Homework 4

Jonathan Zhang EID: jdz357

Problem 7.1:

Prove that for $f \in H^1\left(\mathbb{R}^d\right)$, $\|f\|_{H^1\left(\mathbb{R}^d\right)}$ is equivalent to

$$\left[\int_{\mathbb{R}^d} \left(1 + |\xi|^2\right) |\hat{f}\left(\xi\right)|^2 d\xi\right]^{1/2}$$

Generalize to $H^k\left(\mathbb{R}^d\right)$?

Solution

Problem 7.2:

Prove that if $f \in H_0^1\left(0,1\right)$, then there exists $C > 0$ such that $\|f\|_{L^2(0,1)} \leq C\|f'\|_{L^2(0,1)}$. If instead $f \in \left\{g \in H^1(0,1) : \int_0^1 g(x)dx = 0\right\}$, provide a similar estimate.

Problem 7.3:

Prove that $\delta_0 \notin \left(H^1\left(\mathbb{R}^d\right)\right)^*$ for $d \geq 2$, but that $\delta_0 \in \left(H^1\left(\mathbb{R}\right)\right)^*$. You will need to define that δ_0 applied to $f \in H^1\left(\mathbb{R}\right)$ means.

Problem 7.4:

Prove that $H^1(0,1)$ is continuously embedded in $C_B(0,1)$, the set of bounded and continuous functions on $(0,1)$.