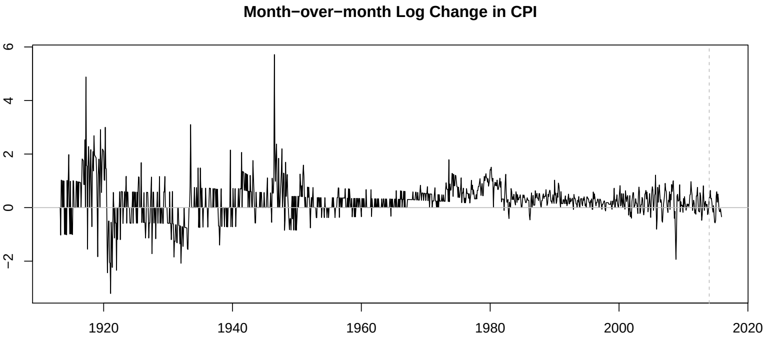
A Simple State Space Model for CPI inflation

Goal and Data

- goal: build a simple linear Gaussian state space model (local level model with seasonal component) for monthly CPI inflation rate in U.S.
- model for $y_t = 100\Delta \log CPI_t$, log difference of Consumer Price Index for All Urban Consumers: All Items, Not Seasonally Adjusted, `FRED/CPIAUCNS`
- data available until 2015M12
- estimation sample: 1955M1-2013M12
- prediction sample: 2014M1-2016M12



State Space Model for CPI Inflation

- log change in CPI y_t, is assumed to consist of a local level component μ_t, a seasonal component γ_t, and an irregular component ε_t

$$
\begin{aligned}
y_t &= \mu_t + \gamma_t + \varepsilon_t & \varepsilon_t &\sim N(0, \sigma^2_\varepsilon) \\
\mu_{t+1} &= \mu_t + \zeta_t & \zeta_t &\sim N(0, \sigma^2_\zeta) \\
(1 + B + B^2 + \ldots + B^{11})\gamma_{t+1} &= \omega_t & \omega_t &\sim N(0, \sigma^2_\omega)
\end{aligned}
$$

State Space Model for CPI Inflation

state-space representation - rewrite the above model in matrix form

$$\underbrace{y_t}_{\boldsymbol{y}_t} = \underbrace{\begin{bmatrix} 1 & 1 & 0 & \dots & 0 \end{bmatrix}}_{\boldsymbol{Z}_t} \underbrace{\begin{bmatrix} \mu_t \\ \gamma_t \\ \gamma_{t-1} \\ \vdots \\ \gamma_{t-10} \end{bmatrix}}_{\boldsymbol{s}_t} + \underbrace{\varepsilon_t}_{\boldsymbol{\varepsilon}_t}$$

$$\underbrace{\begin{bmatrix} \mu_{t+1} \\ \gamma_{t+1} \\ \gamma_t \\ \gamma_{t-1} \\ \vdots \\ \gamma_{t-9} \end{bmatrix}}_{\boldsymbol{s}_{t+1}} = \underbrace{\begin{bmatrix} 1 & 0 & 0 & \dots & 0 & 0 \\ 0 & -1 & -1 & \dots & -1 & -1 \\ 0 & 1 & 0 & \dots & 0 & 0 \\ 0 & 0 & 1 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & 0 & 1 & 0 \end{bmatrix}}_{\boldsymbol{T}_t} \underbrace{\begin{bmatrix} \mu_t \\ \gamma_t \\ \gamma_{t-1} \\ \gamma_{t-2} \\ \vdots \\ \gamma_{t-10} \end{bmatrix}}_{\boldsymbol{s}_t} + \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ \vdots & \vdots \\ 0 & 0 \end{bmatrix}}_{\boldsymbol{R}_t} \underbrace{\begin{bmatrix} \zeta_t \\ \omega_t \end{bmatrix}}_{\boldsymbol{\eta}_t}$$

where

$$\varepsilon_t \sim N(0, \underbrace{\sigma_\varepsilon^2}_{\boldsymbol{H}_t}) \qquad \begin{bmatrix} \zeta_t \\ \xi_t \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \underbrace{\begin{bmatrix} \sigma_\zeta^2 & 0 \\ 0 & \sigma_\omega^2 \end{bmatrix}}_{\boldsymbol{Q}_t} \right)$$

Estimation of the State Space Model for CPI Inflation

- $\sigma_\varepsilon^2, \sigma_\zeta^2, \sigma_\omega^2$ in matrices $\mathbf{H}_t$ and $\mathbf{Q}_t$ are estimated using maximum likelihood

- define the model and set up the update function for the `fitSSM` command

```
# load package for Kalman filtering and smoothing
library(KFAS)
# define state space model - local level with seasonality
y.LLM <- SSModel(y ~ SSMtrend(degree=1, Q=NA)
                         + SSMseasonal(period=12, sea.type="dummy", Q=NA), H=NA)
```

- to incorporate non-negativity constraints for the three variances, parameters estimated are actually log transformed variances

```
# define update function for maximum likelihood estimation
y.updatefn <- function(pars, model) {
    model$H[,,1] <- exp(pars[1])
    model$Q[,,1] <- diag(exp(pars[2:3]))
    model
}
```

- maximum likelihood estimation

```
# initial parameters for maximum likelihood estimation
pars <- log(c(0.01,0.01,0.01))
# maximum likelihood estimation
y.LLM.ML <- fitSSM(y.LLM, inits=pars, updatefn=y.updatefn, method="BFGS")
```

Kalman Filtering and Smoothing

- given the estimated parameters, we have a fully specified state space model, and can run the filtering and smoothing recursions

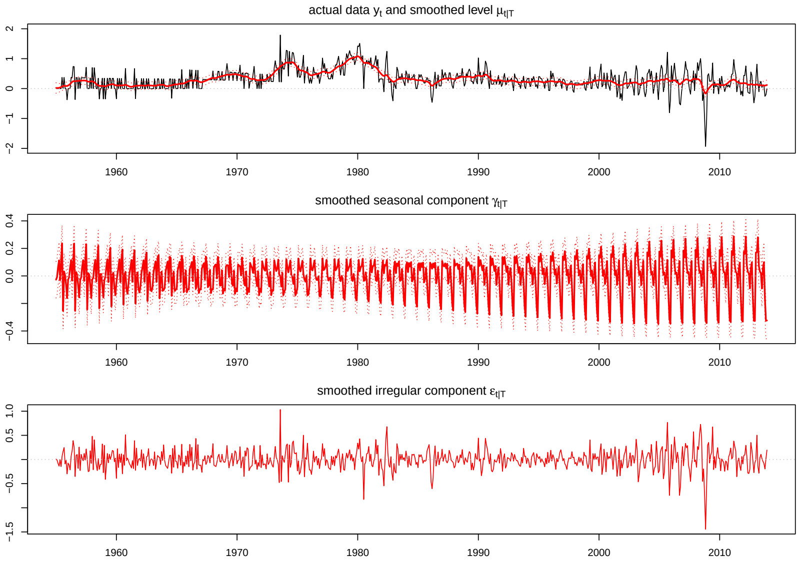
```
# Kalman filtering and smoothing
y.KFS <- KFS(y.LLM.ML$model,filtering=c("state","mean"),smoothing=c("state","mean","disturbance"))
```

- state space model approach allows us to disentangle the three components of CPI inflation:
  - level μ_t capturing the long run tendency in CPI inflation
  - seasonal component γ_t
  - irregular component ε_t capturing short run disturbances

```
# smoothed level and seasonal components + their 90% confidence intervals
y.KS.lvl <- predict(y.KFS$model,states="level",level=0.9,interval="confidence",filtered=FALSE)
y.KS.sea <- predict(y.KFS$model,states="seasonal",level=0.9,interval="confidence",filtered=FALSE)
# smoothed irregular component
y.KS.eps <- residuals(y.KFS,type="response")
```

- in the most recent years of the estimation sample, 2011 to 2013
  - smoothed level component around 0.15
  - smoothed seasonal component lowest in December at -0.35, highest in March at 0.3
  - smoothed irregular component in the -0.5 to 0.5 range

Smoothed Series



Forecast

- forecast for prediction sample 2014M1-2016M12 looks reasonably accurate
- actual inflation after 2014M1 lies in the 90% confidence interval

```
# create forecast
y.fcst <- predict(y.KFS$model, interval="confidence", level=0.9, n.ahead=36)
```