Elastic Distance (Fisher-Rao)

Define the Square Root Velocity Function

$$q : [0,1] \to \mathbb{R}^1, \; q(t) = sign(\dot{f}(t))\sqrt{|\dot{f}(t)|}$$

Fisher Rao Distance is $\mathbb{L}^2$ in SRVF space
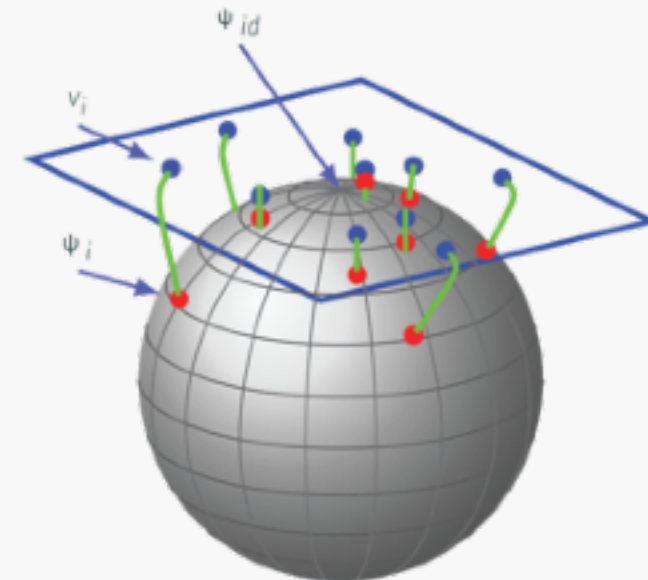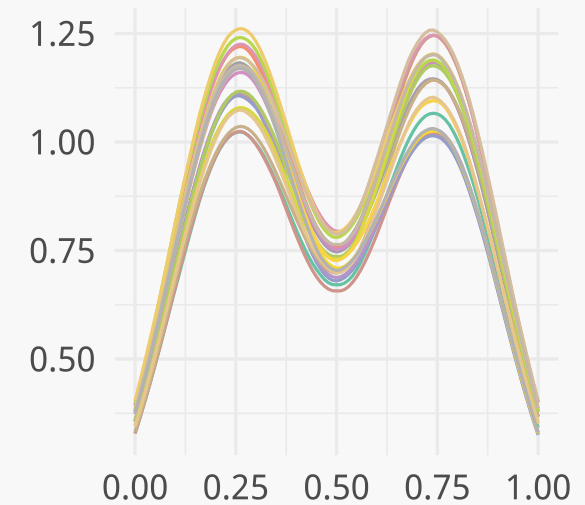
$$d_a(f_1, f_2) = \inf_{\gamma} \|(q_1 \circ \gamma)\sqrt{\dot{\gamma}} - q_2\|$$

Distance is a proper distance

- symmetric, isometric, triangle inequality

Can compute distance on warping functions (how much alignment)

$$d_p(\gamma) = \arccos\left(\int_0^1 \sqrt{\dot{\gamma}}\, dt\right)$$

Analysis of Γ

Γ is a nonlinear manifold and it is infinite dimensional

Represent an element of $\gamma \in \Gamma$ by the square-root of its derivative $\psi = \sqrt{\dot{\gamma}}$

Important advantage of this transformation is the set of all such ψ's is a Hilbert Sphere $\mathbb{S}_\infty$