

# Convolution



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- We know about Convolutional Neural Networks (CNN). Here we will learn about convolution.

## Polynomials

$$(2 + 3x + 4x^2)$$

$$(2 + 3x + 4x^2 + 5x^2 + 6x^4)$$

- Coefficients in polynomials.
- How to find them

$$(K_0 + K_1 + K_2x^2)$$

$$(a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + a_5x^5)$$

- To find the coefficients, we should not multiply all of them. We should progress systematically.

$$(K_2a_2 + K_1a_3 + K_0a_4)x^4$$

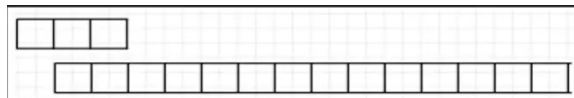
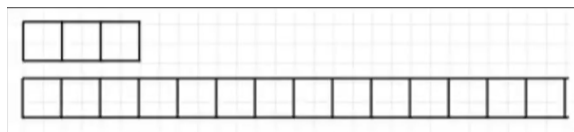
What will be the coefficient of the term?

$$(K_2a_2 + K_1a_3 + K_0a_4)x^4$$

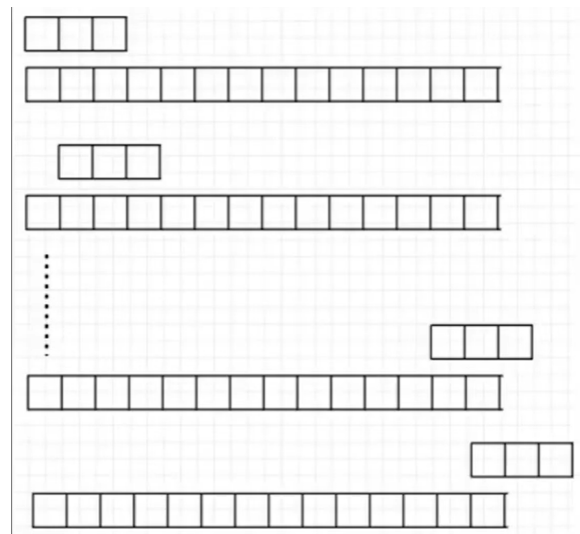
$$\begin{aligned} & (K_0a_0) + \\ & (K_1a_0 + K_0a_1)x + \\ & (K_2a_0 + K_1a_1 + K_0a_2)x^2 + \\ & (K_2a_1 + K_1a_2 + K_0a_3)x^3 + \\ & (K_2a_2 + K_1a_3 + K_0a_4)x^4 + \\ & (K_2a_3 + K_1a_4 + K_0a_5)x^5 + \\ & (K_2a_4 + K_1a_5)x^6 + \\ & (K_2a_5)x^7 \end{aligned}$$

- Iterations

$$\begin{array}{r} (K_0 + K_1x + K_2x^2) \\ (a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + a_5x^5) \\ \hline (K_2x^2 + K_1x + K_0) \\ (a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + a_5x^5) \end{array}$$



The first polynomial is moving with each iteration



$$\begin{pmatrix} K_2x^2 + K_1x + K_0 \\ (a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + a_5x^5) \end{pmatrix}$$

$$\begin{pmatrix} K_2x^2 + K_1x + K_0 \\ (a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + a_5x^5) \end{pmatrix}$$

$$\begin{pmatrix} K_2x^2 + K_1x + K_0 \\ (a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + a_5x^5) \end{pmatrix}$$

$$\begin{pmatrix} K_2x^2 + K_1x + K_0 \\ (a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + a_5x^5) \end{pmatrix}$$

$$\begin{aligned} & (K_0a_0) + \\ & (K_1a_0 + K_0a_1)x + \\ & (K_2a_0 + K_1a_1 + K_0a_2)x^2 + \\ & (K_2a_1 + K_1a_2 + K_0a_3)x^3 + \\ & (K_2a_2 + K_1a_3 + K_0a_4)x^4 + \\ & (K_2a_3 + K_1a_4 + K_0a_5)x^5 + \\ & (K_2a_4 + K_1a_5)x^6 + \\ & (K_2a_5)x^7 \end{aligned}$$

- This type of sliding operation is called convolution. Each term of one polynomial is getting convolved with another term.

## Convolution in one dimension

## Convolution in 2D

## Convolution in 3D

# Convolution in one dimension

$$a = \left[\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right]$$

$$x = [1, 2, 3, 1, 2, 3, 1, 2, 3]$$

$$y(1) = \left\{ \begin{array}{cccccccccccc} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & & & & & & & & & \\ \times & \times & \times & & & & & & & & & \\ 0 & 1 & 2 & 3 & 1 & 2 & 3 & 1 & 2 & 3 & 0 & \end{array} \right\} = 1$$

$$y(2) = \left\{ \begin{array}{cccccccccccc} & & & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & & & & & & \\ & & & \times & \times & \times & & & & & & \\ 0 & 1 & 2 & 3 & 1 & 2 & 3 & 1 & 2 & 3 & 0 & \end{array} \right\} = 2$$

$$y(3) = \left\{ \begin{array}{cccccccccccc} & & & & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & & & & & \\ & & & & \times & \times & \times & & & & & \\ 0 & 1 & 2 & 3 & 1 & 2 & 3 & 1 & 2 & 3 & 0 & \end{array} \right\} = 2$$

$$y(4) = \left\{ \begin{array}{cccccccccccc} & & & & & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & & & & \\ & & & & & \times & \times & \times & & & & \\ 0 & 1 & 2 & 3 & 1 & 2 & 3 & 1 & 2 & 3 & 0 & \end{array} \right\} = 2$$

$$y(5) = \left\{ \begin{array}{cccccccccccc} & & & & & & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & & & \\ & & & & & & \times & \times & \times & & & \\ 0 & 1 & 2 & 3 & 1 & 2 & 3 & 1 & 2 & 3 & 0 & \end{array} \right\} = \frac{5}{3}$$

The array at the bottom is extended with a 0

$$y(1) = \left\{ \begin{array}{cccccccccccc} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & & & & & & & & & \\ \times & \times & \times & & & & & & & & & \\ 1 & 2 & 3 & 1 & 2 & 3 & 1 & 2 & 3 & & & \end{array} \right\} = 2$$

$$y(2) = \left\{ \begin{array}{cccccccccccc} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & & & & & & & & \\ & \times & \times & \times & & & & & & & & \\ 1 & 2 & 3 & 1 & 2 & 3 & 1 & 2 & 3 & & & \end{array} \right\} = 2$$

$$\vdots$$

$$y(7) = \left\{ \begin{array}{cccccccccccc} & & & & & & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & & & \\ & & & & & & \times & \times & \times & & & \\ 1 & 2 & 3 & 1 & 2 & 3 & 1 & 2 & 3 & & & \end{array} \right\} = 2$$

- The upper one is moving two steps

Two choice parameters:

1. Stride: How many steps the filter is moving
2. Padding: Putting extra 0s at the end and beginning

# Convolution in 2D

- For an image, there are two dimensions. Height and width.

Figure 1 shows three 3x3 grids. The first grid contains numbers 1 through 9. The second grid contains numbers -1, 0, and 1, with the cell at row 2, column 2 shaded. The third grid contains numbers -13, -20, -17, -18, -24, -18, 13, 20, and 17.

Images with 9 pixels

- The filter is called the **kernel**. It's convolved with the input image and the output is generated.
- We should decide the shape of the kernel. Usually square shape, but other shapes can be used.
- The kernel average is at the centre.

$$\begin{array}{|c|c|c|} \hline 1 & 2 & 1 \\ \hline 0 & 0 & 0 \\ \hline -1 & -2 & -1 \\ \hline \end{array} \quad \begin{array}{|c|c|c|} \hline & 1 & 2 \\ \hline & 1 & 2 \\ \hline & 4 & 5 \\ \hline \end{array} \quad \begin{array}{|c|} \hline 3 \\ \hline 6 \\ \hline 9 \\ \hline \end{array}$$

$$= 0 \cdot 1 + 0 \cdot 2 + 0 \cdot 1$$

$$+ 0 \cdot 0 + 1 \cdot 0 + 2 \cdot 0$$

$$+ 0 \cdot (-1) + 4 \cdot (-2) + 5 \cdot (-1)$$

$$= -13$$

1

		1	2	3
1	2	4	1	
0	0	7	0	8
-1	-2	-1		

$$\begin{aligned}
 &= 0 \cdot 1 + 4 \cdot 2 + 5 \cdot 1 \\
 &\quad + 0 \cdot 0 + 7 \cdot 0 + 8 \cdot 0 \\
 &\quad + 0 \cdot (-1) + 0 \cdot (-2) + 0 \cdot (-1) \\
 &= 13
 \end{aligned}$$

7

$$\begin{array}{|c|c|c|} \hline 1 & 2 & 1 \\ \hline 0 & 1 & 0 \\ \hline -1 & -2 & -1 \\ \hline \end{array} = \begin{aligned} & 0 \cdot 1 + 0 \cdot 2 + 0 \cdot 1 \\ & + 1 \cdot 0 + 2 \cdot 0 + 3 \cdot 0 \\ & + 4 \cdot (-1) + 5 \cdot (-2) + 6 \cdot (-1) \\ & = -20 \end{aligned}$$

2

	1	2	3
1	4	2	5
0	7	0	8
-1	-2	-1	

$$= 4 \cdot 1 + 5 \cdot 2 + 6 \cdot 1 + 7 \cdot 0 + 8 \cdot 0 + 9 \cdot 0 + 0 \cdot (-1) + 0 \cdot (-2) + 0 \cdot (-1) = 20$$

8

	1	2	1	
1	0	2	0	0
4	-1	5	-2	-1
7		8		9

$$\begin{aligned}
&= 0 \cdot 1 + 0 \cdot 2 + 0 \cdot 1 \\
&\quad + 2 \cdot 0 + 3 \cdot 0 + 0 \cdot 0 \\
&\quad + 5 \cdot (-1) + 6 \cdot (-2) + 0 \cdot (-1) \\
&= -17
\end{aligned}$$

3

1	2	1		
0	0	4	0	5
-1	-2	7	-1	8
				9

$$\begin{aligned}
&= 0 \cdot 1 + 1 \cdot 2 + 2 \cdot 1 \\
&\quad + 0 \cdot 0 + 4 \cdot 0 + 5 \cdot 0 \\
&\quad + 0 \cdot (-1) + 7 \cdot (-2) + 8 \cdot (-1) \\
&= -18
\end{aligned}$$

4

1	2	1		
0	0	4	0	5
-1	-2	7	-1	8
				9

$$\begin{aligned}
&= 1 \cdot 1 + 2 \cdot 2 + 3 \cdot 1 \\
&\quad + 4 \cdot 0 + 5 \cdot 0 + 6 \cdot 0 \\
&\quad + 7 \cdot (-1) + 8 \cdot (-2) + 9 \cdot (-1) \\
&= -24
\end{aligned}$$

5

1	2	1		
0	0	4	0	5
-1	-2	7	-1	8
				9

$$\begin{aligned}
&= 2 \cdot 1 + 3 \cdot 2 + 0 \cdot 1 \\
&\quad + 5 \cdot 0 + 6 \cdot 0 + 0 \cdot 0 \\
&\quad + 8 \cdot (-1) + 9 \cdot (-2) + 0 \cdot (-1) \\
&= -18
\end{aligned}$$

6

1	2	3		
4	1	5	2	1
7	0	8	0	9
	-1	-2	-1	

$$\begin{aligned}
&= 5 \cdot 1 + 6 \cdot 2 + 0 \cdot 1 \\
&\quad + 8 \cdot 0 + 9 \cdot 0 + 0 \cdot 0 \\
&\quad + 0 \cdot (-1) + 0 \cdot (-2) + 0 \cdot (-1) \\
&= 17
\end{aligned}$$

9

-13	-20	-17
-18	-24	-18
13	20	17

Result

# Convolution in 3D

- Usually, we work with 3D colour images(RGB).
- Greyscale images are 2D and only have one channel.
- For colour images, we have 3 channels. Apart from height and width, we will need the depth/channel information.

## How convolution is done

Input	Kernel	Intermediate Output	Output
<div>1 0 1 0 2</div> <div>1 1 3 2 1</div> <div>1 1 0 1 1</div> <div>2 3 2 1 3</div> <div>0 2 0 1 0</div>	<div>0 1 0</div> <div>0 0 2</div> <div>0 1 0</div>	<div>7 5 3</div> <div>4 7 5</div> <div>7 2 8</div>	
<div>1 0 0 1 0</div> <div>2 0 1 2 0</div> <div>3 1 1 3 0</div> <div>0 3 0 3 2</div> <div>1 0 3 2 1</div>	<div>2 1 0</div> <div>0 0 0</div> <div>0 3 0</div>	<div>5 3 10</div> <div>13 1 13</div> <div>7 12 11</div>	<div>19 13 15</div> <div>28 16 20</div> <div>23 18 25</div>
<div>2 0 1 2 1</div> <div>3 3 1 3 2</div> <div>2 1 1 1 0</div> <div>3 1 3 2 0</div> <div>1 1 2 1 1</div>	<div>1 0 0</div> <div>1 0 0</div> <div>0 0 2</div>	<div>7 5 2</div> <div>11 8 2</div> <div>9 4 6</div>	

3D Convolution

- The channels are separated and then the filter/kernel is convolved exactly like 2D convolution.
- Afterwards, the channels are merged to get the final image.