

[Main Page - Back](#)

Alternating Inference Chains

From sudokuwiki.com, the puzzle solver's site

2		
	3	6
5		7

If there is a super-theory of sudoku, or a strategy which encapsulates much of what we've discussed so far, it has to be Alternating Inference Chains (AICs). It's very useful to split out chain-like strategies into X-Wings, XY-Chains, Forcing Chains, XYZ-Wings, X-Cycles, Nice Loops and so on, since they have special characteristics which make them spottable. But it turns out they are all part of a large extended family.

The chief characteristic of all chains is that they start on a candidate which has a strong inference to another candidate, which then has a weak inference to the next candidate, and so on, alternating until the start point is reached. This may not have been apparent with XY-Chains, for example, which were not described in this way, but that was because of their special spottable characteristics. We'll re-cast these previous examples as AICs later on. But AICs can often do tricks that none of the other strategies can hope to replicate, which means that we can use them to generalize and expand our solving armoury.

As we saw in the previous chapter, alternation is just what X-Cycles are about. However, you'll remember that X-Cycles are applied only to a single candidate number. AICs, on the other hand, take everything from an X-Cycle and extend the logic to as many different candidate numbers as necessary.

AICs ask the question "How many ways are there to make a strong or a weak link?" If there is more than one way, we can join them up in an alternating manner and make deductions leading to eliminations. Let's look back on the previous chain-like strategies and note the following:

- We can link two candidates of the same value in a unit - this is called "bi-location".
- We can link two different candidates in the same cell - this is called "bi-value".

There is also a third way ([Almost Locked Sets](#), covered on another page), but for now let's keep it simple and stick to these two dimensions - links between cells and within cells.

But what about grouped nodes used in X-Cycles? They seem to be a mixture of both. To gather grouped nodes under one of the two types, we need to drop "candidate" and think of a "candidate premise". The candidate premise for a single digit X in a cell is telling us that "X is true for this cell". A grouped node is simply a candidate premise that "X is true in cells A, B or C".

We can form a link to or from a grouped node on the same digit (bi-location) or within one or more of the cells in the node to a different digit (bi-value).

More complicated candidate premises are possible, such as “candidates X, Y and Z are all true in cells A, B and C” - which is just what we need for a WXYZ-Wing. Mostly, a candidate premise will refer to a single candidate, certainly in the first part of this chapter.

Once we know how to construct these loops, we can start making deductions. Let’s look at the example on the right.

AIC Rule 1, 1 taken off J9 - chain ends: A9 J3

AIC on 1 (Alternating Inference Chain, length 6):

$1[A9]=3[A9]-3[A7]=3[J7]-3[J3]=1[J3]-$

1	2	3	4	5	6	7	8	9
A	1 2 6	9	4	5	1 2 3	7	3 6 8	1 3
B	1 5	8	7	1 6	4 3	4 6	2	1 3 5 9
C	2 5 6	3	1 2 5	1 8	1 2 8	9	5 6	4 7
D	4	5 7	6	3	9	1	5 7 8	2 5 8
E	8	2	9	4	7	5	1	3 6
F	1 3 5 7	1 5 7	1 5 3	2	6	8	4	7 9 5 9
G	9	4	1 2 5	1 7 8	1 5 8	3	7 8	6 1 2 8
H	1 2 3 5 7	1 5 7	8	1 6 7	1 4 5 6	4 6	9	1 2 3 4 7
J	1 3 7	6	1 3	9	1 4 8	2	7 8	5 1 3 4 8

AIC Example: [Load Example](#) or : [From the Start](#)

Here we have a classic and pleasingly short Alternating Inference Chain with one discontinuity. We’re using Nice Loop Rule 3, which tells us that the candidate between two weak links can be removed, in this case the 1 in J9. How does this loop work?

Let’s start in A9 on number 1. If 1 is not the value of this cell, then 3 must be (there’s a strong link from 1 to 3 in A9, which is bi-value). Next, we go from the 3 in A9 to the 3 in A7. There are more than two 3s in that row so the link must be weak (although even if there were only two, a strong link could be declared as having weak inference).

We stick with 3 at this point to make a link of strong inference down to J7 (there being only two 3s in this column), then a link of weak inference to J3 (there being four 3s in row J). Finally, J3 is bi-value, so a strong link exists to the 1 there. In summary, our logic goes as follows:

- If 1 is in A9, then it’s not in J9.
- If 1 is not in A9, 3 must be, so 3 is not in A7, which forces 3 to be in J7, which means 3 is not in J3, which leaves a 1 in J3. A 1 in J3 means that there can be no 1 in J9.

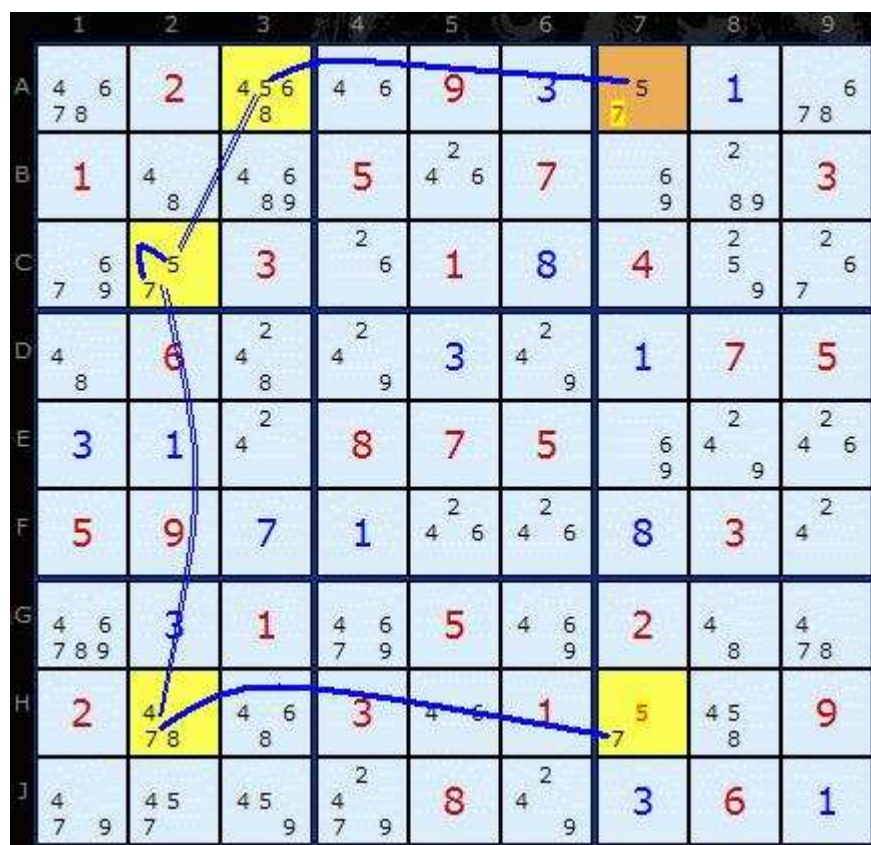
Therefore, there is no 1 in J9.

A nice feature of AICs is that they are bi-directional - they work in both directions. To re-cast this chain the other way, we could argue:

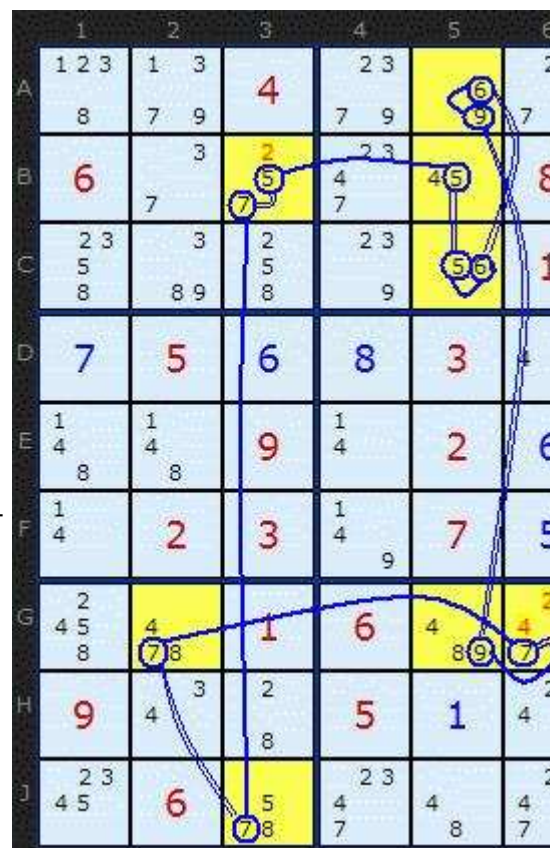
- If 1 is in J3, it is not in J9.
- If 1 is not in J3, then 3 must be, which means 3 is not in J7, which means 3 must be in A7, which means 3 is not in A9, so 1 must be, since it’s the only alternative.

Therefore, 1 can’t be in J9.

So the chain is safely bi-directional and we must conclude that 7 can't be in A7 and 5 can't be in H7.

$$6[A5]=9[A5]-9[G5]=9[G6]-7[G6]=7[G2]-7[J3]=7[B3]-5[B3]=5[B5]-5[C5]=6[C5]-$$


AIC Example 2: [Load Example](#) or : [From the Start](#)



3 von 4

Where you can identify a chain which is closed, any bi-value weak link can lose unused candidates

2		
8		6
		3

	3	6
5		7

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On the aic strategy, is there a limit to how many strong links having weak interference you can use on 1 example? Also, when you go from 1 number to another in the same cell and there are 3 numbers in that cell, is that a weak link or strong? Thank-you

Laura

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