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X-Wing Strategy

This strategy is looking at single numbers in rows and columns. It should be easier to spot in a game as we can concentrate on just one number at a time. The rule is

When there are

only two possible cells for a value in each of two different rows,
and these candidates lie also in the same columns,

then all other candidates for this value in the columns can be eliminated.

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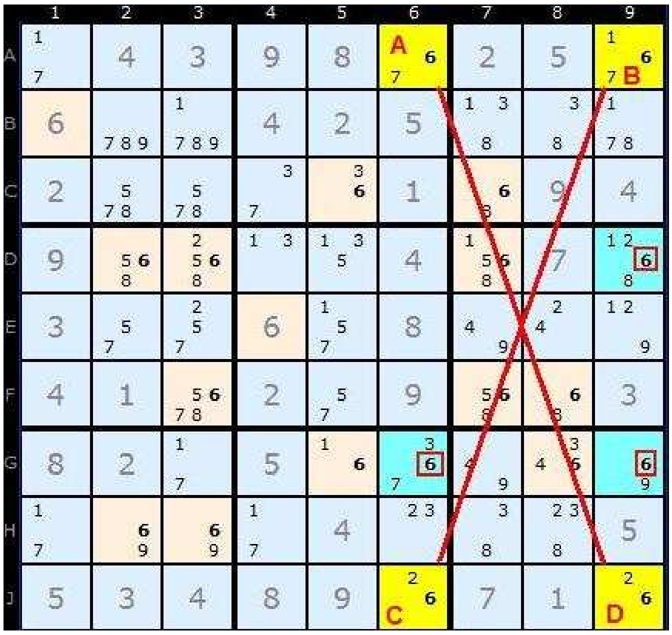
The reverse is also true for 2 columns with 2 common rows.



The above picture shows a classic x-wing, this example being based on the number **six**. The X is formed from the diagonal correspondence of squares marked A, B, C and D. What's special about them? Well, A and B are a locked pair of **6**'s. So is C and D. They are locked because they are the only **6**'s in the first and last rows. We know therefore that if A turns out to be a **6** then B cannot be a **6**, and vice versa. Likewise if **C** turns out to be a **6** then **D** cannot be, and vice versa.

What is interesting is the **6**'s present in the two columns 6 and 9 directly between A and C and B and D. These have been highlighted with yellow boxes. Think about the example this way. A, B, C and D form a rectangle. If A turns out to be a **6** then it rules out a **6** at C as well as B. Because A and CD are 'locked' then D must be a **6** if A is. Or vice versa. So a 6 MUST be present at AD or BC. If this is the case then any other **6**'s along the edge of our rectangle are redundant.

We can remove the **6**'s marked in the cyan squares. This is good news because this leaves only a **9** at **G9** and we can complete.

This strategy works in the other direction as well. If we had two pairs in two columns and those four numbers shared two rows, then we can eliminate any other occurrences of those numbers on the same rows.

Generalising X-Wing

X-Wing is not restricted to rows and columns. We can also extend the idea to boxes as well. If we generalise the rule above we get:

When there are

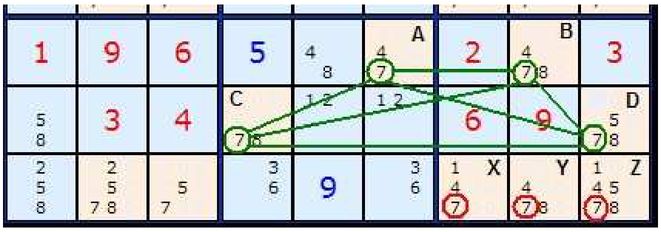
> *only 2 candidates for a value, in each of 2 different units of the same kind,*
> *and these candidates lie also on 2 other units of the same kind,*

then all other candidates for that value can be eliminated from the latter two units.

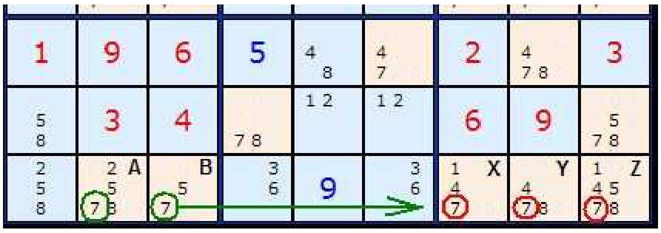
Now we have 6 combinations:

Starting from 2 rows and eliminating in 2 columns	Classic X-Wing
Starting from 2 columns and eliminating in 2 rows	Classic X-Wing
Starting from 2 boxes and eliminating in 2 rows	Same effect as **line/box reduction**
Starting from 2 boxes and eliminating in 2 columns	Same effect as **line/box reduction**
Starting from 2 rows and eliminating in 2 boxes	Same effect as **pointing pairs**
Starting from 2 columns and eliminating in 2 boxes	Same effect as **pointing pairs**

Here is an example of combination 5. Starting from 2 rows and eliminating in 2 boxes, in this case the last two boxes in the Sudoku. The rows are 7 and 8 and they each have two **7**s. Our x-Wing is now a trapeziod but the logic is the same. We can be certain that 7 can be elminated at X, Y and Z.



But HOLD UP one moment. There is a simpler strategy that does the same job!



A and B above are a pointing pair. This removes the same 7s in the same place. Combination 6 is also the complement of a pointing pair. Combinations 3 and 4 are also complements of the Line/Box Reduction. Our generalization of X-Wing to boxes hasn't profited us at all. We learn that

X-Wings containing boxes are the inverse of the *Intersection Removal* strategies

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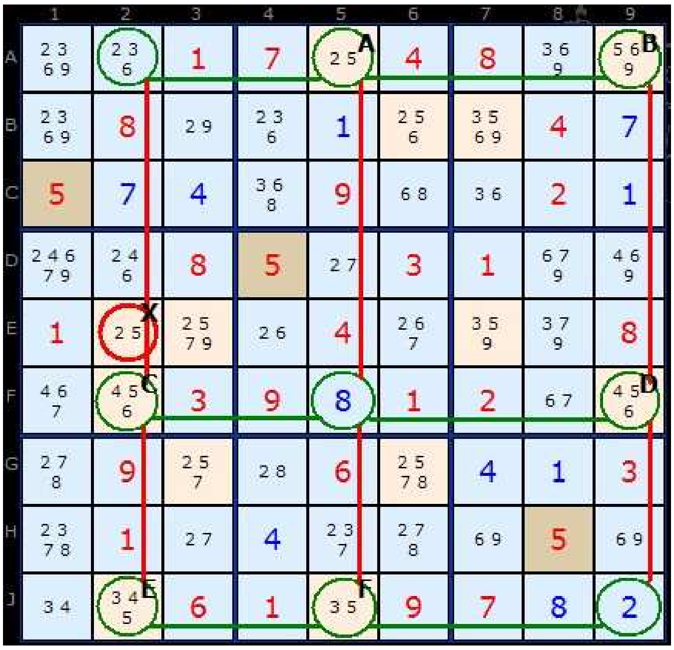
Sword-Fish Strategy

With X-Wing we looked at a rectangle formed by four numbers at the corners. This allowed us to exclude other occurrences of that number in either the row or column. We can extend this pattern to nine cells connected by locked pairs. In the example below (concentrating on the number 5) we have three sets of locked pairs at AB, CD and EF. They are all horizontal pairs but they also lock each vertically in a staircase fashion (I guess this inspired the name).



The vertical pairing is between AF, BD and CE. Now, in this example we can clearly see that the green horizontal lines connect pairs of 5. Because 5 is also locked vertically the red lines represent columns where if a 5 is not on our grid of nice nodes it can be excluded. There is one such 5 on cell X (E2).

Another way of looking at it is to consider any 5 on the Sword-Fish grid. Pretending for a moment its a real 5 the others in the row and column are repressed. What we're left with is an X-Wing. X-Wing logic then applies to exclude the 5s it can see.

Jelly-Fish Strategy

Jelly-Fish extends Swordfish one further row and column. We are looking for either

> four rows such that, *in total*, four cells are occupied in the row by a candidate number; or
> four columns such that, *in total*, four cells are occupied in the column by a candidate number

If this configuration is found then we can look in the opposite direction (if by row then down the column, if by column then across the row. If any candidates are found they can be eliminated. After the elimination both conditions above will hold.

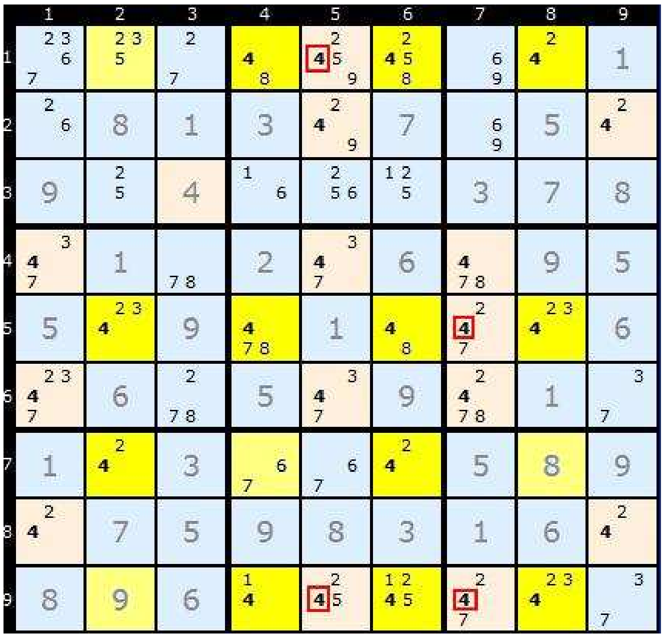


Figure 1

Load This Example **From the start** or **at the required point**

How does it work? Pick any yellow cell in the example above that contains a 4. Keeping an eye on it. Pretend the solution actually is a 4. All others 4s in the row and columns are repressed. What we're left with is a Sword-Fish. The Sword-Fish logic then applies. Pick any 4 in the Sword-Fish and it reduces to an X-Wing. Since any combination of 4s on the grid are possible there is no room for 4s outside the grid - that align on the grid rows and columns.

Squirm-Bag

Jelly-Fish can be extended to five rows and columns. It's been named Squrim-Bag. I know of no necessary example in a solving sequence and thanks to Florian Fischer for explaining why.

Let's take the Jelly-Fish example above. Columns 2,4,6,8 are such that the 4's are restrained to rows 1,5,7,9. The 4 in each column must be placed in one of these rows, and therefore they must occupy all of them and within the yellow cells. So the 4's in rows 1,5,7,9 cannot appear in columns 1,3,5,7,9.

What are we saying? 4's in columns 2,4,6,8 are restricted to rows 1,5,7,9. That is equivalent to saying that there is no 4 in the intersection of columns 2,4,6,8 and rows 2,3,4,6,8. And that is equivalent to say that 4's in the five rows 2,3,4,6,8 are restricted to the five columns 1,3,5,7,9. And that describes a Squirm-Bag. Indeed, you can conclude that the five 4's must occupy all columns 1,3,5,7,9 within rows 2,3,4,6,8 and therefore the 4's in columns 1,3,5,7,9 cannot appear outside of these rows, that is in rows 1,5,7,9.

Generally, a size-N fish has a size-(9-N) fish counterpart. You just need to complement the set of rows and the set of columns and reason the other way round (column-wise or row-wise). Both are equivalent. Therefore you don't need to go further than size 4 Jelly-Fish in a standard 9x9 Sudoku.

Multivalue X-Wing Strategy

I've named this strategy Multivalue because we're dealing with several candidate values but the formation is exactly as an X-Wing, infact it also follows the generalised x-wing as described above.

Take a look at this rectangular formation made from the yellow and brown cells. Connecting the two yellow cells is a conjugate pair of 6, the only two sixes in the row. In the other row connecting the two brown cells is a conjugate pair of 5. What connects the cells in the columns are the additional candidates, in this case 1 in column 1 and 9 in column 9. Note that there are additional 1's and 9's in these columns. These are the candidates we can eliminate and they are highighted in green cells.

The logic goes as follows: 6 must occur in one of the two yellow cells and the 5 must occur in one of the brown cells. No doubt about that. But both 6 and 5 cannot occur in the same column. Lets pretend they do, say 6 and 5 in column 1. That would leave 9 as the only solution in two cells in column 9. Can't have that. So which ever way round 6 is 5 will be in the opposite column. This forces the 1 and 9 to fill the remaining two corners. If 1 and 9 are guaranteed to be in either a yellow or a brown cell apiece then we can't have any more 1s and 9s in those columns. Hense the eliminations.

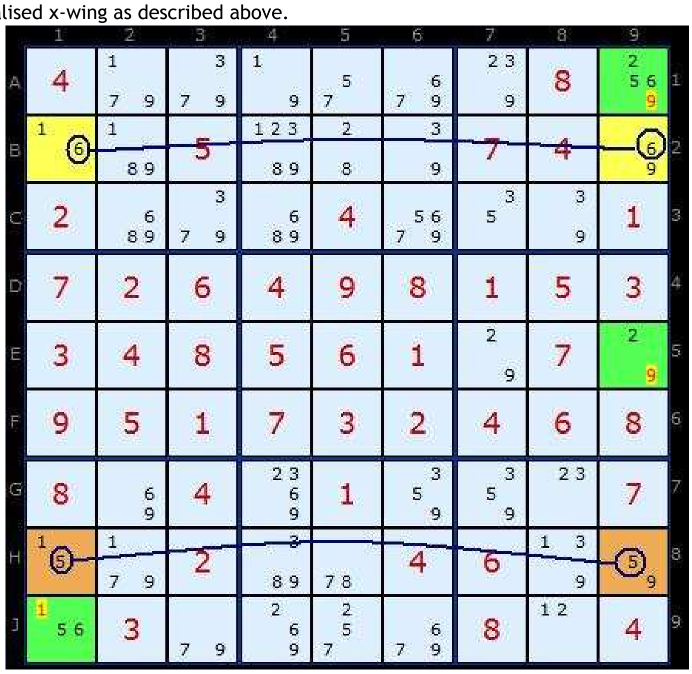


Figure 1

Load This Example **From the start** or **at the required point**

The generalised X-Wing theory says that we can have a distorted X-Wing starting from 2 boxes and eliminating in 2 rows or 2 columns. This next example does just that. We have a strong link between the yellow cells (B7 and H7) using 5. And another string link between brown cells (A9 and J9). Since the top pair share a box and the bottom pair also share a box we don't need exact row alignment.

Using the arguement above we know that one 5 or 3 will occur in B7 or A9 focing the other cell in the top right box to be a 2. We don't know which yet, but of those two cells will be a 2 so all the others in the box can go.

Likewise, a 5 or a 3 will appear one of the cells int the bottom box, H7 or A9. That forces 4 to be the solution to that pair - we just don't know which way round yet. The 4 in H8 can go.

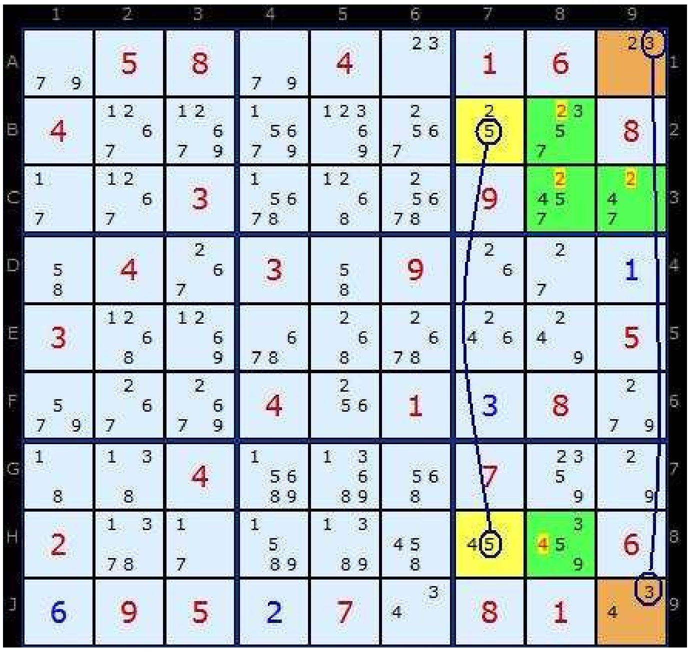


Figure 2

Load This Example **From the start** or **at the required point**

Eliminations such as these can be achieved using Nice Loops and other very advanced strategies but this is well worth looking out for separetely since its both easier to spot and extends the elegance of the familiar X-Wing.

Turbo-Fish Strategy

Documentation coming soon

Finned X-Wing Strategy

Documentation coming soon

Other sources: http://www.setbb.com/sudoku/viewtopic.php?t=750 "Hunting finned X-Wing and other fishy stuff
"

Finned Sashimi X-Wing Strategy

Documentation coming soon

See also **Basic Strategies** and **Advanced Strategies**

If anyone wishes to comment, correct or contribute to these pages please feel free to contact me at andrew@scanraid.com. I'm always interested to hear from other sudoku fanatics.

Andrew Stuart