Cactus Graphs

D-nice Kennon
Eric Veliky
Jay Winkler

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What is a Cactus Graph?

A cactus graph is a connected graph in which any two simple cycles have at most one vertex in common. It also is a connected graph where every edge belongs to at most one cycle, or where every block is an edge or a cycle. Every pseudotree is also considered a cactus graph. There are several classes of graphs

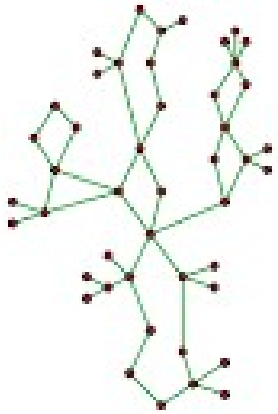

Figure 1: Cactus Graph Example

that are related to a cactus graph such as:

- 3-leaf power
- 4-leaf power
- Helly cactus subtree
- block

There are several other interesting types of cactus graphs such as a Triangular Cactus. This graph is a special type of cactus graph such that each cycle has length three. Triangular cactus graphs are also planar.

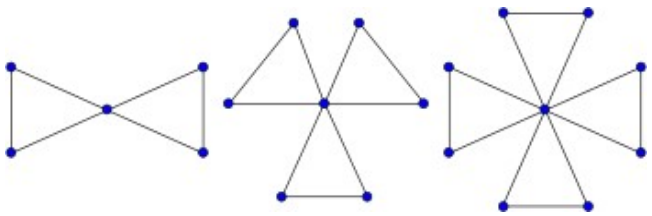

Figure 2: Examples of Triangle Cactus Graphs

Examples of Cactus Graphs

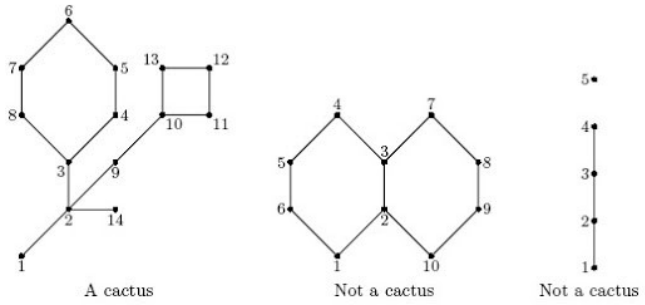

Figure 3: Defining What it Means to be a Cactus Graph

The first graph shown on the left is a cactus because it is a connected graph without any edges being used for more than one circuit. The middle graph is not a cactus because the edge connecting vertex 3 and 4 together is being used to create 2 circuits. The final graph is not a cactus because it is not a connected graph containing at least one cycle.

Real World Examples and Applications

Cactus graphs can be used to model physical setting where a tree would not be the ideal graph for modeling a particular situation. Several examples include:

- Telecommunications
  - For where to lay or place infrastructure for telecommunications, where or how to position the towers or lines so that they can provide the maximum amount of coverage so that they can provide more people more service with minimal resources and without certain towers overlapping each other.
- Transportation

  - Routes such as bus routes that will gather and drop people off at certain locations around a town or city will likely use cactus graphs. To minimize the number of buses needed to cover the entire area while also preventing buses from covering the same ground more than once a company or city might use a cactus graph to make sure buses are driving on their own routes and providing the most efficient coverage.

- Genome Comparison and Assembly
  - Cactus graphs can be used to compare sets of related genomes. They can represent duplications and general genomic rearrangements and can be broken down further to visualize circular genome plots.

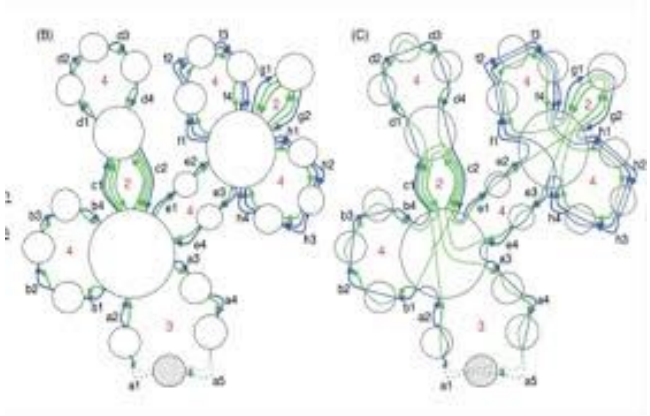

Figure 4: Example of Cactus Graph used in Genome Comparison and Assembly

Importance of Cactus Graph and Recent Study

Cactus graphs in recent years have received extensive studies because of their real world application potential. There are several well-known problems cactus graphs can help solve efficiently in polynomial time:

- All-Pair shortest paths problem
  - Find the shortest paths in a weighted graph with positive or negative edge weights that do not contain negative cycles within the path
- Domination problem
  - A special case of the vertex cover problem related to chess
- Graph Coloring and Labeling problems
- Other forms of cover problems

Resources Used

- https://en.wikipedia.org/wiki/Cactusgraph
- https://mathworld.wolfram.com/CactusGraph.html
- http://poj.org/problem?id=2793
- https://cgl-dev.soe.ucsc.edu/cactus-graphs/
- https://arxiv.org/ftp/arxiv/papers/1408/1408.4005.pdf