Metabolic scaling of Starvation

Chris Kempes

1 Definition of Starvation and Growth

Notation: M denotes adult (full) mass while m is any current mass smaller than M.

Considering that both metabolic, B, and growth, μ rates (derivable from metabolic rate and a concept of repair energy; e.g. Kempes et al. 2012) scale according to

$$B = B_0 M^{\alpha} \tag{1}$$

and (after taking some approximations)

$$\lambda = \lambda_0 M^{\alpha-1} \tag{2}$$

The value of λ is simply given by the specific growth rate (s^{-1}).

For the rate of starvation, a simple assumption is that all of an organisms metabolic energy is coming from digested tissues which implies the simple energy balance

$$\frac{dm}{dt} E_m = -B_0 m^{\alpha} \tag{3}$$

where E_m is the amount of energy stored in a unit of existing body mass. Considering that M is the adult mass of an organism the energy balance prescribes the mass trajectory of a starving organism as

$$m(t) = \left(M^{1-\alpha} - \frac{B_0 (1-\alpha)}{E_m} t \right)^{1/(1-\alpha)} . \tag{4}$$

We can define the timescales for starvation and death as the mass diminishing to specific fractions of normal adult mass. The idea here is that only certain tissues can be digested for energy, for example the brain cannot be degraded to fuel metabolism, and so at some fraction of initial mass the organisms will starve and then die. If we define $m_{starve} = \epsilon M$ where it could be the case that $\epsilon = \epsilon_0 M^{\gamma}$ (e.g. organisms of different size have different amounts of essential tissues, such as the minimal bone mass requirements or overall brain size, and this changes systematically with overall size). Taking all of this together would give the time scale for starvation as

$$\epsilon M = \left(M^{1-\alpha} - \frac{B_0 (1-\alpha)}{E_m} t_s \right)^{1/(1-\alpha)} \tag{5}$$

$$t_s = \frac{M^{1-\alpha} (1-\epsilon) E_m}{B_0 (1-\alpha)}. \tag{6}$$

The starvation rate (or rate to death which would simply involve a different value of ϵ) should then be proportional to $1/t_s$ which implies that σ and μ both scale like $M^{\alpha-1}$ and are proportional to one another. In mammals $\alpha \approx 3/4$.

Another possibility is that the energy harvested from starving simply equates the maintenance energy in which case we have that

$$\frac{dm}{dt} E_m = -B_m m \tag{7}$$

Perhaps a better form would be

$$\sigma = \frac{1}{t_s} \int_0^{t_s} m'(t)\, dt \tag{8}$$

$$\sigma = \frac{1}{t_s} M (1-\epsilon) \tag{9}$$

$$\sigma = \frac{B_0 (1-\alpha) M^{\alpha}}{E_m} \tag{10}$$