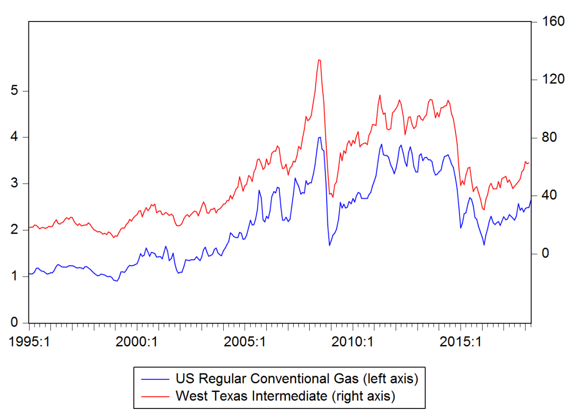
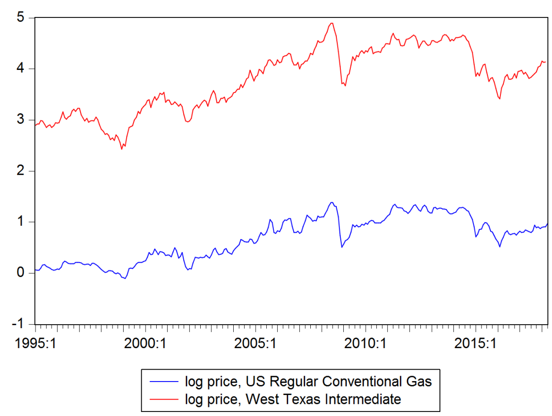
Homework 9

Eco 4306 Economic and Business Forecasting
Spring 2018
Due: Tuesday, May 3, before the class

Problem 1

(a) Left panel below shows the monthly price for US Regular Conventional Gas, $\log p_t^{GAS}$, and the monthly price for West Texas Intermediate Crude Oil, $\log p_t^{OIL}$, during the 1995M1:2018M3 period. The right panel shows these two prices after log transformation.





(b) The log price of US regular conventional gas $\log p_t^{GAS}$ is $I(1)$ time series:

- we can not reject the H_0 of a unit root in $\log p_t^{GAS}$ based on the ADF test, its p-value is 0.4658

Null Hypothesis: LPGAS has a unit root
Exogenous: Constant
Lag Length: 2 (Automatic - based on SIC, maxlag=15)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-1.758725	0.4006
Test critical values:	1% level	-3.453823	
	5% level	-2.871768	
	10% level	-2.572293	

*MacKinnon (1996) one-sided p-values.

- we reject the H_0 of a unit root in $\Delta \log p_t^{GAS}$ based on the ADF test, its p-value is 0.0000

Null Hypothesis: D(LPGAS) has a unit root
Exogenous: Constant
Lag Length: 1 (Automatic - based on SIC, maxlag=15)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-11.68652	0.0000
Test critical values:	1% level	-3.453823	
	5% level	-2.871768	
	10% level	-2.572293	

*MacKinnon (1996) one-sided p-values.

The log price of West Texas Intermediate crude oil $\log p_t^{GAS}$ is $I(1)$ time series:

- we can not reject the H_0 of a unit root in $\log p_t^{OIL}$ based on the ADF test, its p-value is 0.4658

Null Hypothesis: LPOIL has a unit root
Exogenous: Constant
Lag Length: 1 (Automatic - based on SIC, maxlag=15)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-1.743861	0.4080
Test critical values:	1% level	-3.453823	
	5% level	-2.871768	
	10% level	-2.572293	

*MacKinnon (1996) one-sided p-values.

- we reject the H_0 of a unit root in $\Delta \log p_t^{OIL}$ based on the ADF test, its p-value is 0.0000

Null Hypothesis: D(LPOIL) has a unit root
Exogenous: Constant
Lag Length: 0 (Automatic - based on SIC, maxlag=15)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-12.74296	0.0000
Test critical values:	1% level	-3.453823	
	5% level	-2.871768	
	10% level	-2.572293	

*MacKinnon (1996) one-sided p-values.

(c) The Schwarz information criteria suggests that 2 lags should be used in the bivariate VAR

VAR Lag Order Selection Criteria
Endogenous variables: LOG(PGAS) LOG(POIL)
Exogenous variables: C
Date: 05/09/18 Time: 10:26
Sample: 1995M01 2010M12
Included observations: 180

Lag	LogL	LR	FPE	AIC	SC	HQ
0	80.21911	NA	0.001437	-0.869101	-0.833624	-0.854717
1	491.2298	808.3210	1.56e-05	-5.391442	-5.285010	-5.348289
2	522.5107	60.82393	1.15e-05	-5.694563	-5.517177*	-5.622641
3	530.9892	16.29754	1.10e-05	-5.744324	-5.495983	-5.643633
4	533.6751	5.103232	1.11e-05	-5.729723	-5.410427	-5.600262
5	541.8506	15.35175	1.06e-05	-5.776117	-5.385867	-5.617888
6	551.3225	17.57577*	1.00e-05*	-5.836917*	-5.375712	-5.649918*
7	553.2462	3.526704	1.02e-05	-5.813847	-5.281687	-5.598079
8	558.1322	8.849175	1.01e-05	-5.823692	-5.220578	-5.579155
9	561.6891	6.362841	1.02e-05	-5.818768	-5.144699	-5.545462
10	563.9113	3.925859	1.04e-05	-5.799014	-5.053991	-5.496940
11	566.9883	5.367639	1.05e-05	-5.788759	-4.972781	-5.457915
12	568.5506	2.690559	1.08e-05	-5.761673	-4.874740	-5.402060

* indicates lag order selected by the criterion
LR: sequential modified LR test statistic (each test at 5% level)
FPE: Final prediction error
AIC: Akaike information criterion
SC: Schwarz information criterion
HQ: Hannan-Quinn information criterion

(d) In time series plots in (a), $\log p_t^{GAS}$ and $\log p_t^{OIL}$ show only very small upward tendency, and the gap between the two series does not appear to be getting larger over time. Thus Case 2 cointegration test is probably the most suitable, but Case 3 may be also considered as robustness check. Te results of the tests are shown below.

(e) Johansen's trace and maximum eigenvalue cointegration tests for $(\log p_t^{GAS}, \log p_t^{OIL})$, using the sample 1995M1:2010M12, with 2 lags, and Case 2 specification imply that the two series are cointegrated: in both tests we reject H_0 of no cointegration (p- values 0.0006 and 0.0001) and we can not reject the H_0 of one cointegrating relationship (p-values 0.7183 in both tests).

Date: 05/09/18 Time: 10:26
Sample (adjusted): 1995M04 2010M12
Included observations: 189 after adjustments
Trend assumption: No deterministic trend (restricted constant)
Series: LOG(PGAS) LOG(POIL)
Lags interval (in first differences): 1 to 2

Unrestricted Cointegration Rank Test (Trace)

Hypothesized No. of CE(s)	Eigenvalue	Trace Statistic	0.05 Critical Value	Prob.**
None *	0.162415	35.82357	20.26184	0.0002
At most 1	0.012235	2.326637	9.164546	0.7122

Trace test indicates 1 cointegrating eqn(s) at the 0.05 level
* denotes rejection of the hypothesis at the 0.05 level
**MacKinnon-Haug-Michelis (1999) p-values

Unrestricted Cointegration Rank Test (Maximum Eigenvalue)

Hypothesized No. of CE(s)	Eigenvalue	Max-Eigen Statistic	0.05 Critical Value	Prob.**
None *	0.162415	33.49693	15.89210	0.0000
At most 1	0.012235	2.326637	9.164546	0.7122

Max-eigenvalue test indicates 1 cointegrating eqn(s) at the 0.05 level
* denotes rejection of the hypothesis at the 0.05 level
**MacKinnon-Haug-Michelis (1999) p-values

Similar results are obtained with Case 3 specification:

Date: 05/09/18 Time: 10:26
Sample (adjusted): 1995M04 2010M12
Included observations: 189 after adjustments
Trend assumption: Linear deterministic trend
Series: LOG(PGAS) LOG(POIL)
Lags interval (in first differences): 1 to 2

Unrestricted Cointegration Rank Test (Trace)

Hypothesized No. of CE(s)	Eigenvalue	Trace Statistic	0.05 Critical Value	Prob.**
None *	0.162360	34.90013	15.49471	0.0000
At most 1	0.007462	1.415603	3.841466	0.2341

Trace test indicates 1 cointegrating eqn(s) at the 0.05 level
* denotes rejection of the hypothesis at the 0.05 level
**MacKinnon-Haug-Michelis (1999) p-values

Unrestricted Cointegration Rank Test (Maximum Eigenvalue)

Hypothesized No. of CE(s)	Eigenvalue	Max-Eigen Statistic	0.05 Critical Value	Prob.**
None *	0.162360	33.48453	14.26460	0.0000
At most 1	0.007462	1.415603	3.841466	0.2341

Max-eigenvalue test indicates 1 cointegrating eqn(s) at the 0.05 level
* denotes rejection of the hypothesis at the 0.05 level
**MacKinnon-Haug-Michelis (1999) p-values

(f) Below are the results of the estimation of the bivariate VEC for the 1995M1:2010M12 sample. The left panel shows the results for Case 2 specification of cointegration. The estimated model is

$$\Delta \log p_t^{GAS} = -0.327 z_{t-1} + 0.352 \Delta \log p_{t-1}^{GAS} - 0.127 \Delta \log p_{t-2}^{GAS} + 0.103 \Delta \log p_{t-1}^{OIL} + 0.011 \Delta \log p_{t-2}^{OIL} + \varepsilon_{1,t}$$
$$\Delta \log p_t^{OIL} = -0.108 z_{t-1} - 0.115 \Delta \log p_{t-1}^{GAS} - 0.032 \Delta \log p_{t-2}^{GAS} + 0.201 \Delta \log p_{t-1}^{OIL} + 0.081 \Delta \log p_{t-2}^{OIL} + \varepsilon_{2,t}$$

where $z_{t-1} = \log p_{t-1}^{GAS} - 0.597 \log p_{t-1}^{OIL} + 1.587$ is the error terms measuring the deviation in period $t-1$ from the long run equilibrium.

The panel on the right shows the results for Case 3 specification of cointegration, which are very similar.

Vector Error Correction Estimates
Date: 05/09/18 Time: 10:26
Sample (adjusted): 1995M04 2010M12
Included observations: 189 after adjustments
Standard errors in ( ) & t-statistics in [ ]

Cointegrating Eq:	CointEq1	
LOG(PGAS(-1))	1.000000	
LOG(POIL(-1))	-0.631247 (0.01394) [-45.2872]	
C	1.738756 (0.05040) [34.4992]	
Error Correction:	**D(LOG(PGAS))**	**D(LOG(POIL))**
CointEq1	-0.334163 (0.07765) [-4.30353]	-0.029007 (0.12377) [-0.23435]
D(LOG(PGAS(-1)))	0.353684 (0.09534) [3.70974]	-0.138917 (0.15197) [-0.91409]
D(LOG(PGAS(-2)))	-0.143176 (0.09105) [-1.57241]	-0.057373 (0.14514) [-0.39529]
D(LOG(POIL(-1)))	0.135581 (0.06819) [1.98830]	0.275317 (0.10870) [2.53293]
D(LOG(POIL(-2)))	0.017021 (0.06806) [0.25011]	0.170246 (0.10848) [1.56934]
R-squared	0.366524	0.069829
Adj. R-squared	0.352753	0.049607
Sum sq. resids	0.510838	1.297996
S.E. equation	0.052691	0.083990
F-statistic	26.61526	3.453247
Log likelihood	290.6416	202.5181
Akaike AIC	-3.022663	-2.090139
Schwarz SC	-2.936902	-2.004378
Mean dependent	0.005422	0.008309
S.D. dependent	0.065493	0.086154
Determinant resid covariance (dof adj.)		9.40E-06
Determinant resid covariance		8.91E-06
Log likelihood		562.5500
Akaike information criterion		-5.815344
Schwarz criterion		-5.592367

Vector Error Correction Estimates
Date: 05/09/18 Time: 10:26
Sample (adjusted): 1995M04 2010M12
Included observations: 189 after adjustments
Standard errors in ( ) & t-statistics in [ ]

Cointegrating Eq:	CointEq1	
LOG(PGAS(-1))	1.000000	
LOG(POIL(-1))	-0.631302 (0.01398) [-45.1679]	
C	1.740782	
Error Correction:	**D(LOG(PGAS))**	**D(LOG(POIL))**
CointEq1	-0.333087 (0.07779) [-4.28204]	-0.026541 (0.12384) [-0.21432]
D(LOG(PGAS(-1)))	0.354237 (0.09549) [3.70952]	-0.137657 (0.15203) [-0.90547]
D(LOG(PGAS(-2)))	-0.145054 (0.09124) [-1.58977]	-0.061666 (0.14526) [-0.42452]
D(LOG(POIL(-1)))	0.133350 (0.06838) [1.95001]	0.270216 (0.10887) [2.48203]
D(LOG(POIL(-2)))	0.015578 (0.06820) [0.22842]	0.166954 (0.10858) [1.53766]
C	0.003128 (0.00387) [0.80887]	0.005834 (0.00616) [0.94757]
R-squared	0.367973	0.074293
Adj. R-squared	0.350705	0.049000
Sum sq. resids	0.509669	1.291766
S.E. equation	0.052774	0.084017
F-statistic	21.30896	2.937339
Log likelihood	290.8581	202.9727
Akaike AIC	-3.014371	-2.084368
Schwarz SC	-2.911458	-1.981455
Mean dependent	0.005422	0.008309
S.D. dependent	0.065493	0.086154
Determinant resid covariance (dof adj.)		9.46E-06
Determinant resid covariance		8.86E-06
Log likelihood		563.0055
Akaike information criterion		-5.809582
Schwarz criterion		-5.569453

(g) The adjustment parameters are $\gamma_1 = -0.327$ and $\gamma_2 = -0.108$ under Case 2, and $\gamma_1 = -0.326$ and $\gamma_2 = -0.106$ under Case 3. For error correction mechanism to move the system back to the long run equilibrium, whenever there is a disruption and $z_{t-1} \neq 0$ the adjustment parameters need to satisfy $\gamma_1 \leq 0$, $\gamma_2 \geq 0$ and they can not be both equal zero at the same time. In the estimated VEC γ_1 is consistent with the error correction mechanism since it's negative, γ is not consistent with the error correction mechanism because it's negative. But for both cases only γ_1 is statistically significant, γ_2 is not. It thus makes sense to restrict the model and impose $\gamma_2 = 0$.

(h) Below are the results of the estimation of the bivariate VEC with $\gamma_2 = 0$ restriction. The imposed restriction is not rejected by the test, the p-value is 0.4105. The left panel again shows the results for Case 2 specification of cointegration, in which case the model is

$$\Delta \log p_t^{GAS} = -0.282 z_{t-1} + 0.351 \Delta \log p_{t-1}^{GAS} - 0.129 \Delta \log p_{t-2}^{GAS} + 0.104 \Delta \log p_{t-1}^{OIL} + 0.012 \Delta \log p_{t-p}^{OIl} + \varepsilon_{1,t}$$
$$\Delta \log p_t^{OIL} = -0.118 \Delta \log p_{t-1}^{GAS} - 0.036 \Delta \log p_{t-2}^{GAS} + 0.204 \Delta \log p_{t-1}^{OIL} + 0.084 \Delta \log p_{t-2}^{OIL} + \varepsilon_{2,t}$$

For comparison, the panel on the right shows the results for Case 3 specification of cointegration.

Vector Error Correction Estimates
Date: 05/09/18 Time: 10:26
Sample (adjusted): 1995M04 2010M12
Included observations: 189 after adjustments
Standard errors in ( ) & t-statistics in [ ]

Cointegration Restrictions:
B(1,1)=1, A(2,1)=0
Convergence achieved after 2 iterations.
Restrictions identify all cointegrating vectors
LR test for binding restrictions (rank = 1):

Chi-square(1)	0.052801
Probability	0.818259

Cointegrating Eq:	CointEq1
LOG(PGAS(-1))	1.000000
LOG(POIL(-1))	-0.631895 (0.01395) [-45.3004]
C	1.741377 (0.05044) [34.5258]

Error Correction:	D(LOG(PGAS))	D(LOG(POIL))
CointEq1	-0.321047 (0.05379) [-5.96885]	0.000000 (0.00000) [NA]
D(LOG(PGAS(-1)))	0.353400 (0.09536) [3.70577]	-0.139621 (0.15198) [-0.91869]
D(LOG(PGAS(-2)))	-0.143665 (0.09107) [-1.57747]	-0.058384 (0.14514) [-0.40226]
D(LOG(POIL(-1)))	0.135874 (0.06821) [1.99205]	0.276022 (0.10870) [2.53929]
D(LOG(POIL(-2)))	0.017225 (0.06808) [0.25302]	0.170820 (0.10849) [1.57450]
R-squared	0.366211	0.069794
Adj. R-squared	0.352433	0.049572
Sum sq. resids	0.511091	1.298044
S.E. equation	0.052704	0.083992
F-statistic	26.57932	3.451421
Log likelihood	290.5948	202.5146
Akaike AIC	-3.022168	-2.090102
Schwarz SC	-2.936407	-2.004341
Mean dependent	0.005422	0.008309
S.D. dependent	0.065493	0.086154

Determinant resid covariance (dof adj.)	9.40E-06
Determinant resid covariance	8.91E-06
Log likelihood	562.5236
Akaike information criterion	-5.815065
Schwarz criterion	-5.592087

Vector Error Correction Estimates
Date: 05/09/18 Time: 10:26
Sample (adjusted): 1995M04 2010M12
Included observations: 189 after adjustments
Standard errors in ( ) & t-statistics in [ ]

Cointegration Restrictions:
B(1,1)=1, A(2,1)=0
Convergence achieved after 2 iterations.
Restrictions identify all cointegrating vectors
LR test for binding restrictions (rank = 1):

Chi-square(1)	0.045593
Probability	0.830918

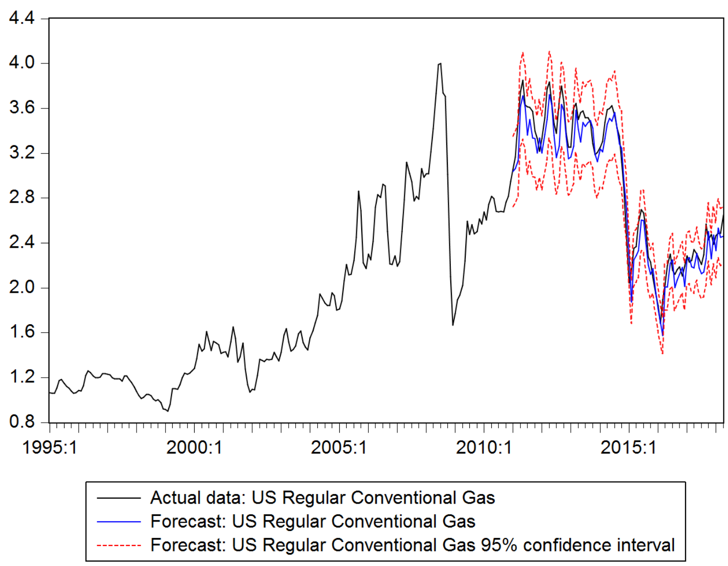
Cointegrating Eq:	CointEq1
LOG(PGAS(-1))	1.000000
LOG(POIL(-1))	-0.631895 (0.01399) [-45.1771]
C	1.742901

Error Correction:	D(LOG(PGAS))	D(LOG(POIL))
CointEq1	-0.321047 (0.05394) [-5.95201]	0.000000 (0.00000) [NA]
D(LOG(PGAS(-1)))	0.354114 (0.09551) [3.70757]	-0.138046 (0.15204) [-0.90798]
D(LOG(PGAS(-2)))	-0.145407 (0.09124) [-1.59362]	-0.062234 (0.14524) [-0.42848]
D(LOG(POIL(-1)))	0.133420 (0.06840) [1.95046]	0.270600 (0.10889) [2.48515]
D(LOG(POIL(-2)))	0.015616 (0.06822) [0.22891]	0.167266 (0.10859) [1.54033]
C	0.003130 (0.00387) [0.80916]	0.005834 (0.00616) [0.94745]
R-squared	0.367799	0.074275
Adj. R-squared	0.350526	0.048982
Sum sq. resids	0.509810	1.291791
S.E. equation	0.052781	0.084018
F-statistic	21.29302	2.936584
Log likelihood	290.8320	202.9709
Akaike AIC	-3.014096	-2.084349
Schwarz SC	-2.911183	-1.981436
Mean dependent	0.005422	0.008309
S.D. dependent	0.065493	0.086154

Determinant resid covariance (dof adj.)	9.46E-06
Determinant resid covariance	8.86E-06
Log likelihood	562.9827
Akaike information criterion	-5.809341
Schwarz criterion	-5.569212

(i) We restricted the model and imposed $\gamma_2 = 0$ since it was not statistically significant. The interpretation of this restriction is that the price of oil does not adjust, when there is a deviation from the long run equilibrium - only price of gas adjusts. So for example if $z_{t-1} > 0$ which means that in $t-1$ price of gas is above the level implied by the long run equilibrium, in the next period it will fall to partially correct the gap z_{t-1}, while no adjustment in price of oil takes place.

(j) The RMSE for the sequence of one step ahead forecasts of the gas price, p_t^{GAS} in 2011M1:2018M3 using the VEC model is 0.1472.



Forecast Evaluation
Date: 05/09/18 Time: 10:26
Sample: 2011M01 2018M04
Included observations: 88

Variable	Inc. obs.	RMSE	MAE	MAPE	Theil
PGAS	88	0.147257	0.117970	4.404665	0.025068
POIL	87	5.288259	4.118257	6.001430	0.033627

RMSE: Root Mean Square Error
MAE: Mean Absolute Error
MAPE: Mean Absolute Percentage Error
Theil: Theil inequality coefficient

(k) The correlogram for $\Delta \log p_t^{GAS}$ shows slow decay in ACF and significant lags 1,2,5,12 in PACF.

Date: 05/09/18 Time: 10:26
Sample: 1995M01 2010M12
Included observations: 191

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.402	0.402	31.313	0.000
		2	-0.082	-0.290	32.623	0.000
		3	-0.084	0.098	33.993	0.000
		4	-0.032	-0.071	34.200	0.000
		5	-0.178	-0.200	40.503	0.000
		6	-0.228	-0.074	50.862	0.000
		7	-0.146	-0.092	55.148	0.000
		8	-0.140	-0.157	59.120	0.000
		9	-0.076	0.009	60.286	0.000
		10	0.151	0.143	64.959	0.000
		11	0.256	0.064	78.414	0.000
		12	0.015	-0.174	78.461	0.000
		13	-0.079	0.015	79.765	0.000
		14	0.078	0.067	81.033	0.000
		15	0.012	-0.141	81.061	0.000
		16	-0.079	0.086	82.374	0.000
		17	-0.090	-0.070	84.104	0.000
		18	-0.100	-0.101	86.244	0.000
		19	-0.107	0.009	88.678	0.000
		20	-0.095	-0.129	90.633	0.000
		21	-0.024	-0.070	90.761	0.000
		22	0.020	0.002	90.848	0.000
		23	-0.010	-0.050	90.870	0.000
		24	0.098	0.103	92.995	0.000
		25	0.251	0.109	106.97	0.000
		26	0.162	-0.001	112.83	0.000
		27	-0.065	-0.135	113.78	0.000
		28	-0.113	-0.033	116.68	0.000
		29	-0.109	-0.069	119.39	0.000
		30	-0.083	0.002	120.96	0.000
		31	-0.106	-0.030	123.54	0.000
		32	-0.097	-0.030	125.72	0.000
		33	-0.048	-0.057	126.26	0.000
		34	0.030	0.031	126.47	0.000
		35	0.168	0.041	133.10	0.000
		36	0.256	0.054	148.63	0.000

Based on this the following AR model can be used to model the change in the price of gas:

$$\Delta \log p_t^{GAS} = \phi_0 + \phi_1 \Delta \log p_{t-1}^{GAS} + \phi_2 \Delta \log p_{t-2}^{GAS} + \phi_5 \Delta \log p_{t-5}^{GAS} + \phi_{11} \Delta \log p_{t-11}^{GAS} + \phi_{12} \Delta \log p_{t-12}^{GAS} + \varepsilon_t$$

Dependent Variable: D(LOG(PGAS))
Method: ARMA Maximum Likelihood (BFGS)
Date: 05/09/18 Time: 10:26
Sample: 1995M02 2010M12
Included observations: 191
Convergence achieved after 9 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.005168	0.004891	1.056728	0.2920
AR(1)	0.515869	0.046975	10.98181	0.0000
AR(2)	-0.260243	0.064205	-4.053294	0.0001
AR(5)	-0.162440	0.065819	-2.467980	0.0145
AR(11)	0.185744	0.066249	2.803728	0.0056
AR(12)	-0.171857	0.058384	-2.943572	0.0037
SIGMASQ	0.002926	0.000250	11.71342	0.0000

R-squared	0.307133	Mean dependent var	0.005331
Adjusted R-squared	0.284539	S.D. dependent var	0.065155
S.E. of regression	0.055112	Akaike info criterion	-2.916979
Sum squared resid	0.558861	Schwarz criterion	-2.797786
Log likelihood	285.5715	Hannan-Quinn criter.	-2.868701
F-statistic	13.59385	Durbin-Watson stat	1.961542
Prob(F-statistic)	0.000000		

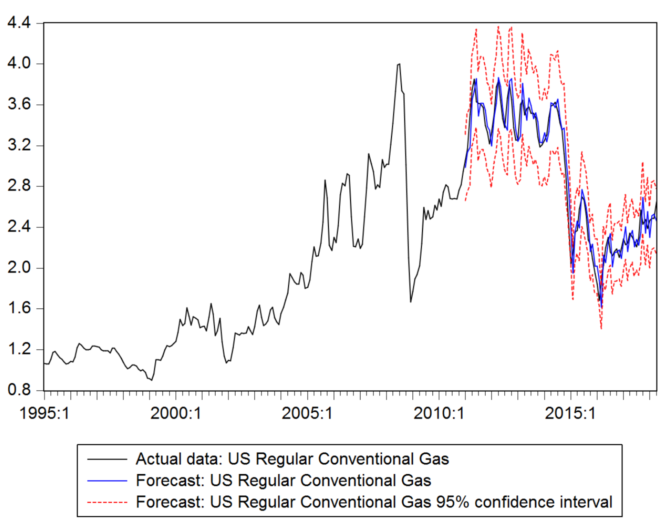
Inverted AR Roots	.75+.18i	.75-.18i	.71-.56i	.71+.56i
	.32-.81i	.32+.81i	-.14+.90i	-.14-.90i
	-.55-.66i	-.55+.66i	-.84+.23i	-.84-.23i

All coefficients of the estimated AR model are statistically significant, the residuals of the model don't show any remaining time dependence and appear to be white noise.

Date: 05/09/18 Time: 10:26
Sample: 1995M01 2010M12
Included observations: 191
Q-statistic probabilities adjusted for 5 ARMA terms

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.018	0.018	0.0648	
		2	-0.038	-0.038	0.3453	
		3	-0.021	-0.020	0.4357	
		4	0.053	0.052	0.9872	
		5	0.014	0.011	1.0271	
		6	-0.101	-0.098	3.0483	0.081
		7	-0.076	-0.070	4.2051	0.122
		8	-0.128	-0.137	7.5266	0.057
		9	-0.060	-0.071	8.2571	0.083
		10	0.049	0.048	8.7425	0.120
		11	0.034	0.035	8.9850	0.174
		12	0.025	0.033	9.1087	0.245
		13	-0.051	-0.053	9.6494	0.291
		14	0.145	0.118	14.035	0.121
		15	-0.110	-0.161	16.555	0.085
		16	-0.013	-0.024	16.589	0.121
		17	-0.005	-0.010	16.594	0.166
		18	-0.115	-0.126	19.431	0.110
		19	-0.049	-0.029	19.954	0.132
		20	-0.039	-0.016	20.278	0.162
		21	-0.041	-0.074	20.647	0.192
		22	0.006	0.018	20.655	0.242
		23	-0.018	-0.036	20.723	0.294
		24	0.061	0.009	21.546	0.307
		25	0.122	0.113	24.860	0.207
		26	0.094	0.057	26.814	0.177
		27	-0.112	-0.133	29.641	0.127
		28	-0.019	-0.055	29.726	0.157
		29	-0.051	-0.061	30.320	0.174
		30	0.011	-0.026	30.346	0.212
		31	-0.052	-0.036	30.973	0.229
		32	-0.054	0.022	31.659	0.245
		33	-0.039	-0.040	32.008	0.274
		34	-0.007	-0.013	32.018	0.319
		35	0.064	0.035	32.996	0.323
		36	0.171	0.101	39.988	0.129

(l) The RMSE for the sequence of one step ahead forecasts of the gas price, p_t^{GAS} in 2011M1:2018M3 using the AR model is 0.1414.



Forecast: PGAS_F_AR12	
Actual: PGAS	
Forecast sample: 2011M01 2018M04	
Included observations: 88	
Root Mean Squared Error	0.141486
Mean Absolute Error	0.113511
Mean Abs. Percent Error	4.193013
Theil Inequality Coefficient	0.023674
Bias Proportion	0.012684
Variance Proportion	0.000360
Covariance Proportion	0.986955
Theil U2 Coefficient	0.972591
Symmetric MAPE	4.169495

(m) The RMSE of the forecast for p_t^{GAS} from (j) based on the VEC model is 0.1472, and the RMSE of the forecast for p_t^{GAS} from (l) based on the AR model is 0.1414. The two models thus appear to produce similarly precise forecasts.

(n) The test for equal predictive ability is performed by running a regression

$$\Delta L_{t+j,1} = \beta_0 + u_{t+j} \quad \text{with } j = 0, 1, 2, \ldots, T - t - 1$$

where $\Delta L_{t,1} = L(e_{t,1}^{(AR)}) - L(e_{t,1}^{(2)}) = (e_{t,1}^{(VEC)})^2 - (e_{t,1}^{(2)})^2$ are the differences in the squared one step ahead forecast errors. The results for this regression are below

Dependent Variable: DLOSS_AR12
Method: Least Squares
Date: 05/09/18 Time: 10:26
Sample: 2011M01 2018M04
Included observations: 88

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.001666	0.002705	-0.615889	0.5396

R-squared	0.000000	Mean dependent var	-0.001666
Adjusted R-squared	0.000000	S.D. dependent var	0.025378
S.E. of regression	0.025378	Akaike info criterion	-4.498561
Sum squared resid	0.056032	Schwarz criterion	-4.470409
Log likelihood	198.9367	Hannan-Quinn criter.	-4.487219
Durbin-Watson stat	1.859767		

Since $\hat{\beta}_0$ is not statistically significant we do not reject the hypothesis of equal predictive power. The mean squared forecast error from the AR model is not statistically significantly larger or smaller than the mean squared forecast error from the VEC model.

(o) There does not appear to be a statistically significant difference in the precision of forecasts from the AR(12) and the VEC(2) models.