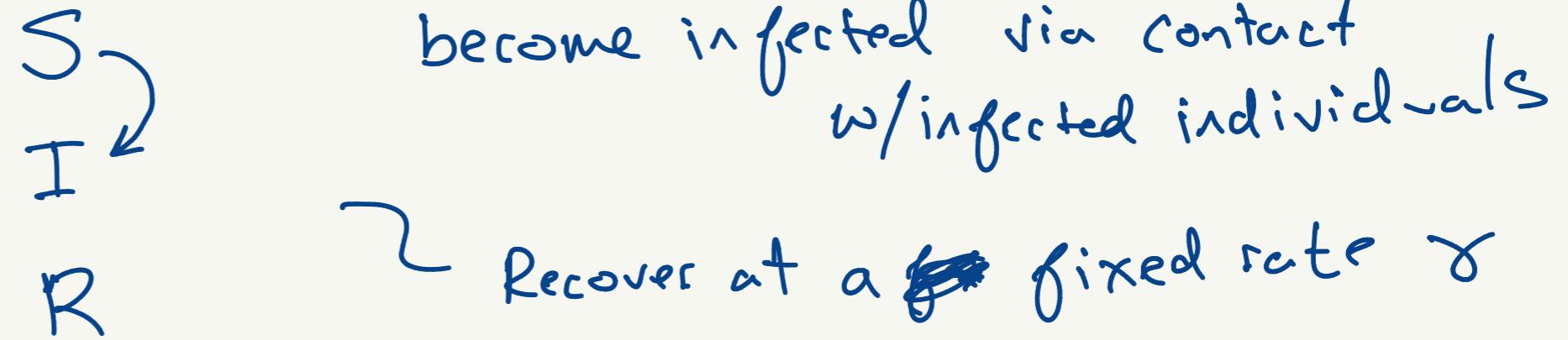


Susceptible individuals
Infected individuals
Recovered individuals



Total number of individuals
does not change over time

$$\frac{dS}{dt}, \frac{dI}{dt}, \frac{dR}{dt}$$

$$S + I + R = N$$

$$\frac{dN}{dt} = \emptyset \Rightarrow \frac{d(S+I+R)}{dt} = \emptyset$$

so: $\frac{1}{\gamma}$ is the average amount
of time an individual is
infected

λ = the force of infection ~ the per-capita rate at which susceptible
individuals acquire infection. ~~⊗⊗~~ λ is not constant
 λ changes with I

$$\lambda(I) = \beta \frac{I}{N}$$

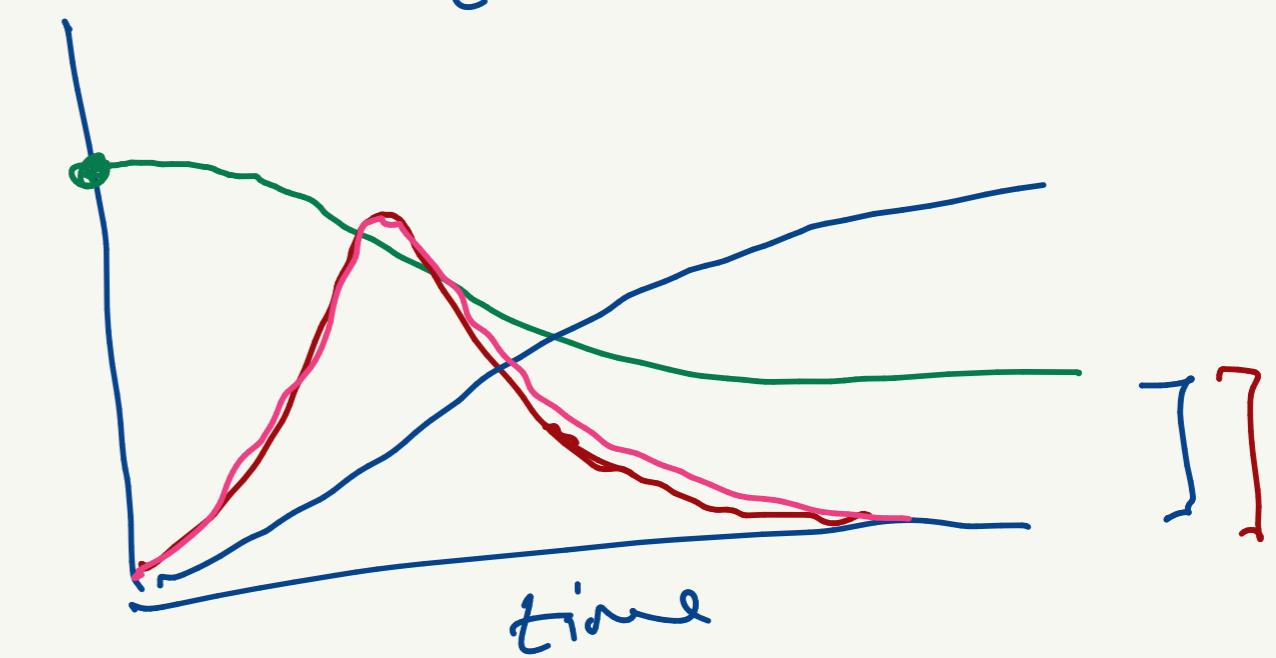
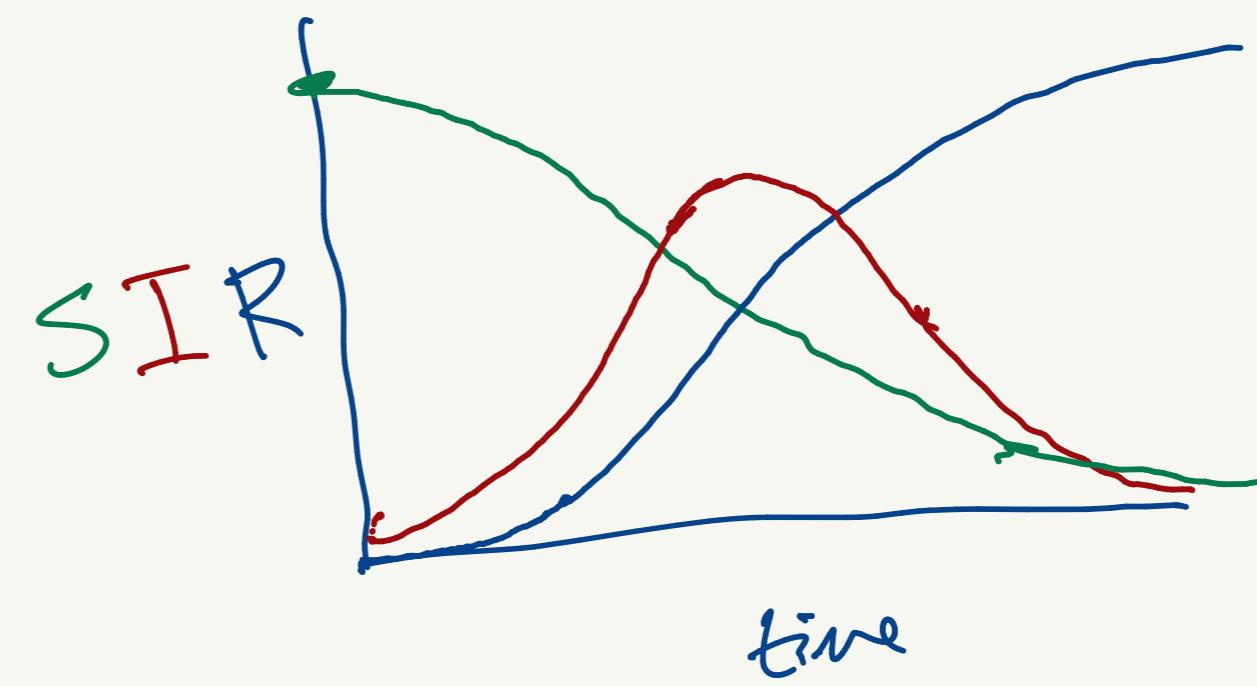
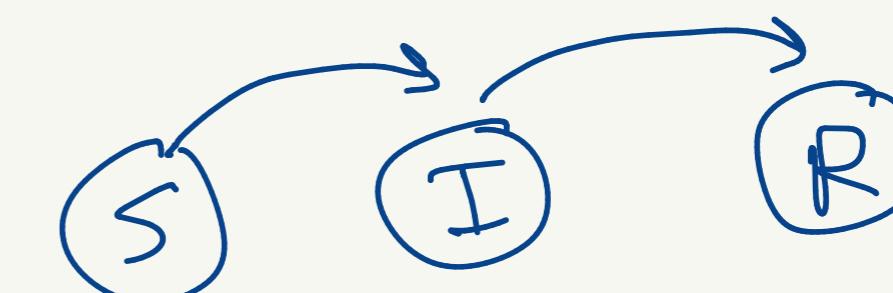
Interaction term
transmission rate
[time]

$$\frac{dS}{dt} = - \underbrace{\lambda(I)S}_{\text{infection}} = - \beta \frac{I}{N} S$$

$$\lambda(I) = \beta \frac{I}{N}$$

$$\frac{dI}{dt} = \underbrace{\lambda(I)S}_{\text{infection}} - \underbrace{\gamma I}_{\text{recovery}} = \beta \frac{I}{N} S - \gamma I$$

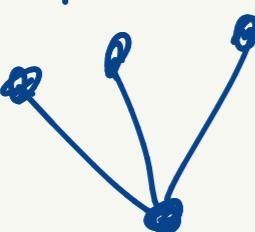
$$\frac{dR}{dt} = \underbrace{\gamma I}_{\text{recovery}}$$



The basic reproductive number R_0

R_0 = the average number of secondary cases arising from a typical primary case in an entirely susceptible population

$$R_0 = 3$$



$$R_0 > 1$$

pathogen expected to spread

$$R_0 < 1$$

pathogen is expected to decline

We assume that the ^{host} population @ $t=0$ has 1 infected individual

$$\begin{aligned} S(0) &= N - 1 \\ I(0) &= 1 \end{aligned}$$

⊗ The disease will fail to spread if $\frac{dI}{dt} < 0$ at $t=0$

$$\frac{dI}{dt} < 0$$

$$\beta \frac{I}{N} S - \gamma I < 0$$

$$\frac{\beta}{\gamma} \frac{N-1}{N} < 1$$

$$\frac{\beta}{\gamma} < 1$$

$$R_0 = \frac{\beta}{\gamma}$$

$$\begin{aligned} \text{transmission rate} &\rightarrow \frac{\beta}{N} S < \gamma I \\ \text{recovery rate} &\rightarrow \frac{\gamma}{\beta} \frac{S}{N} < 1 \end{aligned}$$

if $\beta/\gamma < 1$ no spread
if $\beta/\gamma > 1$ outbreak

$$R_0 = \begin{bmatrix} \beta \\ \gamma \end{bmatrix} \left\{ \begin{array}{l} > 1 \quad \text{outbreak} \\ < 1 \quad \text{pathogen dies out} \end{array} \right.$$

$$\frac{dI}{dt} = \beta \frac{I}{N} S - \gamma I - \nu I$$

$$R_0 = \begin{array}{llll} \hline & \text{Flu: } 2-3 & \text{HIV/AIDS: } 3-5 & \text{Smallpox: } 5-7 \\ \hline & \text{Ebola: } 1.5-2.5 & & \\ & \text{Measles: } 12-18 & & \end{array}$$

$$\delta = \beta \frac{I}{N} S - \gamma I - \nu I$$

$$\beta \frac{I}{N} S = \gamma I + \nu I$$

$$\beta \frac{S}{N} = \gamma + \nu \rightarrow \beta \frac{N-1}{N} = \gamma + \nu$$

$$R_0 =$$

$$\frac{\beta}{\gamma + \nu} = \frac{1}{N-1}$$

$$\frac{N-1}{N} \approx 1 \text{ if } N \text{ is large}$$

R_0 if we consider

Recovery
Mortality

$$\frac{99999}{100000} = 0.99999 \approx 1$$

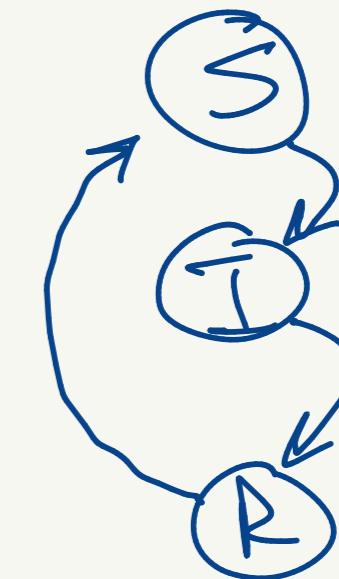
Now let's consider the SIR model w/
demographic processes

Assume

1) All individuals can reproduce at rate b

2) All offspring are susceptible

3) All individuals experience mortality at rate μ
independent of disease!



$$\frac{dS}{dt} = \overbrace{bN} - \beta \frac{I}{N} S - \mu S = b(S+I+R) - \beta \frac{I}{N} S - \mu S$$

$$\frac{dI}{dt} = \beta \frac{I}{N} S - \gamma I - \mu I$$

$$\frac{dR}{dt} = \gamma I - \mu R$$

$R_0?$

$$\boxed{\frac{dI}{dt} > 0}$$

$$\beta \frac{I}{N} S > \gamma I + \mu I$$

$$\beta \frac{S}{N} > \gamma + \mu$$

$$\boxed{\frac{\beta}{\gamma + \mu} > 1}$$

$$R_0 = \frac{\beta}{\gamma + \mu}$$

- Small modification (accounts for an altered rate out of the infected state: Recovered + Dead)

- Not the full story!

Solve for the steady state!

$$\frac{dS}{dt} = bN - \beta \frac{I}{N} S - \gamma S$$

$$bN = \beta \frac{I}{N} S + \gamma S$$

$$bN = \frac{S}{N} (\beta I + N\gamma)$$

$$bN = \left(\frac{\gamma + \nu}{\beta} \right) (\beta I + N\gamma)$$

$$\left(\frac{1}{N}\right) \frac{bN\beta}{\gamma + \nu} = (\beta I + N\gamma) \left(\frac{1}{N}\right)$$

$$\frac{b\beta}{\gamma + \nu} = \beta \frac{I}{N} + \gamma \rightarrow \beta \frac{I}{N} = \frac{b\beta}{(\gamma + \nu)} - \gamma$$

$$\frac{dS}{dt} = \phi, \quad \frac{dI}{dt} = \phi$$

$$\phi = \beta \frac{I}{N} S - \gamma I - \nu I$$

$$\beta \frac{I}{N} S = \gamma I + \nu I$$

$$\beta \frac{S}{N} = \gamma + \nu$$

$$\frac{S}{N} = \frac{\gamma + \nu}{\beta}$$

Proportion of Susceptibles

$$\frac{I}{N} = \frac{b}{(\gamma + \nu)} - \frac{\nu}{\beta}$$

Pop. of Infected

$$\frac{dR}{dt} = \phi$$

$$\phi = \gamma I - \nu R$$

$$\gamma I = \nu R$$

$$R = \frac{\gamma}{\nu} I$$

$$\frac{R}{N} = \frac{\gamma}{\nu} \frac{I}{N}$$

$$\frac{R}{N} = \frac{\gamma}{\nu} \left[\frac{b}{(\gamma + \nu)} - \frac{\nu}{\beta} \right]$$

Pop. of recovered

Not zero... not disease-free.

- 1) This means the disease becomes ENDEMIC ($I^* > 0$)
- 2) Proportions of $\frac{S}{N}$, $\frac{I}{N}$, $\frac{R}{N}$ at steady state depends
on both demographic and disease parameters

What does R_0 mean?

R_0 = the average number of people each sick person infects

$$R_0 = 1 : \textcircled{0} \rightarrow \textcircled{0}$$

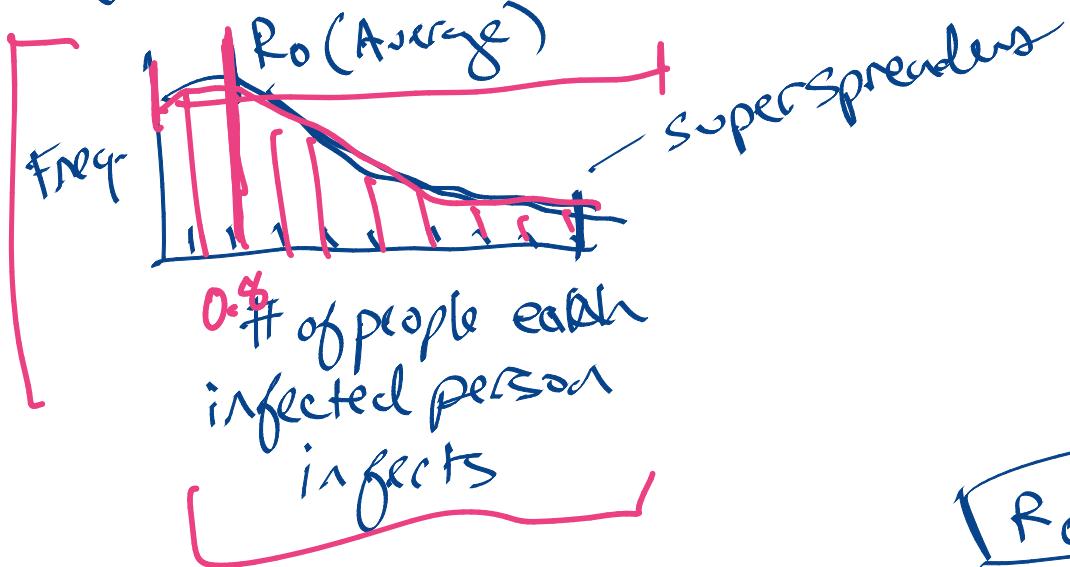
$$R_0 = 2 : \textcircled{0} \xrightarrow{\nearrow} \textcircled{0} \quad \left\{ \begin{array}{l} R_0 > 1 \text{ epidemic continues} \\ \text{exponential growth} \end{array} \right.$$

$$R_0 = < 1 : \textcircled{0.5} \xrightarrow{\nearrow} \textcircled{0} \quad \left\{ \begin{array}{l} R_0 < 1 \text{ epidemic decline} \end{array} \right.$$

What does R_0 mean?

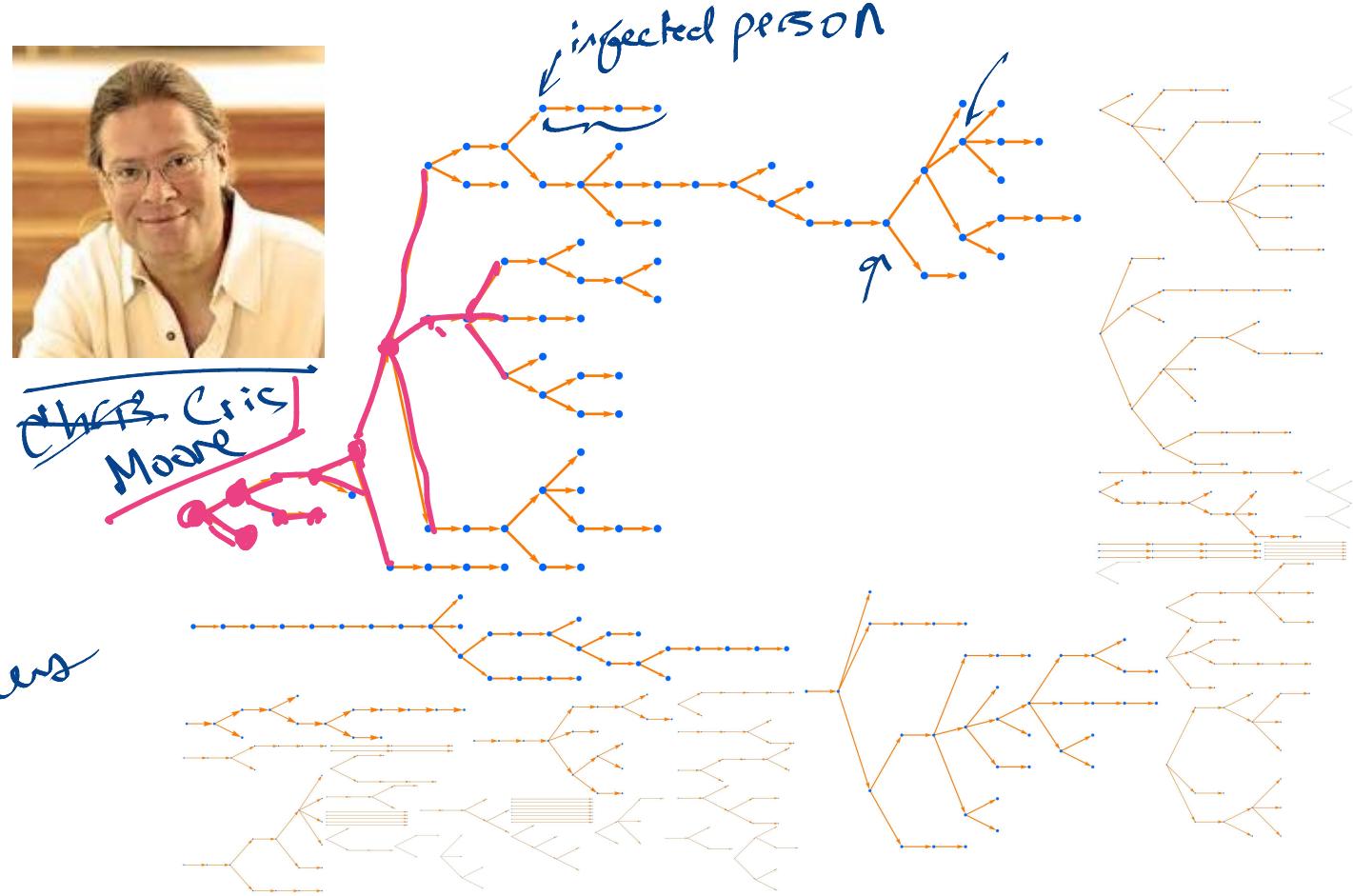
- virus life cycle
- human physiology
- human social dynamics

$R_0 \sim$ describes the AVERAGE



$R_0 < 1$

Superspreaders



➤ A hundred random outbreaks in a scenario where each sick person interacts with 10 others, and infects each one with probability 8 percent. Here $R_0 = 0.8$ and the average outbreak size is five, but 1 percent of the outbreaks have size 50 or larger, and in this run the largest has size 82.

What does R_0 mean?



$$R_0 = 0.8$$

A hundred random outbreaks in a scenario with superspreading, where 1 percent of the cases infect 20 others. As in Figure 1, we have $R_0 = 0.8$ and the average outbreak size is 5, but now the heavy tail of outbreaks is much heavier. In this run the largest outbreak has size 663.

