Bayesian Model Calibration

- Let $z(t, \mathbf{x_i})$ denote an experimental measurement from the i^{th} experiment
- Similarly, let $y(t, \mathbf{x_i}, \mathbf{u})$ denote a simulation of the i^{th} experiment at the with input parameters $\mathbf{u}$
- An approach to Bayesian model calibration with functional response specifies

$$z(t, \mathbf{x_i}) = y(t, \mathbf{x_i}, \theta) + \delta(t, \mathbf{x_i}) + \epsilon_i(t, \mathbf{x_i}), \quad \epsilon(t, \mathbf{x_i}) \sim \mathcal{N}(0, \sigma_\epsilon^2)$$

- Where δ is model discrepancy term and ϵ represents all other error
- This model will suffer from the aforementioned problems with phase variability

Elastic Bayesian Model Calibration

Decompose observation into aligned functions and warping functions

$$z(t, \mathbf{x_i}) = \tilde{z}(t, \mathbf{x_i}) \circ \gamma_{z \to z}(t, \mathbf{x_i})$$

and decompose the simulations

$$y(t, \mathbf{x_j}, \mathbf{u_j}) = \tilde{y}(t, \mathbf{x_j}, \mathbf{u_j}) \circ \gamma_{y \to z}(t, \mathbf{x_j}, \mathbf{u_j})$$

To facilitate modeling, we transform the warping functions into shooting vector space with

$$\mathbf{v}_{z \to z}(\mathbf{x_i}) = \exp_{\psi}^{-1}\left(\sqrt{\dot{\gamma}_{z \to z}(\mathbf{x_i})}\right)$$

$$\mathbf{v}_{y \to z}(\mathbf{x_j}, \mathbf{u_j}) = \exp_{\psi}^{-1}\left(\sqrt{\dot{\gamma}_{y \to z}(\mathbf{x_j}, \mathbf{u_j})}\right)$$