Continuous time search

Farhi & Gutmann 1996

"Analog analogue of digital quantum comp" (6 pages)

Also see N.C. eq. 6.18 & following for alt intro

$$i\frac{d}{dt}|\psi\rangle = H(t)|\psi\rangle$$

$$H_w = E|w\rangle\langle w| \quad \leftarrow E \neq 0 \text{ fixed by } w \text{ (oracle) energy scale}$$

$$H_D = E|s\rangle\langle s|$$

↑ optimal when this energy scale matches H_w

$$H = H_w + H_D$$

$$\psi_{t=0} = |s\rangle = \frac{\sum |x\rangle}{\sqrt{N}}$$

$$\langle s|w\rangle = \frac{1}{\sqrt{N}}$$

$$\langle r|w\rangle = 0 \quad ; \quad |r\rangle = \frac{|s\rangle - x|w\rangle}{\sqrt{1-x^2}} \quad ; \quad |s\rangle = \begin{bmatrix} x \\ \sqrt{1-x^2} \end{bmatrix}$$

$$H = E\begin{bmatrix} 1+x^2 & x\sqrt{1-x^2} \\ x\sqrt{1-x^2} & 1-x^2 \end{bmatrix}$$

$$|\psi_t\rangle = \exp m(iHt)\,|\psi_0\rangle$$

$$= e^{-iEt}\begin{bmatrix} x\cos(Ext) - i\sin(Ext) \\ \sqrt{1-x^2}\cos(Ext) \end{bmatrix}$$

$$P_{meas,w} = \sin^2(Ext) + x^2\cos^2(Ext)$$

$\Rightarrow$ Stopping time $\quad t_f = \dfrac{\pi}{2Ex}$

$$t = \frac{\pi}{2\sqrt{N}}\,\frac{1}{E}$$