Supporting Information for “The dynamics of starvation and recovery”

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Parameter Values and Estimates

Many of the parameter values employed in our model have either been directly measured in previous studies or can be estimated from combining several previous studies. Here we outline previous measurements and simple estimates of the parameters.

Standard synthesis and metabolic parameters Metabolic rate has been generally reported to follow an exponent close to $\eta = 0.75$ (e.g. [1, 2] and the supplement of [3]). We make this assumption in the current paper, although alternate exponents, which are know to vary between roughly 0.25 and 1.5 for single species [2], could be easily incorporated into our framework, and this variation is effectively handled by the 20% variations that we consider around mean trends. It is important to note the exponent, because it not only defines several scalings in our framework but also the value of the metabolic normalization constant, B_0, given a set of data. For mammals the metabolic normalization constant has been reported to vary between 0.018 (W $\text{g}^{-0.75}$) and 0.047 (W $\text{g}^{-0.75}$) [3, 1], where the former value represents basal metabolic rate and the latter represents the field metabolic rate. We employ the field metabolic rate for our NSM model which is appropriate for active mammals (Table 1).

The energy to synthesize a unit of biomass, E_m, has been reported to vary between 1800 to 9500 (J g^{-1}) (e.g. [1, 2, 3]) in mammals with a mean value across many taxonomic groups of $5,774$ (J g^{-1}) [2]. The unit energy available during starvation, E', could range between 7000 (J g^{-1}), the return of the total energy stored during ontogeny [3] to a biochemical upper bound of $E' = 36,000$ (J g^{-1}) for the energetics of palmitate [4, 3]. For our calculations we use the measured value for bulk tissues of 7000 which assumes that the energy stored during ontogeny is returned during starvation [3].

For the scaling of body composition it has been shown that fat mass follows $M_{\text{fat}} = f_0 M^{\gamma}$, with measured relationships following $0.018M^{1.25}$ [5], $0.02M^{1.19}$ [6], and $0.026M^{1.14}$ [7]. We use the values from [6] which falls in the middle of this range. Similarly, the muscle mass follows $M_{\text{musc}} = u_0 M^{\zeta}$ with $u_0 = 0.383$ and $\zeta = 1.00$ [7].

The final parameters that we must consider connect the resource growth rate to the total metabolic rate of an organism. That is, we are interested in the relative rates of resource recovery and consumption by the total population. From [8] the total resource use of a population with an individual body size of M is given by $B_{pop} = 0.00061x^{-0.03}$ (W m^{-2}). Considering an energy density of 18200 (J g^{-1}) of grass [9] and an NPP between and 1.59×10^{-6} and 7.92×10^{-5} (g s^{-1} m^{-2}) would give a range of resource rates between 0.029 and 1.44 (W m^{-2}). This gives a ratio of total resource consumption to supply rates between 0.00042 and 0.021, and we used a value of 0.002 in our calculations and simulations.

Table 1: Parameter values for mammals

Parameter	Value	References
η	$3/4$	(e.g. [1, 2, 3])
E_m	5774 (J gram^{-1})	[2, 1, 3]
E'_m	$36,000$	[4, 3]
B_0	0.047 (W $\text{g}^{-0.75}$)	[3]
γ	1.19	[6]
f_0	0.02	[6]
ζ	1.00	[7]
u_0	0.38	[7]

Rate equations for invaders with modified body mass If an invading subset of the resident population of mass M has an altered mass $M' = M(1+\chi)$ where χ varies between $[-1, 1]$ ($\chi < 0$ denotes a leaner invader; $\chi > 0$ denotes an invader with more endogenous reserves), the invading population will have the following modified rates: $\sigma' = \sigma(M')$, $\rho' = \rho(M')$, $\beta' = \beta(M')$. Because we are assuming that the invading population is only modifying its endogenous energetic stores, we assume that the proportion of body mass that is non-adipose tissue remains the same as the resident population. This assumption leads to the following modified timescales:

$$
\begin{aligned}
t_{\sigma'} &= \frac{-M^{1/4}}{B_0/E'_m} \log\left(\frac{\epsilon_\sigma}{\chi+1}\right), \\
t_{\rho'} &= \frac{-4M^{1/4}}{B_0/E'_m} \log\left(\frac{1-(\epsilon_\lambda(\chi+1))^{1/4}}{1-(\epsilon_\lambda\epsilon_\sigma)^{1/4}}\right), \\
t_{\beta'} &= \xi B_0 \left(M(\chi+1)\right)^{3/4}.
\end{aligned} \quad [1]
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Reserved for Publication Footnotes