Image Processing - I

Created By: Kazi Junaid Mahmud

💡 **Image:** An image is a visual representation of something

💡 **Pixel:** A pixel is the smallest unit of information in an image, and is short for **“picture element”.** Pixels are small squares or dots that represent a single point of colour. When combined with other pixels, they form the images we see on screens, such as computer monitors, smartphones, and televisions.

Pixels of an image

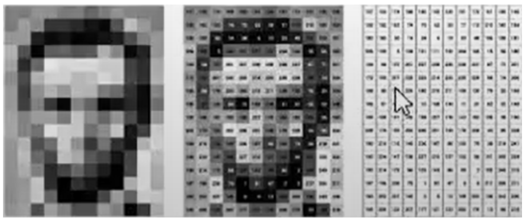
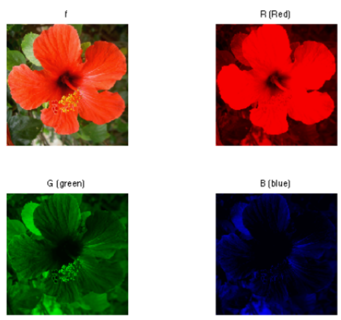


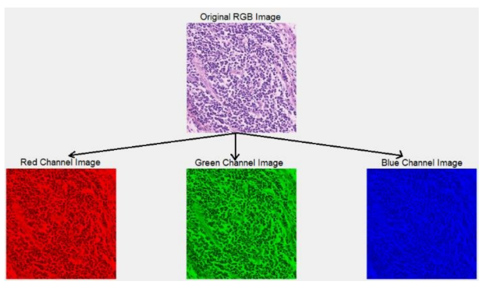
Image and pixel values

Grayscale Image

Grayscale Image

Colour Image





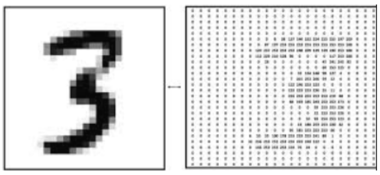
Three channels in a colour image

3D Medical Image

- Medical image will have multiple slices, that will create the 3D image.

Resolution

> 💡 Image resolution is the level of detail in an image. The more pixels there are, the higher the resolution, and the more details the image has. A higher-resolution image will be clearer and have better colour richness and sharpness.



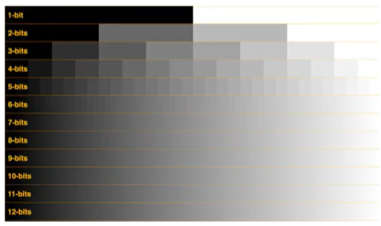
True Label: 3

- The image here is not blurred. If we consider multiple pixels, it may seem blurred. But if we consider a single pixel, then we will understand it’s not. Here the apparent blurriness occurs because of the difference in the pixels’ intensity.

Bit Precision

Bit	Ratio
8	256:1
10	1024:1
12	4096:1
14	16384:1
16	65536:1

Bit Depth	Available Grey Levels	Available Grey Levels
1 bit	2^1	2
2 bit	2^2	4
3 bit	2^3	8
4 bit	2^4	16
6 bit	2^6	64
8 bit	2^8	256
10 bit	2^{10}	1024
12 bit	2^{12}	4096



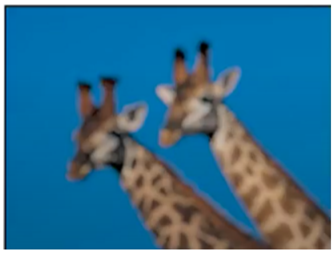
The more bits, the more shades

Image Processing

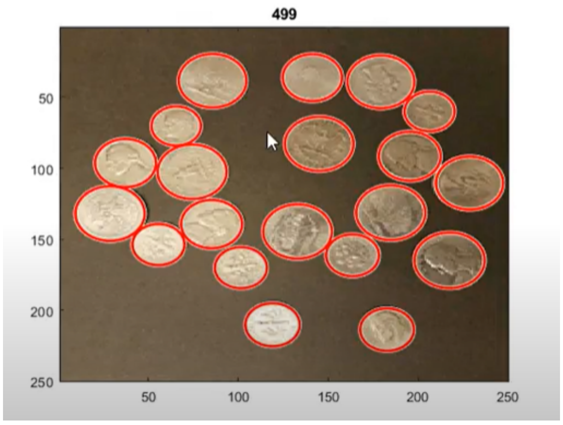
- Transform the image to a new one that is clearer or easier to analyze

Clearer Image or Enhanced Image

- Grainy Image
- Motion Blur
- Defocus Blur



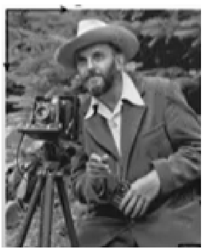
Easier to Analyze



- By finding the features of interest and enhancing them, we can analyze the picture in a better way. In this image, we can enhance the properties of the borders of the coins using image processing so that we can get more information.

Pixel Processing or Point Processing

- Go to the brightness value of each pixel and transform it using some pre-determined mapping.
- Independent of the pixel location.



- f(x, y) is the image intensity at position (x, y)

- Image as a function **f(x, y),** where f is the intensity at the spatial coordinates**(x, y)**.
- If we have a colour image, there will be multiple channels, **red, green, and blue -** each of which will be a function.

$$Transformation\ T\ \ of\ intensity\ \ f\ at\ \ each\ \ pixel\ \ to\ \ intesity\ \ g:$$
$$g(x,y) = T(f(x,y))$$

💡 **Pixel processing/ Point Processing** is the simplest type of processing we can apply to an image. Taking a pixel, we can simply transform its brightness value based on the value itself, and independent of the location of the pixel or the values of other pixels in the image. It is a mapping of one brightness value to another brightness value or one colour to another colour.

Darken Image

$$Darken\ \ (f - value)$$

Lighten Image

$$Lighten\ \ (f + value)$$

Invert Image

$$Invert\ (maxValue - f)$$

Lower Contrast

$$Low\ Contrast\ (f/value)$$

Higher Contrast

$$High\ Contrast\ (f * value)$$

RGB to Grayscale

$$Gray\ (.3\ f_R + 0.6\ f_G + 0.1\ f_B)$$
$$..$$
$$[\text{Here any value can be multiplied with the R, G, B of f. But the values should sum to 1}]$$

> In all the cases, **Pillow**, a Python library for images was used for image processing.

> RGB images were split into three channels, **R, G and B** and then the mathematical processes were done. Then they were merged again for color images.

LSIS System

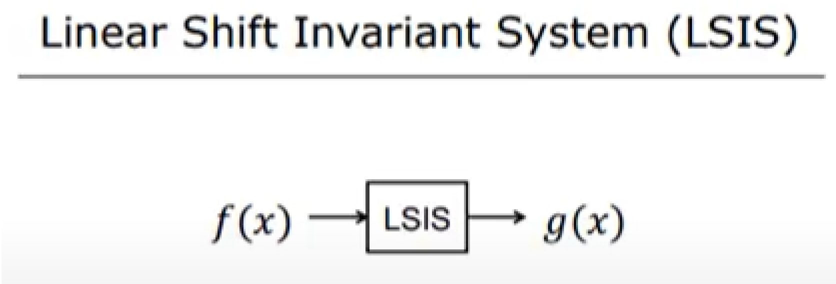
Convolution

LSIS System

💡 **Linear Shift Invariant System(LSIS):** The study of this class of system is important because it leads to many useful image processing algorithms.

LSIS for one-dimensional signals

Here is an LSIS system with input f(x) and output g(x)



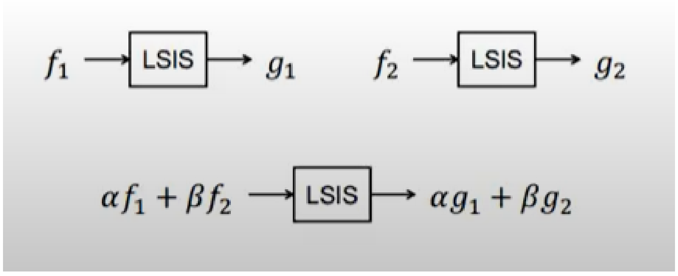
LSIS system has two properties

- Linearity.
- Shift Invariance.

💡 **Linearity**

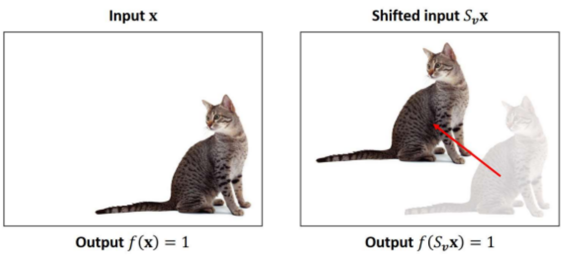
The first property of an LSIS is that it is **linear**.

Imagine we have a system. When we feed it an **input f1** we get an **output g1**, and when we feed it f2 we get g2. If it is a linear system, some linear combination of inputs, should yield the same linear combination of the corresponding outputs. If this condition is satisfied, we say that the system is linear.

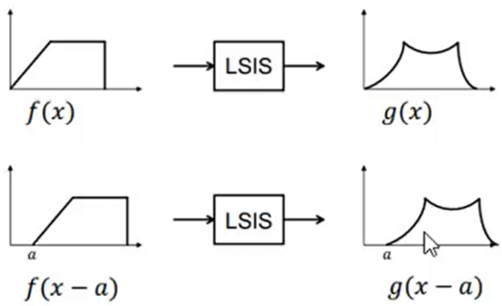


Shift Invariance

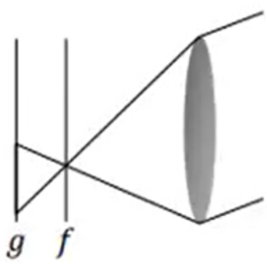
The input is **f(x)** and that the corresponding output is **g(x)**. In the case of a **shift-invariant** system, if we shift the input by **a**, then the output will
also be shifted by
a. If this condition is satisfied, we say that the system is shift-invariant.



The image of the cat is unchanged after shifting in the frame



Relevance of LSIS in Imaging System



- **Linearity**: Brightness variation
- **Shift Invariance:** Scene Movement

- Focused image **(f)**
- Defocused image **(g)**

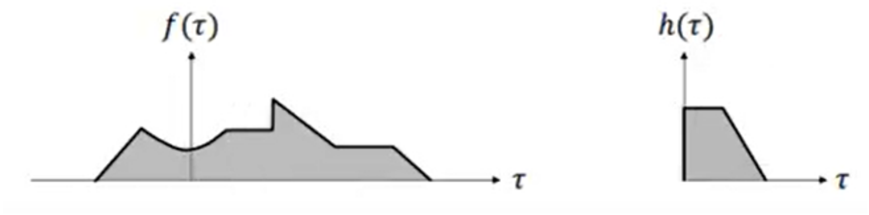
Convolution

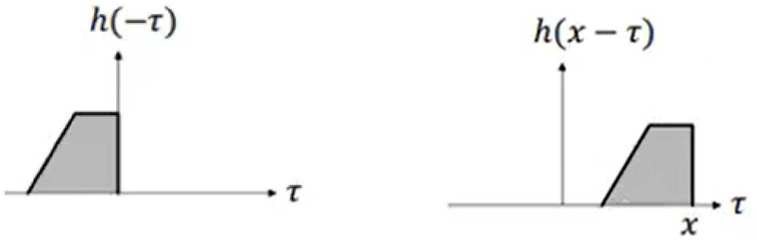
Convolution in one dimension

- Convolution of two functions f(x) and h(x). We have f(x) convolved with h(x) to get the result g(x). Denoted by **asterisk (*).**

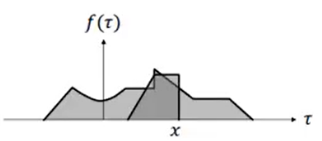
$$g(x) \ = \ f(x) \ * \ h(x)$$

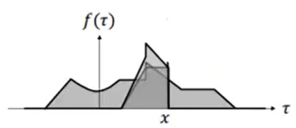
$$f(x) \ * h(x) \ = \int_{\infty}^{\infty} f(\tau)\, h(x-\tau)\, d\tau$$

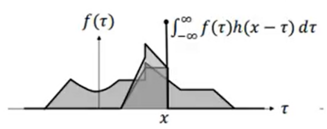




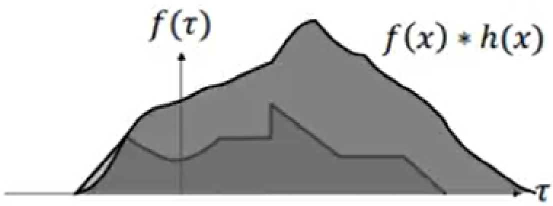
1 **2** **3**







For entire g(x)



The final result of convolution for the 1D function

Where are we?

💡 LSIS implies Convolution and Convolution implies LSIS

> It turns out that any linear shift-invariant system performs a convolution; whenever we do a convolution, we have a linear shift-invariant system.

More on 1D Convolution

Online convolution demo: https://phiresky.github.io/convolution-demo/

Proof

> ### LSIS implies Convolution and Convolution implies LSIS

Linearity

Let's convolve **f1** with **h** and get **g1** as output.

$$g_1(x) = \int_{-\infty}^{\infty} f_1(\tau)\, h(x-\tau)\, d\tau$$

Let's convolve **f2** with **h** and get **g2** as output.

$$g_2(x) = \int_{-\infty}^{\infty} f_2(\tau)\, h(x-\tau)\, d\tau$$

We have found two different outputs for two distinct inputs convolving with the same **h**.

Here, we will find the linear combination of the inputs **f1 and f2** and convolve that with **h**. Then try to find the output of it.

$$\int_{-\infty}^{\infty} \Big(\alpha\ f_1(\tau)\ +\ \beta\ f_2(\tau)\Big)\ h(x-\tau)\, d\tau$$

$$= \alpha\ \int_{-\infty}^{\infty} f_1(\tau)\, h(x-\tau)\, d\tau\ +\ \beta \int_{-\infty}^{\infty} f_2(\tau)\, h(x-\tau)\, d\tau$$

$$= \alpha\ g_1(x)\ +\ \beta\ g_2(x)$$

This proves that convolution is linear.

Shift-Invariance

$$g(x) = \int_{-\infty}^{\infty} f(\mathrm{T})\, h(x-\mathrm{T})\, d\mathrm{T}$$

$$f(\mathrm{T}-\alpha)$$

$$\begin{aligned} g(x) \;&=\; \int_{-\infty}^{\infty} f(\mathrm{T} \;-\; \alpha)\, h(x-\mathrm{T})\, d\mathrm{T} \\ &= \int_{-\infty}^{\infty} f(\mu)\, h(x-\mathrm{T})\, d\mathrm{T} \qquad \left[\, \mu \;=\; \mathrm{T} \;-\; \alpha \,\right] \end{aligned}$$

Again,

$$\mathrm{T} \;=\; \alpha \;+\; \mu$$

So,

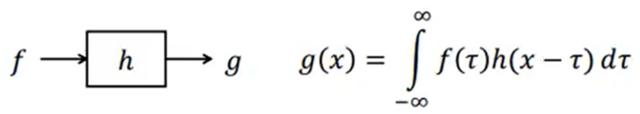
$$\begin{aligned} &\int_{-\infty}^{\infty} f(\mu)\, h(x- \;\alpha\; - \mu)\, d\mathrm{T} \\ &= \; g\;(x-\alpha) \end{aligned}$$

Here,

$$g(x) = \int_{-\infty}^{\infty} f_1(\mathrm{T})\, h(x-\mathrm{T})\, d\mathrm{T}$$

This proves that convolution is shift invariant

Unit Impulse Function

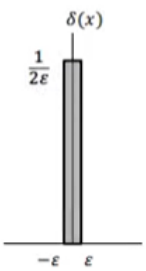


Let’s assume that we are given a system that is linear and shift-invariant. We know that it is doing a convolution, but we do not know what it is
convolving the input with. Let us assume the system is a black box that we cannot
"open up" to determine what the function h(x)

Solution?

$$h(x) = \int_{-\infty}^{\infty} ?(\tau)\ h(x-\tau)\ d\tau$$

$$\delta(x) = \begin{cases} 1/2\ \epsilon, & |x|\ \leq\ \epsilon \\ 0, & |x|\ >\ \epsilon \end{cases}$$
$$\epsilon\ \rightarrow\ 0$$



Unit Impulse Function

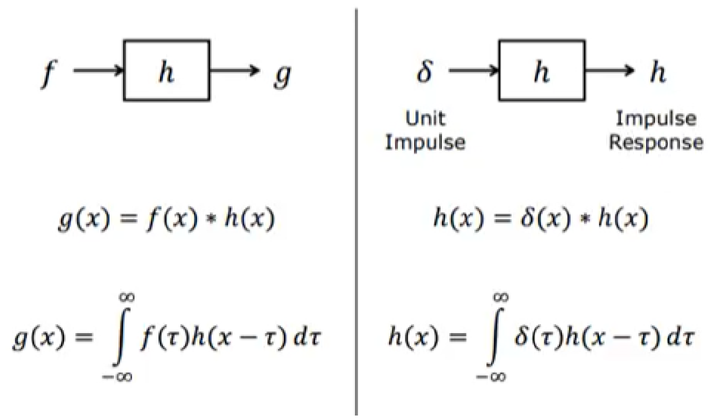
We will pass a unit value through the function to get the value of **h.**

Here the function will be **delta(x).** This is called the **unit impulse function.**

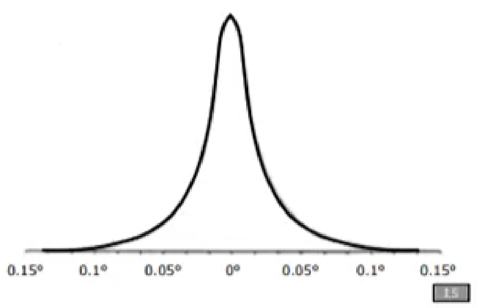
$$\int_{-\infty}^{\infty} \delta(\tau)\ b(x-\tau)\ d\tau = b(x)$$

💡 We take the unit impulse function, flip it, move it to minus infinity, and slide it over **b(x)** while finding the integral of the product of the two functions at each point. Since we are integrating over an infinitesimal width (the width of the impulse function) and the area of the impulse function is one, we simply end up reading out the values of the function **b(x)**.
Thus, any function convolved with the unit impulse function is the original function itself. This is called the **sifting property of the unit impulse function.**

Impulse Response



Point Spread Function (PSF)



Human Eye PSF

Properties of Convolution

$$Commutative\ Law \quad a * b = b * a$$

$$Associative\ Law \quad (a * b) * c = a * (b * c)$$

Cascaded System

Sequence of convolutions