Practice Problems 3

Question 1. Write the equations of a VAR(1) model for two variables, X_t and Y_t.

Question 2. What is Granger causality and how do we test it?

Question 3. What are the impulse-response functions?

Question 4. Explain what spurious regression problem is and give an example.

Question 5. Explain what is means if X_t and Y_t are cointegrated. Give an example.

Question 6. Explain the idea behind error correction model. Draw a diagram illustrating the error correction mechanism.

Question 7. Write the equations of a vector error correction VEC(1) model for two variables, X_t and Y_t.

Question 8. How is cointegration used in pairs trading strategy?

Question 9. Explain what volatility clustering means.

Question 10. Explain the difference between moving average (MA) and exponentially weighted moving average (EWMA) models of the conditional variance.

Question 11. Write the equation for the autoregressive conditional heteroscedasticity ARCH(1) model. Explain the intuition behind this model.

Question 12. Write the equation for the generalized autoregressive conditional heteroscedasticity GARCH(1,1) model. Explain the intuition behind this model.

Question 13. Explain what 1% VaR is and draw a diagram to illustrate this.

Question 14. Why is the Student-t distribution more suitable for ARCH and GARCH models than the normal distribution?

Question 15. Consider a bivariate VAR

$$
\begin{aligned}
y_{1t} &= c_1 + \alpha_{11} y_{1t-1} + \alpha_{12} y_{1t-2} + \beta_{11} y_{2t-1} + \beta_{12} y_{2t-2} + \varepsilon_{1t} \\
y_{2t} &= c_2 + \alpha_{21} y_{1t-1} + \alpha_{22} y_{1t-2} + \beta_{21} y_{2t-1} + \beta_{22} y_{2t-2} + \varepsilon_{2t}
\end{aligned}
$$

where $y_{1,t} = 400\Delta \log GDP_t$ is the growth rate of the U.S. real GDP and $y_{2,t} = 400(\Delta \log SP500_t - \Delta \log p_t^{GDP})$ is the inflation adjusted return of S&P 500. The results of the estimation are shown below.

Are coefficients $\alpha_{11}, \alpha_{12}, \beta_{11}, \beta_{12}$ statistically significant? What does this imply?

Are coefficients $\alpha_{21}, \alpha_{22}, \beta_{21}, \beta_{22}$ statistically significant? What does this imply?

Vector Autoregression Estimates
Date: 05/06/17 Time: 20:10
Sample: 1961Q1 2016Q4
Included observations: 224
Standard errors in ( ) & t-statistics in [ ]

	GRGDP	RRSP500
GRGDP(-1)	0.212529	0.115068
	(0.06396)	(0.17949)
	[3.32266]	[0.64108]
GRGDP(-2)	0.153916	-0.202858
	(0.06236)	(0.17500)
	[2.46802]	[-1.15917]
RRSP500(-1)	0.068634	0.106164
	(0.02401)	(0.06739)
	[2.85804]	[1.57542]
RRSP500(-2)	0.097935	-0.082856
	(0.02446)	(0.06864)
	[4.00359]	[-1.20705]
C	1.799362	1.040329
	(0.28879)	(0.81038)
	[6.23071]	[1.28375]
R-squared	0.234021	0.023754
Adj. R-squared	0.220030	0.005923
Sum sq. resids	1829.631	14407.26
S.E. equation	2.890411	8.110892
F-statistic	16.72712	1.332149
Log likelihood	-553.0672	-784.1925
Akaike AIC	4.982743	7.046361
Schwarz SC	5.058896	7.122514
Mean dependent	3.030036	0.801988
S.D. dependent	3.272810	8.135018
Determinant resid covariance (dof adj.)		534.9741
Determinant resid covariance		511.3579
Log likelihood		-1334.236
Akaike information criterion		12.00211
Schwarz criterion		12.15442

Question 16. Interpret the results of the Granger causality test for a VAR with three variables: $y_{1,t} = 400\Delta \log GDP_t$ is the growth rate of the U.S. real GDP and $y_{2,t} = 400(\Delta \log SP500_t - \Delta \log p_t^{GDP})$ is the inflation adjusted return of S&P 500, and y_{3t} is the Leading Index for the United States.

Discuss what these Granger causality imply about the usefulness of each of the threes variables when it comes to predicting the other ones.

Is there any economic intuition behind these results?

VAR Granger Causality/Block Exogeneity Wald Tests
Date: 05/11/17 Time: 10:58
Sample: 1961Q1 2016Q4
Included observations: 139

Dependent variable: GRGDP

Excluded	Chi-sq	df	Prob.
RRSP500	3.689833	1	0.0547
LI	22.08652	1	0.0000
All	27.70217	2	0.0000

Dependent variable: RRSP500

Excluded	Chi-sq	df	Prob.
GRGDP	0.021518	1	0.8834
LI	0.206983	1	0.6491
All	0.673715	2	0.7140

Dependent variable: LI

Excluded	Chi-sq	df	Prob.
GRGDP	2.487304	1	0.1148
RRSP500	9.320140	1	0.0023
All	12.78463	2	0.0017

Question 17. Interpret the results of the cointegration test for $\log p^{oil}$ and $\log p^{gas}$.

Date: 05/07/17 Time: 20:29
Sample: 1995M01 2010M12
Included observations: 192
Trend assumption: No deterministic trend (restricted constant)
Series: LOG(PGAS) LOG(POIL)
Lags interval (in first differences): 1 to 2

Unrestricted Cointegration Rank Test (Trace)

Hypothesized No. of CE(s)	Eigenvalue	Trace Statistic	0.05 Critical Value	Prob.**
None *	0.146639	32.74225	20.26184	0.0006
At most 1	0.011889	2.296301	9.164546	0.7183

Trace test indicates 1 cointegrating eqn(s) at the 0.05 level
* denotes rejection of the hypothesis at the 0.05 level
**MacKinnon-Haug-Michelis (1999) p-values

Unrestricted Cointegration Rank Test (Maximum Eigenvalue)

Hypothesized No. of CE(s)	Eigenvalue	Max-Eigen Statistic	0.05 Critical Value	Prob.**
None *	0.146639	30.44595	15.89210	0.0001
At most 1	0.011889	2.296301	9.164546	0.7183

Max-eigenvalue test indicates 1 cointegrating eqn(s) at the 0.05 level
* denotes rejection of the hypothesis at the 0.05 level
**MacKinnon-Haug-Michelis (1999) p-values

Question 18. Consider a bivariate VEC

$$\begin{aligned}\Delta \log p_t^{GAS} &= \gamma_1 z_{t-1} + \kappa_{11}\Delta \log p_{t-1}^{GAS} + \kappa_{12}\Delta \log p_{t-2}^{GAS} + \phi_{11}\Delta \log p_{t-1}^{OIL} + \phi_{12}\Delta \log p_{t-2}^{OIL} + \varepsilon_{1,t}\\ \Delta \log p_t^{OIL} &= \gamma_2 z_{t-1} + \kappa_{21}\Delta \log p_{t-1}^{GAS} + \kappa_{22}\Delta \log p_{t-2}^{GAS} + \phi_{21}\Delta \log p_{t-1}^{OIL} + \phi_{22}\Delta \log p_{t-2}^{OIL} + \varepsilon_{2,t}\end{aligned}$$

where $z_{t-1} = \log p_{t-1}^{GAS} - \beta_1 \log p_{t-1}^{OIL} - \beta_0$ is the error terms measuring the deviation in period $t-1$ from the long run equilibrium. The results of the estimation are shown below.

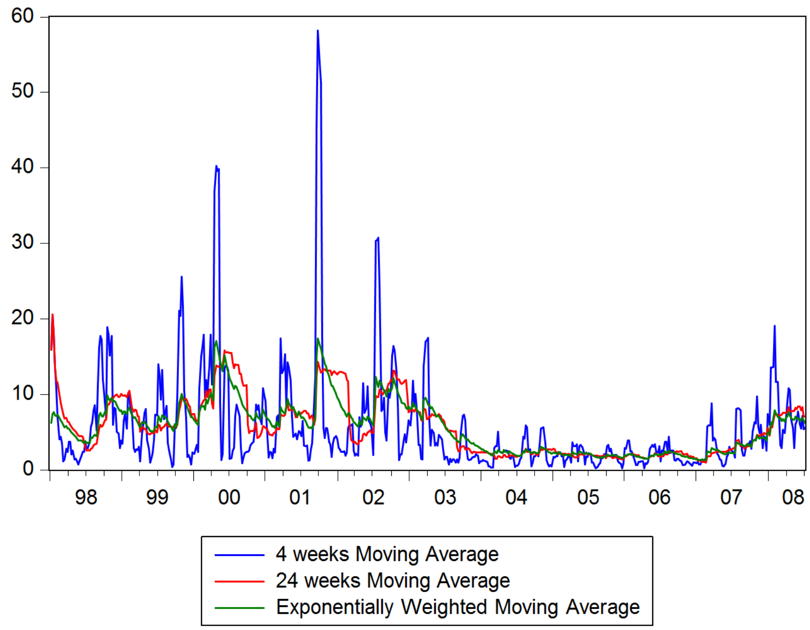
Is the coefficient β_1 statistically significant? Interpret what the estimated value for β_1 means.

Are γ_1 and γ_2 statistically significant? Are the signs of γ_1 and γ_2 in the estimated VEC model consistent with error correction mechanism that moves the system back to the long run equilibrium, whenever there is a disruption and $z_{t-1} \neq 0$?

Vector Error Correction Estimates
Date: 05/07/17 Time: 20:29
Sample: 1995M01 2010M12
Included observations: 192
Standard errors in ( ) & t-statistics in [ ]

Cointegrating Eq:	CointEq1	
LOG(PGAS(-1))	1.000000	
LOG(POIL(-1))	-0.597621	
	(0.01461)	
	[-40.9023]	
C	1.587414	
	(0.05198)	
	[30.5370]	
Error Correction:	D(LOG(PGAS))	D(LOG(POIL))
CointEq1	-0.327074	-0.108005
	(0.07340)	(0.12811)
	[-4.45601]	[-0.84307]
D(LOG(PGAS(-1)))	0.352504	-0.115845
	(0.09747)	(0.17012)
	[3.61644]	[-0.68095]
D(LOG(PGAS(-2)))	-0.127554	-0.032472
	(0.09037)	(0.15772)
	[-1.41151]	[-0.20588]
D(LOG(POIL(-1)))	0.103709	0.201301
	(0.06431)	(0.11223)
	[1.61277]	[1.79359]
D(LOG(POIL(-2)))	0.011155	0.081658
	(0.06267)	(0.10939)
	[0.17799]	[0.74652]
R-squared	0.357266	0.037617
Adj. R-squared	0.343518	0.017032
Sum sq. resids	0.518424	1.579220
S.E. equation	0.052653	0.091897
F-statistic	25.98614	1.827354
Log likelihood	295.3517	188.4180
Akaike AIC	-3.024497	-1.910604
Schwarz SC	-2.939667	-1.825773
Mean dependent	0.005333	0.009102
S.D. dependent	0.064985	0.092690
Determinant resid covariance (dof adj.)		1.10E-05
Determinant resid covariance		1.04E-05
Log likelihood		556.2964
Akaike information criterion		-5.659337
Schwarz criterion		-5.438777

Question 19. Comment on the differences between MA(4), MA(24) and EWMA applied to obtain the 1-week-ahead volatility forecast for the S&P 500 returns.



Question 20. Consider ARCH(9) and GARCHJ(1,1) models for the S&P 500 daily returns. Write the equations for the two estimated models, with estimated parameter values plugged into these equations. Which model would be preferred by Akaike criterion and by Schwarz criterion?

SP500 daily returns—ARCH(9)

Dependent Variable: R
Method: ML - ARCH (BHHH) - Normal distribution
Sample: 5815 8471
Included observations: 2657
Convergence achieved after 16 iterations
Bollerslev-Wooldrige robust standard errors & covariance
Variance backcast: ON
GARCH = C(2) + C(3)*RESID(-1)^2 + C(4)*RESID(-2)^2 + C(5)*RESID(-3)^2 + C(6)*RESID(-4)^2 + C(7)*RESID(-5)^2 + C(8)*RESID(-6)^2 + C(9)*RESID(-7)^2 + C(10)*RESID(-8)^2 + C(11)*RESID(-9)^2

	Coefficient	Std. Error	z-Statistic	Prob.
C	0.037003	0.018214	2.031594	0.0422
Variance Equation				
C	0.271763	0.040891	6.645982	0.0000
RESID(-1)^2	0.029949	0.028081	1.066510	0.2862
RESID(-2)^2	0.149370	0.044623	3.347391	0.0008
RESID(-3)^2	0.095260	0.026377	3.611510	0.0003
RESID(-4)^2	0.101684	0.027620	3.681607	0.0002
RESID(-5)^2	0.082439	0.023397	3.523482	0.0004
RESID(-6)^2	0.060298	0.021251	2.837387	0.0045
RESID(-7)^2	0.090927	0.030511	2.980119	0.0029
RESID(-8)^2	0.142659	0.029601	4.819476	0.0000
RESID(-9)^2	0.082659	0.023815	3.470870	0.0005

R-squared	-0.000565	Mean dependent var	0.009761
Adjusted R-squared	-0.004346	S.D. dependent var	1.146761
S.E. of regression	1.149251	Akaike info criterion	2.910013
Sum squared resid	3494.776	Schwarz criterion	2.934377
Log likelihood	-3854.952	Durbin-Watson stat	2.079077

Dependent Variable: R
Method: ML - ARCH (BHHH) - Normal distribution
Sample: 5815 8471
Included observations: 2657
Convergence achieved after 10 iterations
Bollerslev-Wooldrige robust standard errors & covariance
Variance backcast: ON
GARCH = C(2) + C(3)*RESID(-1)^2 + C(4)*GARCH(-1)

	Coefficient	Std. Error	z-Statistic	Prob.
C	0.036267	0.017439	2.079665	0.0376
Variance Equation				
C	0.010421	0.005245	1.987099	0.0469
RESID(-1)^2	0.065649	0.011338	5.790038	0.0000
GARCH(-1)	0.927400	0.011045	83.96233	0.0000

R-squared	-0.000534	Mean dependent var	0.009761
Adjusted R-squared	-0.001666	S.D. dependent var	1.146761
S.E. of regression	1.147716	Akaike info criterion	2.888638
Sum squared resid	3494.671	Schwarz criterion	2.897498
Log likelihood	-3833.556	Durbin-Watson stat	2.079139

Question 21. Consider a GARCH(1,1) model for daily S&P 500 returns. On April 2, 2008, the 1-day-ahead conditional mean is $\mu_{t|t-1} = 0.036$, the 1-day-ahead conditional standard deviation $\sigma_{t|t-1} = 1.785$. Calculate the 1% VaR and 5% VaR, given that $\Phi^{-1}(0.05) = -1.645$ and $\Phi^{-1}(0.01) = -2.326$. Interpret these numbers, given a portfolio worth 1 million dollars.

Question 22. Consider GARCH(1,1) model for daily S&P 500 returns for the 1/2/1998 to 7/25/2008 sample. With normal innovations, the number of violations $r_t < r_t^{VaR(0.01)}$ is 42 which represents 1.58% of observations. With innovations from Student-t distribution the number of violations $r_t < r_t^{VaR(0.01)}$ is 30 or 1.13% of the sample.

Show where some of these violations can be seen in the figures below. Explain which of these models is more suitable to model volatility of daily S&P 500 returns and why.