

# Neur2SP: Neural Two-Stage Stochastic Programming

**Justin Dumouchelle & Rahul Patel**

Department of Mechanical & Industrial Engineering



Justin Dumouchelle



Rahul Patel



Elias B. Khalil



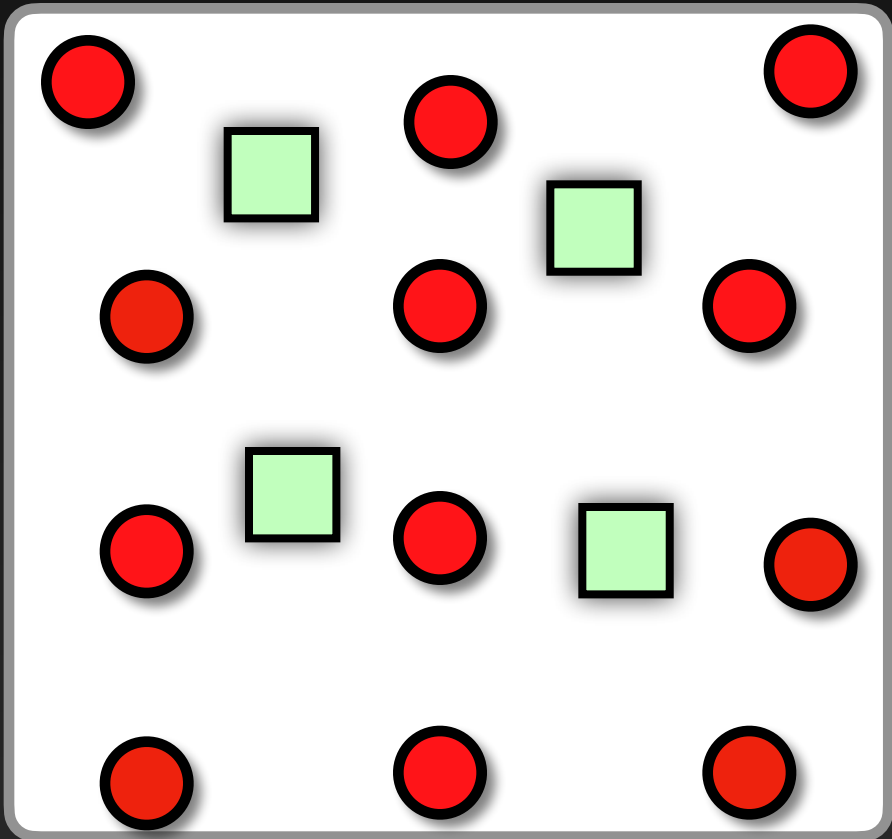
Merve Bodur



UNIVERSITY OF  
TORONTO

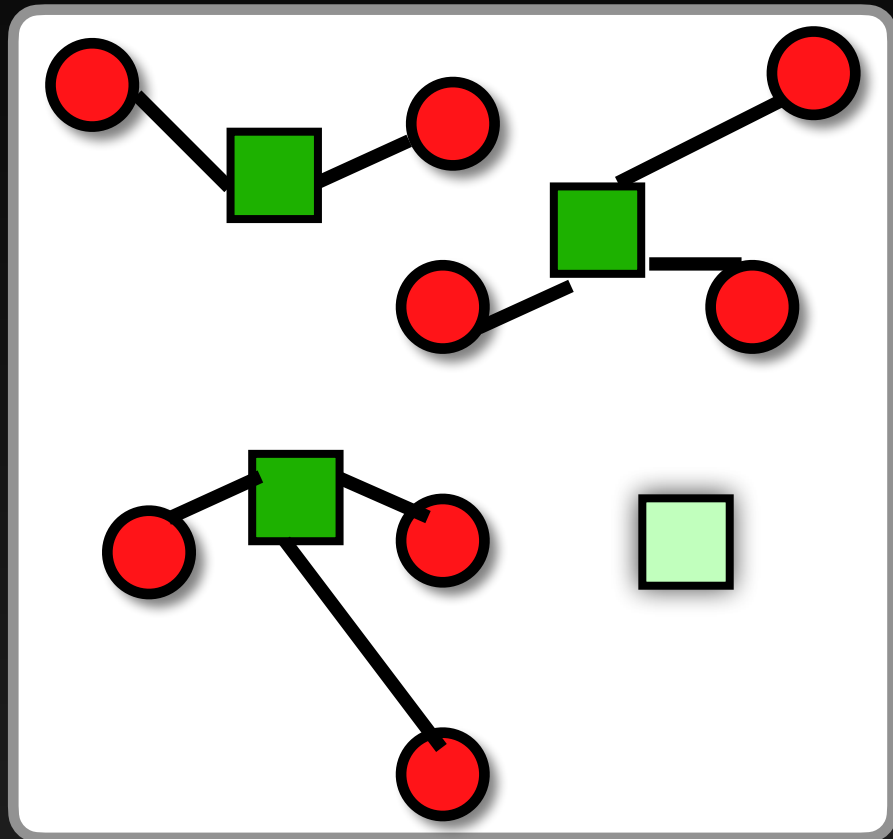
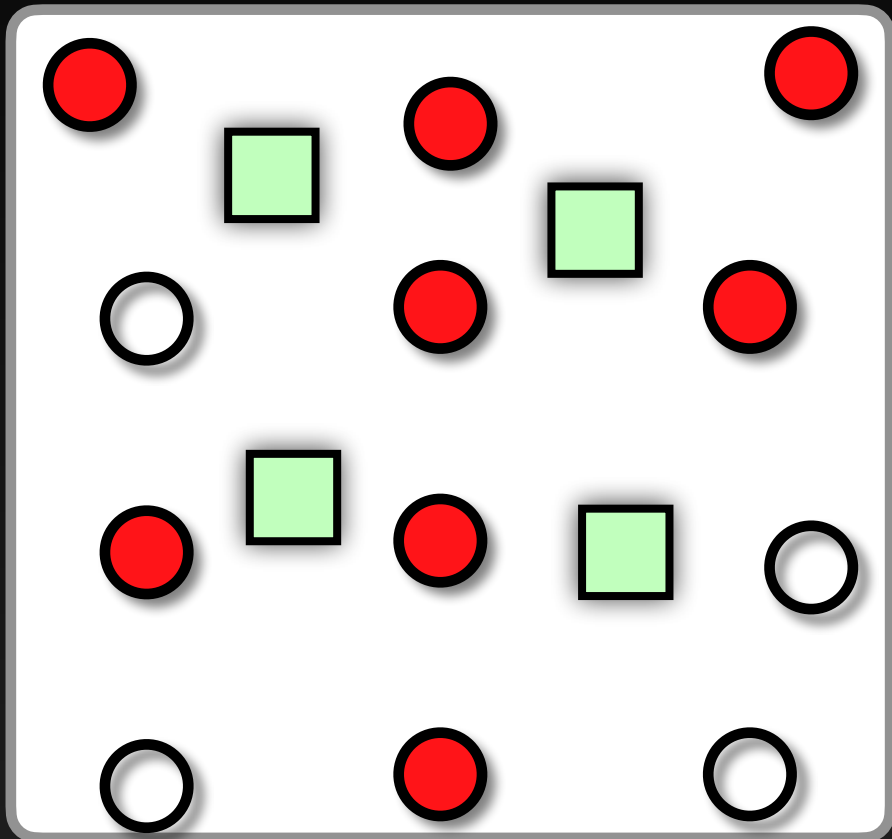
# Stochastic Server Location Problem

- Unavailable Client
- Available Client
- Potential Server
- Actual Server

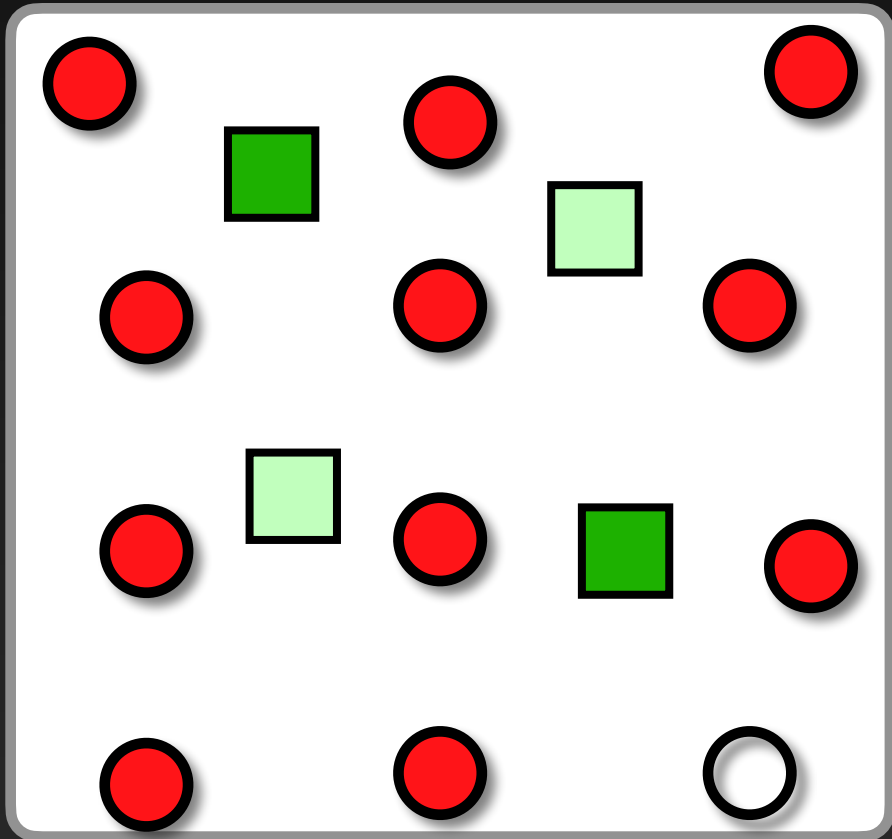
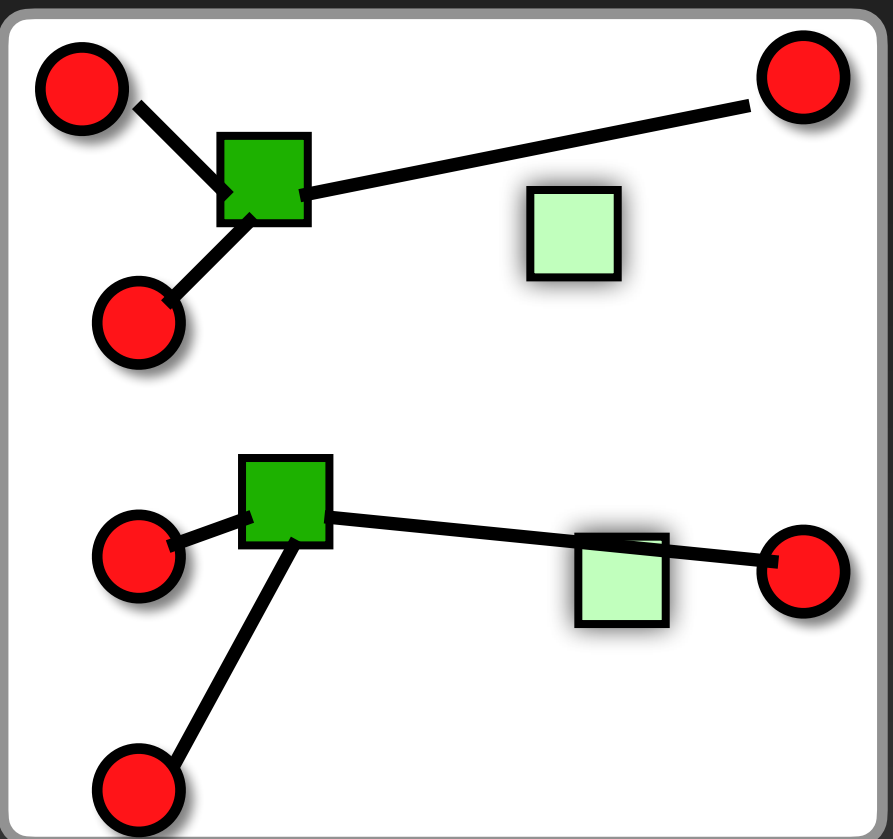
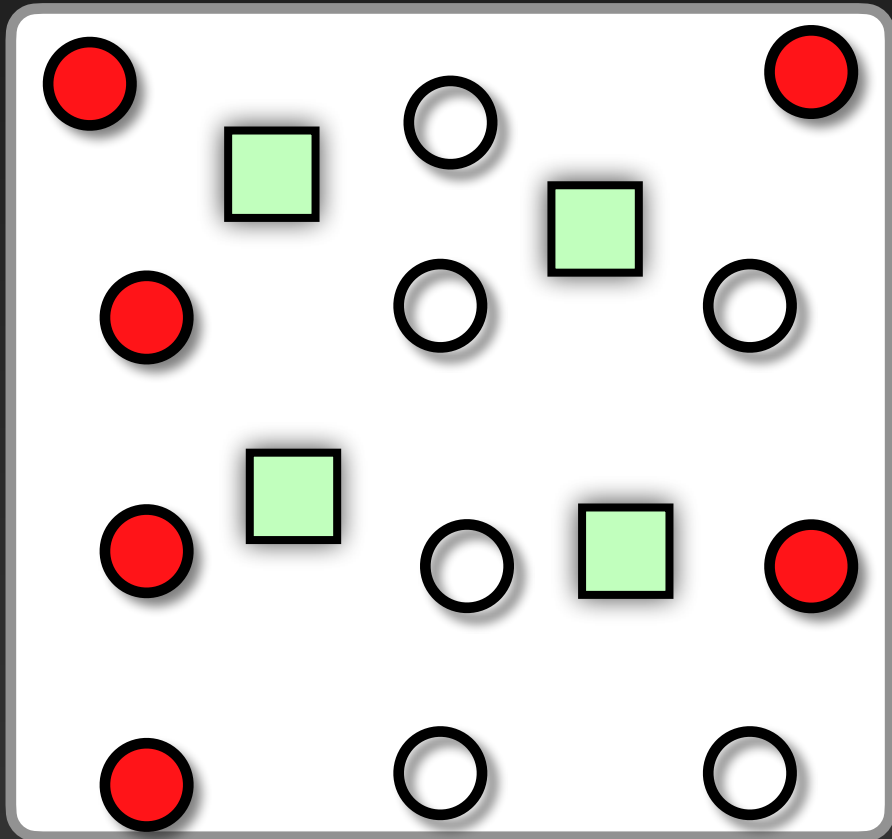


Set of potential  
clients and servers

Scenario 1



Scenario 2



Scenario-optimal  
Location + Assignment

Expected optimal  
Server Location

Scenario  
Realization

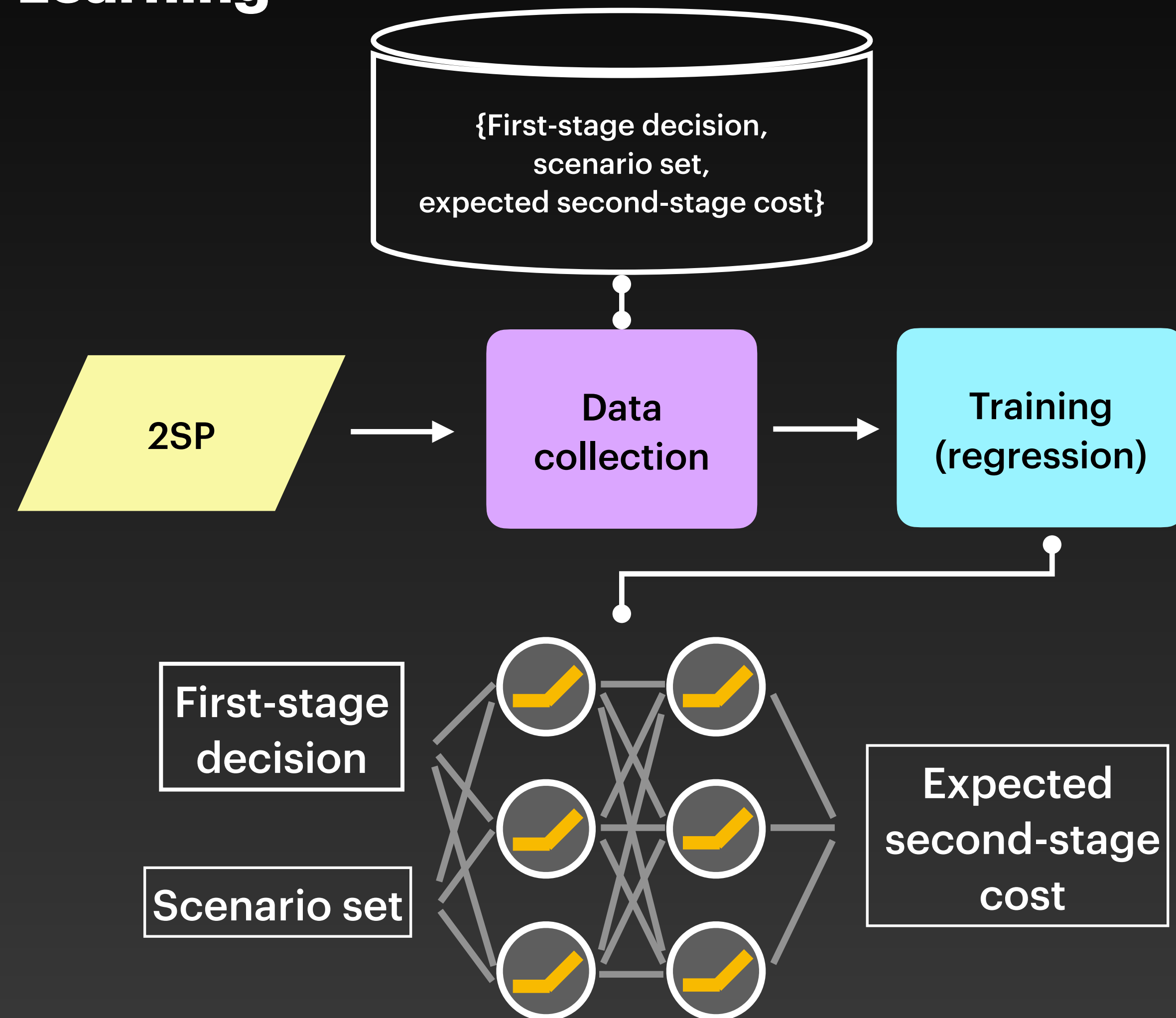
# Two-stage Stochastic Programming

$$\min_{\underbrace{\mathbf{x}}_{\text{First-stage decisions}}} \left\{ \underbrace{\mathbf{c}^T \mathbf{x}}_{\text{First-stage cost}} + \underbrace{\sum_{k=1}^K p_k Q(\mathbf{x}, \xi_k)}_{\text{Expected second-stage cost}} : \underbrace{\mathbf{x} \in \mathcal{X}}_{\text{First-stage feasible set}} \right\}$$

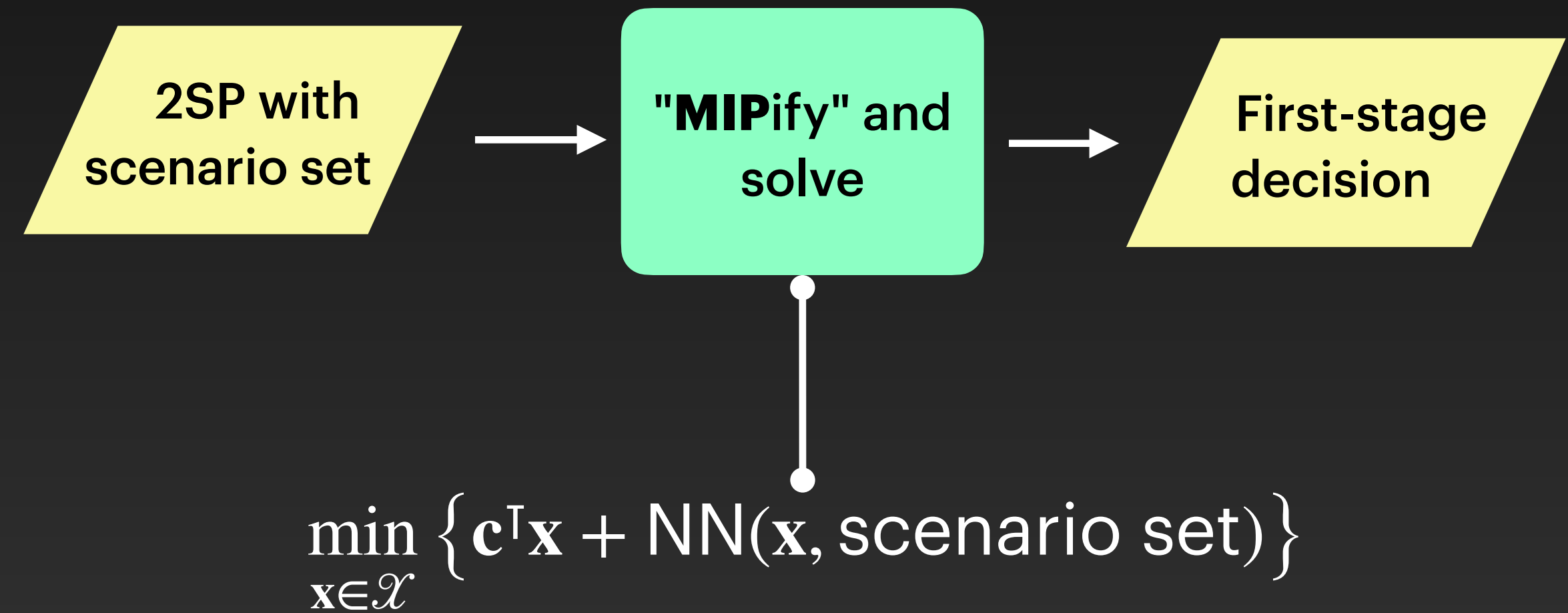
$$Q(\mathbf{x}, \xi) := \min_{\underbrace{\mathbf{y}}_{\text{Second-stage decisions}}} \left\{ \underbrace{F(\mathbf{y}, \xi)}_{\text{Second-stage cost}} : \underbrace{\mathbf{y} \in \mathcal{Y}(\mathbf{x}, \xi)}_{\text{Second-stage feasible set}} \right\}$$

# Overview of Neur2SP

## Learning

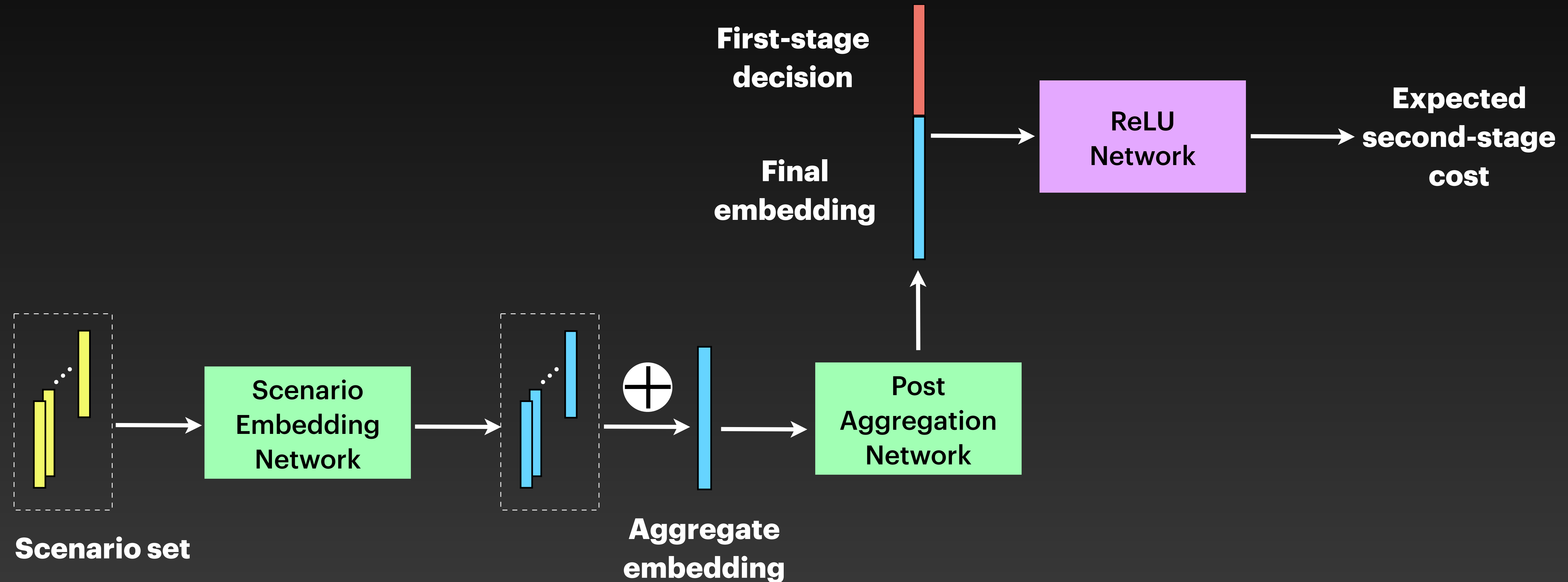


## Deployment



# Neural Architecture

Embed > Aggregate > Predict





# Stochastic Server Location, SIPLib Instances

SSLP\_(# servers)\_(# clients)\_(# scenarios)

Problem	Gap to Optimal (%)		Solving Time	
	Neur2SP	EF	Neur2SP	EF
SSLP_10_50_50	0.00	0.00	0.11	10,801.27
SSLP_10_50_100	0.00	0.00	0.11	10,800.04
SSLP_10_50_500	0.00	0.00	0.11	10,818.23
SSLP_10_50_1000	0.00	28.64	0.12	10,800.26
SSLP_10_50_2000	0.00	51.24	0.13	10,800.20
SSLP_15_45_5	0.46	19.59	0.32	4.17
SSLP_15_45_10	1.57	18.23	0.25	3.71
SSLP_15_45_15	0.53	16.51	0.41	4.74
SSLP_5_25_50	0.00	0.00	0.26	2.35
SSLP_5_25_100	0.00	0.00	0.18	8.87

500K and 1M  
variables in EF

EF times out after 3  
hours with huge  
gaps

Neur2SP finds  
optimal solution in  
~0.1 seconds

# Summary

- **Recipe for fast two-stage stochastic programming:**
  - Learn expected second-stage objective as a ReLU network
  - Embed it in a MIP to obtain a solution heuristically
- **Orders of magnitude speedups** compared to Extensive Form
- **Generic** and works for a variety of 2SP classes

## Neur2SP: Neural Two-Stage Stochastic Programming

Justin Dumouchelle\*, Rahul Patel\*, Elias B. Khalil, Merve Bodur

Contact us at [\*\*khalil@mie.utoronto.ca\*\*](mailto:khalil@mie.utoronto.ca)

**Github:** <https://github.com/khalil-research/Neur2SP>