# Homework 12

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**Exercise (10.2).**

$$F(y) = \int_0^1 \left\{y^2 - yy'\right\} dx, \qquad y \in C^1\left([0,1]\right).$$

1. *Find all extremals.*
2. *If we require $y(0) = 0$, show by example that there is no minimum.*
3. *If we require that $y(0) = y(1) = 0$, show that the extremal is a minimum. (Hint: $yy' = \left(1/2y^2\right)'$).*

**Exercise (10.4).** *Minimize*

$$F(y) = \int_0^1 f(x, y(x), y'(x), y''(x))dx$$

*over the set of $y \in C^2([0,1])$ such that $y(0) = \alpha$, $y'(0) = \beta$, $y(1) = \gamma$, and $y'(1) = \delta$. That is, with $C_0^2([0,1]) = \left\{u \in C^2([0,1]) : u(0) = u'(0) = u(1) = u'(1) = 0\right\}$, and $y \in C_0^2 + p(x)$ where $p$ is the cubic polynomial that matches the BC.*

1. *Find a differential equation, similar to the EL equation, that must be satisfied by the minimum (if it exists).*
2. *Apply you*

**Exercise (10.6).**

**Exercise (10.8).**

**Exercise (10.9).**