A fun problem 2

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Your task in this problem is to establish a relationship between *pairs* of natural numbers and natural numbers. The *set of natural numbers* is the set $\{1, 2, 3, 4, \dots\}$, not including zero. A *pair* of natural numbers is denoted (i, j) where i and j are both natural numbers. The first few pairs of natural numbers are $(1, 1)$, $(1, 2)$, $(2, 1)$, and so on.

Your task is to **find** a formula that takes a pair of natural numbers (i, j) and associates a single, unique, natural number k. For example, you may start like

$$\begin{array}{ccc} (1,1) & \to & 1 \\ (2,1) & \to & 2 \\ (1,2) & \to & 3 \\ (3,1) & \to & 4 \\ \vdots & \vdots & \vdots \end{array}$$

Once you have done so, can you do the reverse? That is, given a natural number k, find the unique pair of natural numbers (i, j) that are associated with it?