Eco 5316 Time Series Econometrics

Lecture 12 Intervention Analysis

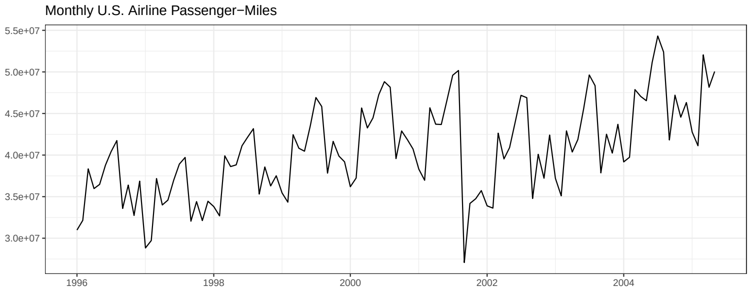
Intervention Analysis

- intervention analysis is used to assess of how the level of a series changes after a special event
- it assumes that the same ARIMA structure for the series holds both before and after the intervention, and models the change in the series as an additional component of the model
- the intervention can change to a law, or policy (speed limits, seatbelts, metal detectors at airports, ...), natural disaster (hurricane Katrina), a terrorist attack (9/11), a large economic shock (1973 oil price shock, Great Depression, Great Recession), a large marketing campaign, ...
- intervention analysis allows to explicitly account for a change in the level of a time series, with potential delay, and/or growing or decaying effect

Intervention Analysis

- example: effect of 9/11 attack on monthly U.S. airline passenger-miles

```
data(airmiles, package="TSA")

library(ggplot2)
library(ggfortify)

theme_set(theme_bw())
autoplot(airmiles) +
    labs(x = "", y = "", title = "Monthly U.S. Airline Passenger-Miles")
```



Intervention Analysis

- model with an intervention

$$Y_t = f_t + Z_t \tag{1}$$

  where f_t is the change in time series due to intervention and Z_t the regular component

- regular component Z_t follows some ARIMA process, potentially seasonal

$$\phi(L)\Phi(L)(1-L)^d(1-L^s)^D Z_t = \theta(L)\Theta(L)\varepsilon_t$$

  which can be written as

$$\hat{\phi}(L)Z_t = \hat{\theta}(L)\varepsilon_t$$

  where $\hat{\phi}(L) = (1-L)^d\phi(L)(1-L^s)^D\Phi(L)$ and $\hat{\theta}(L) = \theta(L)\Theta(L)$

- Z_t is thus given by

$$Z_t = \frac{\hat{\theta}(L)}{\hat{\phi}(L)}\varepsilon_t \tag{2}$$

Intervention Analysis

- f_t captures the *additional* effect of an intervention x_t on y_t through an ARMA process

$$\delta(L)f_t = \omega(L)x_t \tag{3}$$

where x_t will be taking values 0 and 1 to denote nonoccurrence and occurrence of intervention and

$$\delta(L) = 1-\delta_1 L-\ldots-\delta_r L^r$$
$$\omega(L) = \omega_0+\omega_1 L+\ldots+\omega_s L^s$$

allow to capture the dynamic effects of the intervention (gradual decay or growth of the effect over time)

- combining the two components in (2) and (3) our model (1) becomes

$$Y_t = \frac{\omega(L)}{\delta(L)}x_t+\frac{\hat{\theta}(L)}{\hat{\phi}(L)}\varepsilon_t$$

- this is referred to as **transfer function model**

Intervention Analysis

- simple intervention analysis can be based on two types of input series x_t
- step function

$$S_t^{(t_0)} = \begin{cases} 1 & \text{if } t \geq t_0 \\ 0 & \text{otherwise} \end{cases}$$

- pulse function

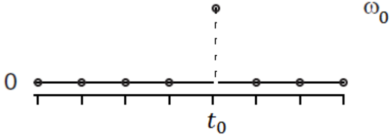
$$P_t^{(t_0)} = \begin{cases} 1 & \text{if } t = t_0 \\ 0 & \text{otherwise} \end{cases}$$

- note that $P_t^{(t_0)} = \Delta S_t^{(t_0)} = (1-L) S_t^{(t_0)}$ or equivalently $S_t^{(t_0)} = \frac{1}{1-L} P_t^{(t_0)}$

Intervention Analysis

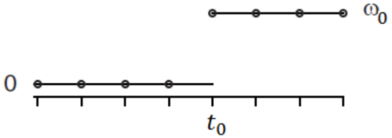
- immediate temporary effect (for example, demand for electricity during a heat wave in summer, sales of beer during Super Bowl week, . . .):
  set $x_t = P_t^{(t_0)}$ and $\frac{\omega(L)}{\delta(L)} = \omega_0$, which implies

  $$f_t = \omega_0 P_t^{(t_0)}$$



- immediate permanent shift: set $x_t = S_t^{(t_0)}$ and $\frac{\omega(L)}{\delta(L)} = \omega_0$, so that

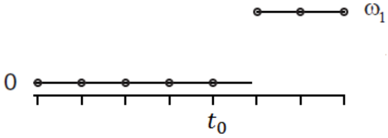
  $$f_t = \omega_0 S_t^{(t_0)}$$



Intervention Analysis

- delayed permanent effect: set $\frac{\omega(L)}{\delta(L)} = L^d \omega_d$, and $x_t = S_t^{(t_0)}$ which yields

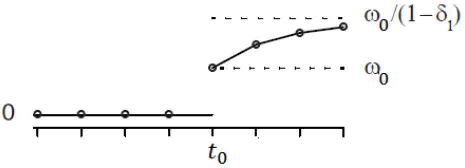
  $f_t = \omega_d S_{t-d}^{(t_0)}$



Intervention Analysis

- gradually growing effect: setting $\delta(L) = 1 - \delta_1 L$, together with $\omega(L) = \omega_0$ and $x_t = S_t^{(t_0)}$ yields $f_t = \delta_1 f_{t-1} + \omega_0 S_t^{(t_0)}$ with initial condition $f_0 = 0$; thus

$$f_t = \frac{\omega_0}{1 - \delta_1 L} S_t^{(t_0)}$$

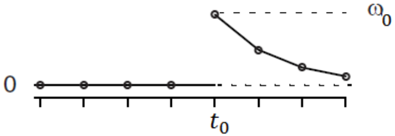


- the immediate impact is ω_0 and the steady state gain is $\frac{\omega_0}{1-\delta_1}$

Intervention Analysis

- gradually decaying effect: setting $\delta(L) = 1 - \delta_1 L$, together with $\omega(L) = \omega_0$ and $x_t = P_t^{(t_0)}$ implies $f_t = \delta_1 f_{t-1} + \omega_0 P_t^{(t_0)}$ with initial condition $f_0 = 0$; thus

$$f_t = \frac{\omega_0}{1 - \delta_1 L} P_t^{(t_0)}$$



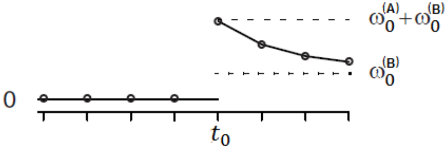
- the immediate impact is ω_0 and the steady state gain is 0

Intervention Analysis

- gradually decaying effect, with non-zero effect in the long run: this can be achieved as combination of two effects, let $f_t = f_t^{(A)} + f_t^{(B)}$

  where $f_t^{(A)} = \frac{\omega_0^{(A)}}{1-\delta_1 L} P_t^{(t_0)}$ and $f_t^{(B)} = \omega_0^{(B)} S_t^{(t_0)}$ so that

$$f_t = \frac{\omega_0^{(A)}}{1-\delta_1 L} P_t^{(t_0)} + \omega_0^{(B)} S_t^{(t_0)}$$



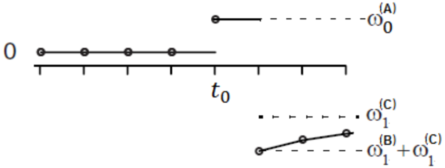
- the immediate impact is $\omega_0^{(A)} + \omega_0^{(B)}$ and the steady state gain is $\omega_0^{(B)}$

Intervention Analysis

- short term and long term effect with different size and signs (announcement of a future tax increase, or a future price increase): can be achieved as combination of three effects, let $f_t = f_t^{(A)} + f_t^{(B)} + f_t^{(C)}$

  where $f_t^{(A)} = \omega_0^{(A)} P_t^{(t_0)}$ and $f_t^{(B)} = \frac{\omega_1^{(B)} L}{1-\delta_1 L} P_t^{(t_0)}$ and $f_t^{(C)} = \omega_1^{(C)} L S_t^{(t_0)}$ so that

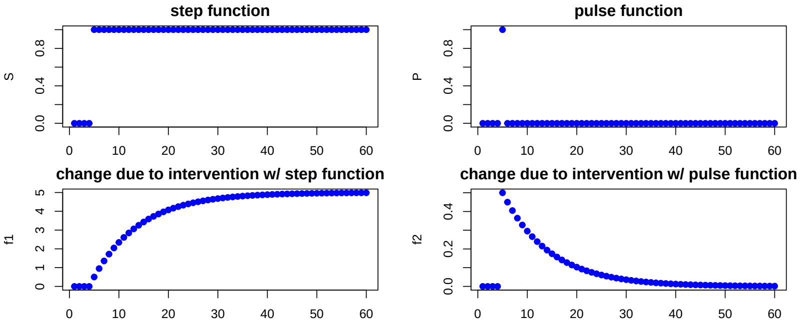
$$f_t = \omega_0^{(A)} P_t^{(t_0)} + \frac{\omega_1^{(B)} L}{1-\delta_1 L} P_t^{(t_0)} + \omega_1^{(C)} L S_t^{(t_0)}$$



- here $\omega_0^{(A)} > 0$ and $\omega_1^{(B)} < 0$, $\omega_1^{(C)} < 0$

Intervention Analysis

```r
# generate step and pulse functions, sample with t periods and intervention at period t0
t <- 60
t0 <- 5
S <- c(rep(0,t0-1), rep(1,t-(t0-1)))
P <- c(rep(0,t0-1), 1, rep(0,t-t0))
# simulate process f_t = delta1*f_{t-1} + omega0*x_t where x_t is either S_t or P_t
omega0 <- 0.5
delta1 <- 0.9
f1 <- filter(x=omega0*S, filter=c(delta1), method='recursive')
f2 <- filter(x=omega0*P, filter=c(delta1), method='recursive')

par(mfrow=c(2,2),  mar=c(2,4,2,2), cex=0.95)
plot(S, col="blue", pch=19, cex=1, main="step function")
plot(P, col="blue", pch=19, cex=1, main="pulse function")
plot(f1, col="blue", type='p', pch=19, cex=1, main='change due to intervention w/ step function')
plot(f2, col="blue", type='p', pch=19, cex=1, main='change due to intervention w/ pulse function')
```



Intervention Analysis

The general modeling procedure for intervention analysis is as follows:

1. specify the model for Z_t using data before intervention $\{y_1, \ldots, y_{t_0-1}\}$
2. use this model to predict Z_t for $t \geq t_0$; denote this by $\hat{Z}_t$
3. examine $Y_t - \hat{Z}_t$ for $t \geq t_0$ to specify $\omega(L)$ and $\delta(L)$
4. perform a joint estimation using all the data
5. check the estimated model for adequacy

Intervention Analysis - Estimation of the Pre-Intervention Model

```r
library(zoo)
library(forecast)
library(TSA)

# Monthly U.S. airline passenger-miles: 01/1996 - 05/2005
data(airmiles)

# whole sample
yall <- airmiles
# pre-intervention period
y <- window(yall, end=c(2001,8))

# estimate model for pre-intervention period
m1 <- Arima(log(y), order = c(0,1,1), seasonal = list(order = c(0,1,1), period = 12))
m1
```

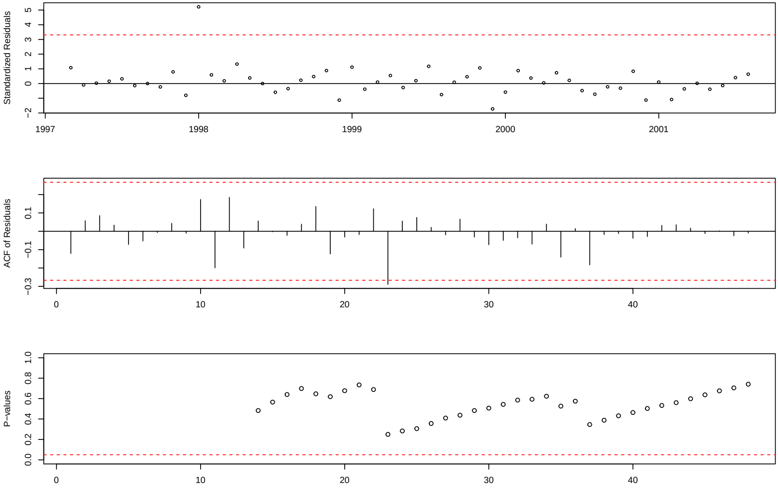
```
## Series: log(y)
## ARIMA(0,1,1)(0,1,1)[12]
##
## Coefficients:
##           ma1     sma1
##       -0.5006  -0.5709
## s.e.   0.1091   0.2298
##
## sigma^2 estimated as 0.001258:  log likelihood=105.82
## AIC=-205.64   AICc=-205.17   BIC=-199.61
```

so the estimated model is

$$\log y_t = \frac{(1-5006L)(1-0.5709L^{12})}{(1-L)(1-L^{12})}\varepsilon_t$$

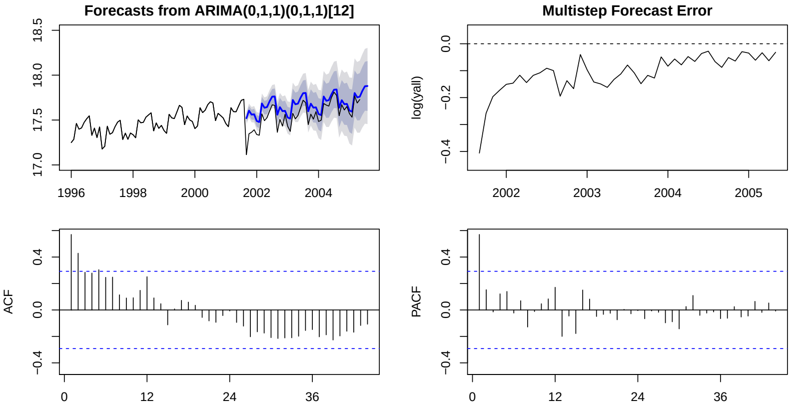
Intervention Analysis - Evaluation of the Pre-Intervention Model

```r
tsdiag(m1, gof.lag=48)
```



Intervention Analysis - Forecast Using Pre-Intervention Model

```
# multistep forecast based on the model for pre-intervention period
m1.f <- forecast(m1, 48)
m1.f.err <- log(yall) - m1.f$mean

# plot forecast, forecast error ACF and PACF for forecast errors
par(mfrow=c(2,2), mar=c(2,4,2,2), cex=0.9)
plot(m1.f, ylim=c(17,18.5))
lines(log(yall))
plot(m1.f.err, ylim=c(-0.45,0.05), main="Multistep Forecast Error")
abline(h=0, lty="dashed")
Acf(m1.f.err, type="correlation", lag.max=48, ylab="ACF", main="")
Acf(m1.f.err, type="partial", lag.max=48, ylab="PACF", main="")
```



Intervention Analysis - Estimation of Transfer Function Models

```r
# next step: estimate model with transfer function
# use 1996/01-2002/12 as estimation sample, keep 2003/01-2005/05 as prediction sample
y <- window(yall, end=c(2002,12))

# create pulse variable for 9/11
P911 <- 1*(index(y)==2001+(9-1)/12)

# estimate model with transfer function which assumes that the effect of 9/11 gradually disappears
#  f_t = delta_1*f_{t-1} + omega_0*P911
m3 <- arimax(log(y), order = c(0,1,1), seasonal = list(order=c(0,1,1), period = 12),
             xtransf = data.frame(P911), transfer = list(c(1,0)), method = "ML")
m3
```

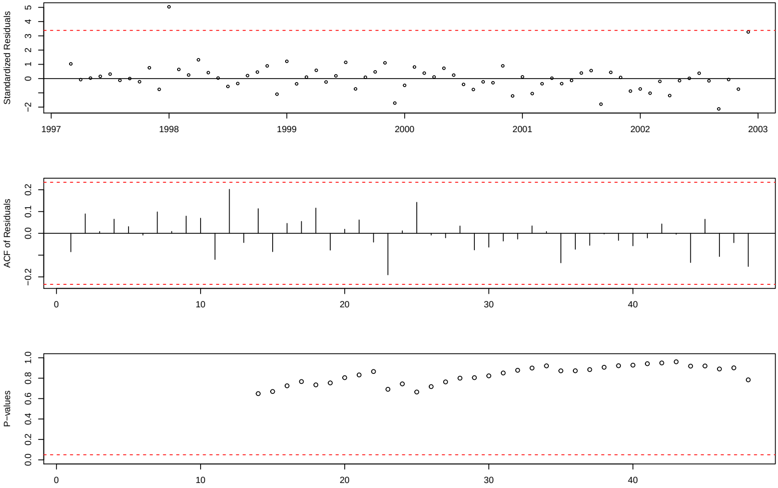
```
##
## Call:
## arimax(x = log(y), order = c(0, 1, 1), seasonal = list(order = c(0, 1, 1), period = 12),
##     method = "ML", xtransf = data.frame(P911), transfer = list(c(1, 0)))
##
## Coefficients:
##           ma1     sma1  P911-AR1  P911-MA0
##       -0.5252  -0.6335    0.7103   -0.3414
## s.e.   0.0971   0.3417    0.0834    0.0363
##
## sigma^2 estimated as 0.001281:  log likelihood = 132.47,  aic = -256.93
```

so the estimated model is

$$\log y_t = -\frac{0.3414}{1-0.7103L} P_t^{911} + \frac{(1-5252L)(1-0.6335L^{12})}{(1-L)(1-L^{12})} \varepsilon_t$$

Intervention Analysis - Model Evaluation

```
tsdiag(m3, gof.lag=48)
```



Intervention Analysis - Estimation of Transfer Function Models

```r
# estimate model with transfer function which assumes that the effect of 9/11 gradually disappears
# but with an additional instantaneous term so that
#  f_t = f^A_t + f^B_t
# where
#  f^A_t = omega_0^A * P911
#  f^B_t = delta_1^B * f^B_{t-1} + omega_0^B * P911
m4 <- arimax(log(y), order=c(0,1,1), seasonal=list(order=c(0,1,1), period=12),
             xtransf=data.frame(P911,P911), transfer=list(c(0,0),c(1,0)), method="ML")
m4
```

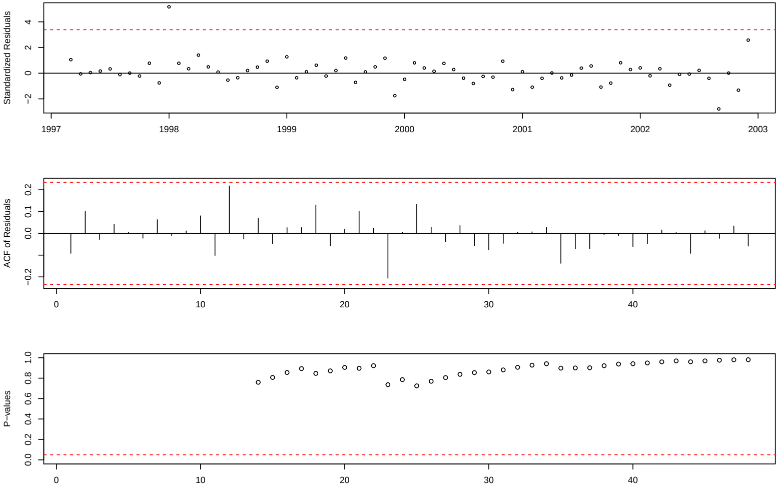
```
##
## Call:
## arimax(x = log(y), order = c(0, 1, 1), seasonal = list(order = c(0, 1, 1), period = 12),
##     method = "ML", xtransf = data.frame(P911, P911), transfer = list(c(0, 0),
##         c(1, 0)))
##
## Coefficients:
##           ma1     sma1  P911-MA0  P911.1-AR1  P911.1-MA0
##       -0.5513  -0.6532   -0.1289      0.8922     -0.2388
## s.e.   0.0973   0.3410    0.0514      0.0819      0.0427
##
## sigma^2 estimated as 0.001195:  log likelihood = 134.66,  aic = -259.33
```

so the estimated model is

$$\log y_t = -0.1289 P_t^{911} - \frac{0.2388}{1-0.8922L} P_t^{911} + \frac{(1-5513L)(1-0.6532L^{12})}{(1-L)(1-L^{12})} \varepsilon_t$$

Intervention Analysis - Model Evaluation

```
tsdiag(m4, gof.lag=48)
```



Intervention Analysis - Forecasting Using Transfer Function Models

```r
# forecast horizon
hmax <- 36
# extend the pulse function
P911 <- c(P911, rep(0,hmax))

# generate the transfer function
tf3 <- m3$coef["P911-MA0"]*filter(P911, filter=m3$coef["P911-AR1"], method='recursive')
# reestimate the model using Arima with tf3 as external regressor
# if tf3 was constructed properly its estimated coefficient will be equal 1
m3x <- Arima(log(y), order=c(0,1,1), seasonal=list(order=c(0,1,1), period=12), xreg=tf3[1:length(y)])
m3x
```

```
## Series: log(y)
## Regression with ARIMA(0,1,1)(0,1,1)[12] errors
##
## Coefficients:
##           ma1     sma1  tf3[1:length(y)]
##       -0.5252  -0.6335            1.0000
## s.e.   0.0957   0.3107            0.0993
##
## sigma^2 estimated as 0.001396:  log likelihood=132.47
## AIC=-256.93   AICc=-256.33   BIC=-247.88
```

```r
# create the forecast
m3x.f.h <- forecast(m3x, h=hmax, xreg=tf3[(length(y)+1):(length(y)+hmax)])
```

Intervention Analysis - Forecasting Using Transfer Function Models

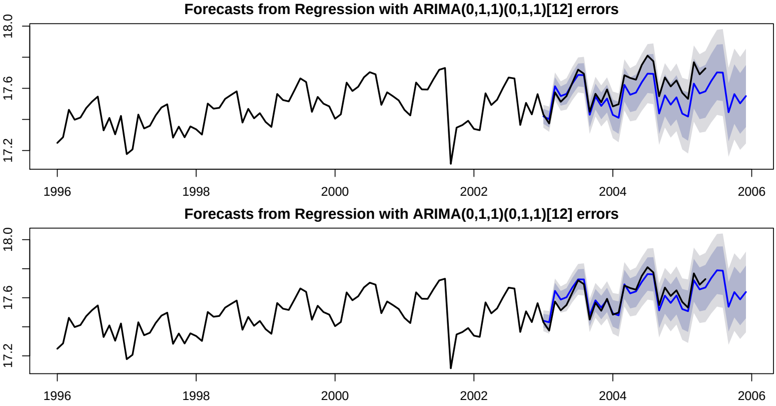
```r
# generate the two components of the transfer function
tf4 <- cbind(m4$coef["P911-MA0"]*P911,
             m4$coef["P911.1-MA0"]*filter(P911, filter=m4$coef["P911.1-AR1"], method='recursive'))
colnames(tf4) <- c("tfA","tfB")
# reestimate the model using Arima with tf4 as matrix of external regressors
# if tf4 was constructed properly its estimated coefficients will be equal 1
m4x <- Arima(log(y), order=c(0,1,1), seasonal=list(order=c(0,1,1), period=12), xreg=tf4[1:length(y),])
m4x
```

```
## Series: log(y)
## Regression with ARIMA(0,1,1)(0,1,1)[12] errors
##
## Coefficients:
##           ma1     sma1    tfA     tfB
##       -0.5513  -0.6532  1.000  1.0000
## s.e.   0.0969   0.3402  0.306  0.1479
##
## sigma^2 estimated as 0.001325:  log likelihood=134.66
## AIC=-259.33   AICc=-258.4   BIC=-248.01
```

```r
# create the forecast
m4x.f.h <- forecast(m4x, h=hmax, xreg=tf4[(length(y)+1):(length(y)+hmax),])
```

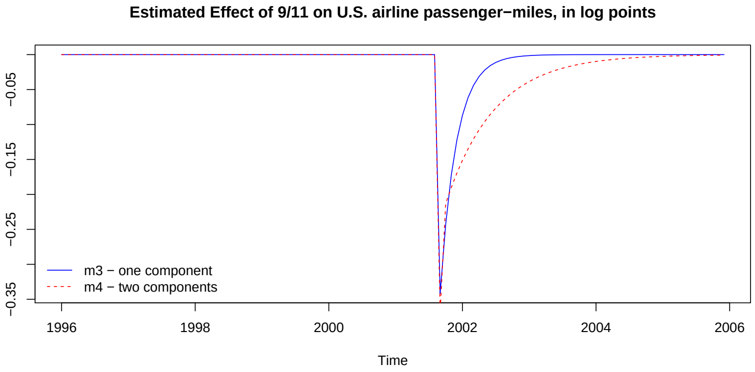
Intervention Analysis - Forecasting Using Transfer Function Models

```r
# plot the forecasts
par(mfrow=c(2,1), mar=c(2,3,2,2), cex=0.9)
plot(m3x.f.h)
lines(log(yall), col="black", lwd=2)
plot(m4x.f.h)
lines(log(yall), col="black", lwd=2)
```



Intervention Analysis - Forecasting Using Transfer Function Models

```r
# construct estimated 9/11 effects on U.S. airline passenger-miles
f3 <- m3$coef["P911-MA0"]*filter(P911, filter=m3$coef["P911-AR1"], method = "recursive")
f4 <- m4$coef["P911-MA0"]*P911 + m4$coef["P911.1-MA0"]*filter(P911, filter=m4$coef["P911.1-AR1"], method='
f3 <- ts(f3, start=1996, frequency=12)
f4 <- ts(f4, start=1996, frequency=12)
plot(f3, type="l", col=4, lty=1,
     ylab="", main="Estimated Effect of 9/11 on U.S. airline passenger-miles, in log points")
lines(f4, type="l", col=2, lty=2)
legend("bottomleft",c("m3 - one component", "m4 - two components"), bty="n", col=c(4,2), lty=c(1,2))
```

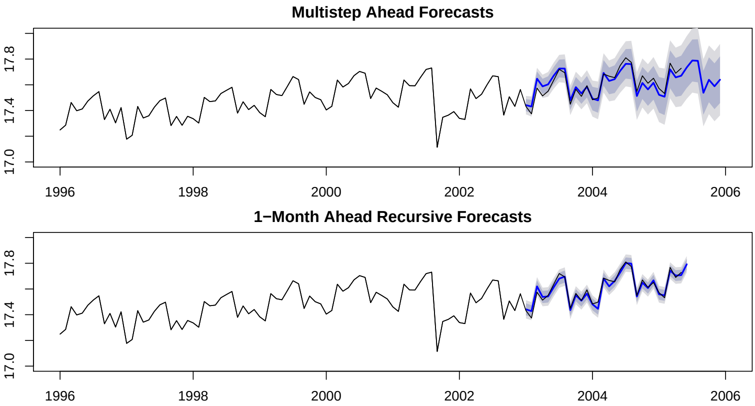


Intervention Analysis - Forecasting Using Transfer Function Models

```r
# create a one month ahead recursive scheme forecast
yall <- airmiles
lstM <- 2002+11/12
y1 <- window(yall, end=lstM)
y2 <- window(yall, start=lstM+1/12)
P911 <- 1*(index(y)==2001+(9-1)/12)

m4x.f.rec <- list()
for(i in 1:(length(y2)+1))
{
    y <- window( yall, end=lstM+(i-1)/12 )
    m4.updt <- arimax(log(y), order=c(0,1,1), seasonal=list(order=c(0,1,1), period=12),
             xtransf=data.frame(P911,P911), transfer=list(c(0,0),c(1,0)), method="ML")
    P911 <- c(P911, 0)
    tf4 <- cbind(m4$coef["P911-MA0"]*P911,
                 m4$coef["P911.1-MA0"]*filter(P911, filter=m4$coef["P911.1-AR1"], method='recursive'))
    m4x.updt <- Arima(log(y),
                      order=c(0,1,1), seasonal=list(order=c(0,1,1), period=12), xreg=tf4[1:length(y),])
    m4x.updt.f.1 <- forecast(m4x.updt, h=1, xreg=t(as.matrix(tf4[(length(y)+1):(length(y)+1),])))
    m4x.f.rec$mean <- rbind(m4x.f.rec$mean, as.zoo(m4x.updt.f.1$mean))
    m4x.f.rec$lower <- rbind(m4x.f.rec$lower, m4x.updt.f.1$lower)
    m4x.f.rec$upper <- rbind(m4x.f.rec$upper, m4x.updt.f.1$upper)
}
m4x.f.rec$mean <- as.ts(m4x.f.rec$mean)
m4x.f.rec$level <- m4x.updt.f.1$level
m4x.f.rec$x <- window(m4x.updt.f.1$x, end=lstM)
class(m4x.f.rec) <- class(m4x.f.h)
```

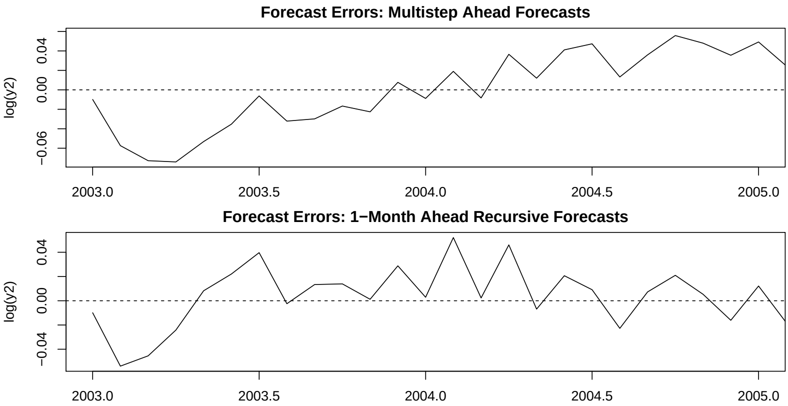
Intervention Analysis - Forecasting Using Transfer Function Models

```
par(mfrow=c(2,1), mar=c(2,4,2,2))

# plot multistep ahead forecasts
plot(m4x.f.h, xlim=c(1996,2006), ylim=c(17,18), main="Multistep Ahead Forecasts")
lines(log(yall))

# plot 1 step ahead rolling forecasts form model m4
plot(m4x.f.rec, xlim=c(1996,2006), ylim=c(17,18), main="1-Month Ahead Recursive Forecasts")
lines(log(yall))
```



Intervention Analysis - Forecasting Using Transfer Function Models

```
par(mfrow=c(2,1), mar=c(2,4,2,2))

# plot multistep ahead forecasts
plot(log(y2)-m4x.f.h$mean, xlim=c(2003,2005), main="Forecast Errors: Multistep Ahead Forecasts")
abline(h=0, lty="dashed")

# plot 1 step ahead rolling forecasts form model m4
plot(log(y2)-m4x.f.rec$mean, xlim=c(2003,2005), main="Forecast Errors: 1-Month Ahead Recursive Forecasts")
abline(h=0, lty="dashed")
```



Intervention Analysis - Forecasting Using Transfer Function Models

```
# multistep forecast
accuracy(m4x.f.h$mean, log(y2))
```

```
##                    ME       RMSE        MAE        MPE      MAPE      ACF1
## Test set 0.004694233 0.03929241 0.03415625 0.02592523 0.1939522 0.7880236
##          Theil's U
## Test set 0.3577295
```

```
# 1 month ahead recursive forecast
accuracy(m4x.f.rec$mean, log(y2))
```

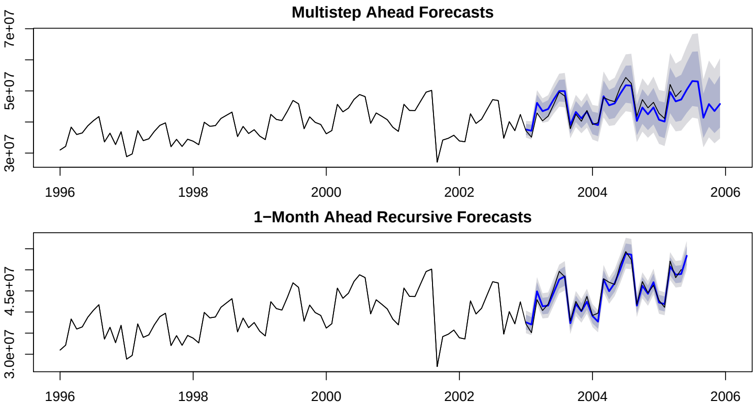
```
##                    ME       RMSE        MAE       MPE      MAPE      ACF1
## Test set 0.004734889 0.02451095 0.01950325 0.0266333 0.1108093 0.1737935
##          Theil's U
## Test set 0.2232651
```

Intervention Analysis - Forecasting Using Transfer Function Models

```
# undo log transformation
m4x.f.h$mean <- exp(m4x.f.h$mean)
m4x.f.h$lower <- exp(m4x.f.h$lower)
m4x.f.h$upper <- exp(m4x.f.h$upper)
m4x.f.h$x <- exp(m4x.f.h$x)

m4x.f.rec$mean <- exp(m4x.f.rec$mean)
m4x.f.rec$lower <- exp(m4x.f.rec$lower)
m4x.f.rec$upper <- exp(m4x.f.rec$upper)
m4x.f.rec$x <- exp(m4x.f.rec$x)
```

Intervention Analysis - Forecasting Using Transfer Function Models

```
par(mfrow=c(2,1), mar=c(2,4,2,2))

# plot multistep ahead forecasts
plot(m4x.f.h, xlim=c(1996,2006), main="Multistep Ahead Forecasts")
lines(yall)

# plot 1 step ahead rolling forecasts form model m4
plot(m4x.f.rec, xlim=c(1996,2006), main="1-Month Ahead Recursive Forecasts")
lines(yall)
```



Intervention Analysis - Forecasting Using Transfer Function Models

```r
# multistep forecast
accuracy(m4x.f.h$mean, y2)
```

```
##              ME    RMSE     MAE      MPE     MAPE      ACF1 Theil's U
## Test set 275761 1760099 1525977 0.392109 3.412685 0.7720393 0.3664354
```

```r
# 1 month ahead recursive forecast
accuracy(m4x.f.rec$mean, y2)
```

```
##                ME    RMSE      MAE       MPE     MAPE      ACF1 Theil's U
## Test set 231974.5 1066582 862657.2 0.4435006 1.944894 0.1122754 0.2285562
```