

Practice Problems 3

Short Questions

1. Write the equations of a VAR(1) model with two variables, X_t and Y_t .

See slide 13 in [lec20slides.pdf](#).

2. What is Granger causality and how do we test it?

See slide 18 in [lec20slides.pdf](#).

3. What are impulse-response functions?

See slides 22, 24, 25 in [lec20slides.pdf](#).

4. Explain what spurious regression problem is and give an example.

See slides 7-9 and 11 in [lec23slides.pdf](#).

5. Explain what it means if X_t and Y_t are cointegrated. Give an example.

See slide 12 in [lec23slides.pdf](#).

6. Explain the idea behind error correction model. Draw a diagram illustrating the error correction mechanism.

See slides 22-26 in [lec23slides.pdf](#).

7. Write the equations of a vector error correction VEC(1) model with two variables, X_t and Y_t .

See slides 27 in [lec23slides.pdf](#).

8. How is cointegration used in pairs trading strategy?

See slides 33-35 in [lec23slides.pdf](#).

9. Explain what volatility clustering means.

See slide 9 in [lec25slides.pdf](#).

10. Explain the difference between moving average (MA) and exponentially weighted moving average (EWMA) models of the conditional variance.

See slide 24, 28 and 31 in [lec25slides.pdf](#).

11. Write the equation for the autoregressive conditional heteroscedasticity ARCH(1) model. Explain the intuition behind this model.

See slide 6 in [lec26slides.pdf](#).

12. What are some weaknesses of ARCH models, and which alternative models have been developed to address them?

See slide 22 in [lec26slides.pdf](#).

13. Write the equation for the generalized autoregressive conditional heteroscedasticity GARCH(1,1) model. Explain the intuition behind this model.

See slide 23 in [lec26slides.pdf](#).

14. Explain the main idea behind TARARCH and PGARCH models.

See slides 36 and 38 in [lec26slides.pdf](#).

15. Why is the Student-t distribution more suitable for GARCH models than the normal distribution?

See slides 11, 13 and 18 in [lec28slides.pdf](#).

16. Explain what 1% VaR is and draw a diagram to illustrate this.

See slides 6, 7 and 10 in [lec28slides.pdf](#).

17. Consider a GARCH(1,1) model for daily S&P 500 returns. On April 2, 2008, the 1-day-ahead conditional mean is $\mu_{t|t-1} = 0.036$, the 1-day-ahead conditional standard deviation $\sigma_{t|t-1} = 1.785$. Calculate the 1% VaR and 5% VaR, given that $\Phi^{-1}(0.05) = -1.645$ and $\Phi^{-1}(0.01) = -2.326$. Interpret these numbers, given a portfolio worth 1 million dollars.

See slides 7 and 11 in [lec28slides.pdf](#).

Question 1. Consider a bivariate VAR

$$\begin{aligned} y_{1t} &= c_1 + \alpha_{11}y_{1t-1} + \alpha_{12}y_{1t-2} + \beta_{11}y_{2t-1} + \beta_{12}y_{2t-2} + \varepsilon_{1t} \\ y_{2t} &= c_2 + \alpha_{21}y_{1t-1} + \alpha_{22}y_{1t-2} + \beta_{21}y_{2t-1} + \beta_{22}y_{2t-2} + \varepsilon_{2t} \end{aligned}$$

where $y_{1,t} = 400\Delta \log GDP_t$ is the growth rate of the U.S. real GDP and $y_{2,t} = 400(\Delta \log SP500_t - \Delta \log p_t^{GDP})$ is the inflation adjusted return of S&P 500. The results of the estimation are shown below.

Are coefficients β_{11}, β_{12} statistically significant? What does this imply?

Are coefficients α_{21}, α_{22} statistically significant? What does this imply?

Vector Autoregression Estimates		
Date: 04/26/18 Time: 16:52		
Sample: 2000Q1 2016Q4		
Included observations: 68		
Standard errors in () & t-statistics in []		
	DLRGDP	DLRSP500
DLRGDP(-1)	0.065707 (0.13856) [0.47422]	0.133711 (0.38951) [0.34328]
DLRGDP(-2)	0.185656 (0.13220) [1.40434]	-0.404508 (0.37164) [-1.08845]
DLRSP500(-1)	0.126714 (0.05048) [2.51016]	0.451046 (0.14191) [3.17844]
DLRSP500(-2)	-0.001633 (0.05241) [-0.03115]	-0.101216 (0.14734) [-0.68698]
C	1.326002 (0.42193) [3.14270]	0.647247 (1.18610) [0.54569]
R-squared	0.228927	0.206599
Adj. R-squared	0.179969	0.156224
Sum sq. resids	313.3873	2476.547
S.E. equation	2.230337	6.269790
F-statistic	4.676069	4.101245
Log likelihood	-148.4375	-218.7217
Akaike AIC	4.512868	6.580049
Schwarz SC	4.676067	6.743248
Mean dependent	1.840825	0.194719
S.D. dependent	2.462949	6.825578

- β_{11} is statistically significant since its t-statistic is 2.51, while β_{12} is not significant since its t-statistic is -0.031

- α_{21} and α_{22} are not statistically significant since their t-statistics are less than 2 in absolute value (0.343 and -1.088)

This implies that current and past real GDP growth rate are not good predictors for future return of S&P 500, but current performance of S&P 500 is a useful predictor for future real GDP growth rate.

Question 2. Interpret the results of the Granger causality test for a VAR with two variables: $y_{1,t} = 400\Delta \log GDP_t$ is the growth rate of the U.S. real GDP and $y_{2,t} = 400(\Delta \log SP500_t - \Delta \log p_t^{GDP})$ is the inflation adjusted return of S&P 500. Discuss what these Granger causality imply about the usefulness of each of the variables when it comes to predicting the other one.

Is there any economic intuition behind these results?

VAR Granger Causality/Block Exogeneity Wald Tests

Date: 04/26/18 Time: 16:57

Sample: 2000Q1 2016Q4

Included observations: 68

Dependent variable: DLRGDP

Excluded	Chi-sq	df	Prob.
DLRSP500	6.679366	2	0.0354
All	6.679366	2	0.0354

Dependent variable: DLRSP500

Excluded	Chi-sq	df	Prob.
DLRGDP	1.201515	2	0.5484
All	1.201515	2	0.5484

The Granger causality tests show that

- we reject the hypothesis that real return of the S&P 500 is not Granger causing real GDP growth rate, since the p-value for the test with $H_0 : \beta_{11} = \beta_{12} = 0$ is 0.0354
- we can not reject the hypothesis that real GDP growth rate is not Granger causing real return of the S&P 500, since the p-value for the test with $H_0 : \alpha_{21} = \alpha_{22} = 0$ is 0.5484

The real returns of the S&P 500 index in the current quarter and the previous quarter are thus useful for predicting next quarter's real GDP growth rate, but real GDP growth in the current quarter and the previous quarter are not useful for predicting next quarter's real return pf the S&P 500 index.

The intuition behind this result is that the financial markets are incorporating news fast, and thus move up or down before the GDP does - they are procyclical but lead the GDP.

Question 3. Interpret the results of the cointegration test for log transformed price of gas $\log p^{gas}$ and log transformed price of oil $\log p^{oil}$.

Date: 05/05/19 Time: 14:53
Sample (adjusted): 1995M04 2010M12
Included observations: 189 after adjustments
Trend assumption: No deterministic trend (restricted constant)
Series: LOG(PGAS) LOG(POIL)
Lags interval (in first differences): 1 to 2

Unrestricted Cointegration Rank Test (Trace)

Hypothesized No. of CE(s)	Eigenvalue	Trace Statistic	0.05 Critical Value	Prob.**
None *	0.148347	32.45985	20.26184	0.0006
At most 1	0.011107	2.110910	9.164546	0.7555

Trace test indicates 1 cointegrating eqn(s) at the 0.05 level

* denotes rejection of the hypothesis at the 0.05 level

**Mackinnon-Haug-Michelis (1999) p-values

Unrestricted Cointegration Rank Test (Maximum Eigenvalue)

Hypothesized No. of CE(s)	Eigenvalue	Max-Eigen Statistic	0.05 Critical Value	Prob.**
None *	0.148347	30.34894	15.89210	0.0001
At most 1	0.011107	2.110910	9.164546	0.7555

Max-eigenvalue test indicates 1 cointegrating eqn(s) at the 0.05 level

* denotes rejection of the hypothesis at the 0.05 level

**Mackinnon-Haug-Michelis (1999) p-values

Johansen's trace and maximum eigenvalue cointegration tests imply that the two series are cointegrated: in both tests we reject H_0 of no cointegration (p-values are 0.0006 and 0.0001) and we can not reject the H_0 of one cointegrating relationship (p-values 0.7555 in both tests).

Question 4. Consider a bivariate VEC for log transformed price of gas $\log p^{gas}$ and log transformed price of oil $\log p^{oil}$.

$$\begin{aligned}\Delta \log p_t^{GAS} &= \gamma_1 z_{t-1} + \kappa_{11} \Delta \log p_{t-1}^{GAS} + \kappa_{12} \Delta \log p_{t-2}^{GAS} + \phi_{11} \Delta \log p_{t-1}^{OIL} + \phi_{12} \Delta \log p_{t-2}^{OIL} + \varepsilon_{1,t} \\ \Delta \log p_t^{OIL} &= \gamma_2 z_{t-1} + \kappa_{21} \Delta \log p_{t-1}^{GAS} + \kappa_{22} \Delta \log p_{t-2}^{GAS} + \phi_{21} \Delta \log p_{t-1}^{OIL} + \phi_{22} \Delta \log p_{t-2}^{OIL} + \varepsilon_{2,t}\end{aligned}$$

where $z_{t-1} = \log p_{t-1}^{GAS} - \beta_1 \log p_{t-1}^{OIL} - \beta_0$ is the error terms measuring the deviation in period $t - 1$ from the long run equilibrium. The results of the estimation are shown below.

Is the coefficient β_1 statistically significant? Interpret what the estimated value for β_1 means.

Are γ_1 and γ_2 statistically significant? Are the signs of γ_1 and γ_2 in the estimated VEC model consistent with error correction mechanism that moves the system back to the long run equilibrium, whenever there is a disruption and $z_{t-1} \neq 0$?

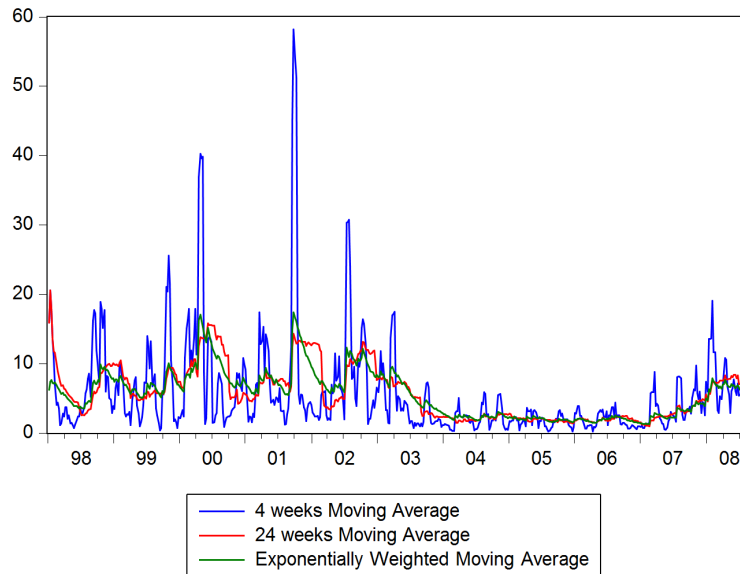
Vector Error Correction Estimates
Date: 05/05/19 Time: 14:53
Sample (adjusted): 1995M04 2010M12
Included observations: 189 after adjustments
Standard errors in () & t-statistics in []

Cointegrating Eq:	CointEq1
LOG(PGAS(-1))	1.000000
LOG(POIL(-1))	-0.596465 (0.01477) [-40.3934]
C	1.582645 (0.05270) [30.0338]
Error Correction:	D(LOG(PGAS)) D(LOG(POIL))
CointEq1	-0.329559 (0.07388) [-4.46050] -0.111602 (0.12907) [-0.86464]
D(LOG(PGAS(-1)))	0.354414 (0.09821) [3.60885] -0.114818 (0.17157) [-0.66924]
D(LOG(PGAS(-2)))	-0.125795 (0.09095) [-1.38320] -0.027945 (0.15888) [-0.17589]
D(LOG(POIL(-1)))	0.104894 (0.06474) [1.62034] 0.201887 (0.11309) [1.78517]
D(LOG(POIL(-2)))	0.008025 (0.06324) [0.12688] 0.079805 (0.11048) [0.72232]

The estimated coefficient $\beta_1 = 0.596$ implies that in the long run a one percent increase in price of oil increases price of gas by 0.596 percent.

The adjustment parameters are $\gamma_1 = -0.329$ and $\gamma_2 = -0.111$. For error correction mechanism to move the system back to the long run equilibrium, whenever there is a disruption and $z_{t-1} \neq 0$ the adjustment parameters need to satisfy $\gamma_1 \leq 0$, $\gamma_2 \geq 0$ and they can not be both equal zero at the same time. In the estimated VEC γ_1 is consistent with the error correction mechanism since it's negative, while γ_2 is not consistent with the error correction mechanism because it's negative. But only γ_1 is statistically significant, γ_2 is not. It thus makes sense to restrict the model and impose $\gamma_2 = 0$ which will make model consistent with the error correction mechanism that moves the system back to the long run equilibrium, whenever there is a disruption and $z_{t-1} \neq 0$.

Question 5. Comment on the differences between MA(4), MA(24) and EWMA applied to obtain the 1-week-ahead volatility forecast for the S&P 500 returns.



See slides 24, 28 and 31 in [lec25slides.pdf](#).

Question 6. Consider ARCH(9) and GARCH(1,1) models for the S&P 500 daily returns. Write the equations for the two estimated models, with estimated parameter values plugged into these equations. Which model would be preferred by Akaike criterion and by Schwarz criterion?

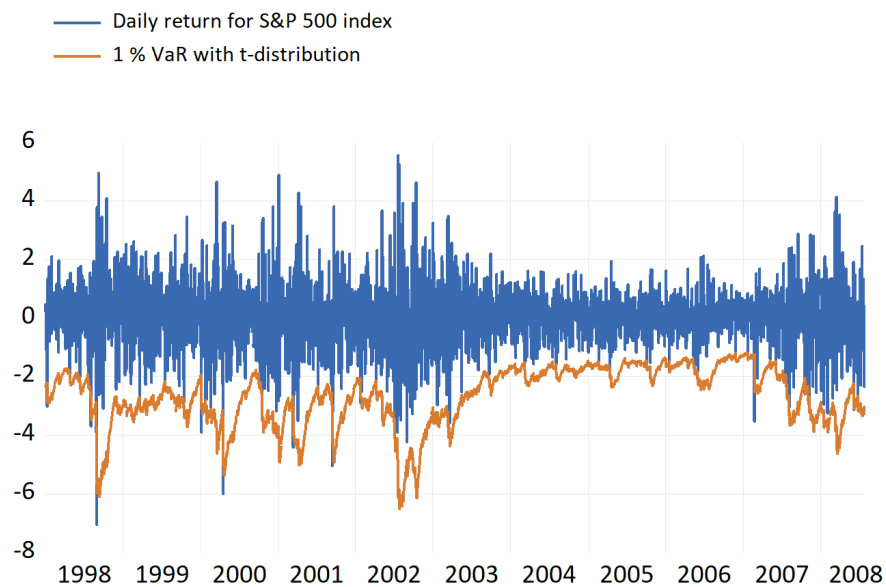
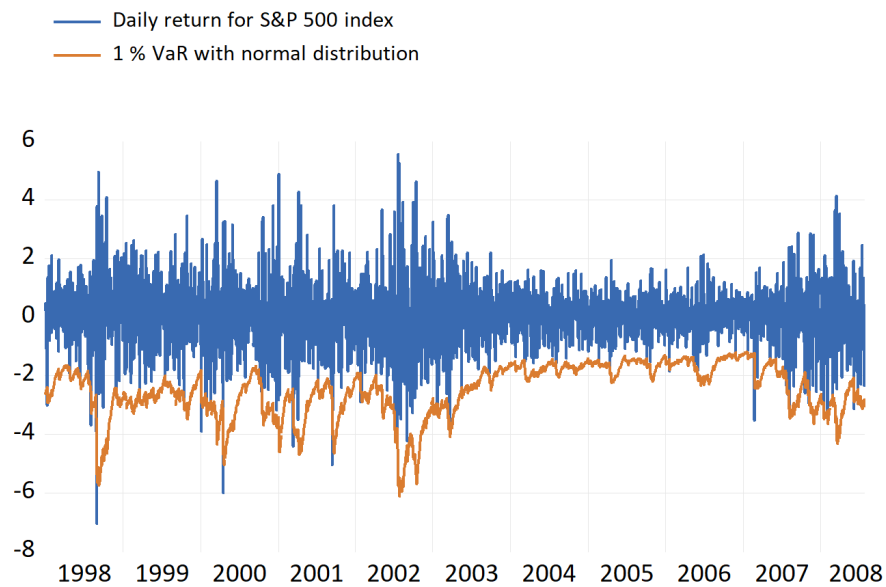
SP500 daily returns—ARCH(9)				
Dependent Variable: R				
Method: ML - ARCH(BHHH) - Normal distribution				
Sample: 5815 8471				
Included observations: 2657				
Convergence achieved after 16 iterations				
Bollerslev-Wooldridge robust standard errors & covariance				
Variance backcast: ON				
GARCH = C(2) + C(3)*RESID(-1)^2 + C(4)*RESID(-2)^2 + C(5)*RESID(-3)^2 + C(6)*RESID(-4)^2 + C(7)*RESID(-5)^2 + C(8)*RESID(-6)^2 + C(9)*RESID(-7)^2 + C(10)*RESID(-8)^2 + C(11)*RESID(-9)^2				
	Coefficient	Std. Error	z-Statistic	Prob.
C	0.037003	0.018214	2.031594	0.0422
Variance Equation				
C	0.271763	0.040891	6.645982	0.0000
RESID(-1)^2	0.029949	0.028081	1.066510	0.2862
RESID(-2)^2	0.149370	0.044623	3.347391	0.0008
RESID(-3)^2	0.095260	0.026377	3.611510	0.0003
RESID(-4)^2	0.101684	0.027620	3.681607	0.0002
RESID(-5)^2	0.082439	0.023397	3.523482	0.0004
RESID(-6)^2	0.060298	0.021251	2.837387	0.0045
RESID(-7)^2	0.090927	0.030511	2.980119	0.0029
RESID(-8)^2	0.142659	0.029601	4.819476	0.0000
RESID(-9)^2	0.082659	0.023815	3.470870	0.0005
R-squared	-0.000565	Mean dependent var	0.009761	
Adjusted R-squared	-0.004346	S.D. dependent var	1.146761	
S.E. of regression	1.149251	Akaike info criterion	2.910013	
Sum squared resid	3494.776	Schwarz criterion	2.934377	
Log likelihood	-3854.952	Durbin-Watson stat	2.079077	

Dependent Variable: R				
Method: ML - ARCH(BHHH) - Normal distribution				
Sample: 5815 8471				
Included observations: 2657				
Convergence achieved after 10 iterations				
Bollerslev-Wooldridge robust standard errors & covariance				
Variance backcast: ON				
GARCH = C(2) + C(3)*RESID(-1)^2 + C(4)*GARCH(-1)				
	Coefficient	Std. Error	z-Statistic	Prob.
C	0.036267	0.017439	2.079665	0.0376
Variance Equation				
C	0.010421	0.005245	1.987099	0.0469
RESID(-1)^2	0.065649	0.011338	5.790038	0.0000
GARCH(-1)	0.927400	0.011045	83.96233	0.0000
R-squared	-0.000534	Mean dependent var	0.009761	
Adjusted R-squared	-0.001666	S.D. dependent var	1.146761	
S.E. of regression	1.147716	Akaike info criterion	2.888638	
Sum squared resid	3494.671	Schwarz criterion	2.897498	
Log likelihood	-3833.556	Durbin-Watson stat	2.079139	

See slides 21 and 32 in [lec26slides.pdf](#).

Question 7. Consider GARCH(1,1) model for daily S&P 500 returns for the 1/2/1998 to 7/25/2008 sample. With normal innovations, the number of violations $r_t < r_t^{VaR(0.01)}$ is 42 which represents 1.58% of observations. With innovations from Student-t distribution the number of violations $r_t < r_t^{VaR(0.01)}$ is 30 or 1.13% of the sample.

Show where some of these violations can be seen in the figures below. Explain which of these models is more suitable to model volatility of daily S&P 500 returns and why.



See slides 11, 13 and 18 in [lec28slides.pdf](#).