Practice Problems 3

Short Questions

1. Write the equations of a VAR(1) model with two variables, X_t and Y_t.

See slide 13 in lec20slides.pdf.

2. What is Granger causality and how do we test it?

See slide 18 in lec20slides.pdf.

3. What are impulse-response functions?

See slides 22, 24, 25 in lec20slides.pdf.

4. Explain what spurious regression problem is and give an example.

See slides 7-9 and 11 in lec23slides.pdf.

5. Explain what is means if X_t and Y_t are cointegrated. Give an example.

See slide 12 in lec23slides.pdf.

6. Explain the idea behind error correction model. Draw a diagram illustrating the error correction mechanism.

See slides 22-26 in lec23slides.pdf.

7. Write the equations of a vector error correction VEC(1) model with two variables, X_t and Y_t.

See slides 27 in lec23slides.pdf.

8. How is cointegration used in pairs trading strategy?

See slides 33-35 in lec23slides.pdf.

9. Explain what volatility clustering means.

See slide 9 in lec25slides.pdf.

10. Explain the difference between moving average (MA) and exponentially weighted moving average (EWMA) models of the conditional variance.

See slide 24, 28 and 31 in lec25slides.pdf.

11. Write the equation for the autoregressive conditional heteroscedasticity ARCH(1) model. Explain the intuition behind this model.

See slide 6 in lec26slides.pdf.

12. What are some weaknesses of ARCH models, and which alternative models have been developed to address them?

See slide 22 in lec26slides.pdf.

13. Write the equation for the generalized autoregressive conditional heteroscedasticity GARCH(1,1) model. Explain the intuition behind this model.

See slide 23 in lec26slides.pdf.

14. Explain the main idea behind TARCH and PGARCH models.

See slides 36 and 38 in lec26slides.pdf.

15. Why is the Student-t distribution more suitable for GARCH models than the normal distribution?

See slides 11, 13 and 18 in lec28slides.pdf.

16. Explain what 1% VaR is and draw a diagram to illustrate this.

See slides 6, 7 and 10 in lec28slides.pdf.

17. Consider a GARCH(1,1) model for daily S&P 500 returns. On April 2, 2008, the 1-day-ahead conditional mean is $\mu_{t|t-1} = 0.036$, the 1-day-ahead conditional standard deviation $\sigma_{t|t-1} = 1.785$. Calculate the 1% VaR and 5% VaR, given that $\Phi^{-1}(0.05) = -1.645$ and $\Phi^{-1}(0.01) = -2.326$. Interpret these numbers, given a portfolio worth 1 million dollars.

See slide 11 in lec28slides.pdf.

Question 1. Consider a bivariate VAR

$$y_{1t} = c_1 + \alpha_{11} y_{1t-1} + \alpha_{12} y_{1t-2} + \beta_{11} y_{2t-1} + \beta_{12} y_{2t-2} + \varepsilon_{1t}$$
$$y_{2t} = c_2 + \alpha_{21} y_{1t-1} + \alpha_{22} y_{1t-2} + \beta_{21} y_{2t-1} + \beta_{22} y_{2t-2} + \varepsilon_{2t}$$

where $y_{1,t} = 400\Delta \log GDP_t$ is the growth rate of the U.S. real GDP and $y_{2,t} = 400(\Delta \log SP500_t - \Delta \log p_t^{GDP})$ is the inflation adjusted return of S&P 500. The results of the estimation are shown below.

Are coefficients $\alpha_{11}, \alpha_{12}, \beta_{11}, \beta_{12}$ statistically significant? What does this imply?

Are coefficients $\alpha_{21}, \alpha_{22}, \beta_{21}, \beta_{22}$ statistically significant? What does this imply?

Vector Autoregression Estimates
Date: 04/26/18 Time: 16:52
Sample: 2000Q1 2016Q4
Included observations: 68
Standard errors in ( ) & t-statistics in [ ]

	DLRGDP	DLRSP500
DLRGDP(-1)	0.065707	0.133711
	(0.13856)	(0.38951)
	[0.47422]	[0.34328]
DLRGDP(-2)	0.185656	-0.404508
	(0.13220)	(0.37164)
	[1.40434]	[-1.08845]
DLRSP500(-1)	0.126714	0.451046
	(0.05048)	(0.14191)
	[2.51016]	[3.17844]
DLRSP500(-2)	-0.001633	-0.101216
	(0.05241)	(0.14734)
	[-0.03115]	[-0.68698]
C	1.326002	0.647247
	(0.42193)	(1.18610)
	[3.14270]	[0.54569]
R-squared	0.228927	0.206599
Adj. R-squared	0.179969	0.156224
Sum sq. resids	313.3873	2476.547
S.E. equation	2.230337	6.269790
F-statistic	4.676069	4.101245
Log likelihood	-148.4375	-218.7217
Akaike AIC	4.512868	6.580049
Schwarz SC	4.676067	6.743248
Mean dependent	1.840825	0.194719
S.D. dependent	2.462949	6.825578

Determinant resid covariance (dof adj.)	131.1124
Determinant resid covariance	112.5401
Log likelihood	-353.5682
Akaike information criterion	10.69318
Schwarz criterion	11.01958

See hw08sol.pdf.

Question 2. Interpret the results of the Granger causality test for a VAR with three variables: $y_{1,t} = 400\Delta \log GDP_t$ is the growth rate of the U.S. real GDP and $y_{2,t} = 400(\Delta \log SP500_t - \Delta \log p_t^{GDP})$ is the inflation adjusted return of S&P 500, and y_{3t} is the Leading Index for the United States.

Discuss what these Granger causality imply about the usefulness of each of the threes variables when it comes to predicting the other ones.

Is there any economic intuition behind these results?

VAR Granger Causality/Block Exogeneity Wald Tests
Date: 05/11/17 Time: 10:58
Sample: 1961Q1 2016Q4
Included observations: 139

Dependent variable: GRGDP

Excluded	Chi-sq	df	Prob.
RRSP500	3.689833	1	0.0547
LI	22.08652	1	0.0000
All	27.70217	2	0.0000

Dependent variable: RRSP500

Excluded	Chi-sq	df	Prob.
GRGDP	0.021518	1	0.8834
LI	0.206983	1	0.6491
All	0.673715	2	0.7140

Dependent variable: LI

Excluded	Chi-sq	df	Prob.
GRGDP	2.487304	1	0.1148
RRSP500	9.320140	1	0.0023
All	12.78463	2	0.0017

See hw08sol.pdf.

Question 3. Interpret the results of the cointegration test for $\log p^{gas}$ and $\log p^{oil}$.

Date: 05/09/18 Time: 10:26
Sample (adjusted): 1995M04 2010M12
Included observations: 189 after adjustments
Trend assumption: No deterministic trend (restricted constant)
Series: LOG(PGAS) LOG(POIL)
Lags interval (in first differences): 1 to 2

Unrestricted Cointegration Rank Test (Trace)

Hypothesized No. of CE(s)	Eigenvalue	Trace Statistic	0.05 Critical Value	Prob.**
None *	0.162415	35.82357	20.26184	0.0002
At most 1	0.012235	2.326637	9.164546	0.7122

Trace test indicates 1 cointegrating eqn(s) at the 0.05 level
* denotes rejection of the hypothesis at the 0.05 level
**MacKinnon-Haug-Michelis (1999) p-values

Unrestricted Cointegration Rank Test (Maximum Eigenvalue)

Hypothesized No. of CE(s)	Eigenvalue	Max-Eigen Statistic	0.05 Critical Value	Prob.**
None *	0.162415	33.49693	15.89210	0.0000
At most 1	0.012235	2.326637	9.164546	0.7122

Max-eigenvalue test indicates 1 cointegrating eqn(s) at the 0.05 level
* denotes rejection of the hypothesis at the 0.05 level
**MacKinnon-Haug-Michelis (1999) p-values

See hw09sol.pdf.

Question 4. Consider a bivariate VEC for $\log p^{gas}$ and $\log p^{oil}$.

$$\Delta \log p_t^{GAS} = \gamma_1 z_{t-1} + \kappa_{11}\Delta \log p_{t-1}^{GAS} + \kappa_{12}\Delta \log p_{t-2}^{GAS} + \phi_{11}\Delta \log p_{t-1}^{OIL} + \phi_{12}\Delta \log p_{t-2}^{OIL} + \varepsilon_{1,t}$$
$$\Delta \log p_t^{OIL} = \gamma_2 z_{t-1} + \kappa_{21}\Delta \log p_{t-1}^{GAS} + \kappa_{22}\Delta \log p_{t-2}^{GAS} + \phi_{21}\Delta \log p_{t-1}^{OIL} + \phi_{22}\Delta \log p_{t-2}^{OIL} + \varepsilon_{2,t}$$

where $z_{t-1} = \log p_{t-1}^{GAS} - \beta_1 \log p_{t-1}^{OIL} - \beta_0$ is the error terms measuring the deviation in period $t-1$ from the long run equilibrium. The results of the estimation are shown below.

Is the coefficient β_1 statistically significant? Interpret what the estimated value for β_1 means.

Are γ_1 and γ_2 statistically significant? Are the signs of γ_1 and γ_2 in the estimated VEC model consistent with error correction mechanism that moves the system back to the long run equilibrium, whenever there is a disruption and $z_{t-1} \neq 0$?

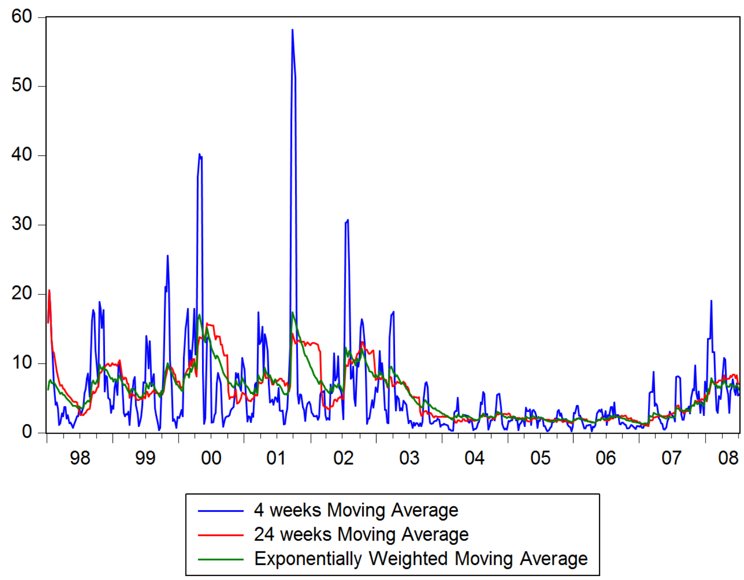
Vector Error Correction Estimates
Date: 05/09/18 Time: 10:26
Sample (adjusted): 1995M04 2010M12
Included observations: 189 after adjustments
Standard errors in ( ) & t-statistics in [ ]

Cointegrating Eq:	CointEq1	
LOG(PGAS(-1))	1.000000	
LOG(POIL(-1))	-0.631247 (0.01394) [-45.2872]	
C	1.738756 (0.05040) [34.4992]	
Error Correction:	**D(LOG(PGAS))**	**D(LOG(POIL))**
CointEq1	-0.334163 (0.07765) [-4.30353]	-0.029007 (0.12377) [-0.23435]
D(LOG(PGAS(-1)))	0.353684 (0.09534) [3.70974]	-0.138917 (0.15197) [-0.91409]
D(LOG(PGAS(-2)))	-0.143176 (0.09105) [-1.57241]	-0.057373 (0.14514) [-0.39529]
D(LOG(POIL(-1)))	0.135581 (0.06819) [1.98830]	0.275317 (0.10870) [2.53293]
D(LOG(POIL(-2)))	0.017021 (0.06806) [0.25011]	0.170246 (0.10848) [1.56934]
R-squared	0.366524	0.069829
Adj. R-squared	0.352753	0.049607
Sum sq. resids	0.510838	1.297996
S.E. equation	0.052691	0.083990
F-statistic	26.61526	3.453247
Log likelihood	290.6416	202.5181
Akaike AIC	-3.022663	-2.090139
Schwarz SC	-2.936902	-2.004378
Mean dependent	0.005422	0.008309
S.D. dependent	0.065493	0.086154

Determinant resid covariance (dof adj.)	9.40E-06
Determinant resid covariance	8.91E-06
Log likelihood	562.5500
Akaike information criterion	-5.815344
Schwarz criterion	-5.592367

See hw09sol.pdf.

Question 5. Comment on the differences between MA(4), MA(24) and EWMA applied to obtain the 1-week-ahead volatility forecast for the S&P 500 returns.



See slides 24, 28 and 31 in lec25slides.pdf.

Question 6. Consider ARCH(9) and GARCHJ(1,1) models for the S&P 500 daily returns. Write the equations for the two estimated models, with estimated parameter values plugged into these equations. Which model would be preferred by Akaike criterion and by Schwarz criterion?

SP500 daily returns—ARCH(9)

Dependent Variable: R
Method: ML - ARCH (BHHH) - Normal distribution
Sample: 5815 8471
Included observations: 2657
Convergence achieved after 16 iterations
Bollerslev-Wooldrige robust standard errors & covariance
Variance backcast: ON
GARCH = C(2) + C(3)*RESID(-1)^2 + C(4)*RESID(-2)^2 + C(5)*RESID(-3)^2 + C(6)*RESID(-4)^2 + C(7)*RESID(-5)^2 + C(8)*RESID(-6)^2 + C(9)*RESID(-7)^2 + C(10)*RESID(-8)^2 + C(11)*RESID(-9)^2

	Coefficient	Std. Error	z-Statistic	Prob.
C	0.037003	0.018214	2.031594	0.0422
Variance Equation				
C	0.271763	0.040891	6.645982	0.0000
RESID(-1)^2	0.029949	0.028081	1.066510	0.2862
RESID(-2)^2	0.149370	0.044623	3.347391	0.0008
RESID(-3)^2	0.095260	0.026377	3.611510	0.0003
RESID(-4)^2	0.101684	0.027620	3.681607	0.0002
RESID(-5)^2	0.082439	0.023397	3.523482	0.0004
RESID(-6)^2	0.060298	0.021251	2.837387	0.0045
RESID(-7)^2	0.090927	0.030511	2.980119	0.0029
RESID(-8)^2	0.142659	0.029601	4.819476	0.0000
RESID(-9)^2	0.082659	0.023815	3.470870	0.0005

R-squared	-0.000565	Mean dependent var	0.009761
Adjusted R-squared	-0.004346	S.D. dependent var	1.146761
S.E. of regression	1.149251	Akaike info criterion	2.910013
Sum squared resid	3494.776	Schwarz criterion	2.934377
Log likelihood	-3854.952	Durbin-Watson stat	2.079077

Dependent Variable: R
Method: ML - ARCH (BHHH) - Normal distribution
Sample: 5815 8471
Included observations: 2657
Convergence achieved after 10 iterations
Bollerslev-Wooldrige robust standard errors & covariance
Variance backcast: ON
GARCH = C(2) + C(3)*RESID(-1)^2 + C(4)*GARCH(-1)

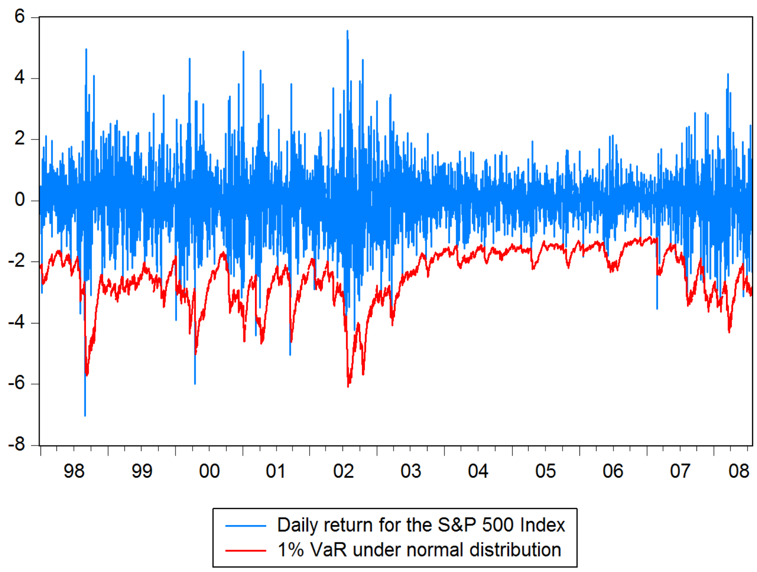
	Coefficient	Std. Error	z-Statistic	Prob.
C	0.036267	0.017439	2.079665	0.0376
Variance Equation				
C	0.010421	0.005245	1.987099	0.0469
RESID(-1)^2	0.065649	0.011338	5.790038	0.0000
GARCH(-1)	0.927400	0.011045	83.96233	0.0000

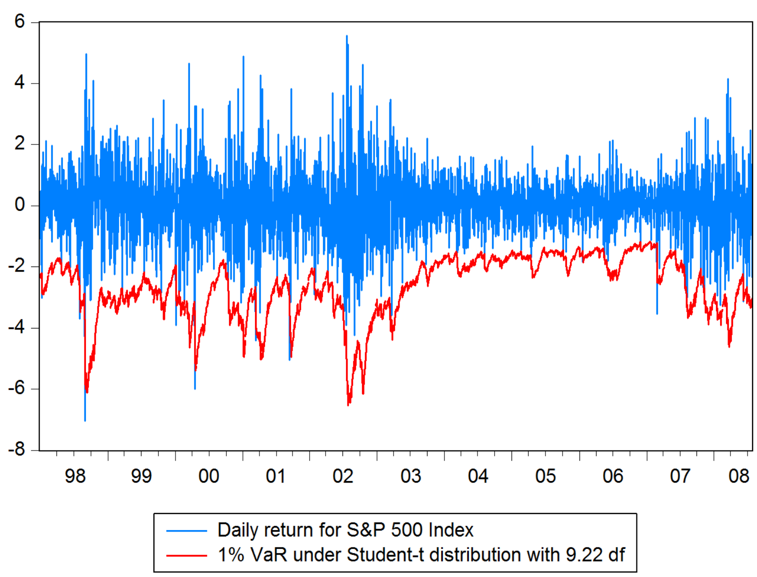
R-squared	-0.000534	Mean dependent var	0.009761
Adjusted R-squared	-0.001666	S.D. dependent var	1.146761
S.E. of regression	1.147716	Akaike info criterion	2.888638
Sum squared resid	3494.671	Schwarz criterion	2.897498
Log likelihood	-3833.556	Durbin-Watson stat	2.079139

See slides 21 and 32 in lec26slides.pdf.

Question 7. Consider GARCH(1,1) model for daily S&P 500 returns for the 1/2/1998 to 7/25/2008 sample. With normal innovations, the number of violations $r_t < r_t^{VaR(0.01)}$ is 42 which represents 1.58% of observations. With innovations from Student-t distribution the number of violations $r_t < r_t^{VaR(0.01)}$ is 30 or 1.13% of the sample.

Show where some of these violations can be seen in the figures below. Explain which of these models is more suitable to model volatility of daily S&P 500 returns and why.





See slides 11, 13 and 18 in lec28slides.pdf.