Homework 1

Jonathan Zhang EID: jdz357

Problem 6.2:

Compute the Fourier transform of $\exp\left(-a|x|^2\right)$, $a > 0$, directly for $x \in \mathbb{R}$.

Solution

Problem 6.3:

If $f \in L^1\left(\mathbb{R}^d\right)$ and $f > 0$, show that for every $\xi \neq 0$, $\left|\hat{f}\left(\xi\right)\right| < \hat{f}\left(0\right)$.

Solution

Problem 6.4:

If $f \in L^1\left(\mathbb{R}^d\right)$ and $f(x) = g\left(|x|\right)$ for some g, show that $\hat{f}\left(\xi\right) = h\left(|\xi|\right)$ for some h. Can you relate g and h?

Solution

Problem 6.6:

Show that the Fourier transform $\mathcal{F} : L^1\left(\mathbb{R}^d\right) \to C_v\left(\mathbb{R}^d\right)$ is not onto. Show that $\mathcal{F}\left(L^1\left(\mathbb{R}^d\right)\right)$ is dense in $C_v\left(\mathbb{R}^d\right)$.

Solution