Eco 5316 Time Series Econometrics

Lecture 4 Moving Average (MA) processes

MA(q) model

- let $\{\varepsilon_t\}$ be a white noise time series; process $\{y_t\}$ follows a moving average model of order 1, or MA(1) model, if

$$y_t = c_0 + \varepsilon_t + \theta_1 \varepsilon_{t-1}$$

  or, using the lag operator we can write $y_t = c_0 + (1 + \theta_1 L)\varepsilon_t$
- the bid-ask bounce in stock trading may introduce an MA(1) structure in a return series
- in general, for $q > 0$ a moving average model of order q, or MA(q) model, is given by

$$y_t = c_0 + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \ldots + \theta_q \varepsilon_{t-q}$$

  or equivalently using the lag operator $y_t = c_0 + (1 + \theta_1 L + \ldots + \theta_q L^q)\varepsilon_t$

Autocorrelation function for MA(q) model

- if $q = 1$ it can be shown that $\gamma_0 = Var(y_t) = (1+\theta_1^2)\sigma_a^2$, and also that $\gamma_1 = \theta_1 \sigma_a^2$ and $\gamma_l = 0$ for $l \geq 2$; thus

$$\rho_1 = \frac{\theta_1}{1+\theta_1^2} \qquad \rho_l = 0 \text{ for } l > 1 \tag{1}$$

- theoretical ACF for MA(1) thus cuts off to zero after lag 1

- if $q = 2$ we have $\gamma_0 = Var(y_t) = (1+\theta_1^2+\theta_2^2)\sigma_a^2$, and the theoretical ACF for MA(2) satisfies

$$\rho_1 = \frac{\theta_1+\theta_1\theta_2}{1+\theta_1^2+\theta_2^2} \qquad \rho_2 = \frac{\theta_2}{1+\theta_2^2+\theta_1^2} \qquad \rho_l = 0 \text{ for } l > 2 \tag{2}$$

- theoretical ACF for MA(1) thus cuts off to zero after lag 2

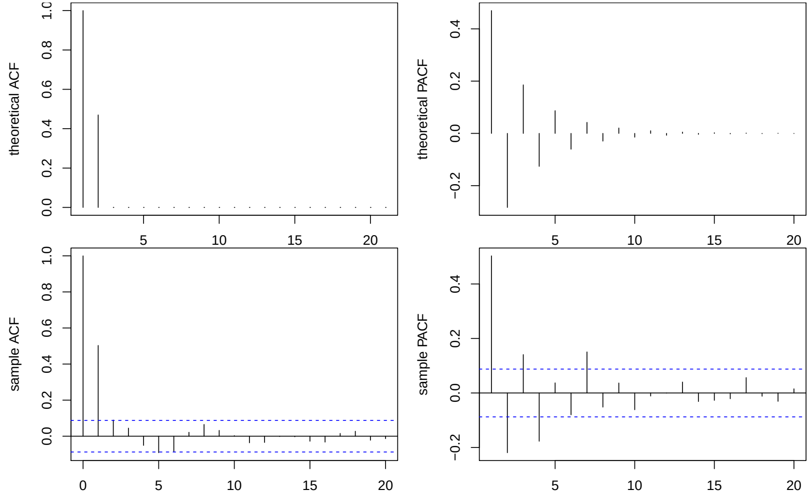
- in general, for MA(q) model $\gamma_0 = Var(y_t) = (1+\theta_1^2+\ldots+\theta_q^2)\sigma_a^2$
- theoretical ACF for MA(q) model satisfies $\rho_l = 0$ for $l > q$

Autocorrelation function for MA(q) model

- ACF is thus useful for identifying the order of an MA model in the same way the PACF is useful in identifying the order of an AR model
- note that MA(q) model is always weakly stationary, unlike AR(p) model which is only weakly stationary if its characteristic roots lie inside the unit circle
- interactive overview of ACF and PACF for simulated MA(q) models is here

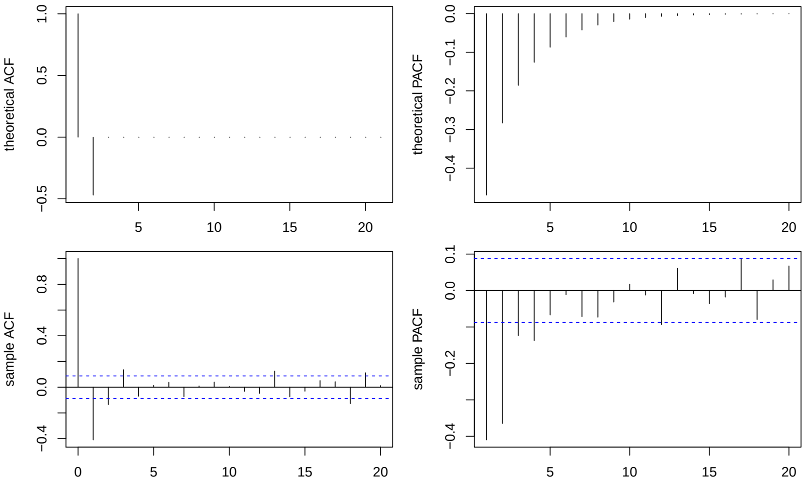
Autocorrelation function for MA(q) model

MA(1) with $\theta_1 = 0.7$



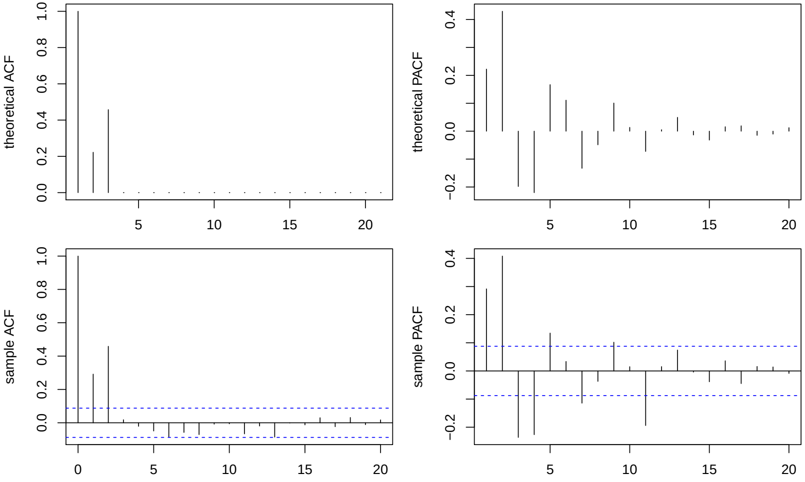
Autocorrelation function for MA(q) model

MA(1) with $\theta_1 = -0.7$



Autocorrelation function for MA(q) model

MA(2) with $\theta_1 = 0.2$, $\theta_2 = 0.7$



Autocorrelation function for MA(q) model

MA(2) with $\theta_1 = 0.2$**,** $\theta_1 = -0.7$