

### Supplementary Note 1: Sensitivity to additional death terms

It should be noted that our set of dynamics (Eqs. 2 and 4, main text) could include a constant death term of the form  $-d_F F$  and  $-d_H H$  to represent death not directly linked to starvation. Adding terms of this form to our model would simply adjust the effective value of  $\lambda$  and  $\mu$ , and we could rewrite Eq. 4 with  $\lambda' = \lambda - d$  and  $\mu' = \mu - d$ . These substitutions would not alter the functional form of our model nor the steady-states and qualitative results, however the quantitative values could shift based on the size of  $d$  relative to  $\lambda$  and  $\mu$ .

Survivorship has a well-known functional form which changes systematically with size (e.g.<sup>17</sup>). Typically survivorship is defined using the Gompertz curve

$$F = F_0 e^{(c_0/c_1)(1-e^{c_1 t})} \quad (1)$$

where the parameters have the following allometric dependencies on adult mass  $c_0 = a_0 M^{b_0}$  and  $c_1 = a_1 M^{b_1}$ , with  $a_0 = 1.88 \times 10^{-8}$  (s g<sup>-b<sub>0</sub></sup>),  $b_0 = -0.56$ ,  $a_1 = 1.45 \times 10^{-7}$  (s g<sup>-b<sub>1</sub></sup>), and  $b_1 = -0.27$  (see<sup>17</sup> for a review).

We are interested in the specific death rate of the form  $\dot{F} = -dF$ , and using the derivative of Eq. 1 we find that  $d = c_0 e^{c_1 t}$ . Our model considers the average rates over a population and lifecycle and the average death rate is given by

$$\bar{d} = \frac{1}{t_{\text{exp}}} \int_0^{t_{\text{exp}}} c_0 e^{c_1 t} dt \quad (2)$$

$$= \frac{c_0 (e^{c_1 t_{\text{exp}}} - 1)}{c_1 t_{\text{exp}}} \quad (3)$$

where  $t_{\text{exp}}$  is the expected lifespan following the allometry of  $t_{\text{exp}} = a_2 M^{b_2}$  with  $a_2 = 4.04 \times 10^6$  (s g<sup>-b<sub>2</sub></sup>) and  $b_2 = 0.30$ <sup>17,18</sup>. Given the allometries above we have that

$$\bar{d} = \frac{a_0 \left( e^{a_1 a_2 M^{b_1+b_2}} - 1 \right) M^{b_0-b_1-b_2}}{a_1 a_2} \quad (4)$$

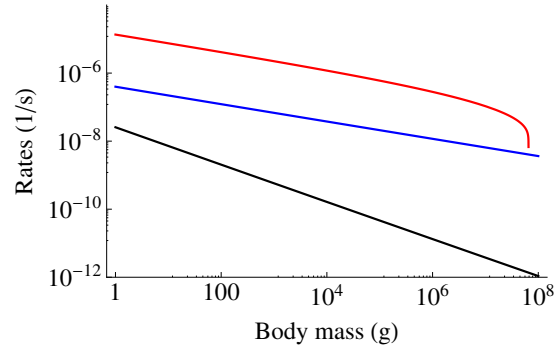
which scales roughly like  $M^{b_0}$  because  $b_1$  and  $b_2$  are close in value but opposite in sign. In Supplementary Figure 1 we compare the value of  $\bar{d}$  to the reproductive,  $\lambda$ , and starvation-based mortality,  $\mu$ , rates. The values of  $\bar{d}$  are orders of magnitude smaller than these rates for all mammalian masses, and thus, adding this non-starvation based death rate to our model does not shift our results within numerical confidence.

### Supplementary Note 2: NSM and the energy equivalence hypothesis

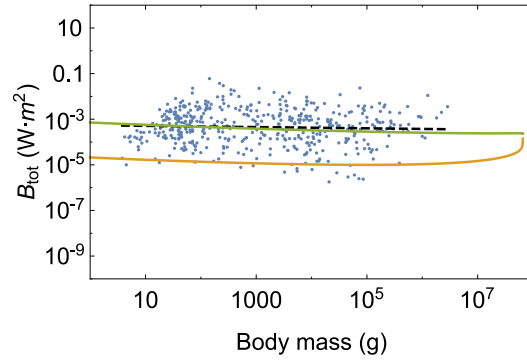
The energy equivalence hypothesis is based on the observation that if one assumes that the total metabolism of an ecosystem  $B_{\text{tot}}$  is equally partitioned between all species ( $B_i$ , the total metabolism of one species, is a constant), then the abundances should follow  $N(M) B(M) = B_i$  implying that  $N(M) \propto M^{-\eta}$ , where  $\eta$  is the metabolic scaling exponent<sup>19,20</sup>. As  $\eta \approx 3/4$  this hypothesis is consistent with Damuth's law<sup>19</sup>. However, the actual equivalence of energy usage of diverse species has not been measured at the population level for a variety of whole populations. Supplementary Figure 2 recasts the results of the NSM in terms of this hypothesis and shows that  $F^* B$  is nearly constant over the same range of mammalian sizes up to the asymptotic behavior for the largest terrestrial mammals.

**Supplementary Note 3: Application of NSM limits to aquatic mammals** A theoretical upper bound on mammalian body size is given by  $\epsilon_\sigma = 0$ , where mammals are entirely composed of metabolic reserves, and this occurs at  $M = 8.3 \times 10^8$  (g), or 120 times the mass of a male African elephant. We note this particular limit as it may have future relevance to considerations of the ultimate constraints on aquatic mammals.

## Supplementary Figures



Supplementary Figure 1: The rates of reproduction  $\lambda$  (blue), starvation-based mortality  $\mu$  (red), and survivorship-based death  $\bar{d}$  (black) as a function of adult mass.



Supplementary Figure 2: Total energetic use  $B_{\text{tot}}$  of consumer populations at the steady state as a function of body mass ( $F^*$  is shown in green and  $H^*$  in orange). The data are from Damuth<sup>21</sup> and have been converted to total population metabolism using the allometric relationships for metabolic rate (e.g. Refs.<sup>3-5</sup>).

## Supplementary References

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