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CSE13S

Assignment 2 Design Document

For Asin, I will use a taylor series. I want to be able to keep track of the current term and multiply it by some factor to get the next term. To find this factor, I simply find the ratio of the n_{th} term to the $(n-1)$th term.

$$a_n = k\,a_{n-1} \Rightarrow k = \frac{a_n}{a_{n-1}}$$

For Asin(x)

$$a_n = \frac{(2n)!}{2^{2n}(n!)^2} \cdot \frac{x^{2n+1}}{2n+1}$$

Therefore,

$$k = \frac{a_n}{a_{n-1}} = \left[\frac{(2n)!}{2^{2n}(n!)^2} \cdot \frac{x^{2n+1}}{2n+1}\right]\left[\frac{2^{2n-2}((n-1)!)^2 \cdot (2n-1)}{(2n-2)! \cdot x^{2n-1}}\right]$$

$$= (2n)(2n-1) \cdot x^2 \cdot 2^{-2} \cdot n^{-2} \cdot \left[\frac{2n-1}{2n+1}\right]$$

$$= \frac{x^2(2n-1)^2}{2n(2n+1)}$$

Now that we have k, the implementation is trivial.

```
Asin(x)
  term = sum = a0
  while(desired_accuracy_not_met)
      term = term · k
      sum = sum + term
  return sum
```

I will find Acos using trig identities, namely

$$\cos^{-1}(x) = \frac{\pi}{2} - \sin^{-1}(x)$$

the implementation is trivial.

I will use the taylor series for Atan. This will require me to find K again.

$$K = \frac{a_n}{a_{n-1}} = \left[\frac{2^{2n}(n!)^2}{(2n+1)!} \cdot \frac{x^{2n+1}}{(1+x^2)^{n+1}}\right]\left[\frac{(2n-1)!\,(1+x^2)^n}{2^{2n-2}\,(n-1)!^2\,x^{2n-2}}\right]$$

$$= \frac{2^2 n^2 x^2}{(2n+1)(2n)\cdot(1+x^2)} = \frac{2nx^2}{(2n+1)(1+x^2)}$$

Atan will be identical to Asin, with this K instead.

Log will be different as the taylor series for log has a small radius of convergence. From the pdf we have:

$$y_{k+1} = y_k + \frac{x - \text{Exp}(y_k)}{\text{Exp}(y_k)}$$

Therefore,

```
Log(x)
  guess = 1
  while (desired_accuracy_not_met)
      guess = guess + (x - Exp(guess))/Exp(guess)
  return guess
```