Goods Market Frictions and the Labor Wedge

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Abstract

I modify the standard real business cycle model by replacing frictionless labor and goods markets with markets that require search effort of market participants to find a match. In this framework I reexamine the shortcoming of the labor search model identified by Shimer (2010) and related to the so called labor wedge - the gap between the firm's marginal product of labor and the household's marginal rate of substitution between consumption and leisure. I show that under the business cycle accounting approach proposed by Chari, Kehoe and McGrattan (2007) goods market frictions in the model manifest themselves as a labor wedge: In an expansion, firms find it easier to sell goods, and consumers benefit from higher availability of goods and smaller disutility from search effort required per unit of consumption purchased; this encourages larger response of the intensive margin of labor supply than in the standard frictionless model. It thus also alleviates the issue arising in model with frictional labor markets, where search frictions act as adjustment costs, and thus result in a labor wedge that resembles a counterfactually procyclical tax on labor income.

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1 Introduction

Business cycle models that incorporate labor market search improve upon frictionless labor market models along several dimensions, as first shown by Andolfatto (1996) and Merz (1995). Shimer (2005) and Shimer (2010) however raises two important issues regarding the labor search and matching model. First, fluctuations in unemployment and vacancies in response to productivity shocks in the calibrated model are much lower than those observed in U.S. data. Second, since search frictions act as adjustment costs, the measured labor wedge (the gap between firm's marginal product of labor and household's marginal rate of substitution) in this model resembles a procyclical labor income tax, contrary to the U.S. data. Thus instead of being able to explain fluctuations in the labor wedge, adding labor market search frictions exacerbates the problem. As a remedy to these issues, Shimer (2010) advocates for wage rigidity, in addition to labor market search frictions, and shows that it helps to explain why unemployment is so volatile and why measured labor wedge resembles a countercyclical tax on labor income. However, the wage rigidity required is that wages of workers in new employment relationships are rigid over the business cycle. Given that the empirical evidence available does not support this claim (see Pissarides, 2009 for a detailed discussion), this solutions is not completely without its own problems. Moreover, Bils, Klenow, and Malin (2014) decompose the labor wedge into product market (price mark-up) and labor market (wage mark-up) components, and argue that product market component is at least as important as labor market component. This implies that sticky wages and labor market matching friction can not fully account for the behavior of the labor wedge, and sticky prices or other frictions that generate countercyclical mark-ups of the product market component deserve more attention in the business cycle research.

In this paper I show that goods market search frictions manifest themselves as a labor wedge. When consumers need to exert effort to purchase goods, value of marginal earnings to a worker are modified by the extra disutility from this search. Similarly, when firm's are only able to sell a fraction of the output supplied to the market because of goods market search frictions, changes in the search effort by consumer's affect the value of the marginal product of the labor. Thus when the economy is subject to technology and preference shocks, a labor wedge equivalent to a countercyclical tax on labor income arises as a consequence of improved ability of firms to sell their goods in expansions, and the lower disutility required per unit of goods purchased by consumers in expansions.

The remainder of the paper is organized as follows. In Section 2 the business cycle accounting approach from Chari, Kehoe, and McGrattan (2007) is discussed, and used to construct the U.S. labor wedge. Section 3 describes the model, Section 4 characterizes the equilibrium and shows that the goods market search frictions present themselves as a labor wedge. In Section 5 I use

Bayesian techniques to parametrize shocks in the model, and then compare the implied business cycle properties of labor wedge generated by the model. Section 6 concludes.

2 The Labor Wedge

As Chari et al. (2007) point out, in many models mechanisms through which different shocks result in business cycle fluctuations manifest themselves as four wedges in the standard growth model - time varying productivity, labor and investment taxes, and government consumption. This motivates them to propose analysis of these wedges as a method to evaluate which mechanisms are promising in explaining business cycle fluctuations. They show that most of the fluctuations in the postwar period can be accounted for by efficiency and labor wedges, and thus stipulate that it is of particular interest to develop models that are able to replicate the behavior of efficiency and labor wedges observed in data.

The equilibrium in the prototype growth model that Chari et al. (2007) is characterized by a following set of conditions

$$Y_t = C_t + K_{t+1} - (1 - \delta_k)K_t + \tilde{G}_t \tag{2.1}$$

$$Y_t = \tilde{z}_t f(K_t, N_t H_t) \tag{2.2}$$

$$-U_H(C_t, H_t) = (1 - \tilde{\tau}_{wt})\tilde{z}_t f_L(K_t, N_t H_t)U_C(C_t, H_t) \tag{2.3}$$

$$(1 + \tilde{\tau}_{it})U_C(C_t, H_t) = \beta\mathbb{E}\Big[\big(\tilde{z}_{t+1}f_K(K_{t+1}, N_{t+1}H_{t+1}) + (1 + \tilde{\tau}_{it+1})(1 - \delta)\big)U_C(C_{t+1}, H_{t+1})\Big] \tag{2.4}$$

where $\tilde{z}$ is the efficiency wedge, $1 - \tilde{\tau}_w$ the labor wedge, $\frac{1}{1+\tilde{\tau}_i}$ the investment wedge, and $\tilde{G}$ the government consumption wedge. The first condition is the resource constraint, second one specifies production technology, third is the intratemporal optimality condition for labor, and the last one is the intertemporal optimality condition for capital.

To construct the time series for labor wedge, it's necessary to make assumptions about the functional forms for preferences and technology. Consider the case where utility and production functions are

$$U(C, H) = \log C - \zeta_n \frac{H^{1+\phi}}{1+\phi}$$

and

$$f(K, NH) = K^{1-\bar{\lambda}}(NH)^{\bar{\lambda}}$$

so that (2.3) yields the following labor wedge

$$1 - \tilde{\tau}_{wt} = \frac{\zeta_n}{\bar{\lambda}} \frac{C_t}{Y_t} H_t^{1+\phi}$$

where $\bar{\lambda}$ the average labor share in the U.S. national income, ζ_n is set to match the average labor wedge of 0.6, and ϕ is in turns chosen to obtain three with Frisch elasticity of labor supply equal

2

to 0.5, 1 and 3. The data used to construct the time series for consumption, output, and hours worked is discussed in Appendix B. The implied time series for labor wedge is shown in Figure 1, and its fluctuations around the long run trend, obtain using the Hodrick-Prescott filter with smoothing parameter 1600, are in Figure 2. In both figures the grey bands represent the NBER recession dates. The procyclical pattern of the labor wedge $1 - \tilde{\tau}_{wt}$ is clearly visible in both figures; for all three values of Frisch elasticity the labor wedge increases in recessions.
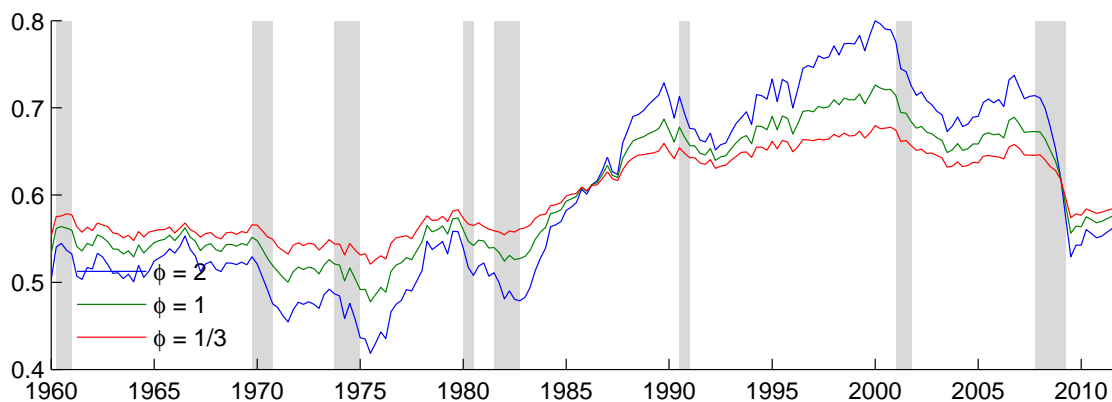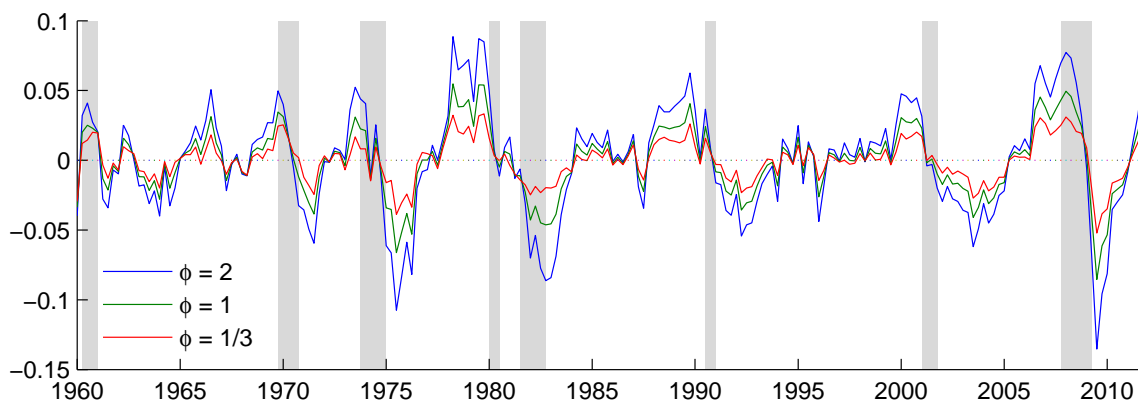
Figure 1: U.S. Labor wedge

Figure 2: U.S. Labor wedge, deviations from the HP trend

3

3 Model

There is a measure one of identical households, each consisting of a continuum of measure one of workers. Goods are sold in market that is subject to search frictions, firms post prices and consumers direct their search effort to acquire goods at a particular price. Workers cannot quit but there is exogenous job destruction. Firms need to open and maintain vacancies to hire new workers. For labor market I employ standard undirected search mechanism with Nash bargaining.

Household

Households are extended families, consisting of a measure one of workers as in Merz (1995). All workers are infinitely lived, ex-ante identical and have preferences

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^t\mathrm{u}(c_t, d_t, h_t, \zeta_t)$$

where c_t is consumption, d_t search effort in goods market, h_t hours worked and ζ_t is the preference shock affecting the marginal disutility of search for consumption good.

Given wealth a_t, held in the form of shares, and the number of members of the household that have a job after separations take place n_t, household decides about goods market search effort of its employed and unemployed workers d_t^n, d_t^u, consumption allocation c_t^n, c_t^u, and about share holdings for next period a_{t+1}. Each member also individually decides in which submarket (p_t, Q_t) to search for consumption goods, and directs the search to the submarket that delivers the biggest contribution to the utility of the household. I incorporate this through a constraint in the problem of a firm which posts priceq and decides about quantity sold. In addition, since in equilibrium only one market is going to be active, in the household's problem price of goods and goods market tightness p_t, Q_t are taken as given.

Each employed worker receives before tax wage w_t and works H_t hours, each unemployed worker receives unemployment benefits $p_t b$; labor income and unemployment benefits are taxed at rate τ_w and the household receives transfers τ_t. Taking prices p_t, wages w_t, hours worked by each employed worker H_t, and dividends R_t as given, the household then faces a budget constraint

$$p_t\big(n_t c_t^n + (1-n_t)c_t^u\big) + a_{t+1} = (1+R_t)a_t + (1-\tau_w)\big(n_t H_t w_t + (1-n_t)p_t b\big) + \tau_t \tag{3.1}$$

with shares acting as the numeraire good.

Search in goods market imposes a constraint

$$n_t c_t^n + (1-n_t)c_t^u = \big(n_t d_t^n + (1-n_t)d_t^u\big)\psi^d(A_t, Q_t, X_t) \tag{3.2}$$

where $\psi^d(A_t, Q_t, X_t)$ is the amount of goods acquired per unit of search effort.

The search in labor market implies that the number of workers employed in the household evolves according to

$$n_{t+1} = (1 - \delta_n)n_t + (1 - n_t)\pi^u(\theta_t) \tag{3.3}$$

where $\pi^u(\theta_t)$ is the probability for an individual to find a match in labor market.

Since the optimal allocation of consumption and search effort among family members in each period solves the problem

$$U(c_t, d_t, n_t, h_t, \zeta_t) = \max_{c_t^n, c_t^u, d_t^n, d_t^u} \left\{ n_t \mathrm{u}(c_t^n, d_t^n, h_t, \zeta_t) + (1 - n_t)\mathrm{u}(c_t^u, d_t^u, 0, \zeta_t) \right\} \tag{3.4}$$

subject to

$$n c_t^n + (1 - n_t)c_t^u = c_t$$
$$n d_t^n + (1 - n_t)d_t^u = d_t$$

where c_t is the total amount of consumption goods available to household and d_t is the overall search effort, I can formally set up the household's problem in which it acts as if it had preferences given by

$$\mathbb{E}\sum_{t=0}^{\infty} \beta^t U(c_t, d_t, n_t, \zeta_t) \tag{3.5}$$

and the household's problem is to choose a set of stochastic processes $\{c_t, d_t, a_{t+1}\}_{t=0}^{\infty}$ to maximize (3.5) subject to (3.1) -(3.4), taking as given stochastic processes $\{\zeta_t, z_t, A_t, p_t, R_t, w_t, b_t, \tau_t, Q_t, \theta_t\}_{t=0}^{\infty}$ and initial conditions a_0, n_0.

Firm

At the beginning of period t a firm has k_t capital, n_t workers employed and i_t stock of inventories. Each firm chooses in which submarket (p_t, Q_t) to sell the goods, and simultaneously also how many vacancies v_t to open and how much of the production to retain and use to add to the capital stock. The production of a firm is given by function $z_t f(k_t, l_t)$ where z_t is the productivity and l_t are the total hours worked in production. The amount of goods x_t that the firm can potentially sell is

$$x_t = z_t A_t f(k_t, n_t h_t - v_t) - k_{t+1} + (1 - \delta_k)k_t + i_t \tag{3.6}$$

where $f_l > 0$, $f_{ll} \leq 0$ which can be interpreted as a case where some of the workers act as recruiters as in Shimer (2010), and thus v_t hours worked are diverted from the production process to hiring. Each vacancy attracts $\pi^v(\theta_t)$ new workers. The firm's workforce thus evolves according to

$$n_{t+1} = (1 - \delta_n)n_t + \pi^v(\theta_t)v_t \tag{3.7}$$

If the firm decides to sell it's output x_t in the (p_t, Q_t) submarket, where the aggregate amount of goods being sold is X_t, then the actual amount of goods for which the firm will be able to find

a customer and sell is given by

$$x_t \psi^x(A_t, Q_t, X_t) = \frac{x_t}{X_t} \frac{\psi^d(A_t, Q_t, X_t)}{Q_t}$$

The firm can store goods that are not sold, in an attempt to sell them in the next period. Let i_{t+1} be the amount of goods carried over to the next period, given by

$$i_{t+1} = (1 - \delta_i)(1 - \psi^x(A_t, Q_t, X_t))x_t \tag{3.8}$$

where $\delta_i \in (0, 1)$ captures the loss of value due to obsoleteness, the fact that some goods will not be demanded at all in the future, storage costs, and the fact that services can not be stored.

As discussed above in section with household's problem, the firm needs to take into account the constraint guaranteeing shoppers in the (p_t, Q_t) submarket at least the equilibrium value of search $W^*_{d,t}$. Let M_t be the marginal value of wealth in terms of utility, then

$$W_{d,t} = U_{d,t} + (U_{c,t} - p_t M_t)\psi^d(A_t, Q_t, X_t) \tag{3.9}$$

is the value to the household of the marginal search effort in the (p, Q) submarket. The expected present value of firm's profits is

$$\mathbb{E} \sum_{t=0}^{\infty} \left(\prod_{j=0}^{t} \frac{1}{1 + R_t} \right) \left[p_t \psi^x(A_t, Q_t, X_t)x_t - (1 + \tau_f)w_t h_t n_t \right] \tag{3.10}$$

and the firm's problem is to choose a set of stochastic processes $\{k_{t+1}, v_t, x_t, i_{t+1}, p_t, Q_t\}_{t=0}^{\infty}$ to maximize (3.10) subject to (3.6)-(3.9), taking as given stochastic processes $\{z_t, A_t, R_t, w_t, h_t, \theta_t\}_{t=0}^{\infty}$ and initial conditions k_0, n_0, i_0.

Government

Government's budget is assumed to be balanced in each period, thus total tax revenues are equal to total government expenditures

$$\tau_w \big(n_t h_t w_t + (1 - n_t)p_t b \big) + \tau_f n_t h_t w_t = (1 - n_t)p_t b + \tau_t \tag{3.11}$$

Labor Market

As in standard labor search model, search in the labor market is not directed, number of matches is given by an aggregate constant returns to scale matching function $m^L(U_t, V_t)$ where U_t are unemployed workers and V_t are the vacancies posted by firms. I denote by $\theta_t = \frac{V_t}{U_t}$ tightness of the market, and by $\pi^u(\theta_t) = m^L(1, \theta_t)$ the probability for an unemployed worker to be hired, and by $\pi^v(\theta_t) = m^L(1/\theta_t, 1)$ the rate at which a recruiter hires workers. If a worker and a recruiter meet, wage w_t and hours worked h_t is set as a solution to the asymmetric Nash bargaining problem that splits the surplus of the match

$$(w_t, h_t) = \operatorname*{argmax}_{\hat{w}, \hat{h}} \hat{W}_{n,t}(\hat{w}, \hat{h})^\mu \hat{\Omega}_{n,t}(\hat{w}, \hat{h})^{1-\mu} \tag{3.12}$$

where $\hat{W}_{n,t}(\hat{w}, \hat{h})$ is household's marginal value of a worker employed under a contract requiring arbitrary hours worked $\hat{h}$ at arbitrary wage $\hat{w}$ in the current period and equilibrium hours h at equilibrium wage w thereafter, until the job is hit by the separation shock δ_n. Similarly $\hat{\Omega}_{n,t}(\hat{w}, \hat{h})$ is firm's marginal value of an employed worker under a contract requiring arbitrary hours $\hat{h}$ at arbitrary wage $\hat{w}$ in the current period and equilibrium hours h and equilibrium wage w thereafter, until the job is hit by the separation shock δ_n.

Goods Market

Acquisition of consumption goods requires active search effort on the side of the consumer to find the goods to be purchased, and I use the competitive search mechanism (Moen, 1997) to model the frictions in the goods market. Goods market is thus divided into submarkets, firm and household can choose in which submarket to search, and the maches in each submarket are determined by the same constant returns to scale matching function $m^G(A_t D_t, T_t X_t)$ with elasticity of substitution σ. Here D_t is aggregate search effort of all consumers in that particular submarket, T_t the measure of firms selling in that particular submarket and X_t is the quantity of goods sold per firm. Submarkets are indexed by (p_t, Q_t) where p_t is the price of consumption good in terms of the shares and $Q_t = \frac{T_t}{D_t}$ is the tightness of the submarket. The amount of goods acquired per unit of search effort by household's shopper is

$$\psi^d(A_t, Q_t, X_t) = m^G(A_t, Q_t X_t)$$

and amount of output successfully sold by a firm trying to sell x_t goods in submarket (p_t, Q_t), where the total amount of goods sold by all firms is X_t is

$$\psi^x(A_t, Q_t, X_t)x_t = m^G\left(\frac{A_t}{Q_t X_t}, 1\right)x_t = \frac{\psi^d(A_t, Q_t, X_t)}{X_t Q_t}x_t$$

Equilibrium

Definition 1. *Given the government's policy (τ_w, τ_f, b), the set of exogenous stochastic processes $\{z_t, A_t, \zeta_t\}_{t=0}^{\infty}$ and initial conditions K_0, N_0, I_0, an equilibrium is a list of stochastic processes $\{C_t, D_t, X_t, I_{t+1}, K_{t+1}, V_t, N_t, H_t, Q_t, \theta_t, p_t, w_t, R_t, \tau_t\}_{t=0}^{\infty}$ such that*

1. *$\{C, D_t\}_{t=0}^{\infty}$ are the optimal choices in the household's problem*

2. *$\{X_t, I_{t+1}, K_{t+1}, V_t, p_t, Q_t\}_{t=0}^{\infty}$ are the optimal choices for firm, $\{R_t\}_{t=0}^{\infty}$ are the associated profits*

3. *Wage w_t and hours worked h_t solves the Nash bargaining problem (3.12)*

4. *Government's budget constraint (3.11) is satisfied*

5. *Goods market tightness is $Q_t = 1/D_t$; labor market tightness $\theta_t = V_t/(1 - N_t)$*

4 Equilibrium Characterization

To obtain conditions that determine the dynamics of model I first derive the optimality conditions for the household and the firm and then use them to obtain the solution for the Nash bargaining problem. This allows to characterize the behavior of the six main variables in the model (K, I, N, H, Q, θ).

4.1 Household's Optimality Conditions

From the first order conditions we get for the value of the marginal unit of income

$$\lambda_{1,t} = \frac{1}{p_t}\left(U_{c,t} + \frac{U_{d,t}}{\psi_t^d}\right)$$

and the following expression for the marginal value of a worker employed under a contract with equilibrium hours of work H_t and equilibrium wage w_t

$$W_{n,t} = U_{n,t} + (1 - \tau_w)\left(\frac{w_t}{p_t}H_t - b\right)\left(U_{c,t} + \frac{U_{d,t}}{\psi_t^d}\right) + (1 - \delta_n - \pi_t^u)\beta\mathbb{E}_t W_{n,t+1} \tag{4.1}$$

Optimal choice of asset accumulation requires that

$$\lambda_{1,t} = \beta\mathbb{E}_t[\lambda_{1,t+1}(1 + R_{t+1})]$$

and yields the following Euler equation equalizing the cost of increasing saving by a marginal unit and the return from this marginal savings

$$\frac{1}{p_t}\left(U_{c,t} + \frac{U_{d,t}}{\psi_t^d}\right) = \beta\mathbb{E}_t\left[(1 + R_{t+1})\frac{1}{p_{t+1}}\left(U_{c,t+1} + \frac{U_{d,t+1}}{\psi_{t+1}^d}\right)\right] \tag{4.2}$$

The left hand side corresponds to the utility cost of extra unit of savings: the household could have instead purchased $\frac{1}{p_t}$ units of good which require utility cost $\frac{U_{d,t}}{\psi_t^d}$ per unit of good because of the search friction, and enjoyed $U_{c,t}$ extra utility per unit of good. The right hand side corresponds to the utility benefit of extra unit of savings: the $1 + R_{t+1}$ monetary flow in the next period can be used to purchase extra consumption in the next period. It will be convenient to denote by M_t the expected discounted utility from marginal unit of share holdings

$$M_t = \beta \mathbb{E}_t \left[(1 + R_{t+1}) \frac{1}{p_{t+1}} \left(U_{c,t+1} + \frac{U_{d,t+1}}{\psi_{t+1}^d} \right) \right] \tag{4.3}$$

The above intertemporal optimality condition thus states that $\lambda_{1,t} = M_t$.

4.2 Firm's Optimality Conditions

Since the household is representative adding the full set of Arrow securities would not affect the allocation, and I can use standard complete markets pricing approach to value the firm. Thus we have for the stochastic discount factor

$$m_{t,t+1} = \beta \frac{p_t}{p_{t+1}} \frac{U_{c,t+1} + \frac{U_{d,t+1}}{\psi_{t+1}^d}}{U_{c,t} + \frac{U_{d,t}}{\psi_t^d}} \tag{4.4}$$

From the first order conditions for firm choosing in which submarket to sell goods we get that the equilibrium price of the consumption good satisfies

$$p_t = \epsilon_{Q,t}^{\psi^d} \frac{U_{c,t}}{M_t} + (1 - \epsilon_Q^{\psi^d})(1 - \delta_i) \mathbb{E}_t[m_{t,t+1}\Omega_{i,t+1}] \tag{4.5}$$

where $\epsilon_{Q,t}^{\psi^d} = \frac{\partial \log \psi^d}{\partial \log \psi^d}$, and the value of the marginal unit of inventories satisfies

$$\Omega_{i,t} = \psi_t^x p_t + (1 - \psi_t^x)(1 - \delta_i) \mathbb{E}_t[m_{t,t+1}\Omega_{i,t+1}] \tag{4.6}$$

The intertemporal condition for optimal capital accumulation requires that

$$\Omega_{i,t} = \mathbb{E}_t \left[m_{t,t+1}\Omega_{i,t+1}(z_{t+1}f_{k,t+1} + 1 - \delta_k) \right] \tag{4.7}$$

and the marginal value of a worker for the firm is given by

$$\Omega_{n,t} = \left(z_t f_{l,t} H_t + \frac{1 - \delta_n}{\pi_t^v} z_t f_{l,t} \right) \Omega_{i,t} - (1 + \tau_f) w_t H_t \tag{4.8}$$

and implies the following job creation condition

$$z_t f_{l,t} \Omega_{i,t} = \pi^v \mathbb{E} \left[m_{t,t+1} \left(\left(H_{t+1} + \frac{1 - \delta_n}{\pi_{t+1}^v} \right) z_{t+1} f_{l,t+1} \Omega_{i,t+1} - (1 + \tau_f) w_{t+1} H_{t+1} \right) \right] \tag{4.9}$$

which equalizes the cost of increasing recruiting to hire an extra worker, as thus resulting in lower production in the present, with expected benefit of having hired an extra worker and thus producing more and requiring lower recruiting in the future.

9

4.3 Goods Market and Capital Accumulation

From (4.5), combined with (4.2) and (4.3), after eliminating p and M one can obtain the following condition

$$-U_{d,t} = (1 - \epsilon_{Q,t}^{\psi^d})\psi_t^d\left[U_{c,t} - (1-\delta_i)\beta\mathbb{E}_t\left[\left(U_{c,t+1} + \frac{U_{d,t+1}}{\psi_{t+1}^d}\right)\Omega_{i,t+1}^r\right]\right] \tag{4.10}$$

which states that the marginal cost and the marginal benefit of search effort in the goods market are equalized.

By plugging the expression for the stochastic discount factor (4.4) into the optimality condition from firm's problem (4.7) we can derive the following Euler equation for optimal capital accumulation

$$\Omega_{i,t}^r\left(U_{c,t} + \frac{U_{d,t}}{\psi_t^d}\right) = \beta\mathbb{E}_t\left[\Omega_{i,t+1}^r(z_{t+1}f_{k,t+1} + 1 - \delta_k)\left(U_{c,t+1} + \frac{U_{d,t+1}}{\psi_{t+1}^d}\right)\right] \tag{4.11}$$

Using the stochastic discount factor (4.4) and the value of the marginal unit of inventories from firm's problem, one can also obtain that

$$\Omega_{i,t}^r = \psi^x + (1 - \psi_t^x)\beta(1-\delta_i)\mathbb{E}_t\left[\frac{U_{c,t+1} + \frac{U_{d,t+1}}{\psi_{t+1}^d}}{U_{c,t} + \frac{U_{d,t}}{\psi_t^d}}\Omega_{i,t+1}^r\right] \tag{4.12}$$

4.4 Labor Market and Employment Determination

Under Nash bargaining protocol, wage w_t and hours worked H_t are jointly determined as a solution to the following problem

$$(w_t, H_t)) = \underset{\hat{w},\hat{H}}{\operatorname{argmax}} \hat{W}_n(\hat{w}, \hat{H})^\mu \hat{\Omega}_n(\hat{w}, \hat{H})^{1-\mu}$$

where $\hat{W}_n(\hat{w}, \hat{H})$ and $\hat{\Omega}_n(\hat{w}, \hat{H})$ are values, to household and firm, of a marginal worker employed under a contract requiring arbitrary hours worked $\hat{H}$ at arbitrary wage $\hat{w}$ in the current period and equilibrium hours H at equilibrium wage w thereafter, until the job is hit by the separation shock δ_n.

By considering a household with n_t members employed for equilibrium wage w_t and working equilibrium hours H_t, and ν members employed for arbitrary wage $\hat{w}$ and working arbitrary hours $\hat{H}$ in the current period and equilibrium wage w and equilibrium hours H thereafter, until the they are hit by the separation shock δ_n, and taking the limit as $\nu \to 0$, we can obtain the value of a marginal member of the household employed for this households

$$\hat{W}_{n,t}(\hat{w}, \hat{H}) = U_{n,t}(\hat{H}) - U_{n,t}(H_t) + (1-\tau_w)\frac{\hat{w}\hat{H} - w_t H_t}{p_t}\left(U_{c,t} + \frac{U_{d,t}}{\psi_t^d}\right) + W_{n,t} \tag{4.13}$$

By considering a firm that employs n_t workers employed at equilibrium wage w_t and equilibrium hours H_t, and ν workers employed at arbitrary wage $\hat{w}$ and arbitrary hours $\hat{H}$ in the current period, and equilibrium w and H thereafter, and taking the limit as $\nu \to 0$, we can obtain the value of an extra worker for the firm

$$\hat{\Omega}_{n,t}(\hat{w}, \hat{H}) = z_t f_{l,t}(\hat{H} - H_t)\Omega_{i,t} + (1 + \tau_f)(w_t H_t - \hat{w}\hat{H}) + \Omega_{n,t} \tag{4.14}$$

The Nash bargaining problem is thus

$$(w_t, H_t) = \operatorname*{argmax}_{\hat{w}, \hat{H}} \hat{W}_n(\hat{w}, \hat{H})^\mu \hat{\Omega}_n(\hat{w}, \hat{H})^{1-\mu}$$

subject to (4.13) and (4.14). The first order condition for w yields a sharing rule

$$W_{n,t} = \frac{\mu}{1-\mu}\left(U_{c,t} + \frac{U_{d,t}}{\psi_t^d}\right)\frac{1 - \tau_w}{1 + \tau_f}\frac{\Omega_{n,t}}{p_t} \tag{4.15}$$

or $\frac{W_{n,t}}{p_t \lambda_{1,t}} = \mu \mathcal{S}_t$ where $\lambda_{i,t}$ is the marginal value of wealth for the household and $\mathcal{S}_t = \frac{\Omega_{n,t}}{p_t} + \frac{W_{n,t}}{p_t \lambda_{1,t}}$ is the total surplus of the match.

From the first order condition for H_t we get, using the first order condition for w_t, the following condition for hours worked

$$-U_{n,h,t} = \frac{1 - \tau_w}{1 + \tau_f}\Omega_{i,t}^r z_t f_{l,t}\left(U_{c,t} + \frac{U_{d,t}}{\psi_t^d}\right) \tag{4.16}$$

To derive the wage equation first plug $W_{n,t}$ from the sharing rule (4.15) into (4.1), use stochastic discount factor (4.4), and the optimality condition for firm (4.8) and (4.9) which after a little bit of algebra yields the equation for wage bill per worker

$$\frac{w_t H_t}{p_t} = \mu\frac{1}{1 + \tau_f}\left(H_t + \theta\right)z_t f_{l,t}\Omega_{i,t}^r + (1 - \mu)\left(b - \frac{1}{1 - \tau_w}\frac{U_{n,t}}{U_{c,t} + \frac{U_{d,t}}{\psi_t^d}}\right) \tag{4.17}$$

Similarly to other search-matching models with Nash bargaining, wage is a weighted average of the value of marginal product of a worker enhanced by the vacancy cost savings, and the marginal rate of substitution between leisure and consumption.

Finally, to get a stochastic difference equation that characterizes the labor market plug in for w' from (4.17) into the job creation equation (4.9), and use stochastic discount factor (4.4) to get

$$\frac{1}{\pi_t^v}z f_{l,t}\Omega_{i,t}^r\left(U_{c,t} + \frac{U_{d,t}}{\psi_t^d}\right)$$

$$= \beta\mathbb{E}_t\left[\left[(1 - \mu)H_{t+1} + \left(\frac{1 - \delta_n}{\pi_{t+1}^v} - \mu\theta_{t+1}\right)\right]z_{t+1}f_{l,t+1}\Omega_{i,t+1}^r\left(U_{c,t+1} + \frac{U_{d,t+1}}{\psi_{t+1}^d}\right)\right.$$

$$\left. - (1 - \mu)\left[(1 + \tau_f)b\left(U_{c,t+1} + \frac{U_{d,t+1}}{\psi_{t+1}^d}\right) - \frac{1 + \tau_f}{1 - \tau_w}U_{n,t+1}\right]\right] \tag{4.18}$$

4.5 Efficiency

The results so far are summarized by following proposition.

Proposition 1. *In equilibrium stochastic processes for market tightness Q_t and θ_t and for allocation H_t, K_t, I_t, N_t satisfy the following system of equations*

$$\Lambda_t = U_{c,t} + \frac{U_{d,t}}{\psi_t^d} \tag{4.19}$$

$$-U_{d,t} = (1 - \epsilon_{Q,t}^{\psi^d})\psi_t^d\Big[U_{c,t} - (1 - \delta_i)\beta\mathbb{E}_t\big[\Lambda_{t+1}\Omega_{i,t+1}^r\big]\Big] \tag{4.20}$$

$$-U_{n,h,t} = \frac{1 - \tau_w}{1 + \tau_f}\Omega_{i,t}^r z_t f_{l,t}\Lambda_t \tag{4.21}$$

$$\frac{1}{\pi_t^v}z_t f_{l,t}\Omega_{i,t}^r\Lambda_t$$

$$= \beta\mathbb{E}_t\Bigg[\Big[(1 - \mu)H_{t+1} + \Big(\frac{1 - \delta_n}{\pi_{t+1}^v} - \mu\theta_{t+1}\Big)\Big]z_{t+1}f_{l,t+1}\Omega_{i,t+1}^r\Lambda_{t+1}$$

$$-(1 - \mu)\Big[(1 + \tau_f)b\Lambda_{t+1} - \frac{1 + \tau_f}{1 - \tau_w}U_{n,t+1}\Big]\Bigg] \tag{4.22}$$

$$\Omega_{i,t}^r\Lambda_t = \beta\mathbb{E}_t\big[\Omega_{i,t+1}^r(z_{t+1}f_{k,t+1} + 1 - \delta_k)\Lambda_{t+1}\big] \tag{4.23}$$

$$\Omega_{i,t}^r = \psi^x + (1 - \psi_t^x)\beta(1 - \delta_i)\mathbb{E}_t\Big[\frac{\Lambda_{t+1}}{\Lambda_t}\Omega_{i,t+1}^r\Big] \tag{4.24}$$

$$N_{t+1} = (1 - \delta_n)N_t + (1 - N_T)\pi_t^u \tag{4.25}$$

$$I_{t+1} = (1 - \delta_i)(1 - \psi_t^x)X_t \tag{4.26}$$

with (C_t, X_t, V_t, D_t) eliminated using

$$C_t = \psi_t^d/_t Q_t$$

$$X_t = z_t f_t + (1 - \delta_k)K_t - K_{t+1} + I_t$$

$$V_t = \theta_t(1 - N_t)$$

$$Q_t = T_t/D_t$$

The first one of the seven equations (4.20)-(4.26) is the intratemporal optimality conditions for goods market with competitive search. The second one the intratemporal optimality conditions for hours worked in equilibrium showing that hours worked optimally equate the utility cots of an extra hour of work with its benefit, which is the utility gain from consumption of goods produced and purchased, adjusted for the utility cost incurred due to extra search needed to purchase these goods. The third equation is the counterpart of the stochastic first order difference equation for labor market tightness θ in the basic labor search model. This condition equates the cost of hiring a worker in terms of utility (fewer goods sold and thus also consumed), with the value of an extra

12

worker hired in terms of utility (increased production and hiring cost saved which both allow to increase consumption in future, adjusted for the value of foregone leisure). In the Euler equation for capital accumulation the left hand side is the cost of marginal unit of output allocated into investment, in the form of foregone utility from consumption, and the right hand side is the benefit of this marginal investment unit in the form of extra utility derived from extra consumption in the next period.

The efficient allocation in this economy is defined as an allocation chosen by a social planner facing the search-matching technological restrictions in the labor and goods markets

Definition 2. *An allocation is efficient if it solves*

$$\max_{\{C_t, D_t, H_t, X_t, V_t, K_{t+1}\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(C_t, D_t, N_t, H_t, \zeta_t)$$

subject to

$$C_t = m^G(D_t, X_t)$$

$$X_t = zf(K_t, N_t H_t - V_t) + (1 - \delta_k)K_t - K_{t+1} + I_t$$

$$I_{t+1} = (1 - \delta_i)(X_t - C_t)$$

$$N_{t+1} = (1 - \delta_n)N_t + m^L(U_t, V_t)$$

Given this definition of efficiency the following proposition establishes condition which guarantees the efficiency of the decentralized economy.

Proposition 2. *If $\tau_w = \tau_f = 0$, $b = 0$ and worker's bargaining power is $\mu = \frac{\partial \log m^L}{\partial \log U}$ equilibrium is efficient.*

This proposition thus implies that the existence of the labor market wedge in this model does not imply inefficiency, as would be the case with wage or price mark-ups due to monopolistic competition or sticky wages and prices.

4.6 Labor Wedge in the Model with Goods and Labor Search

Comparing measured output in the prototype RBC model from Chari et al. (2007) and the RBC model with labor and goods market search, one can see that goods market frictions alter the efficiency wedge; comparing the intratemporal condition for hours worked (4.21) and (2.3) it is clear that they also affect the labor wedge. If in expansion the disutility associated with obtaining a marginal unit of consumption $\frac{U_d}{\psi^d}$ falls, or if the marginal value of an inventory Ω_i^r increases, the labor wedge in the goods and labor market search model will be more procyclical than the labor wedge in the labor search model from Shimer (2010). To show that this is indeed the case, I now turn to the quantitative analysis of the business cycle properties of the model.

5   Quantitative Analysis

5.1   Functional forms

I consider the case with following functional forms for preferences and technology. Utility of an individual worker is given by

$$u(c, d, e, h) = \zeta_c \log c - \zeta_d \frac{d^{1+\varphi}}{1+\varphi} - e\zeta_n \frac{h^{1+\phi}}{1+\phi} - (1-e)\zeta_u$$

Production function is of Cobb-Douglas form $zAf(k, l) = zAk^{1-\lambda}l^{\lambda}$. Matching functions have constant elasticity of substitution form

$$m^L(U, V) = B(\gamma U^{\frac{\nu-1}{\nu}} + (1-\gamma)V^{\frac{\nu-1}{\nu}})^{\frac{\nu}{\nu-1}}$$
$$m^G(AD, X) = \left(\alpha(AD)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)X^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}$$

Since preferences are additively separable between consumption, hours worked, and search effort we immediately get $c_e = c_u = c$ and $d_e = d_u = d$. The household thus acts as if it had preferences

$$U(c, d, n, h, \zeta) = \log c - \zeta_d \frac{d^{1+\varphi}}{1+\varphi} - n\zeta_n \frac{h^{1+\phi}}{1+\phi} - (1-n)\zeta_u$$

The technology is subject to temporary shocks to z and to permanent shocks to the stochastic trend A, which grows at rate $\gamma'_A = \frac{A'}{A}$. To guarantee existence of a balanced growth path, the permanent component of the technology also increases efficiency of the search effort by consumers.

The processes for shocks are assumed to be

$$\log \xi' = (1 - \rho_\xi) \log \bar{\xi} + \rho_\xi \log \xi + \varepsilon'_\xi$$

where $\xi \in \{\gamma_A, z, \zeta_d\}$ and $\varepsilon_\xi \sim N(0, \eta_\xi^2)$.

5.2   Calibration

One period of the model is one quarter, parameter β is chosen to obtain the annual interest rate of 5%. I set $\bar{z}$ to normalize the level of realized consumption $C = 1$ and set λ to target the capital share 0.36. Capital output ratio target is 3.2 as in Shimer (2010), and δ_k is 0.07. I assume a symmetric goods market matching function with $\alpha = 0.5$. Depreciation of inventories is 0.15 for goods and 1 for services, the implied overall quarterly depreciation of inventories is thus 0.83.

For labor market I follow Shimer (2010) by setting $\nu = 0$, $\gamma = 0.5$ which implies a symmetric Cobb-Douglas matching function. I set the value of unemployment benefits b to 0.2 of average labor productivity, target quarterly job separation rate $\delta_n = 0.1$, quarterly job finding rate $\pi^u = 0.733$ and steady state unemployment rate $U = 0.12$. Silva and Toledo (2009) and Hagedorn and Manovskii

(2008) argue for average costs associated with recruiting, screening and interviewing needed to hire a new worker around 4% to 5% of new worker's quarterly wages paid. Since an hour of recruitment in the model attracts π^v workers, to get one worker $\frac{1}{\pi^v}$ hours of recruitment are needed. Thus, if w is the wage in the model, the total costs of a hire are $\frac{1}{\pi^v}w = 0.065 \times wH$ and so I target $\pi^v = \frac{1}{0.065H}$. I set τ_w, τ_f to obtain the steady state measured labor wage of 0.6, consistent with U.S. data as discussed in Section 2. Given the job finding rate and recruitment rates targeted, since $\frac{\pi^u}{\pi^v} = \theta$ and $\pi^u = B\theta^{1-\gamma}$ the matching efficiency parameter is $B = (\pi^u)^\gamma(\pi^v)^{1-\gamma} = 6.13$.

Parameter ϕ is set to get Frisch elasticity 0.7, and φ is to 0. I calibrate ζ_n so that in the steady state hours worked are $H = 0.3$. To set ζ_u notice that for a given bargaining power μ, value of home production and leisure ζ_u affects wage and through that profits of the firms, hiring, labor tightness θ, and also U. I thus proceed as Shimer (2010), set $\mu = \gamma$ and calibrate ζ_u to match the above mentioned target unemployment rate. I set $\bar{\zeta}_c = 1$ and calibrate $\bar{\zeta}_d$ to normalize the steady state goods market tightness to $Q = 1$.

5.3 Estimation

In order to set the parameters of the processes γ_A, z and ζ_d and the elasticity of substitution for the good market matching function σ, I estimate a log-linearized model to match observed quarterly time series for the growth rate of the measured productivity residual $\gamma_{\hat{z}} = \Delta \log \hat{z}$, the growth rate of per capita output γ_Y, and the growth rate of per capita consumption γ_C. The sample used is 1960Q1-2010Q4, ?? describes the data. Table 1 shows the prior distributions for estimation, estimated posterior mode obtained by maximizing the log of the posterior distribution, the approximate standard error based on the corresponding Hessian, and also the mean, mode, 10 and 90 percentile of the posterior distribution of the parameters obtained using the random walk Metropolis-Hastings algorithm with four chains and 500000 draws.

Table 1: Estimation of the model with shocks to γ_A, z, ζ_d

	Prior			Posterior		
	distribution	mean	st.dev.	mode	mean	90 % HPD interval
ρ_d	Beta	0.8	0.1	0.9914	0.9893	[0.9821,0.9968]
ρ_z	Beta	0.8	0.1	0.9205	0.9237	[0.8888,0.9591]
ρ_A	Beta	0.6	0.2	0.1440	0.1563	[0.0621,0.2458]
st.dev. ε_d	Inverse Gamma	0.01	2	0.0096	0.0101	[0.0081,0.0121]
st.dev. ε_z	Inverse Gamma	0.01	2	0.0041	0.0043	[0.0036,0.0050]
st.dev. ε_A	Inverse Gamma	0.01	2	0.0060	0.0060	[0.0055,0.0065]
σ	Gamma	1	1	0.3378	0.3861	[0.2305,0.5309]

5.4 Simulation

Table 2 compares the average standard deviations and correlations of the main variables in 1000 simulations of the model with their counterparts in U.S. data. All variables are in logs, HP filtered with parameter $\lambda = 1600$. The statistics for labor wedge are presented for both the representative agent wedge, and for the intensive margin wedge which allows to distinguish the hours per worker and the employment components. The representative agent wedge is defined as

$$RAW = \frac{MRS}{MP_L} = \frac{\zeta_n(NH)^\phi/\frac{1}{C}}{\bar{\lambda}\frac{Y}{NH}}$$

and the intensive margin wedge as

$$IMW = \frac{MRS_H}{MPN_H} = \frac{N\zeta_n H^\phi/\frac{1}{C}}{\bar{\lambda}\frac{Y}{NH}N}$$

Thus in the representative margin wedge the marginal rate of substitution of consumption for leisure is based on hours per capita, where as in the intensive margin wedge it is based on the hours per worker. Similarly, the marginal product in the representative margin wedge is the marginal product of an hour, where as in the intensive margin wedge it is marginal product of an hour per worker (see Pescatori & Tasci, 2013 and Bils et al., 2014 for a further discussion on this distinction).

Table 2: Summary statistics, based on 1000 simulations of the model

	U.S. data		model	
	st.dev.$(\cdot)$/st.dev.(Y)	corr$(\cdot, Y)$	st.dev.$(\cdot)$/st.dev.(Y)	corr$(\cdot, Y)$
C	0.81	0.88	0.71	0.79
ΔK	3.25	0.93	2.16	0.80
H	0.31	0.79	0.20	0.87
N	0.71	0.83	0.26	0.72
RAW	1.90	0.41	1.01	0.42
IMW	1.05	0.32	0.81	0.22
S	0.90	0.95	0.98	0.99
I	0.86	0.58	0.34	0.76
I/S	0.75	-0.44	0.77	-0.95

All variables in logs, HP filtered $\lambda = 1600$

In the simulations, both the representative agent labor wedge and the intensive margin labor wedge are somewhat less volatile than in the data. Both are however procyclical, with similar correlation with output as observed in the data.

The historical labor wedge and the smoothed labor wedge obtain in the estimation are plotted in Figure 3 and Figure 4. The correlation between the representative agent labor wedge in the U.S. data and the one recovered in the estimation is 0.497, for the intensive margin labor wedge this correlation is 0.324. Table 3 compares the cyclical properties of the actual labor wedge in the U.S. data and the labor wedge recovered in the estimation.

Table 3: Historical vs. smoothed labor wedge

	U.S. data	model
	Standard deviation relative to output	
RAW	1.897	1.382
IMW	1.044	1.022
	Correlation with output	
RAW	0.71	0.69
IMW	0.53	0.51
	Elasticity with respect to output	
RAW	1.349	0.958
IMW	0.554	0.521

All variables in logs, HP filtered $\lambda = 1600$

Table 2 also shows that even though both consumption and investment are somewhat less volatile in the model compared to the data, they are not very far away. The main shortcoming of the model is the implied volatility of employment which is considerably smaller. On the more positive side, even though neither data on inventories nor data on sales was not used as observables in the estimation, the model can replicate the countercyclical inventory-sales ratio, procyclical inventories, and sales which are less volatile than output.
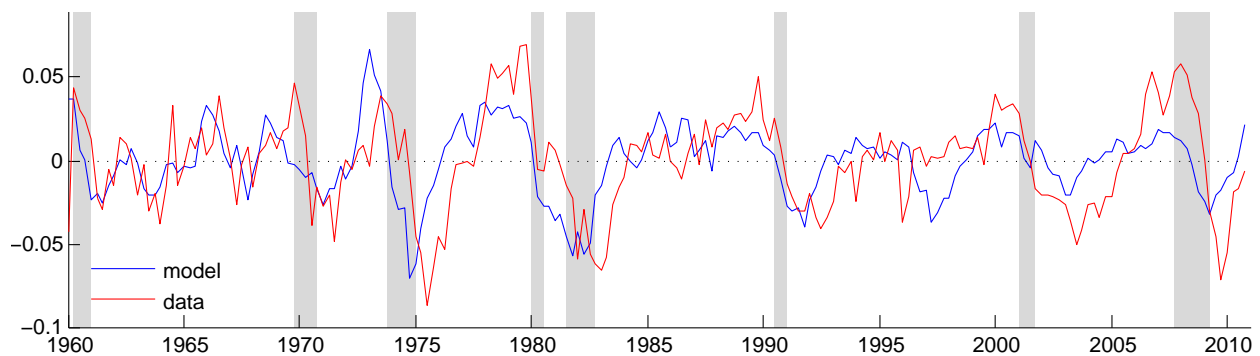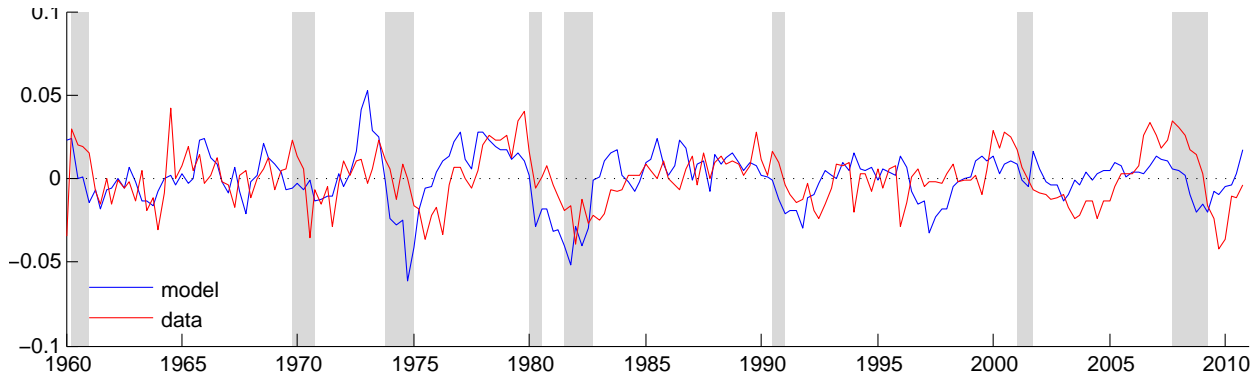
Figure 3: Representative margin wedge

Figure 4: Intensive margin wedge

6 Conclusions

In this paper, I modify the standard real business cycle model by replacing frictionless labor and goods markets with markets that require search effort of market participants to find a match. I use this model to demonstrate that under the business cycle accounting approach proposed by Chari et al. (2007), search frictions in the goods market manifest themselves as a labor wedge. The model is estimated using Bayesian methods to match U.S. Solow residual, output and consumption growth. Both technology and preference shocks to disutility from search are included in the estimation, to allow for supply and demand side disturbances. In the estimated model with search frictions in both labor and goods markets, firms are more likely to sell goods in expansions due to an increase in demand, and the disutility from search effort required per unit of consumption falls in expansion. As a result there is a larger response of the intensive margin of labor supply and the measured labor wedge resembles a countercyclical tax on labor income. This is in stark contrast to the model in Shimer (2010) where only labor market is subject to search frictions, and the labor wedge resembles a counterfactually procyclical tax on labor income. Since inventories naturally arise in an environment where search frictions prevent output from being sold immediately, the developed model also provides a framework to analyze the behavior of inventories and sales. Even though these are not targeted, the model can successfully match the three main facts from U.S. data on inventories that have proved to be quite a challenge to explain - sales that are less volatile than production, inventory investment that are procyclical and inventories-sales ratio which is countercyclical.

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Appendix B

Data sources

Time series used in this paper were retrieved from the following sources:

1. Bureau of Economic Analysis (BEA) www.bea.gov

2. Bureau of Labor Statistics (BLS) www.bls.gov

3. NBER Macrohistory database (NBER) www.nber.org/databases/macrohistory/contents

4. Employment and Earnings data compiled by Cociuba, Prescott, and Ueberfeldt (2012)

In particular, following data was obtained.

National Income and Product Accounts (BEA:NIPA)

1. Table 1.7.5: Gross National Product GNP_t, Consumption of Fixed Capital DEP_t

2. Table 1.12: Compensation of Employees CE_t, Rental Income RI_t, Corporate Profits CP_t, Net Interests NI_t, Current Surplus of Government Enterprises GE_t

3. Table 1.1.4: Price Index for Gross Domestic Product $pGDP_t$

Fixed Assets Accounts Tables (BEA:FAA)

1. Table 1.1: Current Cost Net Private Fixed Assets K_{2005}

2. Table 1.2: Chain-Type Quantity Index for Private Fixed Assets qiK_t

Current Population Survey (BLS:CPS)

1. Civilian Noninstitutional Population, age 16 and more $P16_t$: Series ID LNU00000000

2. Civilian noninstitutional population, 65 years and over $P65_t$: Series ID LNU00000097

3. Employment E_t: Series ID LNU02005053

4. Average Weekly Hours AWH_t: Series ID LNU02005054

NBER Income and Employment (NBER:IE)

1. Average Weekly Hours AWH_t: Series m08354

Cociuba et al. (2012) (CPU)

1. Employment E_t

2. Average Weekly Hours AWH_t

Constructed time series

Labor share: obtained by constructing following time series

$$\lambda_t = 1 - \frac{RI_t + CP_t + NI_t + GE_t + DEP_t}{CE_t + RI_t + CP_t + NI_t + GE_t + DEP_t}$$

See Ríos-Rull and Santaeullia-Llopis (2010) for more details.

Hours: monthly time series for employment E_t and average weekly hours AWH_t were compiled from BLS:CPS, NBER:IE, and CPU sources described above, and were seasonally adjusted using Census X-12 Arima procedure with Easter and Labor day dummies. Then, quarterly averages were constructed, and total hours and hours per person of age 16-64 were obtained using $TH_t = E_t AWH_t$ and $H_t = TH_t/(P16_t - P64_t)$. Finally total hours were annualized and hours per person were expressed relative to 100 hours per week.

Real Capital K_t: obtained by multiplying the chain-type quantity index from BEA:FAA Table 1.2 by the current-cost net stock in 2005 from BEA:FAA Table 1.1, and interpolated to obtain quarterly time series.

Productivity residual: obtained by first taking a logarithm of GNP, real capital and total hours worked, then linearly detrending these time series and finally calculating

$$\log \hat{z}_t = \tilde{y}_t - (1 - \bar{\lambda})\tilde{k}_t - \bar{\lambda}\tilde{th}_t$$

where $\bar{\lambda}$ is the average labor share, and for any variable X_t $\tilde{x}_t = \log X_t - a_X - b_X t$ is the residual from the linear detrending procedure.