

UPLB Eliens ICPC Notebook (C++)

Contents

1 Geometry	1
1.1 Convex Hull	1
1.2 Point inside polygon	1
1.3 Welzian algo	1
2 Graphs	2
2.1 Articulation and Bridge points	2
2.2 Dijkstra	2
2.3 LCA	3
3 Flows	3
3.1 Bipartite	3
3.2 Max-Flow	4
3.3 MCMF	5
4 Tree	7
4.1 BIT	7
4.2 Segment Tree	7
4.3 Lazy-Segment Tree	8
4.4 Policy Tree	10
4.5 Trie	10
5 Math	10
5.1 CRT	10
5.2 DigitDP	10
5.3 DP DNC	11
5.4 Euclidean	11
5.5 Factors in $n-1-3$	12
5.6 Fibo logn	13
5.7 EGaussian Algorithm	13
5.8 Lucas theorem	13
5.9 Matrix expo	13
5.10 Miller-Rabin	14
5.11 Mobius	14
5.12 SQRT CBRT tourist	14
5.13 Euler totient	15
5.14 FFT	15
5.15 FFT-DNC	15
6 Strings	17
6.1 Knuth-Morris-Pratt Algorithm	17
6.2 Suffix array	17
6.3 z-function	18
7 EZPZ	18
7.1 Template	18
7.2 fast io	18
7.3 LIS $n \log n$	18
7.4 MOs	19

1 Geometry

1.1 Convex Hull

```
//convex hull
typedef pair<ll,ll> point;
ll cross (point a, point b, point c) { return (b.x - a.x
) * (c.y - a.y) - (b.y - a.y) * (c.x - a.x);}
```

```
vector<point> ConvexHull(vector<point>&p, ll n) {
    ll sz = 0;
    vector<point> hull(n + n);
    sort(p.begin(), p.end());
    for(ll i = 0; i < n; ++i) {
        while (sz > 1 and cross(hull[sz - 2], hull[sz -
            1], p[i]) <= 0) --sz;
        hull[sz++] = p[i];
    }
    for(ll i = n - 2, j = sz + 1; i >= 0; --i) {
        while (sz >= j and cross(hull[sz - 2], hull[sz -
            1], p[i]) <= 0) --sz;
        hull[sz++] = p[i];
    } hull.resize(sz - 1);
    return hull;
}
```

1.2 Point inside polygon

```
//points inside convex polygon  $O(\log n)$ 
const ll N = 100009;
struct point {
    ll x,y;
}a[N];
ll n;
double cross(const point& p1,const point& p2,const point
& org) {
    return ((p1.x-org.x)*1.0)*(p2.y-org.y)-((p2.x-org.x)
        *1.0)*(p1.y-org.y);
}
inline bool comp(const point& x,const point& y) {
    return cross(x,y,a[0]) >= 0;
}
bool inside(point& p) {
    if (cross(a[0], a[n-1], p)>=0) return false;
    if (cross(a[0], a[1], p) <=0) return false;
    ll l =1;
    ll r =n-1;
    while(l<r) {
        ll m = l + (r-1)/2;
        if(cross(a[m],p,a[0]) >=0)
            l=m+1;
        else
            r=m;
    }
    if(l == 0)
        return false;
    return cross(a[l-1],a[l],p) >0;
}
sort(a+1,a+n,comp);
```

1.3 Welzian algo

```
//welzian algo
struct point {
    long double x;
    long double y;
};
struct circle {
    long double x;
    long double y;
    long double r;
    circle() {}
    circle(long double x, long double y, long double r):
        x(x), y(y), r(r) {}
};
circle b_md(vector<point> R) {
    if (R.size() == 0) {
        return circle(0, 0, -1);
    } else if (R.size() == 1) {
        return circle(R[0].x, R[0].y, 0);
    } else if (R.size() == 2) {
        return circle((R[0].x+R[1].x)/2.0, (R[0].y+R[1].y)/2.0, hypot(R[0].x-R[1].x, R[0].y-R[1].y)/2.0);
    } else {
        long double D = (R[0].x - R[2].x)*(R[1].y - R[2].y) - (R[1].x - R[2].x)*(R[0].y - R[2].y);
        ;
        long double p0 = (((R[0].x - R[2].x)*(R[0].x + R[2].x) + (R[0].y - R[2].y)*(R[0].y + R[2].y)) / 2 * (R[1].y - R[2].y) - ((R[1].x - R[2].x)*(R[1].x + R[2].x) + (R[1].y - R[2].y)*(R[1].y + R[2].y)) / 2 * (R[0].y - R[2].y))/D;
        long double p1 = (((R[1].x - R[2].x)*(R[1].x + R[2].x) + (R[1].y - R[2].y)*(R[1].y + R[2].y)) / 2 * (R[0].x - R[2].x) - ((R[0].x - R[2].x)*(R[0].x + R[2].x) + (R[0].y - R[2].y)*(R[0].y + R[2].y)) / 2 * (R[1].x - R[2].x))/D;
        return circle(p0, p1, hypot(R[0].x - p0, R[0].y - p1));
    }
}
circle b_minidisk(vector<point>& P, int i, vector<point> R) {
    if (i == P.size() || R.size() == 3) {
        return b_md(R);
    } else {
        circle D = b_minidisk(P, i+1, R);
        if (hypot(P[i].x-D.x, P[i].y-D.y) > D.r) {
```

```
            R.push_back(P[i]);
            D = b_minidisk(P, i+1, R);
        }
        return D;
    }
}
// Call this function.
circle minidisk(vector<point> P) {
    random_shuffle(P.begin(), P.end());
    return b_minidisk(P, 0, vector<point>());
}
```

2 Graphs

2.1 Articulation and Bridge points

```
vector<ll>v[100003];
ll disc[100003];
ll low[100003];
ll vis[100003];
set<ll>ap;
set<pair<ll,ll>>br;
ll par[100003];

void dfs(ll p, ll t){
    vis[p]=1;
    disc[p]=t+1;
    low[p]=t+1;
    ll ch=0;
    for(auto e:v[p]){
        if(vis[e]==0){
            ch++;
            par[e]=p;
            dfs(e,t+1);
            low[p]=min(low[p],low[e]);
            if(low[e]>disc[p]){
                br.insert(mp(min(e,p),max(e,p)));
            }
            if(par[p]==0 && ch>1){
                ap.insert(p);
            }else if(par[p]!=0){
                if(low[e]>=disc[p]){
                    ap.insert(p);
                }
            }
        }else if(e!=par[p]){
            low[p]=min(low[p],disc[e]);
        }
    }
}
```

```

}

int main()
{
    par[1]=0;
    dfs(1,0);
}

```

2.2 Dijkstra

```

vector<pair<ll,ll> >v[100003];
ll dist[100003];
int main(){
    ios::sync_with_stdio(0);
    ll n,e,a[1000003];
    cin>>n>>e;
    for(ll i=0;i<e;i++){
        ll p,q,w;
        cin>>p>>q>>w;
        v[p].pb(mp(w,q));
        v[q].pb(mp(w,p));
    }
    for(ll i=0;i<=n;i++){
        dist[i]=100000;
    }
    ll so;
    cin>>so;
    set<pair<ll,ll> >s;
    dist[so]=0;
    s.insert(mp(0,so));
    while(!s.empty()){
        pair<ll,ll>p=(s.begin());
        s.erase(p);
        for(ll i=0;i<v[p.se].size();i++){
            if(dist[v[p.se][i].se]>dist[p.se]+v[p.se][i].fi){
                if(dist[v[p.se][i].se]!=100000){
                    s.erase(v[p.se][i]);
                }
                dist[v[p.se][i].se]=dist[p.se]+v[p.se][i].fi;
                s.insert(mp(dist[v[p.se][i].se],v[p.se][i].se));
            }
        }
    }
}

```

2.3 LCA

```

int parent[MAXN], depth[MAXN], f[MAXN][LOGN + 1];
vector<int> adj[MAXN];
void dfs(int u) {
    if (u != 1) {
        f[u][0] = parent[u];
        for (int i = 1; i <= LOGN; i++)
            f[u][i] = f[f[u][i - 1]][i - 1];
    }
    for (int i = 0; i < (int) adj[u].size(); i++) {
        int v = adj[u][i];
        if (parent[v] == 0) {
            parent[v] = u;
            depth[v] = depth[u] + 1; dfs(v);
        }
    }
}

int lca(int u, int v) {
    if (depth[u] < depth[v]) swap(u, v);
    for (int i = LOGN; i >= 0; i--)
        if (depth[f[u][i]] >= depth[v]) u = f[u][i];
    if (u == v) return v;
    for (int i = LOGN; i >= 0; i--)
        if (f[u][i] != f[v][i])
            u = f[u][i], v = f[v][i];
    return f[u][0];
}

```

3 Flows

3.1 Bipartite

```

//Hopcroft_Karp O(EsqrtV)
struct edge
{
    int from, to, cap, flow, index;
    edge(int from, int to, int cap, int flow, int index):
        from(from), to(to), cap(cap), flow(flow), index(index) {}
};

struct Hopcroft_Karp
{
    static const int inf = 1e9;

    int n;

```

```

vector<int> matchL, matchR, dist;
vector<vector<int>> > g;

Hopcroft_Karp(int n) :
    n(n), matchL(n+1), matchR(n+1), dist(n+1), g(n+1) {}

void addEdge(int u, int v)
{
    g[u].push_back(v);
}

bool bfs()
{
    queue<int> q;
    for(int u=1;u<=n;u++)
    {
        if(!matchL[u])
        {
            dist[u]=0;
            q.push(u);
        }
        else
            dist[u]=inf;
    }
    dist[0]=inf;

    while(!q.empty())
    {
        int u=q.front();
        q.pop();
        for(auto v:g[u])
        {
            if(dist[matchR[v]] == inf)
            {
                dist[matchR[v]] = dist[u] + 1;
                q.push(matchR[v]);
            }
        }
    }

    return (dist[0]!=inf);
}

bool dfs(int u)
{
    if(!u)
        return true;
    for(auto v:g[u])
    {

```

```

        if(dist[matchR[v]] == dist[u]+1 &&dfs(matchR[v]))
        {
            matchL[u]=v;
            matchR[v]=u;
            return true;
        }
    }
    dist[u]=inf;
    return false;
}

int max_matching()
{
    int matching=0;
    while(bfs())
    {
        for(int u=1;u<=n;u++)
        {
            if(!matchL[u])
                if(dfs(u))
                    matching++;
        }
    }
    return matching;
};

int main() {
    Hopcroft_Karp mx(n+m+3);
    mx.addEdge(q,r);
    cout<<mx.max_matching()<<"\n";
}

```

3.2 Max-Flow

```

//Push relabel  $V^2\sqrt{E}$ 
struct edge
{
    int from, to, cap, flow, index;
    edge(int from, int to, int cap, int flow, int index):
        from(from), to(to), cap(cap), flow(flow), index(index) {}
};

struct PushRelabel
{
    static const long long INF=1e18;

    int n;

```

```

vector<vector<edge> > g;
vector<long long> excess;
vector<int> height;

PushRelabel(int n):
    n(n), g(n), excess(n), height(n) {}

void addEdge(int from, int to, int cap)
{
    g[from].push_back(edge(from, to, cap, 0, g[to].
        size()));
    if(from==to)
        g[from].back().index++;
    g[to].push_back(edge(to, from, 0, 0, g[from].
        size()-1));
}

void push(edge &e)
{
    int amt=(int)min(excess[e.from], (long long)e.cap -
        e.flow);
    if(height[e.from]<=height[e.to] || amt==0)
        return;
    e.flow += amt;
    g[e.to][e.index].flow -= amt;
    excess[e.to] += amt;
    excess[e.from] -= amt;
}

void relabel(int u)
{
    int d=2e5;
    for(auto &it:g[u])
    {
        if(it.cap-it.flow>0)
            d=min(d, height[it.to]);
    }
    if(d<INF)
        height[u]=d+1;
}

vector<int> find_max_height_vertices(int source, int
    dest)
{
    vector<int> max_height;
    for(int i=0;i<n;i++)
    {
        if(i!=source && i!=dest && excess[i]>0)
        {
            if(!max_height.empty() && height[i] > height[

```

```

                max_height[0]])
                max_height.clear();
            if(max_height.empty() || height[i] == height[
                max_height[0]])
                max_height.push_back(i);
        }
    }
    return max_height;
}

long long max_flow(int source, int dest)
{
    excess.assign(n, 0);
    height.assign(n, 0);
    height[source]=n;
    excess[source]=INF;
    for(auto &it:g[source])
        push(it);

    vector<int> current;
    while(!(current = find_max_height_vertices(
        source, dest)).empty())
    {
        for(auto i:current)
        {
            bool pushed=false;
            for(auto &e:g[i])
            {
                if(excess[i]==0)
                    break;
                if(e.cap - e.flow>0 && height[e.from] ==
                    height[e.to] + 1)
                {
                    push(e);
                    pushed=true;
                }
            }
            if(!pushed)
            {
                relabel(i);
                break;
            }
        }
    }

    long long max_flow=0;
    for(auto &e:g[source])
        max_flow+=e.flow;

    return max_flow;
}

```

```

}
};

// vector<ll>v[100003];

int main() {
    bolt;
    ll n;
    cin>>n;
    map<char, ll>m;
    m['A']=0;
    m['Z']=1;
    ll ind=2;
    // ll gr[100][100]={0};
    PushRelabel mx(1000);
    forr(i, 0, n) {
        char a, b;
        ll len;
        cin>>a>>b>>len;
        if(m.count(a)==0) {
            m[a]=ind++;
        }
        if(m.count(b)==0) {
            m[b]=ind++;
        }
        // gr[m[a]][m[b]]=len;
        // gr[m[b]][m[a]]=len;
        mx.addEdge(m[a], m[b], len);
    }
    cout<<mx.max_flow(0, 1)<<"\n";
}

```

3.3 MCMF

```

//Works for negative costs, but does not work for
//negative cycles
//Complexity:  $O(\min(E^2 * V \log V, E \log V * \text{flow}))$ 
struct edge
{
    int to, flow, cap, cost, rev;
};

struct MinCostMaxFlow
{
    int nodes;
    vector<int> prio, curflow, prevedge, prevnode, q, pot;
    vector<bool> inqueue;
    vector<vector<edge>> graph;
    MinCostMaxFlow() {}
}

```

```

MinCostMaxFlow(int n): nodes(n), prio(n, 0), curflow(n,
0),
prevedge(n, 0), prevnode(n, 0), q(n, 0), pot(n, 0),
inqueue(n, 0), graph(n) {}

void addEdge(int source, int to, int capacity, int cost)
{
    edge a = {to, 0, capacity, cost, (int)graph[to].
size()};
    edge b = {source, 0, 0, -cost, (int)graph[source
].size()};
    graph[source].push_back(a);
    graph[to].push_back(b);
}

void bellman_ford(int source, vector<int> &dist)
{
    fill(dist.begin(), dist.end(), INT_MAX);
    dist[source] = 0;
    int qt=0;
    q[qt++] = source;
    for(int qh=0; (qh-qt)%nodes!=0; qh++)
    {
        int u = q[qh%nodes];
        inqueue[u] = false;
        for(auto &e : graph[u])
        {
            if(e.flow >= e.cap)
                continue;
            int v = e.to;
            int newDist = dist[u] + e.cost;
            if(dist[v] > newDist)
            {
                dist[v] = newDist;
                if(!inqueue[v])
                {
                    inqueue[v] = true;
                    q[qt++ % nodes] = v;
                }
            }
        }
    }
}

pair<int, int> minCostFlow(int source, int dest, int
maxflow)
{
    bellman_ford(source, pot);
    int flow = 0;
    int flow_cost = 0;
}

```

```

while(flow < maxflow)
{
    priority_queue<pair<int, int>, vector<pair<int,
        int> >, greater<pair<int, int> > > q;
    q.push({0, source});
    fill(prio.begin(), prio.end(), INT_MAX);
    prio[source] = 0;
    curflow[source] = INT_MAX;
    while(!q.empty())
    {
        int d = q.top().first;
        int u = q.top().second;
        q.pop();
        if(d != prio[u])
            continue;
        for(int i=0; i<graph[u].size(); i++)
        {
            edge &e=graph[u][i];
            int v = e.to;
            if(e.flow >= e.cap)
                continue;
            int newPrio = prio[u] + e.cost + pot[u]
                - pot[v];
            if(prio[v] > newPrio)
            {
                prio[v] = newPrio;
                q.push({newPrio, v});
                prevnode[v] = u;
                prevedge[v] = i;
                curflow[v] = min(curflow[u], e.
                    cap - e.flow);
            }
        }
    }
    if(prio[dest] == INT_MAX)
        break;
    for(int i=0; i<nodes; i++)
        pot[i] += prio[i];
    int df = min(curflow[dest], maxflow - flow);
    flow += df;
    for(int v=dest; v!=source; v=prevnode[v])
    {
        edge &e = graph[prevnode[v]][prevedge[v]
            ];
        e.flow += df;
        graph[v][e.rev].flow -= df;
        flow_cost += df * e.cost;
    }
}
return {flow, flow_cost};

```

```

}
};

```

4 Tree

4.1 BIT

1D BIT:

```

int bit[N];

void update(int idx, int val)
{
    while(idx<=n)
    {
        bit[idx] += val;
        idx += idx & -idx;
    }
}

int pref(int idx)
{
    int ans=0;
    while(idx>0)
    {
        ans += bit[idx];
        idx -= idx & -idx;
    }
    return ans;
}

int rsum(int l, int r)
{
    return pref(r) - pref(l-1);
}

Multiple BIT:

int bit[2][N];

void update(int i, int idx, int k)
{
    while(idx<=n)
    {
        bit[i][idx] += k;
        idx += idx & -idx;
    }
}

```

```

int pref(int i, int idx)
{
    int ans=0;
    while(idx>0)
    {
        ans+=bit[i][idx];
        idx-=idx&-idx;
    }
    return ans;
}

int rsum(int i, int l, int r)
{
    return pref(i, r) - pref(i, l-1);
}

```

4.2 Segment Tree

```

void build(ll node, ll a, ll b) { //1, 0, n-1
    if(a>b)
        return;
    if(a==b) {
        tree[node]=arr[a]; //something
        return;
    }
    build(node*2, a, (a+b)/2);
    build(node*2+1, 1+(a+b)/2, b);
    tree[node] = tree[node*2]+tree[node*2+1] //something
}

ll query(ll node, ll a, ll b, ll i, ll j) { //a=0, b=n-1, i=
l, j=r
    if(a > b || a > j || b < i)
        return 0;
    if(a >= i && b <= j) {
        return 0; //something
    }
    ll q1 = query(node*2, a, (a+b)/2, i, j);
    ll q2 = query(1+node*2, 1+(a+b)/2, b, i, j);
    return 0; //something
}

ll update(ll node, ll a, ll b, ll i, ll val) {
    if(a==b) {
        arr[i]=val;
        tree[node]; //something
    }
    else {
        ll mid=(a+b)/2;
        if(a<=i && i<=mid) {

```

```

        update(2*node, a, mid, i, val);
    }
    else {
        update(2*node+1, mid+1, b, i, val);
    }
    tree[node]=(tree[2*node]+tree[2*node+1])%mod; //
    something
}
}

```

4.3 Lazy-Segment Tree

```

int tree[MAX] = {0}; // To store segment tree
int lazy[MAX] = {0}; // To store pending updates

/* si -> index of current node in segment tree
   ss and se -> Starting and ending indexes of elements
   for
       which current nodes stores sum.
   us and ue -> starting and ending indexes of update
   query
   diff -> which we need to add in the range us to ue
   */
void updateRangeUtil(int si, int ss, int se, int us,
                    int ue, int diff)
{
    // If lazy value is non-zero for current node of
    // segment
    // tree, then there are some pending updates. So we
    // need
    // to make sure that the pending updates are done
    // before
    // making new updates. Because this value may be
    // used by
    // parent after recursive calls (See last line of
    // this
    // function)
    if (lazy[si] != 0)
    {
        // Make pending updates using value stored in
        // lazy
        // nodes
        tree[si] += (se-ss+1)*lazy[si];

        // checking if it is not leaf node because if
        // it is leaf node then we cannot go further
        if (ss != se)
        {
            // We can postpone updating children we don't
            t

```



```

        // need their new values now.
        // Since we are not yet updating children of
        // si,
        // we need to set lazy flags for the
        // children
        lazy[si*2 + 1] += lazy[si];
        lazy[si*2 + 2] += lazy[si];
    }

    // Set the lazy value for current node as 0 as
    // it
    // has been updated
    lazy[si] = 0;
}

// out of range
if (ss>se || ss>ue || se<us)
    return ;

// Current segment is fully in range
if (ss>=us && se<=ue)
{
    // Add the difference to current node
    tree[si] += (se-ss+1)*diff;

    // same logic for checking leaf node or not
    if (ss != se)
    {
        // This is where we store values in lazy
        // nodes,
        // rather than updating the segment tree
        // itself
        // Since we don't need these updated values
        // now
        // we postpone updates by storing values in
        // lazy[]
        lazy[si*2 + 1] += diff;
        lazy[si*2 + 2] += diff;
    }
    return;
}

// If not completely in rang, but overlaps, recur
// for
// children,
int mid = (ss+se)/2;
updateRangeUtil(si*2+1, ss, mid, us, ue, diff);
updateRangeUtil(si*2+2, mid+1, se, us, ue, diff);

// And use the result of children calls to update

```

```

        this
        // node
        tree[si] = tree[si*2+1] + tree[si*2+2];
    }

    // Function to update a range of values in segment
    // tree
    /* us and eu -> starting and ending indexes of update
    query
    ue -> ending index of update query
    diff -> which we need to add in the range us to ue
    */
void updateRange(int n, int us, int ue, int diff)
{
    updateRangeUtil(0, 0, n-1, us, ue, diff);
}

/* A recursive function to get the sum of values in
given
range of the array. The following are parameters for
this function.
si --> Index of current node in the segment tree.
Initially 0 is passed as root is always at'
index 0
ss & se --> Starting and ending indexes of the
segment represented by current node,
i.e., tree[si]
qs & qe --> Starting and ending indexes of query
range */
int getSumUtil(int ss, int se, int qs, int qe, int si)
{
    // If lazy flag is set for current node of segment
    // tree,
    // then there are some pending updates. So we need
    // to
    // make sure that the pending updates are done
    // before
    // processing the sub sum query
    if (lazy[si] != 0)
    {
        // Make pending updates to this node. Note that
        // this
        // node represents sum of elements in arr[ss..se
        // ] and
        // all these elements must be increased by lazy[
        // si]
        tree[si] += (se-ss+1)*lazy[si];

        // checking if it is not leaf node because if
    }
}

```

```

// it is leaf node then we cannot go further
if (ss != se)
{
    // Since we are not yet updating children of
    // si,
    // we need to set lazy values for the
    // children
    lazy[si*2+1] += lazy[si];
    lazy[si*2+2] += lazy[si];
}

// unset the lazy value for current node as it
// has
// been updated
lazy[si] = 0;
}

// Out of range
if (ss>se || ss>qe || se<qs)
    return 0;

// At this point we are sure that pending lazy
// updates
// are done for current node. So we can return value
// (same as it was for query in our previous post)

// If this segment lies in range
if (ss>=qs && se<=qe)
    return tree[si];

// If a part of this segment overlaps with the given
// range
int mid = (ss + se)/2;
return getSumUtil(ss, mid, qs, qe, 2*si+1) +
       getSumUtil(mid+1, se, qs, qe, 2*si+2);
}

```

4.4 Policy Tree

```

// policy tree (for o(1) dist in set)
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
typedef tree<int, null_type, less<int>, rb_tree_tag,
            tree_order_statistics_node_update> ordered_set;
xx=(m[v].order_of_key(r+1))-(m[v].order_of_key(l)); //
dist bw with l and r

```

4.5 Trie

```

struct TrieNode
{
    struct TrieNode *children[ALPHABET_SIZE];
    // isEndOfWord is true if the node represents
    // end of a word
    bool isEndOfWord;
};

// Returns new trie node (initialized to NULLs)
struct TrieNode *getNode(void)
{
    struct TrieNode *pNode = new TrieNode;
    pNode->isEndOfWord = false;
    for (int i = 0; i < ALPHABET_SIZE; i++)
        pNode->children[i] = NULL;
    return pNode;
}

// If not present, inserts key into trie
// If the key is prefix of trie node, just
// marks leaf node
void insert(struct TrieNode *root, string key)
{
    struct TrieNode *pCrawl = root;
    for (int i = 0; i < key.length(); i++)
    {
        int index = key[i] - 'a';
        if (!pCrawl->children[index])
            pCrawl->children[index] = getNode();
        pCrawl = pCrawl->children[index];
    }
    // mark last node as leaf
    pCrawl->isEndOfWord = true;
}

// Returns true if key presents in trie, else
// false
bool search(struct TrieNode *root, string key)
{
    struct TrieNode *pCrawl = root;
    for (int i = 0; i < key.length(); i++)
    {
        int index = key[i] - 'a';
        if (!pCrawl->children[index])
            return false;
        pCrawl = pCrawl->children[index];
    }
    return (pCrawl != NULL && pCrawl->isEndOfWord);
}

```

```
// Driver
int main()
{
    string keys[] = {"the", "a", "there",
                    "answer", "any", "by",
                    "bye", "their" };
    int n = sizeof(keys)/sizeof(keys[0]);
    struct TrieNode *root = getNode();
}
```

5 Math

5.1 CRT

```
//crt
ll crt(vector<ll> v, vector<ll> rem){
    int n=v.size();
    ll M=1;
    for(auto e:rem){
        M*=e;
    }
    ll ans=0;
    for(int i=0;i<n;i++){
        ans=ans+v[i]* (inv(M/rem[i], rem[i]) *M/rem
            [i]) %M;
        ans=ans%M;
    }
    return ans;
}
```

5.2 DigitDP

```
vector<int> dig; // contains digits of number
ll dp[24][204][2];
ll get(int pos,int sum,int flag){ //flag checking length
    of prefix
    if(pos==dig.size()){
        if(!pr[sum]){ // end condition
            return 1;
        }
        else return 0;
    }
    if(dp[pos][sum][flag]!=-1){
        return dp[pos][sum][flag];
    }
    int lmt;
    ll ans=0;
    if(!flag){
```

```
        lmt=dig[pos];
    }else{
        lmt=9;
    }
    for(int i=0;i<=lmt;i++){
        int nf=flag;
        if(!flag&&i<lmt){
            nf=1;
        }
        ans+=get(pos+1,sum+i,nf);
    }
    return (dp[pos][sum][flag]=ans);
}
```

5.3 DP DNC

```
#include<bits/stdc++.h>
using namespace std;
typedef long long ll;
ll dp[809][8009],ind[809][8009],c[8009],a[8009];
ll cost(int i,int j){
    if(i>j) return 0;
    ll sum=(c[j]-c[i-1])*(j-i+1);
    return sum;
}
void go(int g,int l,int r,int start_ind,int end_ind){
    if(l>r) return ;
    int mid=(l+r)/2;
    dp[g][mid]=LLONG_MAX;
    for(int i=start_ind;i<=end_ind;i++){
        ll cur=dp[g-1][i]+cost(i+1,mid);
        if(cur<dp[g][mid]){
            dp[g][mid]=cur;
            ind[g][mid]=i;
        }
    }
    go(g,l,mid-1,start_ind,ind[g][mid]);
    go(g,mid+1,r,ind[g][mid],end_ind);
}
int main(){
    int n,G;cin>>n>>G;
    for(int i=1;i<=n;i++){
        cin>>a[i];
        c[i]=a[i]+c[i-1];
    }
    for(ll i=1;i<=n;i++){
        dp[1][i]=c[i]*i;
    }
    for(int i=2;i<=G;i++){
        go(i,0,n,0,n);
    }
```

```

    }
    cout<<dp[G][n];
}

```

5.4 Euclidean

```

ll mod(ll a, ll b)
// return a % b (positive value)
{
    while(a<0) a += b;
    return (a%b); }
ll gcd(ll a, ll b) {ll r; while (b)
    {r = a % b; a = b; b = r;} return a;} // computes
gcd(a,b)
ll lcm(ll a, ll b) {return a / gcd(a, b) * b;} //
computes lcm(a,b)
// returns d = gcd(a,b); finds x,y such that d = ax + by
ll extended_euclid(ll a, ll b, ll x, ll y) {
    ll xx = y = 0; ll yy = x = 1;
    while (b) {
        ll q = a/b, t = b; b = a%b; a = t;
        t = xx; xx = x-q*xx; x = t;
        t = yy; yy = y-q*yy; y = t;
    }
    return a;
}
// finds all solutions to ax = b (mod n)
vector<ll> modular_linear_equation_solver(ll a, ll b, ll
n) {
    ll x,y;
    vector<ll> solutions;
    ll d = extended_euclid(a, n, x, y);
    if (!(b%d)) {
        x = mod (x*(b/d), n);
        for (ll i = 0; i < d; i++)
            solutions.push_back(mod(x + i*(n/d), n));
    }
    return solutions;
}
// computes x and y such that ax + by = c; on failure, x
= y = -1

// Note that solution exists iff c is a multiple of gcd
(a,b)
void linear_diophantine(ll a, ll b, ll c, ll &x, ll &y)
{
    ll d = gcd(a,b);
    if (c%d)
        x = y = -1;
    else {

```

```

        extended_euclid(a,b,x,y);
        x = x*(c/d); y = y*(c/d);
    }
}
// Function to find modulo inverse of a number in log(m)
ll modInverse(ll a, ll m) {
    ll x, y;
    ll g = extended_euclid(a, m, x, y);
    if (g != 1) return -1; // Inverse mod does not
    exist
    ll res = (x%m + m) % m;
    return res;
}

```

5.5 Factors in n-1-3

```

//divisors in cube root n, pr is sieve
inline ll randll() {
    return ( (ll)rand() << 30 ) + ( rand() << 15 ) + rand
    ();
}
inline ll mult(ll a, ll b, ll n) {
    ll res = 0;
    a %= n, b %= n;
    while(b)
    {
        if(b&1) res = ( res + a ) % n;
        a = ( a + a ) % n;
        b >>= 1;
    }
    return res;
}
long long power(long long x, long long p, long long mod) {
    long long s=1, m=x;
    while(p) {
        if(p&1) s=mult(s,m,mod);
        p>>=1;
        m=mult(m,m,mod);
    }
    return s;
}
bool witness(long long a, long long n, long long u, int t) {
    long long x=power(a,u,n);
    for(int i=0; i<t; i++) {
        long long nx=mult(x,x,n);
        if(nx==1&&x!=1&&x!=n-1) return 1;
        x=nx;
    }
    return x!=1;
}

```

```

bool millerRabin(long long n,int s=100) {
    if(n<2) return 0;
    if(!(n&1)) return n==2;
    long long u=n-1;
    int t=0;
    while(u&1) {
        u>>=1;
        t++;
    }
    while(s-->0) {
        long long a=randll()%(n-1)+1;
        if(witness(a,n,u,t)) return 0;
    }
    return 1;
}

inline bool isPr(ll n){
    return millerRabin( n , 1000 );
}

#define K 1000010
ll ans=1;
ll count_div_in_cube_root_n(ll n){
    for( ll i=2;i<K&&i<=n;i++)if(!pr[ i ])
        if(n%i==0){
            ll tcnt = 0;
            while( n % i == 0 )
                tcnt++,n/=i;
            ans*=(tcnt+1ll);
        }
    if(n!=1){
        ll tmp=sqrt( n );
        if( isPr( n ) ) ans*=2ll;
        else if( tmp * tmp == n ) ans*=3ll;
        else ans*=4ll;
    }
    return ans;
}

```

5.6 Fibo logn

```

ll fib(ll n,ll mod){
    ll i,h,j,k,t;i=h=1;j=k=0;
    while(n>0) {
        if(n%2==1)
            t=(j*h)%mod,j=(i*h + j*k +t)%mod,i=(i*k + t)%mod;
        t=(h*h)%mod; h=(2*k*h + t)%mod;
        k=(k*k + t)%mod;n= n/2;
    }
    return j;
}

```

5.7 EGaussian Algorithm

```

//Gaussian elimination
const double EPS = 1e-9;
vector<double> GaussianElimination(const vector<vector<double>> & A, const vector<double> & b) {
    int i, j, k, pivot, n = A.size();
    vector<vector<double>> > B(n, vector<double>(n+1));
    vector<double> x(n);
    for(i = 0; i < n; i++) {
        for(j = 0; j < n; j++) B[i][j] = A[i][j];
        B[i][n] = b[i];
    }
    for(i = 0; i < n; i++) {
        for(pivot = j = i; j < n; ++j) if(fabs(B[j][i]) > fabs(B[pivot][i])) pivot = j;
        swap(B[i], B[pivot]);
        if(fabs(B[i][i]) < EPS) return vector<double>();
        for(j = n; j >= i; --j) B[i][j] /= B[i][i];
        for(j = 0; j < n; j++) if(i != j) for(k = i+1; k <= n; ++k) B[j][k] -= B[j][i] * B[i][k];
    }
    for(i = 0; i < n; i++) x[i] = B[i][n];
    return x;
}

```

5.8 Lucas theorem

```

//lucas thm
ll fact[14258+2];
ll ncr(ll n,ll r, ll MOD){
    if(r>n) return 0;
    ll num=fact[n]%MOD;
    ll den=fact[r]%MOD*fact[n-r]%MOD;
    den=den%MOD;
    return (num*inv(den,MOD))%MOD;
}

ll lucas(ll n, ll r,ll MOD){
    if(r>n) return 0;
    /*
    precompute in main
    ms(fact,0,sz fact);
    fact[0]=fact[1]=1;
    for(int i=2;i<=MOD;i++){
        fact[i]=i*fact[i-1];
        fact[i]%=MOD;
    }*/
    vector<ll> nn,rr;

```

```

11 tn=n, tr=r, rem=0;
while (tn) {
    rem=tn%MOD;
    nn.pb(rem);
    tn=tn/MOD;
}
rem=0;
while (tr) {
    rem=tr%MOD;
    rr.pb(rem);
    tr=tr/MOD;
}
11 ans=1;
for (int i=0; i<rr.size(); i++) {
    ans=ans*ncr(nn[i], rr[i], MOD)%MOD;
    ans=ans%MOD;
}
return ans;
}

```

5.9 Matrix expo

```

// rec relation: Ai=c1*Ai-1+c2*Ai-2+...ck*Ai-k
//A0=a0 A1=a1 ... Ak-1=ak-1
void multiply(11 F[2][2], 11 M[2][2]);
void power(11 F[2][2], 11 n);
11 ini[2];
11 fib(11 n) {
    11 F[2][2] = {{0, -1}, {1, (2*f)%MOD}};
    // F= (0 0 0 .. ck)
    //      (1 0 0 ... ck-1)
    //      (0 1 0 ... ck-2)
    //      (.....)
    //      (0 0 0 ..1 c1)
    if (n == 1) return (ini[1]*I)%MOD; //ini is [a0, a1
    ..., ak]
    power(F, n-1);
    //n-1 => n-k+1
    11 ans=(ini[1]%MOD*F[1][1]%MOD)%MOD+(ini[0]%MOD*F
    [0][1]%MOD)%MOD;
    if (ans<0) ans=(ans+MOD)%MOD;
    ans=(ans*I)%MOD;
    return ans;
}
void power(11 F[2][2], 11 n) {
    if (n == 0 || n == 1)
        return;
    11 M[2][2] = {{0, -1}, {1, (2*f)%MOD}};
    power(F, n/2);
    multiply(F, F);
}

```

```

if (n%2 != 0) multiply(F, M);
}
void multiply(11 F[2][2], 11 M[2][2]) {
    11 x = (F[0][0]%MOD*M[0][0]%MOD + F[0][1]%MOD*M
    [1][0]%MOD)%MOD;
    11 y = (F[0][0]%MOD*M[0][1]%MOD + F[0][1]%MOD*M
    [1][1]%MOD)%MOD;
    11 z = (F[1][0]%MOD*M[0][0]%MOD + F[1][1]%MOD*M
    [1][0]%MOD)%MOD;
    11 w = (F[1][0]%MOD*M[0][1]%MOD + F[1][1]%MOD*M
    [1][1]%MOD)%MOD;
    if (x<0) x=(x+MOD)%MOD;
    if (y<0) y=(y+MOD)%MOD;
    if (z<0) z=(z+MOD)%MOD;
    if (w<0) w=(w+MOD)%MOD;
    F[0][0] = x;
    F[0][1] = y;
    F[1][0] = z;
    F[1][1] = w;
}

```

5.10 Miller-Rabin

```

bool miller_rabin_primality(11 N) {
    static const int p
    [12]={2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37};
    if (N<=1) return false;
    for (int i=0; i<12; ++i) {
        if (p[i]==N) return true;
        if (N%p[i]==0) return false;
    }
    11 c=N-1, g=0;
    while (!(c&1)) c>>=1, ++g;
    for (int i=0; i<12; ++i) {
        11 k=fpow(p[i], c, N);
        for (int j=0; j<g; ++j) {
            11 kk=mult(k, k, N);
            if (kk==1 && k!=1 && k!=N-1)
                return false;
            k=kk;
        }
        if (k!=1)
            return false;
    }
    return true;
}

```

5.11 Mobius

```
//mobius
int mobius(ll n){
    prime.clear(); //primes till n
    pf(n);
    int c[1000000]={0};
    for(int i=0;i<prime.size();i++){
        c[prime[i]]++;
    }
    for(int i=1;i<1000000;i++){
        if(c[i]>=2) return 0;
    }
    if(prime.size() & 1) return -1;
    return 1;
}
```

5.12 SQRT CBRT tourist

```
ll my_sqrt(ll x) {
    assert(x > 0);
    ll y = (ll) (sqrtl((ld) x) + 0.5);
    while (y * y < x)
        y++;
    while (y * y > x)
        y--;
    if (y * y == x)
        return y;
    return -1;
}
ll my_cbrt(ll x) {
    assert(x > 0);
    ll y = (ll) (powl((ld) x, 1.0 / 3.0) + 0.5);
    while (y * y * y < x)
        y++;
    while (y * y * y > x)
        y--;
    if (y * y * y == x)
        return y;
    return -1;
}
```

5.13 Euler totient

```
//phi
int totient[100008];
void phi(){
    for(int i=1;i<=100000;i++){
        int ans=i;
        set<int> s;
```

```
int temp=i;
while(temp!=1){
    s.insert(pr[temp]); //pr is spf
    temp/=pr[temp];
}
for(auto e:s){
    ans-=ans/e;
}
totient[i]=ans;
}
```

5.14 FFT

```
const double PI = 4*atan(1);
const int N=2e5+5;
const int MOD=13313;

int FFT_N=0;
vector<base> omega;

void init_fft(int n)
{
    FFT_N = n;
    omega.resize(n);
    double angle = 2*PI/n;
    for(int i=0;i<n;i++){
        omega[i]=base(cos(i*angle), sin(i*angle));
    }
}

void fft(vector<base> &a)
{
    int n=a.size();
    if(n==1)
        return;
    int half=n>>1;
    vector<base> even(half), odd(half);
    for(int i=0, j=0; i<n; i+=2, j++){
        even[j]=a[i];
        odd[j]=a[i+1];
    }
    fft(even);
    fft(odd);
    int denominator=FFT_N/n;
    for(int i=0;i<half;i++){

```

```

        base cur=odd[i] * omega[i*denominator];
        a[i]=even[i] + cur;
        a[i+half]=even[i] - cur;
    }
}

void multiply(vector<int> &a, vector<int> &b, vector<int>
    > &res)
{
    vector<base> fa(a.begin(), a.end());
    vector<base> fb(b.begin(), b.end());
    int n=1;
    while(n<2*max(a.size(), b.size()))
        n<<=1;
    fa.resize(n);
    fb.resize(n);
    init_fft(n);
    fft(fa);
    fft(fb);
    for(int i=0;i<n;i++)
        fa[i] = conj(fa[i] * fb[i]);
    fft(fa);
    res.resize(n);
    for(int i=0;i<n;i++)
    {
        res[i]=(long long) (fa[i].real()/n + 0.5)
            ;
        res[i]%=MOD;
    }
}

```

5.15 FFT-DNC

```

#include <bits/stdc++.h>
using namespace std;

#define IOS ios::sync_with_stdio(0); cin.tie(0); cout.
    tie(0);
#define endl "\n"
#define int long long

typedef complex<double> base;

const double PI = 4*atan(1);
const int N=2e5+5;
const int MOD=13313;

int FFT_N=0;
vector<base> omega;

```

```

void init_fft(int n)
{
    FFT_N = n;
    omega.resize(n);
    double angle = 2*PI/n;
    for(int i=0;i<n;i++)
    {
        omega[i]=base(cos(i*angle), sin(i*angle)
            );
    }
}

void fft(vector<base> &a)
{
    int n=a.size();
    if(n==1)
        return;
    int half=n>>1;
    vector<base> even(half), odd(half);
    for(int i=0, j=0; i<n; i+=2, j++)
    {
        even[j]=a[i];
        odd[j]=a[i+1];
    }
    fft(even);
    fft(odd);
    int denominator=FFT_N/n;
    for(int i=0;i<half;i++)
    {
        base cur=odd[i] * omega[i*denominator];
        a[i]=even[i] + cur;
        a[i+half]=even[i] - cur;
    }
}

void multiply(vector<int> &a, vector<int> &b, vector<int>
    > &res)
{
    vector<base> fa(a.begin(), a.end());
    vector<base> fb(b.begin(), b.end());
    int n=1;
    while(n<2*max(a.size(), b.size()))
        n<<=1;
    fa.resize(n);
    fb.resize(n);
    init_fft(n);
    fft(fa);
    fft(fb);
    for(int i=0;i<n;i++)
        fa[i] = conj(fa[i] * fb[i]);
}

```



```

fft(fa);
res.resize(n);
for(int i=0;i<n;i++)
{
    res[i]=(long long)(fa[i].real()/n + 0.5)
    ;
    res[i]%=MOD;
}

int n, k, q, curlen, idx=0;
int a[N], f[N];
vector<int> res;
vector<vector<int>> > ans[40];

vector<int> divide(int lo, int hi)
{
    vector<int> ret;
    if(lo==hi)
    {
        ret.resize(f[lo]+1);
        for(int i=0;i<=f[lo];i++)
            ret[i]=1;
        return ret;
    }
    int mid=(lo+hi)>>1;
    vector<int> v1=divide(lo, mid);
    vector<int> v2=divide(mid+1, hi);
    multiply(v1, v2, ret);
    ret.resize((int)v1.size()+(int)v2.size()-1);
    return ret;
}

int32_t main()
{
    IOS;
    cin>>n>>k;
    for(int i=1;i<=n;i++)
    {
        cin>>a[i];
        f[a[i]]++;
    }
    vector<int> ans=divide(1, 2e5);
    cout<<ans[k]<<endl;
    return 0;
}

```

6 Strings

6.1 Knuth-Morris-Pratt Algorithm

```

void compute(string pat, int lps[]) {
    int len=0, m=pat.length();
    lps[0]=0;
    int i=1;
    while(i<m) {
        if(pat[i]==pat[len]) {
            len++;
            lps[i]=len;
            i++;
        } else {
            if(len!=0) {
                len=lps[len-1];
            } else {
                lps[i]=0;
                i++;
            }
        }
    }
}

void kmp(string text, string pat, int lps[]) {
    compute(pat, lps);
    int i=0, j=0;
    while(i<text.length()) {
        if(pat[j]==text[i]) {
            i++;
            j++;
        }
        if(j==pat.length()) {
            cout<<"Found at "<<i-j<<"\n";
            j=lps[j-1];
        } else if(pat[j]!=text[i] && i<text.
            length()) {
            if(j!=0) {
                j=lps[j-1];
            } else {
                i++;
            }
        }
    }
}

int main() {
    int lps[100];
}

```

```

string pat, text;
cin >> text >> pat;
kmp(text, pat, lps);
for (int i = 0; i < pat.length(); i++) {
    cout << lps[i];
}
}

```

6.2 Suffix array

```

struct suffix {
    int index;
    int rank[2];
};

int cmp(struct suffix a, struct suffix b) {
    return (a.rank[0] == b.rank[0]) ? (a.rank[1] < b.rank[1] ? 1 : 0) : (a.rank[0] < b.rank[0] ? 1 : 0);
}

vector<int> buildSuffixArray(string txt) {
    int n = txt.length();
    struct suffix suffixes[n];
    for (int i = 0; i < n; i++) {
        suffixes[i].index = i;
        suffixes[i].rank[0] = txt[i] - 'a';
        suffixes[i].rank[1] = ((i+1) < n) ? (txt[i+1] - 'a') : -1;
    }
    sort(suffixes, suffixes+n, cmp);
    int ind[n];
    for (int k = 4; k < 2*n; k = k*2) {
        int rank = 0;
        int prev_rank = suffixes[0].rank[0];
        suffixes[0].rank[0] = rank;
        ind[suffixes[0].index] = 0;
        for (int i = 1; i < n; i++) {
            if (suffixes[i].rank[0] == prev_rank &&
                suffixes[i].rank[1] == suffixes[i-1].rank[1]) {
                prev_rank = suffixes[i].rank[0];
                suffixes[i].rank[0] = rank;
            }
            else {
                prev_rank = suffixes[i].rank[0];
                suffixes[i].rank[0] = ++rank;
            }
            ind[suffixes[i].index] = i;
        }
        for (int i = 0; i < n; i++) {
            int nextindex = suffixes[i].index + k/2;

```

```

            suffixes[i].rank[1] = (nextindex < n) ?
                suffixes[ind[nextindex]].rank[0] : -1;
        }
        sort(suffixes, suffixes+n, cmp);
    }
    vector<int> suffixArr(n);
    for (int i = 0; i < n; i++)
        suffixArr[i] = suffixes[i].index;
    return suffixArr;
}

```

6.3 z-function

```

//z-function
vector<ll> z(100001, 0);
void calculatez(string &s) { // z[i] is the length of the
    longest substring starting from s[i] which is also
    a prefix of s
    ll n = s.size();
    z[0] = n;
    for (ll i = 1, l = 0, r = 0; i < n; i++) {
        if (i <= r)
            z[i] = min(r - i + 1, z[i - l]);
        while (i + z[i] < n && s[z[i]] == s[i + z[i]])
            z[i]++;
        if (i + z[i] - 1 > r) {
            l = i;
            r = i + z[i] - 1;
        }
    }
}

```

7 EZPZ

7.1 Template

```

#include <bits/stdc++.h>
#define pb push_back
#define mp make_pair
#define fi first
#define se second
#define MOD 1000000007
#define MOD9 1000000009
#define pi 3.1415926535
#define ms(s, n) memset(s, n, sizeof(s))
#define prec(n) fixed<<setprecision(n)
#define eps 0.000001
#define all(v) v.begin(), v.end()

```

```

#define allr(v) v.rbegin(), v.rend()
#define bolt ios::sync_with_stdio(0)
#define light cin.tie(0);cout.tie(0)
#define forr(i,p,n) for(ll i=p;i<n;i++)
#define MAXN 1000003
typedef int ll;
using namespace std;
ll mult(ll a,ll b, ll p=MOD){return ((a%p)*(b%p))%p;}
ll add(ll a, ll b, ll p=MOD){return (a%p + b%p)%p;}
ll fpow(ll n, ll k, ll p = MOD) {ll r = 1; for (; k; k
>>= 1) {if (k & 1) r = r * n%p; n = n * n%p;} return
r;}
ll inv(ll a, ll p = MOD) {return fpow(a, p - 2, p);}
ll inv_euclid(ll a, ll m = MOD){ll m0 = m;ll y = 0, x =
1;if (m == 1)return 0;while (a > 1){ll q = a / m;ll
t = m;m = a % m, a = t;t = y;y = x - q * y;x = t;}if
(x < 0)x += m0;return x;}
//https://www.youtube.com/watch?v=40TRPnvs4JE

```

7.2 fast io

```

void scanint(int &x)
{
    register int c = gc();
    x = 0;
    int neg = 0;
    for(;;(c<48 || c>57) && c != '-');c = gc();
    if(c=='-') {neg=1;c=gc();}
    for(;;c>47 && c<58;c = gc()) {x = (x<<1) + (x<<3) + c
- 48;}
    if(neg) x=-x;
}

```

7.3 LIS nlogn

```

//lis NLOGN
int lis(int a[],int n){
    ll dp[n+3];
    //int lis[n+3];
    //ms(lis,0,sz lis);
    dp[0]=-LLONG_MAX;
    for(int i=1;i<=n;i++){
        dp[i]=LLONG_MAX;
    }
    int anss=-1;
    for(int i=1;i<=n;i++){
        int l=1,r=n,ans;
        while(l<=r){

```

```

            int mid=(l+r)/2;
            if(a[i]<=dp[mid]){
                ans=mid;
                r=mid-1;
            }else{
                l=mid+1;
            }
        }
        dp[ans]=a[i];
        // lis[i]=max(lis[i],ans);
        anss=max(anss,ans);
    }
    return anss;
}

```

7.4 MOs

```

int N, Q;

// Variables, that hold current "state" of computation
long long current_answer;
long long cnt[100];

// Array to store answers (because the order we achieve
them is messed up)
long long answers[100500];
int BLOCK_SIZE;
int arr[100500];

// We will represent each query as three numbers: L, R,
idx. Idx is
// the position (in original order) of this query.
pair< pair<int, int>, int> queries[100500];

```

```

// Essential part of Mo's algorithm: comparator, which
we will
// use with std::sort. It is a function, which must
return True
// if query x must come earlier than query y, and False
otherwise.
inline bool mo_cmp(const pair< pair<int, int>, int> &x,
const pair< pair<int, int>, int> &y)
{
    int block_x = x.first.first / BLOCK_SIZE;
    int block_y = y.first.first / BLOCK_SIZE;
    if(block_x != block_y)
        return block_x < block_y;
    return x.first.second < y.first.second;
}

```

```

// When adding a number, we first nullify it's effect on
// current
// answer, then update cnt array, then account for it's
// effect again.
inline void add(int x)
{
    current_answer -= cnt[x] * cnt[x] * x;
    cnt[x]++;
    current_answer += cnt[x] * cnt[x] * x;
}

// Removing is much like adding.
inline void remove(int x)
{
    current_answer -= cnt[x] * cnt[x] * x;
    cnt[x]--;
    current_answer += cnt[x] * cnt[x] * x;
}

int main()
{
    cin.sync_with_stdio(false);
    cin >> N >> Q;
    BLOCK_SIZE = static_cast<int>(sqrt(N));

    // Read input array
    for(int i = 0; i < N; i++)
        cin >> arr[i];

    // Read input queries, which are 0-indexed. Store
    // each query's
    // original position. We will use it when printing
    // answer.
    for(int i = 0; i < Q; i++) {
        cin >> queries[i].first.first >> queries[i].
            first.second;
        queries[i].second = i;
    }

    // Sort queries using Mo's special comparator we
    // defined.

```

```

sort(queries, queries + Q, mo_cmp);

// Set up current segment [mo_left, mo_right].
int mo_left = 0, mo_right = -1;

for(int i = 0; i < Q; i++) {
    // [left, right] is what query we must answer
    // now.
    int left = queries[i].first.first;
    int right = queries[i].first.second;

    // Usual part of applying Mo's algorithm: moving
    // mo_left
    // and mo_right.
    while(mo_right < right) {
        mo_right++;
        add(arr[mo_right]);
    }
    while(mo_right > right) {
        remove(arr[mo_right]);
        mo_right--;
    }

    while(mo_left < left) {
        remove(arr[mo_left]);
        mo_left++;
    }
    while(mo_left > left) {
        mo_left--;
        add(arr[mo_left]);
    }

    // Store the answer into required position.
    answers[queries[i].second] = current_answer;
}

// We output answers *after* we process all queries.
for(int i = 0; i < Q; i++)
    cout << answers[i] << "\n";
return 0;
}

```

$f(n) = O(g(n))$	iff \exists positive c, n_0 such that $0 \leq f(n) \leq cg(n) \forall n \geq n_0$.	$\sum_{i=1}^n i = \frac{n(n+1)}{2}, \quad \sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}, \quad \sum_{i=1}^n i^3 = \frac{n^2(n+1)^2}{4}.$
$f(n) = \Omega(g(n))$	iff \exists positive c, n_0 such that $f(n) \geq cg(n) \geq 0 \forall n \geq n_0$.	In general:
$f(n) = \Theta(g(n))$	iff $f(n) = O(g(n))$ and $f(n) = \Omega(g(n))$.	$\sum_{i=1}^n i^m = \frac{1}{m+1} \left[(n+1)^{m+1} - 1 - \sum_{i=1}^n ((i+1)^{m+1} - i^{m+1} - (m+1)i^m) \right]$
$f(n) = o(g(n))$	iff $\lim_{n \rightarrow \infty} f(n)/g(n) = 0$.	$\sum_{i=1}^{n-1} i^m = \frac{1}{m+1} \sum_{k=0}^m \binom{m+1}{k} B_k n^{m+1-k}.$
$\lim_{n \rightarrow \infty} a_n = a$	iff $\forall \epsilon > 0, \exists n_0$ such that $ a_n - a < \epsilon, \forall n \geq n_0$.	Geometric series:
$\sup S$	least $b \in \mathbb{R}$ such that $b \geq s, \forall s \in S$.	$\sum_{i=0}^n c^i = \frac{c^{n+1} - 1}{c - 1}, \quad c \neq 1, \quad \sum_{i=0}^{\infty} c^i = \frac{1}{1 - c}, \quad \sum_{i=1}^{\infty} c^i = \frac{c}{1 - c}, \quad c < 1,$
$\inf S$	greatest $b \in \mathbb{R}$ such that $b \leq s, \forall s \in S$.	$\sum_{i=0}^n i c^i = \frac{nc^{n+2} - (n+1)c^{n+1} + c}{(c-1)^2}, \quad c \neq 1, \quad \sum_{i=0}^{\infty} i c^i = \frac{c}{(1-c)^2}, \quad c < 1.$
$\liminf_{n \rightarrow \infty} a_n$	$\lim_{n \rightarrow \infty} \inf \{a_i \mid i \geq n, i \in \mathbb{N}\}.$	Harmonic series:
$\limsup_{n \rightarrow \infty} a_n$	$\lim_{n \rightarrow \infty} \sup \{a_i \mid i \geq n, i \in \mathbb{N}\}.$	$H_n = \sum_{i=1}^n \frac{1}{i}, \quad \sum_{i=1}^n i H_i = \frac{n(n+1)}{2} H_n - \frac{n(n-1)}{4}.$
$\binom{n}{k}$	Combinations: Size k sub-sets of a size n set.	$\sum_{i=1}^n H_i = (n+1)H_n - n, \quad \sum_{i=1}^n \binom{i}{m} H_i = \binom{n+1}{m+1} \left(H_{n+1} - \frac{1}{m+1} \right).$
$[n]$	Stirling numbers (1st kind): Arrangements of an n element set into k cycles.	1. $\binom{n}{k} = \frac{n!}{(n-k)!k!}, \quad 2. \sum_{k=0}^n \binom{n}{k} = 2^n, \quad 3. \binom{n}{k} = \binom{n}{n-k},$
$\left\{ \begin{matrix} n \\ k \end{matrix} \right\}$	Stirling numbers (2nd kind): Partitions of an n element set into k non-empty sets.	4. $\binom{n}{k} = \frac{n}{k} \binom{n-1}{k-1}, \quad 5. \binom{n}{k} = \binom{n-1}{k} + \binom{n-1}{k-1},$
$\langle \begin{matrix} n \\ k \end{matrix} \rangle$	1st order Eulerian numbers: Permutations $\pi_1 \pi_2 \dots \pi_n$ on $\{1, 2, \dots, n\}$ with k ascents.	6. $\binom{n}{m} \binom{m}{k} = \binom{n}{k} \binom{n-k}{m-k}, \quad 7. \sum_{k=0}^n \binom{r+k}{k} = \binom{r+n+1}{n},$
$\langle \langle \begin{matrix} n \\ k \end{matrix} \rangle \rangle$	2nd order Eulerian numbers.	8. $\sum_{k=0}^n \binom{k}{m} = \binom{n+1}{m+1}, \quad 9. \sum_{k=0}^n \binom{r}{k} \binom{s}{n-k} = \binom{r+s}{n},$
C_n	Catalan Numbers: Binary trees with $n+1$ vertices.	10. $\binom{n}{k} = (-1)^k \binom{k-n-1}{k}, \quad 11. \left\{ \begin{matrix} n \\ 1 \end{matrix} \right\} = \left\{ \begin{matrix} n \\ n \end{matrix} \right\} = 1,$
14. $\begin{bmatrix} n \\ 1 \end{bmatrix} = (n-1)!,$	15. $\begin{bmatrix} n \\ 2 \end{bmatrix} = (n-1)!H_{n-1},$	16. $\begin{bmatrix} n \\ n \end{bmatrix} = 1, \quad 17. \begin{bmatrix} n \\ k \end{bmatrix} \geq \left\{ \begin{matrix} n \\ k \end{matrix} \right\},$
18. $\begin{bmatrix} n \\ k \end{bmatrix} = (n-1) \begin{bmatrix} n-1 \\ k \end{bmatrix} + \begin{bmatrix} n-1 \\ k-1 \end{bmatrix},$	19. $\left\{ \begin{matrix} n \\ n-1 \end{matrix} \right\} = \begin{bmatrix} n \\ n-1 \end{bmatrix} = \binom{n}{2},$	20. $\sum_{k=0}^n \begin{bmatrix} n \\ k \end{bmatrix} = n!, \quad 21. C_n = \frac{1}{n+1} \binom{2n}{n},$
22. $\langle \begin{matrix} n \\ 0 \end{matrix} \rangle = \langle \begin{matrix} n \\ n-1 \end{matrix} \rangle = 1,$	23. $\langle \begin{matrix} n \\ k \end{matrix} \rangle = \langle \begin{matrix} n \\ n-1-k \end{matrix} \rangle,$	24. $\langle \begin{matrix} n \\ k \end{matrix} \rangle = (k+1) \langle \begin{matrix} n-1 \\ k \end{matrix} \rangle + (n-k) \langle \begin{matrix} n-1 \\ k-1 \end{matrix} \rangle,$
25. $\langle \begin{matrix} 0 \\ k \end{matrix} \rangle = \begin{cases} 1 & \text{if } k=0, \\ 0 & \text{otherwise} \end{cases}$	26. $\langle \begin{matrix} n \\ 1 \end{matrix} \rangle = 2^n - n - 1,$	27. $\langle \begin{matrix} n \\ 2 \end{matrix} \rangle = 3^n - (n+1)2^n + \binom{n+1}{2},$
28. $x^n = \sum_{k=0}^n \langle \begin{matrix} n \\ k \end{matrix} \rangle \binom{x+k}{n},$	29. $\langle \begin{matrix} n \\ m \end{matrix} \rangle = \sum_{k=0}^m \binom{n+1}{k} (m+1-k)^n (-1)^k,$	30. $m! \left\{ \begin{matrix} n \\ m \end{matrix} \right\} = \sum_{k=0}^n \langle \begin{matrix} n \\ k \end{matrix} \rangle \binom{k}{n-m},$
31. $\langle \begin{matrix} n \\ m \end{matrix} \rangle = \sum_{k=0}^n \left\{ \begin{matrix} n \\ k \end{matrix} \right\} \binom{n-k}{m} (-1)^{n-k-m} k!,$	32. $\langle \langle \begin{matrix} n \\ 0 \end{matrix} \rangle \rangle = 1,$	33. $\langle \langle \begin{matrix} n \\ n \end{matrix} \rangle \rangle = 0 \quad \text{for } n \neq 0,$
34. $\langle \langle \begin{matrix} n \\ k \end{matrix} \rangle \rangle = (k+1) \langle \langle \begin{matrix} n-1 \\ k \end{matrix} \rangle \rangle + (2n-1-k) \langle \langle \begin{matrix} n-1 \\ k-1 \end{matrix} \rangle \rangle,$	35. $\sum_{k=0}^n \langle \langle \begin{matrix} n \\ k \end{matrix} \rangle \rangle = \frac{(2n)n}{2^n},$	
36. $\left\{ \begin{matrix} x \\ x-n \end{matrix} \right\} = \sum_{k=0}^n \langle \langle \begin{matrix} n \\ k \end{matrix} \rangle \rangle \binom{x+n-1-k}{2n},$	37. $\left\{ \begin{matrix} n+1 \\ m+1 \end{matrix} \right\} = \sum_k \binom{n}{k} \left\{ \begin{matrix} k \\ m \end{matrix} \right\} = \sum_{k=0}^n \left\{ \begin{matrix} k \\ m \end{matrix} \right\} (m+1)^{n-k},$	

The Chinese remainder theorem: There exists a number C such that:

$$C \equiv r_1 \pmod{m_1}$$

$$\vdots \quad \vdots \quad \vdots$$

$$C \equiv r_n \pmod{m_n}$$

if m_i and m_j are relatively prime for $i \neq j$.

Euler's function: $\phi(x)$ is the number of positive integers less than x relatively prime to x . If $\prod_{i=1}^n p_i^{e_i}$ is the prime factorization of x then

$$\phi(x) = \prod_{i=1}^n p_i^{e_i-1} (p_i - 1).$$

Euler's theorem: If a and b are relatively prime then

$$1 \equiv a^{\phi(b)} \pmod{b}.$$

Fermat's theorem:

$$1 \equiv a^{p-1} \pmod{p}.$$

The Euclidean algorithm: if $a > b$ are integers then

$$\gcd(a, b) = \gcd(a \bmod b, b).$$

If $\prod_{i=1}^n p_i^{e_i}$ is the prime factorization of x then

$$S(x) = \sum_{d|x} d = \prod_{i=1}^n \frac{p_i^{e_i+1} - 1}{p_i - 1}.$$

Perfect Numbers: x is an even perfect number iff $x = 2^{n-1}(2^n - 1)$ and $2^n - 1$ is prime.

Wilson's theorem: n is a prime iff

$$(n-1)! \equiv -1 \pmod{n}.$$

Möbius inversion:

$$\mu(i) = \begin{cases} 1 & \text{if } i = 1. \\ 0 & \text{if } i \text{ is not square-free.} \\ (-1)^r & \text{if } i \text{ is the product of } r \text{ distinct primes.} \end{cases}$$

If

$$G(a) = \sum_{d|a} F(d),$$

then

$$F(a) = \sum_{d|a} \mu(d) G\left(\frac{a}{d}\right).$$

Prime numbers:

$$p_n = n \ln n + n \ln \ln n - n + n \frac{\ln \ln n}{\ln n}$$

$$+ O\left(\frac{n}{\ln n}\right),$$

$$\pi(n) = \frac{n}{\ln n} + \frac{n}{(\ln n)^2} + \frac{2!n}{(\ln n)^3}$$

$$+ O\left(\frac{n}{(\ln n)^4}\right).$$

Definitions:

Loop An edge connecting a vertex to itself.

Directed Each edge has a direction.

Simple Graph with no loops or multi-edges.

Walk A sequence $v_0 e_1 v_1 \dots e_\ell v_\ell$.

Trail A walk with distinct edges.

Path A trail with distinct vertices.

Connected A graph where there exists a path between any two vertices.

Component A maximal connected subgraph.

Tree A connected acyclic graph.

Free tree A tree with no root.

DAG Directed acyclic graph.

Eulerian Graph with a trail visiting each edge exactly once.

Hamiltonian Graph with a cycle visiting each vertex exactly once.

Cut A set of edges whose removal increases the number of components.

Cut-set A minimal cut.

Cut edge A size 1 cut.

k-Connected A graph connected with the removal of any $k-1$ vertices.

k-Tough $\forall S \subseteq V, S \neq \emptyset$ we have $k \cdot c(G-S) \leq |S|$.

k-Regular A graph where all vertices have degree k .

k-Factor A k -regular spanning subgraph.

Matching A set of edges, no two of which are adjacent.

Clique A set of vertices, all of which are adjacent.

Ind. set A set of vertices, none of which are adjacent.

Vertex cover A set of vertices which cover all edges.

Planar graph A graph which can be embedded in the plane.

Plane graph An embedding of a planar graph.

$$\sum_{v \in V} \deg(v) = 2m.$$

If G is planar then $n - m + f = 2$, so

$$f \leq 2n - 4, \quad m \leq 3n - 6.$$

Any planar graph has a vertex with degree ≤ 5 .

Notation:

$E(G)$ Edge set

$V(G)$ Vertex set

$c(G)$ Number of components

$G[S]$ Induced subgraph

$\deg(v)$ Degree of v

$\Delta(G)$ Maximum degree

$\delta(G)$ Minimum degree

$\chi(G)$ Chromatic number

$\chi_E(G)$ Edge chromatic number

G^c Complement graph

K_n Complete graph

K_{n_1, n_2} Complete bipartite graph

$r(k, \ell)$ Ramsey number

Geometry

Projective coordinates: triples (x, y, z) , not all x, y and z zero.

$$(x, y, z) = (cx, cy, cz) \quad \forall c \neq 0.$$

Cartesian Projective

$$(x, y) \quad (x, y, 1)$$

$$y = mx + b \quad (m, -1, b)$$

$$x = c \quad (1, 0, -c)$$

Distance formula, L_p and L_∞ metric:

$$\sqrt{(x_1 - x_0)^2 + (y_1 - y_0)^2},$$

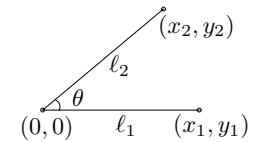
$$[|x_1 - x_0|^p + |y_1 - y_0|^p]^{1/p},$$

$$\lim_{p \rightarrow \infty} [|x_1 - x_0|^p + |y_1 - y_0|^p]^{1/p}.$$

Area of triangle $(x_0, y_0), (x_1, y_1)$ and (x_2, y_2) :

$$\frac{1}{2} \text{abs} \begin{vmatrix} x_1 - x_0 & y_1 - y_0 \\ x_2 - x_0 & y_2 - y_0 \end{vmatrix}.$$

Angle formed by three points:



$$\cos \theta = \frac{(x_1, y_1) \cdot (x_2, y_2)}{l_1 l_2}.$$

Line through two points (x_0, y_0) and (x_1, y_1) :

$$\begin{vmatrix} x & y & 1 \\ x_0 & y_0 & 1 \\ x_1 & y_1 & 1 \end{vmatrix} = 0.$$

Area of circle, volume of sphere:

$$A = \pi r^2, \quad V = \frac{4}{3} \pi r^3.$$

If I have seen farther than others, it is because I have stood on the shoulders of giants.

– Issac Newton

Taylor's series:

$$f(x) = f(a) + (x-a)f'(a) + \frac{(x-a)^2}{2}f''(a) + \dots = \sum_{i=0}^{\infty} \frac{(x-a)^i}{i!} f^{(i)}(a).$$

Expansions:

$$\begin{aligned} \frac{1}{1-x} &= 1 + x + x^2 + x^3 + x^4 + \dots = \sum_{i=0}^{\infty} x^i, \\ \frac{1}{1-cx} &= 1 + cx + c^2x^2 + c^3x^3 + \dots = \sum_{i=0}^{\infty} c^i x^i, \\ \frac{1}{1-x^n} &= 1 + x^n + x^{2n} + x^{3n} + \dots = \sum_{i=0}^{\infty} x^{ni}, \\ \frac{x}{(1-x)^2} &= x + 2x^2 + 3x^3 + 4x^4 + \dots = \sum_{i=0}^{\infty} ix^i, \\ x^k \frac{d^n}{dx^n} \left(\frac{1}{1-x} \right) &= x + 2^n x^2 + 3^n x^3 + 4^n x^4 + \dots = \sum_{i=0}^{\infty} i^n x^i, \\ e^x &= 1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \dots = \sum_{i=0}^{\infty} \frac{x^i}{i!}, \\ \ln(1+x) &= x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + \dots = \sum_{i=1}^{\infty} (-1)^{i+1} \frac{x^i}{i}, \\ \ln \frac{1}{1-x} &= x + \frac{1}{2}x^2 + \frac{1}{3}x^3 + \frac{1}{4}x^4 + \dots = \sum_{i=1}^{\infty} \frac{x^i}{i}, \\ \sin x &= x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \frac{1}{7!}x^7 + \dots = \sum_{i=0}^{\infty} (-1)^i \frac{x^{2i+1}}{(2i+1)!}, \\ \cos x &= 1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 - \frac{1}{6!}x^6 + \dots = \sum_{i=0}^{\infty} (-1)^i \frac{x^{2i}}{(2i)!}, \\ \tan^{-1} x &= x - \frac{1}{3}x^3 + \frac{1}{5}x^5 - \frac{1}{7}x^7 + \dots = \sum_{i=0}^{\infty} (-1)^i \frac{x^{2i+1}}{(2i+1)}, \\ (1+x)^n &= 1 + nx + \frac{n(n-1)}{2}x^2 + \dots = \sum_{i=0}^{\infty} \binom{n}{i} x^i, \\ \frac{1}{(1-x)^{n+1}} &= 1 + (n+1)x + \binom{n+2}{2}x^2 + \dots = \sum_{i=0}^{\infty} \binom{i+n}{i} x^i, \\ \frac{x}{e^x - 1} &= 1 - \frac{1}{2}x + \frac{1}{12}x^2 - \frac{1}{720}x^4 + \dots = \sum_{i=0}^{\infty} \frac{B_i x^i}{i!}, \\ \frac{1}{2x}(1 - \sqrt{1-4x}) &= 1 + x + 2x^2 + 5x^3 + \dots = \sum_{i=0}^{\infty} \frac{1}{i+1} \binom{2i}{i} x^i, \\ \frac{1}{\sqrt{1-4x}} &= 1 + x + 2x^2 + 6x^3 + \dots = \sum_{i=0}^{\infty} \binom{2i}{i} x^i, \\ \frac{1}{\sqrt{1-4x}} \left(\frac{1 - \sqrt{1-4x}}{2x} \right)^n &= 1 + (2+n)x + \binom{4+n}{2}x^2 + \dots = \sum_{i=0}^{\infty} \binom{2i+n}{i} x^i, \\ \frac{1}{1-x} \ln \frac{1}{1-x} &= x + \frac{3}{2}x^2 + \frac{11}{6}x^3 + \frac{25}{12}x^4 + \dots = \sum_{i=1}^{\infty} H_i x^i, \\ \frac{1}{2} \left(\ln \frac{1}{1-x} \right)^2 &= \frac{1}{2}x^2 + \frac{3}{4}x^3 + \frac{11}{24}x^4 + \dots = \sum_{i=2}^{\infty} \frac{H_{i-1} x^i}{i}, \\ \frac{x}{1-x-x^2} &= x + x^2 + 2x^3 + 3x^4 + \dots = \sum_{i=0}^{\infty} F_i x^i, \\ \frac{F_n x}{1 - (F_{n-1} + F_{n+1})x - (-1)^n x^2} &= F_n x + F_{2n} x^2 + F_{3n} x^3 + \dots = \sum_{i=0}^{\infty} F_{ni} x^i. \end{aligned}$$

Ordinary power series:

$$A(x) = \sum_{i=0}^{\infty} a_i x^i.$$

Exponential power series:

$$A(x) = \sum_{i=0}^{\infty} a_i \frac{x^i}{i!}.$$

Dirichlet power series:

$$A(x) = \sum_{i=1}^{\infty} \frac{a_i}{i^x}.$$

Binomial theorem:

$$(x+y)^n = \sum_{k=0}^n \binom{n}{k} x^{n-k} y^k.$$

Difference of like powers:

$$x^n - y^n = (x-y) \sum_{k=0}^{n-1} x^{n-1-k} y^k.$$

For ordinary power series:

$$\alpha A(x) + \beta B(x) = \sum_{i=0}^{\infty} (\alpha a_i + \beta b_i) x^i,$$

$$x^k A(x) = \sum_{i=k}^{\infty} a_{i-k} x^i,$$

$$\frac{A(x) - \sum_{i=0}^{k-1} a_i x^i}{x^k} = \sum_{i=0}^{\infty} a_{i+k} x^i,$$

$$A(cx) = \sum_{i=0}^{\infty} c^i a_i x^i,$$

$$A'(x) = \sum_{i=0}^{\infty} (i+1) a_{i+1} x^i,$$

$$xA'(x) = \sum_{i=1}^{\infty} i a_i x^i,$$

$$\int A(x) dx = \sum_{i=1}^{\infty} \frac{a_{i-1}}{i} x^i,$$

$$\frac{A(x) + A(-x)}{2} = \sum_{i=0}^{\infty} a_{2i} x^{2i},$$

$$\frac{A(x) - A(-x)}{2} = \sum_{i=0}^{\infty} a_{2i+1} x^{2i+1}.$$

Summation: If $b_i = \sum_{j=0}^i a_j$ then

$$B(x) = \frac{1}{1-x} A(x).$$

Convolution:

$$A(x)B(x) = \sum_{i=0}^{\infty} \left(\sum_{j=0}^i a_j b_{i-j} \right) x^i.$$

God made the natural numbers;
all the rest is the work of man.
– Leopold Kronecker

