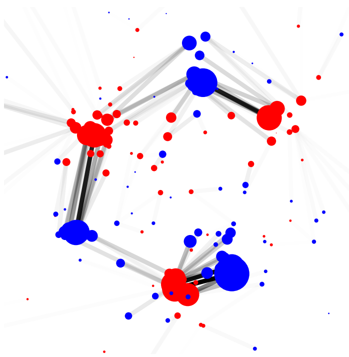
Ask me how far down this rabbit hole goes!

Summary



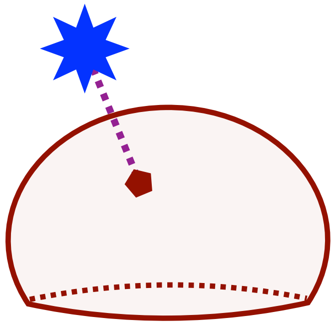
Going with the (Energy) Flow

Restricting our attention to IRC safe information is a theoretically motivated data analysis strategy



The Energy Mover's Distance

Optimal transport allows us to triangulate the space of collider events and define an emergent geometry



Revealing a Hidden Geometry

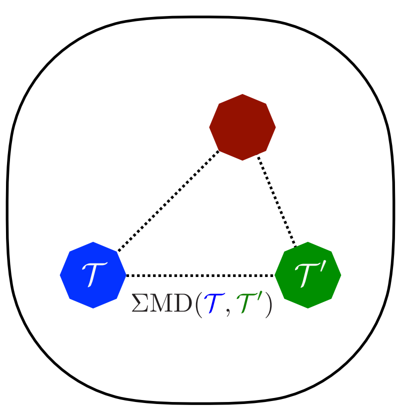
We can gain new perspectives on concepts/techniques in QFT and collider physics from the last half century

How far down does this rabbit hole go?

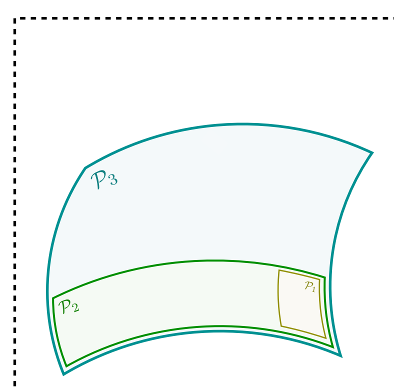
Master Formula for Collider Physics

$$\sigma_{\text{obs}} \simeq \frac{1}{2E_{\text{CM}}^2} \sum_{n=2}^{\infty} \int \mathrm{d}\Phi_n \, \left|\mathcal{M}_{AB\to 12\ldots n}\right|^2 f_{\text{obs}}(\Phi_n)$$

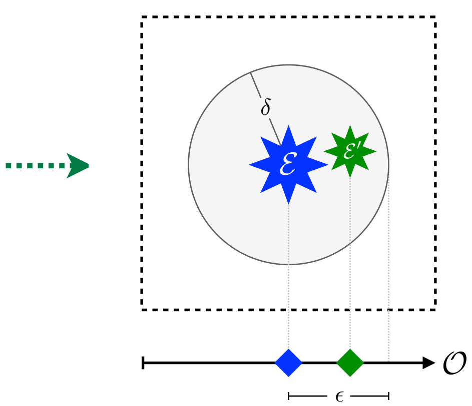
cross section



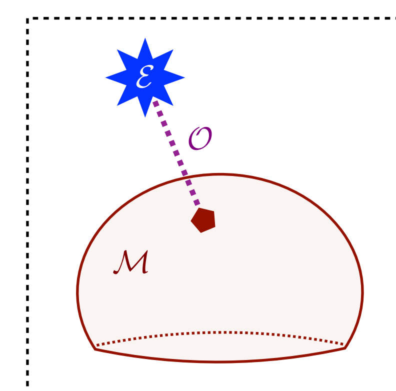
phase space



amplitude

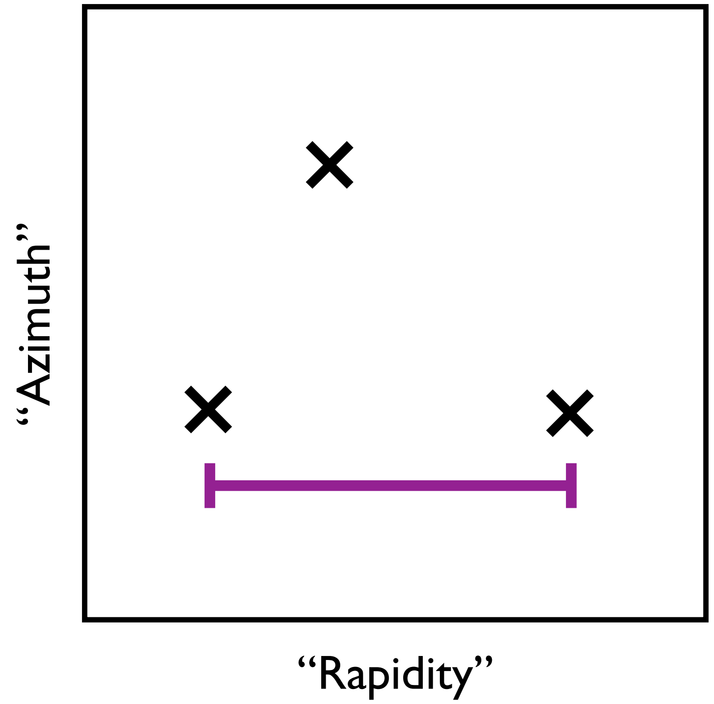


observable



[Komiske, Metodiev, JDT, JHEP 2020]

Direction Space

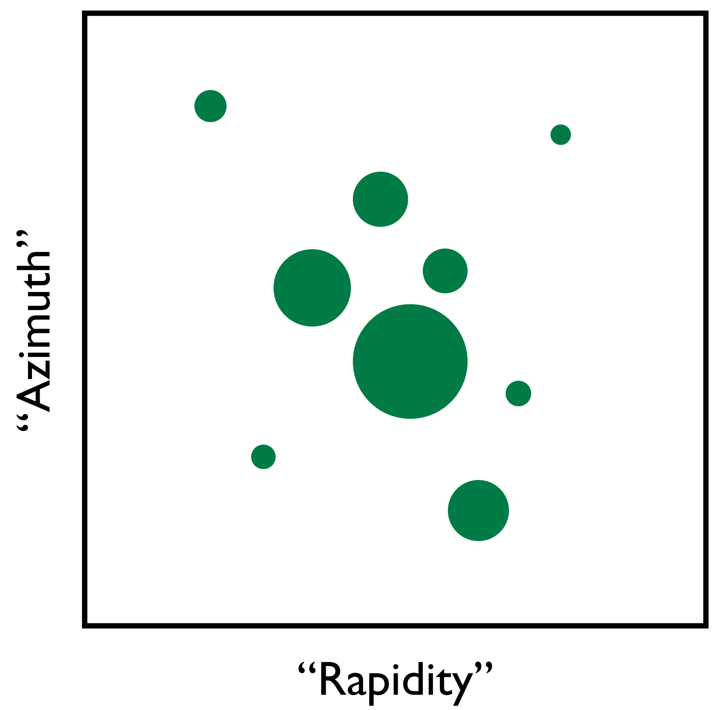


× = Direction $$n_i^\mu = \frac{p_i^\mu}{E_i} = (1, \hat{n})^\mu$$

= Angular Distance $$\theta_{ij} = \sqrt{2 n_i^\mu n_{j\mu}}$$

(for massless particles)

Direction Space Distribution

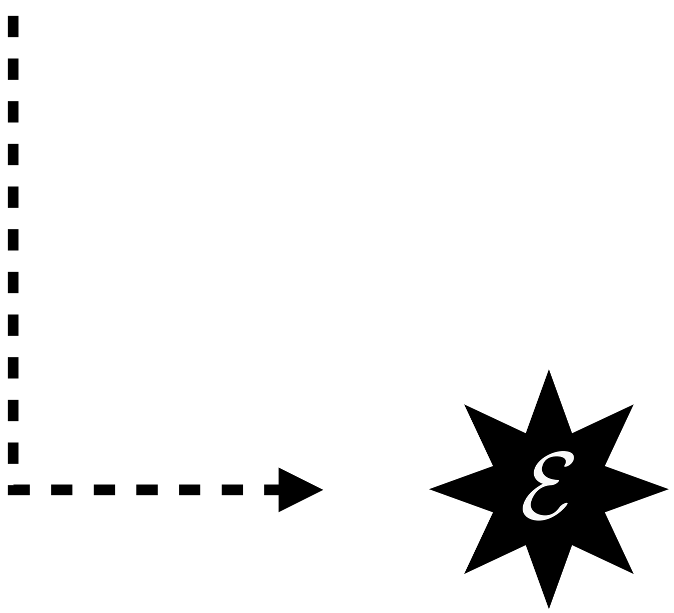


= Weighted Direction $$n_i^\mu = \frac{p_i^\mu}{E_i} = (1, \hat{n})^\mu$$

$$w_i = E_i$$

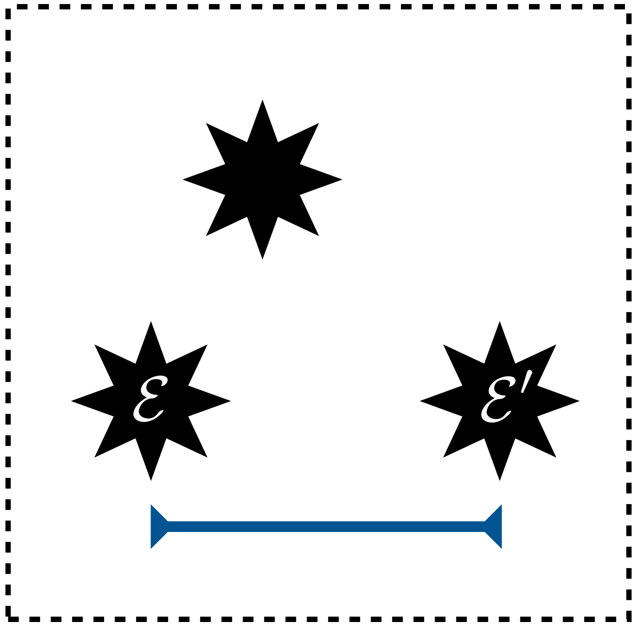
= Angular Distance $$\theta_{ij} = \sqrt{2 n_i^\mu n_{j\mu}}$$

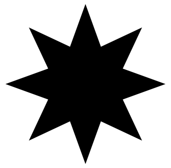
(for massless particles)



= Event

Event Space



 = Event

$$\mathcal{E}(\hat{n}) = \sum_i E_i \, \delta(\hat{n} - \hat{n}_i)$$

 = EMD

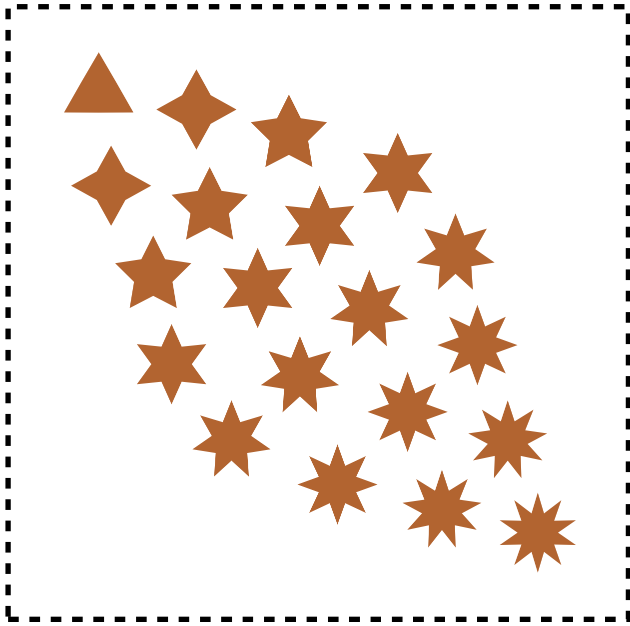
Energy
Mover's Distance

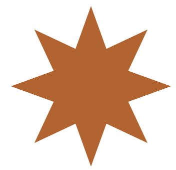
$$\mathrm{EMD}(\mathcal{E}, \mathcal{E}') = \min_{\{f\}} \sum_i \sum_j f_{ij} \theta_{ij}$$

(for equal total energy)

[Komiske, Metodiev, JDT, PRL 2019]

Event Space Distribution



 = Weighted Event

$$\mathcal{E}(\hat{n}) = \sum_i E_i \, \delta(\hat{n} - \hat{n}_i)$$

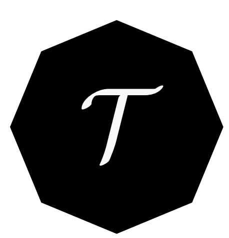
$$w_a = \sigma_a$$

 = EMD

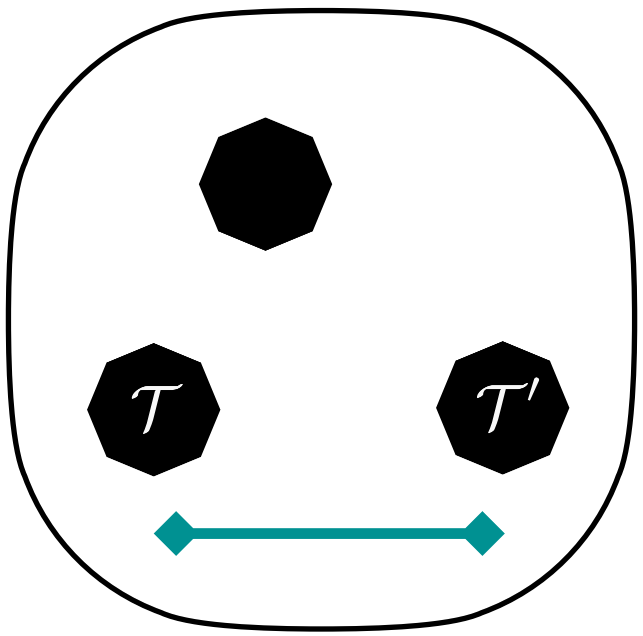
Energy
Mover's Distance

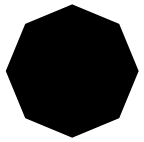
$$\mathrm{EMD}(\mathcal{E}, \mathcal{E}') = \min_{\{f\}} \sum_i \sum_j f_{ij} \theta_{ij}$$

(for equal total energy)

 = Theory

Theory Space



 = Theory

$$\mathcal{T}(\mathcal{E}) = \sum_a \sigma_a \, \delta(\mathcal{E} - \mathcal{E}_a)$$

 = ΣMD

Cross-Section
Mover's Distance

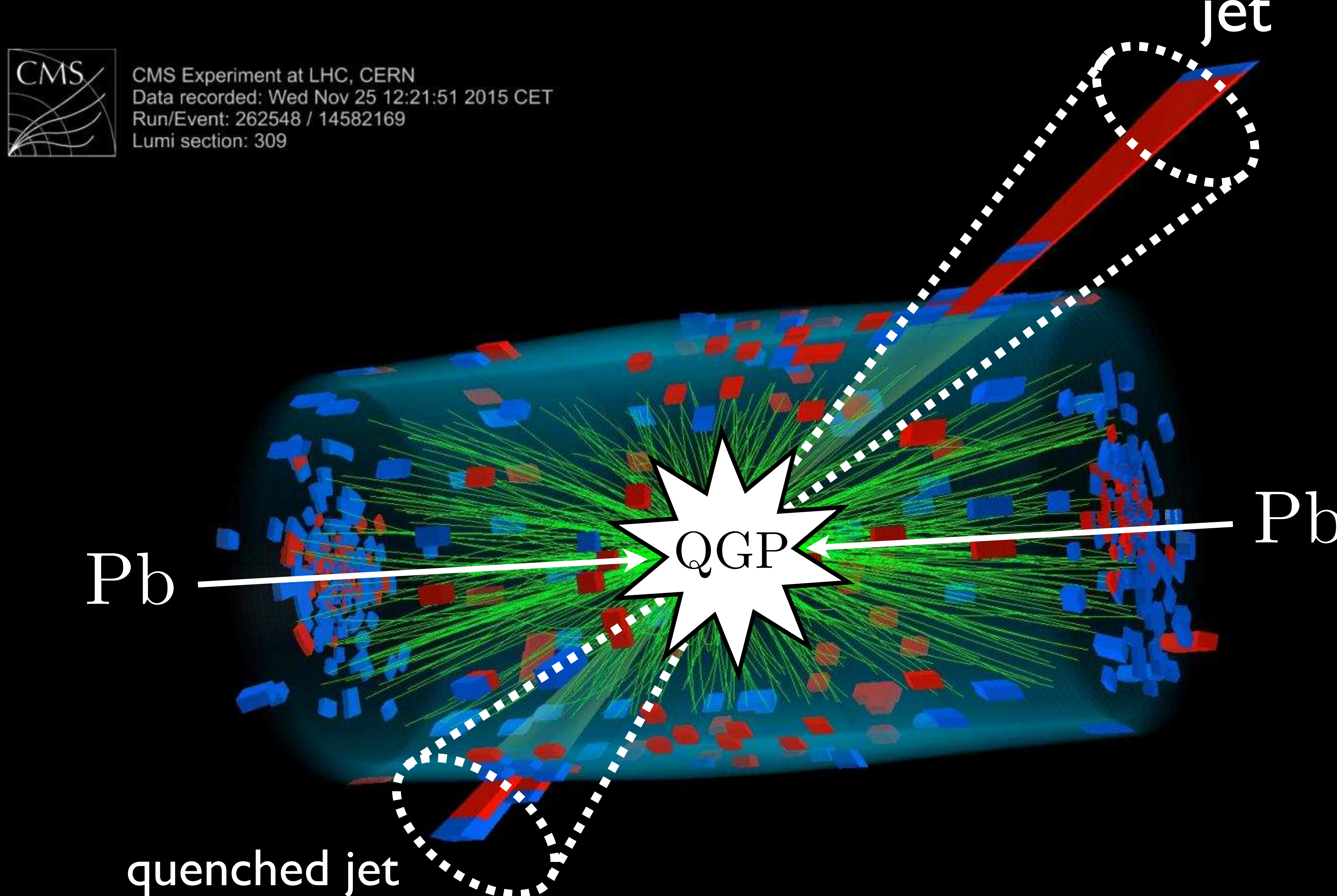
$$\Sigma\mathrm{MD}(\mathcal{T}, \mathcal{T}') = \min_{\{\mathcal{F}\}} \sum_a \sum_b \mathcal{F}_{ab} \mathrm{EMD}(\mathcal{E}_a, \mathcal{E}'_b)$$

(for equal total xsec)

A distance between theories!

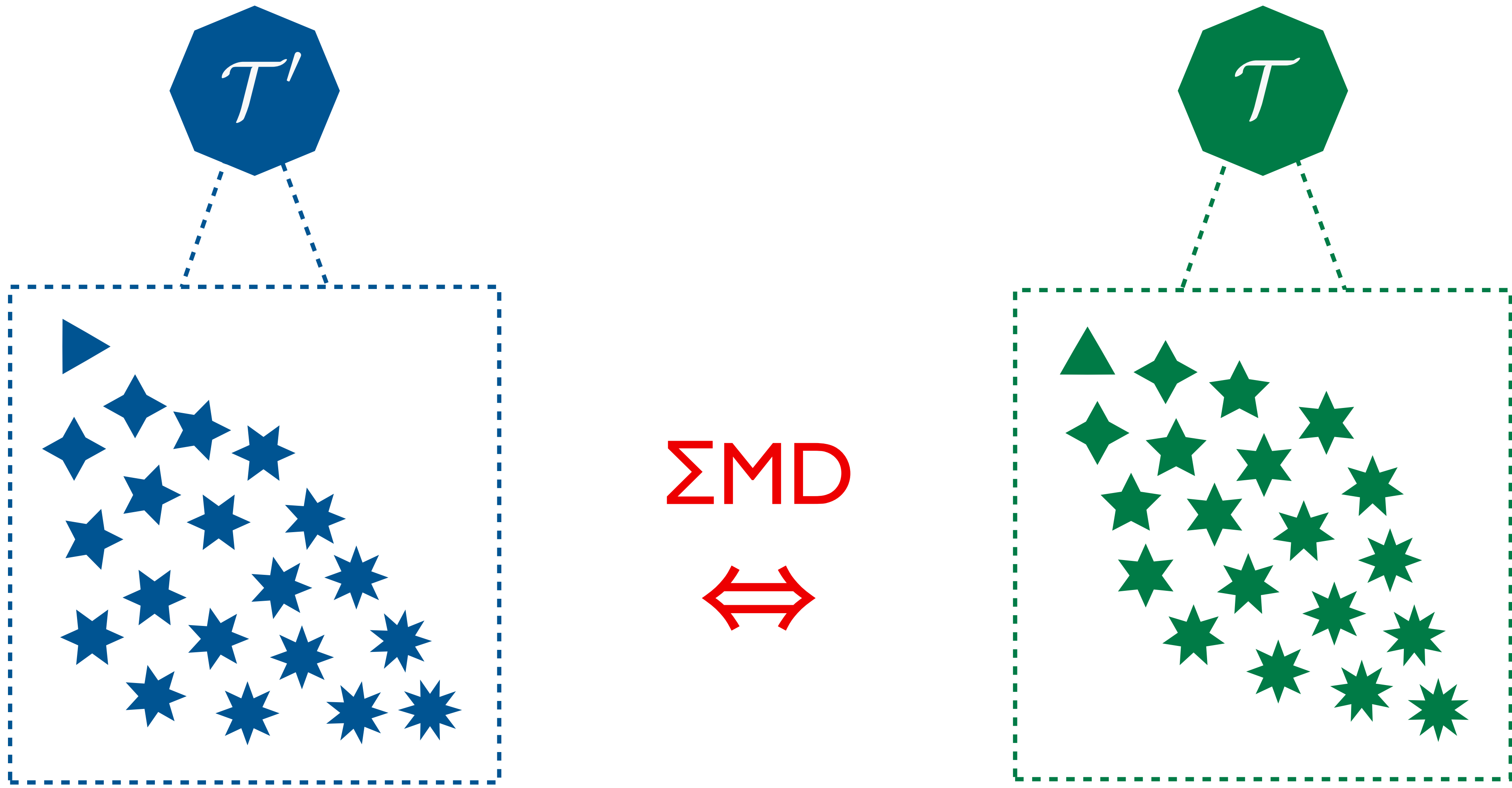
(e.g. EMD : N-jettiness :: ΣMD : k-eventiness)

[Komiske, Metodiev, JDT, JHEP 2020]

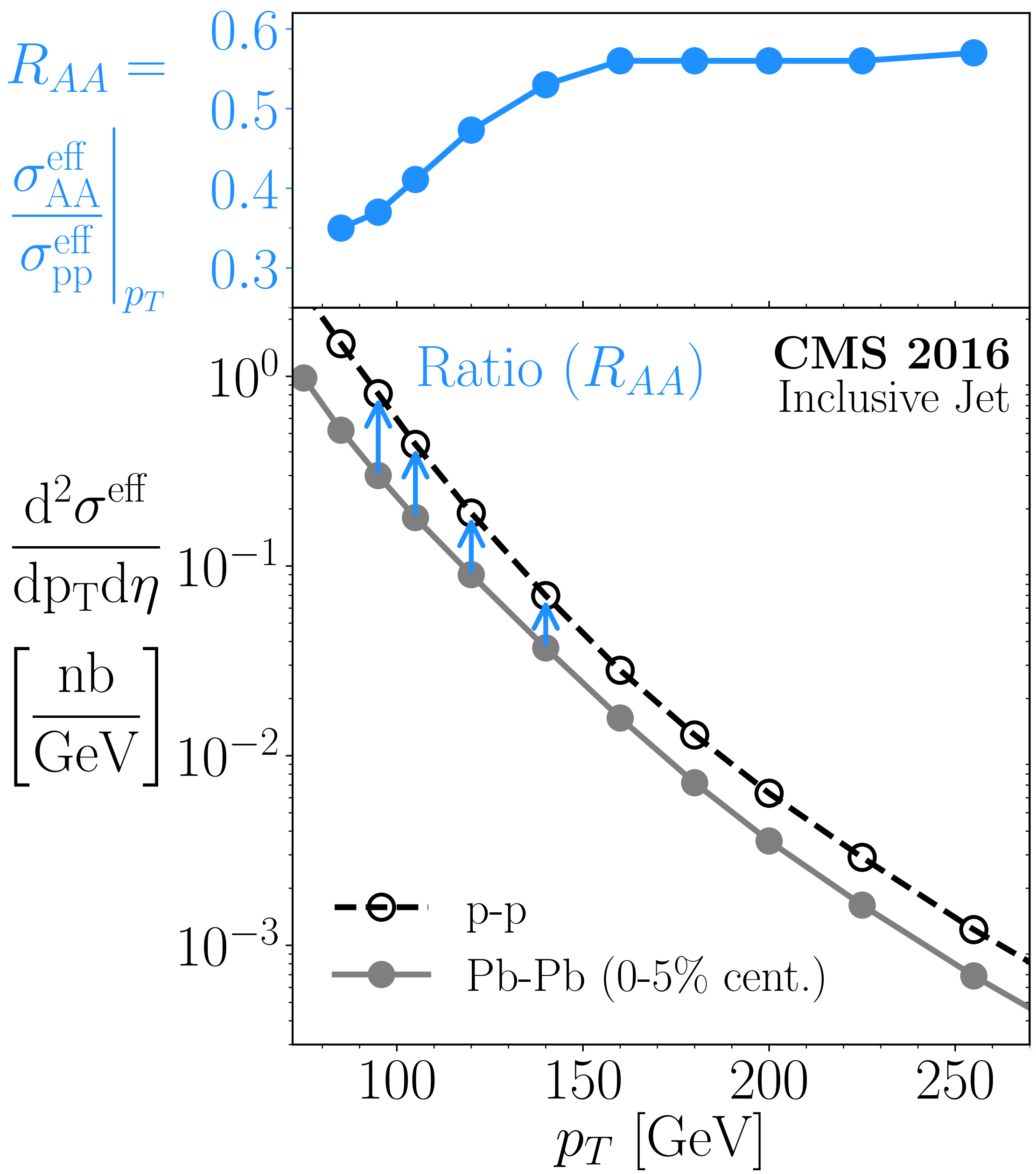

jet
CMS Experiment at LHC, CERN
Data recorded: Wed Nov 25 12:21:51 2015 CET
Run/Event: 262548 / 14582169
Lumi section: 309
Pb
QGP
Pb
quenched jet

Theory Prime:
In-Medium QCD

Theory:
Vacuum QCD



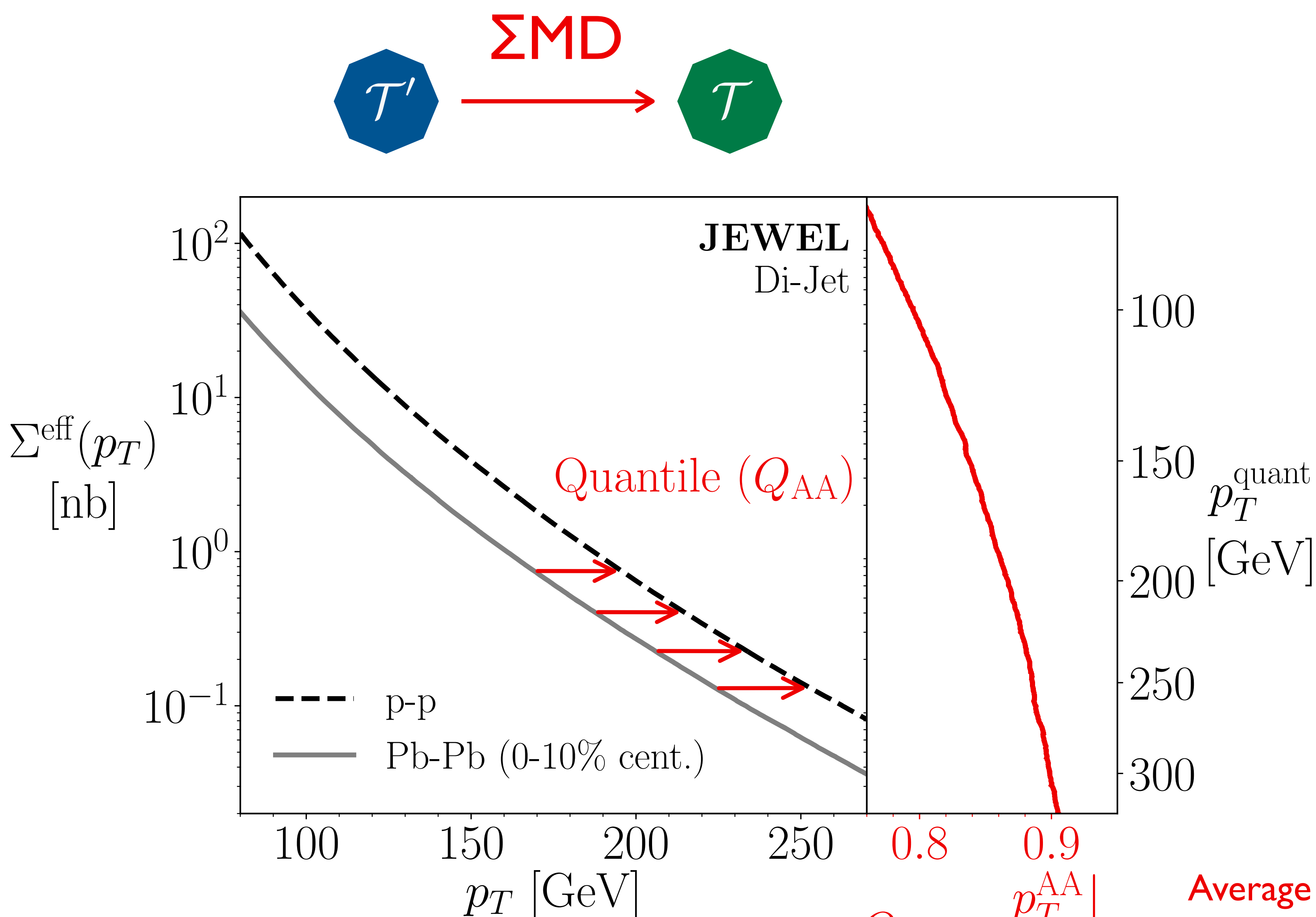
Optimal transportation plan defines mapping between in-medium jets and vacuum jets!



Average
σ loss
per p_T

Jet Quenching via Quantile Matching

Equivalent to following a geodesic in theory space (!)



$$Q_{\rm AA} = \left.\frac{p_T^{\rm AA}}{p_T^{\rm pp}}\right|_{\Sigma^{\rm eff}}$$

Average p_T loss per jet

[Brewer, Milhano, JDT, PRL 2019]

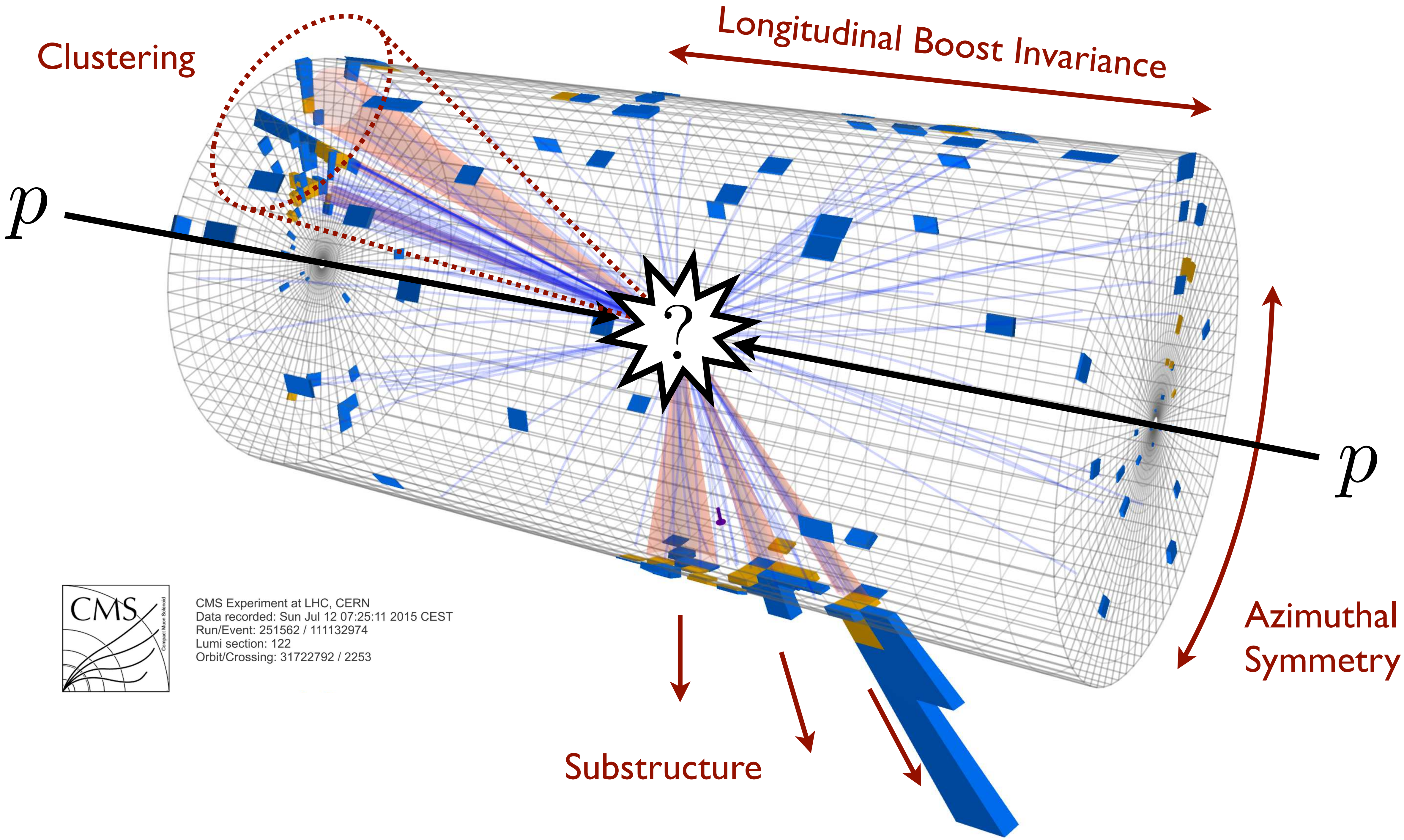
Backup Slides

AI²: Ab Initio Artificial Intelligence

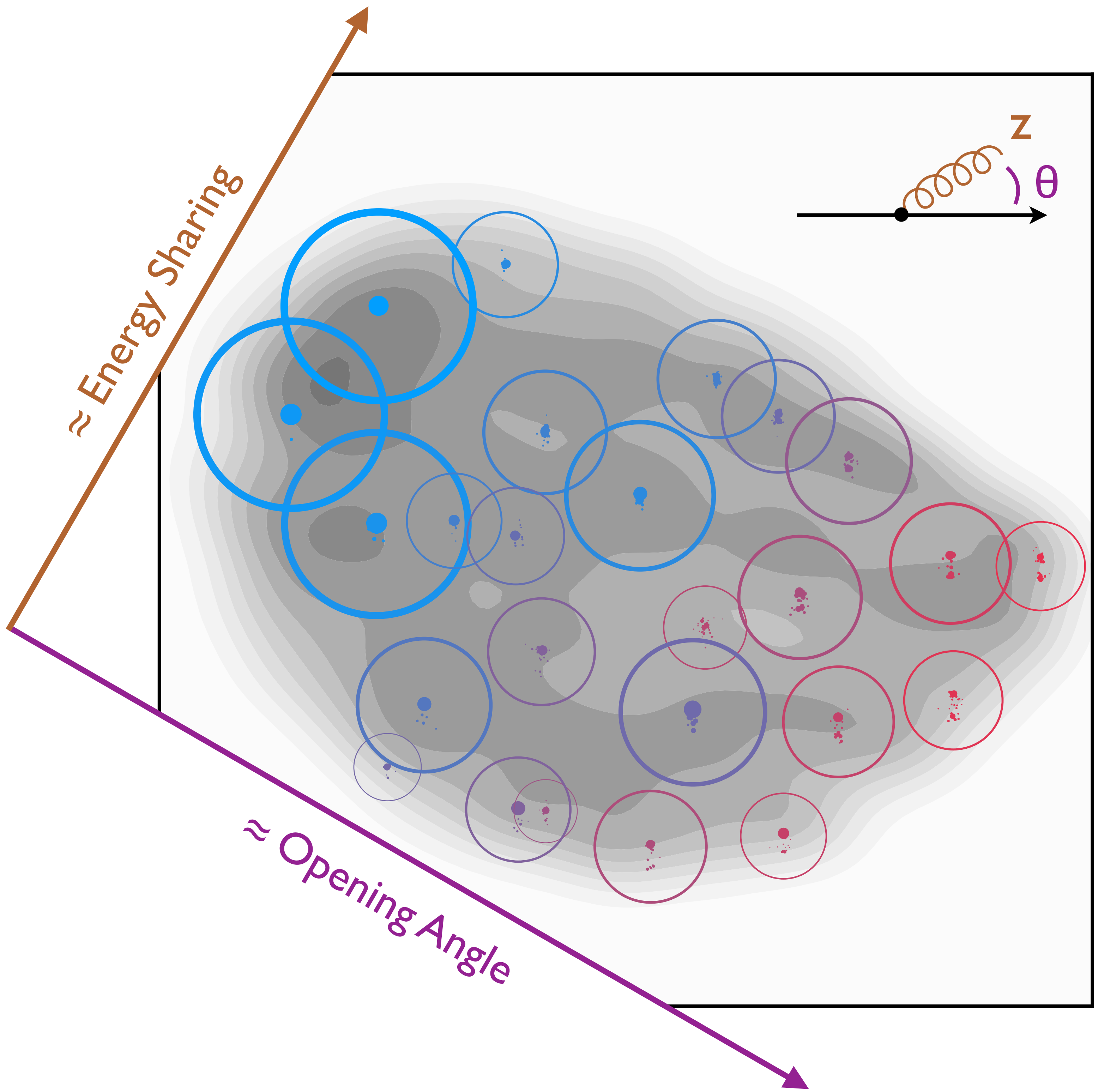
Machine learning that incorporates
first principles, best practices, and domain knowledge
from fundamental physics

Symmetries, conservation laws, scaling relations, limiting behaviors, locality, causality, unitarity, gauge invariance, entropy, least action, factorization, unit tests, exactness, systematic uncertainties, reproducibility, verifiability, …

The Manifest Geometry of One Collision



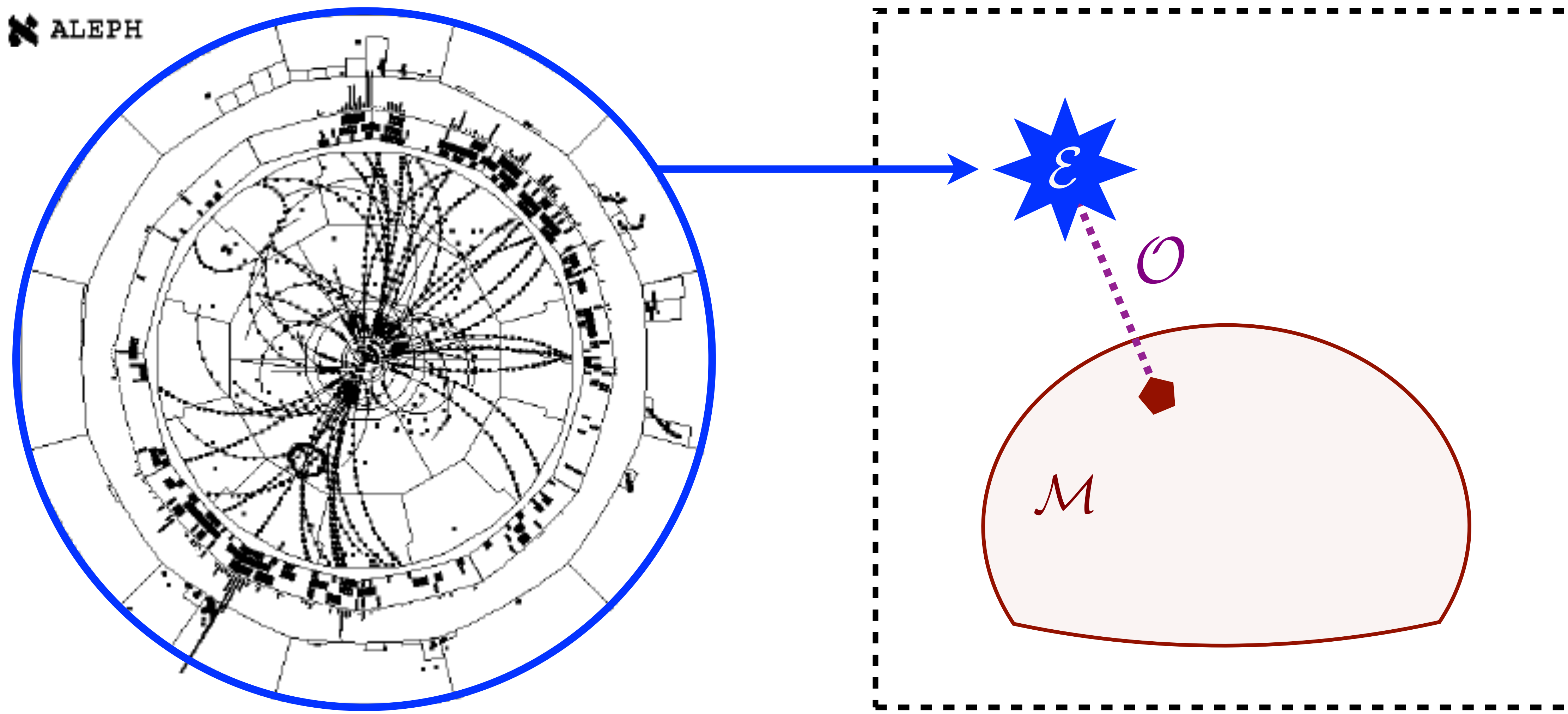
The Emergent Geometry of Many Collisions



[Komiske, Mastandrea, Metodiev, Naik, JDT, PRD 2020;
based on Komiske, Metodiev, JDT, PRL 2019; using EnergyFlow and CMS Open Data]

The Hidden Geometry of Particle Collisions

E.g. Classic QCD Event Shapes



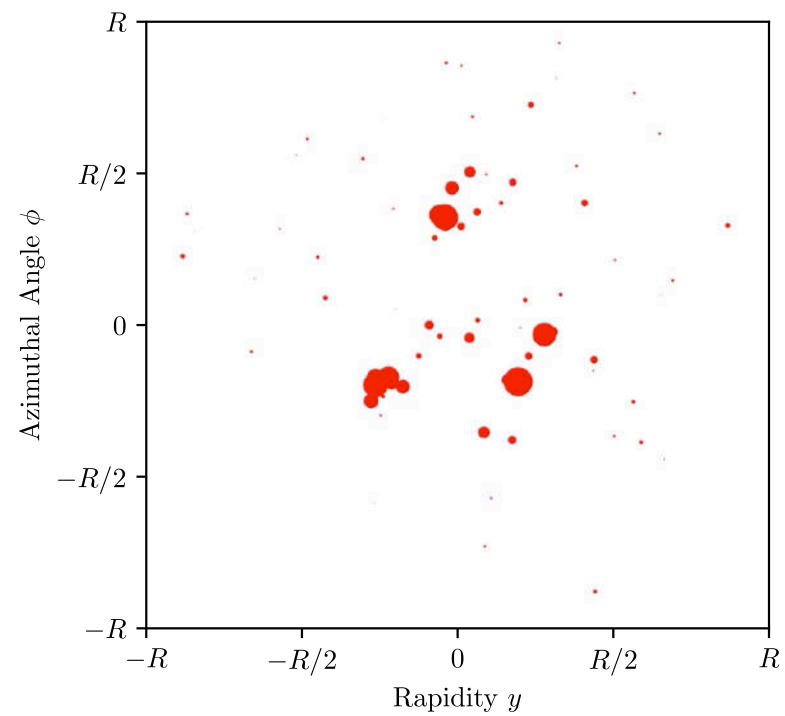
One Electron-Positron Event

Distance to a Manifold in Event Space

[Komiske, Metodiev, JDT, JHEP 2020]
[Brandt, Peyrou, Sosnowski, Wroblewski, PL 1964; Farhi, PRL 1977]

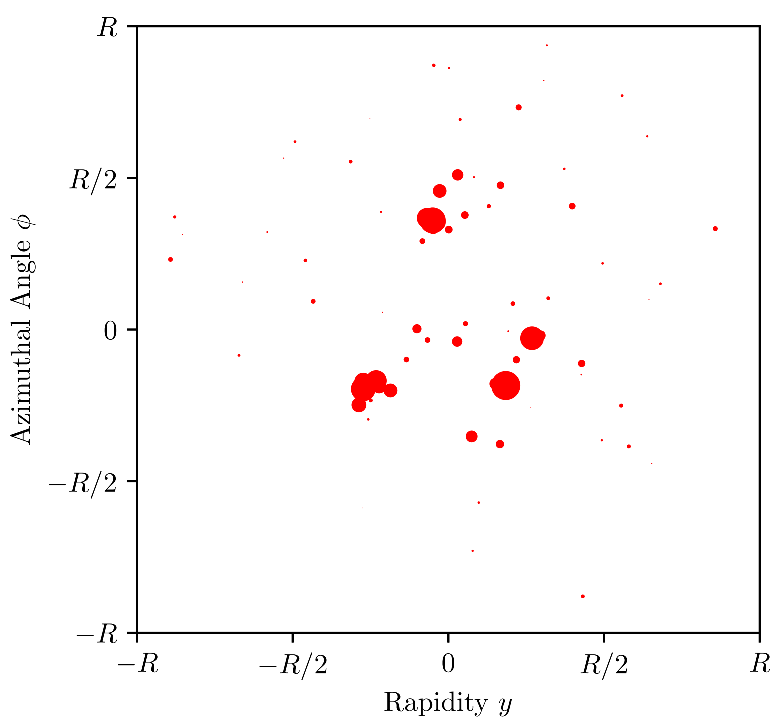
Similarity of Two Energy Flows?

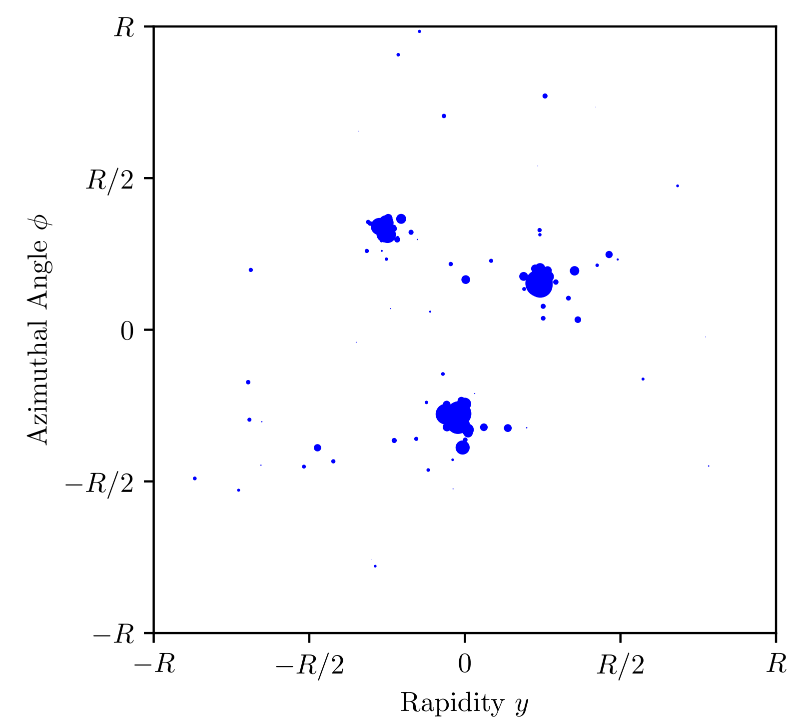
$$\mathcal{E}(\hat{n}) = \sum_i E_i \, \delta(\hat{n} - \hat{n}_i)$$



Optimal Transport:

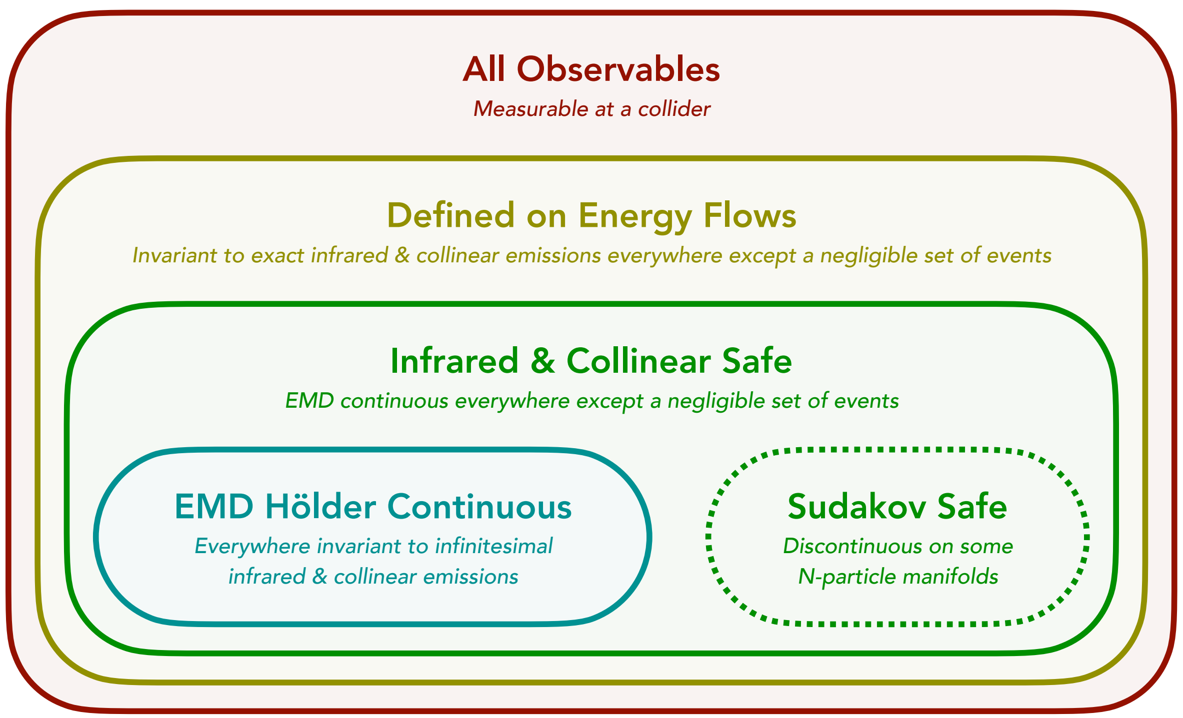
Earth Mover's Distance
a.k.a. 1-Wasserstein metric





[Komiske, Metodiev, JDT, PRL 2019; code at Komiske, Metodiev, JDT, energyflow.network]

Observable Taxonomy



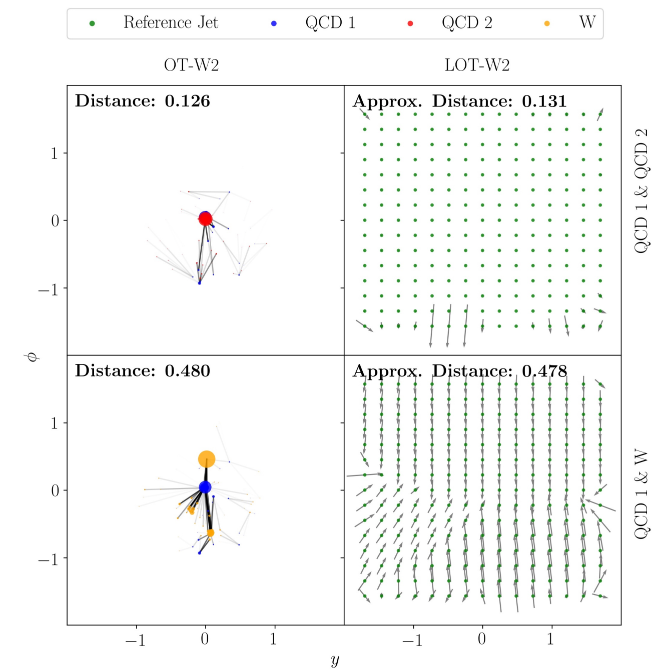
All Observables	Comments
Multiplicity ($\sum_i 1$)	IR unsafe and C unsafe
Momentum Dispersion [65] ($\sum_i E_i^2$)	IR safe but C unsafe
Sphericity Tensor [66] ($\sum_i p_i^\mu p_i^\nu$)	IR safe but C unsafe
Number of Non-Zero Calorimeter Deposits	C safe but IR unsafe
Defined on Energy Flows	
Pseudo-Multiplicity ($\min\{N \mid \mathcal{T}_N = 0\}$)	Robust to exact IR or C emissions
Infrared & Collinear Safe	
Jet Energy ($\sum_i E_i$)	Disc. at jet boundary
Heavy Jet Mass [67]	Disc. at hemisphere boundary
Soft-Dropped Jet Mass [38, 68]	Disc. at grooming threshold
Calorimeter Activity [69] (N_{95})	Disc. at cell boundary
Sudakov Safe	
Groomed Momentum Fraction [39] (z_g)	Disc. on 1-particle manifold
Jet Angularity Ratios [37]	Disc. on 1-particle manifold
N-subjettiness Ratios [47, 48] (τ_{N+1}/τ_N)	Disc. on N-particle manifold
V parameter [36] (Eq. (2.11))	Hölder disc. on 3-particle manifold
EMD Hölder Continuous Everywhere	
Thrust [40, 41]	
Spherocity [42]	
Angularities [70]	
N-jettiness [44] ($\mathcal{T}_N$)	
C parameter [71–74]	Resummation beneficial at $C = \frac{3}{4}$
Linear Sphericity [72] ($\sum_i E_i n_i^\mu n_i^\nu$)	
Energy Correlators [36, 75–77]	
Energy Flow Polynomials [15, 17]	

[Komiske, Metodiev, JDT, JHEP 2020; cf. Sterman, PRD 1979; Banfi, Salam, Zanderighi, JHEP 2005; Larkoski, Marzani, JDT, PRD 2015]

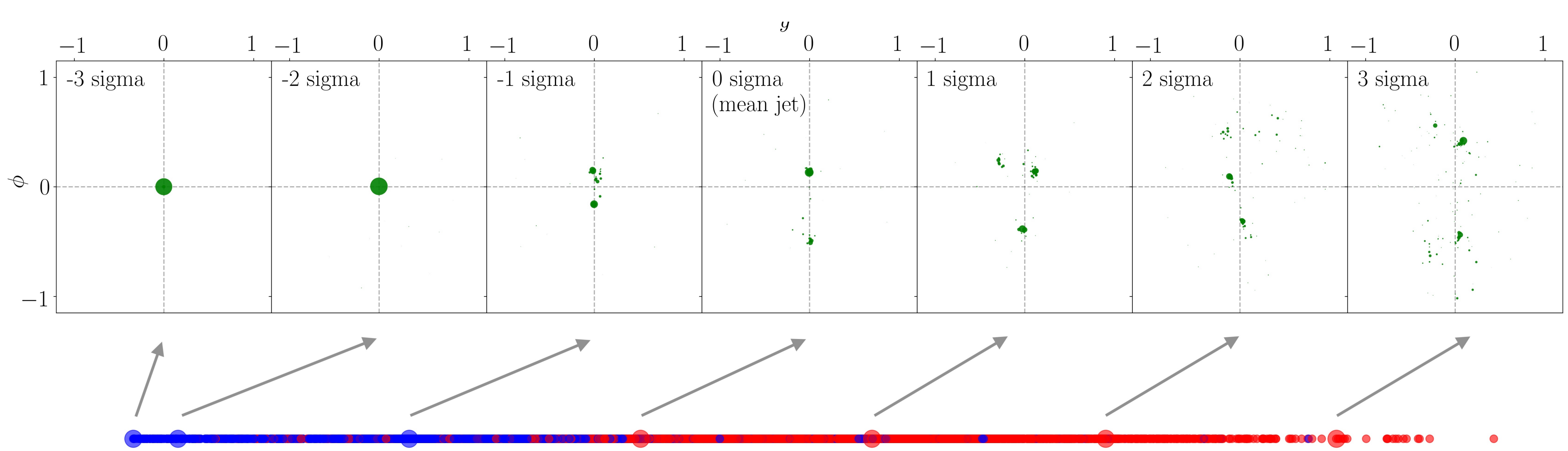
Linearized Optimal Transport

With the help of a reference event, transportation distances* can be efficiently mapped to Euclidean distances

* assuming the 2-Wasserstein measure



Enables coordinate-based techniques like Linear Discriminate Analysis



[Cai, Cheng, Craig, Craig, PRD 2020]