

Functional Data Analysis

Let f be a real valued-function with the domain $[0,1]$, can be extended to any domain

- Only functions that are absolutely continuous on $[0,1]$ will be considered

Let Γ be the group of all warping functions

$$\Gamma = \{\gamma : [0,1] \rightarrow [0,1] \mid \gamma(0) = 0, \gamma(1) = 1, \gamma \text{ is a diffeo}\}$$

It acts on the function space by composition

$$(f, \gamma) = f \circ \gamma$$

It is common to use the following **objective function** for alignment

$$\min_{\gamma \in \Gamma} \|f_1 \circ \gamma - f_2\|$$

Note: It is **not a distance** function since it is not symmetric.

Elastic Distance (Fisher-Rao)

Define the Square Root Velocity Function

$$q : [0,1] \rightarrow \mathbb{R}^1, q(t) = \text{sign}(\dot{f}(t))\sqrt{|\dot{f}(t)|}$$

Fisher Rao Distance is \mathbb{L}^2 in SRVF space

$$d_a(f_1, f_2) = \inf_{\gamma} \|(q_1 \circ \gamma)\sqrt{\dot{\gamma}} - q_2\|$$

Distance is a **proper distance**

- symmetric, isometric, triangle inequality

Can compute distance on warping functions (how much alignment)

$$d_p(\gamma) = \arccos \left(\int_0^1 \sqrt{\dot{\gamma}} dt \right)$$

