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Single's Chains

From sudokuwiki.com, the puzzle solver's site

| | | |
|---|---|---|
| 2 | | |
| | 3 | 6 |
| 5 | | 7 |

Another way of looking at this is the popular technique of Colouring.

We'll go back to our first example. You assign the start of a promising chain with an arbitrary colour, in this case Green (A at D3). Remember, we are only looking at candidate 5 and units with two 5s in them (called **conjugate pairs**).

A has two conjugate pairs, B and X - which are painted in an alternative colour, blue. From cell B I can find two more pairs at C and E which I colour in green. Both these point to D which must be coloured blue, as well as F along the bottom row.

Now we have arrived at the contradiction. X and D are both blue and they are in the same unit (the row in this case).

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|------------|-------------|---------------|--------|-------------|--------|-------------|-------------|-------------|
| A | 3 | 4 | 6 9 | 4 9 | 8 | 4 6 | 2 | 7 | 1 5 |
| B | 2 4 5 6 | 2 5 6 | 7 | 9 | 4 6 | 1 | 8 | 4 2 | 3 |
| C | 4 2 | 8 | 1 | 5 | 3 | 7 | 9 | 4 2 | 6 |
| D | 1 6 | 1 7 | 6 4 5 8 | A | 2 4 | 2 8 | 3 | 2 5 B | 7 6 9 |
| E | 2 5 | 2 7 | 5 3 | 1 | 9 | 6 | 4 | 5 7 E | 8 |
| F | 9 | 4 6 | 4 8 | 7 | 2 5 8 | 4 5 | 2 3 | 3 6 | 1 |
| G | 1 4 5 | 1 4 5 | 9 4 5 9 | X | 3 | 7 | 8 | 6 | 5 9 D |
| H | 8 | 3 9 | 6 | 4 2 | 2 5 | 4 5 | 1 | 3 9 | 7 |
| J | 7 | 5 3 F | 2 | 6 | 1 | 9 | 5 3 C | 8 | 4 |

Colouring Example 1

Our first rule is: *whenever a candidate outside the chain relates by column, row or box to two alternately coloured cells in a singles chain, that 'non-chain' candidate can be excluded.* This applies to X.

Chains form a big part of the advanced strategy armory. Fortunately there is a very simple chain of clues that works with single candidate numbers only. We can scan the board for a configuration looking at one number at a time.

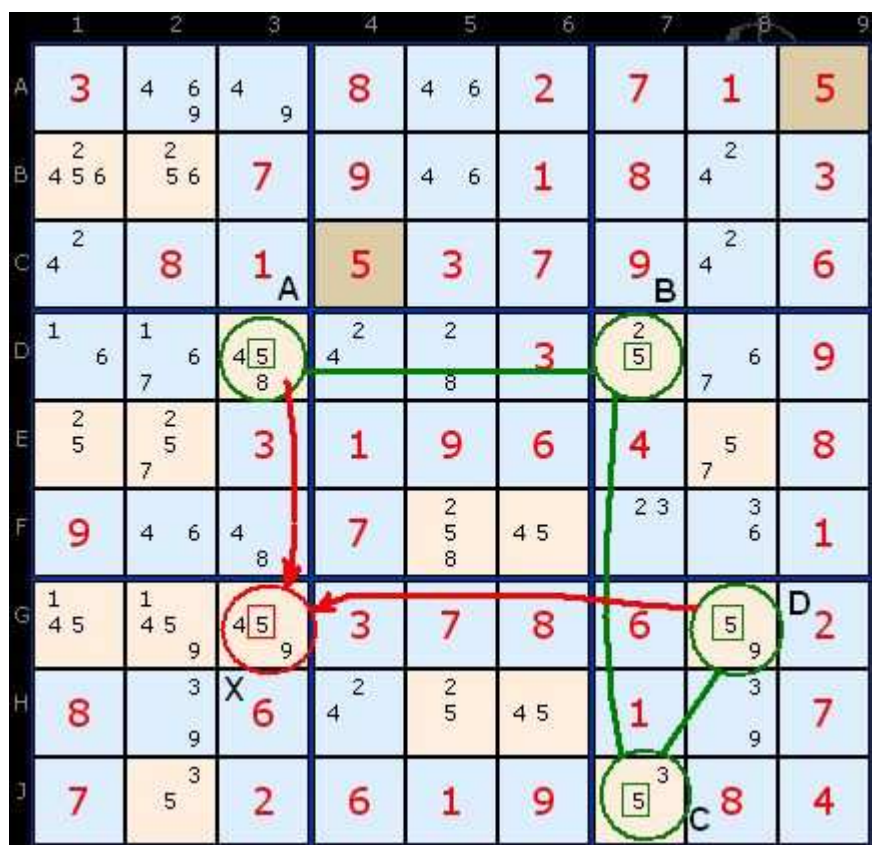
But not only can we trace a chain from A to D which eliminates the 5 at X but we can say something more general and much more interesting. Either ALL the green cells are 5 or ALL the blue cells are 5. Because we have two blue cells in the same row blue must be false. Green must contain ALL the 5s. That gets us a huge number of cells solved in one go.

Our second rule is: *Whenever two cells in a singles chain have the same colour and also share the same unit, that color must be the 'false' color since each unit can only have one of any candidate value.*

Let N be our candidate we're scanning the board for. We are looking for pairs of N in any row, column or box. Having three or more won't do and we must ignore units with more than two of N. If we can join a sequence of these pairs we'll form a chain. Obviously the corners of this chain must change from one type of unit to another - for example a pair in a row followed by a pair in a column and then a row again or a box.

In the example to the right there are only two 5's at A and B IN THAT ROW. Our first pair, B and C link two 5's in their column. And so on to D

We are hoping to make an odd number of links (the green lines in the diagram). If we do then something very useful occurs.



Singles Chain Example 1: [Load Example](#) or : [From the Start](#)

Pretend for one moment that A is 5. B cannot be which forces C to be 5 which eliminates 5 from D. Now think in reverse.

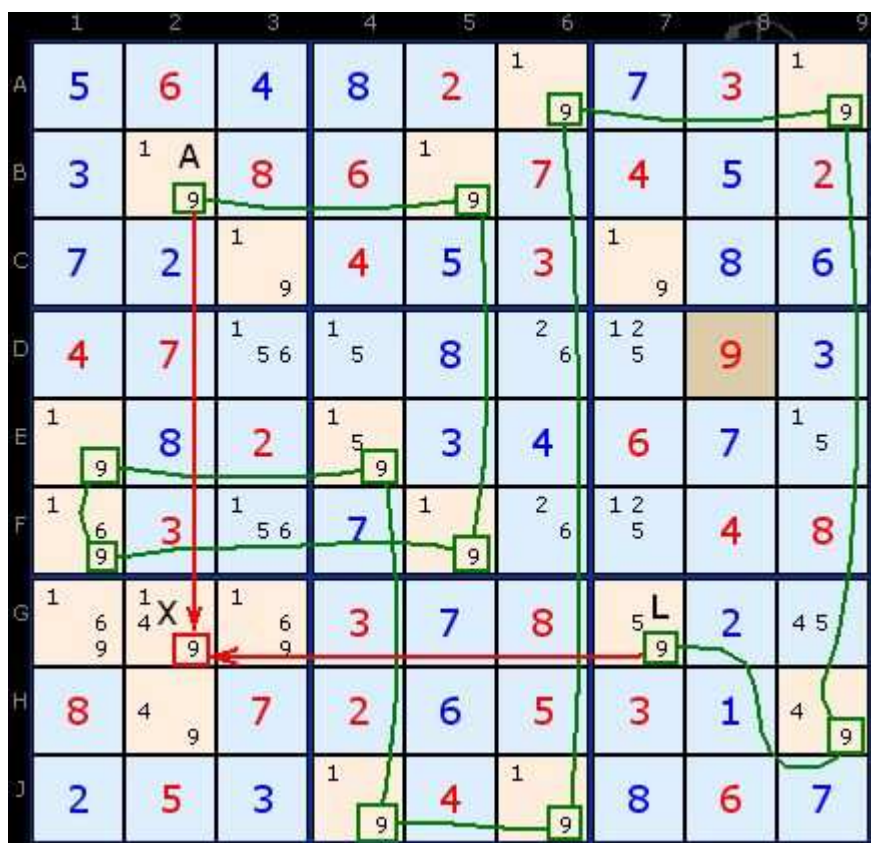
If D is 5 then C cannot be, B must be and A cannot be. Whatever way round you think it through *EITHER A OR D* must be a 5. Any cell that both A and D can see cannot contain a five - in this case G3 and D8. Since there is a 5 at G3 (marked with X) we can remove it. Have a look at the next strategy for an explanation for cells 'seeing' each other.

As long as the chains are linked by an ODD number of links and there is only two candidates in each unit of each link, this strategy will work. We can describe the two options in this way (~ means NOT 5):

$$\begin{aligned}
 &A(5) \rightarrow B(\sim 5) \rightarrow C(5) \rightarrow D(\sim 5) = X(\sim 5) \\
 &\qquad\qquad\qquad\text{OR} \\
 &A(\sim 5) \rightarrow B(5) \rightarrow C(\sim 5) \rightarrow D(5) = X(\sim 5)
 \end{aligned}$$

Chains can be any length. In some cases, ridiculously long as in this example. Here twelve cells (A to L) are joined by eleven links to target the cell at G2.

The minimum number of links is 3. It will be a rare occasion if you need more than 5.



Colouring Example 1:



Comments...

Friday 12-Feb-2010

... by: soooconfused

Why does the solver use coloring when there are 3+ candidates in a unit? I thought this was not allowed for singles chains. <http://i46.tinypic.com/2akfnm.jpg>

Wednesday 10-Feb-2010

... by: Lea Hayes

Thank you for your interesting descriptions on various Sudoku strategies.

What is the difference between single's chains and x-cycles? Do x-cycles always cover the same ground, or are they completely different?

Tuesday 5-Jan-2010

... by: Ben Wearn

The 11-link chain in the last example may be shortened to a 5-link chain by linking B5 to A6.

Tuesday 1-Dec-2009

... by: Steve

Colouring example 1 in fact shows rule 2, not rule 1: A and X are a conjugate pair, and there is no justification for arbitrarily ending the chain at A instead of continuing it to X. If you add 5 as a candidate in F3, then you will have a valid example of rule 1.

Interestingly, though, example 1 also has an X-cycle with a weak link discontinuity at X, which provides a different reason for eliminating 5 at X.

Tuesday 1-Dec-2009**... by: gerryfromktown**

Yeah, this explanation is a mess. It begins with this reference to non-existent text:

"Another way of looking at this is the popular technique of Colouring."

It appears the explanation is a cut-and-paste from here:

http://eric4ever.sudoku.googlepages.com/strategies_advanced.htm#SC

where you can find the missing text.

Thursday 22-Oct-2009**... by: Mike Wallis**

"Another way of looking at this is the popular technique of Colouring". What does "this" refer to?

The explanation for Rule 1 states that 5 can be removed from X (G3) because it's outside the chain and points to A (D3) and D (G8).

However, the explanation for Rule 2 states that X (G3) and D (G8) are the 'false' color because they're in the chain, both are blue, and both in the same row.

How can a square be both in a chain and outside of it at the same time? Moreover, Example 1 shows X (G3) and D (G8) in two separate chains - not in 'a' chain.

There is another chain that runs from F to C, C to D, D to E, E to B, B to A, and A to X. However the example for Rule 2 doesn't show that.

In fact, none of the examples on this page show Rule 2 in action. They only show Rule 1.

The disconnect between Rule 2 and the examples has been very confusing to me and has caused me difficulty in solving puzzles using this technique. For this reason, I think the article for Singles Chains needs to be revised with new examples that more accurately reflect Rule 2.

Sunday 28-Jun-2009**... by: Don**

Cell X cannot be green because cells D and X do not form a conjugate pair. That's because two other cells (G1 and G2) in row G contain 5 as a candidate. In column 3, however, cells A and X are the only two that contain 5 as a candidate; hence they do form a conjugate pair, making cell X blue.

Thursday 18-Jun-2009**... by: Clark**

If you start with E=green, then B=blue, C=green, D=blue, X=green. 'A' now looks like the exclusion. Can you explain this better? Why is X the exclusion and not A?

Tuesday 12-May-2009**... by: Semax**

Every part of the chain links two cells which share one or two units, but these and only these shared units have to fulfill the condition that there can't be any other cells containing the candidate.

So in the first example you can link D7 to E8 because in their shared unit (box 6) there is no other cell containing a 5. In other words, if D7 is 5 then E8 can't be 5, and vice versa. Therefore it doesn't matter what happens in rows D and E or in columns 7 and 8.

Thursday 7-May-2009**... by: Stephen P. Byers**

The Single's Chain business is very confusing. In your article on the subject you write, "... we are looking at candidate 5 and units with two 5's in them (called conjugate pairs)." In the first example, however, you link to Cell E8 that is one of two 5's in Box 6, but appears disqualified to be in the chain because it is one of three 5's in Row E. Is this viable?

A similar problem exists in your example titled "Coloring Example 1" at the end of the article. There are three 9's in Column 1 so how can two of them be included in the chain? Please explain.

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