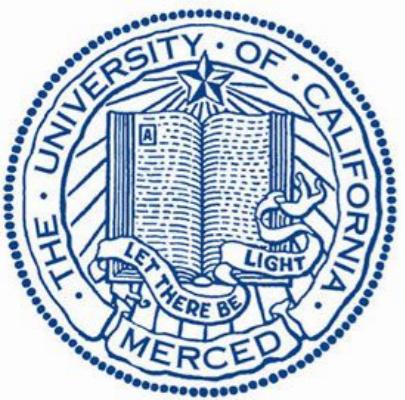


# Extinction, Cope's Rule, and the dynamics of starvation and recovery

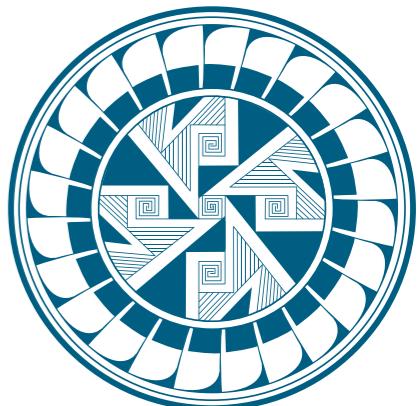


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Santa Fe Institute



Chris Kempes  
Sidney Redner,  
Santa Fe Institute



Organisms condition reproduction on energetic state



Sometimes these activities are partitioned to reinforce the conditional relationship



Via life-history  
-Space  
-Time

Or mediated by behavioral state



## Roadmap for this talk:

- 1) Briefly describe the Nutritional State-structured Model (NSM)
- 2) Implications for extinction risk as a function of mass
- 3) Predictions for contemporary species
- 4) A mechanism for Cope's Rule based on competition dynamics



What is a minimal model to incorporate

- Growth
- Starvation
- Recovery

### **Deriving an *explicit* version:**

**F**ull & **H**ungry consumer class ( $C=F+H$ )

\*\*The transition from  $H \rightarrow F$ ,  $F \rightarrow H$  is driven by resource density

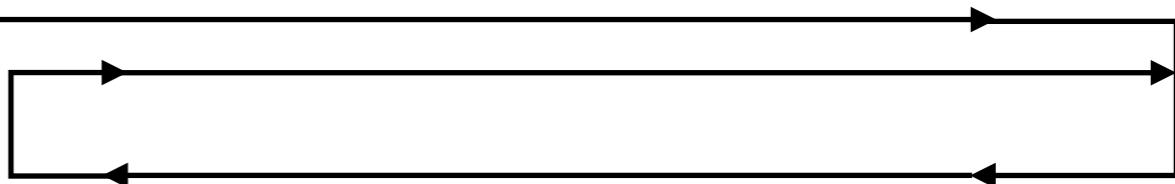
Reproductively active  
Energetically replete

H: Hungry

F: Full

Initial growth

Starved  
Not reproductively active



# Nutritional State Model (NSM)

consumer

growth

recovery

starvation

$$\begin{aligned}\dot{F} &= \lambda F + \xi \rho R H - \sigma (1 - R) F, \\ \dot{H} &= \sigma (1 - R) F - \xi \rho R H - \mu H, \\ \dot{R} &= \alpha R (1 - R) - (\rho R + \delta) H - \beta F\end{aligned}$$

↑  
(NPP)

maintenance

Reproductively active  
Energetically replete

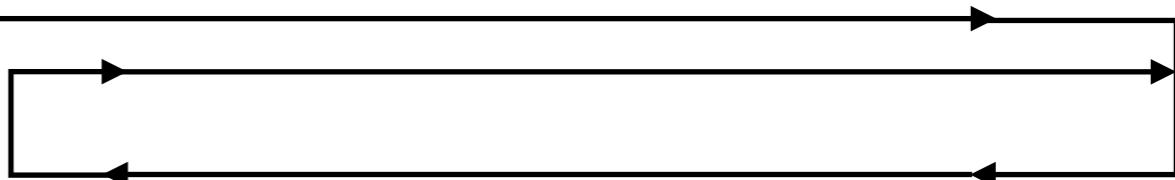
H: Hungry

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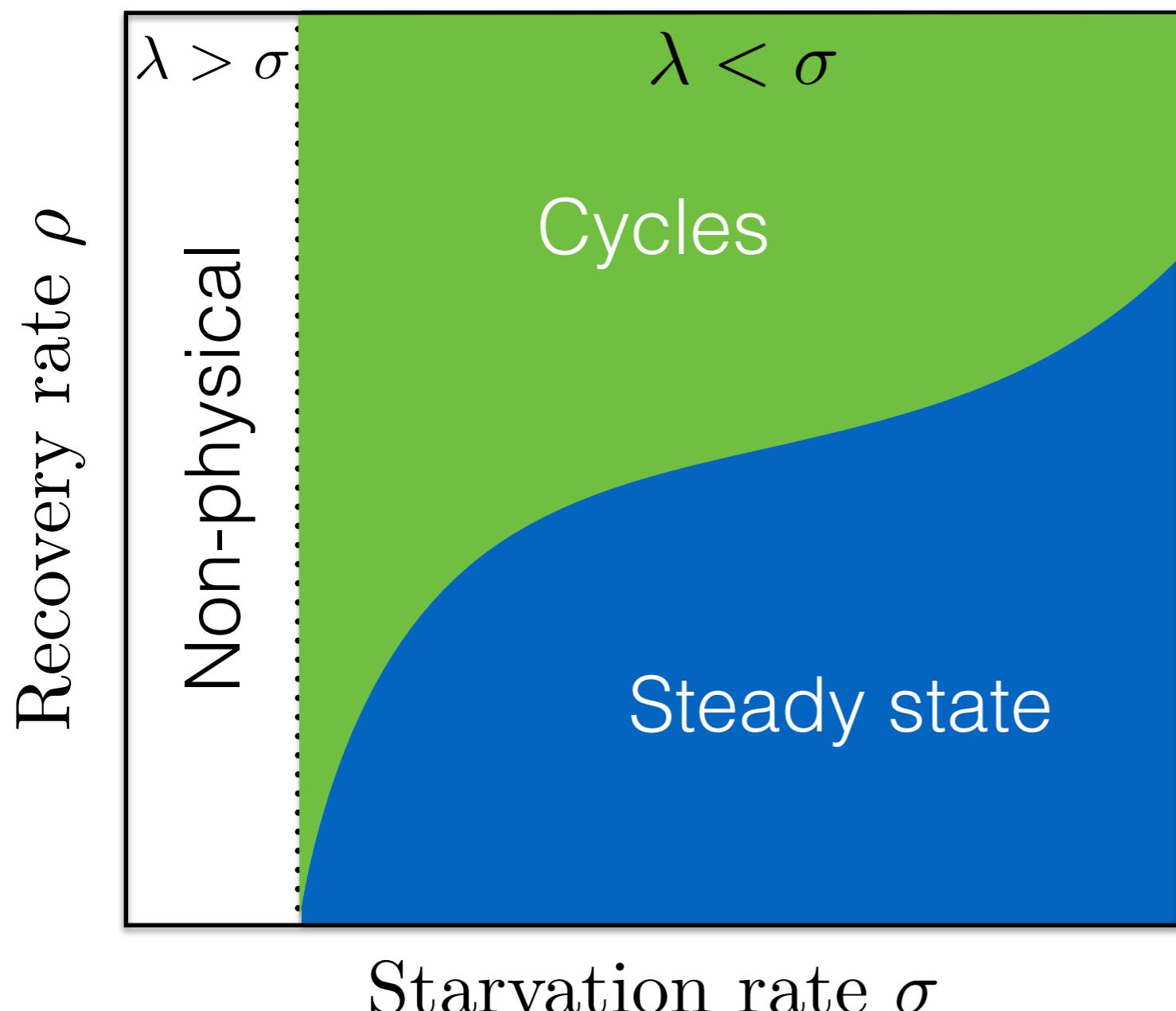


# The general dynamics of the system

$$\dot{F} = \lambda F + \xi \rho R H - \sigma (1 - R) F,$$

$$\dot{H} = \sigma (1 - R) F - \xi \rho R H - \mu H,$$

$$\dot{R} = \alpha R (1 - R) - (\rho R + \delta) H - \beta F$$

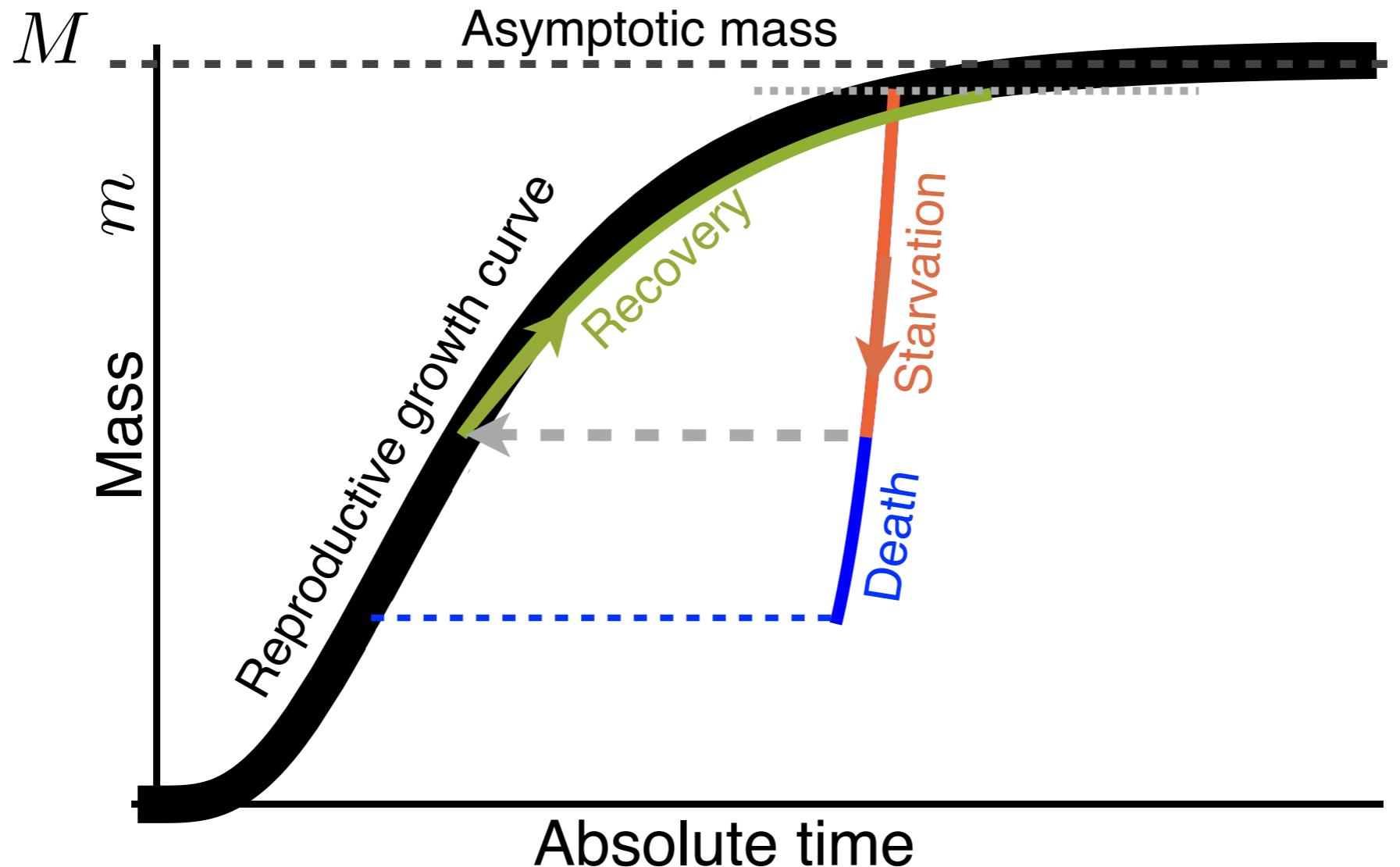


These rates are constrained by metabolism...

Body size!



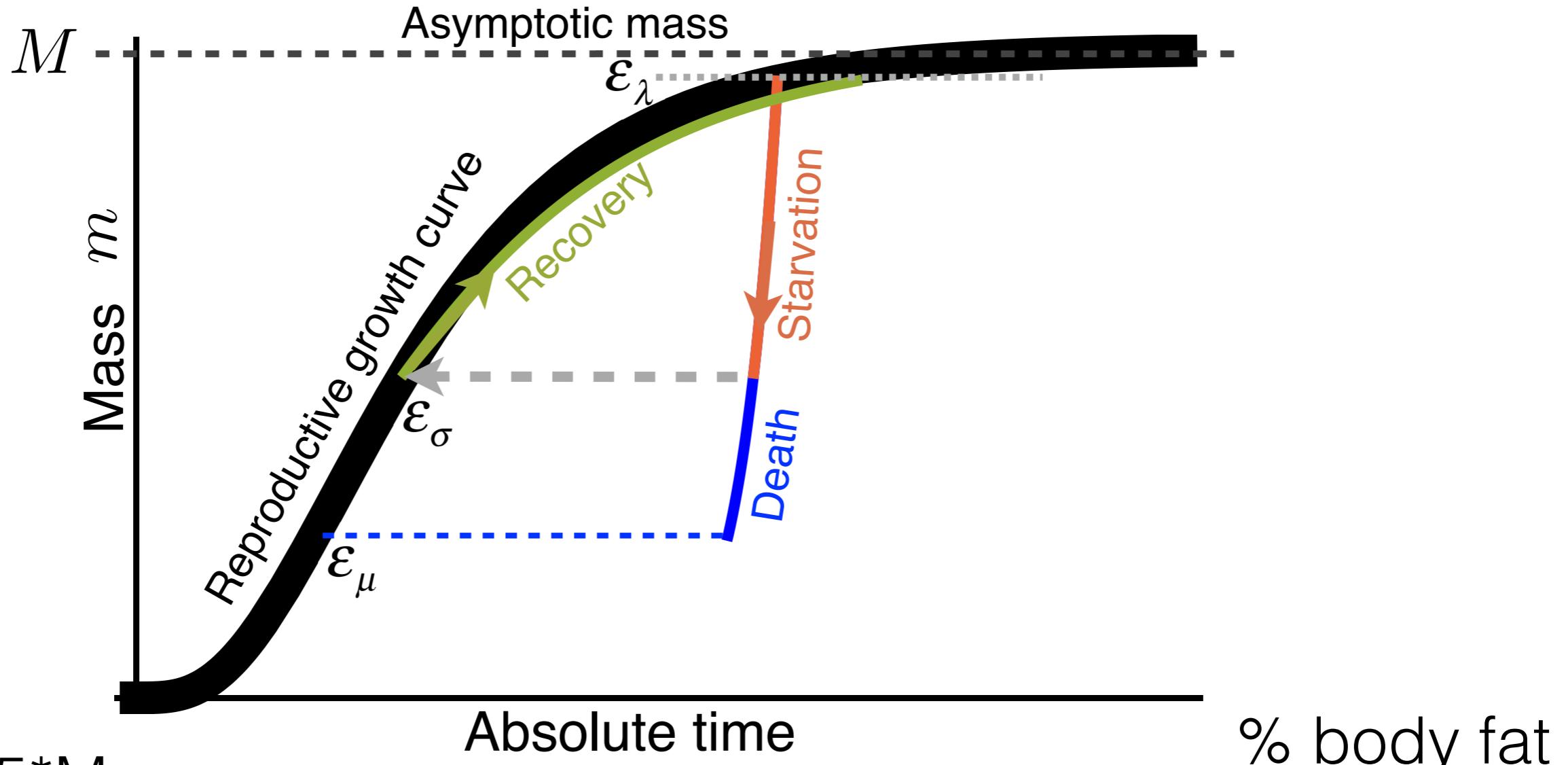
Model for ontogenetic growth supplies nearly all of the rates



$$B_0 m^\eta = E_m \frac{dm}{dt} + B_m m$$

Metabolic rate = Growth + Maintenance

Model for ontogenetic growth supplies nearly all of the rates



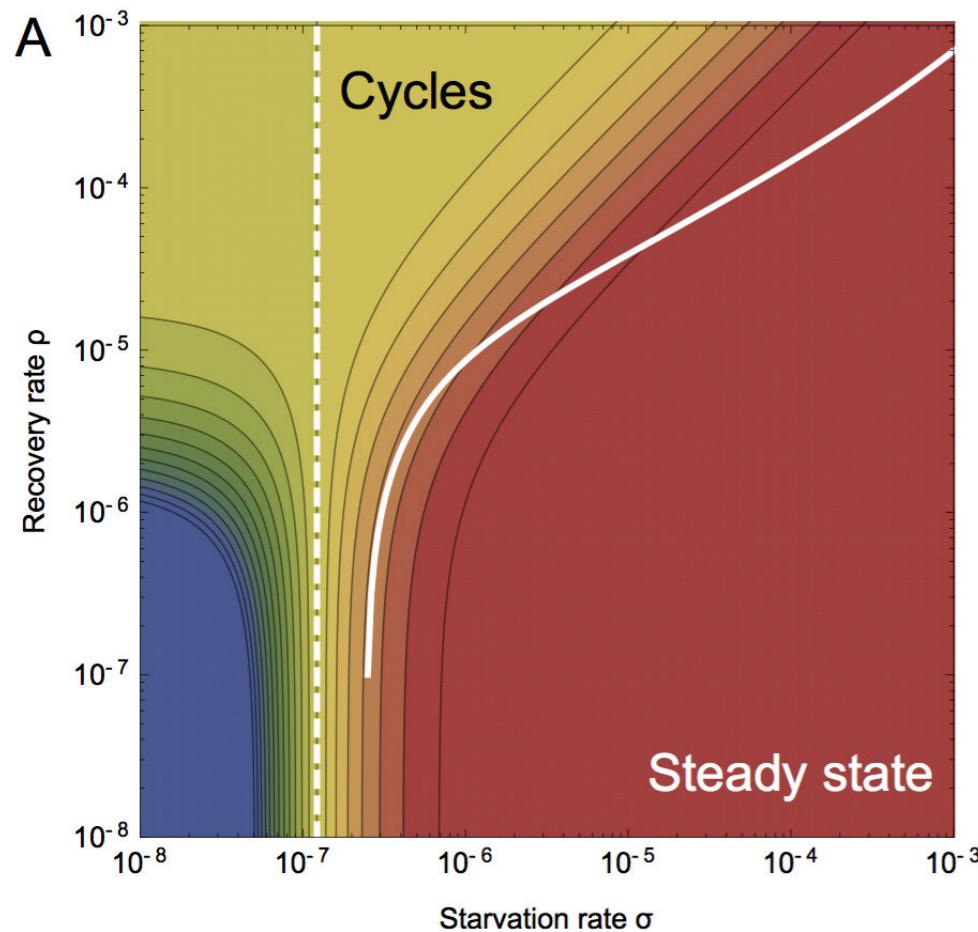
$$\epsilon_\lambda = 0.95 * M$$

$\epsilon_\sigma$  = Organism depleted of fat reserves

$$\epsilon_\sigma = 1 - f_0 M^\gamma / M$$

$\epsilon_\mu$  = Organism depleted of fat+muscle reserves

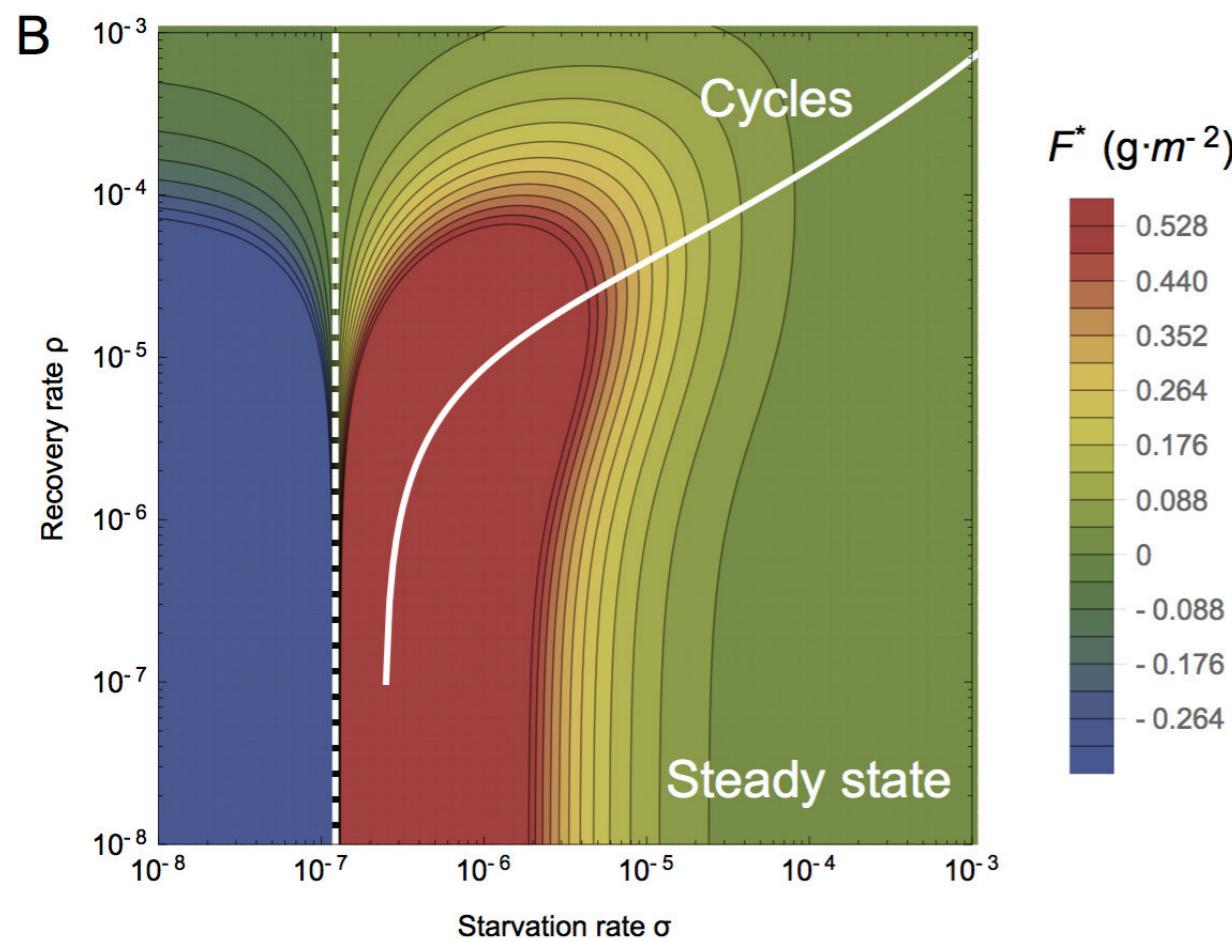
$$\epsilon_\mu = 1 - (f_0 M^\gamma + u_0 M^\zeta) / M$$



Kangaroo rat

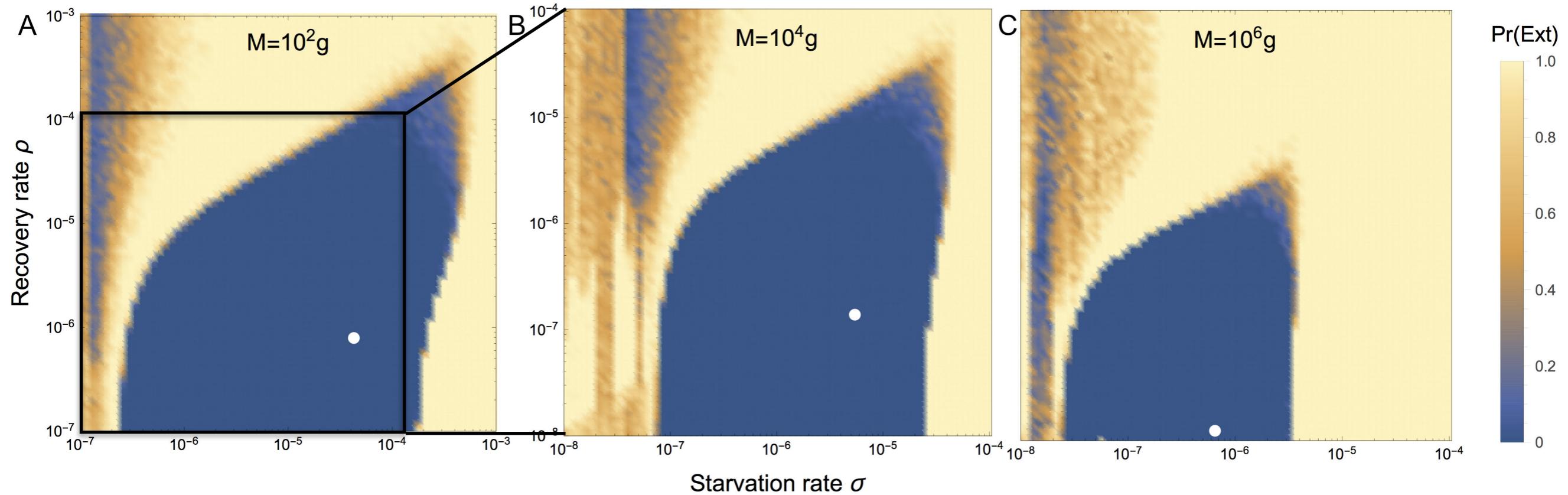
$$M = 100\text{g}$$

Transcritical bifurcation  
Hopf bifurcation



Generally, rates of starvation and recovery close to transitions result in highest biomass densities for the consumer

There are risks in both the cyclic regime and steady state regime

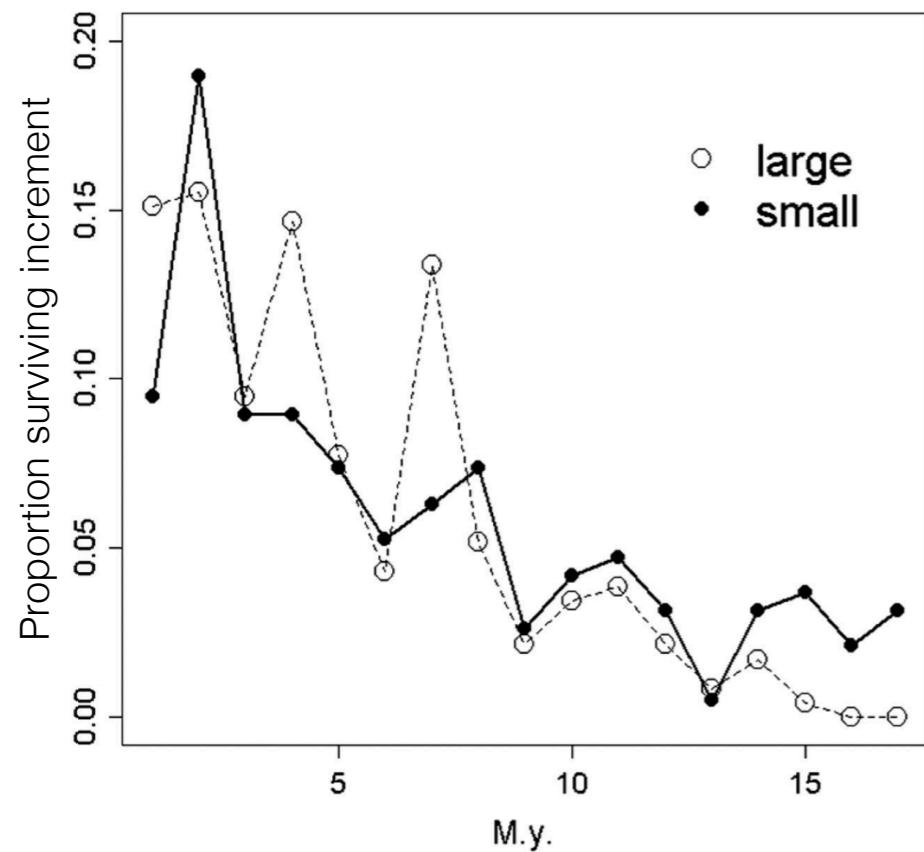


Two boundaries to persistence

- Death by transience
- Death by scarcity

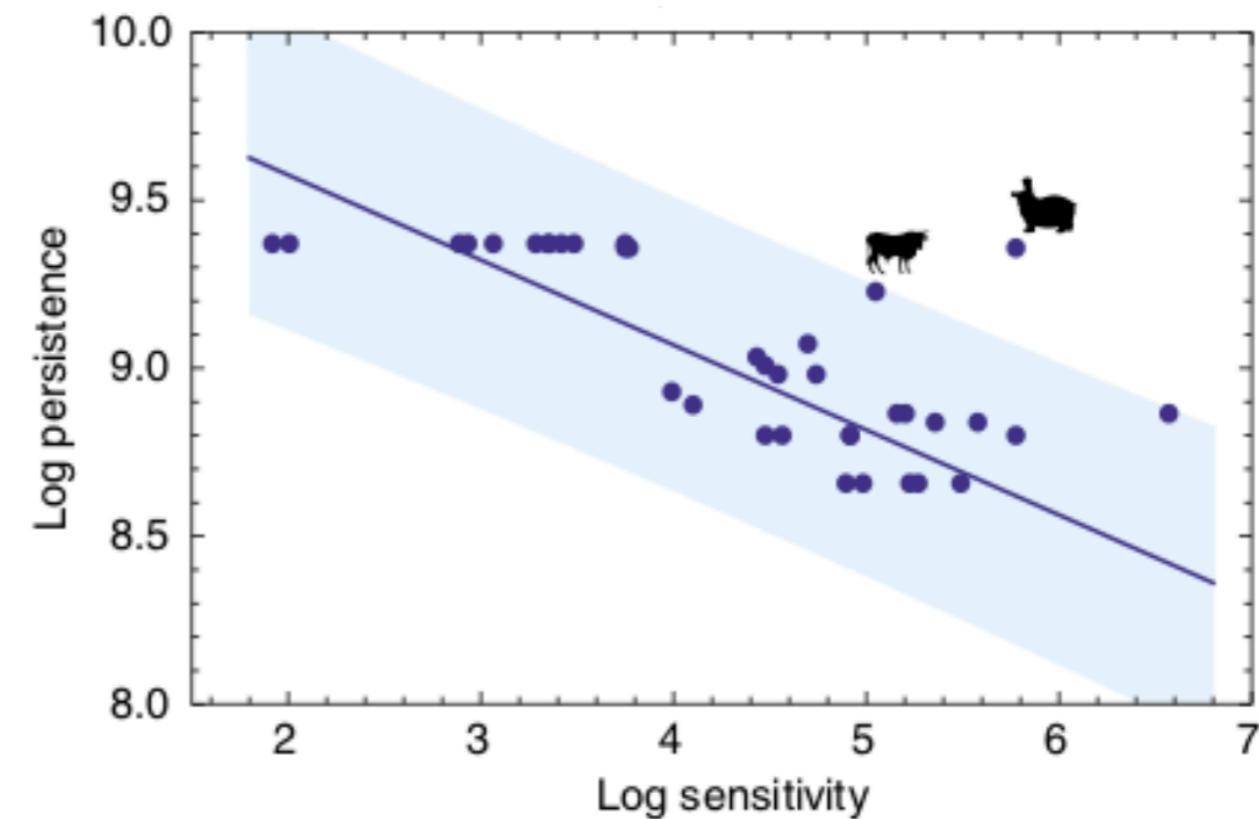
- 1) There is an ‘extinction refuge’, and the expected starvation and recovery rates are found within this range
- 2) This refuge becomes smaller for larger organisms...  
Suggests larger organisms are at greater risk based on the starvation-recovery dynamics in the NSM

# Narrow extinction refuge for larger mammals has qualitative support



Large-bodied genera have shorter durations  
(significant but weak)

Liow et al. 2008

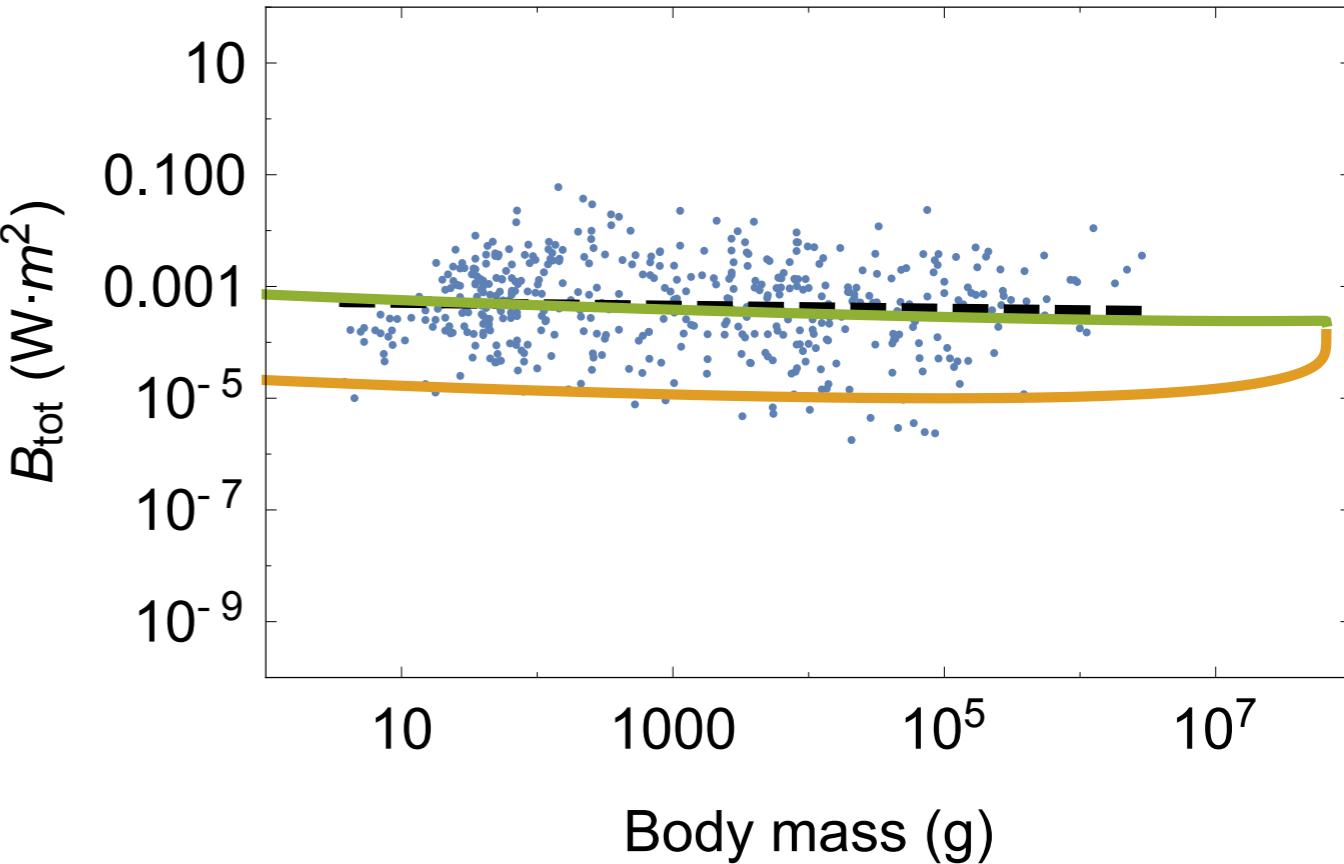
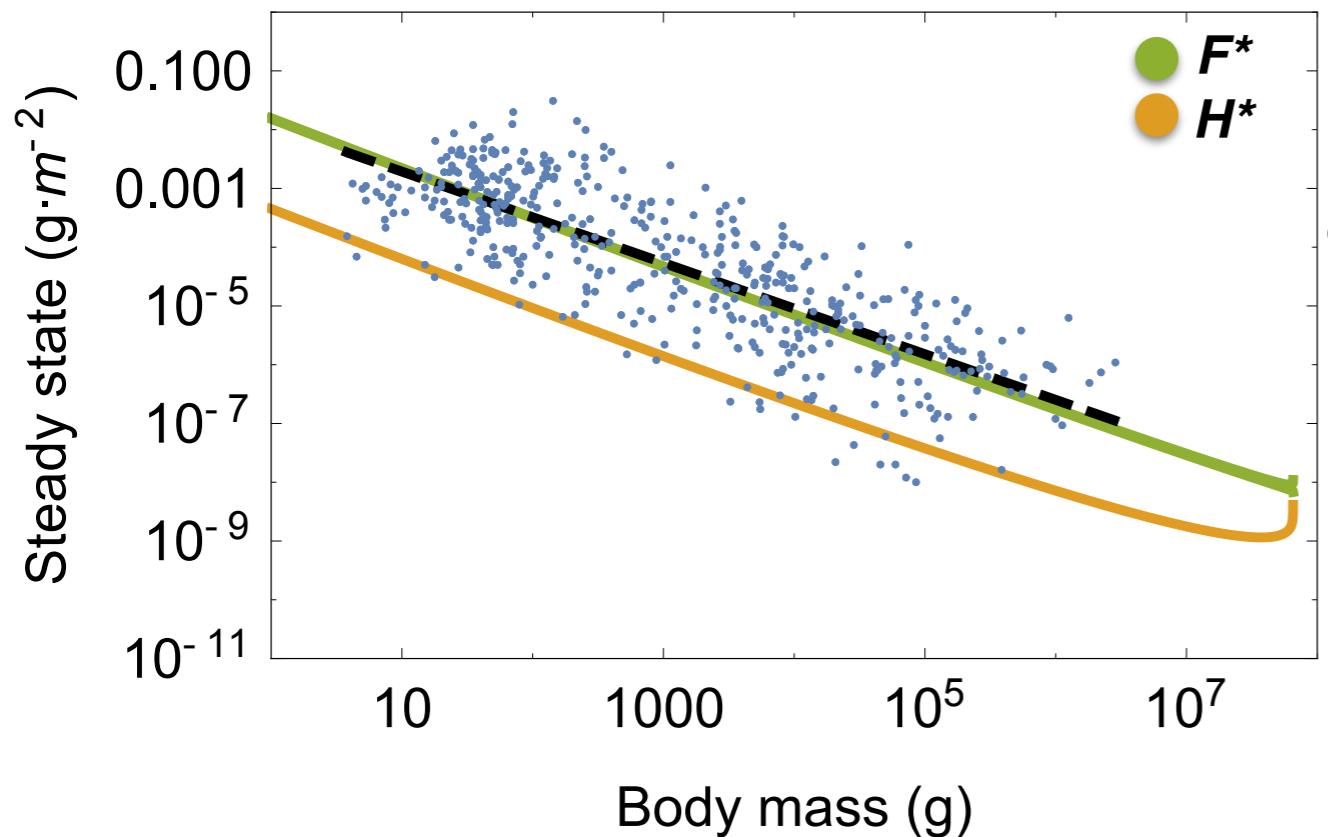


In Egypt, large mammals have persisted for shorter durations over the Holocene

Yeakel et al. 2014

Evolutionary feedback?

# Does the NSM predict anything?

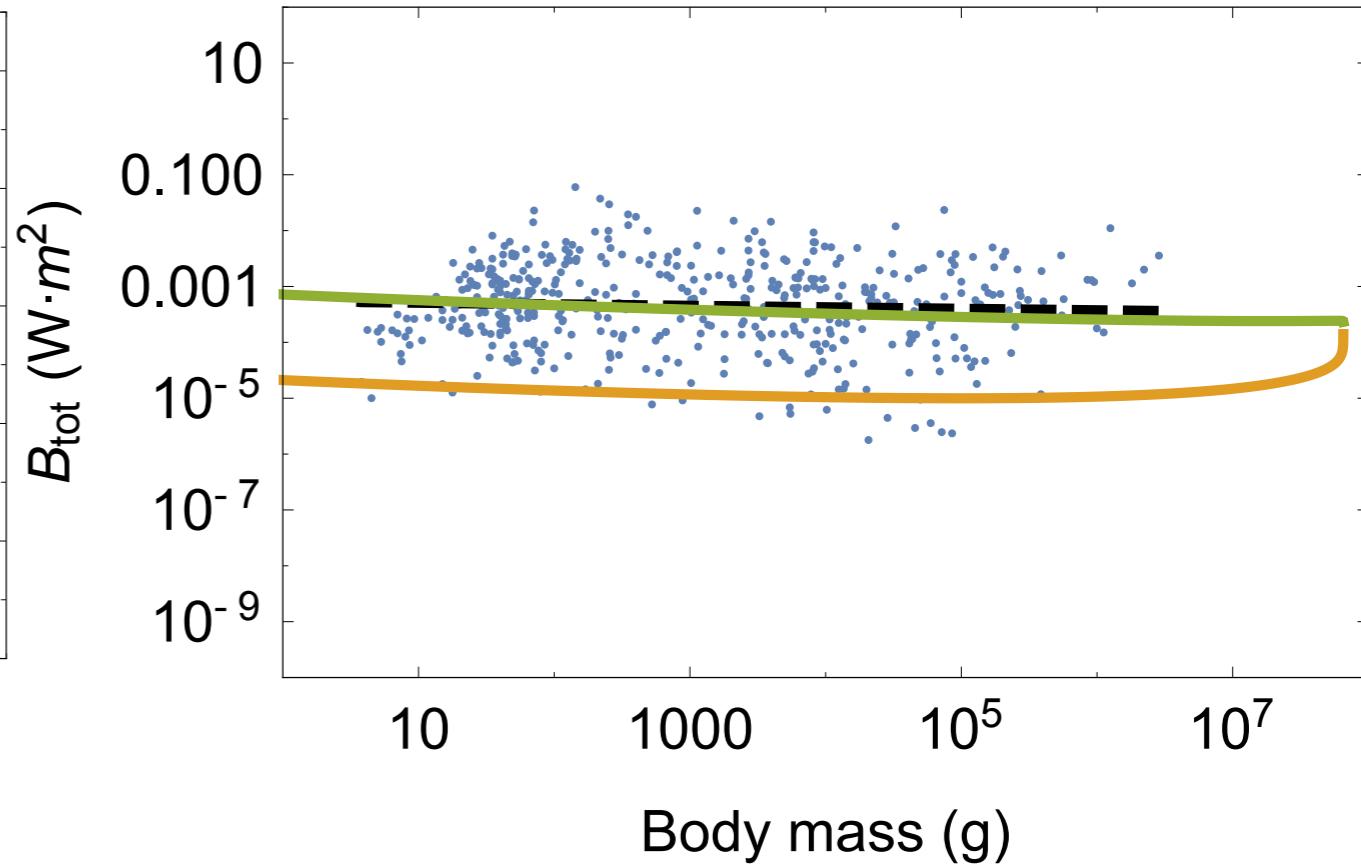
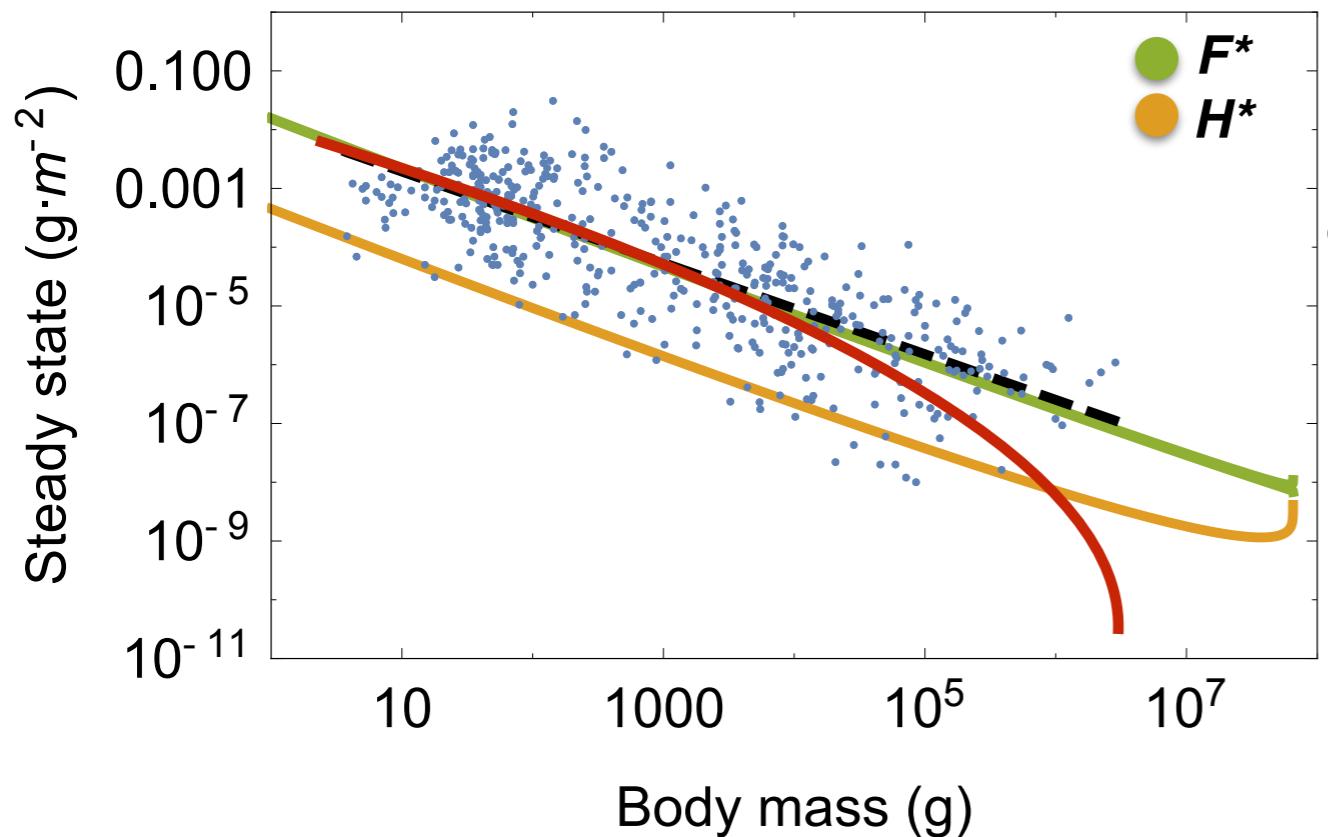


- 1) Intercept scales with resource growth rate
- 2) Slope purely a function of the dynamics and rate equations

$$F^* = (\sigma - \lambda) \frac{\alpha \lambda \mu^2 (\mu + \xi \rho)}{A(\lambda \rho B + \mu \sigma (\beta \mu + \lambda (\delta + \rho)))}$$

- 3) Suggests starvation dynamics are important
- 4) Supports Damuth's Law
- 5) Additional mortality load asymmetrically effects large body sizes ~ extinction risk
- 6) Weird asymptotes at large  $M$

# Does the NSM predict anything?

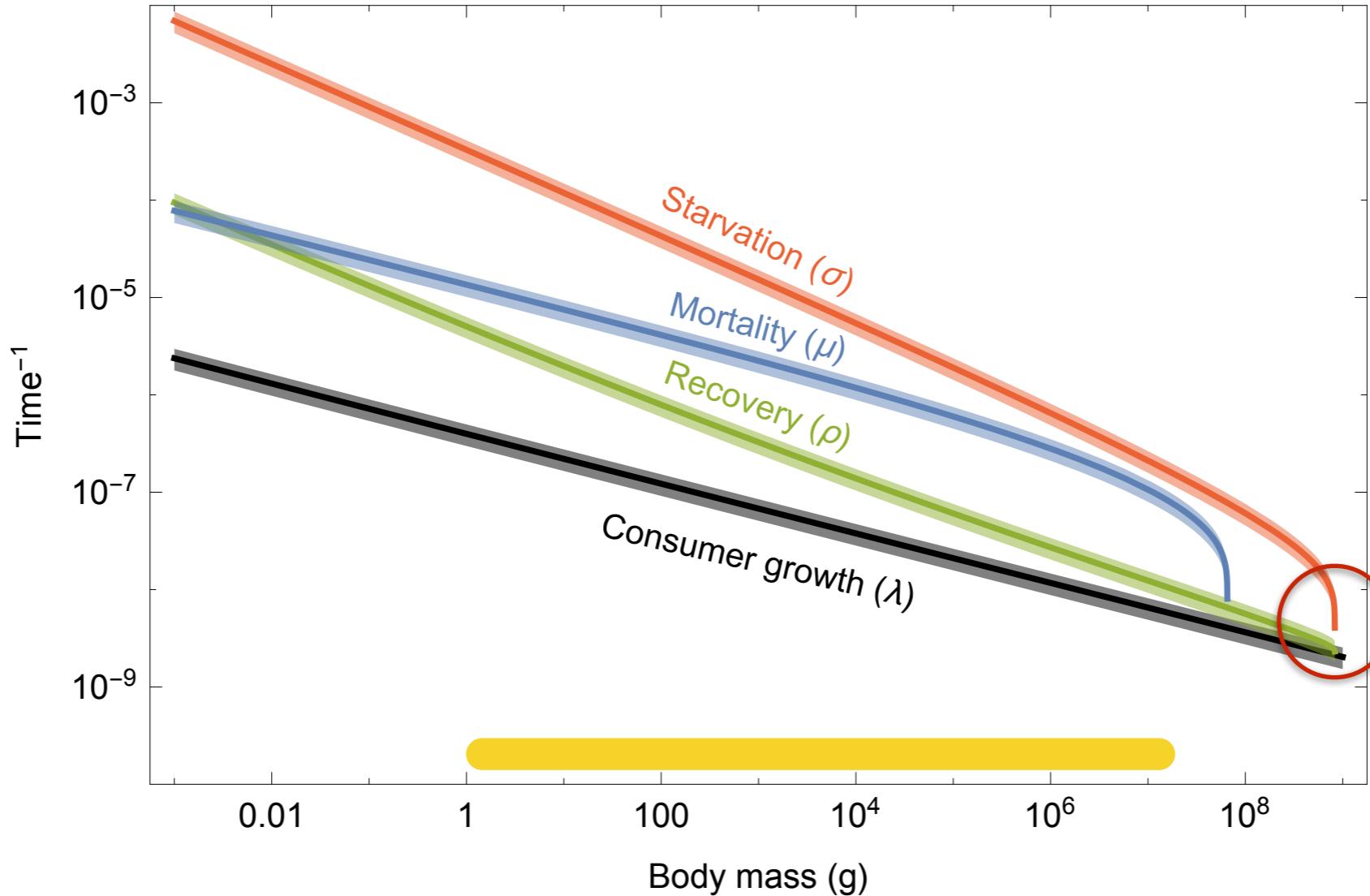


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Does this provide any insight to Cope's Rule, or the evolution of larger body size?



$\lambda < \sigma$  for all realistic values of M

Strong upper-bound to body mass... when body = 100% fat,  
meaning starvation time is infinitely long

$$f_0 M^{\gamma-1} = 1$$

Strong upper-bound to body mass... when body = 100% fat, meaning the rate of starvation become infinite

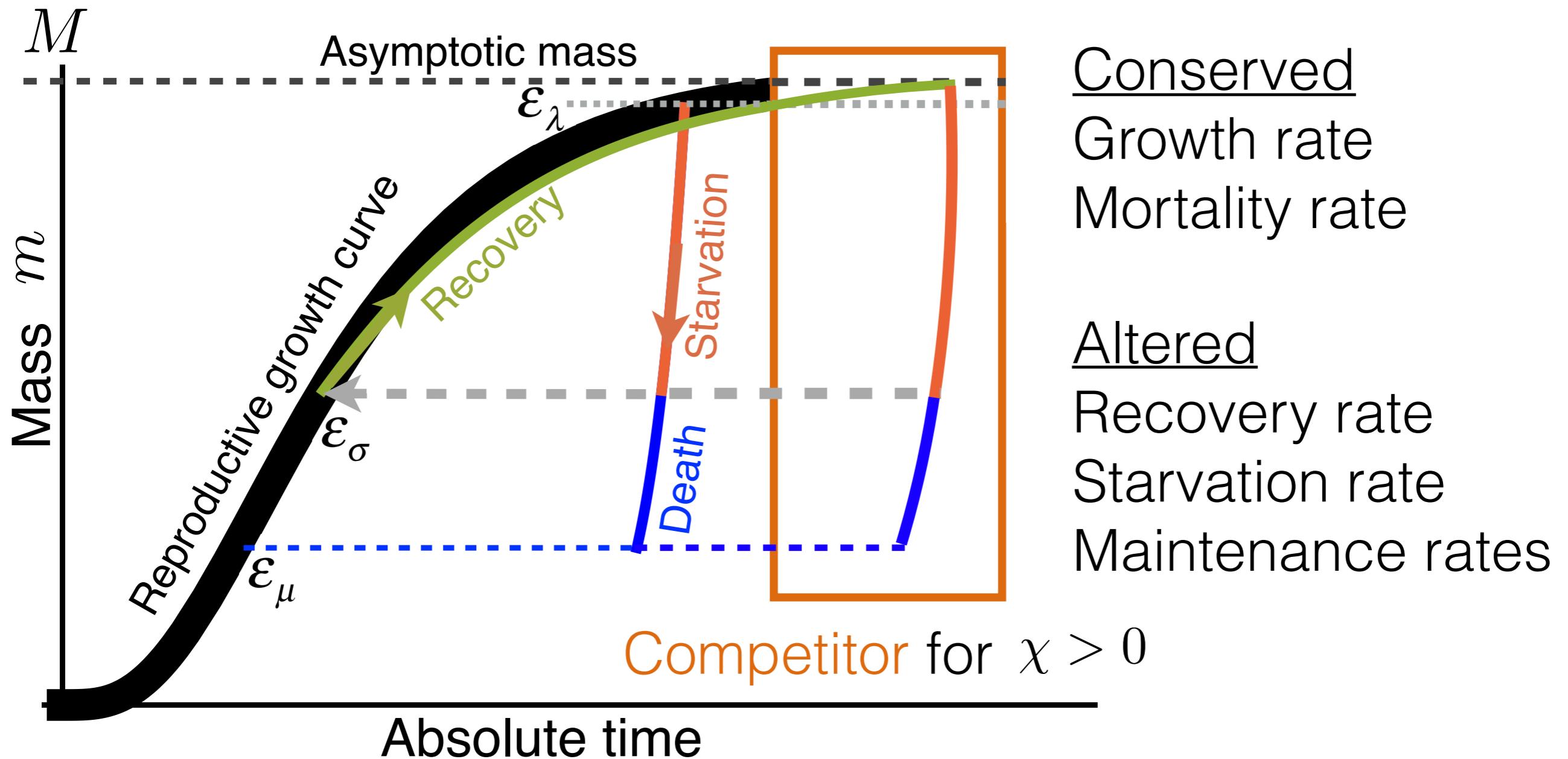
$$f_0 M^{\gamma-1} = 1$$

$M = 8.3 \times 10^8$  ...an organism the size of 120 male African elephants



This is fun, but doesn't introduce a selective mechanism for Cope's rule

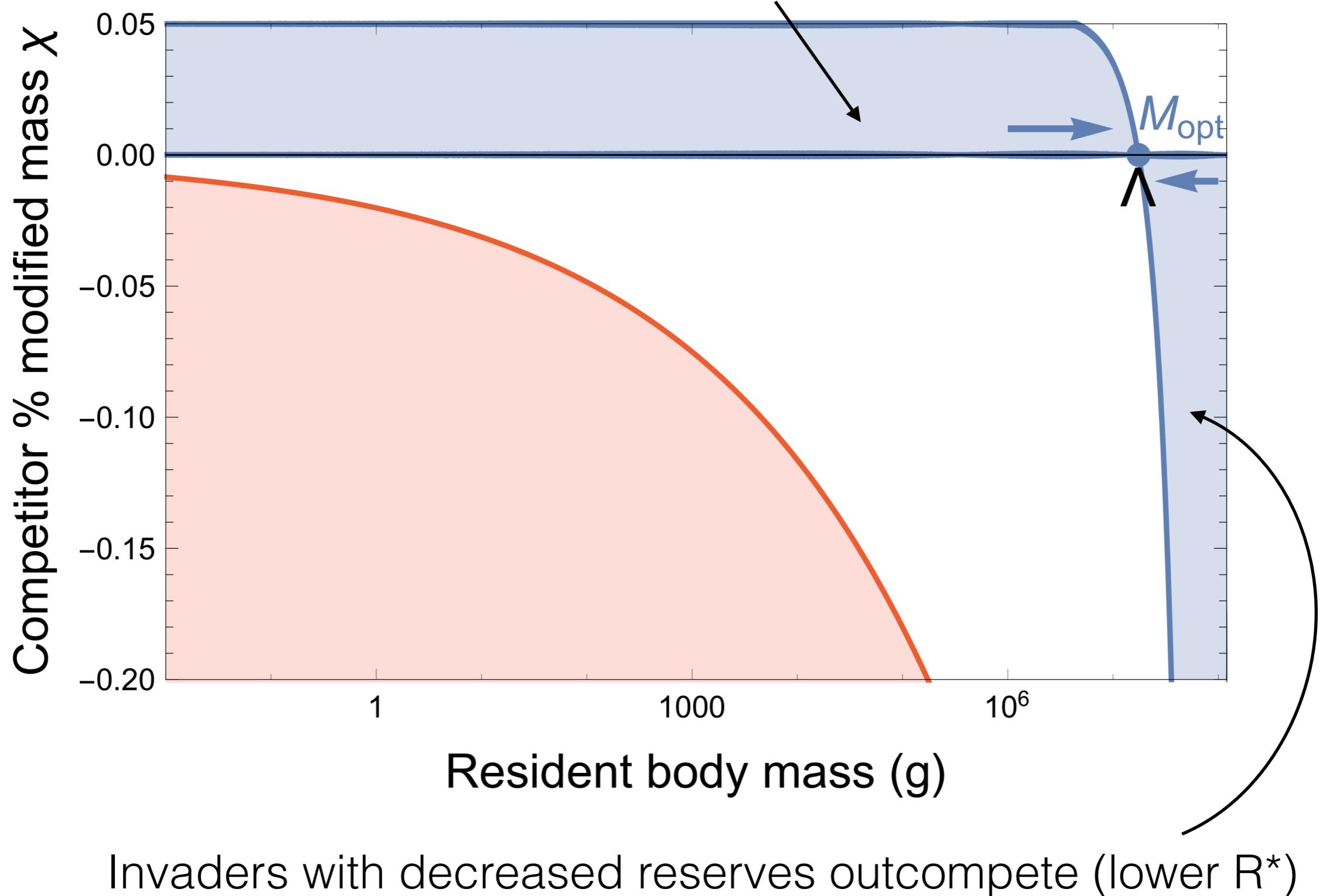
# Characterizing the rates of a competing species



Selection via: who can push resource densities lower?

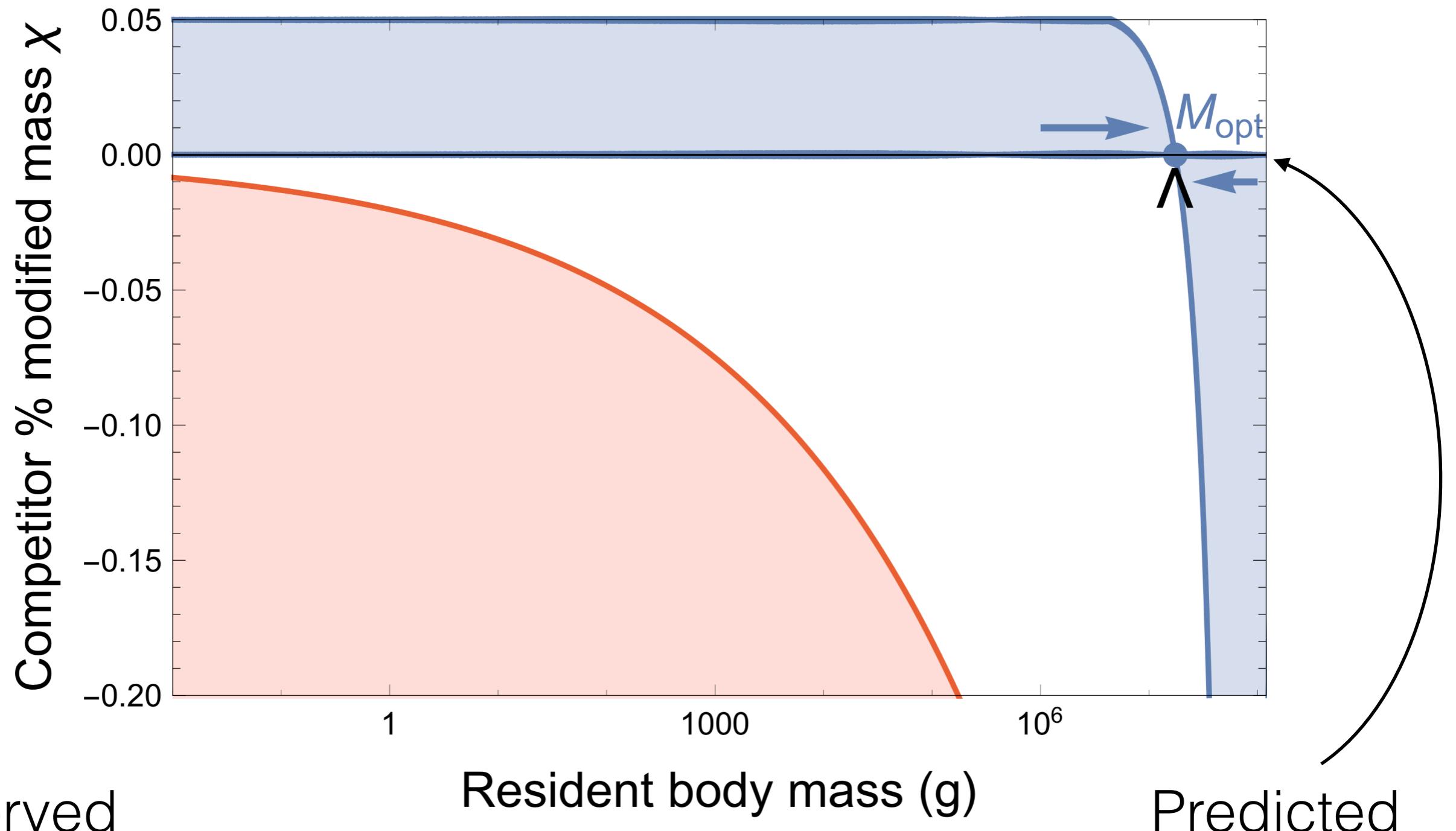
[ $R^*$  theory]

Competitor with increased reserves outcompete (lower  $R^*$ )



Invaders with decreased reserves outcompete (lower  $R^*$ )

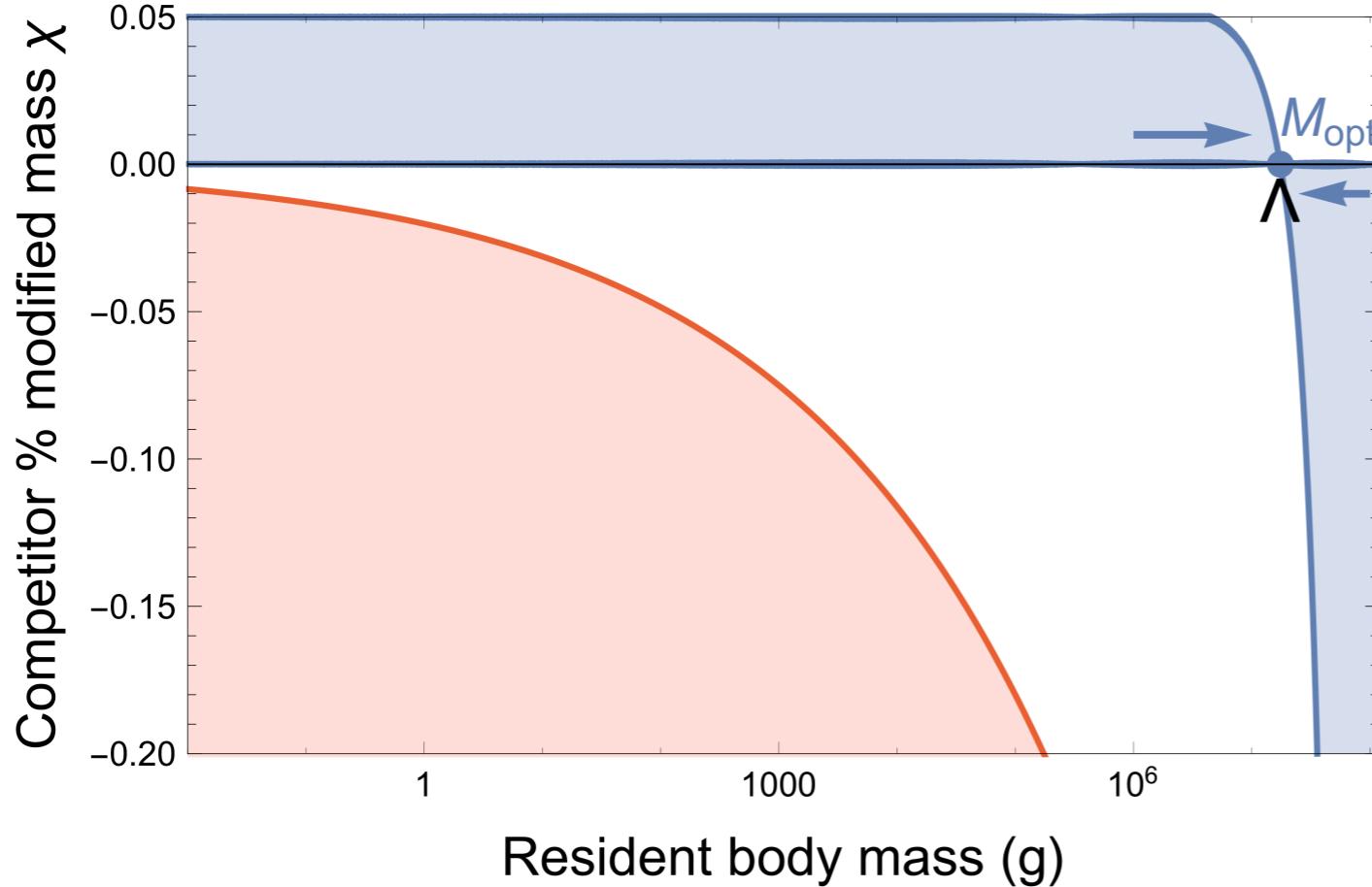
# Evolutionary attractor?



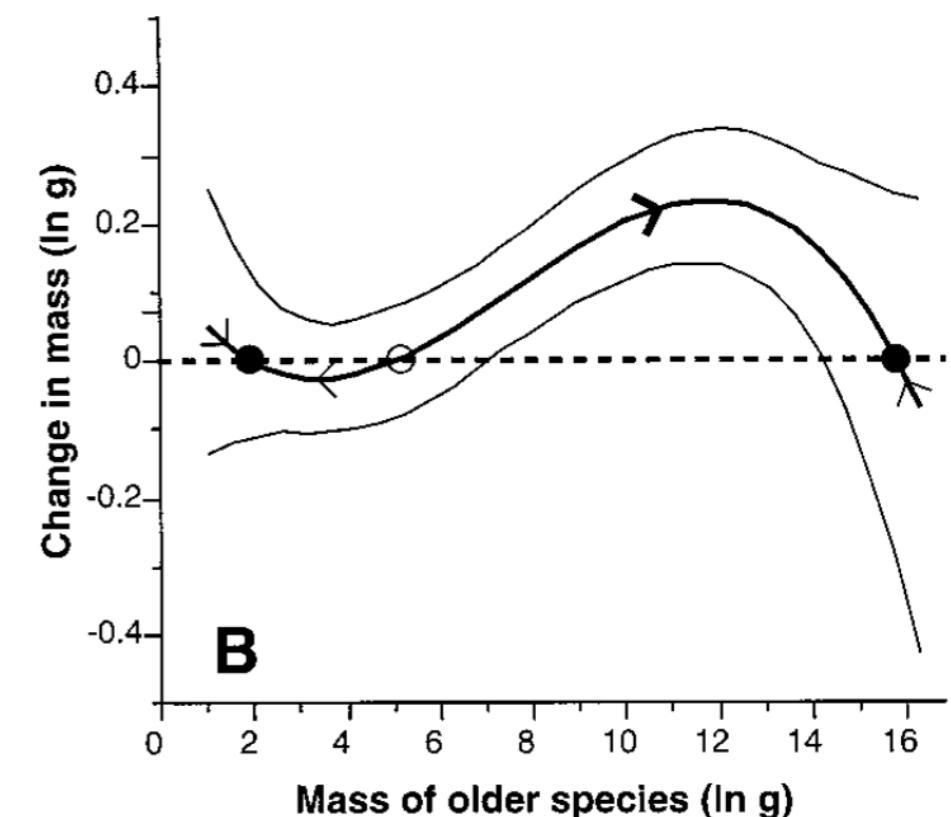
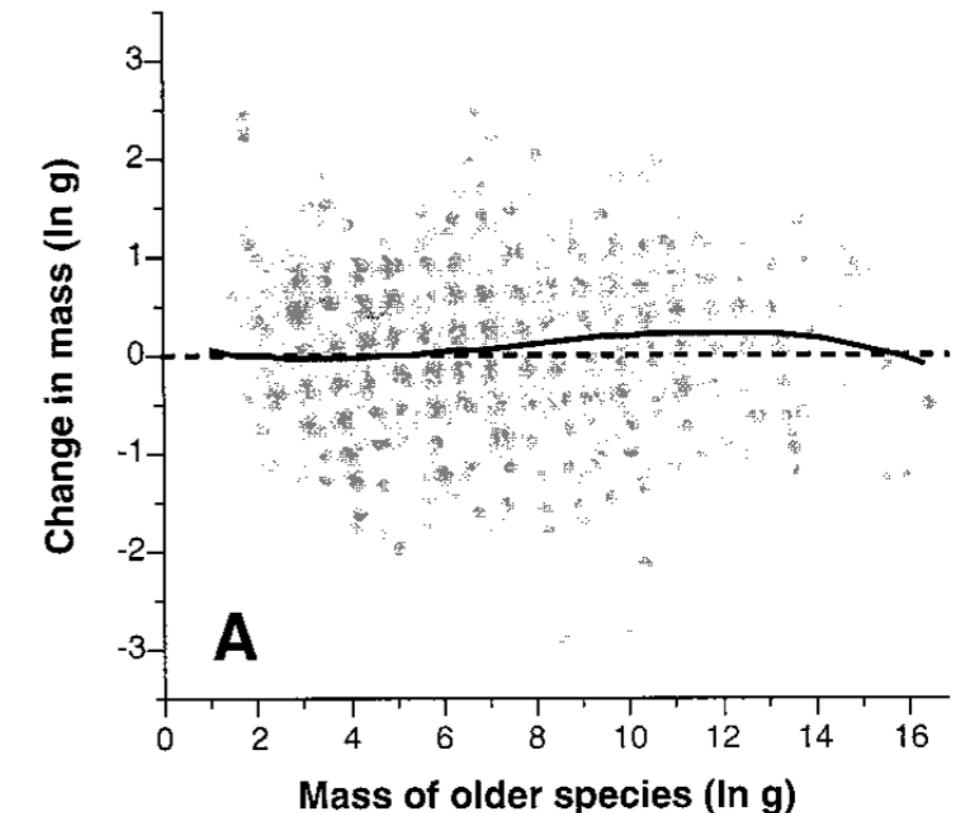
$$M_{Indricotherium} = 1.5 \times 10^7$$
$$M_{Deinotherium} = 1.74 \times 10^7$$

$$M_{opt} = 1.748 \times 10^7$$

# Evolutionary attractor?



No sign of a lower bound, but the NSM is pretty minimal...  
Higher trophic effects?



Alroy, 1998

# Thank you!

