



FACULTY  
OF MATHEMATICS  
AND PHYSICS  
Charles University

# Finite-coupling spectrum of $O(N)$ model in AdS

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# $O(N)$ model

$N$  scalar fields  $\{\phi^i\}$

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$$\text{X}_{\bullet} \sim \frac{\lambda}{N} \left( \text{X}_{\bullet}^{\text{right}} + \text{X}_{\bullet}^{\text{left}} + \text{X}_{\bullet}^{\text{diag3}} \right)$$

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$$\sum_{n=0}^{\infty} \text{X} \underbrace{\text{O} \cdots \text{O}}_{n \text{ bubbles}} \text{X}$$



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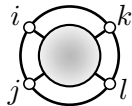
$$\sum_{n=0}^{\infty} \text{X} \underbrace{\text{O} \cdots \text{O}}_{n \text{ bubbles}} \text{X} \sim \frac{1}{N}$$

# $O(N)$ model in **AdS**

fixed AdS background

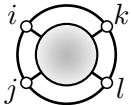
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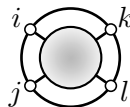


A Feynman diagram representing a bubble with four external legs. The bubble is a shaded circle with a smaller circle inside it. Four external legs, each ending in a small white circle, connect the outer boundary of the bubble to the external labels  $i$ ,  $k$ ,  $j$ , and  $l$ . The legs are arranged such that  $i$  and  $k$  are at the top, and  $j$  and  $l$  are at the bottom.

$$= \langle \mathcal{O}^i \mathcal{O}^j \mathcal{O}^k \mathcal{O}^l \rangle$$

# $O(N)$ model in **AdS**

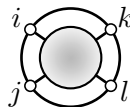
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$$\mathcal{O}^i(P) \sim \lim_{s \rightarrow \infty} s^{\Delta_\phi} \phi^i(X \equiv sP + \dots)$$

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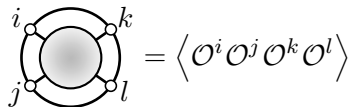

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CFT

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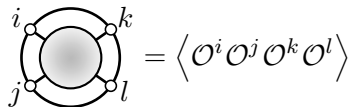

$$\text{Diagram} = \langle \mathcal{O}^i \mathcal{O}^j \mathcal{O}^k \mathcal{O}^l \rangle$$

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QFT in AdS  $\longleftrightarrow$  CFT

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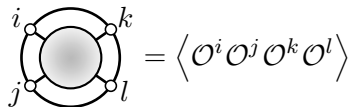
QFT in AdS  $\longleftrightarrow$  CFT

CFT in  $\text{AdS}_{d+1}$



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fixed AdS background


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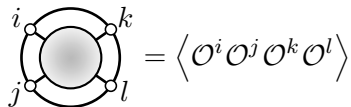
$$\mathcal{O}^i(P) \sim \lim_{s \rightarrow \infty} s^{\Delta_\phi} \phi^i(X \equiv sP + \dots)$$

QFT in AdS  $\longleftrightarrow$  CFT

CFT in  $\text{AdS}_{d+1}$   $\xleftrightarrow{\text{Weyl transformation}}$

# $O(N)$ model in **AdS**

fixed AdS background

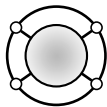

$$= \langle \mathcal{O}^i \mathcal{O}^j \mathcal{O}^k \mathcal{O}^l \rangle$$

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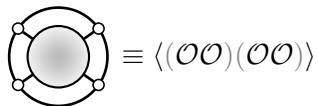
QFT in AdS  $\longleftrightarrow$  CFT

$$\text{CFT in AdS}_{d+1} \xleftrightarrow{\text{Weyl transformation}} \text{BCFT in } \mathbb{R}^d \times \mathbb{R}_{\geq}$$

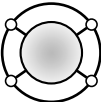
# Conformal Block/Partial Wave Decomposition



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# Conformal Block/Partial Wave Decomposition



$$\equiv \langle (\mathcal{O}\mathcal{O})(\mathcal{O}\mathcal{O}) \rangle = \sum_{\substack{\text{primary } \mathcal{O}_\star \\ \text{with } \Delta_\star, J_\star}} \text{o pe}^2[\mathcal{O}\mathcal{O}\mathcal{O}_\star] \left| G_{\Delta_\star, J_\star}^{(s)} \right\rangle$$

# Conformal Block/Partial Wave Decomposition

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$$\text{Diagram} = \sum_{J=0}^{\infty} \int_{\frac{d}{2} + i\mathbb{R}_{\geq}} \frac{d\Delta}{2\pi i} \text{Spec}_s \left[ \begin{array}{c} \Delta \\ J \end{array} \middle| \text{Diagram} \right] \left| \text{Diagram} \right\rangle$$

# Conformal Block/Partial Wave Decomposition

$$\begin{array}{c} \text{Diagram: A sphere with four small circles on its equator, each connected to the sphere by a line.} \end{array} \equiv \langle (\mathcal{O}\mathcal{O})(\mathcal{O}\mathcal{O}) \rangle = \sum_{\substack{\text{primary } \mathcal{O}_\star \\ \text{with } \Delta_\star, J_\star}} \text{ope}^2[\mathcal{O}\mathcal{O}\mathcal{O}_\star] \left| G_{\Delta_\star, J_\star}^{(s)} \right\rangle$$

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$$\left| \begin{array}{c} \text{Diagram: Two vertices connected by a horizontal line, with two lines extending from each vertex.} \end{array} \right\rangle = K_{\tilde{\Delta}, J} \left| G_{\Delta, J}^{(s)} \right\rangle + K_{\Delta, J} \left| G_{\tilde{\Delta}, J}^{(s)} \right\rangle$$

# Conformal Block/Partial Wave Decomposition

$$\begin{array}{c} \text{Diagram: A sphere with four external legs, each ending in a small circle.} \end{array} \equiv \langle (\mathcal{O}\mathcal{O})(\mathcal{O}\mathcal{O}) \rangle = \sum_{\substack{\text{primary } \mathcal{O}_\star \\ \text{with } \Delta_\star, J_\star}} \text{ope}^2[\mathcal{O}\mathcal{O}\mathcal{O}_\star] \left| G_{\Delta_\star, J_\star}^{(s)} \right\rangle$$

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$$\begin{aligned}
 \left| \begin{array}{c} \Delta, J \end{array} \right\rangle &= K_{\tilde{\Delta}, J} \left| G_{\Delta, J}^{(s)} \right\rangle + K_{\Delta, J} \left| G_{\tilde{\Delta}, J}^{(s)} \right\rangle \\
 \left| \begin{array}{c} \Delta', J' \end{array} \right\rangle &= K_{\tilde{\Delta}', J'} \left| G_{\Delta', J'}^{(t)} \right\rangle + K_{\Delta', J'} \left| G_{\tilde{\Delta}', J'}^{(t)} \right\rangle
 \end{aligned}$$



# Conformal Block/Partial Wave Decomposition

$$\text{Diagram} = \sum_J \int_{\frac{d}{2} + i\mathbb{R}} \frac{d\Delta}{2\pi i} \text{Spec}_s \left[ \frac{\Delta}{J} \middle| \text{Diagram} \right] K_{\tilde{\Delta}, J} \left| G_{\Delta, J}^{(s)} \right\rangle$$

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 &= \sum_{\substack{\text{primary } \mathcal{O}_\star \\ \text{with } \Delta_\star, J_\star}} \text{o pe}^2[\mathcal{O}\mathcal{O}\mathcal{O}_\star] \left| G_{\Delta_\star, J_\star}^{(s)} \right\rangle
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 &= \sum_{\substack{\text{primary } \mathcal{O}_\star \\ \text{with } \Delta_\star, J_\star}} \text{ope}^2[\mathcal{O}\mathcal{O}\mathcal{O}_\star] \left| G_{\Delta_\star, J_\star}^{(s)} \right\rangle \\
 \frac{-C_\star}{\Delta - \Delta_\star} \delta_{J, J_\star} &\in K_{\tilde{\Delta}, J} \text{Spec}_s \left[ \frac{\Delta}{J} \middle| \text{Diagram} \right]
 \end{aligned}$$

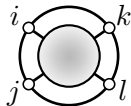
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 \end{aligned}$$

$$\frac{-C_\star}{\Delta - \Delta_\star} \delta_{J, J_\star} \in K_{\tilde{\Delta}, J} \text{Spec}_s \left[ \frac{\Delta}{J} \middle| \text{Diagram} \right]$$

$$\text{ope}^2[\mathcal{O}\mathcal{O}\mathcal{O}_\star] \equiv C_\star = -\text{Res}_{\Delta=\Delta_\star} \left( K_{\tilde{\Delta}, J_\star} \text{Spec}_s \left[ \frac{\Delta}{J_\star} \middle| \text{Diagram} \right] \right)$$

# Boundary 4-point correlator in **AdS**



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$$\text{Diagram with shaded disk} = \left( \text{Diagram 1} + \text{Diagram 2} + \text{Diagram 3} \right)$$

# Boundary 4-point correlator in **AdS**

$$\begin{aligned}
 \text{Diagram 1} &= \left( \text{Diagram 2} + \text{Diagram 3} + \text{Diagram 4} \right) \\
 &+ \frac{1}{N} \left( \text{Diagram 5} + \text{Diagram 6} + \text{Diagram 7} \right)
 \end{aligned}$$

The diagrams represent Feynman diagrams for a boundary 4-point correlator in AdS. The external legs are labeled  $i, k, j, l$ . The first diagram on the left is a bubble with a shaded central region. The first row of diagrams on the right shows three tree-level diagrams: two exchange diagrams and one contact diagram. The second row, multiplied by  $1/N$ , shows three loop-level diagrams: a bubble diagram, a self-energy diagram, and a vertex correction diagram.

# Boundary 4-point correlator in AdS

$$\begin{aligned}
 \text{Bubble with shaded center} &= \left( \text{Diagram 1} + \text{Diagram 2} + \text{Diagram 3} \right) \\
 &+ \frac{1}{N} \left( \text{Diagram 4} + \text{Diagram 5} + \text{Diagram 6} \right)
 \end{aligned}$$

The diagrams in the equation are:

- Diagram 1:** A circle with four external legs labeled  $i, k, j, l$ . The top and bottom arcs are connected by two vertical lines.
- Diagram 2:** A circle with four external legs labeled  $i, k, j, l$ . The left and right arcs are connected by two horizontal lines.
- Diagram 3:** A circle with four external legs labeled  $i, k, j, l$ . The left and right arcs are connected by two diagonal lines crossing in the center.
- Diagram 4:** A circle with four external legs labeled  $i, k, j, l$ . The top and bottom arcs are connected by two vertical lines, each with a shaded vertex in the middle.
- Diagram 5:** A circle with four external legs labeled  $i, k, j, l$ . The left and right arcs are connected by two horizontal lines, each with a shaded vertex in the middle.
- Diagram 6:** A circle with four external legs labeled  $i, k, j, l$ . The left and right arcs are connected by two diagonal lines, each with a shaded vertex in the middle.

$$\text{Bubble with shaded center} \sim \underbrace{\text{Diagram 1} (\dots) \text{Diagram 2}}_{n \text{ bubbles}}$$

The diagram on the right shows a sequence of bubbles connected by vertices, with an ellipsis indicating a continuation of the sequence. The entire sequence is labeled "n bubbles" under a brace.



# Boundary 4-point correlator in AdS

$$\begin{aligned}
 \text{Diagram 1} &= \left( \text{Diagram 2} + \text{Diagram 3} + \text{Diagram 4} \right) \\
 &+ \frac{1}{N} \left( \text{Diagram 5} + \text{Diagram 6} + \text{Diagram 7} \right)
 \end{aligned}$$

The diagrams represent various Feynman-like diagrams in AdS. Diagram 1 is a circle with four external legs labeled  $i, k, j, l$  and a shaded central blob. Diagrams 2, 3, and 4 are circles with four external legs and internal lines connecting the legs. Diagrams 5, 6, and 7 are similar to Diagram 1 but with a shaded internal line and a central blob.

$$\text{Diagram 8} \sim \underbrace{\text{Diagram 9} (\dots \text{Diagram 10})}_{n \text{ bubbles}} \Rightarrow \text{Diagram 11} \sim \delta^{ij}$$

Diagram 8 is a four-point vertex with a shaded internal line. Diagram 9 is a chain of bubbles. Diagram 10 is a single bubble. Diagram 11 is a two-point vertex with a shaded internal line. The equation shows that the four-point vertex is equivalent to a chain of bubbles, which then simplifies to a two-point vertex proportional to  $\delta^{ij}$ .

# Boundary 4-point correlator in AdS

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 \text{Diagram 1} &= \left( \text{Diagram 2} + \text{Diagram 3} + \text{Diagram 4} \right) \\
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 \end{aligned}$$

The diagrams represent various Feynman-like diagrams in AdS. Diagram 1 is a sphere with four external legs labeled  $i, k, j, l$  and a shaded central region. Diagrams 2, 3, and 4 are spheres with four external legs and internal lines. Diagrams 5, 6, and 7 are spheres with four external legs and internal lines, with some lines shaded.

$$\text{Diagram 8} \sim \underbrace{\text{Diagram 9} (\dots \text{Diagram 10})}_{n \text{ bubbles}} \Rightarrow \text{Diagram 11} \sim \delta^{ij}$$

Diagram 8 is a vertex with four external legs and a shaded internal region. Diagram 9 is a vertex with two external legs and a shaded internal region. Diagram 10 is a vertex with two external legs and a shaded internal region. Diagram 11 is a vertex with two external legs and a shaded internal region.

$$\text{Diagram 1} = \delta^{ij} \delta^{kl} \left( \text{Diagram 2} + \frac{1}{N} \text{Diagram 5} \right)$$

# Boundary 4-point correlator in **AdS**

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 \text{Diagram 1} &= \left( \text{Diagram 2} + \text{Diagram 3} + \text{Diagram 4} \right) \\
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 \end{aligned}$$

The diagrams represent various Feynman-like diagrams in AdS. Diagram 1 is a circle with four external legs labeled  $i, k, j, l$  and a shaded central blob. Diagrams 2, 3, and 4 are circles with four external legs and internal lines. Diagrams 5, 6, and 7 are similar to Diagram 1 but with internal shaded blobs and lines.

$$\text{Diagram 8} \sim \underbrace{\text{Diagram 9} (\dots \text{Diagram 10})}_{n \text{ bubbles}} \Rightarrow \text{Diagram 11} \sim \delta^{ij}$$

Diagram 8 is a small diagram with two external legs and a shaded blob. Diagram 9 is a chain of bubbles. Diagram 10 is a single bubble. Diagram 11 is a diagram with two external legs and a shaded blob, representing a delta function  $\delta^{ij}$ .

$$\text{Diagram 1} = \delta^{ij} \delta^{kl} \left( \text{Diagram 2} + \frac{1}{N} \text{Diagram 5} \right) + \text{crossed channels}$$

## Decomposition into $\mathbf{O}(N)$ irreps

$$\begin{array}{c} i \\ \circ \\ \text{---} \circ \text{---} k \\ \backslash \quad / \\ \text{---} \circ \text{---} l \\ / \quad \backslash \\ \circ \\ j \end{array} = \overbrace{N\mathcal{P}_{(\text{S})}^{ijkl}}^{\delta^{ij}\delta^{kl}} \begin{array}{c} \text{S} \\ \circ \\ \text{---} \circ \text{---} \\ \backslash \quad / \\ \text{---} \circ \text{---} \\ / \quad \backslash \\ \circ \end{array}$$

## Decomposition into $\mathbf{O}(N)$ irreps

$$\begin{array}{c} i \\ \circ \\ \text{---} \circ \text{---} k \\ \text{---} \circ \text{---} l \\ \circ \\ j \end{array} = \overbrace{N \mathcal{P}_{(\text{S})}^{ijkl}}^{\delta^{ij} \delta^{kl}} \begin{array}{c} \text{S} \end{array} + \overbrace{\mathcal{P}_{(\text{AS})}^{ijkl}}^{\delta_{[k}^i \delta_{l]}^j} \begin{array}{c} \text{AS} \end{array}$$

## Decomposition into $\mathbf{0}(N)$ irreps

$$\begin{array}{c} i \\ \circ \\ \text{---} \text{---} \text{---} k \\ \circ \\ j \quad \quad l \end{array} = \overbrace{N \mathcal{P}_{(S)}^{ijkl}}^{\delta_{ij} \delta_{kl}} \begin{array}{c} \text{---} \text{---} \text{---} \\ \circ \quad \text{S} \quad \circ \\ \text{---} \text{---} \text{---} \end{array} + \overbrace{\mathcal{P}_{(\text{AS})}^{ijkl}}^{\delta_{[k}^{[i} \delta_{l]}^{j]}} \begin{array}{c} \text{---} \text{---} \text{---} \\ \circ \quad \text{AS} \quad \circ \\ \text{---} \text{---} \text{---} \end{array} + \overbrace{\mathcal{P}_{(\text{ST})}^{ijkl}}^{\delta_{\{k}^{\{i} \delta_{l\}}^{j\}}} \begin{array}{c} \text{---} \text{---} \text{---} \\ \circ \quad \text{ST} \quad \circ \\ \text{---} \text{---} \text{---} \end{array}$$

## Decomposition into $\mathbf{O}(N)$ irreps

$$\begin{array}{c} i \\ \text{---} \\ j \end{array} \begin{array}{c} k \\ \text{---} \\ l \end{array} = \overbrace{N\mathcal{P}_{(\text{S})}^{ijkl}}^{\delta^{ij}\delta^{kl}} \begin{array}{c} \text{S} \end{array} + \overbrace{\mathcal{P}_{(\text{AS})}^{ijkl}}^{\delta_{[k}^i\delta_{l]}^j} \begin{array}{c} \text{AS} \end{array} + \overbrace{\mathcal{P}_{(\text{ST})}^{ijkl}}^{\delta_{\{k}^i\delta_{l\}}^j} \begin{array}{c} \text{ST} \end{array}$$

$$\text{Diagram with central node 'S' and four surrounding nodes} = \left( \text{Diagram with two vertical edges} \right) + \frac{1}{N} \left( \text{Diagram with two horizontal edges} + \text{Diagram with two crossing edges} + \dots \right)$$





# Decomposition into $O(N)$ irreps

$$\begin{array}{c} i \\ \circ \\ \text{---} \circ \quad \circ \quad \circ \text{---} \\ \text{---} \circ \quad \circ \quad \circ \text{---} \\ j \quad \quad \quad l \end{array} = \overbrace{N \mathcal{P}_{(S)}^{ijkl}}^{\delta^{ij} \delta^{kl}} \begin{array}{c} \text{---} \circ \quad \circ \quad \circ \text{---} \\ \text{---} \circ \quad \circ \quad \circ \text{---} \\ \text{S} \end{array} + \overbrace{\mathcal{P}_{(AS)}^{ijkl}}^{\delta_{[k}^i \delta_{l]}^j} \begin{array}{c} \text{---} \circ \quad \circ \quad \circ \text{---} \\ \text{---} \circ \quad \circ \quad \circ \text{---} \\ \text{AS} \end{array} + \overbrace{\mathcal{P}_{(ST)}^{ijkl}}^{\delta^{\{i} \delta_{\{k}^j \}}_{\{l\}}} \begin{array}{c} \text{---} \circ \quad \circ \quad \circ \text{---} \\ \text{---} \circ \quad \circ \quad \circ \text{---} \\ \text{ST} \end{array}$$

$$\begin{array}{c} \text{---} \circ \quad \circ \quad \circ \text{---} \\ \text{---} \circ \quad \circ \quad \circ \text{---} \\ \text{S} \end{array} = \left( \begin{array}{c} \text{---} \circ \quad \circ \quad \circ \text{---} \\ \text{---} \circ \quad \circ \quad \circ \text{---} \end{array} \right) + \frac{1}{N} \left( \begin{array}{c} \text{---} \circ \quad \circ \quad \circ \text{---} \\ \text{---} \circ \quad \circ \quad \circ \text{---} \end{array} + \begin{array}{c} \text{---} \circ \quad \circ \quad \circ \text{---} \\ \text{---} \circ \quad \circ \quad \circ \text{---} \end{array} + \begin{array}{c} \text{---} \circ \quad \circ \quad \circ \text{---} \\ \text{---} \circ \quad \circ \quad \circ \text{---} \end{array} \right)$$

$$\begin{array}{c} \text{---} \circ \quad \circ \quad \circ \text{---} \\ \text{---} \circ \quad \circ \quad \circ \text{---} \\ \text{AS} \end{array} = \left( \begin{array}{c} \text{---} \circ \quad \circ \quad \circ \text{---} \\ \text{---} \circ \quad \circ \quad \circ \text{---} \end{array} - \begin{array}{c} \text{---} \circ \quad \circ \quad \circ \text{---} \\ \text{---} \circ \quad \circ \quad \circ \text{---} \end{array} \right)$$

# Decomposition into $O(N)$ irreps

$$\begin{array}{c} i \\ \circ \\ \text{---} \circ \quad \circ \quad \circ \quad \circ \quad \text{---} k \\ \text{---} \circ \quad \circ \quad \circ \quad \circ \quad \text{---} l \\ j \end{array} = \overbrace{N \mathcal{P}_{(S)}^{ijkl}}^{\delta^{ij} \delta^{kl}} \begin{array}{c} \text{S} \end{array} + \overbrace{\mathcal{P}_{(AS)}^{ijkl}}^{\delta_{[k}^i \delta_{l]}^j} \begin{array}{c} \text{AS} \end{array} + \overbrace{\mathcal{P}_{(ST)}^{ijkl}}^{\delta^{\{i} \delta_{\{k}^j \delta_{l\}}^{\}} \} } \begin{array}{c} \text{ST} \end{array}$$

$$\begin{array}{c} \text{S} \end{array} = \left( \begin{array}{c} \text{---} \circ \quad \circ \quad \circ \quad \circ \quad \text{---} \end{array} \right) + \frac{1}{N} \left( \begin{array}{c} \text{---} \circ \quad \circ \quad \circ \quad \circ \quad \text{---} \end{array} + \begin{array}{c} \text{---} \circ \quad \circ \quad \circ \quad \circ \quad \text{---} \end{array} + \begin{array}{c} \text{---} \circ \quad \circ \quad \circ \quad \circ \quad \text{---} \end{array} \right)$$

$$\begin{array}{c} \text{AS} \end{array} = \left( \begin{array}{c} \text{---} \circ \quad \circ \quad \circ \quad \circ \quad \text{---} \end{array} - \begin{array}{c} \text{---} \circ \quad \circ \quad \circ \quad \circ \quad \text{---} \end{array} \right) + \frac{1}{N} \left( \begin{array}{c} \text{---} \circ \quad \circ \quad \circ \quad \circ \quad \text{---} \end{array} - \begin{array}{c} \text{---} \circ \quad \circ \quad \circ \quad \circ \quad \text{---} \end{array} \right)$$

## Decomposition into $\mathbf{O}(N)$ irreps

$$\begin{array}{c} i \\ \circ \\ \text{---} \end{array} \begin{array}{c} k \\ \circ \\ \text{---} \end{array} = N \overbrace{\mathcal{P}_{(\text{S})}^{ijkl}}^{\delta^{ij}\delta^{kl}} \begin{array}{c} \text{S} \\ \text{---} \end{array} + \overbrace{\mathcal{P}_{(\text{AS})}^{ijkl}}^{\delta_{[k}^{[i}\delta_{l]}^{j]}} \begin{array}{c} \text{AS} \\ \text{---} \end{array} + \overbrace{\mathcal{P}_{(\text{ST})}^{ijkl}}^{\delta_{\{k}^{\{i}\}\delta_{l}^{\{j\]}} \begin{array}{c} \text{ST} \\ \text{---} \end{array}$$

$$\text{Diagram with central node 'S' and four external nodes} = \left( \text{Diagram with two internal nodes and two external nodes} \right) + \frac{1}{N} \left( \text{Diagram with two internal nodes and two external nodes} + \text{Diagram with two internal nodes and two external nodes} + \text{Diagram with two internal nodes and two external nodes} \right)$$

$$\text{AS} = \left( \text{Diagram 1} - \text{Diagram 2} \right) + \frac{1}{N} \left( \text{Diagram 3} - \text{Diagram 4} \right)$$

$$\text{ST} = \left( \text{Diagram 1} + \text{Diagram 2} \right) + \frac{1}{N} \left( \text{Diagram 3} + \text{Diagram 4} \right)$$

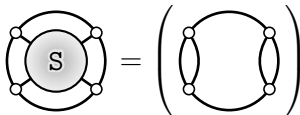
# Singlet sector (MFT)

Analyzed in D. Carmi, L. Di Pietro and S. Komatsu, *A Study of Quantum Field Theories in AdS at Finite Coupling*, *JHEP* **01** (2019) 200 [1810.04185]

The diagram shows an equality between two Feynman diagrams. On the left, a shaded circular blob labeled 'S' is connected to four external vertices (small white circles) arranged in a square. Each vertex is connected to the blob by a single line, and the vertices are also connected to each other by two arcs (top and bottom) forming a square loop. On the right, the same diagram is enclosed in large parentheses. Inside the parentheses, the blob 'S' is replaced by a large empty circle, and the four external vertices are connected to this circle by two arcs each (top and bottom), forming a square loop.

# Singlet sector (MFT)

Analyzed in D. Carmi, L. Di Pietro and S. Komatsu, *A Study of Quantum Field Theories in AdS at Finite Coupling*, *JHEP* **01** (2019) 200 [1810.04185]



$$[\mathcal{O}^i \times \mathcal{O}^j]^{(S)} \stackrel{N \rightarrow \infty}{\sim} \mathbb{1}^{(S)}$$

# Singlet sector (MFT)

Analyzed in D. Carmi, L. Di Pietro and S. Komatsu, *A Study of Quantum Field Theories in AdS at Finite Coupling*, *JHEP* **01** (2019) 200 [1810.04185]

$$\text{Diagram with central } S = \left( \text{Diagram with two internal lines} \right) + \frac{1}{N} \left( \text{Diagram with two internal lines} + \text{Diagram with two crossing internal lines} \right)$$

$$[\mathcal{O}^i \times \mathcal{O}^j](s) \stackrel{N \rightarrow \infty}{\sim} \mathbb{1}(s)$$

# Singlet sector (MFT)

Analyzed in D. Carmi, L. Di Pietro and S. Komatsu, *A Study of Quantum Field Theories in AdS at Finite Coupling*, *JHEP* **01** (2019) 200 [1810.04185]

$$\text{Diagram with central } S = \left( \text{Diagram with two vertical ovals} \right) + \frac{1}{N} \left( \text{Diagram with two horizontal ovals} + \text{Diagram with crossing lines} \right)$$

$$[\mathcal{O}^i \times \mathcal{O}^j]^{(s)} \stackrel{N \rightarrow \infty}{\sim} \mathbb{1}^{(s)}$$

$$[\mathcal{O}^i \times \mathcal{O}^j]^{(s)} \stackrel{\lambda \rightarrow 0}{\sim} \mathbb{1}^{(s)} \oplus \frac{1}{\sqrt{N}} \left[ \mathcal{O}^\bullet \square^n \partial_{\text{even}}^J \mathcal{O}^\bullet \right]^{(s)}$$

# Singlet sector (MFT)

Analyzed in D. Carmi, L. Di Pietro and S. Komatsu, *A Study of Quantum Field Theories in AdS at Finite Coupling*, *JHEP* **01** (2019) 200 [1810.04185]

$$\text{Diagram with shaded circle 'S' and four external legs} = \left( \text{Diagram with two internal lines} \right) + \frac{1}{N} \left( \text{Diagram with two internal lines} + \text{Diagram with two crossing internal lines} \right)$$

$$[\mathcal{O}^i \times \mathcal{O}^j]^{(s)} \stackrel{N \rightarrow \infty}{\sim} \mathbb{1}^{(s)}$$

$$[\mathcal{O}^i \times \mathcal{O}^j]^{(s)} \stackrel{\lambda \rightarrow 0}{\sim} \mathbb{1}^{(s)} \oplus \frac{1}{\sqrt{N}} \left[ \mathcal{O}^\bullet \square^n \partial_{\text{even}}^J \mathcal{O}^\bullet \right]^{(s)}$$

$$\left( \text{Diagram with two internal lines} + \text{Diagram with two crossing internal lines} \right) = \sum_{n=0}^{\infty} \sum_{\substack{J=0 \\ J \text{ even}}}^{\infty} 2C_{n,J}^{(\text{MFT})} \left| G_{2\Delta_\phi+2n+J,J}^{(s)} \right\rangle$$



# Singlet sector (interacting)

Analyzed in D. Carmi, L. Di Pietro and S. Komatsu, *A Study of Quantum Field Theories in AdS at Finite Coupling*, *JHEP* **01** (2019) 200 [1810.04185]

$$\text{Diagram with shaded disk 'S'} = \left( \text{Diagram with two vertical ovals} \right) + \frac{1}{N} \left( \text{Diagram with two horizontal ovals} + \text{Diagram with two crossing lines} + \text{Diagram with two horizontal ovals and shaded disk} \right)$$

$$[\mathcal{O}^i \times \mathcal{O}^j]^{(s)} \stackrel{N \rightarrow \infty}{\sim} \mathbb{1}^{(s)}$$

$$[\mathcal{O}^i \times \mathcal{O}^j]^{(s)} \stackrel{\lambda \rightarrow 0}{\sim} \mathbb{1}^{(s)} \oplus \frac{1}{\sqrt{N}} \left[ \mathcal{O}^\bullet \square^n \partial_{\text{even}}^J \mathcal{O}^\bullet \right]^{(s)}$$

# Singlet sector (interacting)

Analyzed in D. Carmi, L. Di Pietro and S. Komatsu, *A Study of Quantum Field Theories in AdS at Finite Coupling*, *JHEP* **01** (2019) 200 [1810.04185]

$$\text{Diagram with shaded disk } S = \left( \text{Diagram 1} \right) + \frac{1}{N} \left( \text{Diagram 2} + \text{Diagram 3} + \text{Diagram 4} \right)$$

$$[\mathcal{O}^i \times \mathcal{O}^j]^{(S)} \stackrel{N \rightarrow \infty}{\sim} \mathbb{1}^{(S)}$$

$$[\mathcal{O}^i \times \mathcal{O}^j]^{(S)} \stackrel{\lambda \rightarrow 0}{\sim} \mathbb{1}^{(S)} \oplus \frac{1}{\sqrt{N}} \left[ \mathcal{O}^\bullet \square^n \partial_{\text{even}}^J \mathcal{O}^\bullet \right]^{(S)}$$

$$[\mathcal{O}^i \times \mathcal{O}^j]^{(S)} \sim \mathbb{1}^{(S)}$$

# Singlet sector (interacting)

Analyzed in D. Carmi, L. Di Pietro and S. Komatsu, *A Study of Quantum Field Theories in AdS at Finite Coupling*, *JHEP* **01** (2019) 200 [1810.04185]

$$\text{Diagram with shaded disk } S = \left( \text{Diagram with two vertical ovals} \right) + \frac{1}{N} \left( \text{Diagram with two horizontal ovals} + \text{Diagram with two crossing lines} + \text{Diagram with two shaded disks and hatched line} \right)$$

$$[\mathcal{O}^i \times \mathcal{O}^j]^{(S)} \stackrel{N \rightarrow \infty}{\sim} \mathbb{1}^{(S)}$$

$$[\mathcal{O}^i \times \mathcal{O}^j]^{(S)} \stackrel{\lambda \rightarrow 0}{\sim} \mathbb{1}^{(S)} \oplus \frac{1}{\sqrt{N}} \left[ \mathcal{O}^\bullet \square^n \partial_{\text{even}}^J \mathcal{O}^\bullet \right]^{(S)}$$

$$[\mathcal{O}^i \times \mathcal{O}^j]^{(S)} \sim \mathbb{1}^{(S)} \oplus \frac{1}{\sqrt{N}} \left[ \text{“} \mathcal{O}^\bullet \square^n \mathcal{O}^\bullet \text{”} \right]^{(S)}_{O(1) \text{ finite shifts}}$$

# Singlet sector (interacting)

Analyzed in D. Carmi, L. Di Pietro and S. Komatsu, *A Study of Quantum Field Theories in AdS at Finite Coupling*, *JHEP* **01** (2019) 200 [1810.04185]

$$\text{Diagram with } S = \left( \text{Diagram 1} \right) + \frac{1}{N} \left( \text{Diagram 2} + \text{Diagram 3} + \text{Diagram 4} \right)$$

$$[\mathcal{O}^i \times \mathcal{O}^j]^{(S)} \stackrel{N \rightarrow \infty}{\sim} \mathbb{1}^{(S)}$$

$$[\mathcal{O}^i \times \mathcal{O}^j]^{(S)} \stackrel{\lambda \rightarrow 0}{\sim} \mathbb{1}^{(S)} \oplus \frac{1}{\sqrt{N}} \left[ \mathcal{O}^\bullet \square^n \partial_{\text{even}}^J \mathcal{O}^\bullet \right]^{(S)}$$

$$\begin{aligned} [\mathcal{O}^i \times \mathcal{O}^j]^{(S)} &\sim \mathbb{1}^{(S)} \oplus \frac{1}{\sqrt{N}} \left[ \text{“} \mathcal{O}^\bullet \square^n \mathcal{O}^\bullet \text{”} \right]^{(S)}_{O(1) \text{ finite shifts}} \\ &\oplus \frac{1}{\sqrt{N}} \left[ \mathcal{O}^\bullet \square^n \partial_{\text{even}}^{J>0} \mathcal{O}^\bullet \right]^{(S)}_{\text{MFT}} \end{aligned}$$

# Singlet sector — spectral functions

$$\frac{1}{N} \text{Spec}_s \left[ \text{Diagram 1} + \text{Diagram 2} + \text{Diagram 3} \right]$$

The equation shows the spectral function in the singlet sector,  $\frac{1}{N} \text{Spec}_s$ , applied to a sum of three Feynman diagrams. The diagrams are enclosed in large square brackets.

- Diagram 1:** A circle with four white vertices. Two horizontal arcs connect the top and bottom vertices, and two vertical arcs connect the left and right vertices, forming a lens-like structure.
- Diagram 2:** A circle with four white vertices. Two diagonal lines cross each other in the center, connecting the top-left to bottom-right and top-right to bottom-left vertices.
- Diagram 3:** A circle with four white vertices. Two horizontal lines connect the top and bottom vertices. In the center, there is a shaded gray rectangle with diagonal hatching, connected to the two horizontal lines by vertical segments.

# Singlet sector — spectral functions

$$\frac{1}{N} \text{Spec}_s \left[ \text{Diagram 1} + \text{Diagram 2} + \text{Diagram 3} \right]$$

$$\left( \text{Diagram 1} + \text{Diagram 2} \right) = \sum_{n=0}^{\infty} \sum_{\substack{J=0 \\ J \text{ even}}}^{\infty} 2C_{n,J}^{(\text{MFT})} \left| G_{2\Delta_\phi+2n+J,J}^{(s)} \right\rangle$$

# Singlet sector — spectral functions

$$\frac{1}{N} \text{Spec}_s \left[ \text{Diagram 1} + \text{Diagram 2} + \text{Diagram 3} \right]$$

$$\left( \text{Diagram 1} + \text{Diagram 2} \right) = \sum_{n=0}^{\infty} \sum_{\substack{J=0 \\ J \text{ even}}}^{\infty} 2C_{n,J}^{(\text{MFT})} \left| G_{2\Delta_\phi+2n+J,J}^{(s)} \right\rangle$$

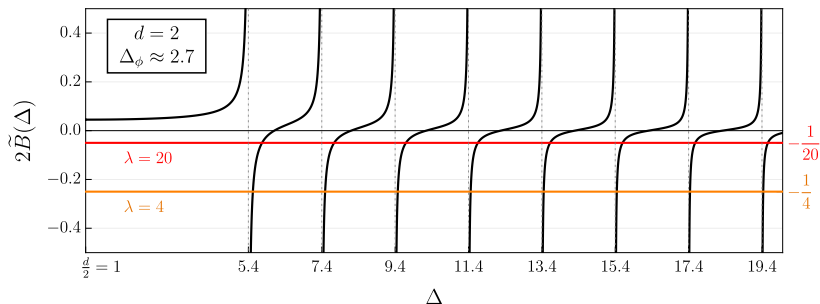
$$\text{Spec}_s \left[ \frac{\Delta}{J} \left| \text{Diagram 3} \right. \right] = -\delta_{J,0} \frac{1}{\lambda^{-1} + 2\widetilde{B}(\Delta)} \times$$

$$\times \frac{\Gamma_{\Delta_\phi-\frac{\Delta}{2}}^2 \Gamma_{\Delta_\phi-\frac{\widetilde{\Delta}}{2}}^2 \Gamma_{\frac{\Delta}{2}}^2 \Gamma_{\frac{\widetilde{\Delta}}{2}}^2}{4\pi^d \Gamma_{\Delta_\phi}^2 \Gamma_{1-\frac{d}{2}+\Delta_\phi}^2 \Gamma_{\Delta-\frac{d}{2}} \Gamma_{\widetilde{\Delta}-\frac{d}{2}}}$$

# Singlet sector — scalar non-MFT operators

$$\text{Spec}_s \left[ \begin{array}{c} \Delta \\ J \end{array} \middle| \text{Diagram} \right] \propto \frac{1}{\lambda^{-1} + 2\tilde{B}(\Delta)}$$

$\Rightarrow \lambda^{-1} + 2\tilde{B}(\Delta_{\bullet,0}^{(s)}) = 0$

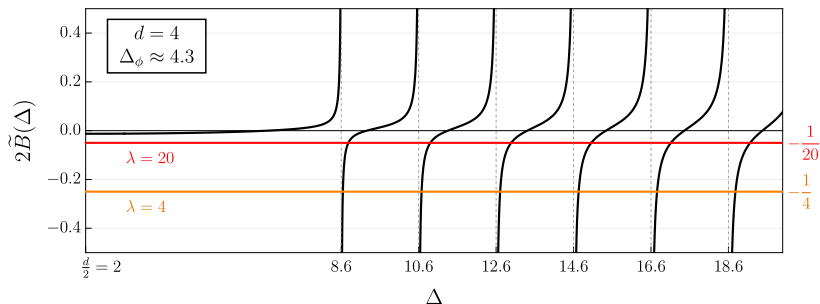




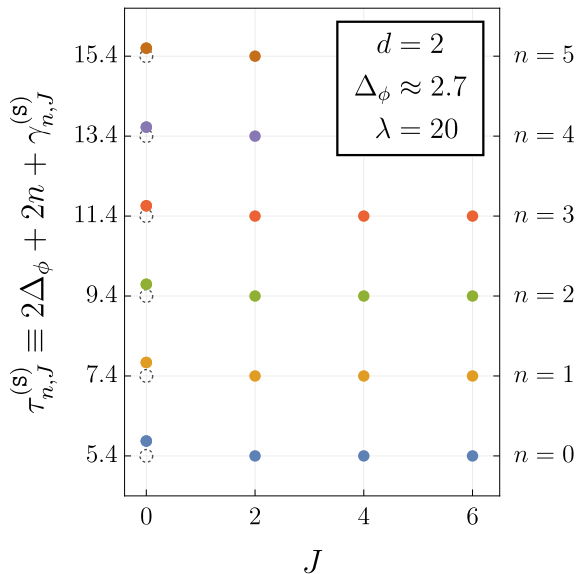
# Singlet sector — scalar non-MFT operators

$$\text{Spec}_s \left[ \begin{array}{c} \Delta \\ J \end{array} \middle| \begin{array}{c} \text{Diagram} \end{array} \right] \propto \frac{1}{\lambda^{-1} + 2\tilde{B}(\Delta)}$$

$$\Rightarrow \lambda^{-1} + 2\tilde{B}(\Delta_{\bullet,0}^{(s)}) = 0$$



# Singlet sector — twist–spin plot



# Non-singlet sector (MFT)

$$\begin{array}{c} \text{ST} \\ \text{AS} \end{array} = \left( \begin{array}{c} \text{Diagram 1} \end{array} \pm \begin{array}{c} \text{Diagram 2} \end{array} \right)$$

The diagram on the left is a circle with four vertices. The top two vertices are connected by an arc above the circle, and the bottom two vertices are connected by an arc below the circle. The interior of the circle is shaded gray and contains the text "ST" above "AS".

The first diagram in the parentheses is a circle with four vertices. The top two vertices are connected by an arc above the circle, and the bottom two vertices are connected by an arc below the circle.

The second diagram in the parentheses is a circle with four vertices. The top two vertices are connected by a straight line segment, and the bottom two vertices are connected by a straight line segment. The two straight line segments cross each other in the center of the circle.

# Non-singlet sector (MFT)

$$\text{Diagram} = \left( \text{Diagram}_1 \pm \text{Diagram}_2 \right)$$

$$[\mathcal{O}^i \times \mathcal{O}^j]^{(\text{ST})} \stackrel{N \rightarrow \infty}{\sim} \left[ \mathcal{O}^{\{i \square^n \partial_{\text{even}}^J \mathcal{O}^j\}} \right]^{(\text{ST})}$$

# Non-singlet sector (MFT)

$$\text{Diagram with central shaded circle labeled ST/AS} = \left( \text{Diagram with two horizontal lines} \pm \text{Diagram with two diagonal lines} \right)$$

$$[\mathcal{O}^i \times \mathcal{O}^j]^{(\text{ST})} \stackrel{N \rightarrow \infty}{\sim} \left[ \mathcal{O}^{\{i \square^n \partial_{\text{even}}^J \mathcal{O}^j\}} \right]^{(\text{ST})}$$

$$[\mathcal{O}^i \times \mathcal{O}^j]^{(\text{AS})} \stackrel{N \rightarrow \infty}{\sim} \left[ \mathcal{O}^{[i \square^n \partial_{\text{odd}}^J \mathcal{O}^j]} \right]^{(\text{AS})}$$

# Non-singlet sector (interacting)

$$\text{Diagram} = \left( \text{Diagram}_1 \pm \text{Diagram}_2 \right) + \frac{1}{N} \left( \text{Diagram}_3 \pm \text{Diagram}_4 \right)$$

$$[\mathcal{O}^i \times \mathcal{O}^j]^{(\text{ST})} \stackrel{N \rightarrow \infty}{\sim} \left[ \mathcal{O}^{\{i \square^n \partial_{\text{even}}^J \mathcal{O}^j\}} \right]^{(\text{ST})}$$

$$[\mathcal{O}^i \times \mathcal{O}^j]^{(\text{AS})} \stackrel{N \rightarrow \infty}{\sim} \left[ \mathcal{O}^{[i \square^n \partial_{\text{odd}}^J \mathcal{O}^j]} \right]^{(\text{AS})}$$

# Non-singlet sector (interacting)

$$\text{Diagram (ST, AS)} = \left( \text{Diagram 1} \pm \text{Diagram 2} \right) + \frac{1}{N} \left( \text{Diagram 3} \pm \text{Diagram 4} \right)$$

$$[\mathcal{O}^i \times \mathcal{O}^j]^{(\text{ST})} \stackrel{N \rightarrow \infty}{\sim} \left[ \mathcal{O}^{\{i \square^n \partial_{\text{even}}^J \mathcal{O}^j\}} \right]^{(\text{ST})}$$

$$[\mathcal{O}^i \times \mathcal{O}^j]^{(\text{AS})} \stackrel{N \rightarrow \infty}{\sim} \left[ \mathcal{O}^{[i \square^n \partial_{\text{odd}}^J \mathcal{O}^j]} \right]^{(\text{AS})}$$

$$[\mathcal{O}^i \times \mathcal{O}^j]^{(\text{ST})} \sim \left[ \mathcal{O}^{\{i \square^n \partial_{\text{even}}^J \mathcal{O}^j\}} \right]^{(\text{ST})} \frac{1}{N}$$

$$[\mathcal{O}^i \times \mathcal{O}^j]^{(\text{AS})} \sim \left[ \mathcal{O}^{[i \square^n \partial_{\text{odd}}^J \mathcal{O}^j]} \right]^{(\text{AS})} \frac{1}{N}$$

# Anomalous dimensions as double poles

$$\frac{-C_\star}{\Delta - \Delta_\star} \delta_{J,J_\star} \in K_{\tilde{\Delta},J} \operatorname{Spec}_s \left[ \frac{\Delta}{J} \mid \text{Diagram} \right]$$



# Anomalous dimensions as double poles

$$\frac{-C_\star}{\Delta - \Delta_\star} \delta_{J,J_\star} \in K_{\tilde{\Delta},J} \operatorname{Spec}_s \left[ \begin{array}{c} \Delta \\ J \end{array} \middle| \begin{array}{c} \text{Diagram} \end{array} \right]$$

$$\begin{array}{c} \text{Diagram 1} \end{array} = \begin{array}{c} \text{Diagram 2} \end{array} + \frac{1}{N} \begin{array}{c} \text{Diagram 3} \end{array} + \dots$$

# Anomalous dimensions as double poles

$$\frac{-C_\star}{\Delta - \Delta_\star} \delta_{J,J_\star} \in K_{\tilde{\Delta},J} \operatorname{Spec}_s \left[ \begin{array}{c} \Delta \\ J \end{array} \middle| \text{Diagram} \right]$$

$$\text{Diagram 1} = \text{Diagram 2} + \frac{1}{N} \text{Diagram 3} + \dots$$

$$\text{ope}^2[\mathcal{O}\mathcal{O}\mathcal{O}_\star] \equiv C_\star\left(\frac{1}{N}\right) = C_\star^{(\text{MFT})} + \frac{1}{N} C_\star^{(1)} + \mathcal{O}\left(\frac{1}{N^2}\right)$$

# Anomalous dimensions as double poles

$$\frac{-C_\star}{\Delta - \Delta_\star} \delta_{J,J_\star} \in K_{\tilde{\Delta},J} \text{Spec}_s \left[ \begin{array}{c} \Delta \\ J \end{array} \middle| \text{Diagram} \right]$$

$$\text{Diagram 1} = \text{Diagram 2} + \frac{1}{N} \text{Diagram 3} + \dots$$

$$\begin{aligned} \text{ope}^2[\mathcal{O}\mathcal{O}\mathcal{O}_\star] &\equiv C_\star\left(\frac{1}{N}\right) = C_\star^{(\text{MFT})} + \frac{1}{N} C_\star^{(1)} + O\left(\frac{1}{N^2}\right) \\ \Delta_\star\left(\frac{1}{N}\right) &= \Delta_\star^{(\text{MFT})} + \frac{1}{N} \gamma_\star^{(1)} + O\left(\frac{1}{N^2}\right) \end{aligned}$$

# Anomalous dimensions as (double) poles

$$\frac{-C_{\star}\left(\frac{1}{N}\right)}{\Delta - \Delta_{\star}\left(\frac{1}{N}\right)}$$

# Anomalous dimensions as (double) poles

$$\frac{-C_{\star}\left(\frac{1}{N}\right)}{\Delta - \Delta_{\star}\left(\frac{1}{N}\right)} = \frac{-C_{\star}^{(\text{MFT})}}{\Delta - \Delta_{\star}^{(\text{MFT})}}$$

# Anomalous dimensions as (double) poles

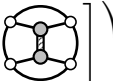
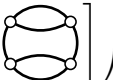
$$\frac{-C_{\star}\left(\frac{1}{N}\right)}{\Delta - \Delta_{\star}\left(\frac{1}{N}\right)} = \frac{-C_{\star}^{(\text{MFT})}}{\Delta - \Delta_{\star}^{(\text{MFT})}} + \frac{1}{N} \left[ \frac{-C_{\star}^{(1)}}{\Delta - \Delta_{\star}^{(\text{MFT})}} \right]$$

# Anomalous dimensions as (double) poles

$$\frac{-C_{\star}\left(\frac{1}{N}\right)}{\Delta - \Delta_{\star}\left(\frac{1}{N}\right)} = \frac{-C_{\star}^{(\text{MFT})}}{\Delta - \Delta_{\star}^{(\text{MFT})}} + \frac{1}{N} \left[ \frac{-C_{\star}^{(1)}}{\Delta - \Delta_{\star}^{(\text{MFT})}} + \frac{-C_{\star}^{(\text{MFT})} \gamma_{\star}^{(1)}}{\left(\Delta - \Delta_{\star}^{(\text{MFT})}\right)^2} \right]$$

# Anomalous dimensions as (double) poles

$$\frac{-C_{\star}\left(\frac{1}{N}\right)}{\Delta - \Delta_{\star}\left(\frac{1}{N}\right)} = \frac{-C_{\star}^{(\text{MFT})}}{\Delta - \Delta_{\star}^{(\text{MFT})}} + \frac{1}{N} \left[ \frac{-C_{\star}^{(1)}}{\Delta - \Delta_{\star}^{(\text{MFT})}} + \frac{-C_{\star}^{(\text{MFT})} \gamma_{\star}^{(1)}}{\left(\Delta - \Delta_{\star}^{(\text{MFT})}\right)^2} \right]$$

$$\gamma_{n,J}^{(1)} = \text{Res}_{\Delta=2\Delta_{\phi}+2n+J} \left( \frac{\text{Spec}_s \left[ \frac{\Delta}{J} \mid \text{Diagram 1} \right]}{\text{Spec}_s \left[ \frac{\Delta}{J} \mid \text{Diagram 2} \right]} \right)$$



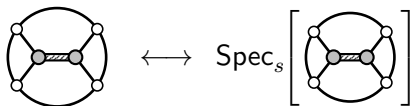


# Crossed channel contributions

Suppose we resolved the “direct”  $s$ -channel spectrum.

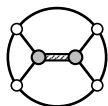
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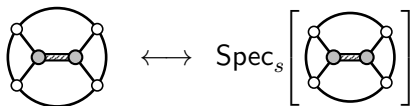
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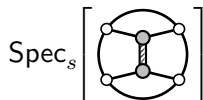
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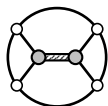
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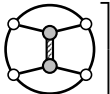


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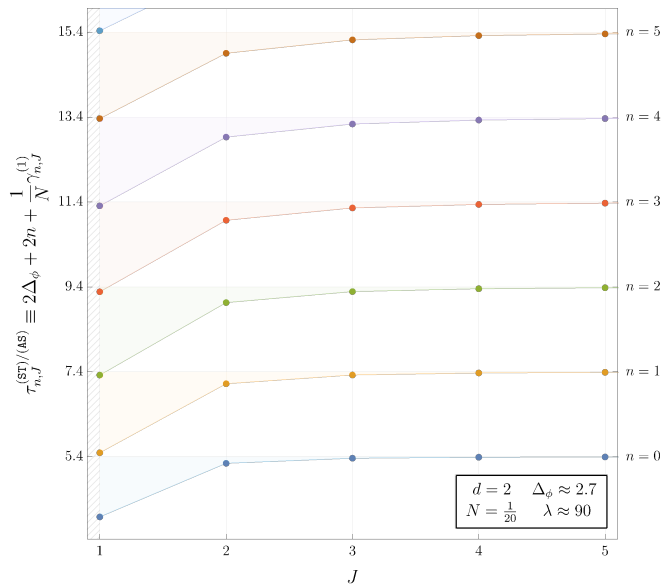
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# Non-singlet sector — twist–spin plot



# Non-singlet sector — large $J$ asymptotics

$$\tau_{n,J} \sim 2\Delta_\phi + 2n - \frac{c_n}{J^{\tau_{\min}}} + \dots$$

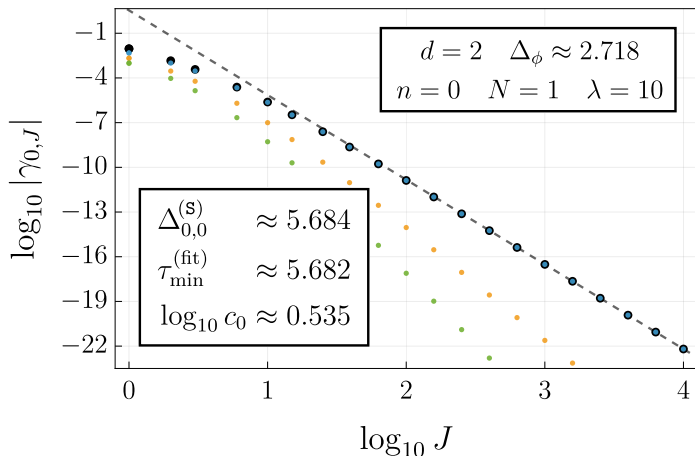
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More details in 2503.16345 [hep-th]

# Extra slides

Some extra slides.

# Hubbard–Stratonovich transformation

$$\mathcal{S}[\phi^\bullet] = \int_{\mathcal{M}} d^{d+1}x \sqrt{g} \left[ \frac{1}{2} (\partial \phi^\bullet)^2 + \frac{1}{2} m^2 (\phi^\bullet)^2 + \frac{\lambda}{2N} \left( (\phi^\bullet)^2 \right)^2 \right]$$

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$$\mathcal{S}_{\text{HS}}[\phi^\bullet, \sigma]$$



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$$\text{---} \equiv -\lambda \mathbb{1} \quad \text{---}^i \text{---}^j \equiv \frac{2}{\sqrt{N}} \delta^{ij}$$

# Exact $\sigma$ -propagator

$$\text{---}\bigcirc\text{---} \equiv -\lambda \mathbb{1} \qquad \begin{array}{c} i \\ \diagup \\ \bullet \\ \diagdown \\ j \end{array} \text{---} \equiv \frac{2}{\sqrt{N}} \delta^{ij}$$

## Exact $\sigma$ -propagator

$$\text{double line} \equiv -\lambda \mathbb{1} \qquad \text{trivalent vertex} \equiv \frac{2}{\sqrt{N}} \delta^{ij}$$

$$\sum_{n=0}^{\infty} \text{Diagram with } n \text{ bubbles}$$

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$$\text{shaded double line} =$$

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$$B(x, y) \equiv \frac{1}{2} x \text{---}\text{---} y \equiv \left[ \frac{1}{(-\square + m_\phi^2) \mathbb{1}} (x, y) \right]^2$$

# Utilizing the spectral representation

$$B(x, y) = \int_{\mathbb{R}} d\nu \tilde{B}(\Delta) \Omega_{\Delta}(x, y) \quad \left( \Delta \equiv \frac{d}{2} + i\nu \right)$$

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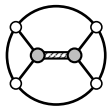
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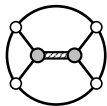




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$$\text{---} \text{---} \text{---} = 4 \int_{\mathbb{R}} d\nu \left( \frac{-1}{\lambda^{-1} + 2\tilde{B}(\Delta)} \right) \text{---} \text{---} \text{---}$$

$$= 4 \int_{\mathbb{R}} d\nu \left( - \frac{1}{\lambda^{-1} + 2\tilde{B}(\Delta)} \right) \sqrt{\mathbf{e}_{\Delta} \mathbf{e}_{\tilde{\Delta}}} \frac{\nu^2}{\pi} \text{---} \text{---} \text{---}$$

$$= \int_{\frac{d}{2} + i\mathbb{R}_{\geq 0}} \frac{d\Delta}{2\pi i} \left( - \frac{1}{\lambda^{-1} + 2\tilde{B}(\Delta)} \right) \left( \frac{\Gamma_{\dots}^2 \dots \Gamma_{\dots}}{\dots \Gamma_{\dots}^2 \dots \Gamma_{\dots}} \right) \text{---} \text{---} \text{---}$$

# Utilizing the spectral representation

$$B(x, y) = \int_{\mathbb{R}} d\nu \tilde{B}(\Delta) \Omega_{\Delta}(x, y) \quad \left( \Delta \equiv \frac{d}{2} + i\nu \right)$$

$$\text{---} \text{---} \text{---} = - \left[ \frac{1}{\lambda} + 2B \right]^{-1} = \int_{\mathbb{R}} d\nu \left( \frac{-1}{\lambda^{-1} + 2\tilde{B}(\Delta)} \right) \Omega_{\Delta}$$

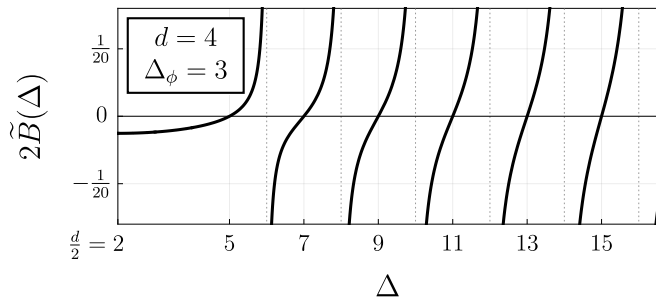
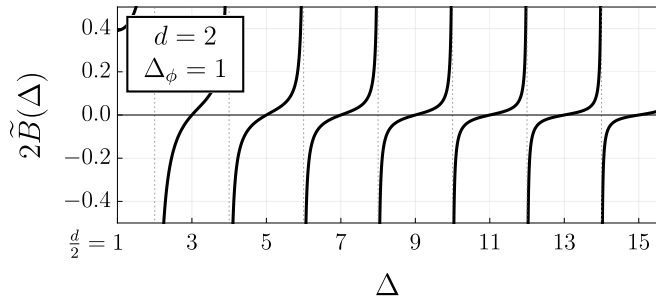
$$\text{---} \text{---} \text{---} = 4 \int_{\mathbb{R}} d\nu \left( \frac{-1}{\lambda^{-1} + 2\tilde{B}(\Delta)} \right) \text{---} \text{---} \text{---}$$

$$= 4 \int_{\mathbb{R}} d\nu \left( - \frac{1}{\lambda^{-1} + 2\tilde{B}(\Delta)} \right) \sqrt{\mathfrak{e}_{\Delta} \tilde{\mathfrak{e}}_{\Delta}} \frac{\nu^2}{\pi} \text{---} \text{---} \text{---}$$

$$= \int_{\frac{d}{2} + i\mathbb{R}_{\geq 0}} \frac{d\Delta}{2\pi i} \left( - \frac{1}{\lambda^{-1} + 2\tilde{B}(\Delta)} \right) \left( \frac{\Gamma_{\dots}^2 \dots \Gamma_{\dots}}{\dots \Gamma_{\dots}^2 \dots \Gamma_{\dots}} \right) \text{---} \text{---} \text{---}$$

$$\equiv \int_{\frac{d}{2} + i\mathbb{R}_{\geq 0}} \frac{d\Delta}{2\pi i} \left( - \frac{1}{\lambda^{-1} + 2\tilde{B}(\Delta)} \right) \left( \frac{\Gamma_{\dots}^2 \dots \Gamma_{\dots}}{\dots \Gamma_{\dots}^2 \dots \Gamma_{\dots}} \right) | \text{---} \text{---} \text{---} \rangle$$

# Criticality in the bulk



# Criticality in the bulk

