Some Notes on Function Spaces

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1 Preliminaries

2 Lebesgue Spaces

3 Distributions

4 Sobolev Spaces

Theorem 4.1. *Here, we use the following notation:* $\left(M^{\alpha} f\right)(x) = x^{\alpha} f(x)$.
If $T \in \mathcal{S}'$ *and* $f \in \mathcal{S}$, *then* $T * f$ *is smooth and polynomially bounded.*

Things to revisit:

- Prop. 6.14: If p is a polynomial, $g \in \mathcal{S}$, and α a multiindex, then the maps $f \mapsto pf$, $f \mapsto gf$, and $f \mapsto D^\alpha f$ are continuous maps from $\mathcal{S}$ into itself.
- Prop 6.15: If $f_j \to f$ in $\mathcal{S}$, then $f_j \to f$ in L^p for $1 \leq p \leq \infty$.