Adiabatic ~~Continuous~~ time search

"How powerful is adiabatic quantum comp" 2002
van Dam, Mosca, Vazirani

"Quantum search by local adiabatic evolution" 2001
Roland, Cerf

$$i \frac{d}{dt} |\psi\rangle = H(t) |\psi\rangle$$

Adiabatic model

$$i \frac{d}{ds} |\psi\rangle = T(s) H(s) |\psi\rangle$$

$H(s)$ is time varying Hamiltonian w/ $H(0) = H_i$ simple

$H(1) = H_f$ grd state encodes ans

$$\boxed{H(s) = s H_f + (1-s) H_i}$$

$$T(s) = \left(\frac{ds}{dt}\right)^{-1}$$

$s \in [0,1]$

rate of change of s along path

If $T(s)$ is sufficiently slow, if $|\psi\rangle$ begin as eig of $H(0)$ remains in eig state of $H(s)$.

$\boxed{\text{From notes on cont. time Grover}}$

$$H_w = E|w\rangle\langle w|$$

$E \neq 0$ fixed by w (oracle) energy scale

$$H_D = E|s\rangle\langle s|$$

optimal when this energy ~~scale~~ matches H_w

$$H = H_w + H_D.$$

$$\psi_{t=0} = |s\rangle = \frac{\sum |x\rangle}{\sqrt{N}}$$

$$H(0) = \mathbb{1} - H_D$$

$$H(1) = \mathbb{1} - H_w$$

Adiabatic speed depends on gap between eigenstate & energetic neighbor ie for grd state, 1st excited state gap $\Delta = E_1 - E_0$

$$H(s) = s H_f + (1-s) H_i$$

$$\Delta(s) = 2\sqrt{(s-\tfrac{1}{2})^2 + s(1-s)/N} = E_1(s) - E_0(s)$$

$$\boxed{\Delta(\tfrac{1}{2}) = \frac{1}{\sqrt{N}}}$$

Taking $T(s) = \dfrac{1}{\varepsilon \Delta^2(s)}$ gives overlap w/ final state w/ target ground state $H_f \varphi_0 = E_0 \varphi_0$

$$|\langle \psi_{s=1} | \varphi_0 \rangle|^2 \geq 1 - \varepsilon$$

Total time

$$T(s) = \left(\frac{ds}{dt}\right)^{-1}$$

$$\frac{dt}{ds} = \frac{1}{\varepsilon \Delta^2(s)}$$

$$\Rightarrow \boxed{\int \frac{ds}{\varepsilon \Delta^2(s)} = \frac{N}{2\sqrt{N-1}} \tan^{-1}\left[\sqrt{N-1}\,(2s-1)\right]}$$

$$\Delta t = \frac{N}{\varepsilon\sqrt{N-1}} \quad \tan^{-1}\sqrt{N-1}$$

$$\approx \frac{\pi}{2\varepsilon}\sqrt{N} \quad \text{for large } N$$