Fitness as result of interaction

Beetles < L ↑ energetic requirements
S ↓ energetic requirements

- when beetles of same size compete, they share what's left
- L Beetles overpower small Beetles

focal Beetle B1 vs. B2

B1 \ B2	L	S
L	3	8
S	1	5

3/3	8/1
1/8	5/5

Beetles don't get to choose strategy... it genetically determined

Fitness differences drive changes in proportion of each phenotype in population $N_S(t)$ vs. $N_L(t)$

Evolutionary stable Strategy ~ Analagous to Nash Equilibrium

= genetically determined strategy that tends to persist once it is prevelant in population

	A	B
A	a	b
B	c	d

Focal

	L	S
L	3	8
S	1	5

$$\phi_A = a\frac{N_A}{N_T} + b\frac{N_B}{N_T}$$

$$\phi_B = c\frac{N_A}{N_T} + d\frac{N_B}{N_T}$$

$\Downarrow$

$$\phi_A = ax + b(1-x)$$

$$\phi_B = cx + d(1-x)$$

$N_A \sim$ number of A inds. in population

$N_B \sim$ " " B "

$$N_T = N_A + N_B$$

$$\frac{N_A}{N_T} + \frac{N_B}{N_T} = 1$$

$\uparrow$ x $\quad$ $\uparrow$ $(1-x)$

$$\phi_L = 3x + 8(1-x) \rightarrow 8 - 5x$$

$$\phi_S = 1x + 5(1-x) \rightarrow 5 - 4x$$

$\phi_L = 8 - 5x$ $\quad$ yint: 8

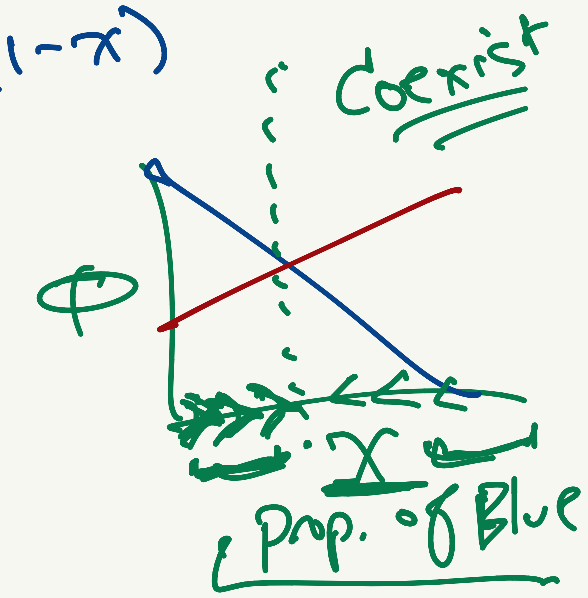
xint: $\emptyset = 8 - 5x \rightarrow 5x = 8$

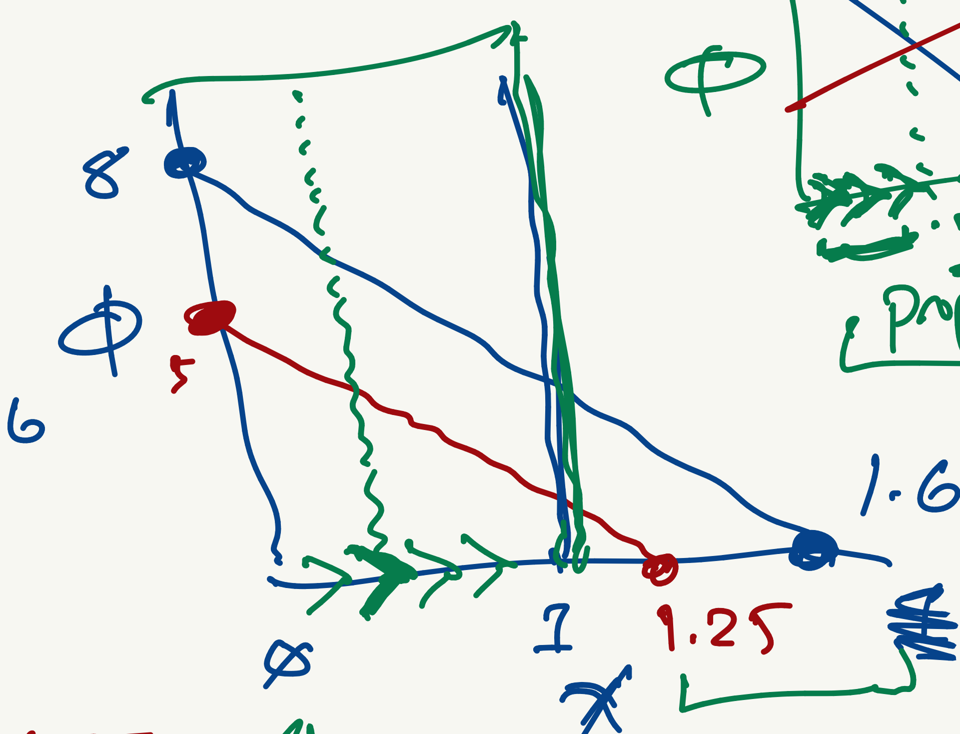
$x = 8/5 = 1.6$

$\phi_S = 5 - 4x$ $\quad$ yint: 5

xint: $\emptyset = 5 - 4x \rightarrow 4x = 5$

$x = 5/4 = 1.25$





(proportion of L individuals)

How do we relate fitness differences to changes in the population?

proportion of L $= x$
" " S $= (1-x)$

Rule 1: If the fitness of a phenotype is better than average that proportion of that phenotype will increase

Rule 2: " " " " " " " worse " "
" " " " " " decline

Average fitness: $\Phi = x\phi_L + (1-x)\phi_S$

$$\rightarrow \quad \Delta x_L = x\left[\phi_L - \Phi\right] \longrightarrow x(1-x)(\phi_L - \phi_S)$$

(+) if $\phi_L > \phi_S$
(−) if $\phi_L < \phi_S$

$$x(t+1) - x(t) = x\left[\phi_L - \Phi\right]$$

$$x(t+1) = x(t) + \underbrace{x\left[\phi_L - \Phi\right]}_{\Delta x}$$

~ Dynamic of the proportion of L phenotype in the population

$N(t)$ ~ Population size @ time t

$$N(t+1) = N(t) + \underset{\text{(births}}{B} - \underset{\text{(deaths)}}{D} + \cancel{I - E}$$

B ~ total number of births

D ~ total number of deaths

$B(N) = bN$ $\quad b = \frac{B}{N}$ per capita birth rate

$D(N) = dN$ $\quad d = \frac{D}{N}$ per capita death rate

$$N(t+1) = N(t) + bN(t) - dN(t)$$

$$N(t+1) = N(t) + \underbrace{(b-d)}_{r_d}N(t)$$

r_d Avg. per-capita discrete growth rate

$$\underbrace{N(t+1) - N(t)}_{\Delta N} = r_d N(t)$$

~ generalize to a time step of size Δt

Examine an interval Δt

$$N(t+\Delta t) - N(t) = \underbrace{r_d \Delta t}\, N(t)$$

Make our time window small

$$\frac{N(t+\Delta t) - N(t)}{\Delta t} = r_d N(t)$$

$r_d \sim$ discrete growth rate

Discrete time

↓

Continuous time

as $\Delta t \to \emptyset$

$$\frac{d}{dt} N = r N$$

"Change in population size over time"

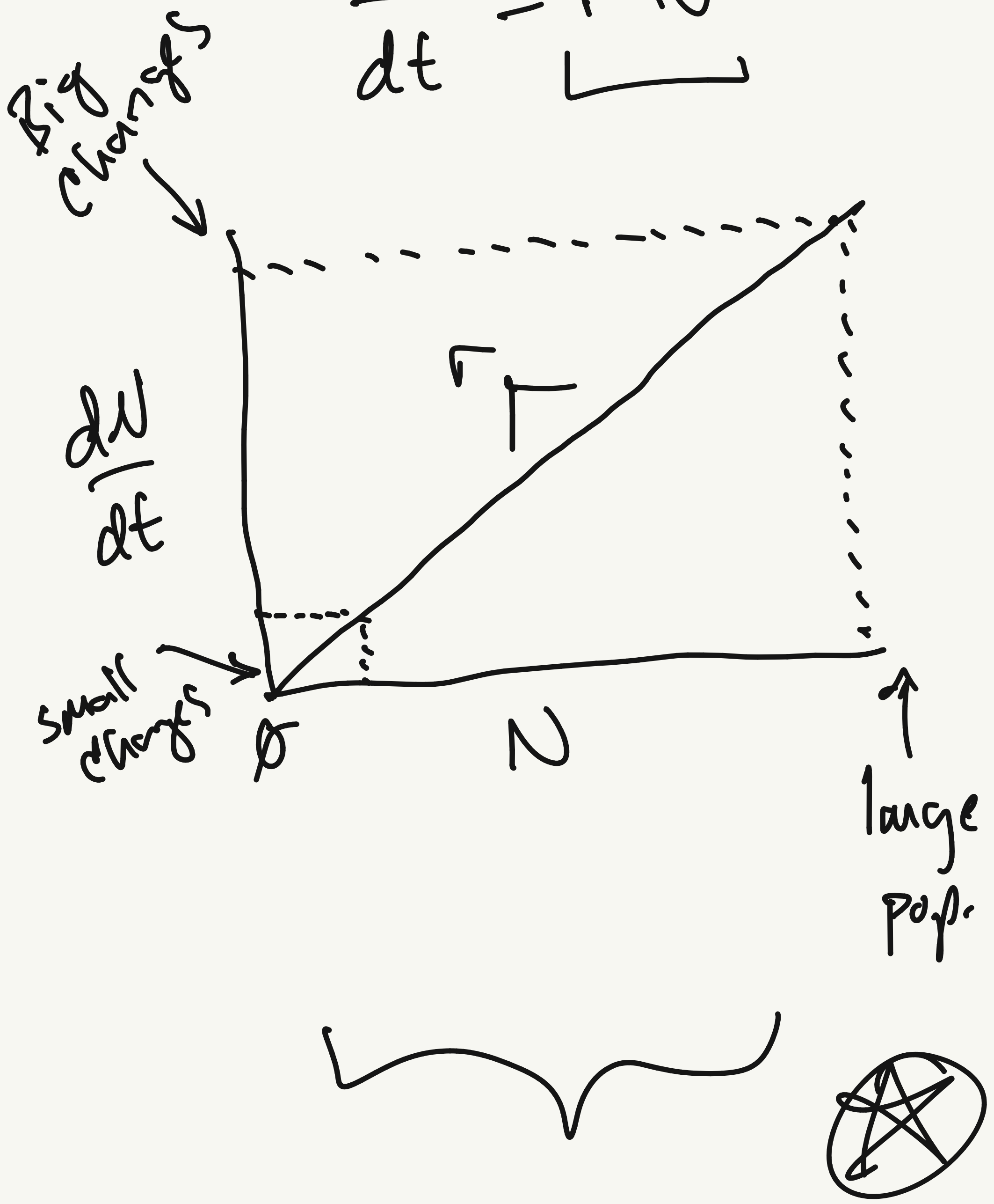
$r \sim$ instantaneous growth rate

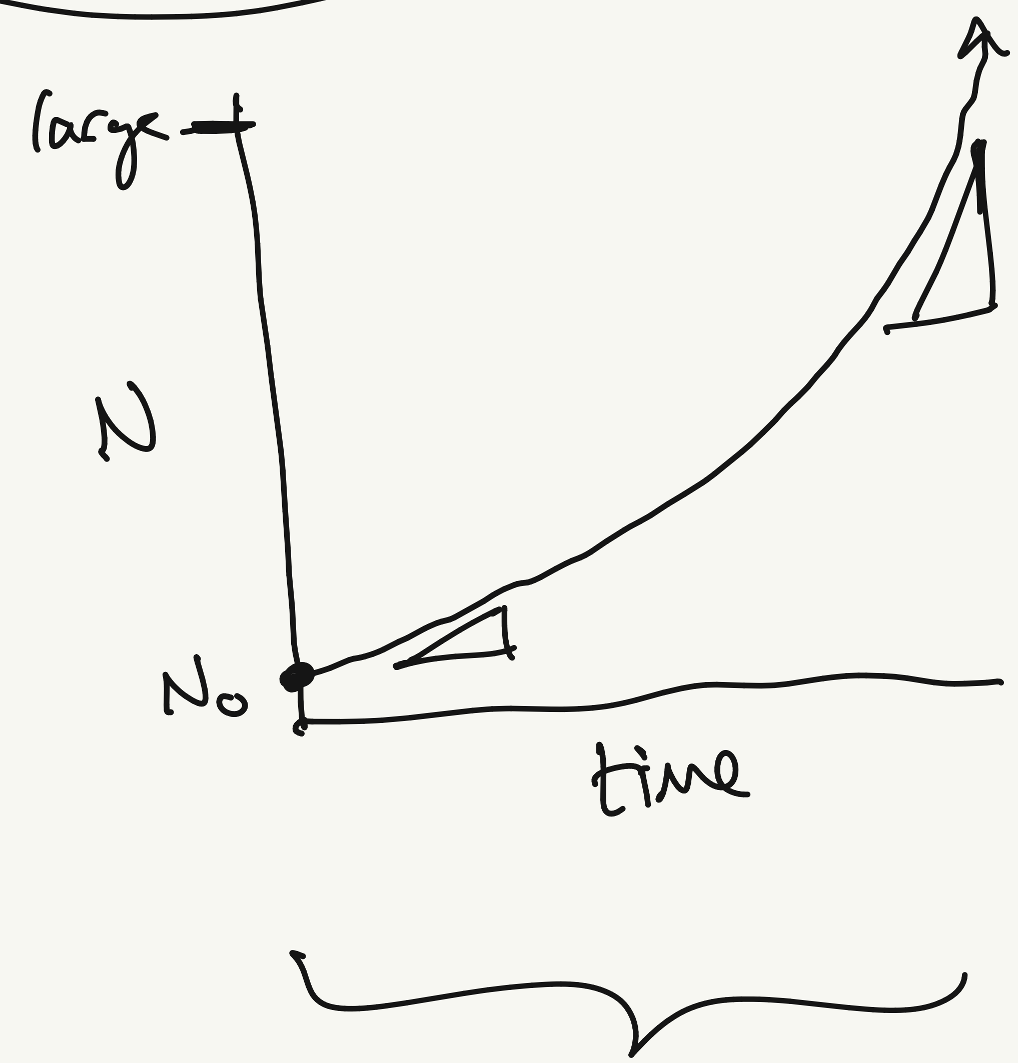
if $r > \emptyset$ $\quad \frac{dN}{dt} > \emptyset$ $\sim$ growth

if $r < \emptyset$ $\quad \frac{dN}{dt} < \emptyset$ $\sim$ decline

if $r = \emptyset$ $\quad \frac{dN}{dt} = \emptyset$ $\sim$ population not changing

$$\frac{dN}{dt} = rN \quad \text{SOLVE} \quad N(t) = N_0 e^{rt}$$





Ex) Doubling time

$$2\cancel{N_0} = \cancel{N_0} e^{rt}$$

$$2 = e^{rt}$$

$$\log(2) = rt$$

$$t = \frac{\log(2)}{r}$$

$$r_{Human} = 0.0067 \text{ yr}^{-1}$$