Elastic Bayesian Model Calibration

Decompose observation into aligned functions and warping functions

$$z(t, \mathbf{x_i}) = \tilde{z}(t, \mathbf{x_i}) \circ \gamma_{z \to z}(t, \mathbf{x_i})$$

and decompose the simulations

$$y(t, \mathbf{x_j}, \mathbf{u_j}) = \tilde{y}(t, \mathbf{x_j}, \mathbf{u_j}) \circ \gamma_{y \to z}(t, \mathbf{x_j}, \mathbf{u_j})$$

To facilitate modeling, we transform the warping functions into shooting vector space with

$$\mathbf{v}_{z \to z}(\mathbf{x_i}) = \exp_{\psi}^{-1}\left(\sqrt{\dot{\gamma}_{z \to z}(\mathbf{x_i})}\right)$$

$$\mathbf{v}_{y \to z}(\mathbf{x_j}, \mathbf{u_j}) = \exp_{\psi}^{-1}\left(\sqrt{\dot{\gamma}_{y \to z}(\mathbf{x_j}, \mathbf{u_j})}\right)$$

Elastic Bayesian Model Calibration

Calibrate the aligned data and shooting vectors using the following model

$$\tilde{z}(t, \mathbf{x_i}) = \tilde{y}(t, \mathbf{x_i}, \theta) + \delta_{\tilde{y}}(t, \mathbf{x_i}) + \epsilon_{\tilde{z}}(t, \mathbf{x_i}), \quad \epsilon_{\tilde{z}}(t, \mathbf{x_i}) \sim \mathcal{N}(0, \sigma_{\tilde{z}}^2)$$

$$\mathbf{v}_{z \to z}(\mathbf{x_i}) = \mathbf{v}_{y \to z}(\mathbf{x_i}, \theta) + \delta_{\mathbf{v}}(\mathbf{x_i}) + \epsilon_{\mathbf{v}}(\mathbf{x_i}), \quad \epsilon_{\mathbf{v}}(\mathbf{x_i}) \sim \mathcal{N}(0, \sigma_v^2 \mathbf{I})$$

Note: The shooting vector will be identity if the data is aligned to the observation (experiment)

Then if θ is calibrated correctly the shooting vectors will be identity