$$\frac{dP}{dt} = \sum_{i=1}^{2} e_i a_i N_i P - dP$$

$$\frac{dN_i}{dt} = r_i N_i \left(1 - \frac{N_i}{k}\right) - a_i N_i P$$
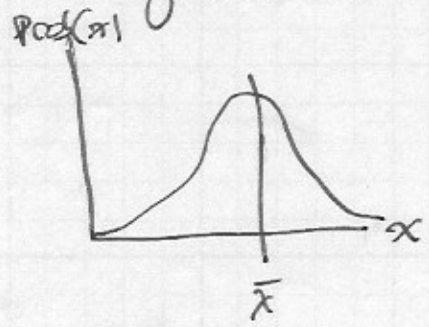
$a_i \sim$ attack rate
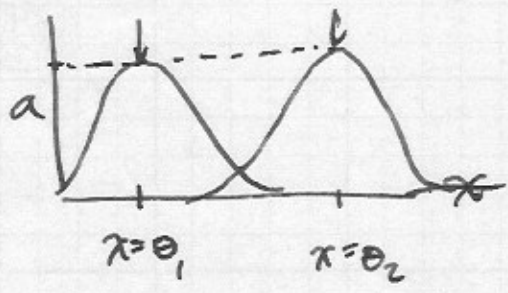
- Assume that it depends on trait x

  high → P is better
  @ attacking N_1

  low → P is better
  @ attacking N_2

- $P(x, \bar{x}) = \text{Normal}(\bar{x}, \sigma)$

  Probability that $x = X$ given mean $\bar{x}$

We can also assume that each prey has an optimal trait value at which attack rate is maximized

$\sim$ tradeoff... you can't
be great @ both

- X is a distribution w/ mean & variance

  $\sim$ many individuals w/ diff. x values (dist~pop ~~behav~~

So: the mean attack rate of the predator on the population i is:

$$\bar{a}_i(\bar{x}_i) = \int_{-\infty}^{\infty} a_i(x)\, p(x,\bar{x})\, dx$$

$\underbrace{}$

sum over attack rates for trait value x given the probability that $x = \bar{x}$

$$\bar{a}_i(\bar{x}_i) = \int_{-\infty}^{\infty} \alpha_i \exp\left[\frac{(x-\theta_i)^2}{2\tau_i^2}\right] p(x,\bar{x})\, dx$$

$$= \frac{\alpha \tau_i}{\sqrt{\sigma^2 + \tau_i^2}} \exp\left[\frac{(\bar{x}-\theta_i)^2}{2(\sigma^2 + \tau_i^2)}\right]$$

Going back to equ.

Fitness: Per capita growth-mortality rate

$$W = \frac{1}{P}\frac{dP}{dt}$$

$$= \sum_{i=1}^{2} e_i a_i(x) N_i - d$$

and mean fitness is

$$\bar{W} = \int_{-\infty}^{\infty} W(x, N_1, N_2)\, p(x,\bar{x})\, dx$$

$$= \sum_{i=1}^{2} e_i \bar{a}_i(\bar{x}) N_i - d$$

$$\frac{dN_i}{dt} = r_i N_i \left(1 - \frac{N_i}{K}\right) - P\bar{a}_i N_i$$

$$\frac{dP}{dt} = P\bar{W} = P\left(\sum_{i=1}^{2} e_i \bar{a}_i(\bar{x}) N_i - d\right)$$

$$\frac{d\bar{x}}{dt} = \sigma_G^2 \frac{d\bar{W}}{d\bar{x}}$$ ← Change in $\overset{mean}{trait}$ value over time
is proportional to

$\left\{ \dfrac{\text{Change in } \overset{mean}{fitness}}{\text{Change in } \overset{mean}{trait \ value}} \right.$

Stabilizing selection
Opposing
force of
selection

$\left\{ \begin{array}{l} \bar{x}\uparrow \text{ but } \downarrow\bar{W}, \text{ then } \frac{d\bar{x}}{dt} < \emptyset \\ \bar{x}\downarrow \text{ but } \bar{W}\downarrow, \text{ then } \frac{d\bar{x}}{dt} > \emptyset \end{array} \right.$

<u>Directional selection</u>

$\left\{ \begin{array}{l} \bar{x}\uparrow \text{ and } \bar{W}\uparrow, \text{ then } \frac{d\bar{x}}{dt} > \emptyset \\ \bar{x}\downarrow \text{ and } \bar{W}\uparrow, \text{ then } \frac{d\bar{x}}{dt} < \emptyset \end{array} \right.$

positive feedback!

Heritability

$h^2 = \frac{\sigma_G^2}{\sigma^2}$ } proportion of $\overset{trait}{variation}$ that is heritable

σ_G^2 (genetic variance) fuels strength of selection.

if ↓ variance, less to select for, and
evol. slows down.

if ↑ variance, more to select for, and
evol. speeds up!