



FACULTY
OF MATHEMATICS
AND PHYSICS
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Finite-coupling spectrum of $O(N)$ model in AdS

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$O(N)$ model

N scalar fields $\{\phi^i\}$

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$$\mathcal{S}[\phi^\bullet] = \int_{\mathcal{M}} d^{d+1}x \sqrt{g} \left[\frac{1}{2} (\partial \phi^\bullet)^2 + \frac{1}{2} m^2 (\phi^\bullet)^2 \right]$$

$O(N)$ model

N scalar fields $\{\phi^i\}$

$$\mathcal{S}[\phi^\bullet] = \int_{\mathcal{M}} d^{d+1}x \sqrt{g} \left[\frac{1}{2} (\partial \phi^\bullet)^2 + \frac{1}{2} m^2 (\phi^\bullet)^2 + \frac{\lambda}{2N} \left((\phi^\bullet)^2 \right)^2 \right]$$

$O(N)$ model

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$$\text{X}_{\bullet} \sim \frac{\lambda}{N} \left(\text{X}_{\bullet}^{\text{left}} + \text{X}_{\bullet}^{\text{right}} + \text{X}_{\bullet}^{\text{diag3}} \right)$$

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$$\text{X} \sim \frac{\lambda}{N} \left(\text{X} + \text{X} + \text{X} \right)$$

large N expansion

$O(N)$ model

N scalar fields $\{\phi^i\}$

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$$\text{four-point vertex} \sim \frac{\lambda}{N} \left(\text{t-channel} + \text{s-channel} + \text{u-channel} \right)$$

large N expansion $\xrightarrow{\text{allows}}$ finite coupling λ

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large N expansion $\xrightarrow{\text{allows}}$ finite coupling λ

$$\sum_{n=0}^{\infty} \text{X} \underbrace{\text{O} \cdots \text{O}}_{n \text{ bubbles}} \text{X}$$

$O(N)$ model

N scalar fields $\{\phi^i\}$

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large N expansion $\xrightarrow{\text{allows}}$ finite coupling λ

$$\sum_{n=0}^{\infty} \text{X} \underbrace{\text{O} \cdots \text{O}}_{n \text{ bubbles}} \text{X} \sim \frac{1}{N}$$

Hubbard–Stratonovich transformation

$$\mathcal{S}[\phi^\bullet] = \int_{\mathcal{M}} d^{d+1}x \sqrt{g} \left[\frac{1}{2} (\partial \phi^\bullet)^2 + \frac{1}{2} m^2 (\phi^\bullet)^2 + \frac{\lambda}{2N} \left((\phi^\bullet)^2 \right)^2 \right]$$

$$\text{X} \sim \frac{\lambda}{N} \left(\text{><} + \text{X} + \text{X} \right)$$

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$$\text{X} \sim \frac{\lambda}{N} \left(\text{><} + \text{X} + \text{X} \right)$$

$$\mathcal{S}_{\text{HS}}[\phi^\bullet, \sigma]$$

Hubbard–Stratonovich transformation

$$\mathcal{S}[\phi^\bullet] = \int_{\mathcal{M}} d^{d+1}x \sqrt{g} \left[\frac{1}{2} (\partial \phi^\bullet)^2 + \frac{1}{2} m^2 (\phi^\bullet)^2 + \frac{\lambda}{2N} \left((\phi^\bullet)^2 \right)^2 \right]$$

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$$\begin{aligned} \mathcal{S}_{\text{HS}}[\phi^\bullet, \sigma] = \int_{\mathcal{M}} d^{d+1}x \sqrt{g} \left[\frac{1}{2} (\partial \phi^\bullet)^2 + \frac{1}{2} m^2 (\phi^\bullet)^2 \right. \\ \left. - \frac{1}{2\lambda} \sigma^2 + \frac{1}{\sqrt{N}} \sigma (\phi^\bullet)^2 \right] \end{aligned}$$

Hubbard–Stratonovich transformation

$$\mathcal{S}[\phi^\bullet] = \int_{\mathcal{M}} d^{d+1}x \sqrt{g} \left[\frac{1}{2} (\partial \phi^\bullet)^2 + \frac{1}{2} m^2 (\phi^\bullet)^2 + \frac{\lambda}{2N} \left((\phi^\bullet)^2 \right)^2 \right]$$

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$$\text{---} \equiv -\lambda \mathbb{1} \qquad \text{---}^i \text{---}^j \equiv \frac{2}{\sqrt{N}} \delta^{ij}$$

Exact σ -propagator

$$\text{---}\bigcirc\text{---} \equiv -\lambda \mathbb{1} \qquad \begin{array}{c} i \\ \diagdown \\ \bullet \\ \diagup \\ j \end{array} \text{---} \equiv \frac{2}{\sqrt{N}} \delta^{ij}$$

Exact σ -propagator

$$\text{---}\equiv -\lambda \mathbb{1} \qquad \begin{array}{c} i \\ \diagdown \\ \text{---}\bullet\text{---} \\ \diagup \\ j \end{array} \equiv \frac{2}{\sqrt{N}} \delta^{ij}$$

$$\sum_{n=0}^{\infty} \text{---}\text{---}\underbrace{\text{---}\text{---}\text{---}\text{---}\text{---}\text{---}\text{---}\text{---}\text{---}\text{---}}_{n \text{ bubbles}}\text{---}\text{---}$$

Exact σ -propagator

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$$\sum_{n=0}^{\infty} \text{---} \underbrace{\text{---} \text{---} \text{---} \text{---} \text{---} \text{---}}_{n \text{ bubbles}} \text{---} = \text{---} \text{---} \text{---} \sim \frac{1}{N}$$

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$$\sum_{n=0}^{\infty} \text{---} \underbrace{\text{---} \cdots \text{---}}_{n \text{ bubbles}} \text{---} = \text{---} \text{---} \text{---} \sim \frac{1}{N}$$

$$\text{---} =$$

Exact σ -propagator

$$\text{double line} \equiv -\lambda \mathbb{1} \qquad \text{trivalent vertex} \equiv \frac{2}{\sqrt{N}} \delta^{ij}$$

$$\sum_{n=0}^{\infty} \text{diagram with } n \text{ bubbles} = \text{diagram with two shaded vertices and a shaded line} \sim \frac{1}{N}$$

$$\text{double line with a hatched segment} = \text{double line}$$

Exact σ -propagator

$$\text{---}\text{---} \equiv -\lambda \mathbb{1} \qquad \begin{array}{c} i \\ \diagdown \\ \text{---}\text{---} \\ \diagup \\ j \end{array} \equiv \frac{2}{\sqrt{N}} \delta^{ij}$$

$$\sum_{n=0}^{\infty} \text{---}\text{---} \underbrace{\text{---}\text{---} \text{---}\text{---} \text{---}\text{---} \text{---}\text{---}}_{n \text{ bubbles}} \text{---}\text{---} = \text{---}\text{---} \text{---}\text{---} \text{---}\text{---} \sim \frac{1}{N}$$

$$\text{---}\text{---} = \text{---}\text{---} + \text{---}\text{---} \text{---}\text{---}$$

Exact σ -propagator

$$\text{---} \equiv -\lambda \mathbb{1} \qquad \begin{array}{c} i \\ \diagup \\ \text{---} \\ \diagdown \\ j \end{array} \equiv \frac{2}{\sqrt{N}} \delta^{ij}$$

$$\sum_{n=0}^{\infty} \text{---} \underbrace{\text{---} \text{---} \text{---} \text{---} \text{---}}_{n \text{ bubbles}} \text{---} = \text{---} \text{---} \text{---} \text{---} \text{---} \sim \frac{1}{N}$$

$$\text{---} = \text{---} + \text{---} \text{---} \text{---} + \text{---} \text{---} \text{---} \text{---} + \dots$$

Exact σ -propagator

$$\text{---}\text{---}\text{---} \equiv -\lambda \mathbb{1} \qquad \begin{array}{c} i \\ \diagdown \\ \text{---}\text{---}\text{---} \\ \diagup \\ j \end{array} \equiv \frac{2}{\sqrt{N}} \delta^{ij}$$

$$\sum_{n=0}^{\infty} \text{---}\text{---}\text{---} \underbrace{\text{---}\text{---}\text{---} \cdots \text{---}\text{---}\text{---}}_{n \text{ bubbles}} \text{---}\text{---}\text{---} = \text{---}\text{---}\text{---} \text{---}\text{---}\text{---} \sim \frac{1}{N}$$

$$\begin{aligned} \text{---}\text{---}\text{---} &= \text{---}\text{---}\text{---} + \text{---}\text{---}\text{---} \text{---}\text{---}\text{---} + \text{---}\text{---}\text{---} \text{---}\text{---}\text{---} \text{---}\text{---}\text{---} + \dots \\ &= (-\lambda \mathbb{1}) + (-\lambda \mathbb{1}) \circ 2B \circ (-\lambda \mathbb{1}) + \dots \end{aligned}$$

Exact σ -propagator

$$\text{---}\text{---}\text{---} \equiv -\lambda \mathbb{1} \qquad \begin{array}{c} i \\ \diagdown \\ \text{---}\text{---}\text{---} \\ \diagup \\ j \end{array} \equiv \frac{2}{\sqrt{N}} \delta^{ij}$$

$$\sum_{n=0}^{\infty} \text{---}\text{---}\text{---} \underbrace{\text{---}\text{---}\text{---} \text{---}\text{---}\text{---} \text{---}\text{---}\text{---}}_{n \text{ bubbles}} \text{---}\text{---}\text{---} = \text{---}\text{---}\text{---} \text{---}\text{---}\text{---} \text{---}\text{---}\text{---} \sim \frac{1}{N}$$

$$\begin{aligned} \text{---}\text{---}\text{---} &= \text{---}\text{---}\text{---} + \text{---}\text{---}\text{---} \text{---}\text{---}\text{---} \text{---}\text{---}\text{---} + \text{---}\text{---}\text{---} \text{---}\text{---}\text{---} \text{---}\text{---}\text{---} \text{---}\text{---}\text{---} + \dots \\ &= (-\lambda \mathbb{1}) + (-\lambda \mathbb{1}) \circ 2B \circ (-\lambda \mathbb{1}) + \dots \\ &= -\lambda \sum_{n=0}^{\infty} (-2\lambda B)^n \end{aligned}$$

Exact σ -propagator

$$\text{---}\text{---}\text{---} \equiv -\lambda \mathbb{1} \qquad \begin{array}{c} i \\ \diagdown \\ \text{---}\text{---}\text{---} \\ \diagup \\ j \end{array} \equiv \frac{2}{\sqrt{N}} \delta^{ij}$$

$$\sum_{n=0}^{\infty} \text{---}\text{---}\text{---} \underbrace{\text{---}\text{---}\text{---} \cdots \text{---}\text{---}\text{---}}_{n \text{ bubbles}} \text{---}\text{---}\text{---} = \text{---}\text{---}\text{---} \text{---}\text{---}\text{---} \sim \frac{1}{N}$$

$$\begin{aligned} \text{---}\text{---}\text{---} &= \text{---}\text{---}\text{---} + \text{---}\text{---}\text{---} \text{---}\text{---}\text{---} + \text{---}\text{---}\text{---} \text{---}\text{---}\text{---} \text{---}\text{---}\text{---} + \dots \\ &= (-\lambda \mathbb{1}) + (-\lambda \mathbb{1}) \circ 2B \circ (-\lambda \mathbb{1}) + \dots \\ &= -\lambda \sum_{n=0}^{\infty} (-2\lambda B)^n = -\left[\frac{\mathbb{1}}{\lambda} + 2B \right]^{-1} \end{aligned}$$

Exact σ -propagator

$$\text{---}\text{---} \equiv -\lambda \mathbb{1} \qquad \begin{array}{c} i \\ \diagdown \\ \text{---}\text{---} \\ \diagup \\ j \end{array} \equiv \frac{2}{\sqrt{N}} \delta^{ij}$$

$$\sum_{n=0}^{\infty} \text{---}\text{---} \underbrace{\text{---}\text{---} \text{---}\text{---} \text{---}\text{---} \text{---}\text{---}}_{n \text{ bubbles}} \text{---}\text{---} = \text{---}\text{---} \text{---}\text{---} \text{---}\text{---} \text{---}\text{---} \sim \frac{1}{N}$$

$$\text{---}\text{---} = \text{---}\text{---} + \text{---}\text{---} \text{---}\text{---} \text{---}\text{---} + \text{---}\text{---} \text{---}\text{---} \text{---}\text{---} \text{---}\text{---} + \dots$$

$$= (-\lambda \mathbb{1}) + (-\lambda \mathbb{1}) \circ 2B \circ (-\lambda \mathbb{1}) + \dots$$

$$= -\lambda \sum_{n=0}^{\infty} (-2\lambda B)^n = -\left[\frac{\mathbb{1}}{\lambda} + 2B \right]^{-1}$$

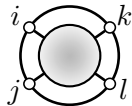
$$B(x, y) \equiv \frac{1}{2} x \text{---}\text{---} y \equiv \left[\frac{1}{(-\square + m_\phi^2) \mathbb{1}} (x, y) \right]^2$$

$O(N)$ model in **AdS**

fixed AdS background

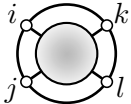
$O(N)$ model in **AdS**

fixed AdS background



$O(N)$ model in **AdS**

fixed AdS background

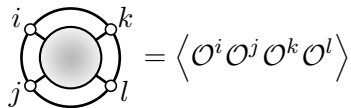


A Feynman diagram representing a bubble with four external legs. The bubble is a shaded circle with a thick black border. Four small white circles are attached to the outer edge of the bubble at the top-left, top-right, bottom-left, and bottom-right positions. These are labeled with italicized letters i , k , j , and l respectively. To the right of the diagram is an equals sign followed by a bracketed expression: $= \langle \mathcal{O}^i \mathcal{O}^j \mathcal{O}^k \mathcal{O}^l \rangle$.

$$= \langle \mathcal{O}^i \mathcal{O}^j \mathcal{O}^k \mathcal{O}^l \rangle$$

$O(N)$ model in **AdS**

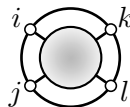
fixed AdS background


$$= \langle \mathcal{O}^i \mathcal{O}^j \mathcal{O}^k \mathcal{O}^l \rangle$$

$$\mathcal{O}^i(P) \sim \lim_{s \rightarrow \infty} s^{\Delta_\phi} \phi^i(X \equiv sP + \dots)$$

$O(N)$ model in **AdS**

fixed AdS background

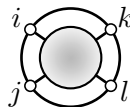

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CFT

$O(N)$ model in **AdS**

fixed AdS background

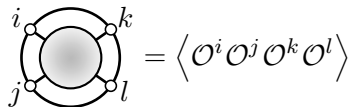

$$= \langle \mathcal{O}^i \mathcal{O}^j \mathcal{O}^k \mathcal{O}^l \rangle$$

$$\mathcal{O}^i(P) \sim \lim_{s \rightarrow \infty} s^{\Delta_\phi} \phi^i(X \equiv sP + \dots)$$

QFT in AdS \longleftrightarrow CFT

$O(N)$ model in **AdS**

fixed AdS background


$$= \langle \mathcal{O}^i \mathcal{O}^j \mathcal{O}^k \mathcal{O}^l \rangle$$

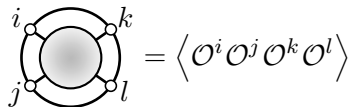
$$\mathcal{O}^i(P) \sim \lim_{s \rightarrow \infty} s^{\Delta_\phi} \phi^i(X \equiv sP + \dots)$$

QFT in AdS \longleftrightarrow CFT

CFT in AdS_{d+1}

$O(N)$ model in **AdS**

fixed AdS background


$$= \langle \mathcal{O}^i \mathcal{O}^j \mathcal{O}^k \mathcal{O}^l \rangle$$

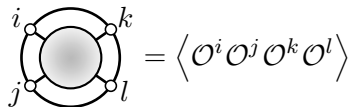
$$\mathcal{O}^i(P) \sim \lim_{s \rightarrow \infty} s^{\Delta_\phi} \phi^i(X \equiv sP + \dots)$$

QFT in AdS \longleftrightarrow CFT

CFT in AdS_{d+1} $\xleftrightarrow{\text{Weyl transformation}}$

$O(N)$ model in **AdS**

fixed AdS background

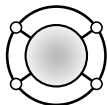

$$= \langle \mathcal{O}^i \mathcal{O}^j \mathcal{O}^k \mathcal{O}^l \rangle$$

$$\mathcal{O}^i(P) \sim \lim_{s \rightarrow \infty} s^{\Delta_\phi} \phi^i(X \equiv sP + \dots)$$

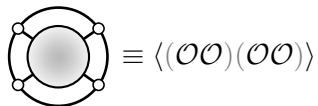
QFT in AdS \longleftrightarrow CFT

$$\text{CFT in AdS}_{d+1} \xleftrightarrow{\text{Weyl transformation}} \text{BCFT in } \mathbb{R}^d \times \mathbb{R}_{\geq}$$

Conformal Block/Partial Wave Decomposition



Conformal Block/Partial Wave Decomposition


$$\equiv \langle (\mathcal{O}\mathcal{O})(\mathcal{O}\mathcal{O}) \rangle$$

Conformal Block/Partial Wave Decomposition

$$\begin{array}{c} \text{Diagram: A central shaded circle with four small white circles at the top, bottom, left, and right. Each small circle is connected to the central circle by a line segment. The entire structure is enclosed within a larger circle.} \end{array} \equiv \langle (\mathcal{O}\mathcal{O})(\mathcal{O}\mathcal{O}) \rangle = \sum_{\substack{\text{primary } \mathcal{O}_\star \\ \text{with } \Delta_\star, J_\star}} \text{ope}^2[\mathcal{O}\mathcal{O}\mathcal{O}_\star] \left| G_{\Delta_\star, J_\star}^{(s)} \right\rangle$$

Conformal Block/Partial Wave Decomposition

$$\text{Diagram} \equiv \langle (\mathcal{O}\mathcal{O})(\mathcal{O}\mathcal{O}) \rangle = \sum_{\substack{\text{primary } \mathcal{O}_\star \\ \text{with } \Delta_\star, J_\star}} \text{ope}^2[\mathcal{O}\mathcal{O}\mathcal{O}_\star] \left| G_{\Delta_\star, J_\star}^{(s)} \right\rangle$$

$$\text{Diagram} = \sum_{J=0}^{\infty} \int_{\frac{d}{2} + i\mathbb{R}_{\geq}} \frac{d\Delta}{2\pi i} \text{Spec}_s \left[\begin{array}{c} \Delta \\ J \end{array} \middle| \text{Diagram} \right] \left| \text{Diagram} \right\rangle$$

Conformal Block/Partial Wave Decomposition

$$\begin{array}{c} \text{Diagram: A sphere with four external legs (two on the left, two on the right) connected by arcs.} \end{array} \equiv \langle (\mathcal{O}\mathcal{O})(\mathcal{O}\mathcal{O}) \rangle = \sum_{\substack{\text{primary } \mathcal{O}_\star \\ \text{with } \Delta_\star, J_\star}} \text{ope}^2[\mathcal{O}\mathcal{O}\mathcal{O}_\star] \left| G_{\Delta_\star, J_\star}^{(s)} \right\rangle$$

$$\begin{array}{c} \text{Diagram: A sphere with four external legs (two on the left, two on the right) connected by arcs.} \end{array} = \sum_{J=0}^{\infty} \int_{\frac{d}{2} + i\mathbb{R}_{\geq}} \frac{d\Delta}{2\pi i} \text{Spec}_s \left[\begin{array}{c} \Delta \\ J \end{array} \middle| \begin{array}{c} \text{Diagram: A sphere with four external legs (two on the left, two on the right) connected by arcs.} \end{array} \right] \left| \begin{array}{c} \text{Diagram: Two external legs connected by a horizontal line with dots at the ends.} \end{array}^{\Delta, J} \right\rangle$$

$$\left| \begin{array}{c} \text{Diagram: Two external legs connected by a horizontal line with dots at the ends.} \end{array}^{\Delta, J} \right\rangle = K_{\tilde{\Delta}, J} \left| G_{\Delta, J}^{(s)} \right\rangle + K_{\Delta, J} \left| G_{\tilde{\Delta}, J}^{(s)} \right\rangle$$

Conformal Block/Partial Wave Decomposition

$$\text{Diagram} \equiv \langle (\mathcal{O}\mathcal{O})(\mathcal{O}\mathcal{O}) \rangle = \sum_{\substack{\text{primary } \mathcal{O}_\star \\ \text{with } \Delta_\star, J_\star}} \text{ope}^2[\mathcal{O}\mathcal{O}\mathcal{O}_\star] \left| G_{\Delta_\star, J_\star}^{(s)} \right\rangle$$

$$\text{Diagram} = \sum_{J=0}^{\infty} \int_{\frac{d}{2} + i\mathbb{R}_{\geq}} \frac{d\Delta}{2\pi i} \text{Spec}_s \left[\begin{matrix} \Delta \\ J \end{matrix} \middle| \text{Diagram} \right] \left| \text{Diagram} \right\rangle$$

$$\begin{aligned}
 \left| \text{Diagram} \right\rangle &= K_{\tilde{\Delta}, J} \left| G_{\Delta, J}^{(s)} \right\rangle + K_{\Delta, J} \left| G_{\tilde{\Delta}, J}^{(s)} \right\rangle \\
 \left| \text{Diagram} \right\rangle &= K_{\tilde{\Delta}', J'} \left| G_{\Delta', J'}^{(t)} \right\rangle + K_{\Delta', J'} \left| G_{\tilde{\Delta}', J'}^{(t)} \right\rangle
 \end{aligned}$$

Conformal Block/Partial Wave Decomposition

$$\text{Diagram} = \sum_J \int_{\frac{d}{2} + i\mathbb{R}} \frac{d\Delta}{2\pi i} \text{Spec}_s \left[\frac{\Delta}{J} \middle| \text{Diagram} \right] K_{\tilde{\Delta}, J} \left| G_{\Delta, J}^{(s)} \right\rangle$$

Conformal Block/Partial Wave Decomposition

$$\begin{aligned}
 \text{Diagram} &= \sum_J \int_{\frac{d}{2} + i\mathbb{R}} \frac{d\Delta}{2\pi i} \text{Spec}_s \left[\frac{\Delta}{J} \middle| \text{Diagram} \right] K_{\tilde{\Delta}, J} \left| G_{\Delta, J}^{(s)} \right\rangle \\
 &= \sum_{\substack{\text{primary } \mathcal{O}_\star \\ \text{with } \Delta_\star, J_\star}} \text{o pe}^2[\mathcal{O}\mathcal{O}\mathcal{O}_\star] \left| G_{\Delta_\star, J_\star}^{(s)} \right\rangle
 \end{aligned}$$

Conformal Block/Partial Wave Decomposition

$$\begin{aligned}
 \text{Diagram} &= \sum_J \int_{\frac{d}{2} + i\mathbb{R}} \frac{d\Delta}{2\pi i} \text{Spec}_s \left[\frac{\Delta}{J} \middle| \text{Diagram} \right] K_{\tilde{\Delta}, J} \left| G_{\Delta, J}^{(s)} \right\rangle \\
 &= \sum_{\substack{\text{primary } \mathcal{O}_\star \\ \text{with } \Delta_\star, J_\star}} \text{ope}^2[\mathcal{O}\mathcal{O}\mathcal{O}_\star] \left| G_{\Delta_\star, J_\star}^{(s)} \right\rangle \\
 \frac{-C_\star}{\Delta - \Delta_\star} \delta_{J, J_\star} &\in K_{\tilde{\Delta}, J} \text{Spec}_s \left[\frac{\Delta}{J} \middle| \text{Diagram} \right]
 \end{aligned}$$

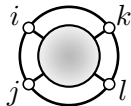
Conformal Block/Partial Wave Decomposition

$$\begin{aligned}
 \text{Diagram} &= \sum_J \int_{\frac{d}{2} + i\mathbb{R}} \frac{d\Delta}{2\pi i} \text{Spec}_s \left[\frac{\Delta}{J} \middle| \text{Diagram} \right] K_{\tilde{\Delta}, J} \left| G_{\Delta, J}^{(s)} \right\rangle \\
 &= \sum_{\substack{\text{primary } \mathcal{O}_\star \\ \text{with } \Delta_\star, J_\star}} \text{ope}^2[\mathcal{O}\mathcal{O}\mathcal{O}_\star] \left| G_{\Delta_\star, J_\star}^{(s)} \right\rangle
 \end{aligned}$$

$$\frac{-C_\star}{\Delta - \Delta_\star} \delta_{J, J_\star} \in K_{\tilde{\Delta}, J} \text{Spec}_s \left[\frac{\Delta}{J} \middle| \text{Diagram} \right]$$

$$\text{ope}^2[\mathcal{O}\mathcal{O}\mathcal{O}_\star] \equiv C_\star = -\text{Res}_{\Delta=\Delta_\star} \left(K_{\tilde{\Delta}, J_\star} \text{Spec}_s \left[\frac{\Delta}{J_\star} \middle| \text{Diagram} \right] \right)$$

Boundary 4-point correlator in **AdS**



Boundary 4-point correlator in **AdS**

$$\text{Diagram 1} = \left(\text{Diagram 2} + \text{Diagram 3} + \text{Diagram 4} \right)$$

The diagram on the left shows a central shaded circle with four external lines labeled i , j , k , and l at the top, bottom, right, and left respectively. The diagram on the right is a sum of three terms in large parentheses. The first term is a circle with four external lines labeled i , j , k , and l at the top, bottom, right, and left respectively, with two internal lines connecting the top and bottom vertices. The second term is a circle with four external lines labeled i , j , k , and l at the top, bottom, right, and left respectively, with two internal lines connecting the top and bottom vertices. The third term is a circle with four external lines labeled i , j , k , and l at the top, bottom, right, and left respectively, with two internal lines connecting the top and bottom vertices.

Boundary 4-point correlator in **AdS**

$$\begin{aligned}
 \text{Diagram 1} &= \left(\text{Diagram 2} + \text{Diagram 3} + \text{Diagram 4} \right) \\
 &+ \frac{1}{N} \left(\text{Diagram 5} + \text{Diagram 6} + \text{Diagram 7} \right)
 \end{aligned}$$

The diagrams represent Feynman diagrams for a boundary 4-point correlator in AdS. The external legs are labeled i, k, j, l . The first diagram on the left is a bubble with a shaded central region. The first row of diagrams on the right shows three tree-level diagrams: two exchange diagrams and one contact diagram. The second row, multiplied by $1/N$, shows three loop-level diagrams: a box diagram and two contact diagrams with internal vertices.

Boundary 4-point correlator in **AdS**

$$\begin{aligned}
 \text{Bubble with shaded center} &= \left(\text{Diagram 1} + \text{Diagram 2} + \text{Diagram 3} \right) \\
 &+ \frac{1}{N} \left(\text{Diagram 4} + \text{Diagram 5} + \text{Diagram 6} \right)
 \end{aligned}$$

The diagrams are:

- Diagram 1: A circle with four external legs labeled i, k, j, l . The top and bottom arcs are connected by two vertical lines.
- Diagram 2: A circle with four external legs labeled i, k, j, l . The left and right arcs are connected by two horizontal lines.
- Diagram 3: A circle with four external legs labeled i, k, j, l . The top and bottom arcs are connected by two diagonal lines crossing in the center.
- Diagram 4: A circle with four external legs labeled i, k, j, l . The top and bottom arcs are connected by two horizontal lines, each with a shaded vertex in the center.
- Diagram 5: A circle with four external legs labeled i, k, j, l . The left and right arcs are connected by two vertical lines, each with a shaded vertex in the center.
- Diagram 6: A circle with four external legs labeled i, k, j, l . The top and bottom arcs are connected by two diagonal lines, each with a shaded vertex in the center.

$$\text{Bubble with shaded center} \sim \underbrace{\text{Diagram 7} (\dots) \text{Diagram 8}}_{n \text{ bubbles}}$$

The diagrams are:

- Diagram 7: A circle with four external legs labeled i, k, j, l . The top and bottom arcs are connected by two horizontal lines, each with a shaded vertex in the center.
- Diagram 8: A circle with four external legs labeled i, k, j, l . The left and right arcs are connected by two vertical lines, each with a shaded vertex in the center.

Boundary 4-point correlator in AdS

$$\begin{aligned}
 \text{Diagram 1} &= \left(\text{Diagram 2} + \text{Diagram 3} + \text{Diagram 4} \right) \\
 &+ \frac{1}{N} \left(\text{Diagram 5} + \text{Diagram 6} + \text{Diagram 7} \right)
 \end{aligned}$$

The diagrams represent various Feynman-like diagrams for a 4-point correlator. Diagram 1 is a circle with four external legs labeled i, k, j, l and a shaded central blob. Diagrams 2, 3, and 4 are circles with four external legs and internal lines connecting the legs. Diagrams 5, 6, and 7 are similar to Diagram 1 but with a shaded internal line and a central blob.

$$\text{Diagram 8} \sim \underbrace{\text{Diagram 9} (\dots \text{Diagram 10})}_{n \text{ bubbles}} \Rightarrow \text{Diagram 11} \sim \delta^{ij}$$

Diagram 8 is a four-point vertex with a shaded internal line. Diagram 9 is a chain of bubbles. Diagram 10 is a single bubble. Diagram 11 is a three-point vertex with a shaded internal line. The equation shows that the four-point correlator is equivalent to a chain of bubbles, which then simplifies to a three-point vertex proportional to δ^{ij} .

Boundary 4-point correlator in **AdS**

$$\begin{aligned}
 \text{Diagram 1} &= \left(\text{Diagram 2} + \text{Diagram 3} + \text{Diagram 4} \right) \\
 &+ \frac{1}{N} \left(\text{Diagram 5} + \text{Diagram 6} + \text{Diagram 7} \right)
 \end{aligned}$$

The diagrams represent various Feynman-like diagrams in AdS. Diagram 1 is a sphere with four external legs labeled i, k, j, l . Diagrams 2, 3, and 4 are spheres with two internal lines. Diagrams 5, 6, and 7 are spheres with two internal lines and a shaded region.

$$\text{Diagram 8} \sim \underbrace{\text{Diagram 9} (\dots \text{Diagram 10})}_{n \text{ bubbles}} \Rightarrow \text{Diagram 11} \sim \delta^{ij}$$

Diagram 8 is a shaded region with four external legs. Diagram 9 is a chain of bubbles. Diagram 10 is a single bubble. Diagram 11 is a shaded region with two external legs labeled i and j .

$$\text{Diagram 1} = \delta^{ij} \delta^{kl} \left(\text{Diagram 2} + \frac{1}{N} \text{Diagram 5} \right)$$

Boundary 4-point correlator in **AdS**

$$\begin{aligned}
 \text{Diagram 1} &= \left(\text{Diagram 2} + \text{Diagram 3} + \text{Diagram 4} \right) \\
 &+ \frac{1}{N} \left(\text{Diagram 5} + \text{Diagram 6} + \text{Diagram 7} \right)
 \end{aligned}$$

The diagrams represent various Feynman-like diagrams in AdS. Diagram 1 is a sphere with four external legs labeled i, k, j, l . Diagrams 2, 3, and 4 are spheres with two internal lines. Diagrams 5, 6, and 7 are spheres with two internal lines and a shaded region.

$$\text{Diagram 8} \sim \underbrace{\text{Diagram 9} (\dots \text{Diagram 10})}_{n \text{ bubbles}} \Rightarrow \text{Diagram 11} \sim \delta^{ij}$$

Diagram 8 is a shaded region with four external legs. Diagram 9 is a chain of bubbles. Diagram 10 is a single bubble. Diagram 11 is a shaded region with two external legs.

$$\text{Diagram 1} = \delta^{ij} \delta^{kl} \left(\text{Diagram 2} + \frac{1}{N} \text{Diagram 5} \right) + \text{crossed channels}$$

Decomposition into $\mathbf{O}(N)$ irreps

$$\begin{array}{c} i \\ \circ \\ \text{---} \circ \text{---} k \\ \backslash \quad / \\ \text{---} \circ \text{---} l \\ / \quad \backslash \\ \circ \\ j \end{array} = \overbrace{N\mathcal{P}_{(\text{S})}^{ijkl}}^{\delta^{ij}\delta^{kl}} \begin{array}{c} \text{S} \\ \circ \\ \text{---} \circ \text{---} \\ \backslash \quad / \\ \text{---} \circ \text{---} \\ / \quad \backslash \\ \circ \end{array}$$

Decomposition into $\mathbf{0}(N)$ irreps

$$\begin{array}{c} i \\ \circ \\ \text{---} \circ \text{---} k \\ \text{---} \circ \text{---} l \\ \circ \\ j \end{array} = \overbrace{N \mathcal{P}_{(\text{S})}^{ijkl}}^{\delta^{ij} \delta^{kl}} \begin{array}{c} \text{S} \end{array} + \overbrace{\mathcal{P}_{(\text{AS})}^{ijkl}}^{\delta_{[k}^{[i} \delta_{l]}^{j]}} \begin{array}{c} \text{AS} \end{array}$$

Decomposition into $\mathbf{0}(N)$ irreps

$$\begin{array}{c} i \\ \circ \\ \text{---} \circ \text{---} k \\ \text{---} \circ \text{---} l \\ \circ \\ j \end{array} = \overbrace{N\mathcal{P}_{(\text{S})}^{ijkl}}^{\delta^{ij}\delta^{kl}} \begin{array}{c} \text{S} \end{array} + \overbrace{\mathcal{P}_{(\text{AS})}^{ijkl}}^{\delta_{[k}^{[i}\delta_{l]}^{j]}} \begin{array}{c} \text{AS} \end{array} + \overbrace{\mathcal{P}_{(\text{ST})}^{ijkl}}^{\delta_{\{k}^{\{i}\}\delta_{l}^{\{j\}}}} \begin{array}{c} \text{ST} \end{array}$$

Decomposition into $O(N)$ irreps

$$\begin{array}{c} i \\ \circ \\ \text{---} \circ \quad \circ \quad \circ \quad \circ \quad \text{---} k \\ \text{---} \circ \quad \circ \quad \circ \quad \circ \quad \text{---} l \\ j \end{array} = \overbrace{N \mathcal{P}_{(S)}^{ijkl}}^{\delta^{ij} \delta^{kl}} \begin{array}{c} \text{S} \end{array} + \overbrace{\mathcal{P}_{(AS)}^{ijkl}}^{\delta_{[k}^i \delta_{l]}^j} \begin{array}{c} \text{AS} \end{array} + \overbrace{\mathcal{P}_{(ST)}^{ijkl}}^{\delta^{\{i} \delta_{\{k}^j\}}_{\{l\}}} \begin{array}{c} \text{ST} \end{array}$$

$$\begin{array}{c} \text{S} \end{array} = \left(\text{Diagram 1} \right) + \frac{1}{N} \left(\text{Diagram 2} + \text{Diagram 3} + \text{Diagram 4} \right)$$

Diagram 1: A circle with four vertices. The top and bottom vertices are connected by two arcs (one on the left, one on the right). The left and right vertices are also connected by two arcs (one on the left, one on the right). This represents the identity operator.

Diagram 2: A circle with four vertices. The top and bottom vertices are connected by two arcs. The left and right vertices are connected by two arcs. This represents the trace of the identity operator.

Diagram 3: A circle with four vertices. The top and bottom vertices are connected by two arcs. The left and right vertices are connected by two arcs. This represents the trace of the identity operator.

Diagram 4: A circle with four vertices. The top and bottom vertices are connected by two arcs. The left and right vertices are connected by two arcs. This represents the trace of the identity operator.

$$\begin{array}{c} \text{AS} \end{array} = \left(\text{Diagram 1} - \text{Diagram 2} \right)$$

Diagram 1: A circle with four vertices. The top and bottom vertices are connected by two arcs. The left and right vertices are connected by two arcs. This represents the trace of the identity operator.

Diagram 2: A circle with four vertices. The top and bottom vertices are connected by two arcs. The left and right vertices are connected by two arcs. This represents the trace of the identity operator.

Decomposition into $O(N)$ irreps

$$\begin{array}{c} i \\ \circ \\ \text{---} \circ \quad \circ \quad \circ \quad \circ \quad \text{---} k \\ \text{---} \circ \quad \circ \quad \circ \quad \circ \quad \text{---} l \\ j \end{array} = \overbrace{N \mathcal{P}_{(S)}^{ijkl}}^{\delta^{ij} \delta^{kl}} \begin{array}{c} \circ \\ \text{---} \circ \quad \circ \quad \circ \quad \circ \quad \text{---} \\ \circ \end{array} + \overbrace{\mathcal{P}_{(AS)}^{ijkl}}^{\delta_{[k}^i \delta_{l]}^j} \begin{array}{c} \circ \\ \text{---} \circ \quad \circ \quad \circ \quad \circ \quad \text{---} \\ \circ \end{array} + \overbrace{\mathcal{P}_{(ST)}^{ijkl}}^{\delta^{\{i} \delta^{j\}}_{\{k} \delta_{l\}}} \begin{array}{c} \circ \\ \text{---} \circ \quad \circ \quad \circ \quad \circ \quad \text{---} \\ \circ \end{array}$$

Diagram showing the decomposition of a four-point function into irreps of $O(N)$. The left side shows a four-point function with external indices i, j, k, l . The right side shows the decomposition into three irreps: S (Symmetric), AS (Antisymmetric), and ST (Traceless Symmetric Tensor).

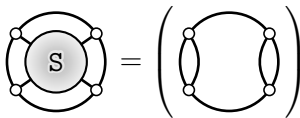
$$\begin{array}{c} \circ \\ \text{---} \circ \quad \circ \quad \circ \quad \circ \quad \text{---} \\ \circ \end{array} = \left(\begin{array}{c} \circ \quad \circ \\ \text{---} \quad \text{---} \\ \circ \quad \circ \end{array} \right) + \frac{1}{N} \left(\begin{array}{c} \circ \quad \circ \\ \text{---} \quad \text{---} \\ \circ \quad \circ \end{array} + \begin{array}{c} \circ \quad \circ \\ \text{---} \quad \text{---} \\ \circ \quad \circ \end{array} + \begin{array}{c} \circ \quad \circ \\ \text{---} \quad \text{---} \\ \circ \quad \circ \end{array} \right)$$

Diagram showing the decomposition of the S irrep into a sum of diagrams. The first term is a diagram with two internal lines. The second term is a sum of three diagrams: a diagram with two internal lines, a diagram with two internal lines, and a diagram with two internal lines.

$$\begin{array}{c} \circ \\ \text{---} \circ \quad \circ \quad \circ \quad \circ \quad \text{---} \\ \circ \end{array} = \left(\begin{array}{c} \circ \quad \circ \\ \text{---} \quad \text{---} \\ \circ \quad \circ \end{array} - \begin{array}{c} \circ \quad \circ \\ \text{---} \quad \text{---} \\ \circ \quad \circ \end{array} \right) + \frac{1}{N} \left(\begin{array}{c} \circ \quad \circ \\ \text{---} \quad \text{---} \\ \circ \quad \circ \end{array} - \begin{array}{c} \circ \quad \circ \\ \text{---} \quad \text{---} \\ \circ \quad \circ \end{array} \right)$$

Diagram showing the decomposition of the AS irrep into a sum of diagrams. The first term is a diagram with two internal lines. The second term is a sum of two diagrams: a diagram with two internal lines and a diagram with two internal lines.

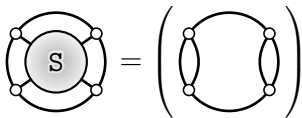
Singlet sector (MFT)



The diagram shows an equality between two Feynman diagrams. On the left, a central shaded circle labeled 'S' is connected to four small white circles at the corners of a square. These four white circles are further connected by four arcs (top, bottom, left, and right) to form an outer ring. On the right, the same outer ring structure is shown, but without the central 'S' circle. The entire right-hand side is enclosed in large parentheses.

$$\text{Diagram with central } S = \left(\text{Diagram without central } S \right)$$

Singlet sector (MFT)



$$[\mathcal{O}^i \times \mathcal{O}^j](s) \stackrel{N \rightarrow \infty}{\sim} \mathbb{1}(s)$$

Singlet sector (MFT)

$$\text{Diagram with central shaded circle 'S' and four external vertices} = \left(\text{Diagram with two vertical arcs and four vertices} \right) + \frac{1}{N} \left(\text{Diagram with two horizontal arcs} + \text{Diagram with two crossing diagonal arcs} \right)$$

$$[\mathcal{O}^i \times \mathcal{O}^j](s) \stackrel{N \rightarrow \infty}{\sim} \mathbb{1}(s)$$

Singlet sector (MFT)

$$\text{Diagram with shaded disk } S = \left(\text{Diagram with two vertical arcs} \right) + \frac{1}{N} \left(\text{Diagram with two horizontal arcs} + \text{Diagram with two crossing diagonal arcs} \right)$$

$$[\mathcal{O}^i \times \mathcal{O}^j]^{(s)} \stackrel{N \rightarrow \infty}{\sim} \mathbb{1}^{(s)}$$

$$[\mathcal{O}^i \times \mathcal{O}^j]^{(s)} \stackrel{\lambda=0}{\sim} \mathbb{1}^{(s)} \oplus \frac{1}{\sqrt{N}} \left[\mathcal{O}^\bullet \square^n \partial_{\text{even}}^J \mathcal{O}^\bullet \right]^{(s)}$$

Singlet sector (MFT)

$$\begin{array}{c} \text{S} \end{array} = \left(\text{Diagram 1} \right) + \frac{1}{N} \left(\text{Diagram 2} + \text{Diagram 3} \right)$$

The diagram on the left is a circle with four vertices. The top two vertices are connected by a horizontal line, and the bottom two vertices are connected by a horizontal line. The central region is shaded gray and labeled 'S'. The first diagram in the parentheses is a circle with four vertices, with two horizontal lines connecting the top and bottom pairs of vertices. The second diagram is a circle with four vertices, with two horizontal lines connecting the top and bottom pairs of vertices. The third diagram is a circle with four vertices, with two diagonal lines connecting the top-left to bottom-right and top-right to bottom-left vertices.

$$[\mathcal{O}^i \times \mathcal{O}^j]^{(s)} \stackrel{N \rightarrow \infty}{\sim} \mathbb{1}^{(s)}$$

$$[\mathcal{O}^i \times \mathcal{O}^j]^{(s)} \stackrel{\lambda=0}{\sim} \mathbb{1}^{(s)} \oplus \frac{1}{\sqrt{N}} \left[\mathcal{O}^\bullet \square^n \partial_{\text{even}}^J \mathcal{O}^\bullet \right]^{(s)}$$

$$\left(\text{Diagram 2} + \text{Diagram 3} \right) = \sum_{n=0}^{\infty} \sum_{\substack{J=0 \\ J \text{ even}}}^{\infty} 2C_{n,J}^{(\text{MFT})} \left| G_{2\Delta_\phi+2n+J,J}^{(s)} \right\rangle$$

Singlet sector (interacting)

$$\text{Diagram with shaded disk } S = \left(\text{Diagram with two vertical loops} \right) + \frac{1}{N} \left(\text{Diagram with two horizontal loops} + \text{Diagram with two crossing lines} + \text{Diagram with two horizontal lines connecting shaded vertices} \right)$$

$$[\mathcal{O}^i \times \mathcal{O}^j]^{(S)} \stackrel{N \rightarrow \infty}{\sim} \mathbb{1}^{(S)}$$

$$[\mathcal{O}^i \times \mathcal{O}^j]^{(S)} \stackrel{\lambda \rightarrow 0}{\sim} \mathbb{1}^{(S)} \oplus \frac{1}{\sqrt{N}} \left[\mathcal{O}^\bullet \square^n \partial_{\text{even}}^J \mathcal{O}^\bullet \right]^{(S)}$$

Singlet sector (interacting)

$$\text{Diagram with shaded disk } S = \left(\text{Diagram with two vertical loops} \right) + \frac{1}{N} \left(\text{Diagram with two horizontal loops} + \text{Diagram with two crossing lines} + \text{Diagram with two horizontal lines and a shaded segment} \right)$$

$$[\mathcal{O}^i \times \mathcal{O}^j]^{(S)} \stackrel{N \rightarrow \infty}{\sim} \mathbb{1}^{(S)}$$

$$[\mathcal{O}^i \times \mathcal{O}^j]^{(S)} \stackrel{\lambda \rightarrow 0}{\sim} \mathbb{1}^{(S)} \oplus \frac{1}{\sqrt{N}} \left[\mathcal{O}^\bullet \square^n \partial_{\text{even}}^J \mathcal{O}^\bullet \right]^{(S)}$$

$$[\mathcal{O}^i \times \mathcal{O}^j]^{(S)} \sim \mathbb{1}^{(S)}$$

Singlet sector (interacting)

$$\text{Diagram with shaded disk } S = \left(\text{Diagram with two vertical ovals} \right) + \frac{1}{N} \left(\text{Diagram with two horizontal ovals} + \text{Diagram with two crossing lines} + \text{Diagram with two horizontal ovals and shaded disk} \right)$$

$$[\mathcal{O}^i \times \mathcal{O}^j]^{(S)} \stackrel{N \rightarrow \infty}{\sim} \mathbb{1}^{(S)}$$

$$[\mathcal{O}^i \times \mathcal{O}^j]^{(S)} \stackrel{\lambda \rightarrow 0}{\sim} \mathbb{1}^{(S)} \oplus \frac{1}{\sqrt{N}} \left[\mathcal{O}^\bullet \square^n \partial_{\text{even}}^J \mathcal{O}^\bullet \right]^{(S)}$$

$$[\mathcal{O}^i \times \mathcal{O}^j]^{(S)} \sim \mathbb{1}^{(S)} \oplus \frac{1}{\sqrt{N}} \left[\text{"}\mathcal{O}^\bullet \square^n \mathcal{O}^\bullet\text{"} \right]^{(S)} \text{ } O(1) \text{ finite shifts}$$

Singlet sector (interacting)

$$\text{Diagram} = \left(\text{Diagram} \right) + \frac{1}{N} \left(\text{Diagram} + \text{Diagram} + \text{Diagram} \right)$$

$$[\mathcal{O}^i \times \mathcal{O}^j]^{(S)} \stackrel{N \rightarrow \infty}{\sim} \mathbb{1}^{(S)}$$

$$[\mathcal{O}^i \times \mathcal{O}^j]^{(S)} \stackrel{\lambda \rightarrow 0}{\sim} \mathbb{1}^{(S)} \oplus \frac{1}{\sqrt{N}} \left[\mathcal{O}^\bullet \square^n \partial_{\text{even}}^J \mathcal{O}^\bullet \right]^{(S)}$$

$$\begin{aligned} [\mathcal{O}^i \times \mathcal{O}^j]^{(S)} &\sim \mathbb{1}^{(S)} \oplus \frac{1}{\sqrt{N}} \left[\text{"}\mathcal{O}^\bullet \square^n \mathcal{O}^\bullet\text{"} \right]^{(S)} \text{O(1) finite shifts} \\ &\oplus \frac{1}{\sqrt{N}} \left[\mathcal{O}^\bullet \square^n \partial_{\text{even}}^{J>0} \mathcal{O}^\bullet \right]^{(S)} \text{MFT} \end{aligned}$$

Utilizing the spectral representation

$$B(x, y) = \int_{\mathbb{R}} d\nu \tilde{B}(\Delta) \Omega_{\Delta}(x, y) \quad \left(\Delta \equiv \frac{d}{2} + i\nu \right)$$

Utilizing the spectral representation

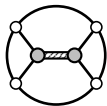
$$B(x, y) = \int_{\mathbb{R}} d\nu \tilde{B}(\Delta) \Omega_{\Delta}(x, y) \quad \left(\Delta \equiv \frac{d}{2} + i\nu \right)$$

$$\text{---}\bigcirc = - \left[\frac{1}{\lambda} + 2B \right]^{-1}$$

Utilizing the spectral representation

$$B(x, y) = \int_{\mathbb{R}} d\nu \tilde{B}(\Delta) \Omega_{\Delta}(x, y) \quad \left(\Delta \equiv \frac{d}{2} + i\nu \right)$$

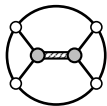
$$\text{---}\text{---}\text{---} = - \left[\frac{1}{\lambda} + 2B \right]^{-1}$$



Utilizing the spectral representation

$$B(x, y) = \int_{\mathbb{R}} d\nu \tilde{B}(\Delta) \Omega_{\Delta}(x, y) \quad \left(\Delta \equiv \frac{d}{2} + i\nu \right)$$

$$\text{---}\text{---}\text{---} = - \left[\frac{\mathbb{1}}{\lambda} + 2B \right]^{-1} = \int_{\mathbb{R}} d\nu \left(\frac{-1}{\lambda^{-1} + 2\tilde{B}(\Delta)} \right) \Omega_{\Delta}$$



Utilizing the spectral representation

$$B(x, y) = \int_{\mathbb{R}} d\nu \tilde{B}(\Delta) \Omega_{\Delta}(x, y) \quad \left(\Delta \equiv \frac{d}{2} + i\nu \right)$$

$$\text{---} \text{---} \text{---} = - \left[\frac{1}{\lambda} + 2B \right]^{-1} = \int_{\mathbb{R}} d\nu \left(\frac{-1}{\lambda^{-1} + 2\tilde{B}(\Delta)} \right) \Omega_{\Delta}$$

$$\text{---} \text{---} \text{---} = 4 \int_{\mathbb{R}} d\nu \left(\frac{-1}{\lambda^{-1} + 2\tilde{B}(\Delta)} \right) \text{---} \text{---} \text{---}$$

Utilizing the spectral representation

$$B(x, y) = \int_{\mathbb{R}} d\nu \tilde{B}(\Delta) \Omega_{\Delta}(x, y) \quad \left(\Delta \equiv \frac{d}{2} + i\nu \right)$$

$$\text{---} \text{---} \text{---} = - \left[\frac{1}{\lambda} + 2B \right]^{-1} = \int_{\mathbb{R}} d\nu \left(\frac{-1}{\lambda^{-1} + 2\tilde{B}(\Delta)} \right) \Omega_{\Delta}$$

$$\begin{aligned} \text{---} \text{---} \text{---} &= 4 \int_{\mathbb{R}} d\nu \left(\frac{-1}{\lambda^{-1} + 2\tilde{B}(\Delta)} \right) \text{---} \text{---} \text{---} \\ &= 4 \int_{\mathbb{R}} d\nu \left(- \frac{1}{\lambda^{-1} + 2\tilde{B}(\Delta)} \right) \sqrt{\mathbf{e}_{\Delta} \tilde{\mathbf{e}}_{\Delta}} \frac{\nu^2}{\pi} \text{---} \text{---} \end{aligned}$$

Utilizing the spectral representation

$$B(x, y) = \int_{\mathbb{R}} d\nu \tilde{B}(\Delta) \Omega_{\Delta}(x, y) \quad \left(\Delta \equiv \frac{d}{2} + i\nu \right)$$

$$\text{---}\text{---}\text{---} = - \left[\frac{1}{\lambda} + 2B \right]^{-1} = \int_{\mathbb{R}} d\nu \left(\frac{-1}{\lambda^{-1} + 2\tilde{B}(\Delta)} \right) \Omega_{\Delta}$$

$$\text{---}\text{---}\text{---} = 4 \int_{\mathbb{R}} d\nu \left(\frac{-1}{\lambda^{-1} + 2\tilde{B}(\Delta)} \right) \text{---}\text{---}\text{---}$$

$$= 4 \int_{\mathbb{R}} d\nu \left(- \frac{1}{\lambda^{-1} + 2\tilde{B}(\Delta)} \right) \sqrt{\mathbf{e}_{\Delta} \mathbf{e}_{\tilde{\Delta}}} \frac{\nu^2}{\pi} \text{---}\text{---}\text{---}$$

$$= \int_{\frac{d}{2} + i\mathbb{R}_{\geq 0}} \frac{d\Delta}{2\pi i} \left(- \frac{1}{\lambda^{-1} + 2\tilde{B}(\Delta)} \right) \left(\frac{\Gamma_{\dots}^2 \dots \Gamma_{\dots}}{\dots \Gamma_{\dots}^2 \dots \Gamma_{\dots}} \right) \text{---}\text{---}\text{---}$$

Utilizing the spectral representation

$$B(x, y) = \int_{\mathbb{R}} d\nu \tilde{B}(\Delta) \Omega_{\Delta}(x, y) \quad \left(\Delta \equiv \frac{d}{2} + i\nu \right)$$

$$\text{---}\text{---}\text{---} = - \left[\frac{1}{\lambda} + 2B \right]^{-1} = \int_{\mathbb{R}} d\nu \left(\frac{-1}{\lambda^{-1} + 2\tilde{B}(\Delta)} \right) \Omega_{\Delta}$$

$$\text{---}\text{---}\text{---} = 4 \int_{\mathbb{R}} d\nu \left(\frac{-1}{\lambda^{-1} + 2\tilde{B}(\Delta)} \right) \text{---}\text{---}\text{---}$$

$$= 4 \int_{\mathbb{R}} d\nu \left(- \frac{1}{\lambda^{-1} + 2\tilde{B}(\Delta)} \right) \sqrt{\mathfrak{e}_{\Delta} \tilde{\mathfrak{e}}_{\Delta}} \frac{\nu^2}{\pi} \text{---}\text{---}\text{---}$$

$$= \int_{\frac{d}{2} + i\mathbb{R}_{\geq 0}} \frac{d\Delta}{2\pi i} \left(- \frac{1}{\lambda^{-1} + 2\tilde{B}(\Delta)} \right) \left(\frac{\Gamma_{\dots}^2 \dots \Gamma_{\dots}}{\dots \Gamma_{\dots}^2 \dots \Gamma_{\dots}} \right) \text{---}\text{---}\text{---}$$

$$\equiv \int_{\frac{d}{2} + i\mathbb{R}_{\geq 0}} \frac{d\Delta}{2\pi i} \left(- \frac{1}{\lambda^{-1} + 2\tilde{B}(\Delta)} \right) \left(\frac{\Gamma_{\dots}^2 \dots \Gamma_{\dots}}{\dots \Gamma_{\dots}^2 \dots \Gamma_{\dots}} \right) | \text{---}\text{---}\text{---} \rangle$$

Singlet sector — spectral functions

$$\frac{1}{N} \text{Spec}_s \left[\text{Diagram 1} + \text{Diagram 2} + \text{Diagram 3} \right]$$

The equation shows the spectral function in the singlet sector, $\frac{1}{N} \text{Spec}_s$, applied to a sum of three Feynman diagrams. The diagrams are enclosed in large square brackets.

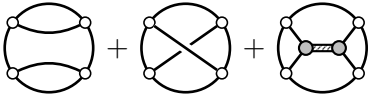
- Diagram 1:** A circle with four white vertices. Two horizontal arcs connect the top and bottom vertices, and two vertical arcs connect the left and right vertices, forming a complete graph K_4 .
- Diagram 2:** A circle with four white vertices. Two diagonal lines connect the top-left to bottom-right and top-right to bottom-left vertices.
- Diagram 3:** A circle with four white vertices. Two horizontal lines connect the top and bottom vertices. In the center, there is a shaded gray circle with two white dots. Two lines connect the top and bottom vertices to these dots, and a horizontal hatched line connects the two dots.

Singlet sector — spectral functions

$$\frac{1}{N} \text{Spec}_s \left[\text{Diagram 1} + \text{Diagram 2} + \text{Diagram 3} \right]$$

$$\left(\text{Diagram 1} + \text{Diagram 2} \right) = \sum_{n=0}^{\infty} \sum_{\substack{J=0 \\ J \text{ even}}}^{\infty} 2C_{n,J}^{(\text{MFT})} \left| G_{2\Delta_\phi+2n+J,J}^{(s)} \right\rangle$$

Singlet sector — spectral functions

$$\frac{1}{N} \text{Spec}_s \left[\text{Diagram 1} + \text{Diagram 2} + \text{Diagram 3} \right]$$


$$\left(\text{Diagram 1} + \text{Diagram 2} \right) = \sum_{n=0}^{\infty} \sum_{\substack{J=0 \\ J \text{ even}}}^{\infty} 2C_{n,J}^{(\text{MFT})} \left| G_{2\Delta_\phi+2n+J,J}^{(s)} \right\rangle$$

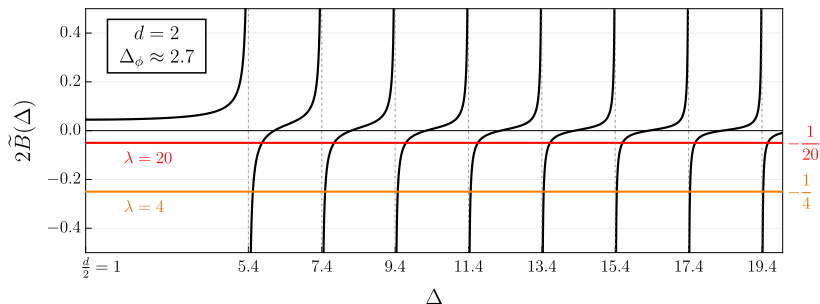
$$\text{Spec}_s \left[\frac{\Delta}{J} \left| \text{Diagram 3} \right. \right] = -\delta_{J,0} \frac{1}{\lambda^{-1} + 2\widetilde{B}(\Delta)} \times$$

$$\times \frac{\Gamma_{\Delta_\phi-\frac{\Delta}{2}}^2 \Gamma_{\Delta_\phi-\frac{\widetilde{\Delta}}{2}}^2 \Gamma_{\frac{\Delta}{2}}^2 \Gamma_{\frac{\widetilde{\Delta}}{2}}^2}{4\pi^d \Gamma_{\Delta_\phi}^2 \Gamma_{1-\frac{d}{2}+\Delta_\phi}^2 \Gamma_{\Delta-\frac{d}{2}} \Gamma_{\widetilde{\Delta}-\frac{d}{2}}}$$

Singlet sector — scalar non-MFT operators

$$\text{Spec}_s \left[\begin{array}{c} \Delta \\ J \end{array} \middle| \text{Diagram} \right] \propto \frac{1}{\lambda^{-1} + 2\tilde{B}(\Delta)}$$

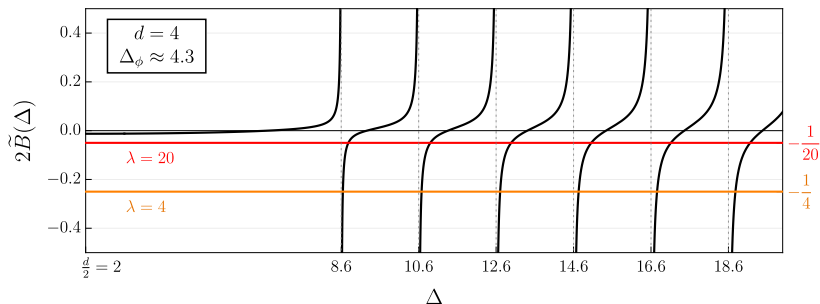
$\Rightarrow \lambda^{-1} + 2\tilde{B}(\Delta_{\bullet,0}^{(s)}) = 0$



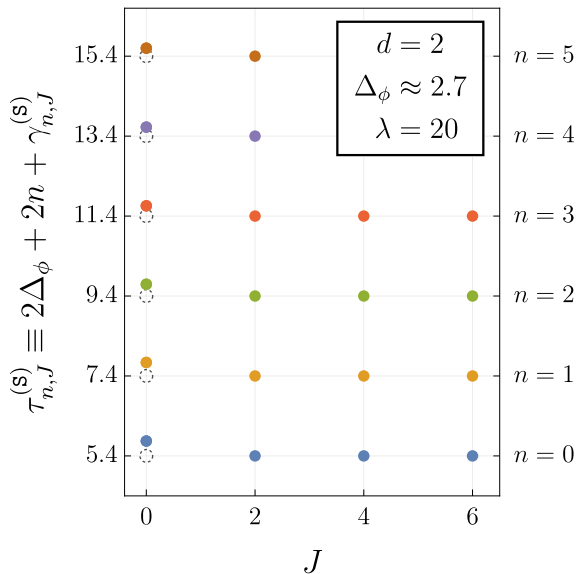
Singlet sector — scalar non-MFT operators

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Singlet sector — twist–spin plot



Non-singlet sector (MFT)

$$\begin{array}{c} \text{ST} \\ \text{AS} \end{array} = \left(\begin{array}{c} \text{Diagram 1} \end{array} \pm \begin{array}{c} \text{Diagram 2} \end{array} \right)$$

The diagram on the left is a circle with four vertices. The top two vertices are connected by an arc above the circle, and the bottom two vertices are connected by an arc below the circle. The interior of the circle is shaded gray and contains the text "ST" above "AS".

The first diagram in the parentheses is a circle with four vertices. The top two vertices are connected by an arc above the circle, and the bottom two vertices are connected by an arc below the circle.

The second diagram in the parentheses is a circle with four vertices. The top two vertices are connected by a diagonal line from the top-left to the bottom-right, and the bottom two vertices are connected by a diagonal line from the bottom-left to the top-right.

Non-singlet sector (MFT)

$$\text{Diagram} = \left(\text{Diagram}_1 \pm \text{Diagram}_2 \right)$$

$$[\mathcal{O}^i \times \mathcal{O}^j]^{(\text{ST})} \stackrel{N \rightarrow \infty}{\sim} \left[\mathcal{O}^{\{i \square^n \partial_{\text{even}}^J \mathcal{O}^j\}} \right]^{(\text{ST})}$$

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$$[\mathcal{O}^i \times \mathcal{O}^j]^{(\text{AS})} \stackrel{N \rightarrow \infty}{\sim} \left[\mathcal{O}^{[i \square^n \partial_{\text{odd}}^J \mathcal{O}^j]} \right]^{(\text{AS})}$$

Non-singlet sector (interacting)

$$\text{Diagram} = \left(\text{Diagram}_1 \pm \text{Diagram}_2 \right) + \frac{1}{N} \left(\text{Diagram}_3 \pm \text{Diagram}_4 \right)$$

$$[\mathcal{O}^i \times \mathcal{O}^j]^{(\text{ST})} \stackrel{N \rightarrow \infty}{\sim} \left[\mathcal{O}^{\{i \square^n \partial_{\text{even}}^J \mathcal{O}^j\}} \right]^{(\text{ST})}$$

$$[\mathcal{O}^i \times \mathcal{O}^j]^{(\text{AS})} \stackrel{N \rightarrow \infty}{\sim} \left[\mathcal{O}^{[i \square^n \partial_{\text{odd}}^J \mathcal{O}^j]} \right]^{(\text{AS})}$$

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Anomalous dimensions as double poles

$$\frac{-C_\star}{\Delta - \Delta_\star} \delta_{J,J_\star} \in K_{\tilde{\Delta},J} \operatorname{Spec}_s \left[\begin{array}{c} \Delta \\ J \end{array} \middle| \begin{array}{c} \text{Diagram} \end{array} \right]$$

Anomalous dimensions as double poles

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$$\begin{array}{c} \text{Diagram 1} \end{array} = \begin{array}{c} \text{Diagram 2} \end{array} + \frac{1}{N} \begin{array}{c} \text{Diagram 3} \end{array} + \dots$$

Anomalous dimensions as double poles

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$$\text{Diagram 1} = \text{Diagram 2} + \frac{1}{N} \text{Diagram 3} + \dots$$

$$\text{ope}^2[\mathcal{O}\mathcal{O}\mathcal{O}_\star] \equiv C_\star\left(\frac{1}{N}\right) = C_\star^{(\text{MFT})} + \frac{1}{N} C_\star^{(1)} + \mathcal{O}\left(\frac{1}{N^2}\right)$$

Anomalous dimensions as double poles

$$\frac{-C_\star}{\Delta - \Delta_\star} \delta_{J,J_\star} \in K_{\tilde{\Delta},J} \text{Spec}_s \left[\begin{array}{c} \Delta \\ J \end{array} \middle| \text{Diagram} \right]$$

$$\text{Diagram 1} = \text{Diagram 2} + \frac{1}{N} \text{Diagram 3} + \dots$$

$$\begin{aligned} \text{ope}^2[\mathcal{O}\mathcal{O}\mathcal{O}_\star] &\equiv C_\star\left(\frac{1}{N}\right) = C_\star^{(\text{MFT})} + \frac{1}{N} C_\star^{(1)} + O\left(\frac{1}{N^2}\right) \\ \Delta_\star\left(\frac{1}{N}\right) &= \Delta_\star^{(\text{MFT})} + \frac{1}{N} \gamma_\star^{(1)} + O\left(\frac{1}{N^2}\right) \end{aligned}$$

Anomalous dimensions as (double) poles

$$\frac{-C_{\star}\left(\frac{1}{N}\right)}{\Delta - \Delta_{\star}\left(\frac{1}{N}\right)}$$

Anomalous dimensions as (double) poles

$$\frac{-C_{\star}\left(\frac{1}{N}\right)}{\Delta - \Delta_{\star}\left(\frac{1}{N}\right)} = \frac{-C_{\star}^{(\text{MFT})}}{\Delta - \Delta_{\star}^{(\text{MFT})}}$$

Anomalous dimensions as (double) poles

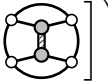
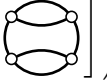
$$\frac{-C_{\star}\left(\frac{1}{N}\right)}{\Delta - \Delta_{\star}\left(\frac{1}{N}\right)} = \frac{-C_{\star}^{(\text{MFT})}}{\Delta - \Delta_{\star}^{(\text{MFT})}} + \frac{1}{N} \left[\frac{-C_{\star}^{(1)}}{\Delta - \Delta_{\star}^{(\text{MFT})}} \right]$$

Anomalous dimensions as (double) poles

$$\frac{-C_{\star}\left(\frac{1}{N}\right)}{\Delta - \Delta_{\star}\left(\frac{1}{N}\right)} = \frac{-C_{\star}^{(\text{MFT})}}{\Delta - \Delta_{\star}^{(\text{MFT})}} + \frac{1}{N} \left[\frac{-C_{\star}^{(1)}}{\Delta - \Delta_{\star}^{(\text{MFT})}} + \frac{-C_{\star}^{(\text{MFT})} \gamma_{\star}^{(1)}}{\left(\Delta - \Delta_{\star}^{(\text{MFT})}\right)^2} \right]$$

Anomalous dimensions as (double) poles

$$\frac{-C_{\star}\left(\frac{1}{N}\right)}{\Delta - \Delta_{\star}\left(\frac{1}{N}\right)} = \frac{-C_{\star}^{(\text{MFT})}}{\Delta - \Delta_{\star}^{(\text{MFT})}} + \frac{1}{N} \left[\frac{-C_{\star}^{(1)}}{\Delta - \Delta_{\star}^{(\text{MFT})}} + \frac{-C_{\star}^{(\text{MFT})} \gamma_{\star}^{(1)}}{\left(\Delta - \Delta_{\star}^{(\text{MFT})}\right)^2} \right]$$

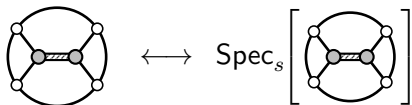
$$\gamma_{n,J}^{(1)} = \text{Res}_{\Delta=2\Delta_{\phi}+2n+J} \left(\frac{\text{Spec}_s \left[\frac{\Delta}{J} \mid \text{Diagram 1} \right]}{\text{Spec}_s \left[\frac{\Delta}{J} \mid \text{Diagram 2} \right]} \right)$$



Crossed channel contributions

Suppose we resolved the “direct” s -channel spectrum.

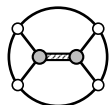
Crossed channel contributions

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$$\text{Diagram} \longleftrightarrow \text{Spec}_s \left[\text{Diagram} \right]$$


What is the contribution of the crossed-channel diagrams?

Crossed channel contributions

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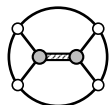
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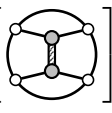
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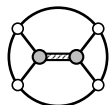
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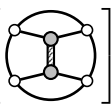
$$\text{Spec}_s \left[\text{Diagram} \right] = \sum_{\mathcal{O}_{\bullet,0}^{(s)}}$$


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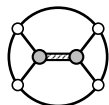
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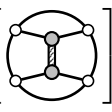
$$\text{Spec}_s \left[\text{Diagram} \right] = \sum_{\mathcal{O}_{\bullet,0}^{(s)}} \text{o pe}^2 \left[\mathcal{O} \mathcal{O} \mathcal{O}_{\bullet,0}^{(s)} \right]$$


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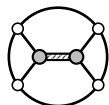
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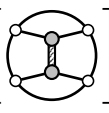
$$\text{Spec}_s \left[\text{Diagram} \right] = \sum_{\mathcal{O}_{\bullet,0}^{(s)}} \text{o pe}^2 \left[\mathcal{O} \mathcal{O} \mathcal{O}_{\bullet,0}^{(s)} \right] \text{CrK} \left\langle \Delta, J \middle| \Delta_{\bullet,0,0}^{(s)} \right\rangle$$


Crossed channel contributions

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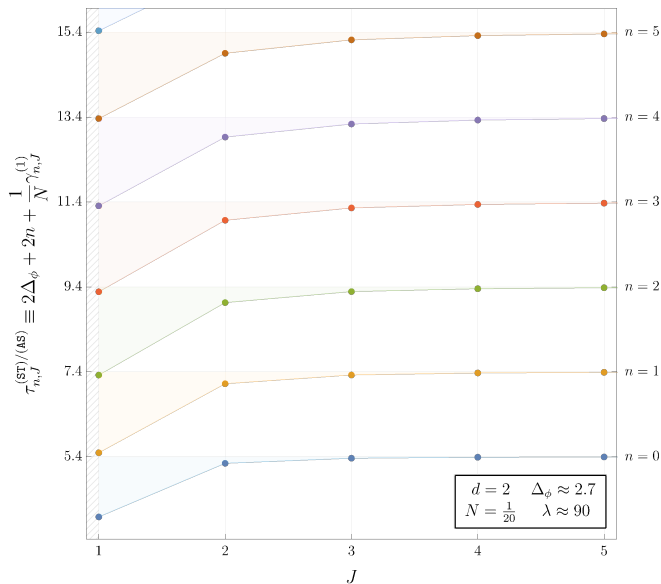
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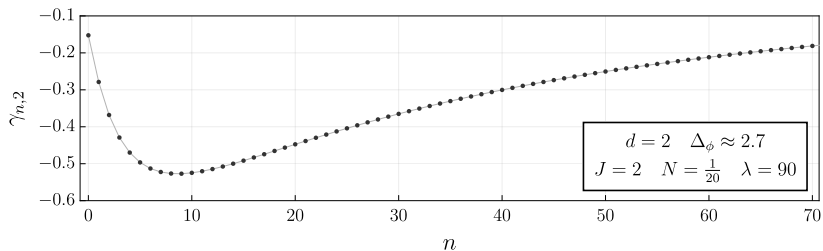
$$\text{Spec}_s \left[\text{Diagram} \right] = \sum_{\mathcal{O}_{\bullet,0}^{(s)}} \text{ope}^2 \left[\mathcal{O} \mathcal{O} \mathcal{O}_{\bullet,0}^{(s)} \right] \text{CrK} \left\langle \Delta, J \left| \Delta_{\bullet,0,0}^{(s)} \right. \right\rangle^{s \leftarrow t}$$


$$\gamma_{n,J}^{(\text{ST})/(\text{AS})} = \sum_{\mathcal{O}_{\bullet,0}^{(s)}} \text{ope}^2 \left[\mathcal{O} \mathcal{O} \mathcal{O}_{\bullet,0}^{(s)} \right] \gamma_{n,J}^{(1)} \Big|_{\substack{t\text{-channel} \\ \text{exchange of } \mathcal{O}_{\bullet,0}^{(s)}}}$$

Non-singlet sector — twist–spin plot



Non-singlet sector — dependence on n



Non-singlet sector — large J asymptotics

$$\tau_{n,J} \sim 2\Delta_\phi + 2n - \frac{c_n}{J^{\tau_{\min}}} + \dots$$

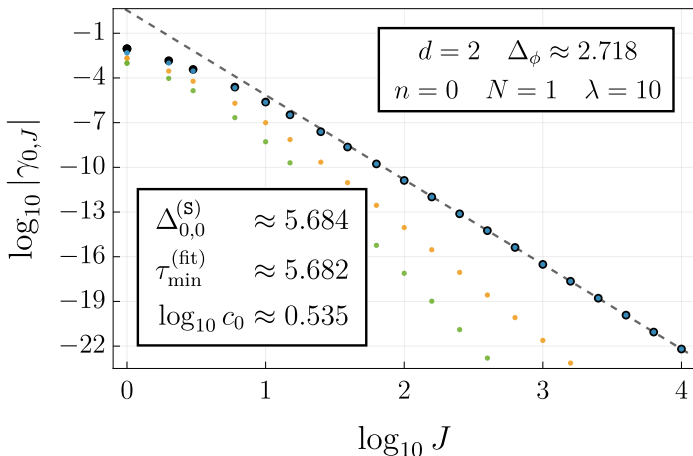
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$$\tau_{n,J} \sim 2\Delta_\phi + 2n - \frac{c_n}{J^{\tau_{\min}}} + \dots$$
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Summary and outlook

Results:

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- Presented general formulas (in $d = 2$ and $d = 4$) for

t -channel conformal block	$\xrightarrow{\text{contribution}}$	anomalous dimensions of s -channel double-twist operators
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Future directions:

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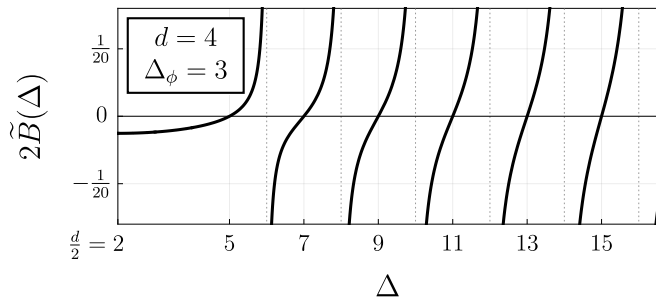
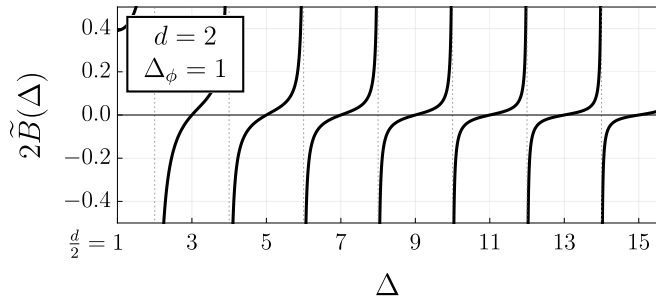
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More details in 2503.16345 [hep-th]

Extra slides

Some extra slides.

Criticality in the bulk



Criticality in the bulk

