Eco 4306 Economic and Business Forecasting

Lecture 29
Chapter 15: Financial Applications of Time Varying Volatility

Motivation

- investors and financial institutions allocate capital among different assets with different amount of risk
- some of the applications of modeling and forecasting the time-varying conditional variance: risk management, portfolio allocation, asset pricing, and option pricing

15.2 Portfolio Allocation

- suppose that we find optimal allocation of financial capital between two risky assets in order to minimize our risk exposure
- the question is how much money we should invest in each asset
- this is the problem of portfolio allocation
- let r_1 denote the return to asset 1 and r_2 the return to asset 2
- portfolio return is a weighted average of both returns $r_p = w_1 r_1 + w_2 r_2$ where w_1 and w_2 are the weights corresponding to asset 1 and 2
- let μ_1, μ_2 and σ_1^2, σ_2^2 be their respective means and variances

15.2 Portfolio Allocation

- to simplify the problem, we assume that both assets are uncorrelated so that their covariance is zero
- under this assumption the mean and variance of this portfolio are

$$\mu_p = w_1\mu_1 + w_2\mu_2$$
$$\sigma_p^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2$$

15.2 Portfolio Allocation

- given that we want to minimize risk exposure, the optimization problem that consists of minimizing the portfolio variance image with respect to the weights w_1 and w_2 subject to a fixed desired portfolio return
- that is, the optimal weights w_1^*, w_2^* are the solution to the following problem

$$\min_{w_1, w_2} w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2$$

subject to

$$\mu_p = w_1 \mu_1 + w_2 \mu_2$$

15.2 Portfolio Allocation

- the optimal weights w_1^*, w_2^* are thus

$$w_1^* = \frac{\mu_1/\sigma_1^2}{\mu_1^2/\sigma_1^2 + \mu_2^2/\sigma_2^2}\mu_p \qquad w_2^* = \frac{\mu_2/\sigma_2^2}{\mu_1^2/\sigma_1^2 + \mu_2^2/\sigma_2^2}\mu_p$$

- optimal weights are proportional to the ratio mean/variance of each asset, the larger the ratio the more capital is allocated to the asset
- ratio mean/variance can be interpreted as a risk-corrected return (i.e., return per unit of variance), which considers the trade-off between profitability and risk

15.2 Portfolio Allocation: Example

- consider two stocks, Apple (AAPL) in the computer industry, and Freeport-McMoRan Copper (FCX) in the mining industry
- sample: daily prices from January 2, 1998, to August 8, 2008, for a total of 2,667 observations
- correlation coefficient of their returns is 0.09, which is practically zero, consistent with our assumptions that the two stocks should be uncorrelated
- daily average return is $\mu_1 = 0.13$ for APPL and $\mu_2 = 0.07$ for FCX

15.2 Portfolio Allocation: Example

- next step: build a model for their conditional means and conditional variances
- for both assets, we estimate a model with GARCH(1,1) process for conditional volatility

$$
\begin{aligned}
r_t &= \mu + \varepsilon_t \\
\varepsilon_t &= \sigma_{t|t-1} z_t \\
\sigma^2_{t|t-1} &= \omega + \alpha_1 \varepsilon^2_{t-1} + \beta_1 \sigma^2_{t-1|t-2}
\end{aligned}
$$

- based on the GARCH model, we then calculate the 1-step-ahead conditional variances $\sigma^2_{t+1|t}$

15.2 Portfolio Allocation: Example

Dependent Variable: R_AAPL_ADJ
Method: ML ARCH - Student's t distribution (BFGS / Marquardt steps)
Date: 05/01/18 Time: 04:10
Sample (adjusted): 1/06/1998 8/08/2008
Included observations: 2665 after adjustments
Convergence achieved after 41 iterations
Coefficient covariance computed using outer product of gradients
Presample variance: backcast (parameter = 0.7)
GARCH = C(3) + C(4)*RESID(-1)^2 + C(5)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.194199	0.048525	4.002057	0.0001
AR(1)	-0.052906	0.019317	-2.738879	0.0062
		Variance Equation		
C	0.401780	0.094963	4.230913	0.0000
RESID(-1)^2	0.083618	0.013466	6.209598	0.0000
GARCH(-1)	0.878571	0.017437	50.38528	0.0000
T-DIST. DOF	5.648556	0.486053	11.62127	0.0000

R-squared	0.002347	Mean dependent var	0.140889
Adjusted R-squared	0.001972	S.D. dependent var	3.454047
S.E. of regression	3.450640	Akaike info criterion	5.010464
Sum squared resid	31708.12	Schwarz criterion	5.023720
Log likelihood	-6670.444	Hannan-Quinn criter.	5.015261
Durbin-Watson stat	1.981548		
Inverted AR Roots	-.05		

15.2 Portfolio Allocation: Example

Dependent Variable: R_FCX_ADJ
Method: ML ARCH - Student's t distribution (BFGS / Marquardt steps)
Date: 05/01/18 Time: 04:10
Sample (adjusted): 1/05/1998 8/08/2008
Included observations: 2666 after adjustments
Convergence achieved after 38 iterations
Coefficient covariance computed using outer product of gradients
Presample variance: backcast (parameter = 0.7)
GARCH = C(2) + C(3)*RESID(-1)^2 + C(4)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.149191	0.051349	2.905425	0.0037
Variance Equation				
C	0.198557	0.068232	2.910031	0.0036
RESID(-1)^2	0.055560	0.010346	5.370078	0.0000
GARCH(-1)	0.924442	0.014194	65.12815	0.0000
T-DIST. DOF	6.948719	0.928432	7.484359	0.0000

R-squared	-0.000581	Mean dependent var	0.076149
Adjusted R-squared	-0.000581	S.D. dependent var	3.030933
S.E. of regression	3.031814	Akaike info criterion	4.950693
Sum squared resid	24496.40	Schwarz criterion	4.961737
Log likelihood	-6594.274	Hannan-Quinn criter.	4.954690
Durbin-Watson stat	2.065892		

15.2 Portfolio Allocation: Example

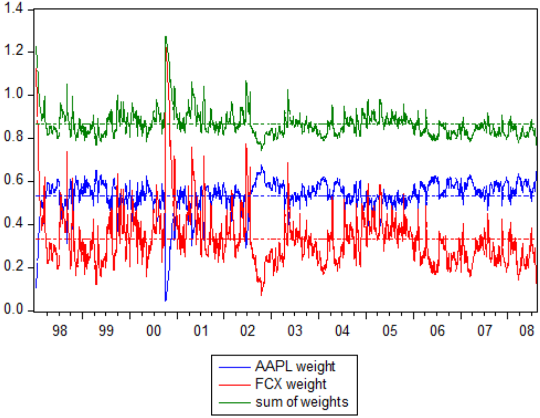
- suppose that our goal is to obtain a daily return of 0.10% so $\mu_p = 0.1$
- note that the desired return is the equal-weighted average of both returns, since $0.10 = 0.5 \times 0.13 + 0.5 \times 0.07$
- thus if we were to put half of the money in AAPL and the other half in FCX, we would obtain an average return of 0.10 over the sample period
- our goal is however to minimize the variance of the portfolio
- using the preceding formulas for optimal weights, we compute the daily weights

15.2 Portfolio Allocation: Example

to calculate the optimal weights for AAPL and FCX in EViews

- first, use the GARCH(1,1) model for AAPL to construct the conditional volatility forecast: click on **Forecast** button, enter **r_aapl_f** into "Forecast name" box and **sigmasq_aapl_f** into "GARCH (optional)" box", change"Method" to "Static forecast"
- use the GARCH(1,1) model for FCX to construct the conditional volatility forecast **sigmasq_fcx_f** in a similar way
- construct w_1^* for AAPL: select **Object** → **Generate Series** and enter **w_aapl = @mean(r_aapl) / sigmasq_aapl_f / (@mean(r_aapl)ˆ2/sigmasq_aapl_f + @mean(r_fcx)ˆ2/sigmasq_fcx_f) *0.1**
- construct w_2^* for FCX: select **Object** → **Generate Series** and enter **w_fcx = @mean(r_fcx) / sigmasq_fcx_f / (@mean(r_aapl)ˆ2/sigmasq_aapl_f + @mean(r_fcx)ˆ2/sigmasq_fcx_f) *0.1**

15.2 Portfolio Allocation: Example



15.2 Portfolio Allocation: Example

Date: 05/01/18 Time: 04:10
Sample: 1/02/1998 8/08/2008

	AAPL weight	FCX weight	sum of weights
Mean	0.532616	0.334275	0.866890
Median	0.544344	0.312719	0.857062
Maximum	0.676902	1.228552	1.274611
Minimum	0.046059	0.069079	0.745982
Std. Dev.	0.074259	0.136485	0.062227
Skewness	-2.368201	2.368201	2.368201
Kurtosis	12.86333	12.86333	12.86333
Jarque-Bera	13293.77	13293.77	13293.77
Probability	0.000000	0.000000	0.000000
Sum	1419.421	890.8416	2310.262
Sum Sq. Dev.	14.69023	49.62566	10.31543
Observations	2665	2665	2665

15.2 Portfolio Allocation: Example

- note that the sum of the weights does not have to equal 1
- in the optimization problem, we did not impose this restriction
- this is why sometimes FCX has a weight larger than one rendering the sum higher than 1
- example 1: on October 2, 2000, FCX weight is 1.36 and AAPL weight is 0.03
- because we do not have the sum restriction, we allow for the possibility of borrowing and lending at the risk-free rate
- thus, for a total weight of 1.39, investor needs to supplement the actual capital by borrowing an additional 39%
- example 2: on August 8, 2008, the FCX weight is 0.12 and APPL weight 0.65 for a sum of 0.77, which means that investor lends 23% of capital
- descriptive summary of the weights shows that on average, weight for FCX is 0.34 and for AAPL 0.54
- so on average 88% of capital is allocated to stocks and 12% to lending

15.3 Asset Pricing

- modern finance theory: expected asset returns is a function of risk
- investors demand a higher return if they are to buy risky assets
- **capital asset pricing model (CAPM)** and the **arbitrage pricing theory (APT)** state that there is a linear relationship between expected returns and risk
- CAPM model defines risk as the covariance of the asset return with the market portfolio return, expected return of an asset is given by

$$E(r_i) = r_f + \frac{cov(r_i, r_m)}{var(r_m)}\big(E(r_m) - r_f\big)$$

  where r_f is the risk-free rate, r_i is the return to asset i, and r_m is the return to the market portfolio
- $\beta = \frac{cov(r_i,r_m)}{var(r_m)}$ is the *beta of an asset*, it captures systematic risk which cannot be diversified away

15.3 Asset Pricing

- β is the expected change in the asset return when a marginal change occurs in the market portfolio return
- $\beta > 1$ classifies the asset as risky because a 1% movement in the market return translates into a change larger than 1% in the asset return
- $\beta < 1$ indicates that the asset return does not fully mimic movements in the market

15.3 Asset Pricing

- arbitrage pricing theory claims that there are more risk factors than the market risk and allows for a richer relationship between other factors and the asset return
- **conditional CAPM** model exploits the information set so that the conditional expected return of the asset is a linear function of its conditional beta

$$E(r_{it}|I_{t-1}) = r_f + \frac{cov(r_{it}, r_{mt}|I_{t-1})}{var(r_{mt}|I_{t-1})}\big(E(r_{mt}|I_{t-1}) - r_f\big)$$

  which makes $\beta_{it} = \frac{cov(r_{it}, r_{mt})}{var(r_{mt})}$ time varying because it is a function of time-varying covariance and variance
- time-varying covariance can be calculated using formula of the correlation coefficient

$$cov(r_{it}, r_{mt}|I_{t-1}) = corr(r_{it}, r_{mt}|I_{t-1})\sqrt{var(r_{it}|I_{t-1})}\sqrt{var(r_{mt}|I_{t-1})}$$

to calculate time varying beta for AAPL in EViews

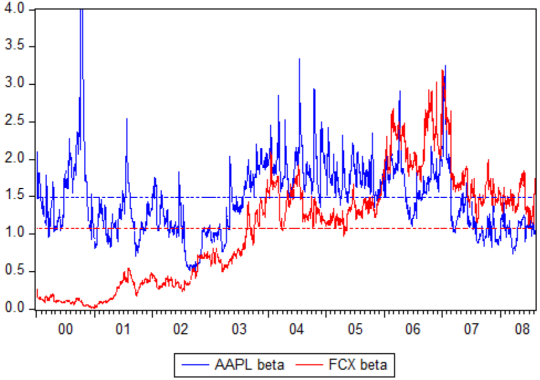
- first construct 200 period moving correlation coefficient select **Object** $\rightarrow$ **Generate Series** and enter **movcorr_aapl_sp500 = @movcor(r_aapl, r_sp500, 200)**
- next, use the GARCH(1,1) model for AAPL to construct the conditional volatility forecast: click on **Forecast** button, enter **r_aapl_f** into "Forecast name" box and **sigmasq_aapl_f** into "GARCH (optional)" box", change"Method" to "Static forecast"
- use the GARCH(1,1) model for S&P500 to construct the conditional volatility forecast **sigmasq_sp500_f** in a similar way
- finally, construct $\beta_{i,t}$ for AAPL by selecting **Object** $\rightarrow$ **Generate Series** and entering **beta_aapl = movcorr_aapl_sp500 * sigmasq_aapl_f^0.5 / sigmasq_sp500_f^0.5**

Dependent Variable: R_SP500_ADJ
Method: ML ARCH - Student's t distribution (BFGS / Marquardt steps)
Date: 05/01/18 Time: 04:10
Sample (adjusted): 1/06/1998 8/08/2008
Included observations: 2665 after adjustments
Convergence achieved after 35 iterations
Coefficient covariance computed using outer product of gradients
Presample variance: backcast (parameter = 0.7)
GARCH = C(3) + C(4)*RESID(-1)^2 + C(5)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.045450	0.016266	2.794136	0.0052
AR(1)	-0.048884	0.021303	-2.294756	0.0217
		Variance Equation		
C	0.006907	0.002768	2.495573	0.0126
RESID(-1)^2	0.066257	0.009329	7.102511	0.0000
GARCH(-1)	0.930759	0.009314	99.93263	0.0000
T-DIST. DOF	9.258718	1.345500	6.881246	0.0000

R-squared	0.000741	Mean dependent var	0.010609
Adjusted R-squared	0.000366	S.D. dependent var	1.150129
S.E. of regression	1.149919	Akaike info criterion	2.869154
Sum squared resid	3521.319	Schwarz criterion	2.882410
Log likelihood	-3817.148	Hannan-Quinn criter.	2.873951
Durbin-Watson stat	1.985377		

Inverted AR Roots	-.05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AAPL beta
FCX beta

15.3 Asset Pricing

- AAPL is riskier than FCX: for AAPL the average beta is 1.48, the average beta of FCX is 1.07
- over time beta for FCX increased significantly, due to an increase in its covariance $cov(r_{it}, r_{mt})$
- AAPL's beta is quite volatile, largest betas occured during the 2001 tech bubble