

Practice Problems 2

Question 1. Write down the equation for a pure seasonal S-AR(1) model. Describe how its AC and PAC functions look like.

See [lec11slides.pdf](#) slides 9 to 13.

Question 2. Write down the equation for a model with regular AR(1) and seasonal S-AR(1) components and describe how its AC and PAC functions look like.

See [lec11slides.pdf](#) slides 16 to 18.

Question 3. Explain the difference between estimation sample and prediction sample.

See [lec13slides.pdf](#) slide 2 and also [lec06slides.pdf](#) slide 15.

Question 4. Explain the difference between in-sample evaluation and out-of-sample evaluation.

See [lec13slides.pdf](#) slide 2 and also [lec06slides.pdf](#) slide 15.

Question 5. Explain how Mean Squared Error, Mean Absolute Error, and Mean Loss are used in the assessment of forecasts.

See [lec13slides.pdf](#) slides 14 and 15.

Question 6. Give an example of a deterministic trend $g(t)$ other than a linear trend and plot its graph. Write the model equation for this trend.

See [lec16slides.pdf](#) slides 10 to 12.

Question 7. Write the equation for pure random walk process and the equation for a random walk process with a drift. Explain the main difference between the two.

See [lec18slides.pdf](#) slides 4, 6 and 7.

Question 8. Draw a typical correlogram for a random walk process.

See [lec18slides.pdf](#) slide 12.

Question 9. Explain the difference between a trend stationary time series and a difference stationary time series.

See [lec18slides.pdf](#) slide 15.

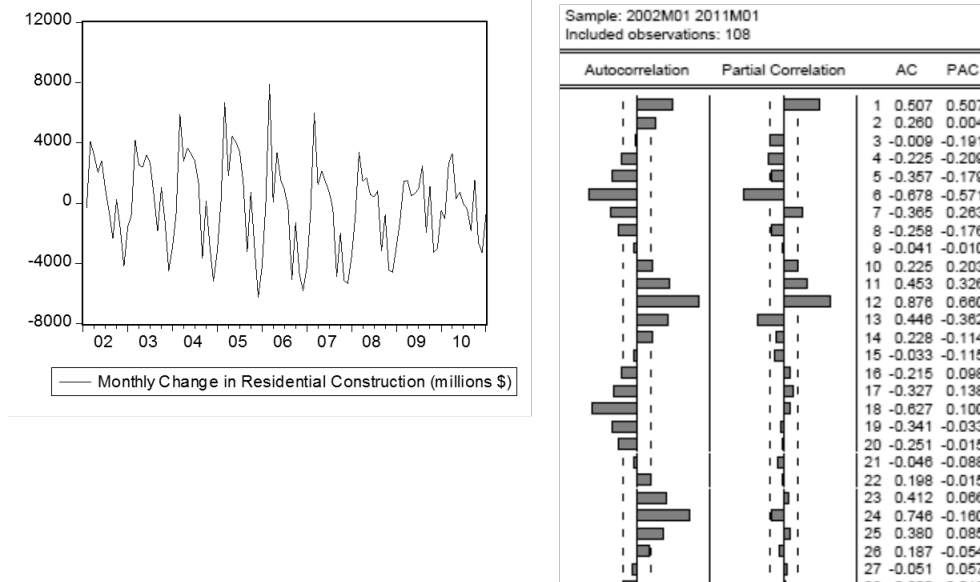
Question 10. Explain the idea behind the Dickey Fuller unit root test.

See [lec18slides.pdf](#) slide 19.

Question 11. Explain what it means that a time series process is $I(1)$, and what it means that a process is $I(0)$.

See [lec18slides.pdf](#) slide 14.

Question 12. Consider the data for monthly changes in U.S. residential construction for the period January 2002-January 2011 shown below. Discuss what kind of model would you estimate for this time series and explain why.



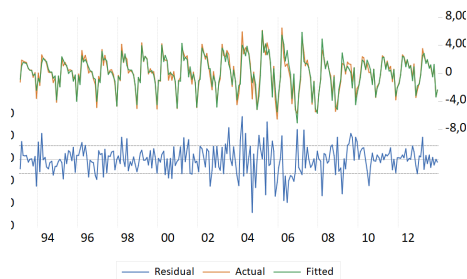
See [lec11slides.pdf](#) slide 16 to 20.

Question 13. Consider two candidate models for change in private residential construction spending, AR(1)+SAR(1) and AR(2)+SAR(1), the results for which are below. Discuss which of these models would be preferred based on plots and correlograms of residuals, AIC and BIC, and statistical significance of coefficients.

Dependent Variable: DCONST
Method: ARMA Maximum Likelihood (BFGS)
Date: 03/08/19 Time: 17:29
Sample: 1993M02 2013M12
Included observations: 251
Convergence achieved after 5 iterations
Coefficient covariance computed using outer product of gradients

| Variable | Coefficient | Std. Error | t-Statistic | Prob. |
|----------|-------------|------------|-------------|--------|
| C | 218.5690 | 1235.621 | 0.176890 | 0.8597 |
| AR(1) | 0.563509 | 0.044817 | 12.57349 | 0.0000 |
| SAR(12) | 0.844410 | 0.013547 | 69.71517 | 0.0000 |
| SIGMASQ | 379202.0 | 27048.41 | 14.01938 | 0.0000 |

| | | | |
|--------------------|-----------|-----------------------|----------|
| R-squared | 0.933686 | Mean dependent var | 49.44223 |
| Adjusted R-squared | 0.932881 | S.D. dependent var | 2396.078 |
| S.E. of regression | 620.7600 | Akaike info criterion | 15.82347 |
| Sum squared resid | 95179704 | Schwarz criterion | 15.87965 |
| Log likelihood | -1981.845 | Hannan-Quinn criter. | 15.84608 |
| F-statistic | 1159.242 | Durbin-Watson stat | 2.130377 |
| Prob(F-statistic) | 0.000000 | | |



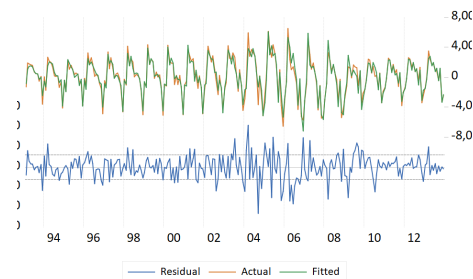
Date: 03/08/19 Time: 17:29
Sample: 1993M01 2013M12
Included observations: 251
Q-statistic probabilities adjusted for 2 ARMA terms

| Autocorrelation | Partial Correlation | AC | PAC | Q-Stat | Prob |
|-----------------|---------------------|--------|--------|--------|------|
| 1 | -0.066 | -0.066 | 1.1109 | | |
| 2 | 0.136 | 0.132 | 5.8246 | | |
| 3 | -0.022 | -0.005 | 5.9460 | 0.015 | |
| 4 | -0.010 | -0.030 | 5.9704 | 0.051 | |
| 5 | 0.058 | 0.061 | 6.8484 | 0.077 | |
| 6 | -0.075 | -0.065 | 8.3212 | 0.080 | |
| 7 | -0.081 | -0.109 | 10.030 | 0.074 | |
| 8 | 0.116 | 0.132 | 13.549 | 0.035 | |
| 9 | -0.088 | -0.054 | 15.602 | 0.029 | |
| 10 | -0.017 | -0.073 | 15.675 | 0.047 | |
| 11 | -0.044 | -0.013 | 16.196 | 0.063 | |
| 12 | -0.004 | 0.010 | 16.201 | 0.094 | |
| 13 | -0.016 | -0.046 | 16.267 | 0.132 | |
| 14 | -0.008 | 0.010 | 16.284 | 0.179 | |
| 15 | 0.004 | 0.030 | 16.288 | 0.234 | |
| 16 | 0.009 | -0.027 | 16.308 | 0.295 | |
| 17 | 0.050 | 0.054 | 16.997 | 0.319 | |
| 18 | -0.093 | -0.085 | 19.357 | 0.251 | |
| 19 | 0.047 | 0.020 | 19.956 | 0.276 | |
| 20 | 0.068 | 0.098 | 21.224 | 0.268 | |
| 21 | -0.041 | -0.048 | 21.684 | 0.300 | |
| 22 | 0.029 | -0.011 | 21.918 | 0.345 | |
| 23 | -0.020 | 0.022 | 22.033 | 0.398 | |
| 24 | 0.152 | 0.147 | 28.514 | 0.159 | |
| 25 | -0.000 | -0.029 | 28.514 | 0.197 | |
| 26 | -0.059 | -0.051 | 29.501 | 0.202 | |
| 27 | -0.028 | -0.033 | 29.724 | 0.235 | |
| 28 | 0.025 | 0.020 | 29.903 | 0.272 | |
| 29 | -0.047 | -0.041 | 30.532 | 0.291 | |
| 30 | -0.033 | -0.026 | 30.842 | 0.324 | |
| 31 | -0.043 | 0.000 | 31.369 | 0.348 | |
| 32 | 0.058 | 0.025 | 32.353 | 0.351 | |
| 33 | 0.002 | 0.019 | 32.354 | 0.400 | |
| 34 | 0.033 | 0.043 | 32.675 | 0.434 | |
| 35 | -0.022 | -0.003 | 32.818 | 0.476 | |
| 36 | -0.072 | -0.129 | 34.329 | 0.452 | |

Dependent Variable: DCONST
Method: ARMA Maximum Likelihood (BFGS)
Date: 03/08/19 Time: 17:29
Sample: 1993M02 2013M12
Included observations: 251
Convergence achieved after 7 iterations
Coefficient covariance computed using outer product of gradients

| Variable | Coefficient | Std. Error | t-Statistic | Prob. |
|----------|-------------|------------|-------------|--------|
| C | 214.4219 | 1374.109 | 0.156044 | 0.8761 |
| AR(1) | 0.497140 | 0.054595 | 9.105947 | 0.0000 |
| AR(2) | 0.116143 | 0.052147 | 2.227211 | 0.0268 |
| SAR(12) | 0.944592 | 0.013249 | 71.29646 | 0.0000 |
| SIGMASQ | 373960.6 | 26485.07 | 14.11968 | 0.0000 |

| | | | |
|--------------------|-----------|-----------------------|----------|
| R-squared | 0.934603 | Mean dependent var | 49.44223 |
| Adjusted R-squared | 0.933540 | S.D. dependent var | 2396.078 |
| S.E. of regression | 617.7066 | Akaike info criterion | 15.81783 |
| Sum squared resid | 93864109 | Schwarz criterion | 15.88805 |
| Log likelihood | -1980.137 | Hannan-Quinn criter. | 15.84609 |
| F-statistic | 878.9103 | Durbin-Watson stat | 1.975678 |
| Prob(F-statistic) | 0.000000 | | |



Date: 03/08/19 Time: 17:29
Sample: 1993M01 2013M12
Included observations: 251
Q-statistic probabilities adjusted for 3 ARMA terms

| Autocorrelation | Partial Correlation | AC | PAC | Q-Stat | Prob |
|-----------------|---------------------|--------|--------|--------|------|
| 1 | 0.011 | 0.011 | 0.0322 | | |
| 2 | 0.054 | 0.054 | 0.7743 | | |
| 3 | -0.054 | -0.056 | 1.5282 | | |
| 4 | -0.032 | -0.034 | 1.7972 | 0.180 | |
| 5 | 0.042 | 0.049 | 2.2539 | 0.324 | |
| 6 | -0.091 | -0.092 | 4.3953 | 0.222 | |
| 7 | -0.081 | -0.089 | 6.1091 | 0.191 | |
| 8 | 0.112 | 0.132 | 9.3965 | 0.094 | |
| 9 | -0.073 | -0.078 | 10.799 | 0.095 | |
| 10 | -0.031 | -0.065 | 11.048 | 0.137 | |
| 11 | -0.041 | -0.009 | 11.501 | 0.175 | |
| 12 | -0.005 | -0.001 | 11.507 | 0.243 | |
| 13 | -0.013 | -0.053 | 11.553 | 0.316 | |
| 14 | -0.007 | 0.014 | 11.565 | 0.397 | |
| 15 | 0.006 | 0.021 | 11.575 | 0.480 | |
| 16 | 0.019 | -0.024 | 11.670 | 0.555 | |
| 17 | 0.040 | 0.045 | 12.109 | 0.598 | |
| 18 | -0.095 | -0.096 | 14.580 | 0.482 | |
| 19 | 0.042 | 0.037 | 15.068 | 0.520 | |
| 20 | 0.070 | 0.082 | 16.416 | 0.495 | |
| 21 | -0.041 | -0.061 | 16.875 | 0.532 | |
| 22 | 0.010 | -0.006 | 16.904 | 0.596 | |
| 23 | -0.005 | 0.036 | 16.912 | 0.659 | |
| 24 | 0.158 | 0.146 | 23.933 | 0.296 | |
| 25 | 0.008 | -0.038 | 23.952 | 0.350 | |
| 26 | -0.075 | -0.038 | 25.527 | 0.324 | |
| 27 | -0.033 | -0.021 | 25.837 | 0.362 | |
| 28 | 0.024 | 0.019 | 25.999 | 0.408 | |
| 29 | -0.044 | -0.047 | 26.564 | 0.432 | |
| 30 | -0.043 | -0.024 | 27.097 | 0.459 | |
| 31 | -0.037 | -0.000 | 27.498 | 0.491 | |
| 32 | 0.064 | 0.024 | 28.673 | 0.482 | |
| 33 | 0.022 | 0.028 | 28.816 | 0.527 | |
| 34 | 0.044 | 0.058 | 29.388 | 0.549 | |
| 35 | -0.016 | -0.004 | 29.459 | 0.596 | |
| 36 | -0.071 | -0.119 | 30.955 | 0.569 | |

See [hw05sol.pdf](#) Problem 1

Question 14. Consider a regression for the test of equal predictive ability for fixed scheme forecast vs simple four quarter moving average forecast

$$\Delta L_{t+j,1} = \beta_0 + u_{t+j} \quad \text{with } j = 0, 1, 2, \dots, T - t - 1$$

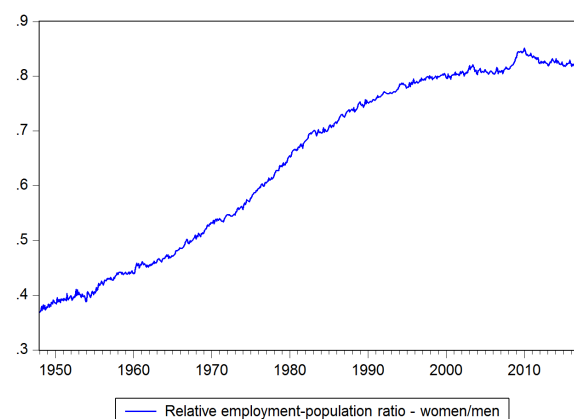
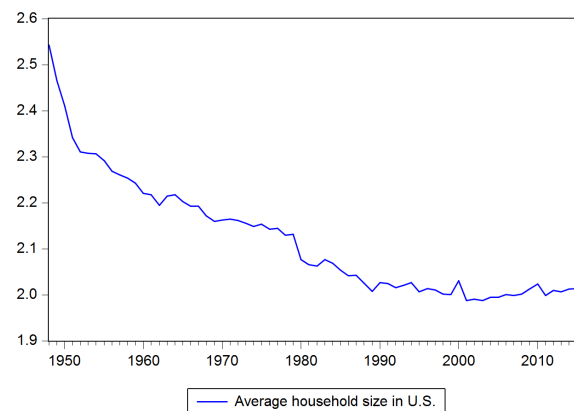
where $\Delta L_{t+j,1} = (e_{t+j,1}^{ma})^2 - (e_{t+j,1}^{fixed})^2$, the results of which are below. Explain the idea behind this test and interpret the results below.

Dependent Variable: DL_MA
Method: Least Squares
Date: 04/08/17 Time: 19:21
Sample: 2009Q1 2016Q4
Included observations: 32
HAC standard errors & covariance (Bartlett kernel, Newey-West fixed bandwidth = 4.0000)

| Variable | Coefficient | Std. Error | t-Statistic | Prob. |
|--------------------|-------------|-----------------------|-------------|-----------|
| C | 8.81E-05 | 3.96E-05 | 2.224905 | 0.0335 |
| R-squared | 0.000000 | Mean dependent var | | 8.81E-05 |
| Adjusted R-squared | 0.000000 | S.D. dependent var | | 0.000237 |
| S.E. of regression | 0.000237 | Akaike info criterion | | -13.82388 |
| Sum squared resid | 1.75E-06 | Schwarz criterion | | -13.77808 |
| Log likelihood | 222.1821 | Hannan-Quinn criter. | | -13.80870 |
| Durbin-Watson stat | 2.025517 | | | |

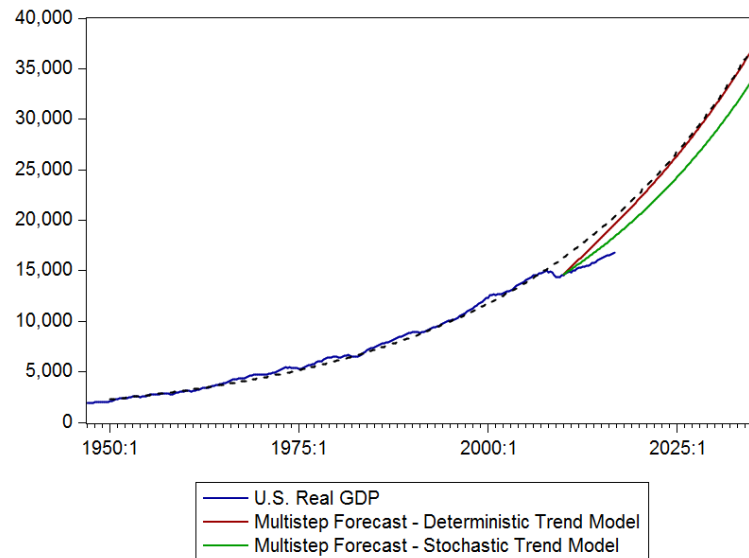
See [lec13slides.pdf](#) slide 18 to 21.

Question 15. Discuss the choice of a trend when developing models for the two series plotted below.



See [lec16slides.pdf](#).

Question 16. The following figure shows the multistep forecasts for the U.S. real GDP, from the deterministic model and from the stochastic trend model, both for the period 2010Q1-2035Q4. Discuss the main difference in the behavior of the forecast and explain the reason for this difference.

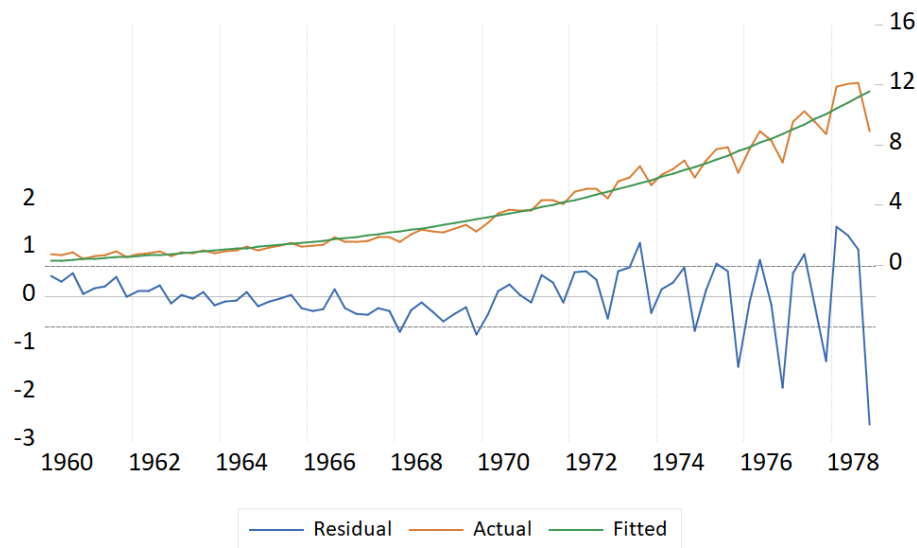


See [lec18slides.pdf](#) slides 17, 18 and 44.

Question 17. Consider a model for quarterly earnings per share of the Johnson and Johnson company

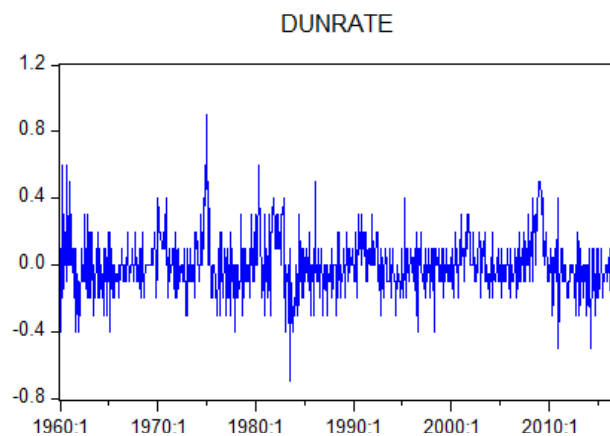
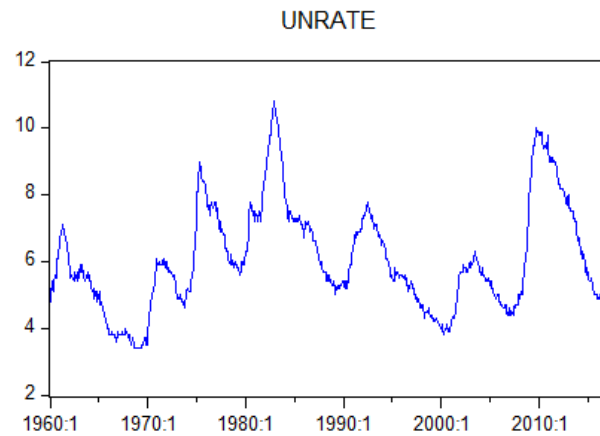
$$JNJ_t = \beta_0 + \beta_1 e^{\beta_2 t} + \varepsilon_t$$

Given the plot with actual values, fitted values, and residuals below, explain how you would proceed with modifying/developing the model further.



See [lec16slides.pdf](#) slide 21.

Question 18. Interpret the below results of the Augmented Dickey-Fuller test for unemployment rate UNRATE and its first difference DUNRATE, determine whether unemployment is $I(0)$ or $I(1)$. Explain why only constant was used in both tests.



Null Hypothesis: UNRATE has a unit root
Exogenous: Constant
Lag Length: 4 (Automatic - based on SIC, maxlag=19)

| | t-Statistic | Prob.* |
|---|------------------|---------------|
| Augmented Dickey-Fuller test statistic | -3.057041 | 0.0304 |
| Test critical values: 1% level | -3.439682 | |
| 5% level | -2.865549 | |
| 10% level | -2.568961 | |

*Mackinnon (1996) one-sided p-values.

Null Hypothesis: D(UNRATE) has a unit root
Exogenous: Constant
Lag Length: 3 (Automatic - based on SIC, maxlag=19)

| | t-Statistic | Prob.* |
|---|------------------|---------------|
| Augmented Dickey-Fuller test statistic | -7.891697 | 0.0000 |
| Test critical values: 1% level | -3.439682 | |
| 5% level | -2.865549 | |
| 10% level | -2.568961 | |

*Mackinnon (1996) one-sided p-values.

See [lec18slides.pdf](#) slide 26.

Question 19. Below are the results for the Augmented Dickey-Fuller unit root test for log transformed earnings per share $\log JNJ_t$, and for the first difference of the log transformed earnings per share $\Delta \log JNJ_t$. Interpret the results, and determine whether $\log JNJ_t$ is $I(0)$ or $I(1)$. Explain why trend and constant were used in the test for $\log JNJ_t$ but only constant was used in the test for $\Delta \log JNJ_t$.

| Null Hypothesis: LNJ has a unit root Exogenous: Constant, Linear Trend Lag Length: 3 (Automatic - based on SIC, maxlag=11) | | |
|---|------------------|---------------|
| | t-Statistic | Prob.* |
| Augmented Dickey-Fuller test statistic | -1.696535 | 0.7428 |
| Test critical values: 1% level | -4.090602 | |
| 5% level | -3.473447 | |
| 10% level | -3.163967 | |
| *Mackinnon (1996) one-sided p-values. | | |
| Null Hypothesis: D(LNJ) has a unit root Exogenous: Constant, Linear Trend Lag Length: 2 (Automatic - based on SIC, maxlag=11) | | |
| | t-Statistic | Prob.* |
| Augmented Dickey-Fuller test statistic | -19.93554 | 0.0001 |
| Test critical values: 1% level | -4.090602 | |
| 5% level | -3.473447 | |
| 10% level | -3.163967 | |
| *Mackinnon (1996) one-sided p-values. | | |

See [lec18slides.pdf](#).

Question 20. Consider two models for U.S. real GDP, used to construct forecast for the period 2010Q1-2016Q4:

(A) deterministic trend model

$$\log rGDP_t = \beta_0 + \beta_1 t + u_t$$

$$u_t = \phi_1 u_{t-1} + \phi_2 u_{t-2} + \varepsilon_t$$

for which the sequence of 1-step ahead forecasts has RMSE=103.459 and the multistep forecast has RMSE=1649.069

(B) stochastic trend model

$$\Delta \log rGDP_t = \beta_0 + u_t$$

$$u_t = \phi_1 u_{t-1} + \varepsilon_t$$

for which the sequence of 1-step ahead forecasts has RMSE=77.3231 and the multistep forecast has RMSE=905.1898.

Discuss how we would choose which model is preferred based on this information. How would we conduct a formal test that one of the models produces more precise forecasts?

See [lec13slides.pdf](#) slides 45 to 48.