University of Warsaw

UW3

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Contest (1)

sol.cpp

27 lines

```
#include <bits/stdc++.h>
using namespace std;
#define rep(i, a, b) for (int i = (a); i < (b); i++)
#define all(x) begin(x), end(x)
#define sz(x) int((x).size())
using ll = long long;
using pii = pair<int, int>;
using vi = vector<int>;
#ifdef LOCAL
auto operator<<(auto& o, auto x) -> decltype(x.first, o);
auto operator<<(auto& o, auto x) -> decltype(x.end(), o) {
  o << "{";
  for (int i = 0; auto y : x) o << ", " + !i++ * 2 << y;
  return o << "}"; }
auto operator<<(auto& o, auto x) -> decltype(x.first, o) {
  return o << "(" << x.first << ", " << x.second << ")"; }
void __print(auto... x) { ((cerr << x << " "), ...) << endl; }
#define debug(x...) __print("[" #x "]:", x)
#else
#define debug(...) 2137
#endif
int main() {
  cin.tie(0)->sync_with_stdio(0);
}
```

.bashrc

8 lines

```
c() {
  g++ -std=c++20 -fsanitize=address,undefined -g \
    -DLOCAL -Wall -Wextra -Wshadow $1.cpp -o $1;
}
nc() { g++ -std=c++20 -O2 $1.cpp -o $1; }
alias rm='trash'
alias mv='mv -i'
alias cp='cp -i'
```

.vimrc

8 lines

```
set nu et ts=2 sw=2
filetype indent on
syntax on
colorscheme habamax
hi MatchParen ctermfg=66 ctermbg=234 cterm=underline
nnoremap ; :
nnoremap : ;
inoremap {<cr> {<cr>}<esc>O <bs>
```

hash.sh

3 lines

```
# Hashes a file, ignoring all whitespace and comments. Use for
# verifying that code was correctly typed.
cpp -dD -P -fpreprocessed | tr -d '[:space:]'| md5sum | cut -c-6
```

test.sh

5 lines

```
for((i=1;i>0;i++)) do
  echo "$i"
  echo "$i" | ./gen > int
  diff -w <(./sol < int) <(./slow < int) || break
done
```

troubleshoot.txt

17 lines

```
Czy na wejsciu pojawic sie moga long longi, np. m<=n^2?
Czy tresc jest w 100% jasna?
Czy bardzo dokladnie przeczytana jest sekcja input?
Daj komus innemu przeczytac tresc niezaleznie,
zadaj pytanie.
Czy dziala dla brzegowych, np. n/m = 0/1,
wszystkie a_i = 0?
Czy tablice za male?
Czy wszedzie modulujesz? Czy dobre modulo?
Czy na wejsciu moga byc liczby poza [0, mod)?
Czy zle parsujesz wejscie, np. zamiast wczytac double?
WA z double nie oznacza bledu precyzji,
tym bardziej z long double.
Czy napisales cos, co mogles przepisac z biblioteczki?
Czy interakcja jest poprawna, np. brak znaku zapytania?
Czy format wyjscia jest poprawny, np. brak YES?
Czy uzywasz double gdzies, gdzie mozna tego uniknac?
```

Mathematics (2)

2.1 Equations

$$ax^2+bx+c=0 \Rightarrow x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

The extremum is given by $x=-b/2a$.

$$\begin{aligned} ax+by=e \\ cx+dy=f \end{aligned} \Rightarrow \begin{aligned} x=\frac{ed-bf}{ad-bc} \\ y=\frac{af-ec}{ad-bc} \end{aligned}$$

In general, given an equation $Ax=b$, the solution to a variable x_i is given by

$$x_i=\frac{\det A_i'}{\det A}$$

where A_i' is A with the i'th column replaced by b.

2.2 Recurrences

If $a_n=c_1a_{n-1}+\dots+c_ka_{n-k}$, and $r_1,\dots,r_k$ are distinct roots of $x^k-c_1x^{k-1}-\dots-c_k$, there are $d_1,\dots,d_k$ s.t.

$$a_n=d_1r_1^n+\dots+d_kr_k^n.$$

Non-distinct roots r become polynomial factors, e.g. $a_n=(d_1n+d_2)r^n$.

2.3 Trigonometry

$$\sin(v+w)=\sin v\cos w+\cos v\sin w$$
$$\cos(v+w)=\cos v\cos w-\sin v\sin w$$

$$\tan(v+w)=\frac{\tan v+\tan w}{1-\tan v\tan w}$$
$$\sin v+\sin w=2\sin\frac{v+w}{2}\cos\frac{v-w}{2}$$
$$\cos v+\cos w=2\cos\frac{v+w}{2}\cos\frac{v-w}{2}$$

$$(V+W)\tan(v-w)/2=(V-W)\tan(v+w)/2$$

where V,W are lengths of sides opposite angles v,w.

$$a\cos x+b\sin x=r\cos(x-\phi)$$
$$a\sin x+b\cos x=r\sin(x+\phi)$$

where $r=\sqrt{a^2+b^2}$, $\phi=\operatorname{atan2}(b,a)$.

2.4 Geometry

2.4.1 Triangles

Side lengths: a,b,c

Semiperimeter: $p=\dfrac{a+b+c}{2}$

Area: $A=\sqrt{p(p-a)(p-b)(p-c)}$

Circumradius: $R=\dfrac{abc}{4A}$

Inradius: $r=\dfrac{A}{p}$

Length of median (divides triangle into two equal-area triangles): $m_a=\frac{1}{2}\sqrt{2b^2+2c^2-a^2}$

Length of bisector (divides angles in two):

$$s_a=\sqrt{bc\left[1-\left(\frac{a}{b+c}\right)^2\right]}$$

Law of sines: $\dfrac{\sin\alpha}{a}=\dfrac{\sin\beta}{b}=\dfrac{\sin\gamma}{c}=\dfrac{1}{2R}$

Law of cosines: $a^2=b^2+c^2-2bc\cos\alpha$

Law of tangents: $\dfrac{a+b}{a-b}=\dfrac{\tan\frac{\alpha+\beta}{2}}{\tan\frac{\alpha-\beta}{2}}$

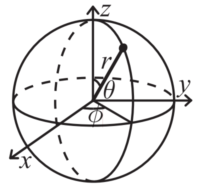
2.4.2 Quadrilaterals

With side lengths a,b,c,d, diagonals e,f, diagonals angle θ, area A and magic flux $F=b^2+d^2-a^2-c^2$:

$$4A=2ef\cdot\sin\theta=F\tan\theta=\sqrt{4e^2f^2-F^2}$$

For cyclic quadrilaterals the sum of opposite angles is 180°, $ef=ac+bd$, and $A=\sqrt{(p-a)(p-b)(p-c)(p-d)}$.

2.4.3 Spherical coordinates



$$\begin{aligned} x&=r\sin\theta\cos\phi & r&=\sqrt{x^2+y^2+z^2} \\ y&=r\sin\theta\sin\phi & \theta&=\operatorname{acos}(z/\sqrt{x^2+y^2+z^2}) \\ z&=r\cos\theta & \phi&=\operatorname{atan2}(y,x) \end{aligned}$$

2.4.4 Pick's theorem

Let i be the number of integer points interior to a polygon, and b be the number of points on the boundary (including both vertices and points along the sides). Then the area A of the polygon is:

$$A=i+\frac{b}{2}+1.$$

2.5 Derivatives/Integrals

$$\frac{d}{dx}\arcsin x=\frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}\arccos x=-\frac{1}{\sqrt{1-x^2}}$$
$$\frac{d}{dx}\tan x=1+\tan^2 x \qquad \frac{d}{dx}\arctan x=\frac{1}{1+x^2}$$
$$\int\tan ax=-\frac{\ln|\cos ax|}{a} \qquad \int x\sin ax=\frac{\sin ax-ax\cos ax}{a^2}$$
$$\int e^{-x^2}=\frac{\sqrt{\pi}}{2}\operatorname{erf}(x) \qquad \int xe^{ax}dx=\frac{e^{ax}}{a^2}(ax-1)$$

Polish Mafia:

$$\int\sqrt{1+x^2}=\frac{1}{2}\left(x\sqrt{1+x^2}+\operatorname{arcsinh}x\right) \quad (\operatorname{arcsinh}=\texttt{asi}$$
$$\int\sqrt{1-x^2}=\frac{1}{2}\left(x\sqrt{1-x^2}+\arcsin x\right)$$
$$\int\frac{1}{ax^2+bx+c}=\frac{2}{\sqrt{4ac-b^2}}\arctan\frac{2ax+b}{\sqrt{4ac-b^2}} \quad (\Delta<0)$$
$$\int\frac{x}{ax^2+bx+c}=\frac{1}{2a}\ln|ax^2+bx+c|-\frac{b}{2a}\int\frac{1}{ax^2+bx+}$$
$$\texttt{tgamma}(t)=\Gamma(t)=\int_0^\infty x^{t-1}e^{-x}\,dx$$

Integration by parts:

$$\int_a^b f(x)g(x)dx=[F(x)g(x)]_a^b-\int_a^b F(x)g'(x)dx$$

2.6 Sums

$$c^a + c^{a+1} + \cdots + c^b = \frac{c^{b+1} - c^a}{c-1}, c \neq 1$$

$$1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$$

$$1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(2n+1)(n+1)}{6}$$

$$1^3 + 2^3 + 3^3 + \cdots + n^3 = \frac{n^2(n+1)^2}{4}$$

$$1^4 + 2^4 + 3^4 + \cdots + n^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$$

2.7 Series

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots, (-\infty < x < \infty)$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots, (-1 < x \leq 1)$$

$$\sqrt{1+x} = 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{2x^3}{32} - \frac{5x^4}{128} + \ldots, (-1 \leq x \leq 1)$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots, (-\infty < x < \infty)$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots, (-\infty < x < \infty)$$

2.8 Probability theory

Let X be a discrete random variable with probability $p_X(x)$ of assuming the value x. It will then have an expected value (mean) $\mu = \mathbb{E}(X) = \sum_x x p_X(x)$ and variance $\sigma^2 = V(X) = \mathbb{E}(X^2) - (\mathbb{E}(X))^2 = \sum_x (x - \mathbb{E}(X))^2 p_X(x)$ where σ is the standard deviation. If X is instead continuous it will have a probability density function $f_X(x)$ and the sums above will instead be integrals with $p_X(x)$ replaced by $f_X(x)$.

Expectation is linear:

$$\mathbb{E}(aX + bY) = a\mathbb{E}(X) + b\mathbb{E}(Y)$$

For independent X and Y,

$$V(aX + bY) = a^2V(X) + b^2V(Y).$$

2.8.1 Discrete distributions

Binomial distribution

The number of successes in n independent yes/no experiments, each which yields success with probability p is $\mathrm{Bin}(n,p)$, $n = 1, 2, \ldots$, $0 \leq p \leq 1$.

$$p(k) = \binom{n}{k} p^k (1-p)^{n-k}$$

$$\mu = np, \sigma^2 = np(1-p)$$

$\mathrm{Bin}(n,p)$ is approximately $\mathrm{Po}(np)$ for small p.

First success distribution

The number of trials needed to get the first success in independent yes/no experiments, each which yields success with probability p is $\mathrm{Fs}(p)$, $0 \leq p \leq 1$.

$$p(k) = p(1-p)^{k-1}, k = 1, 2, \ldots$$

$$\mu = \frac{1}{p}, \sigma^2 = \frac{1-p}{p^2}$$

Poisson distribution

The number of events occurring in a fixed period of time t if these events occur with a known average rate κ and independently of the time since the last event is $\mathrm{Po}(\lambda)$, $\lambda = t\kappa$.

$$p(k) = e^{-\lambda} \frac{\lambda^k}{k!}, k = 0, 1, 2, \ldots$$

$$\mu = \lambda, \sigma^2 = \lambda$$

2.8.2 Continuous distributions

Uniform distribution

If the probability density function is constant between a and b and 0 elsewhere it is $\mathrm{U}(a,b)$, $a < b$.

$$f(x) = \begin{cases} \frac{1}{b-a} & a < x < b \\ 0 & \text{otherwise} \end{cases}$$

$$\mu = \frac{a+b}{2}, \sigma^2 = \frac{(b-a)^2}{12}$$

Exponential distribution

The time between events in a Poisson process is $\mathrm{Exp}(\lambda)$, $\lambda > 0$.

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

$$\mu = \frac{1}{\lambda}, \sigma^2 = \frac{1}{\lambda^2}$$

Normal distribution

Most real random values with mean μ and variance σ^2 are well described by $\mathcal{N}(\mu, \sigma^2)$, $\sigma > 0$.

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

If $X_1 \sim \mathcal{N}(\mu_1, \sigma_1^2)$ and $X_2 \sim \mathcal{N}(\mu_2, \sigma_2^2)$ then

$$aX_1 + bX_2 + c \sim \mathcal{N}(\mu_1 + \mu_2 + c, a^2\sigma_1^2 + b^2\sigma_2^2)$$

2.9 Markov chains

A *Markov chain* is a discrete random process with the property that the next state depends only on the current state. Let $X_1, X_2, \ldots$ be a sequence of random variables generated by the Markov process. Then there is a transition matrix $\mathbf{P} = (p_{ij})$, with $p_{ij} = \Pr(X_n = i | X_{n-1} = j)$, and $\mathbf{p}^{(n)} = \mathbf{P}^n \mathbf{p}^{(0)}$ is the probability distribution for X_n (i.e., $p_i^{(n)} = \Pr(X_n = i)$), where $\mathbf{p}^{(0)}$ is the initial distribution.

π is a stationary distribution if $\pi = \pi\mathbf{P}$. If the Markov chain is *irreducible* (it is possible to get to any state from any state), then $\pi_i = \frac{1}{\mathbb{E}(T_i)}$ where $\mathbb{E}(T_i)$ is the expected time between two visits in state i. π_j/π_i is the expected number of visits in state j between two visits in state i.

For a connected, undirected and non-bipartite graph, where the transition probability is uniform among all neighbors, π_i is proportional to node i's degree.

A Markov chain is *ergodic* if the asymptotic distribution is independent of the initial distribution. A finite Markov chain is ergodic iff it is irreducible and *aperiodic* (i.e., the gcd of cycle lengths is 1). $\lim_{k\to\infty} \mathbf{P}^k = \mathbf{1}\pi$.

A Markov chain is an A-chain if the states can be partitioned into two sets $\mathbf{A}$ and $\mathbf{G}$, such that all states in $\mathbf{A}$ are absorbing ($p_{ii} = 1$), and all states in $\mathbf{G}$ leads to an absorbing state in $\mathbf{A}$. The probability for absorption in state $i \in \mathbf{A}$, when the initial state is j, is $a_{ij} = p_{ij} + \sum_{k \in \mathbf{G}} a_{ik} p_{kj}$. The expected time until absorption, when the initial state is i, is $t_i = 1 + \sum_{k \in \mathbf{G}} p_{ki} t_k$.

2.9.1 Optimization

Lagrange multipliers

To optimize $f(x_1, \ldots, x_n)$ subject to the constraints $g_k(x_1, \ldots, x_n) = 0$, a necessary condition for $(x_1, \ldots, x_n)$ to be a local extremum is that the gradient $\nabla f(x_1, \ldots x_n)$ must be a linear combination of the gradients $\nabla g_k(x_1, \ldots, x_n)$.

Data structures (3)

OrderStatisticTree.h

Description: A set (not multiset!) with support for finding the n'th element, and finding the index of an element. To get a map, change `null_type`.

Time: $\mathcal{O}(\log N)$ b9b97b, 17 lines

```cpp
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;
template<class T>
using Tree = tree<T, null_type, less<T>, rb_tree_tag,
    tree_order_statistics_node_update>;
void example() {
  Tree<int> t, t2; t.insert(8);
  auto it = t.insert(10).first;
  assert(it == t.lower_bound(9));
  assert(t.order_of_key(10) == 1);
  assert(t.order_of_key(11) == 2);
  assert(*t.find_by_order(0) == 8);
  t.join(t2); // assuming T < T2 or T > T2, merge t2
      into t
}
```

HashMap.h

Description: Hash map with mostly the same API as unordered_map, but ~3x faster. Uses 1.5x memory. Initial capacity must be a power of 2 (if provided). 481ffe, 7 lines

```cpp
#include <ext/pb_ds/assoc_container.hpp>
// To use most bits rather than just the lowest ones:
struct chash { // large odd number for C
  const uint64_t C = ll(4e18 * acos(0)) | 71;
  ll operator()(ll x) const { return __builtin_bswap64(
      x*C); }
};
__gnu_pbds::gp_hash_table<ll,ll,chash> h({},{},{},{},{
    1<<16});
```

SegmentTree.h

Description: Zero-indexed max-tree. Bounds are inclusive to the left and exclusive to the right. Can be changed by modifying T, f and unit.

Time: $\mathcal{O}(\log N)$ ceb5d1, 19 lines

```cpp
struct Tree {
  typedef int T;
  static constexpr T unit = INT_MIN;
  T f(T a, T b) { return max(a, b); } // (any
      associative fn)
  vector<T> s; int n;
  Tree(int N = 0, T def = unit) : s(2*N, def), n(N) {}
  void update(int pos, T val) {
    for (s[pos += n] = val; pos /= 2;)
      s[pos] = f(s[pos * 2], s[pos * 2 + 1]);
  }
  T query(int b, int e) { // query [b, e)
    T ra = unit, rb = unit;
    for (b += n, e += n; b < e; b /= 2, e /= 2) {
      if (b % 2) ra = f(ra, s[b++]);
      if (e % 2) rb = f(s[--e], rb);
    }
    return f(ra, rb);
  }
};
```

LazySegmentTree.h

Description: Segment tree with ability to add or set values of large intervals, and compute max of intervals. Can be changed to other things. Use with a bump allocator for better performance, and SmallPtr or implicit indices to save memory.

Usage: `Node* tr = new Node(v, 0, sz(v));`

Time: $\mathcal{O}(\log N)$.

"../various/BumpAllocator.h" 34ecf5, 50 lines

```cpp
const int inf = 1e9;
struct Node {
  Node *l = 0, *r = 0;
  int lo, hi, mset = inf, madd = 0, val = -inf;
  Node(int lo,int hi):lo(lo),hi(hi){} // Large interval
      of -inf
  Node(vi& v, int lo, int hi) : lo(lo), hi(hi) {
    if (lo + 1 < hi) {
      int mid = lo + (hi - lo)/2;
      l = new Node(v, lo, mid); r = new Node(v, mid, hi
          );
      val = max(l->val, r->val);
    }
    else val = v[lo];
  }
  int query(int L, int R) {
    if (R <= lo || hi <= L) return -inf;
    if (L <= lo && hi <= R) return val;
    push();
    return max(l->query(L, R), r->query(L, R));
  }
  void set(int L, int R, int x) {
    if (R <= lo || hi <= L) return;
    if (L <= lo && hi <= R) mset = val = x, madd = 0;
    else {
      push(), l->set(L, R, x), r->set(L, R, x);
      val = max(l->val, r->val);
    }
  }
  void add(int L, int R, int x) {
    if (R <= lo || hi <= L) return;
    if (L <= lo && hi <= R) {
      if (mset != inf) mset += x;
      else madd += x;
```

```
    val += x;
  }
  else {
    push(), l->add(L, R, x), r->add(L, R, x);
    val = max(l->val, r->val);
  }
}
void push() {
  if (!l) {
    int mid = lo + (hi - lo)/2;
    l = new Node(lo, mid); r = new Node(mid, hi);
  }
  if (mset != inf)
    l->set(lo,hi,mset), r->set(lo,hi,mset), mset = inf;
  else if (madd)
    l->add(lo,hi,madd), r->add(lo,hi,madd), madd = 0;
}
};
```

LiChao.h

Description: Extended Li Chao tree (segment tree for functions). Let F be a family of functions closed under function addition, such that for every $f \neq g$ from the family F there exists x such that $f(z) \leq g(z)$ for $z \leq x$ else $f(z) \geq g(z)$ or the other way around (intersect at one point). Typically F is the family of linear functions. DS maintains a sequence $c_0, c_1 \ldots c_{n-1}$ under operations max, add. b88a40, 74 lines

```
struct LiChao {
  struct Func {
    ll a, b; // a*x + b
    // Evaluate function in point x
    ll operator()(ll x) const { return a*x+b; }
    Func operator+(Func r) const {
      return {a+r.a, b+r.b};
    } // Sum of two functions
  }; // ID_ADD/MAX neutral elements for add/max
  static constexpr Func ID_ADD{0, 0};
  static constexpr Func ID_MAX{0, ll(-1e9)};
  vector<Func> val, lazy;
  int len;
  // Initialize tree for n elements; time: O(n)
  LiChao(int n = 0) {
    for (len = 1; len < n; len *= 2);
    val.resize(len*2, ID_MAX);
    lazy.resize(len*2, ID_ADD);
  }
  void push(int i) {
    if (i < len) rep(j, 2) {
      lazy[i*2+j] = lazy[i*2+j] + lazy[i];
      val[i*2+j] = val[i*2+j] + lazy[i];
    }
    lazy[i] = ID_ADD;
  } // For each x in [vb;ve)
    // set c[x] = max(c[x], f(x));
    // time: O(log^2 n) in general case,
    //       O(log n) if [vb;ve) = [0;len)
  void max(int vb, int ve, Func f,
           int i = 1, int b = 0, int e = -1) {
    if (e < 0) e = len;
    if (vb >= e || b >= ve || i >= len*2)
      return;
    int m = (b+e) / 2;
    push(i);
    if (b >= vb && e <= ve) {
      auto& g = val[i];
      if (g(m) < f(m)) swap(g, f);
      if (g(b) < f(b))
        max(vb, ve, f, i*2, b, m);
      else
        max(vb, ve, f, i*2+1, m, e);
    } else {
      max(vb, ve, f, i*2, b, m);
      max(vb, ve, f, i*2+1, m, e);
    }
  } // For each x in [vb;ve)
    // set c[x] = c[x] + f(x);
    // time: O(log^2 n) in general case,
    //       O(1) if [vb;ve) = [0;len)
  void add(int vb, int ve, Func f,
           int i = 1, int b = 0, int e = -1) {
    if (e < 0) e = len;
    if (vb >= e || b >= ve) return;
    if (b >= vb && e <= ve) {
      lazy[i] = lazy[i] + f;
      val[i] = val[i] + f;
    } else {
      int m = (b+e) / 2;
      push(i);
      max(b, m, val[i], i*2, b, m);
      max(m, e, val[i], i*2+1, m, e);
      val[i] = ID_MAX;
      add(vb, ve, f, i*2, b, m);
      add(vb, ve, f, i*2+1, m, e);
    }
  } // Get value of c[x]; time: O(log n)
  auto query(int x) {
    int i = x+len;
    auto ret = val[i](x);
    while (i /= 2)
      ret = ::max(ret+lazy[i](x), val[i](x));
    return ret; } };
```

UnionFind.h

Description: Disjoint-set data structure.
Time: $\mathcal{O}(\alpha(N))$ 7aa27c, 14 lines

```
struct UF {
  vi e;
  UF(int n) : e(n, -1) {}
  bool sameSet(int a, int b) { return find(a) == find(b); }
  int size(int x) { return -e[find(x)]; }
  int find(int x) { return e[x] < 0 ? x : e[x] = find(e[x]); }
  bool join(int a, int b) {
    a = find(a), b = find(b);
    if (a == b) return false;
    if (e[a] > e[b]) swap(a, b);
    e[a] += e[b]; e[b] = a;
    return true;
  }
};
```

UnionFindRollback.h

Description: Disjoint-set data structure with undo. If undo is not needed, skip st, time() and rollback().
Usage: int t = uf.time(); ...; uf.rollback(t);
Time: $\mathcal{O}(\log(N))$ de4ad0, 21 lines

```
struct RollbackUF {
  vi e; vector<pii> st;
  RollbackUF(int n) : e(n, -1) {}
  int size(int x) { return -e[find(x)]; }
  int find(int x) { return e[x] < 0 ? x : find(e[x]); }
  int time() { return sz(st); }
  void rollback(int t) {
    for (int i = time(); i --> t;)
      e[st[i].first] = st[i].second;
    st.resize(t);
  }
  bool join(int a, int b) {
    a = find(a), b = find(b);
    if (a == b) return false;
    if (e[a] > e[b]) swap(a, b);
    st.push_back({a, e[a]});
    st.push_back({b, e[b]});
    e[a] += e[b]; e[b] = a;
    return true;
  }
};
```

DequeRollback.h

Description: Deque-like undoing on data structures with amortized O(log n) overhead for operations. Maintains a deque of objects alongside a data structure that contains all of them. The data structure only needs to support insertions and undoing of last insertion using the following interface: - insert(...) - insert an object to DS - time() - returns current version number - rollback(t) - undo all operations after t Assumes time() == 0 for empty DS. 57bab5, 38 lines

```
struct DequeUndo {
  // Argument for insert(...) method of DS.
  using T = tuple<int, int>;
  DataStructure ds; // Configure DS type here.
  vector<T> elems[2];
  vector<pii> his = {{0,0}};
  // Push object to front or back of deque, depending on side arg.
  void push(T val, bool side) {
    elems[side].push_back(val);
    doPush(0, side);
  }
  // Pop object from front or back of deque, depending on side arg.
  void pop(int side) {
    auto &A = elems[side], &B = elems[!side];
    int cnt[2] = {};
    if (A.empty()) {
      assert(!B.empty());
      auto it = B.begin() + sz(B)/2 + 1;
      A.assign(B.begin(), it);
      B.erase(B.begin(), it);
      reverse(all(A));   his.resize(1);
      cnt[0] = sz(A);    cnt[1] = sz(B);
    } else{
      do {
        cnt[his.back().y ^ side]++;
        his.pop_back();
      } while (cnt[0]*2 < cnt[1] && cnt[0] < sz(A));
    }
    cnt[0]--; A.pop_back();
    ds.rollback(his.back().x);
    for (int i : {1, 0})
      while (cnt[i]) doPush(--cnt[i], i^side);
  }
  void doPush(int i, bool s) {
    apply([&](auto... x) { ds.insert(x...); },elems[s].rbegin()[i]);
    his.push_back({ds.time(), s});
  }
};
```

SubMatrix.h

Description: Calculate submatrix sums quickly, given upper-left and lower-right corners (half-open).
Usage: SubMatrix<int> m(matrix);
m.sum(0, 0, 2, 2); // top left 4 elements
Time: $\mathcal{O}\left(N^2+Q\right)$ c59ada, 13 lines

```
template<class T>
struct SubMatrix {
  vector<vector<T>> p;
  SubMatrix(vector<vector<T>>& v) {
    int R = sz(v), C = sz(v[0]);
    p.assign(R+1, vector<T>(C+1));
    rep(r,0,R) rep(c,0,C)
      p[r+1][c+1] = v[r][c] + p[r][c+1] + p[r+1][c] - p[r][c];
  }
  T sum(int u, int l, int d, int r) {
    return p[d][r] - p[d][l] - p[u][r] + p[u][l];
  }
};
```

Matrix.h

Description: Basic operations on square matrices.
Usage: Matrix<int, 3> A;
A.d = {{{{1,2,3}}, {{4,5,6}}, {{7,8,9}}}};
array<int, 3> vec = {1,2,3};
vec = (A^N) * vec; 6ab5db, 26 lines

```
template<class T, int N> struct Matrix {
  typedef Matrix M;
  array<array<T, N>, N> d{};
  M operator*(const M& m) const {
    M a;
    rep(i,0,N) rep(j,0,N)
      rep(k,0,N) a.d[i][j] += d[i][k]*m.d[k][j];
    return a;
  }
  array<T, N> operator*(const array<T, N>& vec) const {
    array<T, N> ret{};
    rep(i,0,N) rep(j,0,N) ret[i] += d[i][j] * vec[j];
    return ret;
  }
  M operator^(ll p) const {
    assert(p >= 0);
    M a, b(*this);
    rep(i,0,N) a.d[i][i] = 1;
    while (p) {
      if (p&1) a = a*b;
      b = b*b;
      p >>= 1;
    }
    return a;
  }
};
```

LineContainer.h

Description: Container where you can add lines of the form kx+m, and query maximum values at points x. Useful for dynamic programming ("convex hull trick").
Time: $\mathcal{O}(\log N)$ 8ec1c7, 30 lines

```
struct Line {
  mutable ll k, m, p;
  bool operator<(const Line& o) const { return k < o.k; }
  bool operator<(ll x) const { return p < x; }
};
struct LineContainer : multiset<Line, less<>> {
  // (for doubles, use inf = 1/.0, div(a,b) = a/b)
  static const ll inf = LLONG_MAX;
  ll div(ll a, ll b) { // floored division
    return a / b - ((a ^ b) < 0 && a % b); }
  bool isect(iterator x, iterator y) {
    if (y == end()) return x->p = inf, 0;
    if (x->k == y->k) x->p = x->m > y->m ? inf : -inf;
    else x->p = div(y->m - x->m, x->k - y->k);
    return x->p >= y->p;
  }
  void add(ll k, ll m) {
    auto z = insert({k, m, 0}), y = z++, x = y;
    while (isect(y, z)) z = erase(z);
    if (x != begin() && isect(--x, y)) isect(x, y = erase(y));
    while ((y = x) != begin() && (--x)->p >= y->p)
      isect(x, erase(y));
  }
  ll query(ll x) {
    assert(!empty());
    auto l = *lower_bound(x);
    return l.k * x + l.m;
  }
};
```

Treap.h

Description: A short self-balancing tree. It acts as a sequential container with log-time splits/joins, and is easy to augment with additional data.
Time: $\mathcal{O}(\log N)$ e8e28b, 77 lines

```
mt19937 rng(2137);
struct Node {
  Node *l = 0, *r = 0, *p = 0;
  int val, y, c = 1;
  Node(int val) : val(val), y(rng()) {}
  friend int cnt(Node* n) { return n ? n->c : 0; }
  void recalc() { c = cnt(l) + cnt(r) + 1; }
  void push() {}
};
void each(Node* n, auto f) {
  if (n) { each(n->l, f); f(n->val); each(n->r, f); }
}
pair<Node*, Node*> split(Node* n, int k) {
  if (!n) return {};
  n->push(); n->p = 0;
  if (cnt(n->l) >= k) { // "n->val >= k" for lower_bound(k)
    auto [L,R] = split(n->l, k);
    if (n->l = R) n->l->p = n;
    return n->recalc(), pair(L, n);
  } else {
    auto [L,R] = split(n->r,k - cnt(n->l) - 1); // and just "k"
    if (n->r = L) n->r->p = n;
    return n->recalc(), pair(n, R);
  }
}
Node* merge(Node* l, Node* r) {
  if (!l || !r) return l ?: r;
  if (l->y > r->y) {
    l->push();
    if (l->r = merge(l->r, r)) l->r->p = l;
    return l->recalc(), l;
  } else {
    r->push();
    if (r->l = merge(l, r->l)) r->l->p = r;
    return r->recalc(), r;
  }
}
Node* ins(Node* t, Node* n, int pos) {
  auto [l,r] = split(t, pos);
  return merge(merge(l, n), r);
}
// Union of two sorted treaps, O(m log(n/m)) where m<=n
// Makes small-to-large O(n log n) instead of log^2.
// Requires lower_bound split (not the default one).
Node* unite(Node* a, Node* b) {
  if (!a || !b) return a ?: b;
  if (a->y < b->y) swap(a, b);
  a->push(); a->p = 0;
  auto [l, r] = split(b, a->val); // lower_bound split
  if (a->l = unite(l, a->l)) a->l->p = a;
  if (a->r = unite(r, a->r)) a->r->p = a;
```

```
  return a->recalc(), a;
}
// Number of elements before n. If there are range
// reverse queries, recursively push the path to n.
int idx(Node* n) {
  int c = cnt(n->l);
  while (n->p) {
    if (n->p->l != n) c += cnt(n->p->l) + 1;
    n = n->p;
  }
  return c;
}
// Example application: move the range [l, r) to index k
void move(Node*& t, int l, int r, int k) {
  Node *a, *b, *c;
  tie(a,b) = split(t, l); tie(b,c) = split(b, r - l);
  if (k <= l) t = merge(ins(a, b, k), c);
  else t = merge(a, ins(c, b, k - r));
}
```

IntSet.h

Description: bitset with fast predecessor and successor queries. Assumes x86 shift overflows. Extremely fast (50-200mln operations in 1 second).

85cd6f, 32 lines

```
template<int N>
struct IntSet {
  static constexpr int B = 64;
  uint64_t V[N / B + 1] = {};
  IntSet<(N < B + 1 ? 0 : N / B + 1)> up;
  bool has(int i) { return (V[i / B] >> i) & 1; }
  void add(int i) {
    if (!V[i / B]) up.add(i / B);
    V[i / B] |= 1ull << i;
  }
  void del(int i) {
    if (!(V[i / B] &= ~(1ull << i))) up.del(i / B);
  }
  int next(int i) { // j > i such that j inside or -1
    auto x = V[i / B] >> i;
    if (x &= ~1) return i + __builtin_ctzll(x);
    return (i = up.next(i / B)) < 0 ? i :
      i * B + __builtin_ctzll(V[i]);
  }
  int prev(int i) { // j < i such that j inside or -1
    auto x = V[i / B] << (B - i - 1);
    if (x &= INT64_MAX)
      return i-__builtin_clzll(x);
    return (i = up.prev(i / B)) < 0 ? i :
      i * B + B - 1 - __builtin_clzll(V[i]);
  }
};
template<>
struct IntSet<0> {
  void add(int) {} void del(int) {}
  int next(int) { return -1; }
  int prev(int) { return -1; } };
```

FenwickTree.h

Description: Computes partial sums a[0] + a[1] + ... + a[pos - 1], and updates single elements a[i], taking the difference between the old and new value.

Time: Both operations are $\mathcal{O}(\log N)$.

e62fac, 22 lines

```
struct FT {
  vector<ll> s;
  FT(int n) : s(n) {}
  void update(int pos, ll dif) { // a[pos] += dif
    for (; pos < sz(s); pos |= pos + 1) s[pos] += dif;
  }
  ll query(int pos) { // sum of values in [0, pos)
    ll res = 0;
    for (; pos > 0; pos &= pos - 1) res += s[pos-1];
    return res;
  }
  int lower_bound(ll sum) {// min pos st sum of [0, pos] >= sum
    // Returns n if no sum is >= sum, or -1 if empty sum is.
    if (sum <= 0) return -1;
    int pos = 0;
    for (int pw = 1 << 25; pw; pw >>= 1) {
      if (pos + pw <= sz(s) && s[pos + pw-1] < sum)
        pos += pw, sum -= s[pos-1];
    }
    return pos;
  }
};
```

FenwickTree2d.h

Description: Computes sums a[i,j] for all i<I, j<J, and increases single elements a[i,j]. Requires that the elements to be updated are known in advance (call fakeUpdate() before init()).

Time: $\mathcal{O}\left(\log^2 N\right)$. (Use persistent segment trees for $\mathcal{O}(\log N)$.)

"FenwickTree.h" 157f07, 22 lines

```
struct FT2 {
  vector<vi> ys; vector<FT> ft;
  FT2(int limx) : ys(limx) {}
  void fakeUpdate(int x, int y) {
    for (; x < sz(ys); x |= x + 1) ys[x].push_back(y);
  }
  void init() {
    for (vi& v : ys) sort(all(v)), ft.emplace_back(sz(v));
  }
  int ind(int x, int y) {
    return (int)(lower_bound(all(ys[x]), y) - ys[x].begin()); }
  void update(int x, int y, ll dif) {
    for (; x < sz(ys); x |= x + 1)
      ft[x].update(ind(x, y), dif);
  }
  ll query(int x, int y) {
    ll sum = 0;
    for (; x; x &= x - 1)
      sum += ft[x-1].query(ind(x-1, y));
    return sum;
  }
};
```

RMQ.h

Description: Range Minimum Queries on an array. Returns min(V[a], V[a + 1], ... V[b - 1]) in constant time.

Usage: `RMQ rmq(values);`
`rmq.query(inclusive, exclusive);`

Time: $\mathcal{O}(|V|\log|V| + Q)$

510c32, 16 lines

```
template<class T>
struct RMQ {
  vector<vector<T>> jmp;
  RMQ(const vector<T>& V) : jmp(1, V) {
    for (int pw = 1, k = 1; pw * 2 <= sz(V); pw *= 2, ++k) {
      jmp.emplace_back(sz(V) - pw * 2 + 1);
      rep(j,0,sz(jmp[k]))
        jmp[k][j] = min(jmp[k - 1][j], jmp[k - 1][j + pw]);
    }
  }
  T query(int a, int b) {
    assert(a < b); // or return inf if a == b
    int dep = 31 - __builtin_clz(b - a);
    return min(jmp[dep][a], jmp[dep][b - (1 << dep)]);
  }
};
```

MoQueries.h

Description: Answer interval or tree path queries by finding an approximate TSP through the queries, and moving from one query to the next by adding/removing points at the ends. If values are on tree edges, change `step` to add/remove the edge (a, c) and remove the initial `add` call (but keep `in`).

Time: $\mathcal{O}\left(N\sqrt{Q}\right)$

a12ef4, 49 lines

```
void add(int ind, int end) { ... } // add a[ind] (end = 0 or 1)
void del(int ind, int end) { ... } // remove a[ind]
int calc() { ... } // compute current answer
vi mo(vector<pii> Q) {
  int L = 0, R = 0, blk = 350; // ~N/sqrt(Q)
  vi s(sz(Q)), res = s;
#define K(x) pii(x.first/blk, x.second ^ -(x.first/blk & 1))
  iota(all(s), 0);
  sort(all(s), [&](int s, int t){ return K(Q[s]) < K(Q[t]); });
  for (int qi : s) {
    pii q = Q[qi];
    while (L > q.first) add(--L, 0);
    while (R < q.second) add(R++, 1);
    while (L < q.first) del(L++, 0);
    while (R > q.second) del(--R, 1);
    res[qi] = calc();
  }
  return res;
}
vi moTree(vector<array<int, 2>> Q, vector<vi>& ed, int root=0){
  int N = sz(ed), pos[2] = {}, blk = 350; // ~N/sqrt(Q)
  vi s(sz(Q)), res = s, I(N), L(N), R(N), in(N), par(N);
  add(0, 0), in[0] = 1;
  auto dfs = [&](int x, int p, int dep, auto& f) -> void {
    par[x] = p;
    L[x] = N;
    if (dep) I[x] = N++;
    for (int y : ed[x]) if (y != p) f(y, x, !dep, f);
    if (!dep) I[x] = N++;
    R[x] = N;
  };
  dfs(root, -1, 0, dfs);
#define K(x) pii(I[x[0]] / blk, I[x[1]] ^ -(I[x[0]] / blk & 1))
  iota(all(s), 0);
  sort(all(s), [&](int s, int t){ return K(Q[s]) < K(Q[t]); });
  for (int qi : s) rep(end,0,2) {
    int &a = pos[end], b = Q[qi][end], i = 0;
#define step(c) { if (in[c]) { del(a, end); in[a] = 0; } \
                  else { add(c, end); in[c] = 1; } a = c; }
    while (!(L[b] <= L[a] && R[a] <= R[b]))
      I[i++] = b, b = par[b];
    while (a != b) step(par[a]);
    while (i--) step(I[i]);
    if (end) res[qi] = calc();
  }
  return res;
}
```

WaveletTree.h

Description: Wavelet tree. Can be sped up with bitset. Easily extendable to support sum.

Time: $\mathcal{O}((n + q)\log n)$

c63f2b, 45 lines

```
struct Node {
  int lo, hi;
  vector<int> s;
  Node *l = 0, *r = 0;
  Node(auto st, auto ed, auto sst) {
    int n = ed - st;
    lo = sst[0];
    hi = sst[n - 1] + 1;
    if (lo + 1 < hi) {
      int mid = sst[n / 2];
      if (mid==sst[0])mid=*upper_bound(sst,sst+n,mid);
      s.reserve(n + 1);
      s.push_back(0);
      for (auto it = st; it != ed; it++) {
        s.push_back(s.back() + (*it < mid));
      }
      auto k = stable_partition(st, ed, [&](int x) {
        return x < mid;
      });
      auto sm = lower_bound(sst, sst + n, mid);
      if (k != st) l = new Node(st, k, sst);
      if (k != ed) r = new Node(k, ed, sm);
    }
  }
  int kth(int a, int b, int k) {
    if (lo + 1 == hi) return lo;
    int x = s[a], y = s[b];
    return k < y - x ? l->kth(x, y, k)
                     : r->kth(a-x,b-y,k-(y-x));
  }
  int count(int a, int b, int k) {
    if (lo >= k) return 0;
    if (hi <= k) return b - a;
    int x = s[a], y = s[b];
    return (l ? l->count(x, y, k) : 0) +
           (r ? r->count(a - x, b - y, k) : 0);
  }
  int freq(int a, int b, int k) {
    if (k < lo || hi <= k) return 0;
    if (lo + 1 == hi) return b - a;
    int x = s[a], y = s[b];
    return (l ? l->freq(x, y, k) : 0) +
           (r ? r->freq(a - x, b - y, k) : 0);
  }
};
```

Numerical (4)

4.1 Polynomials and recurrences

Polynomial.h

c9b7b0, 17 lines

```
struct Poly {
  vector<double> a;
  double operator()(double x) const {
    double val = 0;
    for (int i = sz(a); i--;) (val *= x) += a[i];
    return val;
  }
  void diff() {
    rep(i,1,sz(a)) a[i-1] = i*a[i];
    a.pop_back();
  }
  void divroot(double x0) {
    double b = a.back(), c; a.back() = 0;
    for(int i=sz(a)-1; i--;) c = a[i], a[i] = a[i+1]*x0+b, b=c;
    a.pop_back();
  }
};
```

PolyRoots.h

Description: Finds the real roots to a polynomial.

Usage: `polyRoots({{2,-3,1}},-1e9,1e9) // solve x^2-3x+2 = 0`

Time: $\mathcal{O}\left(n^2\log(1/\epsilon)\right)$

"Polynomial.h" b00bfe, 23 lines

```
vector<double> polyRoots(Poly p, double xmin, double xmax) {
  if (sz(p.a) == 2) { return {-p.a[0]/p.a[1]}; }
  vector<double> ret;
  Poly der = p;
  der.diff();
  auto dr = polyRoots(der, xmin, xmax);
  dr.push_back(xmin-1);
  dr.push_back(xmax+1);
  sort(all(dr));
  rep(i,0,sz(dr)-1) {
    double l = dr[i], h = dr[i+1];
    bool sign = p(l) > 0;
    if (sign ^ (p(h) > 0)) {
      rep(it,0,60) { // while (h - l > 1e-8)
        double m = (l + h) / 2, f = p(m);
        if ((f <= 0) ^ sign) l = m;
        else h = m;
      }
      ret.push_back((l + h) / 2);
    }
  }
  return ret;
}
```

PolyInterpolate.h

Description: 1. Interpolate set of points (i, vec[i]) and return it evaluated at x; 2. Given n points (x, f(x)) compute n-1-degree polynomial f that passes through them;

Time: $\mathcal{O}(n)$ and $\mathcal{O}\left(n^2\right)$

174038, 33 lines

```
template<class T>
T polyExtend(vector<T>& vec, T x) {
  int n = sz(vec);
  vector<T> fac(n, 1), suf(n, 1);
  rep(i, 1, n) fac[i] = fac[i-1] * i;
  for (int i=n; --i;) suf[i-1] = suf[i]*(x-i);
  T pref = 1, ret = 0;
  rep(i, 0, n) {
    T d = fac[i] * fac[n-i-1] * ((n-i)%2*2-1);
    ret += vec[i] * suf[i] * pref / d;
    pref *= x-i;
  }
  return ret;
}
template<class T>
vector<T> polyInterp(vector<pair<T, T>> P) {
  int n = sz(P);
  vector<T> ret(n), tmp(n);
  T last = 0;
  tmp[0] = 1;
  rep(k, 0, n-1) rep(i, k+1, n)
    P[i].second = (P[i].second-P[k].second) / (P[i].first-P[k].first);
  rep(k, 0, n) rep(i, 0, n) {
    ret[i] += P[k].second * tmp[i];
    swap(last, tmp[i]);
```

```
    tmp[i] -= last * P[k].first;
  }
  return ret;
}
```

BerlekampMassey.h

Description: Recovers any n-order linear recurrence relation from the first $2n$ terms of the recurrence. Useful for guessing linear recurrences after brute-forcing the first terms. Should work on any field, but numerical stability for floats is not guaranteed. Output will have size $\leq n$.
Usage: berlekampMassey({0, 1, 1, 3, 5, 11}) // {1, 2}
Time: $\mathcal{O}\left(N^2\right)$

"../number-theory/ModPow.h" 96548b, 20 lines

```
vector<ll> berlekampMassey(vector<ll> s) {
  int n = sz(s), L = 0, m = 0;
  vector<ll> C(n), B(n), T;
  C[0] = B[0] = 1;
  ll b = 1;
  rep(i,0,n) { ++m;
    ll d = s[i] % mod;
    rep(j,1,L+1) d = (d + C[j] * s[i - j]) % mod;
    if (!d) continue;
    T = C; ll coef = d * modpow(b, mod-2) % mod;
    rep(j,m,n) C[j] = (C[j] - coef * B[j - m]) % mod;
    if (2 * L > i) continue;
    L = i + 1 - L; B = T; b = d; m = 0;
  }
  C.resize(L + 1); C.erase(C.begin());
  for (ll& x : C) x = (mod - x) % mod;
  return C;
}
```

LinearRecurrence.h

Description: Generates the k'th term of an n-order linear recurrence $S[i] = \sum_j S[i-j-1]tr[j]$, given $S[0 \ldots \geq n-1]$ and $tr[0 \ldots n-1]$. Faster than matrix multiplication. Useful together with Berlekamp–Massey.
Usage: linearRec({0, 1}, {1, 1}, k) // k'th Fibonacci number
Time: $\mathcal{O}\left(n^2 \log k\right)$

f4e444, 26 lines

```
typedef vector<ll> Poly;
ll linearRec(Poly S, Poly tr, ll k) {
  int n = sz(tr);
  auto combine = [&](Poly a, Poly b) {
    Poly res(n * 2 + 1);
    rep(i,0,n+1) rep(j,0,n+1)
      res[i + j] = (res[i + j] + a[i] * b[j]) % mod;
    for (int i = 2 * n; i > n; --i) rep(j,0,n)
      res[i - 1 - j] = (res[i - 1 - j] + res[i] * tr[j
          ]) % mod;
    res.resize(n + 1);
    return res;
  };
  Poly pol(n + 1), e(pol);
  pol[0] = e[1] = 1;
  for (++k; k; k /= 2) {
    if (k % 2) pol = combine(pol, e);
    e = combine(e, e);
  }
  ll res = 0;
  rep(i,0,n) res = (res + pol[i + 1] * S[i]) % mod;
  return res;
}
```

FastMulDet.h

Description: Given a matrix M, s.t. we can quickly compute f(v) = Mv for any vector v, computes det(M). Single iteration fails on identity matrix with probability around n^2/mod. For small mod you can modify this to use a field extension.
Time: 4n calls to f

"BerlekampMassey.h" 84262e, 30 lines

```
mt19937_64 rnd{2137};
vector<ll> rndVec(int n) {
  vector<ll> r(n);
  rep(i, 0, n) r[i] = rnd() % mod;
  return r;
}
ll dot(vector<ll> &a, vector<ll> &b) {
  ll r = 0;
  rep(i, 0, sz(a)) r += a[i] * b[i] % mod;
  return r % mod;
}
void pointwise(vector<ll> &a, vector<ll> &b) {
  rep(i, 0, sz(a)) a[i] = a[i] * b[i] % mod;
}
ll detOnce(int n, auto f) {
  auto v = rndVec(n), r = rndVec(n), a = rndVec(n);
  vector<ll> vals;
  rep(_, 0, n*2) {
    pointwise(a, r);
    vals.push_back(dot(v, a = f(a)));
  }
  auto ber = berlekampMassey(vals);
  if (sz(ber) != n) return 0;
  ll prod = 1;
  for (ll x : r) prod = prod * x % mod;
  int sg = n % 2 ? 1 : -1;
  return (mod + ber[n-1] * sg) * modpow(prod, mod-2) %
      mod;
}
ll det(int n, auto f) {
  return detOnce(n, f) ?: detOnce(n, f); }
```

4.2 Optimization

GoldenSectionSearch.h

Description: Finds the argument minimizing the function f in the interval $[a, b]$ assuming f is unimodal on the interval, i.e. has only one local minimum and no local maximum. The maximum error in the result is eps. Works equally well for maximization with a small change in the code. See TernarySearch.h in the Various chapter for a discrete version.
Usage: double func(double x) { return 4+x+.3*x*x; }
double xmin = gss(-1000,1000,func);
Time: $\mathcal{O}\left(\log((b-a)/\epsilon)\right)$

31d45b, 14 lines

```
double gss(double a, double b, double (*f)(double)) {
  double r = (sqrt(5)-1)/2, eps = 1e-7;
  double x1 = b - r*(b-a), x2 = a + r*(b-a);
  double f1 = f(x1), f2 = f(x2);
  while (b-a > eps)
    if (f1 < f2) { //change to > to find maximum
      b = x2; x2 = x1; f2 = f1;
      x1 = b - r*(b-a); f1 = f(x1);
    } else {
      a = x1; x1 = x2; f1 = f2;
      x2 = a + r*(b-a); f2 = f(x2);
    }
  return a;
}
```

HillClimbing.h

Description: Poor man's optimization for unimodal functions.

8eeeaf, 14 lines

```
typedef array<double, 2> P;
template<class F> pair<double, P> hillClimb(P start, F
    f) {
  pair<double, P> cur(f(start), start);
  for (double jmp = 1e9; jmp > 1e-20; jmp /= 2) {
    rep(j,0,100) rep(dx,-1,2) rep(dy,-1,2) {
      P p = cur.second;
      p[0] += dx*jmp;
      p[1] += dy*jmp;
      cur = min(cur, make_pair(f(p), p));
    }
  }
  return cur;
}
```

Integrate.h

Description: Simple integration of a function over an interval using Simpson's rule. The error should be proportional to h^4, although in practice you will want to verify that the result is stable to desired precision when epsilon changes.

4756fc, 7 lines

```
template<class F>
double quad(double a, double b, F f, const int n =
    1000) {
  double h = (b - a) / 2 / n, v = f(a) + f(b);
  rep(i,1,n*2)
    v += f(a + i*h) * (i&1 ? 4 : 2);
  return v * h / 3;
}
```

IntegrateAdaptive.h

Description: Fast integration using an adaptive Simpson's rule.
Usage: double sphereVolume = quad(-1, 1, [](double x) {
return quad(-1, 1, [&](double y) {
return quad(-1, 1, [&](double z) {
return x*x + y*y + z*z < 1; });});});

92dd79, 15 lines

```
typedef double d;
#define S(a,b) (f(a) + 4*f((a+b) / 2) + f(b)) * (b-a) /
    6
template <class F>
d rec(F& f, d a, d b, d eps, d S) {
  d c = (a + b) / 2;
  d S1 = S(a, c), S2 = S(c, b), T = S1 + S2;
  if (abs(T - S) <= 15 * eps || b - a < 1e-10)
    return T + (T - S) / 15;
  return rec(f, a, c, eps / 2, S1) + rec(f, c, b, eps /
      2, S2);
}
template<class F>
d quad(d a, d b, F f, d eps = 1e-8) {
  return rec(f, a, b, eps, S(a, b));
}
```

Simplex.h

Description: Solves a general linear maximization problem: maximize $c^T x$ subject to $Ax \leq b$, $x \geq 0$. Returns -inf if there is no solution, inf if there are arbitrarily good solutions, or the maximum value of $c^T x$ otherwise. The input vector is set to an optimal x (or in the unbounded case, an arbitrary solution fulfilling the constraints). Numerical stability is not guaranteed. For better performance, define variables such that $x = 0$ is viable.
Usage: vvd A = {{1,-1}, {-1,1}, {-1,-2}};
vd b = {1,1,-4}, c = {-1,-1}, x;
T val = LPSolver(A, b, c).solve(x);
Time: $\mathcal{O}\left(NM * \#pivots\right)$, where a pivot may be e.g. an edge relaxation. $\mathcal{O}\left(2^n\right)$ in the general case.

aa8530, 68 lines

```
typedef double T; // long double, Rational, double +
    mod<P>...
typedef vector<T> vd;
typedef vector<vd> vvd;
const T eps = 1e-8, inf = 1/.0;
#define MP make_pair
#define ltj(X) if(s == -1 || MP(X[j],N[j]) < MP(X[s],N[
    s])) s=j
struct LPSolver {
  int m, n;
  vi N, B;
  vvd D;
  LPSolver(const vvd& A, const vd& b, const vd& c) :
    m(sz(b)), n(sz(c)), N(n+1), B(m), D(m+2, vd(n+2)) {
      rep(i,0,m) rep(j,0,n) D[i][j] = A[i][j];
      rep(i,0,m) { B[i] = n+i; D[i][n] = -1; D[i][n+1]
          = b[i];}
      rep(j,0,n) { N[j] = j; D[m][j] = -c[j]; }
      N[n] = -1; D[m+1][n] = 1;
    }
  void pivot(int r, int s) {
    T *a = D[r].data(), inv = 1 / a[s];
    rep(i,0,m+2) if (i != r && abs(D[i][s]) > eps) {
      T *b = D[i].data(), inv2 = b[s] * inv;
      rep(j,0,n+2) b[j] -= a[j] * inv2;
      b[s] = a[s] * inv2;
    }
    rep(j,0,n+2) if (j != s) D[r][j] *= inv;
    rep(i,0,m+2) if (i != r) D[i][s] *= -inv;
    D[r][s] = inv;
    swap(B[r], N[s]);
  }
  bool simplex(int phase) {
    int x = m + phase - 1;
    for (;;) {
      int s = -1;
      rep(j,0,n+1) if (N[j] != -phase) ltj(D[x]);
      if (D[x][s] >= -eps) return true;
      int r = -1;
      rep(i,0,m) {
        if (D[i][s] <= eps) continue;
        if (r == -1 || MP(D[i][n+1] / D[i][s], B[i])
                     < MP(D[r][n+1] / D[r][s], B[r])) r
                         = i;
      }
      if (r == -1) return false;
      pivot(r, s);
    }
  }
  T solve(vd &x) {
    int r = 0;
    rep(i,1,m) if (D[i][n+1] < D[r][n+1]) r = i;
    if (D[r][n+1] < -eps) {
      pivot(r, n);
      if (!simplex(2) || D[m+1][n+1] < -eps) return -
          inf;
      rep(i,0,m) if (B[i] == -1) {
        int s = 0;
        rep(j,1,n+1) ltj(D[i]);
        pivot(i, s);
      }
    }
    bool ok = simplex(1); x = vd(n);
    rep(i,0,m) if (B[i] < n) x[B[i]] = D[i][n+1];
    return ok ? D[m][n+1] : inf;
  }
};
```

4.3 Matrices

Determinant.h

Description: Calculates determinant of a matrix. Destroys the matrix.
Time: $\mathcal{O}\left(N^3\right)$

4583fb, 16 lines

```
template<class T>
T det(vector<vector<T>>& a) {
  int n = sz(a); T res = 1;
  rep(i,0,n) {
    int b = i;
    rep(j,i+1,n) if (abs(a[j][i]) > abs(a[b][i])) b = j
        ;
    if (i != b) swap(a[i], a[b]), res *= -1;
    res *= a[i][i];
    if (res == 0) return 0;
    rep(j,i+1,n) {
      T v = a[j][i] / a[i][i];
      if (v != 0) rep(k,i+1,n) a[j][k] -= v * a[i][k];
    }
  }
  return res;
}
```

IntDeterminant.h

Description: Calculates determinant using modular arithmetics. Modulos can also be removed to get a pure-integer version.
Time: $\mathcal{O}\left(N^3\right)$

3313dc, 18 lines

```
const ll mod = 12345;
ll det(vector<vector<ll>>& a) {
  int n = sz(a); ll ans = 1;
  rep(i,0,n) {
    rep(j,i+1,n) {
      while (a[j][i] != 0) { // gcd step
        ll t = a[i][i] / a[j][i];
        if (t) rep(k,i,n)
          a[i][k] = (a[i][k] - a[j][k] * t) % mod;
        swap(a[i], a[j]);
        ans *= -1;
      }
    }
    ans = ans * a[i][i] % mod;
    if (!ans) return 0;
  }
  return (ans + mod) % mod;
}
```

SolveLinear.h

Description: Solves $Ax = b$. If no solutions exist, returns -1. Otherwise, returns the rank of A and transforms it s.t. $\{A'_1, A'_2, \ldots\}$ is a basis of the kernel of A.
Time: $\mathcal{O}\left(n^2 m\right)$

4f0aa8, 41 lines

```
const double eps = 1e-12;
template<class T>
int solveLinear(vector<vector<T>>& A, vector<T>& b,
    vector<T>& x) {
  int n = sz(A), m = sz(x), rank = 0, br, bc;
  if (n) assert(sz(A[0]) == m);
  vi col(m); iota(all(col), 0);
  rep(i,0,n) {
    T v, bv = 0;
    rep(r,i,n) rep(c,i,m)
      if ((v = abs(A[r][c])) > bv)
        br = r, bc = c, bv = v;
    if (bv <= eps) {
      rep(j,i,n) if (abs(b[j]) > eps) return -1;
      break;
```

```
    }
    swap(A[i], A[br]);
    swap(b[i], b[br]);
    swap(col[i], col[bc]);
    rep(j,0,n) swap(A[j][i], A[j][bc]);
    bv = 1/A[i][i];
    rep(j,0,n) if (j != i) {
      T fac = A[j][i] * bv;
      b[j] -= fac * b[i];
      rep(k,i+1,m) A[j][k] -= fac*A[i][k];
    }
    rank++;
  }
  x.assign(m, 0);
  for (int i = rank - 1; i >= 0; i--) {
    b[i] /= A[i][i];
    x[col[i]] = b[i];
  }
  vector<vector<T>> ker(m - rank, vector<T>(m));
  rep(i, rank, m) {
    ker[i - rank][col[i]] = 1;
    rep(j, 0, rank) ker[i - rank][col[j]] -= A[j][i] /
        A[j][j];
  }
  return A = ker, rank;
}
```

SolveLinearBinary.h

Description: Solves $Ax = b$ over $\mathbb{F}_2$. If there are multiple solutions, one is returned arbitrarily. Returns rank, or -1 if no solutions. Destroys A and b.

Time: $\mathcal{O}\left(n^2 m\right)$ fa2d7a, 34 lines

```
typedef bitset<1000> bs;
int solveLinear(vector<bs>& A, vi& b, bs& x, int m) {
  int n = sz(A), rank = 0, br;
  assert(m <= sz(x));
  vi col(m); iota(all(col), 0);
  rep(i,0,n) {
    for (br=i; br<n; ++br) if (A[br].any()) break;
    if (br == n) {
      rep(j,i,n) if(b[j]) return -1;
      break;
    }
    int bc = (int)A[br]._Find_next(i-1);
    swap(A[i], A[br]);
    swap(b[i], b[br]);
    swap(col[i], col[bc]);
    rep(j,0,n) if (A[j][i] != A[j][bc]) {
      A[j].flip(i); A[j].flip(bc);
    }
    rep(j,i+1,n) if (A[j][i]) {
      b[j] ^= b[i];
      A[j] ^= A[i];
    }
    rank++;
  }
  x = bs();
  for (int i = rank; i--;) {
    if (!b[i]) continue;
    x[col[i]] = 1;
    rep(j,0,i) b[j] ^= A[j][i];
  }
  return rank; // (multiple solutions if rank < m)
}
```

MatrixInverse.h

Description: Invert matrix A. Returns rank; result is stored in A unless singular (rank < n). Can easily be extended to prime moduli; for prime powers, repeatedly set $A^{-1} = A^{-1}(2I - AA^{-1}) \pmod{p^k}$ where A^{-1} starts as the inverse of A mod p, and k is doubled in each step.

Time: $\mathcal{O}\left(n^3\right)$ d43579, 36 lines

```
template<class T>
int matInv(vector<vector<T>>& A) {
  int n = sz(A); vi col(n);
  vector<vector<T>> tmp(n, vector<T>(n));
  rep(i,0,n) tmp[i][i] = 1, col[i] = i;
  rep(i,0,n) {
    int r = i, c = i;
    rep(j,i,n) rep(k,i,n)
      if (abs(A[j][k]) > abs(A[r][c]))
        r = j, c = k;
    if (abs(A[r][c]) < 1e-12) return i;
    A[i].swap(A[r]); tmp[i].swap(tmp[r]);
    rep(j,0,n)
      swap(A[j][i], A[j][c]), swap(tmp[j][i], tmp[j][c
          ]);
    swap(col[i], col[c]);
    T v = A[i][i];
    rep(j,i+1,n) {
      T f = A[j][i] / v;
      A[j][i] = 0;
      rep(k,i+1,n) A[j][k] -= f*A[i][k];
      rep(k,0,n) tmp[j][k] -= f*tmp[i][k];
    }
    rep(j,i+1,n) A[i][j] /= v;
    rep(j,0,n) tmp[i][j] /= v;
    A[i][i] = 1;
  }
  for (int i = n-1; i > 0; --i) rep(j,0,i) {
    T v = A[j][i];
    rep(k,0,n) tmp[j][k] -= v*tmp[i][k];
  }
  rep(i,0,n) rep(j,0,n) A[col[i]][col[j]] = tmp[i][j];
  return n;
}
```

Tridiagonal.h

Description: $x = \text{tridiagonal}(d, p, q, b)$ solves the equation system

$$\begin{pmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \\ \vdots \\ b_{n-1} \end{pmatrix} = \begin{pmatrix} d_0 & p_0 & 0 & 0 & \cdots & 0 \\ q_0 & d_1 & p_1 & 0 & \cdots & 0 \\ 0 & q_1 & d_2 & p_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & q_{n-3} & d_{n-2} & p_{n-2} \\ 0 & 0 & \cdots & 0 & q_{n-2} & d_{n-1} \end{pmatrix} \begin{pmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_{n-1} \end{pmatrix}.$$

This is useful for solving problems on the type

$$a_i = b_i a_{i-1} + c_i a_{i+1} + d_i, \, 1 \le i \le n,$$

where a_0, a_{n+1}, b_i, c_i and d_i are known. a can then be obtained from

$$\{a_i\} = \text{tridiagonal}(\{1, -1, -1, ..., -1, 1\}, \{0, c_1, c_2, \ldots, c_n\}, \{b_1, b_2, \ldots, b_n, 0\}, \{a_0, d_1, d_2, \ldots, d_n, a_{n+1}\}).$$

Fails if the solution is not unique.

If $|d_i| > |p_i| + |q_{i-1}|$ for all i, or $|d_i| > |p_{i-1}| + |q_i|$, or the matrix is positive definite, the algorithm is numerically stable and neither `tr` nor the check for `diag[i] == 0` is needed.

Time: $\mathcal{O}(N)$ 8f9fa8, 26 lines

```
typedef double T;
vector<T> tridiagonal(vector<T> diag, const vector<T>&
    super,
    const vector<T>& sub, vector<T> b) {
  int n = sz(b); vi tr(n);
  rep(i,0,n-1) {
    if (abs(diag[i]) < 1e-9 * abs(super[i])) { // diag[
        i] == 0
      b[i+1] -= b[i] * diag[i+1] / super[i];
      if (i+2 < n) b[i+2] -= b[i] * sub[i+1] / super[i
          ];
      diag[i+1] = sub[i]; tr[++i] = 1;
    } else {
      diag[i+1] -= super[i]*sub[i]/diag[i];
      b[i+1] -= b[i]*sub[i]/diag[i];
    }
  }
  for (int i = n; i--;) {
    if (tr[i]) {
      swap(b[i], b[i-1]);
      diag[i-1] = diag[i];
      b[i] /= super[i-1];
    } else {
      b[i] /= diag[i];
      if (i) b[i-1] -= b[i]*super[i-1];
    }
  }
  return b;
}
```

4.4 Fourier transforms

FastFourierTransform.h

Description: fft(a) computes $\hat{f}(k) = \sum_x a[x] \exp(2\pi i \cdot kx/N)$ for all k. N must be a power of 2. Useful for convolution: `conv(a, b) = c`, where $c[x] = \sum a[i]b[x-i]$. For convolution of complex numbers or more than two vectors: FFT, multiply pointwise, divide by n, reverse(start+1, end), FFT back. Rounding is safe if $(\sum a_i^2 + \sum b_i^2) \log_2 N < 9 \cdot 10^{14}$ (in practice 10^{16}; higher for random inputs). Otherwise, use NTT/FFTMod.

Time: $\mathcal{O}(N \log N)$ with $N = |A| + |B|$ ($\sim 1s$ for $N = 2^{22}$) 00ced6, 35 lines

```
typedef complex<double> C;
typedef vector<double> vd;
void fft(vector<C>& a) {
  int n = sz(a), L = 31 - __builtin_clz(n);
  static vector<complex<long double>> R(2, 1);
  static vector<C> rt(2, 1);  // (^ 10% faster if
      double)
  for (static int k = 2; k < n; k *= 2) {
    R.resize(n); rt.resize(n);
    auto x = polar(1.0L, acos(-1.0L) / k);
    rep(i,k,2*k) rt[i] = R[i] = i&1 ? R[i/2] * x : R[i
        /2];
  }
  vi rev(n);
  rep(i,0,n) rev[i] = (rev[i / 2] | (i & 1) << L) / 2;
  rep(i,0,n) if (i < rev[i]) swap(a[i], a[rev[i]]);
  for (int k = 1; k < n; k *= 2)
    for (int i = 0; i < n; i += 2 * k) rep(j,0,k) {
      C z = rt[j+k] * a[i+j+k]; // (25% faster if hand-
          rolled)
      a[i + j + k] = a[i + j] - z;
      a[i + j] += z;
    }
}
vd conv(const vd& a, const vd& b) {
  if (a.empty() || b.empty()) return {};
  vd res(sz(a) + sz(b) - 1);
  int L = 32 - __builtin_clz(sz(res)), n = 1 << L;
  vector<C> in(n), out(n);
  copy(all(a), begin(in));
  rep(i,0,sz(b)) in[i].imag(b[i]);
  fft(in);
  for (C& x : in) x *= x;
  rep(i,0,n) out[i] = in[-i & (n - 1)] - conj(in[i]);
  fft(out);
  rep(i,0,sz(res)) res[i] = imag(out[i]) / (4 * n);
  return res;
}
```

FastFourierTransformMod.h

Description: Higher precision FFT, can be used for convolutions modulo arbitrary integers as long as $N \log_2 N \cdot \text{mod} < 8.6 \cdot 10^{14}$ (in practice 10^{16} or higher). Inputs must be in $[0, \text{mod})$.

Time: $\mathcal{O}(N \log N)$, where $N = |A| + |B|$ (twice as slow as NTT or FFT)

"FastFourierTransform.h" b82773, 22 lines

```
typedef vector<ll> vl;
template<int M> vl convMod(const vl &a, const vl &b) {
  if (a.empty() || b.empty()) return {};
  vl res(sz(a) + sz(b) - 1);
  int B=32-__builtin_clz(sz(res)), n=1<<B, cut=int(sqrt
      (M));
  vector<C> L(n), R(n), outs(n), outl(n);
  rep(i,0,sz(a)) L[i] = C((int)a[i] / cut, (int)a[i] %
      cut);
  rep(i,0,sz(b)) R[i] = C((int)b[i] / cut, (int)b[i] %
      cut);
  fft(L), fft(R);
  rep(i,0,n) {
    int j = -i & (n - 1);
    outl[j] = (L[i] + conj(L[j])) * R[i] / (2.0 * n);
    outs[j] = (L[i] - conj(L[j])) * R[i] / (2.0 * n) /
        1i;
  }
  fft(outl), fft(outs);
  rep(i,0,sz(res)) {
    ll av = ll(real(outl[i])+.5), cv = ll(imag(outs[i])
        +.5);
    ll bv = ll(imag(outl[i])+.5) + ll(real(outs[i])+.5)
        ;
    res[i] = ((av % M * cut + bv) % M * cut + cv) % M;
  }
  return res;
}
```

NumberTheoreticTransform.h

Description: ntt(a) computes $\hat{f}(k) = \sum_x a[x] g^{xk}$ for all k, where $g = \text{root}^{(mod-1)/N}$. N must be a power of 2. Useful for convolution modulo specific nice primes of the form $2^a b + 1$, where the convolution result has size at most 2^a. For arbitrary modulo, see FFTMod. `conv(a, b) = c`, where $c[x] = \sum a[i]b[x-i]$. For manual convolution: NTT the inputs, multiply pointwise, divide by n, reverse(start+1, end), NTT back. Inputs must be in [0, mod).

Time: $\mathcal{O}(N \log N)$

"../number-theory/ModPow.h" ced03d, 35 lines

```
const ll mod = (119 << 23) + 1, root = 62; // =
    998244353
// For p < 2^30 there is also e.g. 5 << 25, 7 << 26,
    479 << 21
// and 483 << 21 (same root). The last two are > 10^9.
typedef vector<ll> vl;
void ntt(vl &a) {
  int n = sz(a), L = 31 - __builtin_clz(n);
  static vl rt(2, 1);
  for (static int k = 2, s = 2; k < n; k *= 2, s++) {
    rt.resize(n);
    ll z[] = {1, modpow(root, mod >> s)};
    rep(i,k,2*k) rt[i] = rt[i / 2] * z[i & 1] % mod;
  }
  vi rev(n);
  rep(i,0,n) rev[i] = (rev[i / 2] | (i & 1) << L) / 2;
  rep(i,0,n) if (i < rev[i]) swap(a[i], a[rev[i]]);
  for (int k = 1; k < n; k *= 2)
    for (int i = 0; i < n; i += 2 * k) rep(j,0,k) {
      ll z = rt[j + k] * a[i + j + k] % mod, &ai = a[i
          + j];
      a[i + j + k] = ai - z + (z > ai ? mod : 0);
      ai += (ai + z >= mod ? z - mod : z);
    }
}
vl conv(const vl &a, const vl &b) {
  if (a.empty() || b.empty()) return {};
  int s = sz(a) + sz(b) - 1, B = 32 - __builtin_clz(s),
      n = 1 << B;
  int inv = modpow(n, mod - 2);
  vl L(a), R(b), out(n);
  L.resize(n), R.resize(n);
  ntt(L), ntt(R);
  rep(i,0,n)
    out[-i & (n - 1)] = (ll)L[i] * R[i] % mod * inv %
        mod;
  ntt(out);
  return {out.begin(), out.begin() + s};
}
```

FastSubsetTransform.h

Description: Transform to a basis with fast convolutions of the form $c[z] = \sum_{z=x \oplus y} a[x] \cdot b[y]$, where $\oplus$ is one of AND, OR, XOR. The size of a must be a power of two.

Time: $\mathcal{O}(N \log N)$ 464cf3, 16 lines

```
void FST(vi& a, bool inv) {
  for (int n = sz(a), step = 1; step < n; step *= 2) {
    for (int i = 0; i < n; i += 2 * step) rep(j,i,i+
        step) {
      int &u = a[j], &v = a[j + step]; tie(u, v) =
        inv ? pii(v - u, u) : pii(v, u + v); // AND
        inv ? pii(v, u - v) : pii(u + v, u); // OR
        pii(u + v, u - v);                   // XOR
    }
  }
  if (inv) for (int& x : a) x /= sz(a); // XOR only
}
vi conv(vi a, vi b) {
  FST(a, 0); FST(b, 0);
  rep(i,0,sz(a)) a[i] *= b[i];
  FST(a, 1); return a;
}
```

FFT.h

Description: Multiply polynomials for any modulus. Works for $n + m \le 2^{24}$ and $c_k \le 5 \cdot 10^{25}$.

Time: $\mathcal{O}((n + m) \log(n + m))$

"../number-theory/ModInt.h" 0ef504, 45 lines

```
template<class T>
void ntt(vector<T>& a, bool inv) {
  int n = sz(a); vector<T> b(n);
  for (int i = n / 2; i; i /= 2, swap(a, b)) {
    T w = T(T::rt).pow((T::mod - 1) / n * i), m = 1;
    for (int j=0; j<n; j += 2 * i, m *= w) rep(k,0,i) {
      T u = a[j + k], v = a[j + k + i] * m;
```

```
      b[j/2 + k] = u + v, b[j/2 + k + n/2] = u - v;
    }
  }
  if (inv) {
    reverse(1 + all(a));
    T z = T(n).inv(); rep(i, 0, n) a[i] *= z;
  }
}
template<class T>
vector<T> conv(vector<T> a, vector<T> b) {
  int s = sz(a) + sz(b) - 1, n = 1 << __lg(2 * s - 1);
  a.resize(n); ntt(a, 0); b.resize(n); ntt(b, 0);
  rep(i, 0, n) a[i] *= b[i];
  ntt(a, 1); a.resize(s);
  return a;
}
template<class T>
vector<T> mconv(const auto& x, const auto& y) {
  auto con = [&](const auto& v) {
    vector<T> w(sz(v)); rep(i, 0, sz(v)) w[i] = v[i].x;
    return w; };
  return conv(con(x), con(y));
}
template<class T>
vector<T> conv3(const vector<T>& a,const vector<T>& b){
  using m0=Mod<754974721, 11>; auto c0=mconv<m0>(a, b);
  using m1=Mod<167772161, 3>;  auto c1=mconv<m1>(a, b);
  using m2=Mod<469762049, 3>;  auto c2=mconv<m2>(a, b);
  int n = sz(c0); vector<T> d(n);
  m1 r01 = m1(m0::mod).inv();
  m2 r02 = m2(m0::mod).inv(), r12 = m2(m1::mod).inv();
  rep(i, 0, n) {
    int x = c0[i].x, y = ((c1[i] - x) * r01).x,
        z = (((c2[i] - x) * r02 - y) * r12).x;
    d[i] = (T(z) * m1::mod + y) * m0::mod + x;
  }
  return d;
}
```

FFTPoly.h

Description: Fast operations on polynomials.

Time: $\mathcal{O}(n \log n)$, eval and interp are $\mathcal{O}\left(n \log^2 n\right)$

"FFT.h" 6ec538, 145 lines

```
using Poly = vector<mint>;
Poly& operator+=(Poly& a, const Poly& b) {
  a.resize(max(sz(a), sz(b)));
  rep(i, 0, sz(b)) a[i] += b[i];
  return a;
}
Poly& operator-=(Poly& a, const Poly& b) {
  a.resize(max(sz(a), sz(b)));
  rep(i, 0, sz(b)) a[i] -= b[i];
  return a;
}
Poly& operator*=(Poly& a, const Poly& b) {
  if (min(sz(a), sz(b)) < 50) {
    Poly c(sz(a) + sz(b) - 1);
    rep(i,0,sz(a)) rep(j,0,sz(b)) c[i+j] += a[i]*b[j];
    return a = c;
  }
  return a = conv(move(a), b);
}
Poly operator+(Poly a, const Poly& b){ return a += b; }
Poly operator-(Poly a, const Poly& b){ return a -= b; }
Poly operator*(Poly a, const Poly& b){ return a *= b; }
Poly modK(Poly a, int k) {
  return a.resize(min(sz(a), k)), a;
}
Poly inv(const Poly& a) { // a[0] != 0
  Poly b = {1 / a[0]};
  while (sz(b) < sz(a))
    b = modK(b*(Poly{2} - modK(a,2*sz(b))*b), 2*sz(b));
  return modK(b, sz(a));
}
Poly deriv(Poly a) {
  if (!sz(a)) return {};
  rep(i, 1, sz(a)) a[i - 1] = a[i] * i;
  return a.pop_back(), a;
}
Poly integr(const Poly& a) {
  if (!sz(a)) return {};
  Poly b(sz(a) + 1);
  rep(i, 1, sz(b)) b[i] = a[i - 1] / i;
  return b;
}
Poly shift(Poly p, mint c) { // p(x + c)
  int n = sz(p);
  Poly q(n, 1); mint fac = 1;
  rep(i, 1, n) {
    p[i] *= (fac *= i);
    q[n - 1 - i] = q[n - i] * c / i;
  }
  p *= q;
  p.erase(p.begin(), p.begin() + n - 1);
  fac = 1;
  rep(i, 1, n) p[i] /= (fac *= i);
  return p;
}
Poly log(const Poly& a) { // a[0] = 1
  Poly b = integr(deriv(a) * inv(a));
  return b.resize(sz(a)), b;
}
Poly exp(const Poly& a) { // a[0] = 0
  Poly b = {1};
  if (!sz(a)) return b;
  while (sz(b) < sz(a)) {
    b.resize(sz(b) * 2);
    b *= Poly{1} + modK(a, sz(b)) - log(b);
    b.resize(sz(b) / 2 + 1);
  }
  return modK(b, sz(a));
}
Poly pow(Poly a, ll m) {
  int p = 0, n = sz(a);
  if (!m) { a.assign(n, 0); a[0] = 1; return a; }
  while (p < n && !a[p].x) p++;
  if (p >= (n + m - 1) / m) return Poly(n);
  mint j = a[p];
  a = Poly(p + all(a)) * Poly{1 / j};
  a.resize(n);
  Poly res = exp(log(a) * Poly{m}) * Poly{j.pow(m)};
  res.insert(res.begin(), p * m, 0);
  return modK(res, n);
}
Poly& operator/=(Poly& a, Poly b) {
  if (sz(a) < sz(b)) return a = {};
  int s = sz(a) - sz(b) + 1;
  reverse(all(a)), reverse(all(b));
  a.resize(s), b.resize(s);
  a *= inv(b);
  a.resize(s), reverse(all(a));
  return a;
}
Poly operator/(Poly a, Poly b) { return a /= b; }
Poly& operator%=(Poly& a, const Poly& b) {
  if (sz(a) < sz(b)) return a;
  return a = modK(a - (a / b) * b, sz(b) - 1);
}
Poly operator%(Poly a,const Poly& b) { return a %= b; }
vector<mint> eval(const Poly& a,const vector<mint>& x){
  int n = sz(x);
  if (!n) return {};
  vector<Poly> up(2 * n);
  rep(i, 0, n) up[i + n] = Poly{0 - x[i], 1};
  for (int i = n - 1; i > 0; i--)
    up[i] = up[2 * i] * up[2 * i + 1];
  vector<Poly> down(2 * n);
  down[1] = a % up[1];
  rep(i, 2, 2 * n) down[i] = down[i / 2] % up[i];
  vector<mint> y(n);
  rep(i, 0, n) y[i] = down[i + n][0];
  return y;
}
Poly interp(vector<mint> x, vector<mint> y) {
  int n = sz(x);
  vector<Poly> up(2 * n);
  rep(i, 0, n) up[i + n] = Poly{0 - x[i], 1};
  for (int i = n - 1; i > 0; i--)
    up[i] = up[2 * i] * up[2 * i + 1];
  vector<mint> a = eval(deriv(up[1]), x);
  vector<Poly> down(2 * n);
  rep(i, 0, n) down[i + n] = Poly{y[i] / a[i]};
  for(int i = n - 1; i > 0; i--)
    down[i] = down[2*i]*up[2*i+1]+down[2*i+1]*up[2*i];
  return down[1];
}
// B(x) = product of (1 + x^k)^{a_k} for k=1..inf
Poly subsetSum(Poly a) { // a[0] = 0
  int n = sz(a);
  Poly b(n);
  rep(i, 1, n) b[i] = mint(i).inv() * (i % 2 ? 1 : -1);
  for (int i = n - 2; i > 0; i--)
    for (int j = 2; i * j < n; j++)
      a[i * j] += b[j] * a[i];
  return exp(a);
}
// B(x) = product of 1 / (1 - a_k*x^k) for k=1..inf
Poly eulerTransform(Poly a) { // a[0] = 0
  int n = sz(a);
  Poly b(n);
  rep(i, 1, n) b[i] = mint(i).inv();
  for (int i = n - 1; i > 0; i--) {
    mint m = a[i];
    for (int j = 2; i * j < n; j++)
      m *= a[i], a[i * j] += b[j] * m;
  }
  return exp(a);
}
```

Number theory (5)

5.1 Modular arithmetic

ModInt.h

Description: Operators for modular arithmetic. 3f0e63, 29 lines

```
template<int M, int R>
struct Mod {
  static const int mod = M, rt = R;
  int x;
  Mod(ll y = 0) : x(y % M) { x += (x < 0) * M; }
  Mod& operator+=(Mod o) {
    if ((x += o.x) >= M) x -= M;
    return *this; }
  Mod& operator-=(Mod o) {
    if ((x -= o.x) < 0) x += M;
    return *this; }
  Mod& operator*=(Mod o) {
    x = 1ll * x * o.x % M;
    return *this; }
  Mod& operator/=(Mod o) { return *this *= o.inv(); }
  friend Mod operator+(Mod a, Mod b) { return a += b; }
  friend Mod operator-(Mod a, Mod b) { return a -= b; }
  friend Mod operator*(Mod a, Mod b) { return a *= b; }
  friend Mod operator/(Mod a, Mod b) { return a /= b; }
  auto operator<=>(const Mod&) const = default;
  Mod pow(ll n) const {
    Mod a = x, b = 1;
    for (; n; n /= 2, a *= a) if (n % 2) b *= a;
    return b; }
  Mod inv() const { assert(x); return pow(M - 2); }
  friend ostream& operator<<(ostream& os, Mod x) {
    return os << x.x; }
};
using mint = Mod<998244353, 3>;
```

ModInverse.h

Description: Pre-computation of modular inverses. Assumes LIM $\leq$ mod and that mod is a prime. 6f684f, 3 lines

```
const ll mod = 1000000007, LIM = 200000;
ll* inv = new ll[LIM] - 1; inv[1] = 1;
rep(i,2,LIM) inv[i] = mod - (mod / i) * inv[mod % i] %
      mod;
```

ModPow.h

b83e45, 8 lines

```
const ll mod = 1000000007; // faster if const
ll modpow(ll b, ll e) {
  ll ans = 1;
  for (; e; b = b * b % mod, e /= 2)
    if (e & 1) ans = ans * b % mod;
  return ans;
}
```

ModLog.h

Description: Returns the smallest $x > 0$ s.t. $a^x = b \pmod{m}$, or -1 if no such x exists. modLog(a,1,m) can be used to calculate the order of a.

Time: $\mathcal{O}\left(\sqrt{m}\right)$ c040b8, 11 lines

```
ll modLog(ll a, ll b, ll m) {
  ll n = (ll) sqrt(m) + 1, e = 1, f = 1, j = 1;
  unordered_map<ll, ll> A;
  while (j <= n && (e = f = e * a % m) != b % m)
    A[e * b % m] = j++;
  if (e == b % m) return j;
  if (__gcd(m, e) == __gcd(m, b))
    rep(i,2,n+2) if (A.count(e = e * f % m))
      return n * i - A[e];
  return -1;
}
```

ModSum.h

Description: Sums of mod'ed arithmetic progressions.

modsum(to, c, k, m) $= \sum_{i=0}^{\mathrm{to}-1} (ki + c)\%m$. divsum is similar but for floored division.

Time: $\log(m)$, with a large constant. 5c5bc5, 16 lines

```
typedef unsigned long long ull;
ull sumsq(ull to) { return to / 2 * ((to-1) | 1); }
ull divsum(ull to, ull c, ull k, ull m) {
  ull res = k / m * sumsq(to) + c / m * to;
  k %= m; c %= m;
  if (!k) return res;
  ull to2 = (to * k + c) / m;
  return res + (to - 1) * to2 - divsum(to2, m-1 - c, m,
        k);
}
ll modsum(ull to, ll c, ll k, ll m) {
  c = ((c % m) + m) % m;
  k = ((k % m) + m) % m;
  return to * c + k * sumsq(to) - m * divsum(to, c, k,
       m);
}
```

ModLinear.h

Description: Statistics on a mod'ed arithmetic sequence.

Time: $\mathcal{O}(\log m)$

"euclid.h" 45f202, 34 lines

```
ll cdiv(ll x, ll y) {
  return x / y + ((x ^ y) > 0 && x % y); }
// min (ax + b) % m for 0 <= x <= n
ll minRemainder(ll a, ll b, ll m, ll n) {
  assert(a >= 0 && m > 0 && b >= 0 && n >= 0);
  a %= m, b %= m; n = min(n, m - 1);
  if (a == 0) return b;
  if (b >= a) {
    ll ad = cdiv(m - b, a);
    n -= ad; if (n < 0) return b;
    b += ad * a - m;
  }
  ll q = m / a, m2 = m % a;
  if (m2 == 0) return b;
  if (b / m2 > n / q) return b - n / q * m2;
  n -= b / m2 * q; b %= m2;
  ll y2 = (n * a + b) / m;
  ll x2 = cdiv(m2 * y2 - b, a);
  if (x2 * a - m2 * y2 + b >= m2) --x2;
  return minRemainder(a, b, m2, x2);
}
// min x >= 0 s.t. l <= (ax + b) % m <= r
ll minBetween(ll a, ll b, ll m, ll l, ll r) {
  ll x, y, g = euclid(a, m, x, y);
  if (g > 1)
    return minBetween(a/g,b/g,m/g,
                     l/g+(l%g>b%g),r/g-(r%g<b%g));
  if (l > r) return -1; // no solution
  if ((x %= m) < 0) x += m;
  ll b2 = (l - b) * x % m;
  return minRemainder(x, b2 < 0 ? b2 + m : b2, m, r-l);
}
```

ModMulLL.h

Description: Calculate $a \cdot b \bmod c$ (or $a^b \bmod c$) for $0 \leq a, b \leq c \leq 7.2 \cdot 10^{18}$.

Time: $\mathcal{O}(1)$ for modmul, $\mathcal{O}(\log b)$ for modpow bbbd8f, 11 lines

```
typedef unsigned long long ull;
ull modmul(ull a, ull b, ull M) {
  ll ret = a * b - M * ull(1.L / M * a * b);
  return ret + M * (ret < 0) - M * (ret >= (ll)M);
}
ull modpow(ull b, ull e, ull mod) {
  ull ans = 1;
  for (; e; b = modmul(b, b, mod), e /= 2)
    if (e & 1) ans = modmul(ans, b, mod);
  return ans;
}
```

ModSqrt.h

Description: Tonelli-Shanks algorithm for modular square roots. Finds x s.t. $x^2 = a \pmod{p}$ ($-x$ gives the other solution) or -1 if no such x exists.

Time: $\mathcal{O}\left(\log^2 p\right)$ worst case, $\mathcal{O}(\log p)$ for most p

"ModMulLL.h" b7cab4, 24 lines

```
ll sqrt(ll a, ll p) {
  a %= p; if (a < 0) a += p;
  if (a == 0) return 0;
  if (modpow(a, (p-1)/2, p) != 1) return -1;
  if (p % 4 == 3) return modpow(a, (p+1)/4, p);
  // a^(n+3)/8 or 2^(n+3)/8 * 2^(n-1)/4 works if p % 8
       == 5
  ll s = p - 1, n = 2;
  int r = 0, m;
  while (s % 2 == 0)
    ++r, s /= 2;
```

```
  while (modpow(n, (p - 1) / 2, p) != p - 1) ++n;
  ll x = modpow(a, (s + 1) / 2, p);
  ll b = modpow(a, s, p), g = modpow(n, s, p);
  for (;; r = m) {
    ll t = b;
    for (m = 0; m < r && t != 1; ++m)
      t = t * t % p;
    if (m == 0) return x;
    ll gs = modpow(g, 1LL << (r - m - 1), p);
    g = gs * gs % p;
    x = x * gs % p;
    b = b * g % p;
  }
}
```

PrimitiveRoot.h

Description: Finds a primitive root modulo p.

"Factor.h", "ModMulLL.h" d63925, 9 lines

```
mt19937_64 rng(2137);
ll primitiveRoot(ll p) {
  auto f = factor(p - 1); sort(all(f));
  f.resize(unique(all(f)) - f.begin()); rep:
  ll g = rng() % (p - 1) + 1;
  for (auto x : f) if (modpow(g, (p - 1) / x, p) == 1)
    goto rep;
  return g;
}
```

5.2 Primality

FastEratosthenes.h

Description: Prime sieve for generating all primes smaller than LIM.

Time: LIM=1e9 $\approx$ 1.5s 6b2912, 20 lines

```
const int LIM = 1e6;
bitset<LIM> isPrime;
vi eratosthenes() {
  const int S = (int)round(sqrt(LIM)), R = LIM / 2;
  vi pr = {2}, sieve(S+1); pr.reserve(int(LIM/log(LIM)
      *1.1));
  vector<pii> cp;
  for (int i = 3; i <= S; i += 2) if (!sieve[i]) {
    cp.push_back({i, i * i / 2});
    for (int j = i * i; j <= S; j += 2 * i) sieve[j] =
        1;
  }
  for (int L = 1; L <= R; L += S) {
    array<bool, S> block{};
    for (auto &[p, idx] : cp)
      for (int i=idx; i < S+L; idx = (i+=p)) block[i-L]
          = 1;
    rep(i,0,min(S, R - L))
      if (!block[i]) pr.push_back((L + i) * 2 + 1);
  }
  for (int i : pr) isPrime[i] = 1;
  return pr;
}
```

MillerRabin.h

Description: Deterministic Miller-Rabin primality test. Guaranteed to work for numbers up to $7 \cdot 10^{18}$; for larger numbers, use Python and extend A randomly.

Time: 7 times the complexity of $a^b \mod c$.

"ModMulLL.h" 60dcd1, 12 lines

```
bool isPrime(ull n) {
  if (n < 2 || n % 6 % 4 != 1) return (n | 1) == 3;
  ull A[] = {2, 325, 9375, 28178, 450775, 9780504,
      1795265022},
      s = __builtin_ctzll(n-1), d = n >> s;
  for (ull a : A) {   // ^ count trailing zeroes
    ull p = modpow(a%n, d, n), i = s;
    while (p != 1 && p != n - 1 && a % n && i--)
      p = modmul(p, p, n);
    if (p != n-1 && i != s) return 0;
  }
  return 1;
}
```

Factor.h

Description: Pollard-rho randomized factorization algorithm. Returns prime factors of a number, in arbitrary order (e.g. 2299 -> {11, 19, 11}).

Time: $\mathcal{O}\left(n^{1/4}\right)$, less for numbers with small factors.

"ModMulLL.h", "MillerRabin.h" d8d98d, 18 lines

```
ull pollard(ull n) {
  ull x = 0, y = 0, t = 30, prd = 2, i = 1, q;
  auto f = [&](ull x) { return modmul(x, x, n) + i; };
  while (t++ % 40 || __gcd(prd, n) == 1) {
    if (x == y) x = ++i, y = f(x);
    if ((q = modmul(prd, max(x,y) - min(x,y), n))) prd
        = q;
    x = f(x), y = f(f(y));
  }
  return __gcd(prd, n);
}
vector<ull> factor(ull n) {
  if (n == 1) return {};
  if (isPrime(n)) return {n};
  ull x = pollard(n);
  auto l = factor(x), r = factor(n / x);
  l.insert(l.end(), all(r));
  return l;
}
```

5.3 Divisibility

euclid.h

Description: Finds two integers x and y, such that $ax+by = \gcd(a, b)$. If you just need gcd, use the built in `__gcd` instead. If a and b are coprime, then x is the inverse of $a \pmod b$. 33ba8f, 5 lines

```
ll euclid(ll a, ll b, ll &x, ll &y) {
  if (!b) return x = 1, y = 0, a;
  ll d = euclid(b, a % b, y, x);
  return y -= a/b * x, d;
}
```

CRT.h

Description: Chinese Remainder Theorem.

`crt(a, m, b, n)` computes x such that $x \equiv a \pmod m$, $x \equiv b \pmod n$. If $|a| < m$ and $|b| < n$, x will obey $0 \le x < \text{lcm}(m, n)$. Assumes $mn < 2^{62}$.

Time: $\log(n)$

"euclid.h" 04d93a, 7 lines

```
ll crt(ll a, ll m, ll b, ll n) {
  if (n > m) swap(a, b), swap(m, n);
  ll x, y, g = euclid(m, n, x, y);
  assert((a - b) % g == 0); // else no solution
  x = (b - a) % n * x % n / g * m + a;
  return x < 0 ? x + m*n/g : x;
}
```

5.3.1 Bézout's identity

For $a \neq$, $b \neq 0$, then $d = gcd(a, b)$ is the smallest positive integer for which there are integer solutions to

$$ax + by = d$$

If (x, y) is one solution, then all solutions are given by

$$\left(x + \frac{kb}{\gcd(a,b)}, y - \frac{ka}{\gcd(a,b)}\right), \quad k \in \mathbb{Z}$$

phiFunction.h

Description: *Euler's* ϕ function is defined as $\phi(n) := \#$ of positive integers $\le n$ that are coprime with n. $\phi(1) = 1$, p prime $\Rightarrow \phi(p^k) = (p-1)p^{k-1}$, m, n coprime $\Rightarrow \phi(mn) = \phi(m)\phi(n)$. If $n = p_1^{k_1} p_2^{k_2} \ldots p_r^{k_r}$ then $\phi(n) = (p_1 - 1)p_1^{k_1-1} \ldots (p_r - 1)p_r^{k_r-1}$. $\phi(n) = n \cdot \prod_{p|n}(1 - 1/p)$. $\sum_{d|n} \phi(d) = n$, $\sum_{1 \le k \le n, \gcd(k,n)=1} k = n\phi(n)/2, n > 1$

Euler's thm: a, n coprime $\Rightarrow a^{\phi(n)} \equiv 1 \pmod n$.

Fermat's little thm: p prime $\Rightarrow a^{p-1} \equiv 1 \pmod p\ \forall a$. cf7d6d, 8 lines

```
const int LIM = 5000000;
int phi[LIM];
void calculatePhi() {
  rep(i,0,LIM) phi[i] = i&1 ? i : i/2;
  for (int i = 3; i < LIM; i += 2) if(phi[i] == i)
    for (int j = i; j < LIM; j += i) phi[j] -= phi[j] /
        i;
}
```

Min25.h

Description: Fast multiplicative function prefix sums. Requires `isPrime` calculated up to $\sqrt{n}$.

Time: $\mathcal{O}\left(\frac{n^{3/4}}{\log n}\right)$

"FastEratosthenes.h" 4c2ea6, 47 lines

```
template<class T>
struct Min25 {
  ll n, sq, s, hls; vi p;
  Min25(ll N) : n(N) {
    sq = sqrt(n) + 5;
    while (sq * sq > n) sq--;
    hls = quo(n, sq);
    if (hls != 1 && quo(n, hls - 1) == sq) hls--;
    s = hls + sq;
    rep(i, 2, sq + 1) if (isPrime[i]) p.push_back(i);
  }
  vector<T> sieve(auto f) {
    vector<T> h(s);
    rep(i, 1, hls) h[i] = f(quo(n, i)) - 1;
    rep(i, 1, sq + 1) h[s - i] = f(i) - 1;
    for (ll x : p) {
      T xt = f(x) - f(x - 1), pi = h[s - x + 1];
      ll x2=x*x, imax=min(hls, quo(n, x2) + 1), ix=x;
      for (int i = 1; i < imax; i++, ix += x)
        h[i] -= ((ix<hls?h[ix]:h[s-quo(n,ix)])-pi)*xt;
      for (int i = sq; i >= x2; i--)
        h[s - i] -= (h[s - quo(i, x)] - pi) * xt;
    }
    return h;
  }
  vector<T> unsieve(vector<T>& fp, auto f) {
    vector<ll> ns(s);
    rep(i, 1, hls) ns[i] = quo(n, i);
    rep(i, 1, sq + 1) ns[s - i] = i;
    auto F = fp, G = fp;
    for (ll P : p | views::reverse) {
      for (ll pk=P, k=1; quo(n,P)>=pk; k++, pk*=P) {
        T x=fp[idx(P)], y=f(P,k,pk), z=f(P,k+1,pk*P);
        rep(i, 1, s) {
          ll m = ns[i];
          if (P * pk > m) break;
          G[i] += y * (F[idx(quo(m, pk))] - x) + z;
        }
      }
      copy_n(G.begin(),min(s,idx(P*P)+1),F.begin());
    }
    rep(i, 1, sz(ns)) F[i] += 1;
    return F;
  }
  ll quo(ll x, ll y) { return (double)x / y; }
  ll idx(ll x) { return x <= sq ? s - x : quo(n, x); }
};
```

SameDiv.h

Description: Divides the interval $[1, \infty)$ into constant division intervals. For a significant speedup, get rid of `v` and do the calculations directly in the loop.

Time: $\mathcal{O}\left(\sqrt{n}\right)$ 24617c, 13 lines

```
vector<ll> sameFloor(ll n) {
  vector<ll> v;
  for (ll i = 1; i <= n; i = n/(n/i)+1) v.push_back(i);
  return v.push_back(n + 1), v;
}
vector<ll> sameCeil(ll n) {
  vector<ll> v;
  for (ll i = 1, j; i < n; i = (n + j - 2) / (j - 1)) {
    j = (n + i - 1) / i;
    v.push_back(i);
  }
  return v.push_back(n), v;
}
```

5.4 Fractions

ContinuedFractions.h

Description: Given N and a real number $x \ge 0$, finds the closest rational approximation p/q with $p, q \le N$. It will obey $|p/q - x| \le 1/qN$.

For consecutive convergents, $p_{k+1}q_k - q_{k+1}p_k = (-1)^k$. ($p_k/q_k$ alternates between $> x$ and $< x$.) If x is rational, y eventually becomes ∞; if x is the root of a degree 2 polynomial the a's eventually become cyclic.

Time: $\mathcal{O}(\log N)$ dd6c5e, 21 lines

```
typedef double d; // for N ~ 1e7; long double for N ~ 1
    e9
pair<ll, ll> approximate(d x, ll N) {
  ll LP = 0, LQ = 1, P = 1, Q = 0, inf = LLONG_MAX; d y
      = x;
  for (;;) {
    ll lim = min(P ? (N-LP) / P : inf, Q ? (N-LQ) / Q :
        inf),
       a = (ll)floor(y), b = min(a, lim),
       NP = b*P + LP, NQ = b*Q + LQ;
    if (a > b) {
      // If b > a/2, we have a semi-convergent that
          gives us a
      // better approximation; if b = a/2, we *may*
          have one.
      // Return {P, Q} here for a more canonical
          approximation.
      return (abs(x - (d)NP / (d)NQ) < abs(x - (d)P / (
          d)Q)) ?
        make_pair(NP, NQ) : make_pair(P, Q);
    }
    if (abs(y = 1/(y - (d)a)) > 3*N) {
      return {NP, NQ};
    }
    LP = P; P = NP;
    LQ = Q; Q = NQ;
  }
}
```

FracBinarySearch.h

Description: Given f and N, finds the smallest fraction $p/q \in [0, 1]$ such that $f(p/q)$ is true, and $p, q \le N$. You may want to throw an exception from f if it finds an exact solution, in which case N can be removed.

Usage: `fracBS([](Frac f) { return f.p>=3*f.q; }, 10);` `// {1,3}`

Time: $\mathcal{O}(\log(N))$ 27ab3e, 25 lines

```
struct Frac { ll p, q; };
template<class F>
Frac fracBS(F f, ll N) {
  bool dir = 1, A = 1, B = 1;
  Frac lo{0, 1}, hi{1, 1}; // Set hi to 1/0 to search
      (0, N]
  if (f(lo)) return lo;
  assert(f(hi));
  while (A || B) {
    ll adv = 0, step = 1; // move hi if dir, else lo
    for (int si = 0; step; (step *= 2) >>= si) {
      adv += step;
      Frac mid{lo.p * adv + hi.p, lo.q * adv + hi.q};
      if (abs(mid.p) > N || mid.q > N || dir == !f(mid)
          ) {
        adv -= step; si = 2;
      }
    }
    hi.p += lo.p * adv;
    hi.q += lo.q * adv;
    dir = !dir;
    swap(lo, hi);
    A = B; B = !!adv;
  }
  return dir ? hi : lo;
}
```

5.5 Pythagorean Triples

The Pythagorean triples are uniquely generated by

$$a = k \cdot (m^2 - n^2), \;\; b = k \cdot (2mn), \;\; c = k \cdot (m^2 + n^2),$$

with $m > n > 0$, $k > 0$, $m \perp n$, and either m or n even.

5.6 Primes

$p = 962592769$ is such that $2^{21} \mid p - 1$, which may be useful. For hashing use 970592641 (31-bit number), 31443539979727 (45-bit), 3006703054056749 (52-bit). There are 78498 primes less than 1 000 000.

Primitive roots exist modulo any prime power p^a, except for $p = 2, a > 2$, and there are $\phi(\phi(p^a))$ many. For $p = 2, a > 2$, the group $\mathbb{Z}_{2^a}^\times$ is instead isomorphic to $\mathbb{Z}_2 \times \mathbb{Z}_{2^{a-2}}$.

5.7 Estimates

$\sum_{d|n} d = O(n \log \log n)$.

The number of divisors of n is at most around 100 for $n < 5e4$, 500 for $n < 1e7$, 2000 for $n < 1e10$, 200 000 for $n < 1e19$.

5.8 Mobius Function

$$\mu(n) = \begin{cases} 0 & n \text{ is not square free} \\ 1 & n \text{ has even number of prime factors} \\ -1 & n \text{ has odd number of prime factors} \end{cases}$$

Mobius Inversion:

$$g(n) = \sum_{d|n} f(d) \Leftrightarrow f(n) = \sum_{d|n} \mu(d) g(n/d)$$

Other useful formulas/forms:

$\sum_{d|n} \mu(d) = [n = 1]$ (very useful)

$g(n) = \sum_{n|d} f(d) \Leftrightarrow f(n) = \sum_{n|d} \mu(d/n) g(d)$

$g(n) = \sum_{1 \le m \le n} f(\lfloor \frac{n}{m} \rfloor) \Leftrightarrow f(n) = \sum_{1 \le m \le n} \mu(m) g(\lfloor \frac{n}{m} \rfloor)$

Combinatorial (6)

6.1 Permutations

6.1.1 Factorial

n	1	2	3	4	5	6	7	8	9	10
$n!$	1	2	6	24	120	720	5040	40320	362880	3628800
n	11	12	13	14	15	16	17			
$n!$	4.0e7	4.8e8	6.2e9	8.7e10	1.3e12	2.1e13	3.6e14			
n	20	25	30	40	50	100	150	171		
$n!$	2e18	2e25	3e32	8e47	3e64	9e157	6e262	>DBL_MAX		

IntPerm.h

Description: Permutation -> integer conversion. (Not order preserving.) Integer -> permutation can use a lookup table.
Time: $\mathcal{O}(n)$ 044568, 6 lines

```
int permToInt(vi& v) {
  int use = 0, i = 0, r = 0;
  for(int x:v) r = r * ++i + __builtin_popcount(use &
      -(1<<x)),
    use |= 1 << x;                       // (note: minus,
         not ~!)
  return r;
}
```

6.1.2 Cycles

Let $g_S(n)$ be the number of n-permutations whose cycle lengths all belong to the set S. Then

$$\sum_{n=0}^{\infty} g_S(n) \frac{x^n}{n!} = \exp\left(\sum_{n \in S} \frac{x^n}{n} \right)$$

6.1.3 Derangements

Permutations of a set such that none of the elements appear in their original position.

$$D(n) = (n-1)(D(n-1) + D(n-2)) = nD(n-1) + (-1)^n = \left\lfloor \frac{n!}{e} \right\rceil$$

6.1.4 Burnside's lemma

Given a group G of symmetries and a set X, the number of elements of X *up to symmetry* equals

$$\frac{1}{|G|} \sum_{g \in G} |X^g|,$$

where X^g are the elements fixed by g ($g.x = x$).

If $f(n)$ counts "configurations" (of some sort) of length n, we can ignore rotational symmetry using $G = \mathbb{Z}_n$ to get

$$g(n) = \frac{1}{n} \sum_{k=0}^{n-1} f(\gcd(n, k)) = \frac{1}{n} \sum_{k|n} f(k) \phi(n/k).$$

6.2 Partitions and subsets

6.2.1 Partition function

Number of ways of writing n as a sum of positive integers, disregarding the order of the summands.

$$p(0) = 1, \; p(n) = \sum_{k \in \mathbb{Z} \setminus \{0\}} (-1)^{k+1} p(n - k(3k-1)/2)$$

$$p(n) \sim 0.145/n \cdot \exp(2.56\sqrt{n})$$

n	0	1	2	3	4	5	6	7	8	9	20	50	100
$p(n)$	1	1	2	3	5	7	11	15	22	30	627	~2e5	~2e8

6.2.2 Lucas' Theorem

Let n, m be non-negative integers and p a prime. Write $n = n_k p^k + ... + n_1 p + n_0$ and $m = m_k p^k + ... + m_1 p + m_0$. Then $\binom{n}{m} \equiv \prod_{i=0}^{k} \binom{n_i}{m_i} \pmod{p}$.

6.2.3 Binomials

multinomial.h

Description: Computes $\binom{k_1 + \cdots + k_n}{k_1, k_2, \ldots, k_n} = \frac{(\sum k_i)!}{k_1! k_2! \ldots k_n!}$. a0a312, 5 lines

```
ll multinomial(vi& v) {
  ll c = 1, m = v.empty() ? 1 : v[0];
  rep(i,1,sz(v)) rep(j,0,v[i]) c = c * ++m / (j+1);
  return c;
}
```

6.3 General purpose numbers

6.3.1 Bernoulli numbers

EGF of Bernoulli numbers is $B(t) = \frac{t}{e^t - 1}$ (FFT-able).
$B[0, \ldots] = [1, -\frac{1}{2}, \frac{1}{6}, 0, -\frac{1}{30}, 0, \frac{1}{42}, \ldots]$

Sums of powers:

$$\sum_{i=1}^{n} n^m = \frac{1}{m+1} \sum_{k=0}^{m} \binom{m+1}{k} B_k \cdot (n+1)^{m+1-k}$$

Euler-Maclaurin formula for infinite sums:

$$\sum_{i=m}^{\infty} f(i) = \int_m^{\infty} f(x)dx - \sum_{k=1}^{\infty} \frac{B_k}{k!} f^{(k-1)}(m)$$

$$\approx \int_m^{\infty} f(x)dx + \frac{f(m)}{2} - \frac{f'(m)}{12} + \frac{f'''(m)}{720} + O(f^{(5)}(m))$$

6.3.2 Stirling numbers of the first kind

Number of permutations on n items with k cycles.

$$c(n, k) = c(n-1, k-1) + (n-1)c(n-1, k), \; c(0, 0) = 1$$

$\sum_{k=0}^{n} c(n, k) x^k = x(x+1) \ldots (x+n-1)$

$c(8, k) = 8, 0, 5040, 13068, 13132, 6769, 1960, 322, 28, 1$
$c(n, 2) = 0, 0, 1, 3, 11, 50, 274, 1764, 13068, 109584, \ldots$

6.3.3 Eulerian numbers

Number of permutations $\pi \in S_n$ in which exactly k elements are greater than the previous element. k j:s s.t. $\pi(j) > \pi(j+1)$, $k+1$ j:s s.t. $\pi(j) \ge j$, k j:s s.t. $\pi(j) > j$.

$$E(n, k) = (n-k)E(n-1, k-1) + (k+1)E(n-1, k)$$

$$E(n, 0) = E(n, n-1) = 1$$

$$E(n, k) = \sum_{j=0}^{k} (-1)^j \binom{n+1}{j} (k+1-j)^n$$

6.3.4 Stirling numbers of the second kind

Partitions of n distinct elements into exactly k groups.

$$S(n, k) = S(n-1, k-1) + kS(n-1, k)$$

$$S(n, 1) = S(n, n) = 1$$

$$S(n, k) = \frac{1}{k!} \sum_{j=0}^{k} (-1)^{k-j} \binom{k}{j} j^n$$

6.3.5 Bell numbers

Total number of partitions of n distinct elements. $B(n) = 1, 1, 2, 5, 15, 52, 203, 877, 4140, 21147, \ldots$. For p prime,

$$B(p^m + n) \equiv mB(n) + B(n+1) \pmod{p}$$

6.3.6 Labeled unrooted trees

\# on n vertices: n^{n-2}
\# on k existing trees of size n_i: $n_1 n_2 \cdots n_k n^{k-2}$
\# with degrees d_i: $(n-2)!/((d_1-1)! \cdots (d_n-1)!)$

6.3.7 Catalan numbers

$$C_n = \frac{1}{n+1}\binom{2n}{n} = \binom{2n}{n} - \binom{2n}{n+1} = \frac{(2n)!}{(n+1)!n!}$$

$$C_0 = 1, \; C_{n+1} = \frac{2(2n+1)}{n+2} C_n, \; C_{n+1} = \sum C_i C_{n-i}$$

$C_n = 1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786, \ldots$

- sub-diagonal monotone paths in an $n \times n$ grid.
- strings with n pairs of parenthesis, correctly nested.
- binary trees with with $n+1$ leaves (0 or 2 children).
- ordered trees with $n+1$ vertices.
- ways a convex polygon with $n+2$ sides can be cut into triangles by connecting vertices with straight lines.
- permutations of $[n]$ with no 3-term increasing subseq.

6.3.8 LGV lemma

Let $A = (a_1, \ldots, a_n)$, $B = (b_1, \ldots, b_n)$ be subsets of vertices of a DAG. By $\omega(P)$ denote a path weight, the product of edge weights in that path. Let $M_{i,j}$ be the sum of path weights over all possible paths from a_i to b_j (when unit weights, note this is the number of paths).

$$\det(M) = \sum_{(P_1, \ldots, P_n) \in S_\pi(A, B)} \operatorname{sgn}(\pi) \prod_{i=1}^{n} \omega(P_i)$$

Where $S_\pi(A, B)$ is the set of n-tuples of vertex disjoint paths (including endpoints) where the k-th path is from a_k to $b_{\pi(k)}$. Particularly useful when only identity permutation is possible.

6.4 Other

DeBruijn.h

Description: Recursive FKM, given alphabet $[0, k)$ constructs cyclic string of length k^n that contains every length n string as substr. a7faa5, 13 lines

```
vi dseq(int k, int n) {
  if (k == 1) return {0};
  vi res, aux(n+1);
  function<void(int,int)> gen = [&](int t, int p) {
    if (t > n) { // consider lyndon word of len p
      if (n%p == 0) FOR(i,1,p+1) res.pb(aux[i]);
    } else {
      aux[t] = aux[t-p]; gen(t+1,p);
      FOR(i,aux[t-p]+1,k) aux[t] = i, gen(t+1,t);
    }
  };
  gen(1,1); return res;
}
```

PermGroup.h

Description: Schreier-Sims lets you add a permutation to a group, count number of permutations in a group, test whether a permutation is a member of a group. Works well for $n \le 15$, maybe for larger too. Construct PermGroup() and run order() to get order of the group.
Time: $\mathcal{O}\left(n^6\right)$ d6edf4, 54 lines

```
vi inv(vi v) { vi V(sz(v)); rep(i,sz(v)) V[v[i]]=i;
     return V; }
vi id(int n) { vi v(n); iota(all(v),0); return v; }
vi operator*(const vi& a, const vi& b) {
  vi c(sz(a)); rep(i,sz(a)) c[i] = a[b[i]];
  return c;
}
struct PermGroup {
  struct Group {
    vi flag;
    vector<vi> gen, sigma;
    Group(int n, int p) : flag(n), sigma(n) {
        flag[p] = 1; sigma[p] = id(n);
    }
  };
  int n = 0; vector<Group> g;
  PermGroup() {}
  bool check(const vi& cur, int k) {
    if (!k) return 1;
```

```
    int t = cur[k];
    return g[k].flag[t] ? check(inv(g[k].sigma[t])*cur,
        k-1) : 0;
  }
  void updateX(const vi& cur, int k) {
    int t = cur[k]; // if flag, fixes k -> k
    if (g[k].flag[t]) ins(inv(g[k].sigma[t])*cur,k-1);
    else {
        g[k].flag[t] = 1, g[k].sigma[t] = cur;
        for(auto x: g[k].gen)
          updateX(x*cur,k);
    }
  }
  void ins(const vi& cur, int k) {
    if (check(cur,k)) return;
    g[k].gen.pb(cur);
    rep(i,n) if (g[k].flag[i]) updateX(cur*g[k].sigma[i
        ],k);
  }
  ll order(vector<vi> gen) {
    if(sz(gen) == 0) return 1;
    n = sz(gen[0]);
    rep(i,n) g.pb(Group(n,i));
    for(auto a: gen)
        ins(a, n-1); // insert perms into group one by
            one
    ll tot = 1; // watch out for overflows, can be up
        to n!
    rep(i,n) {
        int cnt = 0;
        rep(j,i+1) cnt += g[i].flag[j];
        tot *= cnt;
    }
    return tot;
  }
};
```

GrayCode.h

Description: Gray code: $\text{gray}(0), \ldots, \text{gray}(2^n - 1)$ - permutation in which each two consecutive (cyclically) numbers. differ in exactly one bit.

b4cc82, 6 lines

```
using ull = unsigned long long;
ull gray(ull i) { return i^i>>1; }
ull invg(ull i) { // i=invg(gray(i))=gray(invg(i))
  i^=i>>1; i^=i>>2; i^=i>>4;
  i^=i>>8; i^=i>>16; i^=i>>32; return i;
}
```

MatroidIsect.h

Description: Given two matroids, finds the largest common independent set. Pass the matroid with more expensive add/clear operations to M1.

Time: R^2N (M2.add + M1.check + M2.check) + R^3 M1.add + R^2 M1.clear + RN M2.clear, where R is the size of the largest independent set.

"../data-structures/UnionFind.h" 9812a7, 60 lines

```
struct ColorMat {
  vi cnt, clr;
  ColorMat(int n, vector<int> clr) : cnt(n), clr(clr) {
      }
  bool check(int x) { return !cnt[clr[x]]; }
  void add(int x) { cnt[clr[x]]++; }
  void clear() { fill(all(cnt), 0); }
};
struct GraphMat {
  UF uf;
  vector<array<int, 2>> e;
  GraphMat(int n, vector<array<int, 2>> e) : uf(n), e(e
      ) {}
  bool check(int x) { return !uf.sameSet(e[x][0], e[x
      ][1]); }
  void add(int x) { uf.join(e[x][0], e[x][1]); }
  void clear() { uf = UF(sz(uf.e)); }
};
template <class M1, class M2> struct MatroidIsect {
  int n;
  vector<char> iset;
  M1 m1; M2 m2;
  MatroidIsect(M1 m1, M2 m2, int n) : n(n), iset(n + 1)
      , m1(m1), m2(m2) {}
  vi solve() {
    rep(i,0,n) if (m1.check(i) && m2.check(i))
      iset[i] = true, m1.add(i), m2.add(i);
    while (augment());
    vi ans;
    rep(i,0,n) if (iset[i]) ans.push_back(i);
    return ans;
  }
  bool augment() {
    vector<int> frm(n, -1);
    queue<int> q({n}); // starts at dummy node
    auto fwdE = [&](int a) {
      vi ans;
      m1.clear();
      rep(v, 0, n) if (iset[v] && v != a) m1.add(v);
      rep(b, 0, n) if (!iset[b] && frm[b] == -1 && m1.
          check(b))
        ans.push_back(b), frm[b] = a;
      return ans;
    };
    auto backE = [&](int b) {
      m2.clear();
      rep(cas, 0, 2) rep(v, 0, n)
        if ((v == b || iset[v]) && (frm[v] == -1) ==
            cas) {
          if (!m2.check(v))
            return cas ? q.push(v), frm[v] = b, v : -1;
          m2.add(v);
        }
      return n;
    };
    while (!q.empty()) {
      int a = q.front(), c; q.pop();
      for (int b : fwdE(a))
        while((c = backE(b)) >= 0) if (c == n) {
          while (b != n) iset[b] ^= 1, b = frm[b];
          return true;
        }
    }
    return false;
  }
};
```

WeightedMatroidIsect.h

Description: Given two matroids, finds a largest common independent set of maximal weight. To minimize weight, multiply costs by -1. Returns vector V such that `V[i]=1` iff i-th element is included in found set.

Time: $\mathcal{O}\left(r^2(init + n \cdot add) + rn^3\right)$, where r is the size of the largest common independent set. Here rn^3 comes from running Bellman-Ford r times, which is practically faster.

76ecaa, 150 lines

```
const ll INF = 1e18;
template<class T, class U>
vector<bool> maxMatroidIsect(T& A, U& B, vector<ll>& c,
      int n){
  vector<bool> ans(n);
  bool ok = 1;
  while(ok) {
    vector<vector<pair<int, ll>>> G(n);
    vector<bool> good(n);
    vector<pair<ll, ll>> d(n, {INF, INF});
    vi prev(n, -1);
    A.init(ans); B.init(ans); ok = 0;
    rep(i, 0, n) if(!ans[i]) {
      if(A.canAdd(i)) d[i] = {-c[i], 0}, prev[i] = -2;
      good[i] = B.canAdd(i);
    }
    rep(i, 0, n) if(ans[i]) {
      ans[i] = 0;
      A.init(ans); B.init(ans);
      rep(j, 0, n) if(i != j && !ans[j]) {
        if(A.canAdd(j)) G[i].push_back({j, -c[j]});
        if(B.canAdd(j)) G[j].push_back({i, c[i]});
      }
      ans[i] = 1;
    }
    bool ford = 1;
    while(ford) {
      ford = 0;
      rep(i, 0, n) if(prev[i] != -1)
        for(auto [to, cst] : G[i]) {
          pair<ll, ll> nd = {d[i].first + cst, d[i].
              second+1};
          if(nd < d[to]) d[to] = nd, prev[to] = i, ford
              = 1;
        }
    }
    int e = -1;
    pair<ll, ll> dst = {INF, INF};
    rep(i, 0, n)
      if(good[i] && d[i] < dst) dst = d[i], e = i;
    if(e == -1) break;
    ans[e] = 1, ok = 1;
    while(prev[e] >= 0) ans[e=prev[e]] = 0, ans[e=prev[
        e]] = 1;
  }
  return ans;
}
// Matroid where each element has color
// and set is independent iff for each color c
// #{elements of color c} <= maxAllowed[c].
struct LimOracle {
  vi color; // color[i] = color of i-th element
  vi maxAllowed; // Limits for colors
  vi tmp;
  // Init oracle for independent set S; O(n)
  void init(vector<bool>& S) {
    tmp = maxAllowed;
    rep(i, 0, sz(S)) tmp[color[i]] -= S[i];
  }
  // Check if S+{k} is independent; time: O(1)
  bool canAdd(int k) { return tmp[color[k]] > 0;}
};
// Graphic matroid - each element is edge,
// set is independent iff subgraph is acyclic.
struct GraphOracle {
  vector<pii> elems; // Ground set: graph edges
  int n; // Number of vertices, indexed [0;n-1]
  vi par;
  int find(int i) {
    return par[i] == -1 ? i : par[i] = find(par[i]);
  }
  // Init oracle for independent set S; ~O(n)
  void init(vector<bool>& S) {
    par.assign(n, -1);
    rep(i, 0, sz(S)) if (S[i])
      par[find(elems[i].first)] = find(elems[i].second)
          ;
  }
  // Check if S+{k} is independent; time: ~O(1)
  bool canAdd(int k) {
    return find(elems[k].first) != find(elems[k].second
        );
  }
};
// Co-graphic matroid - each element is edge,
// set is independent iff after removing edges
// from graph number of connected components
// doesn't change.
// Works for multiedges and loops.
struct CographOracle {
  vector<pii> elems; // Ground set: graph edges
  int n; // Number of vertices, indexed [0;n-1]
  vector<vi> G;
  vi pre, low;
  int cnt;
  int dfs(int v, int p) {
    pre[v] = low[v] = ++cnt;
    bool skip = 0;
    for(auto e : G[v]) {
      if (e == p && !skip) {
        skip = 1;
        continue;
      }
      low[v] = min(low[v], pre[e] ?: dfs(e,v));
    }
    return low[v];
  }
  // Init oracle for independent set S; O(n)
  void init(vector<bool>& S) {
    G.assign(n, {});
    pre.assign(n, 0);
    low.resize(n);
    cnt = 0;
    rep(i, 0, sz(S)) if (!S[i]) {
      pii e = elems[i];
      if(e.first == e.second) continue;
      G[e.first].push_back(e.second);
      G[e.second].push_back(e.first);
    }
    rep(v, 0, n) if (!pre[v]) dfs(v, -1);
  }
  // Check if S+{k} is independent; time: O(1)
  bool canAdd(int k) {
    auto [u, v] = elems[k];
    if (u == v) return 1;
    return max(pre[u], pre[v]) != max(low[u], low[v]);
  }
};
// Matroid equivalent to linear space with XOR
struct XorOracle {
  vector<ll> elems; // Ground set: numbers
  vector<ll> base;
  // Init for independent set S; O(n+r^2)
  void init(vector<bool>& S) {
    base.assign(63, 0);
    rep(i, 0, sz(S)) if (S[i]) {
      ll e = elems[i];
      rep(j, 0, sz(base)) if ((e >> j) & 1) {
        if (!base[j]) {
          base[j] = e;
          break;
        }
        e ^= base[j];
      }
    }
  }
  // Check if S+{k} is independent; time: O(r)
  bool canAdd(int k) {
    ll e = elems[k];
    rep(i, 0, sz(base)) if ((e >> i) & 1) {
      if (!base[i]) return 1;
      e ^= base[i];
    }
    return 0;
  }
};
```

Graph (7)

7.1 Fundamentals

BellmanFord.h

Description: Calculates shortest paths from s in a graph that might have negative edge weights. Unreachable nodes get dist = inf; nodes reachable through negative-weight cycles get dist = -inf. Assumes $V^2 \max |w_i| < \sim 2^{63}$.

Time: $\mathcal{O}(VE)$

830a8f, 23 lines

```
const ll inf = LLONG_MAX;
struct Ed { int a, b, w, s() { return a < b ? a : -a; }
    };
struct Node { ll dist = inf; int prev = -1; };
void bellmanFord(vector<Node>& nodes, vector<Ed>& eds,
    int s) {
  nodes[s].dist = 0;
  sort(all(eds), [](Ed a, Ed b) { return a.s() < b.s();
      });
  int lim = sz(nodes) / 2 + 2; // /3+100 with shuffled
      vertices
  rep(i,0,lim) for (Ed ed : eds) {
    Node cur = nodes[ed.a], &dest = nodes[ed.b];
    if (abs(cur.dist) == inf) continue;
    ll d = cur.dist + ed.w;
    if (d < dest.dist) {
      dest.prev = ed.a;
      dest.dist = (i < lim-1 ? d : -inf);
    }
  }
  rep(i,0,lim) for (Ed e : eds) {
    if (nodes[e.a].dist == -inf)
      nodes[e.b].dist = -inf;
  }
}
```

FloydWarshall.h

Description: Calculates all-pairs shortest path in a directed graph that might have negative edge weights. Input is an distance matrix m, where $m[i][j] = \texttt{inf}$ if i and j are not adjacent. As output, $m[i][j]$ is set to the shortest distance between i and j, `inf` if no path, or `-inf` if the path goes through a negative-weight cycle.

Time: $\mathcal{O}\left(N^3\right)$

531245, 12 lines

```
const ll inf = 1LL << 62;
void floydWarshall(vector<vector<ll>>& m) {
  int n = sz(m);
  rep(i,0,n) m[i][i] = min(m[i][i], 0LL);
  rep(k,0,n) rep(i,0,n) rep(j,0,n)
    if (m[i][k] != inf && m[k][j] != inf) {
      auto newDist = max(m[i][k] + m[k][j], -inf);
      m[i][j] = min(m[i][j], newDist);
    }
  rep(k,0,n) if (m[k][k] < 0) rep(i,0,n) rep(j,0,n)
    if (m[i][k] != inf && m[k][j] != inf) m[i][j] = -
        inf;
}
```

TopoSort.h

Description: Topological sorting. Given is an oriented graph. Output is an ordering of vertices, such that there are edges only from left to right. If there are cycles, the returned list will have size smaller than n – nodes reachable from cycles will not be returned.

Time: $\mathcal{O}(|V| + |E|)$

d678d8, 8 lines

```
vi topoSort(const vector<vi>& gr) {
  vi indeg(sz(gr)), q;
  for (auto& li : gr) for (int x : li) indeg[x]++;
  rep(i,0,sz(gr)) if (indeg[i] == 0) q.push_back(i);
  rep(j,0,sz(q)) for (int x : gr[q[j]])
    if (--indeg[x] == 0) q.push_back(x);
  return q;
}
```

SPFA.h

Description: SPFA with subtree erasure heuristic. Returns array of distances or empty array if negative cycle is reachable from source. par[v] = parent in shortest path tree

Time: $\mathcal{O}(VE)$ but fast on random — bf4a3f, 33 lines

```
using Edge = pair<int, ll>;
vector<ll> spfa(vector<vector<Edge>>& G,
                vi& par, int src) {
  int n = sz(G); vi que, prv(n+1);
  iota(all(prv), 0); vi nxt = prv;
  vector<ll> dist(n, INT64_MAX);
  par.assign(n, -1);
  auto add = [&](int v, int p, ll d) {
    par[v] = p; dist[v] = d;
    prv[n] = nxt[prv[v] = prv[nxt[v] = n]] = v;
  };
  auto del = [&](int v) {
    nxt[prv[nxt[v]] = prv[v]] = nxt[v];
    prv[v] = nxt[v] = v;
  };
  for (add(src, -2, 0); nxt[n] != n;) {
    int v = nxt[n]; del(v);
    for (auto e : G[v]) {
      ll alt = dist[v] + e.y;
      if (alt < dist[e.x]) {
        que = {e.x};
        rep(i, sz(que)) {
          int w = que[i]; par[w] = -1;
          del(w);
          for (auto f : G[w])
            if (par[f.x] == w) que.pb(f.x);
        }
        if (par[v] == -1) return {};
        add(e.x, v, alt);
      }
    }
  }
  return dist; }
```

Shapes.h

Description: Counts all subgraph shapes with at most 4 edges. No multiedges / loops allowed;

Time: $\mathcal{O}(m\sqrt{m})$ — d96152, 39 lines

```
struct Shapes {
  ll tri = 0, rect = 0, path3 = 0, path4 = 0, star3 =
      0, p = 0;
  __int128_t y = 0, star4 = 0;
  Shapes(vector<vi> &g) {
    int n = sz(g);
    vector<vi> h(n);
    vector<ll> f(n), c(n), s(n);
    rep(v, 0, n) f[v] = (s[v] = sz(g[v])) * n + v;
    rep(v, 0, n) {
      ll x = 0;
      star3 += s[v] * (s[v] - 1) * (s[v] - 2);
      star4 += __int128_t(s[v] - 1) * s[v] * (s[v] - 2)
          * (s[v] - 3);
      for (auto u : g[v]) {
        path4 += s[u] * x - x; x += s[u] - 1;
        y += (s[v] - 1) * (s[u] - 1) * (s[u] - 2) / 2;
        if (f[u] < f[v]) h[v].push_back(u);
      }
    }
    rep(v, 0, n) {
      for (int u : h[v])
        for (int w : g[u]) if (f[v] > f[w])
          rect += c[w] ++;
      for(int u : h[v]) {
        tri += c[u]; c[u] *= -1;
        path3 += (s[v] - 1) * (s[u] - 1);
        for(int w : g[u])
          if (c[w] < 0)
            p += s[v] + s[u] + s[w] - 6, c[w] ++;
          else if (c[w] > 0)
            c[w] --;
      }
    }
    path3 -= 3 * tri;
    y -= 2 * p;
    path4 -= 4 * rect + 2 * p + 3 * tri;
    star3 /= 6;
    star4 /= 24;
  }
};
```

7.2 Network flow

PushRelabel.h

Description: Push-relabel using the highest label selection rule and the gap heuristic. Quite fast in practice. To obtain the actual flow, look at positive values only.

Time: $\mathcal{O}\left(V^2\sqrt{E}\right)$ — 0ae1d4, 48 lines

```
struct PushRelabel {
  struct Edge {
    int dest, back;
    ll f, c;
  };
  vector<vector<Edge>> g;
  vector<ll> ec;
  vector<Edge*> cur;
  vector<vi> hs; vi H;
  PushRelabel(int n) : g(n), ec(n), cur(n), hs(2*n), H(
      n) {}
  void addEdge(int s, int t, ll cap, ll rcap=0) {
    if (s == t) return;
    g[s].push_back({t, sz(g[t]), 0, cap});
    g[t].push_back({s, sz(g[s])-1, 0, rcap});
  }
  void addFlow(Edge& e, ll f) {
    Edge &back = g[e.dest][e.back];
    if (!ec[e.dest] && f) hs[H[e.dest]].push_back(e.
        dest);
    e.f += f; e.c -= f; ec[e.dest] += f;
    back.f -= f; back.c += f; ec[back.dest] -= f;
  }
  ll calc(int s, int t) {
    int v = sz(g); H[s] = v; ec[t] = 1;
    vi co(2*v); co[0] = v-1;
    rep(i,0,v) cur[i] = g[i].data();
    for (Edge& e : g[s]) addFlow(e, e.c);
    for (int hi = 0;;) {
      while (hs[hi].empty()) if (!hi--) return -ec[s];
      int u = hs[hi].back(); hs[hi].pop_back();
      while (ec[u] > 0)  // discharge u
        if (cur[u] == g[u].data() + sz(g[u])) {
          H[u] = 1e9;
          for (Edge& e : g[u]) if (e.c && H[u] > H[e.
              dest]+1)
            H[u] = H[e.dest]+1, cur[u] = &e;
          if (++co[H[u]], !--co[hi] && hi < v)
            rep(i,0,v) if (hi < H[i] && H[i] < v)
              --co[H[i]], H[i] = v + 1;
          hi = H[u];
        } else if (cur[u]->c && H[u] == H[cur[u]->dest
            ]+1)
          addFlow(*cur[u], min(ec[u], cur[u]->c));
        else ++cur[u];
    }
  }
  bool leftOfMinCut(int a) { return H[a] >= sz(g); }
};
```

MinCostMaxFlow.h

Description: Min-cost max-flow. If costs can be negative, call setpi before maxflow, but note that negative cost cycles are not supported. To obtain the actual flow, look at positive values only.

Time: $\mathcal{O}(FE\log(V))$ where F is max flow. $\mathcal{O}(VE)$ for setpi. — f8c012, 79 lines

```
#include <ext/pb_ds/priority_queue.hpp>
const ll INF = numeric_limits<ll>::max() / 4;
struct MCMF {
  struct edge {
    int from, to, rev;
    ll cap, cost, flow;
  };
  int N;
  vector<vector<edge>> ed;
  vi seen;
  vector<ll> dist, pi;
  vector<edge*> par;
  MCMF(int N) : N(N), ed(N), seen(N), dist(N), pi(N),
      par(N) {}
  void addEdge(int from, int to, ll cap, ll cost) {
    if (from == to) return;
    ed[from].push_back(edge{ from,to,sz(ed[to]),cap,
        cost,0 });
    ed[to].push_back(edge{ to,from,sz(ed[from])-1,0,-
        cost,0 });
  }
  void path(int s) {
    fill(all(seen), 0);
    fill(all(dist), INF);
    dist[s] = 0; ll di;
    __gnu_pbds::priority_queue<pair<ll, int>> q;
    vector<decltype(q)::point_iterator> its(N);
    q.push({ 0, s });
    while (!q.empty()) {
      s = q.top().second; q.pop();
      seen[s] = 1; di = dist[s] + pi[s];
      for (edge& e : ed[s]) if (!seen[e.to]) {
        ll val = di - pi[e.to] + e.cost;
        if (e.cap - e.flow > 0 && val < dist[e.to]) {
          dist[e.to] = val;
          par[e.to] = &e;
          if (its[e.to] == q.end())
            its[e.to] = q.push({ -dist[e.to], e.to });
          else
            q.modify(its[e.to], { -dist[e.to], e.to });
        }
      }
    }
    rep(i,0,N) pi[i] = min(pi[i] + dist[i], INF);
  }
  pair<ll, ll> maxflow(int s, int t) {
    ll totflow = 0, totcost = 0;
    while (path(s), seen[t]) {
      ll fl = INF;
      for (edge* x = par[t]; x; x = par[x->from])
        fl = min(fl, x->cap - x->flow);
      totflow += fl;
      for (edge* x = par[t]; x; x = par[x->from]) {
        x->flow += fl;
        ed[x->to][x->rev].flow -= fl;
      }
    }
    rep(i,0,N) for(edge& e : ed[i]) totcost += e.cost *
        e.flow;
    return {totflow, totcost/2};
  }
  // If some costs can be negative, call this before
      maxflow:
  void setpi(int s) { // (otherwise, leave this out)
    fill(all(pi), INF); pi[s] = 0;
    int it = N, ch = 1; ll v;
    while (ch-- && it--)
      rep(i,0,N) if (pi[i] != INF)
        for (edge& e : ed[i]) if (e.cap)
          if ((v = pi[i] + e.cost) < pi[e.to])
            pi[e.to] = v, ch = 1;
    assert(it >= 0); // negative cost cycle
  }
};
```

EdmondsKarp.h

Description: Flow algorithm with guaranteed complexity $O(VE^2)$. To get edge flow values, compare capacities before and after, and take the positive values only. — 482fe0, 36 lines

```
template<class T> T edmondsKarp(vector<unordered_map<
    int, T>>&
    graph, int source, int sink) {
  assert(source != sink);
  T flow = 0;
  vi par(sz(graph)), q = par;
  for (;;) {
    fill(all(par), -1);
    par[source] = 0;
    int ptr = 1;
    q[0] = source;
    rep(i,0,ptr) {
      int x = q[i];
      for (auto e : graph[x]) {
        if (par[e.first] == -1 && e.second > 0) {
          par[e.first] = x;
          q[ptr++] = e.first;
          if (e.first == sink) goto out;
        }
      }
    }
    return flow;
out:
    T inc = numeric_limits<T>::max();
    for (int y = sink; y != source; y = par[y])
      inc = min(inc, graph[par[y]][y]);
    flow += inc;
    for (int y = sink; y != source; y = par[y]) {
      int p = par[y];
      if ((graph[p][y] -= inc) <= 0) graph[p].erase(y);
      graph[y][p] += inc;
    }
  }
}
```

Dinic.h

Description: Flow algorithm with complexity $O(VE\log U)$ where $U = \max|\text{cap}|$. $O(\min(E^{1/2}, V^{2/3})E)$ if $U = 1$; $O(\sqrt{V}E)$ for bipartite matching. — d7f0f1, 42 lines

```
struct Dinic {
  struct Edge {
    int to, rev;
    ll c, oc;
    ll flow() { return max(oc - c, 0LL); } // if you
        need flows
  };
  vi lvl, ptr, q;
  vector<vector<Edge>> adj;
  Dinic(int n) : lvl(n), ptr(n), q(n), adj(n) {}
  void addEdge(int a, int b, ll c, ll rcap = 0) {
    adj[a].push_back({b, sz(adj[b]), c, c});
    adj[b].push_back({a, sz(adj[a]) - 1, rcap, rcap});
  }
  ll dfs(int v, int t, ll f) {
    if (v == t || !f) return f;
    for (int& i = ptr[v]; i < sz(adj[v]); i++) {
      Edge& e = adj[v][i];
      if (lvl[e.to] == lvl[v] + 1)
        if (ll p = dfs(e.to, t, min(f, e.c))) {
          e.c -= p, adj[e.to][e.rev].c += p;
          return p;
        }
    }
    return 0;
  }
  ll calc(int s, int t) {
    ll flow = 0; q[0] = s;
    rep(L,0,31) do { // 'int L=30' maybe faster for
        random data
      lvl = ptr = vi(sz(q));
      int qi = 0, qe = lvl[s] = 1;
      while (qi < qe && !lvl[t]) {
        int v = q[qi++];
        for (Edge e : adj[v])
          if (!lvl[e.to] && e.c >> (30 - L))
            q[qe++] = e.to, lvl[e.to] = lvl[v] + 1;
      }
      while (ll p = dfs(s, t, LLONG_MAX)) flow += p;
    } while (lvl[t]);
    return flow;
  }
  bool leftOfMinCut(int a) { return lvl[a] != 0; }
};
```

MinCut.h

Description: After running max-flow, the left side of a min-cut from s to t is given by all vertices reachable from s, only traversing edges with positive residual capacity.

GlobalMinCut.h

Description: Find a global minimum cut in an undirected graph, as represented by an adjacency matrix.

Time: $\mathcal{O}\left(V^3\right)$ — 8b0e19, 21 lines

```
pair<int, vi> globalMinCut(vector<vi> mat) {
  pair<int, vi> best = {INT_MAX, {}};
  int n = sz(mat);
  vector<vi> co(n);
  rep(i,0,n) co[i] = {i};
  rep(ph,1,n) {
    vi w = mat[0];
    size_t s = 0, t = 0;
    rep(it,0,n-ph) { // O(V^2) -> O(E log V) with prio.
        queue
      w[t] = INT_MIN;
      s = t, t = max_element(all(w)) - w.begin();
      rep(i,0,n) w[i] += mat[t][i];
    }
    best = min(best, {w[t] - mat[t][t], co[t]});
    co[s].insert(co[s].end(), all(co[t]));
    rep(i,0,n) mat[s][i] += mat[t][i];
    rep(i,0,n) mat[i][s] = mat[s][i];
    mat[0][t] = INT_MIN;
  }
  return best;
```

```
}
```

GomoryHu.h

Description: Given a list of edges representing an undirected flow graph, returns edges of the Gomory-Hu tree. The max flow between any pair of vertices is given by minimum edge weight along the Gomory-Hu tree path.

Time: $\mathcal{O}(V)$ Flow Computations

"PushRelabel.h" 0418b3, 13 lines

```
typedef array<ll, 3> Edge;
vector<Edge> gomoryHu(int N, vector<Edge> ed) {
  vector<Edge> tree;
  vi par(N);
  rep(i,1,N) {
    PushRelabel D(N); // Dinic also works
    for (Edge t : ed) D.addEdge(t[0], t[1], t[2], t[2])
        ;
    tree.push_back({i, par[i], D.calc(i, par[i])});
    rep(j,i+1,N)
      if (par[j] == par[i] && D.leftOfMinCut(j)) par[j]
          = i;
  }
  return tree;
}
```

7.3 Matching

Matching.h

Description: Fast bipartite matching algorithm. Graph g should be a list of neighbors of the left partition, and r should be a vector full of -1's of the same size as the right partition. Returns the size of the matching. $r[i]$ will be the match for vertex i on the right side, or -1 if it's not matched.

Time: $\mathcal{O}\left(E\sqrt{V}\right)$

daf32c, 21 lines

```
int matching(vector<vi>& g, vi& r) {
  int n = sz(g), res = 0;
  vi l(n, -1), q(n), d(n);
  auto dfs = [&](auto f, int u) -> bool {
    int t = exchange(d[u], 0) + 1;
    for (int v : g[u])
      if (r[v] == -1 || (d[r[v]] == t && f(f, r[v])))
        return l[u] = v, r[v] = u, 1;
    return 0;
  };
  for (int t = 0, f = 0;; t = f = 0, d.assign(n, 0)) {
    rep(i, 0, n) if (l[i] == -1) q[t++] = i, d[i] = 1;
    rep(i, 0, t) for (int v : g[q[i]]) {
      if (r[v] == -1) f = 1;
      else if (!d[r[v]])
        d[r[v]] = d[q[i]] + 1, q[t++] = r[v];
    }
    if (!f) return res;
    rep(i, 0, n) if (l[i] == -1) res += dfs(dfs, i);
  }
}
```

MinimumVertexCover.h

Description: Finds a minimum vertex cover in a bipartite graph. The size is the same as the size of a maximum matching, and the complement is a maximum independent set.

"Matching.h" fc69c6, 20 lines

```
vi cover(vector<vi>& g, int n, int m) {
  vi match(m, -1);
  int res = matching(g, match);
  vector<bool> lfound(n, true), seen(m);
  for (int it : match) if (it != -1) lfound[it] = false
      ;
  vi q, cover;
  rep(i,0,n) if (lfound[i]) q.push_back(i);
  while (!q.empty()) {
    int i = q.back(); q.pop_back();
    lfound[i] = 1;
    for (int e : g[i]) if (!seen[e] && match[e] != -1)
        {
      seen[e] = true;
      q.push_back(match[e]);
    }
  }
  rep(i,0,n) if (!lfound[i]) cover.push_back(i);
  rep(i,0,m) if (seen[i]) cover.push_back(n+i);
  assert(sz(cover) == res);
  return cover;
}
```

WeightedMatching.h

Description: Given a weighted bipartite graph, matches every node on the left with a node on the right such that no nodes are in two matchings and the sum of the edge weights is minimal. Takes cost[N][M], where cost[i][j] = cost for L[i] to be matched with R[j] and returns (min cost, match), where L[i] is matched with R[match[i]]. Negate costs for max cost. Requires $N \leq M$.

Time: $\mathcal{O}\left(N^2M\right)$

541052, 31 lines

```
pair<ll, vi> hungarian(const vector<vector<ll>> &a) {
  if (a.empty()) return {0, {}};
  int n = sz(a) + 1, m = sz(a[0]) + 1;
  vi p(m), ans(n - 1); vector<ll> u(n), v(m);
  rep(i,1,n) {
    p[0] = i;
    int j0 = 0; // add "dummy" worker 0
    vi pre(m, -1); vector<ll> dist(m, LLONG_MAX);
    vector<bool> done(m + 1);
    do { // dijkstra
      done[j0] = true;
      int i0 = p[j0], j1; ll delta = LLONG_MAX;
      rep(j,1,m) if (!done[j]) {
        auto cur = a[i0 - 1][j - 1] - u[i0] - v[j];
        if (cur < dist[j]) dist[j] = cur, pre[j] = j0;
        if (dist[j] < delta) delta = dist[j], j1 = j;
      }
      rep(j,0,m) {
        if (done[j]) u[p[j]] += delta, v[j] -= delta;
        else dist[j] -= delta;
      }
      j0 = j1;
    } while (p[j0]);
    while (j0) { // update alternating path
      int j1 = pre[j0];
      p[j0] = p[j1], j0 = j1;
    }
  }
  rep(j,1,m) if (p[j]) ans[p[j] - 1] = j - 1;
  return {-v[0], ans}; // min cost
}
```

Blossom.h

Description: Matching for general graphs using Blossom algorithm.

Time: $\mathcal{O}(NM)$, fast in practice

df28db, 46 lines

```
int blossom(vector<vi>& G, vi& match) {
  int n = sz(G), cnt = -1, ans = 0;match.assign(n, -1);
  vi lab(n), par(n), orig(n), aux(n, -1), q;
  auto blos = [&](int v, int w, int a) {
    while (orig[v] != a) {
      par[v] = w; w = match[v];
      if (lab[w] == 1) lab[w] = 0, q.push_back(w);
      orig[v] = orig[w] = a; v = par[w];
    }
  };
  rep(i, 0, n) if (match[i] == -1)
    for (auto e : G[i]) if (match[e] == -1) {
      match[match[e] = i] = e; ans++; break;
    }
  rep(root, 0, n) if (match[root] == -1) {
    fill(all(lab), -1);
    iota(all(orig), 0);
    lab[root] = 0;
    q = {root};
    rep(i, 0, sz(q)) {
      int v = q[i];
      for (auto x : G[v]) if (lab[x] == -1) {
        lab[x] = 1; par[x] = v;
        if (match[x] == -1) {
          for (int y = x; y+1;) {
            int p = par[y], w = match[p];
            match[match[p] = y] = p; y = w;
          }
          ans++;
          goto nxt;
        }
        lab[match[x]] = 0; q.push_back(match[x]);
      } else if (lab[x] == 0 && orig[v]!=orig[x]) {
        int a = orig[v], b = orig[x];
        for (cnt++;; swap(a, b)) if (a+1) {
          if (aux[a] == cnt) break;
          aux[a] = cnt;
          a = (match[a]+1 ?
            orig[par[match[a]]] : -1);
        }
        blos(x, v, a); blos(v, x, a);
      }
    }
    nxt:;
  }
  return ans; }
```

WeightedBlossom.h

Description: Edmond's Blossom algorithm for weighted maximum matching in general graphs. Weights must be positive.

Time: $\mathcal{O}\left(N^3\right)$

998ff6, 228 lines

```
struct WeightedBlossom {
  struct edge { int u, v, w; };
  int n, s, nx;
  vector<vector<edge>> g;
  vi lab, match, slack, st, pa, S, vis;
  vector<vi> flo, floFrom;
  queue<int> q;
  // Initialize for k vertices
  WeightedBlossom(int k)
      : n(k), s(n*2+1),
        g(s, vector<edge>(s)),
        lab(s), match(s), slack(s), st(s),
        pa(s), S(s), vis(s), flo(s),
        floFrom(s, vi(n+1)) {
    rep(u, 1, n+1) rep(v, 1, n+1)
      g[u][v] = {u, v, 0};
  }
  // Add edge between u and v with weight w
  void addEdge(int u, int v, int w) {
    u++; v++;
    g[u][v].w = g[v][u].w = max(g[u][v].w, w);
  }
  // Compute max weight matching.
  // 'count' is set to matching size,
  // 'weight' is set to matching weight.
  // Returns vector 'match' such that:
  // match[v] = vert matched to v or -1
  vi solve(int& count, ll& weight) {
    fill(all(match), 0);
    nx = n;
    weight = count = 0;
    rep(u, 0, n+1) flo[st[u] = u].clear();
    int tmp = 0;
    rep(u, 1, n+1) rep(v, 1, n+1) {
      floFrom[u][v] = (u-v ? 0 : v);
      tmp = max(tmp, g[u][v].w);
    }
    rep(u, 1, n+1) lab[u] = tmp;
    while (matching()) count++;
    rep(u, 1, n+1)
      if (match[u] && match[u] < u)
        weight += g[u][match[u]].w;
    vi ans(n);
    rep(i, 0, n) ans[i] = match[i+1]-1;
    return ans;
  }
  int delta(edge& e) {
    return lab[e.u]+lab[e.v]-g[e.u][e.v].w*2;
  }
  void updateSlack(int u, int x) {
    if (!slack[x] || delta(g[u][x]) <
      delta(g[slack[x]][x])) slack[x] = u;
  }
  void setSlack(int x) {
    slack[x] = 0;
    rep(u, 1, n+1) if (g[u][x].w > 0 &&
      st[u] != x && !S[st[u]])
        updateSlack(u, x);
  }
  void push(int x) {
    if (x <= n) q.push(x);
    else rep(i, 0, sz(flo[x])) push(flo[x][i]);
  }
  void setSt(int x, int b) {
    st[x] = b;
    if (x > n) rep(i, 0, sz(flo[x]))
      setSt(flo[x][i],b);
  }
  int getPr(int b, int xr) {
    int pr = int(find(all(flo[b]), xr) -
      flo[b].begin());
    if (pr % 2) {
      reverse(flo[b].begin()+1, flo[b].end());
      return sz(flo[b]) - pr;
    } else return pr;
  }
  void setMatch(int u, int v) {
    match[u] = g[u][v].v;
    if (u <= n) return;
    edge e = g[u][v];
    int xr = floFrom[u][e.u], pr = getPr(u,xr);
    rep(i, 0, pr)
      setMatch(flo[u][i], flo[u][i^1]);
    setMatch(xr, v);
    rotate(flo[u].begin(), flo[u].begin()+pr,
      flo[u].end());
  }
  void augment(int u, int v) {
    while (1) {
      int xnv = st[match[u]];
      setMatch(u, v);
      if (!xnv) return;
      setMatch(xnv, st[pa[xnv]]);
      u = st[pa[xnv]], v = xnv;
    }
  }
  int getLca(int u, int v) {
    static int t = 0;
    for (++t; u||v; swap(u, v)) {
      if (!u) continue;
      if (vis[u] == t) return u;
      vis[u] = t;
      u = st[match[u]];
      if (u) u = st[pa[u]];
    }
    return 0;
  }
  void blossom(int u, int lca, int v) {
    int b = n+1;
    while (b <= nx && st[b]) ++b;
    if (b > nx) ++nx;
    lab[b] = S[b] = 0;
    match[b] = match[lca];
    flo[b].clear();
    flo[b].push_back(lca);
    for (int x=u, y; x != lca; x = st[pa[y]]) {
      flo[b].push_back(x);
      flo[b].push_back(y = st[match[x]]);
      push(y);
    }
    reverse(flo[b].begin()+1, flo[b].end());
    for (int x=v, y; x != lca; x = st[pa[y]]) {
      flo[b].push_back(x);
      flo[b].push_back(y = st[match[x]]);
      push(y);
    }
    setSt(b, b);
    rep(x, 1, nx+1) g[b][x].w = g[x][b].w = 0;
    rep(x, 1, n+1) floFrom[b][x] = 0;
    rep(i, 0, sz(flo[b])) {
      int xs = flo[b][i];
      rep(x, 1, nx+1) if (!g[b][x].w ||
        delta(g[xs][x]) < delta(g[b][x]))
          g[b][x]=g[xs][x], g[x][b]=g[x][xs];
      rep(x, 1, n+1) if (floFrom[xs][x])
        floFrom[b][x] = xs;
    }
    setSlack(b);
  }
  void blossom(int b) {
    for (auto &e : flo[b]) setSt(e, e);
    int xr = floFrom[b][g[b][pa[b]].u];
    int pr = getPr(b, xr);
    for (int i = 0; i < pr; i += 2) {
      int xs = flo[b][i], xns = flo[b][i+1];
      pa[xs] = g[xns][xs].u;
      S[xs] = 1; S[xns] = slack[xs] = 0;
      setSlack(xns); push(xns);
    }
    S[xr] = 1; pa[xr] = pa[b];
    rep(i, pr+1, sz(flo[b])) {
      int xs = flo[b][i];
      S[xs] = -1; setSlack(xs);
    }
    st[b] = 0;
  }
  bool found(const edge& e) {
    int u = st[e.u], v = st[e.v];
    if (S[v] == -1) {
      pa[v] = e.u; S[v] = 1;
      int nu = st[match[v]];
      slack[v] = slack[nu] = S[nu] = 0;
      push(nu);
    } else if (!S[v]) {
      int lca = getLca(u, v);
      if (!lca) return augment(u, v),
        augment(v, u), 1;
      else blossom(u, lca, v);
    }
    return 0;
  }
  bool matching() {
```

```
    fill(S.begin(), S.begin()+nx+1, -1);
    fill(slack.begin(), slack.begin()+nx+1, 0);
    q = {};
    rep(x, 1, nx+1)
      if (st[x] == x && !match[x])
        pa[x] = S[x] = 0, push(x);
    if (q.empty()) return 0;
    while (1) {
      while (q.size()) {
        int u = q.front(); q.pop();
        if (S[st[u]] == 1) continue;
        rep(v, 1, n+1)
          if (g[u][v].w > 0 && st[u] != st[v]){
            if (!delta(g[u][v])) {
              if (found(g[u][v])) return 1;
            } else updateSlack(u, st[v]);
          }
      }
      int d = INT_MAX;
      rep(b, n+1, nx+1)
        if (st[b] == b && S[b] == 1)
          d = min(d, lab[b]/2);
      rep(x, 1, nx+1)
        if (st[x] == x && slack[x]) {
          if (S[x] == -1)
            d = min(d, delta(g[slack[x]][x]));
          else if (!S[x])
            d = min(d,delta(g[slack[x]][x])/2);
        }
      rep(u, 1, n+1) {
        if (!S[st[u]]) {
          if (lab[u] <= d) return 0;
          lab[u] -= d;
        } else if (S[st[u]] == 1) lab[u] += d;
      }
      rep(b, n+1, nx+1) if (st[b] == b) {
        if (!S[st[b]]) lab[b] += d*2;
        else if (S[st[b]] == 1) lab[b] -= d*2;
      }
      q = {};
      rep(x, 1, nx+1)
        if (st[x] == x && slack[x] &&
          st[slack[x]] != x &&
          !delta(g[slack[x]][x]) &&
          found(g[slack[x]][x])) return 1;
      rep(b, n+1, nx+1)
        if (st[b] == b && S[b] == 1 && !lab[b])
          blossom(b);
    }
    return 0;
  }
};
```

7.4 DFS algorithms

SCC.h

Description: Finds strongly connected components in a directed graph. If vertices u, v belong to the same component, we can reach u from v and vice versa.

Usage: scc(graph, [&](vi& v) { ... }) visits all components
in reverse topological order. comp[i] holds the component
index of a node (a component only has edges to components with
lower index). ncomps will contain the number of components.

Time: $\mathcal{O}(E+V)$ 76b5c9, 24 lines

```
vi val, comp, z, cont;
int Time, ncomps;
template<class G, class F> int dfs(int j, G& g, F& f) {
  int low = val[j] = ++Time, x; z.push_back(j);
  for (auto e : g[j]) if (comp[e] < 0)
    low = min(low, val[e] ?: dfs(e,g,f));
  if (low == val[j]) {
    do {
      x = z.back(); z.pop_back();
      comp[x] = ncomps;
      cont.push_back(x);
    } while (x != j);
    f(cont); cont.clear();
    ncomps++;
  }
  return val[j] = low;
}
template<class G, class F> void scc(G& g, F f) {
  int n = sz(g);
  val.assign(n, 0); comp.assign(n, -1);
  Time = ncomps = 0;
  rep(i,0,n) if (comp[i] < 0) dfs(i, g, f);
}
```

BiconnectedComponents.h

Description: Finds all biconnected components in an undirected graph, and runs a callback for the edges in each. In a biconnected component there are at least two distinct paths between any two nodes. Note that a node can be in several components. An edge which is not in a component is a bridge, i.e., not part of any cycle.

Usage: int eid = 0; ed.resize(N);
for each edge (a,b) {
ed[a].emplace_back(b, eid);
ed[b].emplace_back(a, eid++); }
bicomps([&](const vi& edgelist) {...});

Time: $\mathcal{O}(E+V)$ c6b7c7, 32 lines

```
vi num, st;
vector<vector<pii>> ed;
int Time;
template<class F>
int dfs(int at, int par, F& f) {
  int me = num[at] = ++Time, top = me;
  for (auto [y, e] : ed[at]) if (e != par) {
    if (num[y]) {
      top = min(top, num[y]);
      if (num[y] < me)
        st.push_back(e);
    } else {
      int si = sz(st);
      int up = dfs(y, e, f);
      top = min(top, up);
      if (up == me) {
        st.push_back(e);
        f(vi(st.begin() + si, st.end()));
        st.resize(si);
      }
      else if (up < me) st.push_back(e);
      else { /* e is a bridge */ }
    }
  }
  return top;
}
template<class F>
void bicomps(F f) {
  num.assign(sz(ed), 0);
  rep(i,0,sz(ed)) if (!num[i]) dfs(i, -1, f);
}
```

2sat.h

Description: Calculates a valid assignment to boolean variables a, b, c,... to a 2-SAT problem, so that an expression of the type $(a||b)\&\&(!a||c)\&\&(d||!b)\&\&\ldots$ becomes true, or reports that it is unsatisfiable. Negated variables are represented by bit-inversions (~x).

Usage: TwoSat ts(number of boolean variables);
ts.either(0, ~3); // Var 0 is true or var 3 is false
ts.setValue(2); // Var 2 is true
ts.atMostOne({0,~1,2}); // <= 1 of vars 0, ~1 and 2 are true
ts.solve(); // Returns true iff it is solvable
ts.values[0..N-1] holds the assigned values to the vars

Time: $\mathcal{O}(N+E)$, where N is the number of boolean variables, and E is the number of clauses. 5f9706, 56 lines

```
struct TwoSat {
  int N;
  vector<vi> gr;
  vi values; // 0 = false, 1 = true
  TwoSat(int n = 0) : N(n), gr(2*n) {}
  int addVar() { // (optional)
    gr.emplace_back();
    gr.emplace_back();
    return N++;
  }
  void either(int f, int j) {
    f = max(2*f, -1-2*f);
    j = max(2*j, -1-2*j);
    gr[f].push_back(j^1);
    gr[j].push_back(f^1);
  }
  void setValue(int x) { either(x, x); }
  void atMostOne(const vi& li) { // (optional)
    if (sz(li) <= 1) return;
    int cur = ~li[0];
    rep(i,2,sz(li)) {
      int next = addVar();
      either(cur, ~li[i]);
      either(cur, next);
      either(~li[i], next);
      cur = ~next;
    }
    either(cur, ~li[1]);
  }
  vi val, comp, z; int time = 0;
  int dfs(int i) {
    int low = val[i] = ++time, x; z.push_back(i);
    for(int e : gr[i]) if (!comp[e])
      low = min(low, val[e] ?: dfs(e));
    if (low == val[i]) do {
      x = z.back(); z.pop_back();
      comp[x] = low;
      if (values[x>>1] == -1)
        values[x>>1] = x&1;
    } while (x != i);
    return val[i] = low;
  }
  bool solve() {
    values.assign(N, -1);
    val.assign(2*N, 0); comp = val;
    rep(i,0,2*N) if (!comp[i]) dfs(i);
    rep(i,0,N) if (comp[2*i] == comp[2*i+1]) return 0;
    return 1;
  }
};
```

EulerWalk.h

Description: Eulerian undirected/directed path/cycle algorithm. Input should be a vector of (dest, global edge index), where for undirected graphs, forward/backward edges have the same index. Returns a list of pairs (node, incoming edge) in the Eulerian path/cycle with src at both start and end, or empty list if no cycle/path exists.

Time: $\mathcal{O}(V+E)$ c62d93, 16 lines

```
vector<pii> eulerWalk(vector<vector<pii>>& gr, int
      nedges, int src=0) {
  int n = sz(gr);
  vi D(n), its(n), eu(nedges); vector<pii> ret, s = {{
       src, -1}};
  D[src]++; // to allow Euler paths, not just cycles
  while (!s.empty()) {
    int x = s.back().first, y, e, &it = its[x], end =
         sz(gr[x]);
    if (it == end) {
      ret.push_back(s.back()); s.pop_back(); continue;
           }
    tie(y, e) = gr[x][it++];
    if (!eu[e]) {
      D[x]--, D[y]++;
      eu[e] = 1; s.push_back({y, e});
    }}
  for (int x : D) if (x < 0 || sz(ret) != nedges+1)
       return {};
  return {ret.rbegin(), ret.rend()};
}
```

Dominators.h

Description: Tarjan's dominators in directed graph. Returns tree (as array of parents) of immediate dominators idom. idom[root] = root, idom[v] = -1 if v is unreachable from root.

Time: $\mathcal{O}(|E|log|V|)$ 50d914, 32 lines

```
vi dominators(vector<vi>& G, int root) {
  int n = sz(G); vector<vi> in(n), bucket(n);
  vi pre(n, -1), anc(n, -1), par(n), best(n);
  vi ord, idom(n, -1), sdom(n, n), rdom(n);
  auto dfs = [&] (auto f, int v, int p) -> void {
    if (pre[v] == -1) {
      par[v] = p; pre[v] = sz(ord);
      ord.push_back(v);
      for (auto e :  G[v])
        in[e].push_back(v), f(f, e, v);
    }
  };
  auto find = [&](auto f, int v) -> pii {
    if (anc[v] == -1) return {best[v], v};
    int b; tie(b, anc[v]) = f(f, anc[v]);
    if (sdom[b] < sdom[best[v]]) best[v] = b;
    return {best[v], anc[v]};
  };
  rdom[root] = idom[root] = root;
  iota(all(best), 0); dfs(dfs, root, -1);
  rep(i, 0, sz(ord)) {
    int v = ord[sz(ord)-i-1], b = pre[v];
    for (auto e : in[v])
      b = min(b, pre[e] < pre[v] ? pre[e] :
      sdom[find(find, e).first]);
    for (auto u : bucket[v])rdom[u]=find(find,u).first;
    sdom[v] = b; anc[v] = par[v];
    bucket[ord[sdom[v]]].push_back(v);
  }
  for (auto v : ord) idom[v] = (rdom[v] == v ?
    ord[sdom[v]] : idom[rdom[v]]);
  return idom; }
```

KthShortest.h

Description: Given directed weighted graph with non-negative edge weights gets K-th shortest walk (not necessarily simple) or -1 if no next path (can only happen in DAG).

Memory: $\mathcal{O}(m\log m+k\log m)$ (uses persistent heaps)

Time: $\mathcal{O}(m\log m+k\log m)$ e29f5a, 57 lines

```
constexpr ll INF = 1e18;
struct Eppstein {
  using T = ll; using Edge = pair<int, T>;
  struct Node { int E[2] = {}, s = 0; Edge x; };
  T shortest; // Shortest path length
  priority_queue<pair<T, int>> Q;
  vector<Node> P{1}; vi h;
  Eppstein(vector<vector<Edge>>& G, int s, int t) {
    int n = sz(G); vector<vector<Edge>> H(n);
    rep(i,0,n) for(auto &[j, w] : G[i])
      H[j].push_back({i,w});
    vi ord, par(n, -1); vector<T> d(n, -INF);
    Q.push({d[t] = 0, t});
    while (!Q.empty()) {
      auto [dd, v] = Q.top(); Q.pop();
      if (d[v] == dd) {
        ord.push_back(v);
        for(auto &[u, w] : H[v])
        if (dd-w > d[u]) {
          Q.push({d[u] = dd-w, u});
          par[u] = v;
        }
      }
    }
    if ((shortest = -d[s]) >= INF) return;
    h.resize(n);
    for(auto &v : ord) {
      int p = par[v]; if (p+1) h[v] = h[p];
      for(auto &[u, w] : G[v]) if (d[u] > -INF) {
        T k = w - d[u] + d[v];
        if (k || u != p)
          h[v] = push(h[v], {u, k});
        else p = -1;
      }
    }
    P[0].x.first = s; Q.push({0, 0});
  }
  int push(int t, Edge x) {
    P.push_back(P[t]);
    if (!P[t = sz(P)-1].s || P[t].x.second >= x.second)
      swap(x, P[t].x);
    if (P[t].s) {
      int i = P[t].E[0], j = P[t].E[1];
      int d = P[i].s > P[j].s;
      int k = push(d ? j : i, x);
      P[t].E[d] = k; // Don't inline k!
    }
    P[t].s++; return t;
  }
  ll nextPath() { // next length, -1 if no next path
    if (Q.empty()) return -1;
    auto [d, v] = Q.top(); Q.pop();
    for (int i : P[v].E) if (i)
      Q.push({ d-P[i].x.second+P[v].x.second, i });
    int t = h[P[v].x.first];
    if (t) Q.push({d - P[t].x.second, t });
    return shortest - d; } };
```

DenseDFS.h

Description: DFS over dense graph. Suddenly DFS over N <= 1000 graph many times becomes feasible 6e9645, 68 lines

```
// DFS over bit-packed adjacency matrix
// G = NxN adjacency matrix of graph
//     G(i,j) <=> (i,j) is edge
// V = 1xN matrix containing unvisited vertices
//     V(0,i) <=> i-th vertex is not visited
// Total DFS time: O(n^2/64)
using ull = uint64_t;
// Matrix over Z_2 (bits and xor)
// TODO: arithmetic operations //!HIDE
```

```cpp
struct BitMatrix {
  vector<ull> M;
  int rows, cols, stride;
  // Create matrix with n rows and m columns
  BitMatrix(int n = 0, int m = 0) {
    rows = n; cols = m;
    stride = (m+63)/64;
    M.resize(n*stride);
  }
  // Get pointer to bit-packed data of i-th row
  ull* row(int i) { return &M[i*stride]; }
  // Get value in i-th row and j-th column
  bool operator()(int i, int j) {
    return (row(i)[j/64] >> (j%64)) & 1;
  }
  // Set value in i-th row and j-th column
  void set(int i, int j, bool val) {
    ull &w = row(i)[j/64], m = 1ull << (j%64);
    if (val) w |= m;
    else w &= ~m;
  }
};
struct DenseDFS {
  BitMatrix G, V; // space: O(n^2/64)
  // Initialize structure for n vertices
  DenseDFS(int n = 0) : G(n, n), V(1, n) {
    reset();
  }
  // Mark all vertices as unvisited
  void reset() { for(auto &x : V.M) x = -1; }
  // Get/set visited flag for i-th vertex
  void setVisited(int i) { V.set(0, i, 0); }
  bool isVisited(int i)  { return !V(0, i); }
  // DFS step: func is called on each unvisited
  // neighbour of i. You need to manually call
  // setVisited(child) to mark it visited
  // or this function will call the callback
  // with the same vertex again.
  void step(int i, auto func) {
    ull* E = G.row(i);
    for (int w = 0; w < G.stride;) {
      ull x = E[w] & V.row(0)[w];
      if (x) func((w<<6) | __builtin_ctzll(x));
      else   w++;
    }
  }
};
```

PlanarFaces.h

Description: Finds the faces of a simple planar graph and returns the vertex indices for each face in either clockwise (inner) or counterclockwise (outer) order. Disconnected graphs may have multiple outer faces and require careful handling.
Time: $\mathcal{O}(n \log n)$

"../geometry/Point.h", "../geometry/AngleCmp.h" 2c9685, 24 lines

```cpp
template<class P>
vector<vi> planarFaces(vector<vi>& g, vector<P>& p) {
  int n = sz(g); P o;
  auto cmp = [&](int x,int y) {
    return angleCmp(p[x] - o, p[y] - o); };
  vector<vi> vis(n);
  rep(i, 0, n) {
    o = p[i], sort(all(g[i]), cmp);
    vis[i].resize(sz(g[i]));
  }
  vector<vi> f;
  rep(i, 0, n) rep(j, 0, sz(adj[i])) {
    if (vis[i][j]) continue;
    vi s; int u = i, k = j;
    while (!vis[u][k]) {
      vis[u][k] = 1; s.push_back(u);
      int v = adj[u][k]; o = p[v];
      int kk=lower_bound(all(g[v]),u,cmp)-g[v].begin();
      u = v, k = (kk + 1) % sz(adj[u]);
    }
    f.push_back(s);
  }
  return f;
}
```

ChordalGraph.h

Description: A graph is chordal if any cycle C >= 4 has a chord i.e. an edge (u, v) where u and v is in the cycle but (u, v) is not A perfect elimination ordering (PEO) in a graph is an ordering of the vertices of the graph such that, $\forall v$: v and its neighbors that occur after v in the order (later) form a clique. A graph is chordal if and only if it has a perfect elimination ordering. Optimal vertex coloring of the graph: first fit: col[i] = smallest color that is not used by any of the neighbours earlier in PEO. Max clique = Chromatic number = 1+max over number of later neighbours for all vertices. Chromatic polynomial $= (x - d_1)(x - d_2) \dots (x - d_n)$ where d_i = number of neighbors of i later in PEO.
Time: $\mathcal{O}(n + m)$ 3c9cbb, 42 lines

```cpp
// 0-indexed, adj list
vi perfectEliminationOrder(vector<vi>& g) {
  int top = 0, n = sz(g);
  vi ord, vis(n), indeg(n);
  vector<vi> bucket(n);
  rep(i, 0, n) bucket[0].push_back(i);
  for(int i = 0; i < n; ) {
    while(bucket[top].empty()) --top;
    int u = bucket[top].back();
    bucket[top].pop_back();
    if(vis[u]) continue;
    ord.push_back(u);
    vis[u] = 1;
    ++i;
    for(int v : g[u]) {
      if(vis[v]) continue;
      bucket[++indeg[v]].push_back(v);
      top = max(top, indeg[v]);
    }
  }
  reverse(all(ord));
  return ord;
}
//ord = perfectEliminationOrder(g)
bool isChordal(vector<vi>& g, vi ord) {
  int n = sz(g);
  set<pii> edg;
  rep(i, 0, n) for(auto v:g[i]) edg.insert({i,v});
  vi pos(n); rep(i, 0, n) pos[ord[i]] = i;
  rep(u, 0, n){
    int mn = n;
    for(auto v : g[u]) if(pos[u] < pos[v])
      mn=min(mn,pos[v]);
    if(mn != n) {
      int p = ord[mn];
      for(auto v : g[u])
        if(pos[v]>pos[u] && v!=p && !edg.count({v, p}))
          return 0;
    }
  }
  return 1;
}
```

7.5 Coloring

EdgeColoring.h

Description: Given a simple, undirected graph with max degree D, computes a $(D + 1)$-coloring of the edges such that no neighboring edges share a color. (D-coloring is NP-hard, but can be done for bipartite graphs by repeated matchings of max-degree nodes.)
Time: $\mathcal{O}(NM)$ e210e2, 31 lines

```cpp
vi edgeColoring(int N, vector<pii> eds) {
  vi cc(N + 1), ret(sz(eds)), fan(N), free(N), loc;
  for (pii e : eds) ++cc[e.first], ++cc[e.second];
  int u, v, ncols = *max_element(all(cc)) + 1;
  vector<vi> adj(N, vi(ncols, -1));
  for (pii e : eds) {
    tie(u, v) = e;
    fan[0] = v;
    loc.assign(ncols, 0);
    int at = u, end = u, d, c = free[u], ind = 0, i =
         0;
    while (d = free[v], !loc[d] && (v = adj[u][d]) !=
         -1)
      loc[d] = ++ind, cc[ind] = d, fan[ind] = v;
    cc[loc[d]] = c;
    for (int cd = d; at != -1; cd ^= c ^ d, at = adj[at
         ][cd])
      swap(adj[at][cd], adj[end = at][cd ^ c ^ d]);
    while (adj[fan[i]][d] != -1) {
      int left = fan[i], right = fan[++i], e = cc[i];
      adj[u][e] = left;
      adj[left][e] = u;
      adj[right][e] = -1;
      free[right] = e;
    }
    adj[u][d] = fan[i];
    adj[fan[i]][d] = u;
    for (int y : {fan[0], u, end})
      for (int& z = free[y] = 0; adj[y][z] != -1; z++);
  }
  rep(i,0,sz(eds))
    for (tie(u, v) = eds[i]; adj[u][ret[i]] != v;) ++
         ret[i];
  return ret;
}
```

ChromaticNumber.h

Description: Calculates chromatic number of a graph represented by a vector of bitmasks. Self loops are not allowed.
Usage: `chromaticNumber({6, 5, 3}) // 3-clique`
Time: $\mathcal{O}(2^n n)$ 07ea3d, 20 lines

```cpp
const int MOD = 1000500103; // big prime
int chromaticNumber(vi g) {
  int n = sz(g);
  if (!n) return 0;
  vi ind(1 << n, 1), s(1 << n);
  rep(i, 0, 1 << n) s[i] = __popcount(i) & 1 ? -1 : 1;
  rep(i, 1, 1 << n) {
    int ctz = __builtin_ctz(i);
    ind[i] = ind[i - (1 << ctz)] + ind[(i - (1 << ctz))
         & ~g[ctz]];
    if (ind[i] >= MOD) ind[i] -= MOD;
  }
  rep(k, 1, n) {
    ll sum = 0;
    rep(i, 0, 1 << n) {
      s[i] = int((ll)s[i] * ind[i] % MOD);
      sum += s[i];
    }
    if (sum % MOD) return k;
  }
  return n; }
```

7.6 Heuristics

MaximalCliques.h

Description: Runs a callback for all maximal cliques in a graph (given as a symmetric bitset matrix; self-edges not allowed). Callback is given a bitset representing the maximal clique.
Time: $\mathcal{O}\left(3^{n/3}\right)$, much faster for sparse graphs b0d5b1, 12 lines

```cpp
typedef bitset<128> B;
template<class F>
void cliques(vector<B>& eds, F f, B P = ~B(), B X={}, B
      R={}) {
  if (!P.any()) { if (!X.any()) f(R); return; }
  auto q = (P | X)._Find_first();
  auto cands = P & ~eds[q];
  rep(i,0,sz(eds)) if (cands[i]) {
    R[i] = 1;
    cliques(eds, f, P & eds[i], X & eds[i], R);
    R[i] = P[i] = 0; X[i] = 1;
  }
}
```

MaximumClique.h

Description: Quickly finds a maximum clique of a graph (given as symmetric bitset matrix; self-edges not allowed). Can be used to find a maximum independent set by finding a clique of the complement graph.
Time: Runs in about 1s for n=155 and worst case random graphs (p=.90). Runs faster for sparse graphs. f7c0bc, 49 lines

```cpp
typedef vector<bitset<200>> vb;
struct Maxclique {
  double limit=0.025, pk=0;
  struct Vertex { int i, d=0; };
  typedef vector<Vertex> vv;
  vb e;
  vv V;
  vector<vi> C;
  vi qmax, q, S, old;
  void init(vv& r) {
    for (auto& v : r) v.d = 0;
    for (auto& v : r) for (auto j : r) v.d += e[v.i][j.
         i];
    sort(all(r), [](auto a, auto b) { return a.d > b.d;
         });
    int mxD = r[0].d;
    rep(i,0,sz(r)) r[i].d = min(i, mxD) + 1;
  }
  void expand(vv& R, int lev = 1) {
    S[lev] += S[lev - 1] - old[lev];
    old[lev] = S[lev - 1];
    while (sz(R)) {
      if (sz(q) + R.back().d <= sz(qmax)) return;
      q.push_back(R.back().i);
      vv T;
      for(auto v:R) if (e[R.back().i][v.i]) T.push_back
           ({v.i});
      if (sz(T)) {
        if (S[lev]++ / ++pk < limit) init(T);
        int j = 0, mxk = 1, mnk = max(sz(qmax) - sz(q)
             + 1, 1);
        C[1].clear(), C[2].clear();
        for (auto v : T) {
          int k = 1;
          auto f = [&](int i) { return e[v.i][i]; };
          while (any_of(all(C[k]), f)) k++;
          if (k > mxk) mxk = k, C[mxk + 1].clear();
          if (k < mnk) T[j++].i = v.i;
          C[k].push_back(v.i);
        }
        if (j > 0) T[j - 1].d = 0;
        rep(k,mnk,mxk + 1) for (int i : C[k])
          T[j].i = i, T[j++].d = k;
        expand(T, lev + 1);
      } else if (sz(q) > sz(qmax)) qmax = q;
      q.pop_back(), R.pop_back();
    }
  }
  vi maxClique() { init(V), expand(V); return qmax; }
  Maxclique(vb conn) : e(conn), C(sz(e)+1), S(sz(C)),
       old(S) {
    rep(i,0,sz(e)) V.push_back({i});
  }
};
```

MaximumIndependentSet.h

Description: To obtain a maximum independent set of a graph, find a max clique of the complement. If the graph is bipartite, see MinimumVertexCover.

7.7 Trees

BinaryLifting.h

Description: Calculate power of two jumps in a tree, to support fast upward jumps and LCAs. Assumes the root node points to itself.
Time: construction $\mathcal{O}(N \log N)$, queries $\mathcal{O}(\log N)$ bfce85, 25 lines

```cpp
vector<vi> treeJump(vi& P){
  int on = 1, d = 1;
  while(on < sz(P)) on *= 2, d++;
  vector<vi> jmp(d, P);
  rep(i,1,d) rep(j,0,sz(P))
    jmp[i][j] = jmp[i-1][jmp[i-1][j]];
  return jmp;
}
int jmp(vector<vi>& tbl, int nod, int steps){
  rep(i,0,sz(tbl))
    if(steps&(1<<i)) nod = tbl[i][nod];
  return nod;
}
int lca(vector<vi>& tbl, vi& depth, int a, int b) {
  if (depth[a] < depth[b]) swap(a, b);
  a = jmp(tbl, a, depth[a] - depth[b]);
  if (a == b) return a;
  for (int i = sz(tbl); i--;) {
    int c = tbl[i][a], d = tbl[i][b];
    if (c != d) a = c, b = d;
  }
  return tbl[0][a];
}
```

LCA.h

Description: Data structure for computing lowest common ancestors in a tree (with 0 as root). C should be an adjacency list of the tree, either directed or undirected.
Time: $\mathcal{O}(N \log N + Q)$

"../data-structures/RMQ.h" 0f62fb, 21 lines

```cpp
struct LCA {
  int T = 0;
  vi time, path, ret;
  RMQ<int> rmq;
  LCA(vector<vi>& C) : time(sz(C)), rmq((dfs(C,0,-1),
       ret)) {}
  void dfs(vector<vi>& C, int v, int par) {
```

```
    time[v] = T++;
    for (int y : C[v]) if (y != par) {
      path.push_back(v), ret.push_back(time[v]);
      dfs(C, y, v);
    }
  }
  int lca(int a, int b) {
    if (a == b) return a;
    tie(a, b) = minmax(time[a], time[b]);
    return path[rmq.query(a, b)];
  }
  //dist(a,b){return depth[a] + depth[b] - 2*depth[lca(
      a,b)];}
};
```

CompressTree.h

Description: Given a rooted tree and a subset S of nodes, compute the minimal subtree that contains all the nodes by adding all (at most $|S|-1$) pairwise LCA's and compressing edges. Returns a list of (par, orig_index) representing a tree rooted at 0. The root points to itself.

Time: $\mathcal{O}(|S| \log |S|)$

"LCA.h" 9775a0, 21 lines

```
typedef vector<pair<int, int>> vpi;
vpi compressTree(LCA& lca, const vi& subset) {
  static vi rev; rev.resize(sz(lca.time));
  vi li = subset, &T = lca.time;
  auto cmp = [&](int a, int b) { return T[a] < T[b]; };
  sort(all(li), cmp);
  int m = sz(li)-1;
  rep(i,0,m) {
    int a = li[i], b = li[i+1];
    li.push_back(lca.lca(a, b));
  }
  sort(all(li), cmp);
  li.erase(unique(all(li)), li.end());
  rep(i,0,sz(li)) rev[li[i]] = i;
  vpi ret = {pii(0, li[0])};
  rep(i,0,sz(li)-1) {
    int a = li[i], b = li[i+1];
    ret.emplace_back(rev[lca.lca(a, b)], b);
  }
  return ret;
}
```

HLD.h

Description: Decomposes a tree into vertex disjoint heavy paths and light edges such that the path from any leaf to the root contains at most log(n) light edges. Code does additive modifications and max queries, but can support commutative segtree modifications/queries on paths and subtrees. Takes as input the full adjacency list. VALS_EDGES being true means that values are stored in the edges, as opposed to the nodes. All values initialized to the segtree default. Root must be 0.

Time: $\mathcal{O}\left((\log N)^2\right)$

"../data-structures/LazySegmentTree.h" 9547af, 46 lines

```
template <bool VALS_EDGES> struct HLD {
  int N, tim = 0;
  vector<vi> adj;
  vi par, siz, rt, pos;
  Node *tree;
  HLD(vector<vi> adj_)
    : N(sz(adj_)), adj(adj_), par(N, -1), siz(N, 1),
      rt(N),pos(N),tree(new Node(0, N)){ dfsSz(0);
          dfsHld(0); }
  void dfsSz(int v) {
    for (int& u : adj[v]) {
      adj[u].erase(find(all(adj[u]), v));
      par[u] = v;
      dfsSz(u);
      siz[v] += siz[u];
      if (siz[u] > siz[adj[v][0]]) swap(u, adj[v][0]);
    }
  }
  void dfsHld(int v) {
    pos[v] = tim++;
    for (int u : adj[v]) {
      rt[u] = (u == adj[v][0] ? rt[v] : u);
      dfsHld(u);
    }
  }
  template <class B> void process(int u, int v, B op) {
    for (;; v = par[rt[v]]) {
      if (pos[u] > pos[v]) swap(u, v);
      if (rt[u] == rt[v]) break;
      op(pos[rt[v]], pos[v] + 1);
    }
    op(pos[u] + VALS_EDGES, pos[v] + 1);
  }
  void modifyPath(int u, int v, int val) {
    process(u, v, [&](int l, int r) { tree->add(l, r,
        val); });
  }
  int queryPath(int u, int v) { // Modify depending on
      problem
    int res = -1e9;
    process(u, v, [&](int l, int r) {
        res = max(res, tree->query(l, r));
    });
    return res;
  }
  int querySubtree(int v) { // modifySubtree is similar
    return tree->query(pos[v] + VALS_EDGES, pos[v] +
        siz[v]);
  }
};
```

LCT.h

Description: Link-cut tree with path and subtree queries. Path operations can be arbitrary, but subtree operations need to be reversible. Current implementation supports subtree addition and sum.

Time: $\mathcal{O}(\log n)$

6758b4, 104 lines

```
struct Node {
  Node *p, *c[2];
  Node() { p = c[0] = c[1] = 0; }
  bool rev = 0;
  ll v, s, vs = 0;
  int sz, vsz = 0;
  ll d = 0, vd = 0, cc = 0;
  void add(ll x) {
    d += x, vd += x;
    v += x, s += sz * x, vs += vsz * x;
  }
  void flip() { swap(c[0], c[1]), rev ^= 1; }
  void push() {
    if (rev) {
      rep(i, 0, 2) if (c[i]) c[i]->flip();
      rev = 0;
    }
    if (d) {
      rep(i, 0, 2) if (c[i]) c[i]->add(d);
      d = 0;
    }
  }
  void pull() {
    s = v + vs;
    sz = 1 + vsz;
    if (c[0]) s += c[0]->s, sz += c[0]->sz;
    if (c[1]) s += c[1]->s, sz += c[1]->sz;
  }
  // Add a virtual edge, cancel current virtual delta
  void vadd(Node* x) {
    vs += x->s;
    vsz += x->sz;
    x->cc = vd;
  }
  // Delete a virutal edge, push the virtual delta
  void vdel(Node* x) {
    x->add(vd - x->cc);
    vs -= x->s;
    vsz -= x->sz;
    x->cc = 0;
  }
  // Swap the cancels because of a splay rotation
  void vswap(Node* x, Node* y) {
    swap(x->cc, y->cc);
  }
  int up() {
    if (!p) return -2;
    rep(i, 0, 2) if (p->c[i] == this) return i;
    return -1;
  }
  bool isRoot() { return up() < 0; }
  friend void setLink(Node* x, Node* y, int d) {
    if (y) y->p = x;
    if (d >= 0) x->c[d] = y;
  }
  void rot() {
    int x = up(); Node* pp = p;
    setLink(pp->p, this, pp->up());
    setLink(pp, c[x ^ 1], x); setLink(this, pp, x ^ 1);
    if (pp->p) pp->p->vswap(pp, this);
    pp->pull();
  }
  void fix() { if (!isRoot()) p->fix(); push(); }
  void splay() {
    for (fix(); !isRoot();) {
      if (p->isRoot()) rot();
      else if (up() == p->up()) p->rot(), rot();
      else rot(), rot();
    }
    pull();
  }
};
struct LinkCut {
  vector<Node> t;
  LinkCut(int n) : t(n) {}
  void link(int u, int v) {
    makeRoot(&t[v]); access(&t[u]);
    setLink(&t[v], &t[u], 0); t[v].pull();
  }
  void cut(int u, int v) {
    makeRoot(&t[u]); access(&t[v]);
    t[v].c[0] = t[u].p = 0; t[v].pull();
  }
  bool connected(int u, int v) {
    return lca(&t[u], &t[v]);
  }
  Node* lca(Node* u, Node* v) {
    if (u == v) return u;
    access(u); access(v); if (!u->p) return 0;
    u->splay(); return u->p ?: u;
  }
  void access(Node* u) {
    for (Node* x = u, *y = 0; x; x = x->p) {
      x->splay();
      if (y) x->vdel(y);
      if (x->c[1]) x->vadd(x->c[1]);
      x->c[1] = y; x->pull(); y = x;
    }
    u->splay();
  }
  void makeRoot(Node* u) {
    access(u), u->flip(), u->push();
  }
};
```

DirectedMST.h

Description: Finds a minimum spanning tree/arborescence of a directed graph, given a root node. If no MST exists, returns -1.

Time: $\mathcal{O}(E \log V)$

"../data-structures/UnionFindRollback.h" 39e620, 60 lines

```
struct Edge { int a, b; ll w; };
struct Node {
  Edge key;
  Node *l, *r;
  ll delta;
  void prop() {
    key.w += delta;
    if (l) l->delta += delta;
    if (r) r->delta += delta;
    delta = 0;
  }
  Edge top() { prop(); return key; }
};
Node *merge(Node *a, Node *b) {
  if (!a || !b) return a ?: b;
  a->prop(), b->prop();
  if (a->key.w > b->key.w) swap(a, b);
  swap(a->l, (a->r = merge(b, a->r)));
  return a;
}
void pop(Node*& a) { a->prop(); a = merge(a->l, a->r);
    }
pair<ll, vi> dmst(int n, int r, vector<Edge>& g) {
  RollbackUF uf(n);
  vector<Node*> heap(n);
  for (Edge e : g) heap[e.b] = merge(heap[e.b], new
      Node{e});
  ll res = 0;
  vi seen(n, -1), path(n), par(n);
  seen[r] = r;
  vector<Edge> Q(n), in(n, {-1,-1}), comp;
  deque<tuple<int, int, vector<Edge>>> cycs;
  rep(s,0,n) {
    int u = s, qi = 0, w;
    while (seen[u] < 0) {
      if (!heap[u]) return {-1,{}};
      Edge e = heap[u]->top();
      heap[u]->delta -= e.w, pop(heap[u]);
      Q[qi] = e, path[qi++] = u, seen[u] = s;
      res += e.w, u = uf.find(e.a);
      if (seen[u] == s) {
        Node* cyc = 0;
        int end = qi, time = uf.time();
        do cyc = merge(cyc, heap[w = path[--qi]]);
        while (uf.join(u, w));
        u = uf.find(u), heap[u] = cyc, seen[u] = -1;
        cycs.push_front({u, time, {&Q[qi], &Q[end]}});
      }
    }
    rep(i,0,qi) in[uf.find(Q[i].b)] = Q[i];
  }
  for (auto& [u,t,comp] : cycs) { // restore sol (
      optional)
    uf.rollback(t);
    Edge inEdge = in[u];
    for (auto& e : comp) in[uf.find(e.b)] = e;
    in[uf.find(inEdge.b)] = inEdge;
  }
  rep(i,0,n) par[i] = in[i].a;
  return {res, par};
}
```

CentroidTree.h

Description: Centroid decomposition tree. Example usage can be extended for weighted trees with a BST.

Time: $\mathcal{O}\left(n \log n + q \log^2 n\right)$

"../data-structures/FenwickTree.h" fd433b, 60 lines

```
struct CT {
  vi sub, cp, d; // centroid subtree, parent, depth
  vector<vi> g, dst; // dst[depth][descendant]
  CT(vector<vi>& G) : sub(sz(G)), cp(sz(G), -2),
    d(sz(G)), g(G), dst(__lg(sz(G)) + 1, vi(sz(G))) {
    rec(0, 0);
  }
  void dfs(int u, int p) {
    sub[u] = 1;
    for (int v : g[u]) if (v != p && cp[v] == -2)
      dfs(v, u), sub[u] += sub[v];
  }
  void gen(int u, int p, int lev) {
    dst[lev][u] = dst[lev][p] + 1;
    for (int v : g[u]) if (v != p && cp[v] == -2)
      gen(v, u, lev);
  }
  int rec(int u, int dd) {
    dfs(u, -1);
    int p = -1, s = sub[u]; rep:
    for (int v : g[u])
      if (v != p && cp[v] == -2 && sub[v] > s / 2) {
        p = u, u = v; goto rep; }
    sub[u] = s, d[u] = dd, cp[u] = -1;
    for (int v : g[u]) if (cp[v] == -2)
      gen(v, u, d[u]), cp[rec(v, dd + 1)] = u;
    return u;
  }
  void path(int u, auto f) { // f(centroid, son, dist)
    for (int x = u, y = -1; x != -1; y = x, x = cp[x])
      f(x, y, dst[d[x]][u]);
  }
};
struct ContourAdd : CT {
  vector<FT> d, c;
  ContourAdd(vector<vi>&G) : CT(G),d(sz(g),FT(0)),c(d){
    rep(i, 0, sz(g)) d[i] = c[i] = FT(sub[i] + 1);
  }
  // Add x to verts whose distance from p is in [l, r)
  void add(int p, int l, int r, int x) {
    path(p, [&](int u, int v, int dd) {
      d[u].update(max(0, l - dd), x);
      if (r - dd < sub[u])
        d[u].update(max(0, r - dd), -x);
      if (v != -1) {
        c[v].update(max(0, l - dd), x);
        if (r - dd < sub[u])
          c[v].update(max(0, r - dd), -x);
      }
    });
  }
  ll get(int p) {
    ll ans = 0;
    path(p, [&](int u, int v, int dd) {
      ans += d[u].query(dd + 1);
      if (v != -1) ans -= c[v].query(dd + 1);
    });
    return ans;
  }
};
```

RerootDP.h

Description: Calculates a DP from every root in a tree. Use `dp`, `rdp` and `p` for edge dp.
Time: $\mathcal{O}\left(\sum d \log d\right)$ c0a8b6, 40 lines

```
struct S {
  void init(int u) {}
  void join(int u, int i, const S& c) {}
  void push(int u, int i) {} // i = -1 if root
};
vector<S> reroot(vector<vi>& g) {
  int n = sz(g), t = 1;
  vi q(n), p(n);
  for (int u : q) for (int v : g[u]) if (p[u] != v) {
    p[v] = u, q[t++] = v;
  }
  vector<S> dp(n), rdp(n), ans(n);
  for (int i = n - 1; i >= 0; i--) {
    int u = q[i], k = -1;
    dp[u].init(u);
    rep(j, 0, sz(g[u])) {
      if (g[u][j] != p[u]) dp[u].join(u,j,dp[g[u][j]]);
      else k = j;
    }
    ans[u] = dp[u], dp[u].push(u, k);
  }
  if (n == 1) return dp;
  for (int u : q) {
    int d = sz(g[u]); vector<S> e(d);
    rep(i, 0, d) e[i].init(u);
    for (int b = __lg(d); b >= 0; b--) {
      for (int i = d - 1; i >= 0; i--) e[i] = e[i / 2];
      rep(i, 0, d - (d & !b)) {
        S& s = g[u][i] != p[u] ? dp[g[u][i]] : rdp[u];
        e[(i >> b) ^ 1].join(u, i, s);
      }
    }
    rep(i, 0, sz(g[u])) {
      if (p[u]!=g[u][i]) (rdp[g[u][i]]=e[i]).push(u,i);
      else ans[u].join(u, i, rdp[u]);
    }
    ans[u].push(u, -1);
  }
  return ans;
}
```

7.8 Math

7.8.1 Number of Spanning Trees

Create an $N \times N$ matrix `mat`, and for each edge $a \to b \in G$, do `mat[a][b]--, mat[b][b]++` (and `mat[b][a]--, mat[a][a]++` if G is undirected). Remove the ith row and column and take the determinant; this yields the number of directed spanning trees rooted at i (if G is undirected, remove any row/column).

7.8.2 Erdős–Gallai theorem

A simple graph with node degrees $d_1 \geq \cdots \geq d_n$ exists iff $d_1 + \cdots + d_n$ is even and for every $k = 1 \ldots n$,

$$\sum_{i=1}^{k} d_i \leq k(k-1) + \sum_{i=k+1}^{n} \min(d_i, k).$$

7.8.3 Gale-Ryser theorem

A simple bipartite graph with degree sequences $a_1 \geq \cdots \geq a_n$ and $b_1, \ldots, b_m$ exists iff $\sum a_i = \sum b_i$ and for every $1 \leq k \leq n$

$$\sum_{i=1}^{k} a_i \leq \sum_{i=1}^{m} \min(b_i, k).$$

7.8.4 BEST theorem

The number of Eulerian circuits on an Eulerian graph equals

$$t(v) \prod_u (\deg(u) - 1)!$$

where $t(v)$ is the number of spanning trees directed towards an arbitrary root v, and deg(u) is the outdegree of vertex u.

Geometry (8)

8.1 Geometric primitives

Point.h

Description: Class to handle points in the plane. T can be e.g. double or long long. (Avoid int.) 3e64f3, 26 lines

```
template <class T> int sgn(T x) { return (x > 0) - (x < 0); }
template<class T>
struct Point {
  typedef Point P;
  T x, y;
  auto operator<=>(const P&) const = default;
  P operator+(P p) const { return P(x+p.x, y+p.y); }
  P operator-(P p) const { return P(x-p.x, y-p.y); }
  P operator*(T d) const { return P(x*d, y*d); }
  P operator/(T d) const { return P(x/d, y/d); }
  T dot(P p) const { return x*p.x + y*p.y; }
  T cross(P p) const { return x*p.y - y*p.x; }
  T cross(P a, P b) const { return (a-*this).cross(b-*this); }
  T dist2() const { return x*x + y*y; }
  double dist() const { return sqrt((double)dist2()); }
  // angle to x-axis in interval (-pi, pi]
  double angle() const { return atan2(y, x); }
  P unit() const { return *this/dist(); } // makes dist()=1
  P perp() const { return P(-y, x); } // rotates +90 degrees
  P normal() const { return perp().unit(); }
  // returns point rotated 'a' radians ccw around the origin
  P rotate(double a) const {
    return P(x*cos(a)-y*sin(a),x*sin(a)+y*cos(a)); }
  friend ostream& operator<<(ostream& os, P p) {
    return os << "(" << p.x << "," << p.y << ")"; }
};
```

lineDistance.h

Description: Returns the signed distance between point p and the line containing points a and b. Positive value on left side and negative on right as seen from a towards b. a==b gives nan. P is supposed to be Point<T> or Point3D<T> where T is e.g. double or long long. It uses products in intermediate steps so watch out for overflow if using int or long long. Using Point3D will always give a non-negative distance. For Point3D, call .dist on the result of the cross product.
"Point.h" f6bf6b, 4 lines

```
template<class P>
double lineDist(const P& a, const P& b, const P& p) {
  return (double)(b-a).cross(p-a)/(b-a).dist();
}
```

SegmentDistance.h

Description: Returns the shortest distance between point p and the line segment from point s to e.
Usage: `Point<double> a, b(2,2), p(1,1);`
`bool onSegment = segDist(a,b,p) < 1e-10;`
"Point.h" 5c88f4, 6 lines

```
typedef Point<double> P;
double segDist(P& s, P& e, P& p) {
  if (s==e) return (p-s).dist();
  auto d = (e-s).dist2(), t = min(d,max(.0,(p-s).dot(e-s)));
  return ((p-s)*d-(e-s)*t).dist()/d;
}
```

SegmentIntersection.h

Description: If a unique intersection point between the line segments going from s1 to e1 and from s2 to e2 exists then it is returned. If no intersection point exists an empty vector is returned. If infinitely many exist a vector with 2 elements is returned, containing the endpoints of the common line segment. The wrong position will be returned if P is Point<ll> and the intersection point does not have integer coordinates. Products of three coordinates are used in intermediate steps so watch out for overflow if using int or long long.
Usage: `vector<P> inter = segInter(s1,e1,s2,e2);`
`if (sz(inter)==1)`
`cout << "segments intersect at " << inter[0] << endl;`
"Point.h", "OnSegment.h" 9d57f2, 13 lines

```
template<class P> vector<P> segInter(P a, P b, P c, P d) {
  auto oa = c.cross(d, a), ob = c.cross(d, b),
       oc = a.cross(b, c), od = a.cross(b, d);
  // Checks if intersection is single non-endpoint point.
  if (sgn(oa) * sgn(ob) < 0 && sgn(oc) * sgn(od) < 0)
    return {(a * ob - b * oa) / (ob - oa)};
  set<P> s;
  if (onSegment(c, d, a)) s.insert(a);
  if (onSegment(c, d, b)) s.insert(b);
  if (onSegment(a, b, c)) s.insert(c);
  if (onSegment(a, b, d)) s.insert(d);
  return {all(s)};
}
```

lineIntersection.h

Description: If a unique intersection point of the lines going through s1,e1 and s2,e2 exists {1, point} is returned. If no intersection point exists {0, (0,0)} is returned and if infinitely many exists {-1, (0,0)} is returned. The wrong position will be returned if P is Point<ll> and the intersection point does not have integer coordinates. Products of three coordinates are used in intermediate steps so watch out for overflow if using int or ll.
Usage: `auto res = lineInter(s1,e1,s2,e2);`
`if (res.first == 1)`
`cout << "intersection point at " << res.second << endl;`
"Point.h" a01f81, 8 lines

```
template<class P>
pair<int, P> lineInter(P s1, P e1, P s2, P e2) {
  auto d = (e1 - s1).cross(e2 - s2);
  if (d == 0) // if parallel
    return {-(s1.cross(e1, s2) == 0), P(0, 0)};
  auto p = s2.cross(e1, e2), q = s2.cross(e2, s1);
  return {1, (s1 * p + e1 * q) / d};
}
```

sideOf.h

Description: Returns where p is as seen from s towards e. 1/0/-1 ⇔ left/on line/right. If the optional argument *eps* is given 0 is returned if p is within distance *eps* from the line. P is supposed to be Point<T> where T is e.g. double or long long. It uses products in intermediate steps so watch out for overflow if using int or long long.
Usage: `bool left = sideOf(p1,p2,q)==1;`
"Point.h" 3af81c, 9 lines

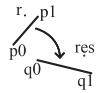
```
template<class P>
int sideOf(P s, P e, P p) { return sgn(s.cross(e, p)); }
template<class P>
int sideOf(const P& s, const P& e, const P& p, double eps) {
  auto a = (e-s).cross(p-s);
  double l = (e-s).dist()*eps;
  return (a > l) - (a < -l);
}
```

OnSegment.h

Description: Returns true iff p lies on the line segment from s to e. Use `(segDist(s,e,p)<=epsilon)` instead when using Point<double>.
"Point.h" c597e8, 3 lines

```
template<class P> bool onSegment(P s, P e, P p) {
  return p.cross(s, e) == 0 && (s - p).dot(e - p) <= 0;
}
```

linearTransformation.h



Description:
Apply the linear transformation (translation, rotation and scaling) which takes line p0-p1 to line q0-q1 to point r.
"Point.h" 03a306, 6 lines

```
typedef Point<double> P;
P linearTransformation(const P& p0, const P& p1,
    const P& q0, const P& q1, const P& r) {
  P dp = p1-p0, dq = q1-q0, num(dp.cross(dq), dp.dot(dq));
  return q0 + P((r-p0).cross(num), (r-p0).dot(num))/dp.dist2();
}
```

LineProjectionReflection.h

Description: Projects point p onto line ab. Set refl=true to get reflection of point p across line ab instead. The wrong point will be returned if P is an integer point and the desired point doesn't have integer coordinates. Products of three coordinates are used in intermediate steps so watch out for overflow.
"Point.h" b5562d, 5 lines

```
template<class P>
P lineProj(P a, P b, P p, bool refl=false) {
  P v = b - a;
  return p - v.perp()*(1+refl)*v.cross(p-a)/v.dist2();
}
```

Angle.h

Description: A class for ordering angles (as represented by int points and a number of rotations around the origin). Useful for rotational sweeping. Sometimes also represents points or vectors.
Usage: `vector<Angle> v = {w[0], w[0].t360() ...}; // sorted`
`int j = 0; rep(i,0,n) { while (v[j] < v[i].t180()) ++j; }`
`// sweeps j such that (j-i) represents the number of positively oriented triangles with vertices at 0 and i`
0f0602, 35 lines

```
struct Angle {
  int x, y;
  int t;
  Angle(int x, int y, int t=0) : x(x), y(y), t(t) {}
  Angle operator-(Angle b) const { return {x-b.x, y-b.y, t}; }
  int half() const {
    assert(x || y);
    return y < 0 || (y == 0 && x < 0);
  }
  Angle t90() const { return {-y, x, t + (half() && x >= 0)}; }
  Angle t180() const { return {-x, -y, t + half()}; }
  Angle t360() const { return {x, y, t + 1}; }
};
bool operator<(Angle a, Angle b) {
  // add a.dist2() and b.dist2() to also compare distances
  return make_tuple(a.t, a.half(), a.y * (ll)b.x) <
         make_tuple(b.t, b.half(), a.x * (ll)b.y);
}
// Given two points, this calculates the smallest angle between
// them, i.e., the angle that covers the defined line segment.
pair<Angle, Angle> segmentAngles(Angle a, Angle b) {
  if (b < a) swap(a, b);
  return (b < a.t180() ?
          make_pair(a, b) : make_pair(b, a.t360()));
}
Angle operator+(Angle a, Angle b) { // point a + vector b
  Angle r(a.x + b.x, a.y + b.y, a.t);
  if (a.t180() < r) r.t--;
  return r.t180() < a ? r.t360() : r;
}
Angle angleDiff(Angle a, Angle b) { // angle b - angle a
  int tu = b.t - a.t; a.t = b.t;
  return {a.x*b.x + a.y*b.y, a.x*b.y - a.y*b.x, tu - (b < a)};
}
```

AngleCmp.h

Description: Sorts points in ascending order by angle within the interval $(-\pi, \pi]$. The point $(0, 0)$ has an angle of 0. Equivalent to sorting by atan2(y, x).

"Point.h" 496e9b, 6 lines

```
template<class P>
bool angleCmp(P a, P b) {
  auto half = [](P p){ return sgn(p.y) ?: -sgn(p.x); };
  int A = half(a), B = half(b);
  return A == B ? a.cross(b) > 0 : A < B;
}
```

directedSegment.h

Description: Segment representation usefull for sweeping. Compares two disjoint (can touch) segments on the sweep line (OY projection). Transitivity breaks if three segments touch in one point and are on different side of the sweep line. Can be easily fixed by comparing mean X coordinates.

"Point.h" d144f8, 15 lines

```
template<class P>
struct dirSeg {
  P s, e; int rev;
  dirSeg(P _s, P _e) : s(_s), e(_e), rev(0) {
    if(e < s) swap(s, e), rev = 1;
  }
  P getY(P X) { // takes x * 2, returns y * 2 as a
       fraction
    P d = (e - s);
    return !sgn(d.x) ? P(s.y+e.y, 1) : P(d.cross(s*2-X)
         , d.x);
  }
  int cmp(dirSeg b) { // needs ~64 * M^3 !
    P X(max(s.x, b.s.x) + min(e.x, b.e.x), 0);
    return sgn(getY(X).cross(b.getY(X)));
  }
};
```

8.2 Circles

CircleIntersection.h

Description: Computes the pair of points at which two circles intersect. Returns false in case of no intersection.

"Point.h" 84d6d3, 11 lines

```
typedef Point<double> P;
bool circleInter(P a,P b,double r1,double r2,pair<P, P
    >* out) {
  if (a == b) { assert(r1 != r2); return false; }
  P vec = b - a;
  double d2 = vec.dist2(), sum = r1+r2, dif = r1-r2,
         p = (d2 + r1*r1 - r2*r2)/(d2*2), h2 = r1*r1 -
             p*p*d2;
  if (sum*sum < d2 || dif*dif > d2) return false;
  P mid = a + vec*p, per = vec.perp() * sqrt(fmax(0, h2
      ) / d2);
  *out = {mid + per, mid - per};
  return true;
}
```

CircleTangents.h

Description: Finds the external tangents of two circles, or internal if r2 is negated. Can return 0, 1, or 2 tangents – 0 if one circle contains the other (or overlaps it, in the internal case, or if the circles are the same); 1 if the circles are tangent to each other (in which case .first = .second and the tangent line is perpendicular to the line between the centers). .first and .second give the tangency points at circle 1 and 2 respectively. To find the tangents of a circle with a point set r2 to 0.

"Point.h" b0153d, 13 lines

```
template<class P>
vector<pair<P, P>> tangents(P c1, double r1, P c2,
     double r2) {
  P d = c2 - c1;
  double dr = r1 - r2, d2 = d.dist2(), h2 = d2 - dr *
      dr;
  if (d2 == 0 || h2 < 0)  return {};
  vector<pair<P, P>> out;
  for (double sign : {-1, 1}) {
    P v = (d * dr + d.perp() * sqrt(h2) * sign) / d2;
    out.push_back({c1 + v * r1, c2 + v * r2});
  }
  if (h2 == 0) out.pop_back();
  return out;
}
```

CircleLine.h

Description: Finds the intersection between a circle and a line. Returns a vector of either 0, 1, or 2 intersection points. P is intended to be Point<double>.

"Point.h" e0cfba, 9 lines

```
template<class P>
vector<P> circleLine(P c, double r, P a, P b) {
  P ab = b - a, p = a + ab * (c-a).dot(ab) / ab.dist2()
      ;
  double s = a.cross(b, c), h2 = r*r - s*s / ab.dist2()
      ;
  if (h2 < 0) return {};
  if (h2 == 0) return {p};
  P h = ab.unit() * sqrt(h2);
  return {p - h, p + h};
}
```

CirclePolygonIntersection.h

Description: Returns the area of the intersection of a circle with a ccw polygon.
Time: $\mathcal{O}(n)$

"../../content/geometry/Point.h" 19add1, 19 lines

```
typedef Point<double> P;
#define arg(p, q) atan2(p.cross(q), p.dot(q))
double circlePoly(P c, double r, vector<P> ps) {
  auto tri = [&](P p, P q) {
    auto r2 = r * r / 2;
    P d = q - p;
    auto a = d.dot(p)/d.dist2(), b = (p.dist2()-r*r)/d.
        dist2();
    auto det = a * a - b;
    if (det <= 0) return arg(p, q) * r2;
    auto s = max(0., -a-sqrt(det)), t = min(1., -a+sqrt
        (det));
    if (t < 0 || 1 <= s) return arg(p, q) * r2;
    P u = p + d * s, v = q + d * (t-1);
    return arg(p,u) * r2 + u.cross(v)/2 + arg(v,q) * r2
        ;
  };
  auto sum = 0.0;
  rep(i,0,sz(ps))
    sum += tri(ps[i] - c, ps[(i + 1) % sz(ps)] - c);
  return sum;
}
```

circumcircle.h

Description: The circumcirle of a triangle is the circle intersecting all three vertices. ccRadius returns the radius of the circle going through points A, B and C and ccCenter returns the center of the same circle.

"Point.h" 1caa3a, 9 lines

```
typedef Point<double> P;
double ccRadius(const P& A, const P& B, const P& C) {
  return (B-A).dist()*(C-B).dist()*(A-C).dist()/
      abs((B-A).cross(C-A))/2;
}
P ccCenter(const P& A, const P& B, const P& C) {
  P b = C-A, c = B-A;
  return A + (b*c.dist2()-c*b.dist2()).perp()/b.cross(c
      )/2;
}
```

MinimumEnclosingCircle.h

Description: Computes the minimum circle that encloses a set of points.
Time: expected $\mathcal{O}(n)$

"circumcircle.h" 09dd0a, 17 lines

```
pair<P, double> mec(vector<P> ps) {
  shuffle(all(ps), mt19937(time(0)));
  P o = ps[0];
  double r = 0, EPS = 1 + 1e-8;
  rep(i,0,sz(ps)) if ((o - ps[i]).dist() > r * EPS) {
    o = ps[i], r = 0;
    rep(j,0,i) if ((o - ps[j]).dist() > r * EPS) {
      o = (ps[i] + ps[j]) / 2;
      r = (o - ps[i]).dist();
      rep(k,0,j) if ((o - ps[k]).dist() > r * EPS) {
        o = ccCenter(ps[i], ps[j], ps[k]);
        r = (o - ps[i]).dist();
      }
    }
  }
  return {o, r};
}
```

CirclesUnionArea.h

Description: Returns the area of the sum of circles.
Time: $\mathcal{O}\left(n^2 \log n\right)$

"CircleIntersection.h" 2be987, 34 lines

```
template<typename T> // double or long double
T circlesArea(vector<pair<P, T>> c) {
  const T PI = acos((T)-1);
  sort(all(c)); c.erase(unique(all(c)), c.end());
  T res = 0;
  for(auto &[p, r]: c) {
    int cnt = 0, cover = 0;
    vector<pair<T, int>> eve = {{-PI, 0}};
    for(auto &[q, s]: c) if(make_pair(p, r) !=
         make_pair(q, s)) {
      T dst = (p - q).dist();
      if(r + dst <= s) { cover = 1; break; }
      pair<P, P> inters;
      if(!circleInter(p, q, r, s, &inters)) continue;
      T le = (inters.first - p).angle();
      T re = (inters.second - p).angle();
      cnt += le > re;
      eve.pb({le, 1}), eve.pb({re, -1});
    }
    if(cover) continue;
    sort(eve.begin() + 1, eve.end());
    eve.pb({PI, 0});
    T loc = 0;
    rep(i, 1, SZ(eve)) {
      if(!cnt) {
        T a = eve[i-1].first, b = eve[i].first;
        loc += r * (b - a) +
          p.cross(P(cos(b)-cos(a), sin(b)-sin(a)));
      }
      cnt += eve[i].second;
    }
    res += r * loc;
  }
  return res / 2;
}
```

8.3 Polygons

InsidePolygon.h

Description: Returns true if p lies within the polygon. If strict is true, it returns false for points on the boundary. The algorithm uses products in intermediate steps so watch out for overflow.
Usage: vector<P> v = {P{4,4}, P{1,2}, P{2,1}};
bool in = inPolygon(v, P{3, 3}, false);
Time: $\mathcal{O}(n)$

"Point.h", "OnSegment.h", "SegmentDistance.h" 2bf504, 11 lines

```
template<class P>
bool inPolygon(vector<P> &p, P a, bool strict = true) {
  int cnt = 0, n = sz(p);
  rep(i,0,n) {
    P q = p[(i + 1) % n];
    if (onSegment(p[i], q, a)) return !strict;
    //or: if (segDist(p[i], q, a) <= eps) return !
         strict;
    cnt ^= ((a.y<p[i].y) - (a.y<q.y)) * a.cross(p[i], q
         ) > 0;
  }
  return cnt;
}
```

PolygonArea.h

Description: Returns twice the signed area of a polygon. Clockwise enumeration gives negative area. Watch out for overflow if using int as T!

"Point.h" f12300, 6 lines

```
template<class T>
T polygonArea2(vector<Point<T>>& v) {
  T a = v.back().cross(v[0]);
  rep(i,0,sz(v)-1) a += v[i].cross(v[i+1]);
  return a;
}
```

PolygonCenter.h

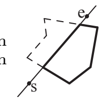
Description: Returns the center of mass for a polygon.
Time: $\mathcal{O}(n)$

"Point.h" 9706dc, 9 lines

```
typedef Point<double> P;
P polygonCenter(const vector<P>& v) {
  P res(0, 0); double A = 0;
  for (int i = 0, j = sz(v) - 1; i < sz(v); j = i++) {
    res = res + (v[i] + v[j]) * v[j].cross(v[i]);
    A += v[j].cross(v[i]);
  }
  return res / A / 3;
}
```

PolygonCut.h

Description:
Returns a vector with the vertices of a polygon with everything to the left of the line going from s to e cut away.



Usage: vector<P> p = ...;
p = polygonCut(p, P(0,0), P(1,0));

"Point.h" d07181, 13 lines

```
typedef Point<double> P;
vector<P> polygonCut(const vector<P>& poly, P s, P e) {
  vector<P> res;
  rep(i,0,sz(poly)) {
    P cur = poly[i], prev = i ? poly[i-1] : poly.back()
        ;
    auto a = s.cross(e, cur), b = s.cross(e, prev);
    if ((a < 0) != (b < 0))
      res.push_back(cur + (prev - cur) * (a / (a - b)))
          ;
    if (a < 0)
      res.push_back(cur);
  }
  return res;
}
```

PolygonUnion.h

Description: Calculates the area of the union of n polygons (not necessarily convex). The points within each polygon must be given in CCW order. (Epsilon checks may optionally be added to sideOf/sgn, but shouldn't be needed.)
Time: $\mathcal{O}\left(N^2\right)$, where N is the total number of points

"Point.h", "sideOf.h" 3931c6, 33 lines

```
typedef Point<double> P;
double rat(P a, P b) { return sgn(b.x) ? a.x/b.x : a.y/
     b.y; }
double polyUnion(vector<vector<P>>& poly) {
  double ret = 0;
  rep(i,0,sz(poly)) rep(v,0,sz(poly[i])) {
    P A = poly[i][v], B = poly[i][(v + 1) % sz(poly[i])
        ];
    vector<pair<double, int>> segs = {{0, 0}, {1, 0}};
    rep(j,0,sz(poly)) if (i != j) {
      rep(u,0,sz(poly[j])) {
        P C = poly[j][u], D = poly[j][(u + 1) % sz(poly
             [j])];
        int sc = sideOf(A, B, C), sd = sideOf(A, B, D);
        if (sc != sd) {
          double sa = C.cross(D, A), sb = C.cross(D, B)
              ;
          if (min(sc, sd) < 0)
            segs.emplace_back(sa / (sa - sb), sgn(sc -
                 sd));
        } else if (!sc && !sd && j<i && sgn((B-A).dot(D
             -C))>0){
          segs.emplace_back(rat(C - A, B - A), 1);
          segs.emplace_back(rat(D - A, B - A), -1);
        }
      }
    }
    sort(all(segs));
    for (auto& s : segs) s.first = min(max(s.first,
         0.0), 1.0);
    double sum = 0;
    int cnt = segs[0].second;
    rep(j,1,sz(segs)) {
      if (!cnt) sum += segs[j].first - segs[j - 1].
           first;
      cnt += segs[j].second;
    }
    ret += A.cross(B) * sum;
  }
  return ret / 2;
}
```

PolygonTangents.h

Description: Polygon tangents from a given point. The polygon must be ccw and have no collinear points. Returns a pair of indices of the given polygon. Should work for a point on border (for a point being polygon vertex returns previous and next one).
Time: $\mathcal{O}(\log n)$

"Point.h" 096fab, 21 lines

```
#define pdir(i) (ph ? p - poly[(i)%n] : poly[(i)%n] - p)
#define cmp(i,j) sgn(pdir(i).cross(poly[(i)%n]-poly[(j)%n]))
#define extr(i) cmp(i + 1, i) >= 0 && cmp(i, i - 1 + n) < 0
template <class P>
array<int, 2> polygonTangents(vector<P>& poly, P p) {
  auto bs = [&](int ph) {
    int n = sz(poly), lo = 0, hi = n;
    if(extr(0)) return 0;
    while(lo + 1 < hi) {
      int m = (lo + hi) / 2;
      if(extr(m)) return m;
      int ls = cmp(lo + 1, lo), ms = cmp(m + 1, m);
      (ls < ms || (ls == ms && ls == cmp(lo, m)) ? hi: lo) = m;
    }
    return lo;
  };
  array<int, 2> res = {bs(0), bs(1)};
  if(res[0] == res[1]) res[0] = (res[0] + 1) % sz(poly);
  if(poly[res[0]] == p) res[0] = (res[0] + 1) % sz(poly);
  return res;
}
```

ConvexHull.h

Description: Returns a vector of the points of the convex hull in counter-clockwise order. Points on the edge of the hull between two other points are not considered part of the hull.
Time: $\mathcal{O}(n \log n)$

"Point.h" 310954, 13 lines

```
typedef Point<ll> P;
vector<P> convexHull(vector<P> pts) {
  if (sz(pts) <= 1) return pts;
  sort(all(pts));
  vector<P> h(sz(pts)+1);
  int s = 0, t = 0;
  for (int it = 2; it--; s = --t, reverse(all(pts)))
    for (P p : pts) {
      while (t >= s + 2 && h[t-2].cross(h[t-1], p) <= 0) t--;
      h[t++] = p;
    }
  return {h.begin(), h.begin() + t - (t == 2 && h[0] == h[1])};
}
```

ConvexHullOnline.h

Description: Allows online point insertion. If exists, left vertical segment is included; right one is excluded. To get a lower hull add (-x, -y) instead of (x, y).
Time: amortized $\mathcal{O}(\log n)$ per add

"Point.h" 10c55b, 16 lines

```
using P = Point<ll>;
struct UpperHull : set<P> {
  bool rm(auto it) {
    if (it==begin() || it==end() || next(it)==end() ||
        it->cross(*prev(it), *next(it)) > 0)
      return false;
    erase(it); return true;
  }
  bool add(P p) { // true iff added
    auto [it, ok] = emplace(p);
    if (!ok || rm(it)) return false;
    while (rm(next(it)));
    while (it != begin() && rm(prev(it)));
    return true;
  }
};
```

ConvexHullOnline2.h

Description: Fully dynamic upper / lower convex hull, can be used for computing onion layers. All points should be known in advance. Points on the edges are included in the hull. Return indices are the same as in the input.
Time: $\mathcal{O}\left(\log^2 n\right)$, as fast as other $\mathcal{O}(\log n)$ hulls

"Point.h" 78db53, 70 lines

```
template<class T>
struct DynHull {
  using P = Point<T>;
  struct Node { int l, r; }; vector<P> ps;
  int n; vi in, id; int s; vector<Node> t; vector<T> m;
  DynHull(vector<P> _ps, bool lower = 0, int start = -1)
    : ps(_ps), n(sz(ps)), in(n), id(n) {
    if(start == -1) start = n;
    s = 1; while(s < n) s *= 2;
    t.resize(s * 2, {-1, -1}); m.resize(s);
    vector<pair<P, int>> pts;
    rep(i, 0, n) pts.push_back({ps[i] * (lower ? -1 : 1), i});
    sort(all(pts));
    rep(i, 0, n) {
      tie(ps[i], id[i]) = pts[i]; in[id[i]] = i;
      int p = i + s; while((p & 1) ^ 1) p >>= 1;
      m[p >> 1] = ps[i].x;
    }
    rep(i, 0, start) t[s + in[i]] = {in[i], in[i]};
    for (int i = s - 1; i >= 1; i --) pull(i);
  }
  int go(int v) {
    while(t[v].l < 0) v = v * 2 + t[v].l + 3;
    return v;
  }
  void pull(int v) {
    auto crossNegX = [](P a, P b, P c, P d, T x) {
      // change __int128 if using doubles!
      __int128 p = a.cross(b, c), q = b.cross(a, d);
      return p + q == 0 || (d.x - x) * p + (c.x - x) * q <= 0;
    };
    int p = v * 2, q = p + 1;
    if(t[p].l == -1 && t[q].l == -1) t[v] = {-1, -1};
    else if(t[p].l == -1) t[v] = {-2, -2};
    else if(t[q].l == -1) t[v] = {-3, -3};
    else {
      p = go(p), q = go(q);
      while(p < s || q < s) {
        auto [a, b] = t[p]; auto [c, d] = t[q];
        if(a != b && ps[a].cross(ps[b], ps[c]) > 0) {
          p = go(p * 2); }
        else if(c != d && ps[b].cross(ps[c], ps[d]) > 0) {
          q = go(q * 2 + 1); }
        else if(a == b) q = go(q * 2);
        else if(c == d ||
          crossNegX(ps[a], ps[b], ps[c], ps[d], m[v])) {
          p = go(p * 2 + 1); }
        else q = go(q * 2);
      }
      t[v] = {p - s, q - s};
    }
  }
  void add(int i) {
    i = in[i]; int v = i + s; t[v] = {i, i};
    while(v >>= 1) pull(v);
  }
  void del(int i) {
    i = in[i]; int v = i + s; t[v] = {-1, -1};
    while(v >>= 1) {
      if(t[v].l < 0 || t[v].l == i || t[v].r == i) pull(v); }
  }
  void dfs(int v, int l, int r, vi &h) {
    if(v >= s) return h.push_back(id[t[v].l]);
    if(l <= t[v].l) dfs(go(v * 2), l, min(t[v].l, r), h);
    if(t[v].r <= r) dfs(go(v * 2 + 1), max(t[v].r, l), r,  h);
  }
  vi hull() {
    vi h; if(~t[1].l) dfs(go(1), 0, n - 1, h); return h;
  }
};
```

HullDiameter.h

Description: Returns the two points with max distance on a convex hull (ccw, no duplicate/collinear points).
Time: $\mathcal{O}(n)$

"Point.h" c571b8, 12 lines

```
typedef Point<ll> P;
array<P, 2> hullDiameter(vector<P> S) {
  int n = sz(S), j = n < 2 ? 0 : 1;
  pair<ll, array<P, 2>> res({0, {S[0], S[0]}});
  rep(i,0,j)
    for (;; j = (j + 1) % n) {
      res = max(res, {(S[i] - S[j]).dist2(), {S[i], S[j]}});
      if ((S[(j + 1) % n] - S[j]).cross(S[i + 1] - S[i]) >= 0)
        break;
    }
  return res.second;
}
```

PointInsideHull.h

Description: Determine whether a point t lies inside a convex hull (CCW order, with no collinear points). Returns true if point lies within the hull. If strict is true, points on the boundary aren't included.
Time: $\mathcal{O}(\log N)$

"Point.h", "sideOf.h", "OnSegment.h" 71446b, 14 lines

```
typedef Point<ll> P;
bool inHull(const vector<P>& l, P p, bool strict = true) {
  int a = 1, b = sz(l) - 1, r = !strict;
  if (sz(l) < 3) return r && onSegment(l[0], l.back(), p);
  if (sideOf(l[0], l[a], l[b]) > 0) swap(a, b);
  if (sideOf(l[0], l[a], p) >= r || sideOf(l[0], l[b], p)<= -r)
    return false;
  while (abs(a - b) > 1) {
    int c = (a + b) / 2;
    (sideOf(l[0], l[c], p) > 0 ? b : a) = c;
  }
  return sgn(l[a].cross(l[b], p)) < r;
}
```

LineHullIntersection.h

Description: Line-convex polygon intersection. The polygon must be ccw and have no collinear points. lineHull(line, poly) returns a pair describing the intersection of a line with the polygon: • $(-1,-1)$ if no collision, • $(i,-1)$ if touching the corner i, • (i,i) if along side $(i,i+1)$, • (i,j) if crossing sides $(i,i+1)$ and $(j,j+1)$. In the last case, if a corner i is crossed, this is treated as happening on side $(i,i+1)$. The points are returned in the same order as the line hits the polygon. extrVertex returns the point of a hull with the max projection onto a line.
Time: $\mathcal{O}(\log n)$

"Point.h" 7cf45b, 39 lines

```
#define cmp(i,j) sgn(dir.perp().cross(poly[(i)%n]-poly[(j)%n]))
#define extr(i) cmp(i + 1, i) >= 0 && cmp(i, i - 1 + n) < 0
template <class P> int extrVertex(vector<P>& poly, P dir) {
  int n = sz(poly), lo = 0, hi = n;
  if (extr(0)) return 0;
  while (lo + 1 < hi) {
    int m = (lo + hi) / 2;
    if (extr(m)) return m;
    int ls = cmp(lo + 1, lo), ms = cmp(m + 1, m);
    (ls < ms || (ls == ms && ls == cmp(lo, m)) ? hi : lo) = m;
  }
  return lo;
}
#define cmpL(i) sgn(a.cross(poly[i], b))
template <class P>
array<int, 2> lineHull(P a, P b, vector<P>& poly) {
  int endA = extrVertex(poly, (a - b).perp());
  int endB = extrVertex(poly, (b - a).perp());
  if (cmpL(endA) < 0 || cmpL(endB) > 0)
    return {-1, -1};
  array<int, 2> res;
  rep(i,0,2) {
    int lo = endB, hi = endA, n = sz(poly);
    while ((lo + 1) % n != hi) {
      int m = ((lo + hi + (lo < hi ? 0 : n)) / 2) % n;
      (cmpL(m) == cmpL(endB) ? lo : hi) = m;
    }
    res[i] = (lo + !cmpL(hi)) % n;
    swap(endA, endB);
  }
  if (res[0] == res[1]) return {res[0], -1};
  if (!cmpL(res[0]) && !cmpL(res[1]))
    switch ((res[0] - res[1] + sz(poly) + 1) % sz(poly)) {
      case 0: return {res[0], res[0]};
      case 2: return {res[1], res[1]};
    }
  return res;
}
```

Minkowski.h

Description: Computes Minkowski sum of two convex polygons in ccw order. Vertices are required to be in ccw order.
Time: $\mathcal{O}(n+m)$

"Point.h", "Angle.h" 7d8a28, 18 lines

```
P edgeSeq(vector<P> p, vector<P>& edges) {
  int i = 0, n = sz(p);
  rep(j, 0, n) if (tie(p[i].y, p[i].x) > tie(p[j].y, p[j].x)) i = j;
  rep(j, 0, n) edges.push_back(p[(i+j+1)%n] - p[(i+j)%n]);
  return p[i];
}
vector<P> hullSum(vector<P> A, vector<P> B) {
  vector<P> sum, e1, e2, es(sz(A) + sz(B));
  P pivot = edgeSeq(A, e1) + edgeSeq(B, e2);
  merge(all(e1), all(e2), es.begin(), [&](P a, P b){
    return Angle(a.x, a.y) < Angle(b.x,b.y);
  });
  sum.push_back(pivot);
  for(auto e: es) sum.push_back(sum.back() + e);
  sum.pop_back();
  return sum; //can have collinear vertices!
}
```

HalfplaneIntersection.h

Description: Online half plane intersection. Works both for ll and long double. Bounding box is optional, but needed for distinguishing bounded vs unbounded. Halfplanes are sorted ccw in HPI.s. Time: O(log n) per add.

"Point.h" 5e6600, 98 lines

```
using T = ll; // has to fit 2*|pts|**2
using P = Point<T>; // only cross needed
using SuperT = __int128_t; // has to fit 6*|pts|**3
const SuperT EPS = 1e-12; // |pts| <= 10^6 (for T=dbl)
struct Line {
  T a,b,c;
  Line(T a_=0, T b_=0, T c_=0): a(a_), b(b_), c(c_) {}
      //ax + by + c >= 0 (coords <= 10^9)
  Line(P p, P q): a(p.y-q.y), b(q.x-p.x), c(p.cross(q)) {} //p->q ccw (coords <= 10^6)
  Line operator- () const {return Line(-a, -b, -c); }
  bool up() const { return a?(a<0):(b>0);}
  P v() const {return P(a,b);}
  P vx() {return P(b,c);} P vy() {return P(a,c);}
  T wek(Line p) const {return v().cross(p.v());}
  bool operator<(Line b) const {
    if (up() != b.up()) return up() > b.up();
    return wek(b) > 0;
  }
};
bool parallel(Line a, Line b) {return !a.wek(b);}
bool same(Line a, Line b) {
  return parallel(a,b) && !a.vy().cross(b.vy()) && !a.vx().cross(b.vx());
}
T weaker (Line a, Line b) {
  if (abs(a.a) > abs(a.b)) return a.c*abs(b.a) - b.c*abs(a.a);
  return a.c*abs(b.b) - b.c*abs(a.b);
}
array<SuperT, 3> intersect(Line a, Line b) {
  SuperT det = a.wek(b);
  SuperT x = a.vx().cross(b.vx());
  SuperT y = a.vy().cross(b.vy());
  // if (T=dbl) return {x / det, -y / det, 1.0};
  if (det > 0) return {x, -y, det};
  return {-x, y, -det};
}
struct HPI {
  bool empty=0, pek=0;
  set<Line> s;
  typedef set<Line>::iterator iter;
  iter next(iter it){return ++it == s.end() ? s.begin() : it;}
  iter prev(iter it){return it == s.begin() ? --s.end() : --it;}
  bool hide(Line a, Line b, Line c) { // do a,b hide c?
    if (parallel(a,b)) {
      if (weaker(a, -b) < 0) empty = 1;
```

```
      return 0;
    }
    if (a.wek(b) < 0) swap(a,b);
    auto [rx, ry, rdet] = intersect(a,b);
    auto v = rx*c.a + ry*c.b + rdet*c.c;
    if (a.wek(c) >=0  && c.wek(b) >=0 && v >= -EPS)
        return 1;
    if (a.wek(c) < 0  && c.wek(b) < 0) {
      if (v < -EPS) empty = 1;
      else if (v <= EPS) pek = 1;
    }
    return 0;
  }
  void delAndMove(iter& i, int nxt) {
    iter j = i;
    if(nxt==1) i = next(i);
    else i = prev(i);
    s.erase(j);
  }
  void add(Line l) {
    if (empty) return;
    if (l.a == 0 && l.b == 0) {
      if (l.c < 0) empty = 1;
      return;
    }
    iter it = s.lower_bound(l);  //parallel
    if(it != s.end() && parallel(*it, l) && it->up() == l.up()) {
      if (weaker(l, *it)>=0) return;
      delAndMove(it,1);
    }
    if(it == s.end()) it = s.begin(); //*it>p
    while(sz(s) >= 2 && hide(l, *next(it), *it))
      delAndMove(it,1);
    if(sz(s)) it = prev(it); //*it<p
    while(sz(s) >= 2 && hide(l, *prev(it), *it))
      delAndMove(it,0);
    if(sz(s) < 2 || !hide(*it, *next(it), l)) s.insert(l);
  }
  int type() { // 0=empty, 1=point, 2=segment,
    if(empty) return 0; // 3=halfline, 4=line,
    if(sz(s) <= 4) { // 5=polygon or unbounded
      vector<Line> r(all(s));
      if(sz(r) == 2 && parallel(r[0], r[1]) && weaker(r[0],-r[1])<0)
        return 0;
      rep(i, 0, sz(r)) rep(j, 0, i) if(same(r[i], r[j])) {
        if(sz(r) == 2) return 4;
        if(sz(r) == 3) return 3;
        if(sz(r) == 4 && same(r[0], r[2]) && same(r[1], r[3])) return 1;
        return 2;
      }
      if(sz(r) == 3 && pek) return 1;
    }
    return 5;
  }
};
```

PointLocation.h

Description: Computes (not necessarily convex) polygon tree structure. Also for each query point computes its location (including boundaries).
Time: $\mathcal{O}(n \log n)$

"directedSegment.h" 0f077f, 47 lines

```
template<class P>
pair<vi,vi> pointLoc(vector<vector<P>> polys, vector<P> pts) {
  vector<tuple<P, int, int>> eve; // {point, event_type, id}
  vector<pair<dirSeg<P>, int>> segs;  // {s, e, poly_id}
  rep(i, 0, sz(polys)) rep(j, 0, sz(polys[i])) {
    dirSeg<P> seg(polys[i][j], polys[i][(j+1)%sz(polys[i])]);
    eve.push_back({seg.s,0,sz(segs)}), eve.push_back({seg.e,2,sz(segs)});
    segs.push_back({seg, i});
  }
  rep(i, 0, sz(pts)) eve.push_back({pts[i], 1, i});
  sort(all(eve));
  vi par(sz(polys), -2), ans(sz(pts), -1);
  auto cmp = [](auto a, auto b) {
    return make_pair(a.first.cmp(b.first), a.second) < make_pair(0, b.second);
  };
  set<pair<dirSeg<P>, int>, decltype(cmp)> s(cmp);
  for(auto &[_, eve_tp, id]: eve) {
    if(eve_tp == 1) { // point query
      P p = pts[id];
      auto it = s.lower_bound({dirSeg(p, p), 0});
      if(it != s.begin()) { // on vertical segment?
        auto prv = prev(it);
        if(!sgn(p.cross(prv->first.s, prv->first.e)))
          it--;
      }
      if(it == s.end()) ans[id] = -1;
      else {
        auto [seg, seg_id] = *it;
        int poly_id = segs[seg_id].second; // strictness there!
        ans[id] = !seg.rev && sgn(p.cross(seg.s, seg.e))
          ? par[poly_id] : poly_id;
      }
    }
    if(eve_tp == 0) { // add segment
      auto it = next(s.insert({segs[id].first, id}).first);
      int poly_id = segs[id].second;
      if(par[poly_id] == -2) {
        if(it == s.end()) par[poly_id] = -1;
        else {
          int up_rev = it->first.rev, up_id = segs[it->second].second;
          par[poly_id] = !up_rev ? par[up_id] : up_id;
        }
      }
    }
    if(eve_tp == 2) s.erase({segs[id].first, id});  // del segment
  }
  return {par, ans};
}
```

8.4 Misc. Point Set Problems

ClosestPair.h

Description: Finds the closest pair of points.
Time: $\mathcal{O}(n \log n)$

"Point.h" ac41a6, 17 lines

```
typedef Point<ll> P;
pair<P, P> closest(vector<P> v) {
  assert(sz(v) > 1);
  set<P> S;
  sort(all(v), [](P a, P b) { return a.y < b.y; });
  pair<ll, pair<P, P>> ret{LLONG_MAX, {P(), P()}};
  int j = 0;
  for (P p : v) {
    P d{1 + (ll)sqrt(ret.first), 0};
    while (v[j].y <= p.y - d.x) S.erase(v[j++]);
    auto lo = S.lower_bound(p - d), hi = S.upper_bound(p + d);
    for (; lo != hi; ++lo)
      ret = min(ret, {(*lo - p).dist2(), {*lo, p}});
    S.insert(p);
  }
  return ret.second;
}
```

ManhattanMST.h

Description: Given N points, returns up to 4*N edges, which are guaranteed to contain a minimum spanning tree for the graph with edge weights w(p, q) = —p.x - q.x— + —p.y - q.y—. Edges are in the form (distance, src, dst). Use a standard MST algorithm on the result to find the final MST.
Time: $\mathcal{O}(N \log N)$

"Point.h" df6f59, 23 lines

```
typedef Point<int> P;
vector<array<int, 3>> manhattanMST(vector<P> ps) {
  vi id(sz(ps));
  iota(all(id), 0);
  vector<array<int, 3>> edges;
  rep(k,0,4) {
    sort(all(id), [&](int i, int j) {
         return (ps[i]-ps[j]).x < (ps[j]-ps[i]).y;});
    map<int, int> sweep;
    for (int i : id) {
      for (auto it = sweep.lower_bound(-ps[i].y);
                it != sweep.end(); sweep.erase(it++)) {
        int j = it->second;
        P d = ps[i] - ps[j];
        if (d.y > d.x) break;
        edges.push_back({d.y + d.x, i, j});
      }
      sweep[-ps[i].y] = i;
    }
    for (P& p : ps) if (k & 1) p.x = -p.x; else swap(p.x, p.y);
  }
  return edges;
}
```

kdTree.h

Description: KD-tree (2d, can be extended to 3d)

"Point.h" bac5b0, 63 lines

```
typedef long long T;
typedef Point<T> P;
const T INF = numeric_limits<T>::max();
bool on_x(const P& a, const P& b) { return a.x < b.x; }
bool on_y(const P& a, const P& b) { return a.y < b.y; }
struct Node {
  P pt; // if this is a leaf, the single point in it
  T x0 = INF, x1 = -INF, y0 = INF, y1 = -INF; // bounds
  Node *first = 0, *second = 0;
  T distance(const P& p) { // min squared distance to a point
    T x = (p.x < x0 ? x0 : p.x > x1 ? x1 : p.x);
    T y = (p.y < y0 ? y0 : p.y > y1 ? y1 : p.y);
    return (P(x,y) - p).dist2();
  }
  Node(vector<P>&& vp) : pt(vp[0]) {
    for (P p : vp) {
      x0 = min(x0, p.x); x1 = max(x1, p.x);
      y0 = min(y0, p.y); y1 = max(y1, p.y);
    }
    if (vp.size() > 1) {
      // split on x if width >= height (not ideal...)
      sort(all(vp), x1 - x0 >= y1 - y0 ? on_x : on_y);
      // divide by taking half the array for each child (not
      // best performance with many duplicates in the middle)
      int half = sz(vp)/2;
      first = new Node({vp.begin(), vp.begin() + half});
      second = new Node({vp.begin() + half, vp.end()});
    }
  }
};
struct KDTree {
  Node* root;
  KDTree(const vector<P>& vp) : root(new Node({all(vp)})) {}
  pair<T, P> search(Node *node, const P& p) {
    if (!node->first) {
      // uncomment if we should not find the point itself:
      // if (p == node->pt) return {INF, P()};
      return make_pair((p - node->pt).dist2(), node->pt);
    }
    Node *f = node->first, *s = node->second;
    T bfirst = f->distance(p), bsec = s->distance(p);
    if (bfirst > bsec) swap(bsec, bfirst), swap(f, s);
    // search closest side first, other side if needed
    auto best = search(f, p);
    if (bsec < best.first)
      best = min(best, search(s, p));
    return best;
  }
  // find nearest point to a point, and its squared distance
  // (requires an arbitrary operator< for Point)
  pair<T, P> nearest(const P& p) {
    return search(root, p);
  }
};
```

DelaunayTriangulation.h

Description: Computes the Delaunay triangulation of a set of points. Each circumcircle contains none of the input points. If any three points are collinear or any four are on the same circle, behavior is undefined.
Time: $\mathcal{O}\left(n^2\right)$

"Point.h", "3dHull.h" c0e7bc, 10 lines

```
template<class P, class F>
void delaunay(vector<P>& ps, F trifun) {
  if (sz(ps) == 3) { int d = (ps[0].cross(ps[1], ps[2]) < 0);
    trifun(0,1+d,2-d); }
  vector<P3> p3;
  for (P p : ps) p3.emplace_back(p.x, p.y, p.dist2());
  if (sz(ps) > 3) for(auto t:hull3d(p3)) if ((p3[t.b]-p3[t.a]).
      cross(p3[t.c]-p3[t.a]).dot(P3(0,0,1)) < 0)
    trifun(t.a, t.c, t.b);
}
```

FastDelaunay.h

Description: Fast Delaunay triangulation. Each circumcircle contains none of the input points. There must be no duplicate points. If all points are on a line, no triangles will be returned. Should work for doubles as well, though there may be precision issues in 'circ'. Returns triangles in order {t[0][0], t[0][1], t[0][2], t[1][0], …}, all counter-clockwise.
Time: $\mathcal{O}(n \log n)$

"Point.h" eefdf5, 88 lines

```
typedef Point<ll> P;
typedef struct Quad* Q;
typedef __int128_t lll; // (can be ll if coords are < 2e4)
P arb(LLONG_MAX,LLONG_MAX); // not equal to any other point

struct Quad {
  Q rot, o; P p = arb; bool mark;
  P& F() { return r()->p; }
  Q& r() { return rot->rot; }
  Q prev() { return rot->o->rot; }
  Q next() { return r()->prev(); }
} *H;

bool circ(P p, P a, P b, P c) { // is p in the circumcircle?
  lll p2 = p.dist2(), A = a.dist2()-p2,
      B = b.dist2()-p2, C = c.dist2()-p2;
  return p.cross(a,b)*C + p.cross(b,c)*A + p.cross(c,a)*B > 0;
}
Q makeEdge(P orig, P dest) {
  Q r = H ? H : new Quad{new Quad{new Quad{new Quad{0}}}};
  H = r->o; r->r()->r() = r;
  rep(i,0,4) r = r->rot, r->p = arb, r->o = i & 1 ? r : r->r();
  r->p = orig; r->F() = dest;
  return r;
}
void splice(Q a, Q b) {
  swap(a->o->rot->o, b->o->rot->o); swap(a->o, b->o);
}
Q connect(Q a, Q b) {
  Q q = makeEdge(a->F(), b->p);
  splice(q, a->next());
  splice(q->r(), b);
  return q;
}

pair<Q,Q> rec(const vector<P>& s) {
  if (sz(s) <= 3) {
    Q a = makeEdge(s[0], s[1]), b = makeEdge(s[1], s.back());
    if (sz(s) == 2) return { a, a->r() };
    splice(a->r(), b);
    auto side = s[0].cross(s[1], s[2]);
    Q c = side ? connect(b, a) : 0;
    return {side < 0 ? c->r() : a, side < 0 ? c : b->r() };
  }

#define H(e) e->F(), e->p
#define valid(e) (e->F().cross(H(base)) > 0)
  Q A, B, ra, rb;
  int half = sz(s) / 2;
  tie(ra, A) = rec({all(s) - half});
  tie(B, rb) = rec({sz(s) - half + all(s)});
  while ((B->p.cross(H(A)) < 0 && (A = A->next())) ||
         (A->p.cross(H(B)) > 0 && (B = B->r()->o)));
  Q base = connect(B->r(), A);
  if (A->p == ra->p) ra = base->r();
  if (B->p == rb->p) rb = base;

#define DEL(e, init, dir) Q e = init->dir; if (valid(e)) \
    while (circ(e->dir->F(), H(base), e->F())) { \
      Q t = e->dir; \
      splice(e, e->prev()); \
      splice(e->r(), e->r()->prev()); \
      e->o = H; H = e; e = t; \
    }
  for (;;) {
    DEL(LC, base->r(), o);  DEL(RC, base, prev());
    if (!valid(LC) && !valid(RC)) break;
    if (!valid(LC) || (valid(RC) && circ(H(RC), H(LC))))
      base = connect(RC, base->r());
    else
```

```
      base = connect(base->r(), LC->r());
  }
  return { ra, rb };
}
vector<P> triangulate(vector<P> pts) {
  sort(all(pts));  assert(unique(all(pts)) == pts.end()
      );
  if (sz(pts) < 2) return {};
  Q e = rec(pts).first;
  vector<Q> q = {e};
  int qi = 0;
  while (e->o->F().cross(e->F(), e->p) < 0) e = e->o;
#define ADD { Q c = e; do { c->mark = 1; pts.push_back(
    c->p); \
  q.push_back(c->r()); c = c->next(); } while (c != e);
      }
  ADD; pts.clear();
  while (qi < sz(q)) if (!(e = q[qi++])->mark) ADD;
  return pts;
}
```

SegmentsIntersection.h

Description: Finds one of segments intersections. You should change dirSeg's comparator, to compare segments at their left end.
Time: $\mathcal{O}(N \log N)$

"SegmentIntersection.h", "directedSegment.h" 85935a, 34 lines

```
template<class P>
pii segmentsIntersect(vector<pair<P, P>> segments) {
  vector<tuple<P, int, int>> eve; // {point, event_type
      , id}
  vector<dirSeg<P>> segs;
  for(auto &[s, e]: segments) {
    dirSeg<P> seg(s, e);
    eve.push_back({seg.s,0,sz(segs)});
    eve.push_back({seg.e,1,sz(segs)});
    segs.push_back(seg);
  }
  sort(all(eve));
  auto inter = [](auto a, auto b) {
    return sz(segInter(a->first.s, a->first.e,b->first.
        s, b->first.e));
  };
  auto cmp = [](auto a, auto b) {
    return make_pair(a.first.cmp(b.first),a.second)<
        make_pair(0, b.second);
  };
  set<pair<dirSeg<P>, int>, decltype(cmp)> s(cmp);
  for(auto &[_, eve_tp, id]: eve) {
    if(eve_tp == 0) { // add segment
      auto it = s.insert({segs[id], id}).first;
      if(next(it) != s.end() && inter(it, next(it)))
        return {it->second, next(it)->second};
      if(it != s.begin() && inter(it, prev(it)))
        return {it->second, prev(it)->second};
    }
    if(eve_tp == 1) { // del segment
      auto it = s.erase(s.find({segs[id], id}));
      if(it!=s.begin() && it!=s.end() && inter(it, prev
          (it)))
        return {it->second, prev(it)->second};
    }
  }
  return {-1, -1};
}
```

8.5 3D

PolyhedronVolume.h

Description: Magic formula for the volume of a polyhedron. Faces should point outwards. 3058c3, 6 lines

```
template<class V, class L>
double signedPolyVolume(const V& p, const L& trilist) {
  double v = 0;
  for (auto i : trilist) v += p[i.a].cross(p[i.b]).dot(
      p[i.c]);
  return v / 6;
}
```

Point3D.h

Description: Class to handle points in 3D space. T can be e.g. double or long long. 8058ae, 32 lines

```
template<class T> struct Point3D {
  typedef Point3D P;
  typedef const P& R;
  T x, y, z;
  explicit Point3D(T x=0, T y=0, T z=0) : x(x), y(y), z
      (z) {}
  bool operator<(R p) const {
    return tie(x, y, z) < tie(p.x, p.y, p.z); }
  bool operator==(R p) const {
    return tie(x, y, z) == tie(p.x, p.y, p.z); }
  P operator+(R p) const { return P(x+p.x, y+p.y, z+p.z
      ); }
  P operator-(R p) const { return P(x-p.x, y-p.y, z-p.z
      ); }
  P operator*(T d) const { return P(x*d, y*d, z*d); }
  P operator/(T d) const { return P(x/d, y/d, z/d); }
  T dot(R p) const { return x*p.x + y*p.y + z*p.z; }
  P cross(R p) const {
    return P(y*p.z - z*p.y, z*p.x - x*p.z, x*p.y - y*p.
        x);
  }
  T dist2() const { return x*x + y*y + z*z; }
  double dist() const { return sqrt((double)dist2()); }
  //Azimuthal angle (longitude) to x-axis in interval
      [-pi, pi]
  double phi() const { return atan2(y, x); }
  //Zenith angle (latitude) to the z-axis in interval
      [0, pi]
  double theta() const { return atan2(sqrt(x*x+y*y),z);
      }
  P unit() const { return *this/(T)dist(); } //makes
      dist()=1
  //returns unit vector normal to *this and p
  P normal(P p) const { return cross(p).unit(); }
  //returns point rotated 'angle' radians ccw around
      axis
  P rotate(double angle, P axis) const {
    double s = sin(angle), c = cos(angle); P u = axis.
        unit();
    return u*dot(u)*(1-c) + (*this)*c - cross(u)*s;
  }
};
```

3dHull.h

Description: Computes all faces of the 3-dimension hull of a point set. *No four points must be coplanar*, or else random results will be returned. All faces will point outwards.
Time: $\mathcal{O}\left(n^2\right)$

"Point3D.h" 5b45fc, 49 lines

```
typedef Point3D<double> P3;
struct PR {
  void ins(int x) { (a == -1 ? a : b) = x; }
  void rem(int x) { (a == x ? a : b) = -1; }
  int cnt() { return (a != -1) + (b != -1); }
  int a, b;
};
struct F { P3 q; int a, b, c; };
vector<F> hull3d(const vector<P3>& A) {
  assert(sz(A) >= 4);
  vector<vector<PR>> E(sz(A), vector<PR>(sz(A), {-1, -1
      }));
#define E(x,y) E[f.x][f.y]
  vector<F> FS;
  auto mf = [&](int i, int j, int k, int l) {
    P3 q = (A[j] - A[i]).cross((A[k] - A[i]));
    if (q.dot(A[l]) > q.dot(A[i]))
      q = q * -1;
    F f{q, i, j, k};
    E(a,b).ins(k); E(a,c).ins(j); E(b,c).ins(i);
    FS.push_back(f);
  };
  rep(i,0,4) rep(j,i+1,4) rep(k,j+1,4)
    mf(i, j, k, 6 - i - j - k);
  rep(i,4,sz(A)) {
    rep(j,0,sz(FS)) {
      F f = FS[j];
      if(f.q.dot(A[i]) > f.q.dot(A[f.a])) {
        E(a,b).rem(f.c);
        E(a,c).rem(f.b);
        E(b,c).rem(f.a);
        swap(FS[j--], FS.back());
        FS.pop_back();
      }
    }
    int nw = sz(FS);
    rep(j,0,nw) {
      F f = FS[j];
#define C(a, b, c) if (E(a,b).cnt() != 2) mf(f.a, f.b,
    i, f.c);
      C(a, b, c); C(a, c, b); C(b, c, a);
    }
  }
  for (F& it : FS) if ((A[it.b] - A[it.a]).cross(
    A[it.c] - A[it.a]).dot(it.q) <= 0) swap(it.c, it.b)
        ;
  return FS;
};
```

sphericalDistance.h

Description: Returns the shortest distance on the sphere with radius radius between the points with azimuthal angles (longitude) f1 (ϕ_1) and f2 (ϕ_2) from x axis and zenith angles (latitude) t1 (θ_1) and t2 (θ_2) from z axis (0 = north pole). All angles measured in radians. The algorithm starts by converting the spherical coordinates to cartesian coordinates so if that is what you have you can use only the two last rows. dx*radius is then the difference between the two points in the x direction and d*radius is the total distance between the points. 611f07, 8 lines

```
double sphericalDistance(double f1, double t1,
    double f2, double t2, double radius) {
  double dx = sin(t2)*cos(f2) - sin(t1)*cos(f1);
  double dy = sin(t2)*sin(f2) - sin(t1)*sin(f1);
  double dz = cos(t2) - cos(t1);
  double d = sqrt(dx*dx + dy*dy + dz*dz);
  return radius*2*asin(d/2);
}
```

Strings (9)

KMP.h

Description: pi[x] computes the length of the longest prefix of s that ends at x, other than s[0...x] itself (abacaba -> 0010123). Can be used to find all occurrences of a string.
Time: $\mathcal{O}(n)$ d4375c, 16 lines

```
vi pi(const string& s) {
  vi p(sz(s));
  rep(i,1,sz(s)) {
    int g = p[i-1];
    while (g && s[i] != s[g]) g = p[g-1];
    p[i] = g + (s[i] == s[g]);
  }
  return p;
}
vi match(const string& s, const string& pat) {
  vi p = pi(pat + '\0' + s), res;
  rep(i,sz(p)-sz(s),sz(p))
    if (p[i] == sz(pat)) res.push_back(i - 2 * sz(pat))
        ;
  return res;
}
```

Zfunc.h

Description: z[i] computes the length of the longest common prefix of s[i:] and s (abacaba -> 7010301).
Time: $\mathcal{O}(n)$ 584523, 13 lines

```
vi Z(const string& S) {
  vi z(sz(S));
  int l = -1, r = -1;
  rep(i,1,sz(S)) {
    z[i] = i >= r ? 0 : min(r - i, z[i - l]);
    while (i + z[i] < sz(S) && S[i + z[i]] == S[z[i]])
      z[i]++;
    if (i + z[i] > r)
      l = i, r = i + z[i];
  }
  if (sz(S)) z[0] = sz(S);
  return z;
}
```

Manacher.h

Description: For each position in a string, computes p[0][i] = half length of longest even palindrome around pos i, p[1][i] = longest odd (half rounded down).
Time: $\mathcal{O}(N)$ e7ad79, 13 lines

```
array<vi, 2> manacher(const string& s) {
  int n = sz(s);
  array<vi,2> p = {vi(n+1), vi(n)};
  rep(z,0,2) for (int i=0,l=0,r=0; i < n; i++) {
    int t = r-i+!z;
    if (i<r) p[z][i] = min(t, p[z][l+t]);
    int L = i-p[z][i], R = i+p[z][i]-!z;
    while (L>=1 && R+1<n && s[L-1] == s[R+1])
      p[z][i]++, L--, R++;
    if (R>r) l=L, r=R;
  }
  return p;
}
```

MainLorentz.h

Description: Main-Lorentz algorithm for finding all squares in given word; Results are in compressed form: (b, e, l) means that for each b <= i < e there is square at position i of size 2l. Each square is present in only one interval.
Time: $\mathcal{O}(nlgn)$

"Zfunc.h" e01cd1, 46 lines

```
struct Sqr {
  int begin, end, len;
};
vector<Sqr> lorentz(const string &s) {
  vector<Sqr> ans;
  vi pos(sz(s) / 2 + 2, -1);
  rep(mid, 1, sz(s)) {
    int part = mid & ~(mid - 1), off = mid - part;
    int end = min(mid + part, sz(s));
    auto a = s.substr(off, part);
    auto b = s.substr(mid, end - mid);
    string ra(a.rbegin(), a.rend());
    string rb(b.rbegin(), b.rend());
    rep(j, 0, 2) {
      // Set # to some unused character!
      vi z1 = Z(ra);
      vi z2 = Z(b + "#" + a);
      z1.push_back(0);
      z2.push_back(0);
      rep(c, 0, sz(a)) {
        int l = sz(a) - c;
        int x = c - min(l - 1, z1[l]);
        int y = c - max(l - z2[sz(b) + c + 1], j);
        if (x > y)
          continue;
        int sb = (j ? end - y - l * 2 : off + x);
        int se = (j ? end - x - l * 2 + 1 : off + y +
            1);
        int &p = pos[l];
        if (p != -1 && ans[p].end == sb)
          ans[p].end = se;
        else
          p = sz(ans), ans.push_back({sb, se, l});
      }
      a.swap(rb);
      b.swap(ra);
    }
  }
  return ans;
}
```

Lyndon.h

Description: Compute Lyndon factorization for s; Word is simple iff it's stricly smaller than any of it's nontrivial suffixes. Lyndon factorization is division of string into non-increasing simple words. It is unique.
Time: $\mathcal{O}(n)$ fa3adf, 12 lines

```
vector<string> duval(const string &s) {
  int n = sz(s), i = 0;
  vector<string> ret;
  while (i < n) {
    int j = i + 1, k = i;
    while (j < n && s[k] <= s[j])
      k = (s[k] < s[j] ? i : k + 1), j++;
    while (i <= k)
      ret.push_back(s.substr(i, j - k)), i += j - k;
  }
  return ret;
}
```

MinRotation.h

Description: Finds the lexicographically smallest rotation of a string.
Usage: `rotate(v.begin(), v.begin()+minRotation(v), v.end());`
Time: $\mathcal{O}(N)$ d07a42, 8 lines

```
int minRotation(string s) {
  int a=0, N=sz(s); s += s;
  rep(b,0,N) rep(k,0,N) {
    if (a+k == b || s[a+k] < s[b+k]) {b += max(0, k-1);
        break;}
    if (s[a+k] > s[b+k]) { a = b; break; }
  }
  return a;
}
```

SuffixArray.h

Description: Builds suffix array for a string. `sa[i]` is the starting index of the suffix which is i'th in the sorted suffix array. The returned vector is of size $n+1$, and `sa[0] = n`. The `lcp` array contains longest common prefixes for neighbouring strings in the suffix array: `lcp[i] = lcp(sa[i], sa[i-1])`, `lcp[0] = 0`. The input string must not contain any nul chars.

Time: $\mathcal{O}(n \log n)$ 635552, 22 lines

```
struct SuffixArray {
  vi sa, lcp;
  SuffixArray(string s, int lim=256) { // or vector<int>
    s.push_back(0); int n = sz(s), k = 0, a, b;
    vi x(all(s)), y(n), ws(max(n, lim));
    sa = lcp = y, iota(all(sa), 0);
    for (int j = 0, p = 0; p < n; j = max(1, j * 2), lim = p) {
      p = j, iota(all(y), n - j);
      rep(i,0,n) if (sa[i] >= j) y[p++] = sa[i] - j;
      fill(all(ws), 0);
      rep(i,0,n) ws[x[i]]++;
      rep(i,1,lim) ws[i] += ws[i - 1];
      for (int i = n; i--;) sa[--ws[x[y[i]]]] = y[i];
      swap(x, y), p = 1, x[sa[0]] = 0;
      rep(i,1,n) a = sa[i - 1], b = sa[i], x[b] =
        (y[a] == y[b] && y[a + j] == y[b + j]) ? p - 1 : p++;
    }
    for (int i = 0, j; i < n - 1; lcp[x[i++]] = k)
      for (k && k--, j = sa[x[i] - 1];
          s[i + k] == s[j + k]; k++);
  }
};
```

SuffixTree.h

Description: Ukkonen's algorithm for online suffix tree construction. Each node contains indices [l, r) into the string, and a list of child nodes. Suffixes are given by traversals of this tree, joining [l, r) substrings. The root is 0 (has l = -1, r = 0), non-existent children are -1. To get a complete tree, append a dummy symbol – otherwise it may contain an incomplete path (still useful for substring matching, though).

Time: $\mathcal{O}(26N)$ aae0b8, 50 lines

```
struct SuffixTree {
  enum { N = 200010, ALPHA = 26 }; // N ~ 2*maxlen+10
  int toi(char c) { return c - 'a'; }
  string a; // v = cur node, q = cur position
  int t[N][ALPHA],l[N],r[N],p[N],s[N],v=0,q=0,m=2;
  void ukkadd(int i, int c) { suff:
    if (r[v]<=q) {
      if (t[v][c]==-1) { t[v][c]=m;  l[m]=i;
        p[m++]=v; v=s[v]; q=r[v];  goto suff; }
      v=t[v][c]; q=l[v];
    }
    if (q==-1 || c==toi(a[q])) q++; else {
      l[m+1]=i;  p[m+1]=m;  l[m]=l[v];  r[m]=q;
      p[m]=p[v];  t[m][c]=m+1;  t[m][toi(a[q])]=v;
      l[v]=q;  p[v]=m;  t[p[m]][toi(a[l[m]])]=m;
      v=s[p[m]];  q=l[m];
      while (q<r[m]) { v=t[v][toi(a[q])];  q+=r[v]-l[v]; }
      if (q==r[m])  s[m]=v;  else s[m]=m+2;
      q=r[v]-(q-r[m]);  m+=2;  goto suff;
    }
  }
  SuffixTree(string a) : a(a) {
    fill(r,r+N,sz(a));
    memset(s, 0, sizeof s);
    memset(t, -1, sizeof t);
    fill(t[1],t[1]+ALPHA,0);
    s[0] = 1; l[0] = l[1] = -1; r[0] = r[1] = p[0] = p[1] = 0;
    rep(i,0,sz(a)) ukkadd(i, toi(a[i]));
  }
  // example: find longest common substring (uses ALPHA = 28)
  pii best;
  int lcs(int node, int i1, int i2, int olen) {
    if (l[node] <= i1 && i1 < r[node]) return 1;
    if (l[node] <= i2 && i2 < r[node]) return 2;
    int mask = 0, len = node ? olen + (r[node] - l[node]) : 0;
    rep(c,0,ALPHA) if (t[node][c] != -1)
      mask |= lcs(t[node][c], i1, i2, len);
    if (mask == 3)
      best = max(best, {len, r[node] - len});
    return mask;
  }
  static pii LCS(string s, string t) {
    SuffixTree st(s + (char)('z' + 1) + t + (char)('z' + 2));
    st.lcs(0, sz(s), sz(s) + 1 + sz(t), 0);
    return st.best;
  }
};
```

Hashing.h

Description: Self-explanatory methods for string hashing. 2d2a67, 44 lines

```
// Arithmetic mod 2^64-1. 2x slower than mod 2^64 and more
// code, but works on evil test data (e.g. Thue-Morse, where
// ABBA... and BAAB... of length 2^10 hash the same mod 2^64).
// "typedef ull H;" instead if you think test data is random,
// or work mod 10^9+7 if the Birthday paradox is not a problem.
typedef uint64_t ull;
struct H {
  ull x; H(ull x=0) : x(x) {}
  H operator+(H o) { return x + o.x + (x + o.x < x); }
  H operator-(H o) { return *this + ~o.x; }
  H operator*(H o) { auto m = (__uint128_t)x * o.x;
    return H((ull)m) + (ull)(m >> 64); }
  ull get() const { return x + !~x; }
  bool operator==(H o) const { return get() == o.get(); }
  bool operator<(H o) const { return get() < o.get(); }
};
static const H C = (ll)1e11+3; // (order ~ 3e9; random also ok)
struct HashInterval {
  vector<H> ha, pw;
  HashInterval(string& str) : ha(sz(str)+1), pw(ha) {
    pw[0] = 1;
    rep(i,0,sz(str))
      ha[i+1] = ha[i] * C + str[i],
      pw[i+1] = pw[i] * C;
  }
  H hashInterval(int a, int b) { // hash [a, b)
    return ha[b] - ha[a] * pw[b - a];
  }
};
vector<H> getHashes(string& str, int length) {
  if (sz(str) < length) return {};
  H h = 0, pw = 1;
  rep(i,0,length)
    h = h * C + str[i], pw = pw * C;
  vector<H> ret = {h};
  rep(i,length,sz(str)) {
    ret.push_back(h = h * C + str[i] - pw * str[i-length]);
  }
  return ret;
}
H hashString(string& s){H h{}; for(char c:s) h=h*C+c; return h;}
```

Hash61.h

Description: Arithmetic for fast hashing modulo $2^{61}-1$ (prime).

Time: About 30% faster than naive modulo. 51cf65, 8 lines

```
const ll MOD = (1ll << 61) - 1;
ll add(ll a, ll b) {
  return a+b >= MOD ? a+b - MOD : a+b; }
ll sub(ll a, ll b) { return add(a, MOD - b); }
ll mul(ll a, ll b) {
  auto c = (__int128)a * b;
  return add(c & MOD, c >> 61);
}
```

AhoCorasick.h

Description: Aho-Corasick automaton, used for multiple pattern matching. Initialize with AhoCorasick ac(patterns); the automaton start node will be at index 0. find(word) returns for each position the index of the longest word that ends there, or -1 if none. findAll(−, word) finds all words (up to $N\sqrt{N}$ many if no duplicate patterns) that start at each position (shortest first). Duplicate patterns are allowed; empty patterns are not. To find the longest words that start at each position, reverse all input. For large alphabets, split each symbol into chunks, with sentinel bits for symbol boundaries.

Time: construction takes $\mathcal{O}(26N)$, where N = sum of length of patterns. find(x) is $\mathcal{O}(N)$, where N = length of x. findAll is $\mathcal{O}(NM)$. f35677, 66 lines

```
struct AhoCorasick {
  enum {alpha = 26, first = 'A'}; // change this!
  struct Node {
    // (nmatches is optional)
    int back, next[alpha], start = -1, end = -1, nmatches = 0;
    Node(int v) { memset(next, v, sizeof(next)); }
  };
  vector<Node> N;
  vi backp;
  void insert(string& s, int j) {
    assert(!s.empty());
    int n = 0;
    for (char c : s) {
      int& m = N[n].next[c - first];
      if (m == -1) { n = m = sz(N); N.emplace_back(-1); }
      else n = m;
    }
    if (N[n].end == -1) N[n].start = j;
    backp.push_back(N[n].end);
    N[n].end = j;
    N[n].nmatches++;
  }
  AhoCorasick(vector<string>& pat) : N(1, -1) {
    rep(i,0,sz(pat)) insert(pat[i], i);
    N[0].back = sz(N);
    N.emplace_back(0);
    queue<int> q;
    for (q.push(0); !q.empty(); q.pop()) {
      int n = q.front(), prev = N[n].back;
      rep(i,0,alpha) {
        int &ed = N[n].next[i], y = N[prev].next[i];
        if (ed == -1) ed = y;
        else {
          N[ed].back = y;
          (N[ed].end == -1 ? N[ed].end : backp[N[ed].start])
            = N[y].end;
          N[ed].nmatches += N[y].nmatches;
          q.push(ed);
        }
      }
    }
  }
  vi find(string word) {
    int n = 0;
    vi res; // ll count = 0;
    for (char c : word) {
      n = N[n].next[c - first];
      res.push_back(N[n].end);
      // count += N[n].nmatches;
    }
    return res;
  }
  vector<vi> findAll(vector<string>& pat, string word) {
    vi r = find(word);
    vector<vi> res(sz(word));
    rep(i,0,sz(word)) {
      int ind = r[i];
      while (ind != -1) {
        res[i - sz(pat[ind]) + 1].push_back(ind);
        ind = backp[ind];
      }
    }
    return res;
  }
};
```

PalTree.h

Description: Palindrome tree. Can be used for counting number of occurrences, just add 1 to suffix link path. Replace array with map if ML is tight.

Time: $\mathcal{O}(26N)$, split is $\mathcal{O}(n \log n)$ 2b16ac, 44 lines

```
const int A = 26;
struct PalTree {
  int last = 0;
  vi len = {0, -1}, link = {1, 0}, s = {-1};
  vector<array<int, A>> to = {{}, {}};
  int find(int u) {
    while (s.back() != s[sz(s)-len[u]-2]) u = link[u];
    return u;
  }
  int add(int x) { // x in [0, A)
    s.push_back(x); last = find(last);
    if (!to[last][x]) {
      to.push_back({});
      len.push_back(len[last] + 2);
      link.push_back(to[find(link[last])][x]);
      to[last][x] = sz(to) - 1;
    }
    return last = to[last][x];
  }
};
// min even/odd palindromic split of prefix of size i
const int INF = 1e9;
struct F { int e, o; };
F op(F x, F y) { return {min(x.e,y.e), min(x.o,y.o)}; }
vector<F> split(vi v) {
  PalTree t;
  vector<F> s(2), ans(sz(v) + 1, {INF, INF});
  vi go(2), d(2); ans[0] = s[0] = s[1] = {0, INF};
  rep(i, 0, sz(v)) {
    int x = t.add(v[i]), y = t.link[x];
    if (x >= sz(go)) {
      d.push_back(t.len[x] - t.len[y]);
      go.push_back(d[x] == d[y] ? go[y] : y);
      s.push_back(ans[0]);
    }
    for (int u = x; t.len[u] > 0; u = go[u]) {
      s[u] = ans[i + 1 - t.len[go[u]] - d[u]];
      if (d[u] == d[t.link[u]])
        s[u] = op(s[u], s[t.link[u]]);
      ans[i+1] = op(ans[i+1], {s[u].o+1, s[u].e+1});
    }
  }
  return ans;
}
```

WildcardMatching.h

Description: Finds all occurrences of T in S over an alphabet with wildcards. Requires $|T| \leq |S|$.

Time: $\mathcal{O}(|S| \log |S|)$

"../numerical/FFT.h" 639ceb, 20 lines

```
mt19937 rng(2137);
vector<bool> match(string s, string t, char w = '*') {
  int n = sz(s), m = sz(t); mint d = rng();
  vector<mint> f1(n),f2(n),f3(n),g1(m),g2(m),g3(m);
  rep(i, 0, n) f1[i] = s[i] == w ? 0 : s[i] + d;
  rep(i, 0, n) f2[i]=f1[i]*f1[i], f3[i]=f2[i]*f1[i];
  rep(i, 0, m) g1[i] = t[i] == w ? 0 : t[i] + d;
  rep(i, 0, m) g2[i]=g1[i]*g1[i], g3[i]=g2[i]*g1[i];
  auto mul = [&](auto a, auto b) {
    int sz = 1 << __lg(2 * n - 1); reverse(all(b));
    a.resize(sz); ntt(a, 0); b.resize(sz); ntt(b, 0);
    rep(i, 0, sz) a[i] *= b[i];
    ntt(a, 1); a.erase(a.begin(), a.begin() + m - 1);
    return a;
  };
  auto a = mul(f1,g3), b = mul(f2,g2), c = mul(f3,g1);
  vector<bool> ans(n - m + 1);
  rep(i, 0, n - m + 1) ans[i] = a[i]-b[i]-b[i]+c[i]==0;
  return ans;
}
```

SuffixAutomaton.h

Description: Suffix automaton. 0d0b7f, 51 lines

```
struct SuffixAutomaton {
  static constexpr int sigma = 26;
  using Node = array<int, sigma>; // map<int, int>
  Node new_node;
  vector<Node> edges;
  vector<int> link = {-1}, length = {0};
  int last = 0;
  SuffixAutomaton() {
    // -1 - stan nieistniejacy. dodajemy stan startowy,
    // ktory reprezentuje puste slowo
    new_node.fill(-1);
    edges = {new_node};
  }
  void add_letter(int c) {
    edges.emplace_back(new_node);
    length.emplace_back(length[last] + 1);
    link.emplace_back(0);
    int r = ssize(edges) - 1, p = last;
    while(p != -1 && edges[p][c] == -1) {
      edges[p][c] = r;
      p = link[p];
    }
    if(p != -1) {
      int q = edges[p][c];
      if(length[p] + 1 == length[q])
        link[r] = q;
      else {
```

```
      edges.emplace_back(edges[q]);
      length.emplace_back(length[p] + 1);
      link.emplace_back(link[q]);
      int q_prim = ssize(edges) - 1;
      link[q] = link[r] = q_prim;
      while(p != -1 && edges[p][c] == q) {
        edges[p][c] = q_prim;
        p = link[p];
      }
    }
  }
  last = r;
}
bool is_inside(vector<int> &s) {
  int q = 0;
  for(int c : s) {
    if(edges[q][c] == -1)
      return false;
    q = edges[q][c];
  }
  return true;
}
};
```

Various (10)

10.1 Intervals

IntervalContainer.h

Description: Add and remove intervals from a set of disjoint intervals. Will merge the added interval with any overlapping intervals in the set when adding. Intervals are [inclusive, exclusive).

Time: $\mathcal{O}(\log N)$ edce47, 23 lines

```
set<pii>::iterator addInterval(set<pii>& is, int L, int
    R) {
  if (L == R) return is.end();
  auto it = is.lower_bound({L, R}), before = it;
  while (it != is.end() && it->first <= R) {
    R = max(R, it->second);
    before = it = is.erase(it);
  }
  if (it != is.begin() && (--it)->second >= L) {
    L = min(L, it->first);
    R = max(R, it->second);
    is.erase(it);
  }
  return is.insert(before, {L,R});
}
void removeInterval(set<pii>& is, int L, int R) {
  if (L == R) return;
  auto it = addInterval(is, L, R);
  auto r2 = it->second;
  if (it->first == L) is.erase(it);
  else (int&)it->second = L;
  if (R != r2) is.emplace(R, r2);
}
```

IntervalCover.h

Description: Compute indices of smallest set of intervals covering another interval. Intervals should be [inclusive, exclusive). To support [inclusive, inclusive], change (A) to add `|| R.empty()`. Returns empty set on failure (or if G is empty).

Time: $\mathcal{O}(N \log N)$ 9e9d8d, 19 lines

```
template<class T>
vi cover(pair<T, T> G, vector<pair<T, T>> I) {
  vi S(sz(I)), R;
  iota(all(S), 0);
  sort(all(S), [&](int a, int b) { return I[a] < I[b];
      });
  T cur = G.first;
  int at = 0;
  while (cur < G.second) { // (A)
    pair<T, int> mx = make_pair(cur, -1);
    while (at < sz(I) && I[S[at]].first <= cur) {
      mx = max(mx, make_pair(I[S[at]].second, S[at]));
      at++;
    }
    if (mx.second == -1) return {};
    cur = mx.first;
    R.push_back(mx.second);
  }
  return R;
}
```

ConstantIntervals.h

Description: Split a monotone function on [from, to) into a minimal set of half-open intervals on which it has the same value. Runs a callback g for each such interval.

Usage: `constantIntervals(0, sz(v), [&](int x){return v[x];}, [&](int lo, int hi, T val){...});`

Time: $\mathcal{O}\left(k \log \frac{n}{k}\right)$ 753a4c, 19 lines

```
template<class F, class G, class T>
void rec(int from, int to, F& f, G& g, int& i, T& p, T
    q) {
  if (p == q) return;
  if (from == to) {
    g(i, to, p);
    i = to; p = q;
  } else {
    int mid = (from + to) >> 1;
    rec(from, mid, f, g, i, p, f(mid));
    rec(mid+1, to, f, g, i, p, q);
  }
}
template<class F, class G>
void constantIntervals(int from, int to, F f, G g) {
  if (to <= from) return;
  int i = from; auto p = f(i), q = f(to-1);
  rec(from, to-1, f, g, i, p, q);
  g(i, to, q);
}
```

10.2 Dynamic programming

KnuthDP.h

Description: When doing DP on intervals: $a[i][j] = \min_{i<k<j}(a[i][k] + a[k][j]) + f(i,j)$, where the (minimal) optimal k increases with both i and j, one can solve intervals in increasing order of length, and search $k = p[i][j]$ for $a[i][j]$ only between $p[i][j-1]$ and $p[i+1][j]$. This is known as Knuth DP. Sufficient criteria for this are if $f(b,c) \leq f(a,d)$ and $f(a,c) + f(b,d) \leq f(a,d) + f(b,c)$ for all $a \leq b \leq c \leq d$. Consider also: LineContainer (ch. Data structures), monotone queues, ternary search.

Time: $\mathcal{O}\left(N^2\right)$

DivideAndConquerDP.h

Description: Given $a[i] = \min_{lo(i) \leq k < hi(i)}(f(i,k))$ where the (minimal) optimal k increases with i, computes $a[i]$ for $i = L..R-1$.

Time: $\mathcal{O}((N + (hi - lo)) \log N)$ d38d2b, 18 lines

```
struct DP { // Modify at will:
  int lo(int ind) { return 0; }
  int hi(int ind) { return ind; }
  ll f(int ind, int k) { return dp[ind][k]; }
  void store(int ind, int k, ll v) { res[ind] = pii(k,
      v); }
  void rec(int L, int R, int LO, int HI) {
    if (L >= R) return;
    int mid = (L + R) >> 1;
    pair<ll, int> best(LLONG_MAX, LO);
    rep(k, max(LO,lo(mid)), min(HI,hi(mid)))
      best = min(best, make_pair(f(mid, k), k));
    store(mid, best.second, best.first);
    rec(L, mid, LO, best.second+1);
    rec(mid+1, R, best.second, HI);
  }
  void solve(int L, int R) { rec(L, R, INT_MIN, INT_MAX
      ); }
};
```

AliensTrick.h

Description: Optimize dp where you want "k things with minimal cost". The slope of f(k) must be non increasing. Provide a function g(lambda) that computes the best answer for any k with costs increased by lambda. 71bca3, 10 lines

```
ll aliens(ll k, auto g) { // returns f(k)
  // make sure [l, r) is ok (r > max slope etc)
  ll l = 0, r = 1e11;
  while (l + 1 < r) {
    ll m = (l + r) / 2;
    (g(m - 1) + k <= g(m) ? l : r) = m;
  }
  // return l if you want the optimal lambda
  return g(l) - l * k;
}
```

10.3 Misc. algorithms

TernarySearch.h

Description: Find the smallest i in $[a,b]$ that maximizes $f(i)$, assuming that $f(a) < \dots < f(i) \geq \dots \geq f(b)$. To reverse which of the sides allows non-strict inequalities, change the < marked with (A) to <=, and reverse the loop at (B). To minimize f, change it to >, also at (B).

Usage: `int ind = ternSearch(0,n-1,[&](int i){return a[i];});`

Time: $\mathcal{O}(\log(b-a))$ 9155b4, 11 lines

```
template<class F>
int ternSearch(int a, int b, F f) {
  assert(a <= b);
  while (b - a >= 5) {
    int mid = (a + b) / 2;
    if (f(mid) < f(mid+1)) a = mid; // (A)
    else b = mid+1;
  }
  rep(i,a+1,b+1) if (f(a) < f(i)) a = i; // (B)
  return a;
}
```

LIS.h

Description: Compute indices for the longest increasing subsequence.

Time: $\mathcal{O}(N \log N)$ 2932a0, 17 lines

```
template<class I> vi lis(const vector<I>& S) {
  if (S.empty()) return {};
  vi prev(sz(S));
  typedef pair<I, int> p;
  vector<p> res;
  rep(i,0,sz(S)) {
    // change 0 -> i for longest non-decreasing
        subsequence
    auto it = lower_bound(all(res), p{S[i], 0});
    if (it == res.end()) res.emplace_back(), it = res.
        end()-1;
    *it = {S[i], i};
    prev[i] = it == res.begin() ? 0 : (it-1)->second;
  }
  int L = sz(res), cur = res.back().second;
  vi ans(L);
  while (L--) ans[L] = cur, cur = prev[cur];
  return ans;
}
```

FastKnapsack.h

Description: Given N non-negative integer weights w and a non-negative target t, computes the maximum S <= t such that S is the sum of some subset of the weights.

Time: $\mathcal{O}(N \max(w_i))$ b20ccc, 16 lines

```
int knapsack(vi w, int t) {
  int a = 0, b = 0, x;
  while (b < sz(w) && a + w[b] <= t) a += w[b++];
  if (b == sz(w)) return a;
  int m = *max_element(all(w));
  vi u, v(2*m, -1);
  v[a+m-t] = b;
  rep(i,b,sz(w)) {
    u = v;
    rep(x,0,m) v[x+w[i]] = max(v[x+w[i]], u[x]);
    for (x = 2*m; --x > m;) rep(j, max(0,u[x]), v[x])
      v[x-w[j]] = max(v[x-w[j]], j);
  }
  for (a = t; v[a+m-t] < 0; a--) ;
  return a;
}
```

SubsetSum.h

Description: Finds all subset sums of a multiset with sum s. `cnt[k]` should be the number of elements with value k.

Usage: `subsetSum<MAXN>(s, cnt);`

Time: $\mathcal{O}\left(\frac{s\sqrt{s}}{32}\right)$ 6aa4ae, 13 lines

```
template<int N>
vi subsetSum(int s, vi& cnt) {
  if (s < 3*N/4) return subsetSum<3*N/4>(s, cnt);
  bitset<N> b; b[0] = 1;
  rep(i, 1, sz(cnt)) {
    for (int j = 1; j <= cnt[i]; j *= 2)
      b |= b << (j * i), cnt[i] -= j;
    if (cnt[i]) b |= b << (cnt[i] * i);
  }
  vi ans;
  rep(i, 0, s + 1) if (b[i]) ans.push_back(i);
  return ans;
}
```

SA.py

Description: Simulated annealing. Neighbor function shouldn't change the answer by too much, but it should be possible to get from any state to another in a small number of steps. 22 lines

```
def P(old, new, temp):
  if new < old:
    return 1.0
  return exp((old-new)/temp)
# tweak these:
temp_start = 10.0
temp_end = 0.1
def temperature(elapsed_frac):
  return temp_start * (temp_end / temp_start).pow(
        elapsed_frac)
s = random(state)
best = s
while elapsed_time() <= time_limit:
  t = temperature(elapsed_time()/time_limit)
  next = neighbor(s)
  if value(s) < value(best):
    best = s # important
  if P(value(s), value(next), t) >= random(0, 1):
    s = next
print(value(best))
```

FastMod.h

Description: Compute $a\%b$ about 5 times faster than usual, where b is constant but not known at compile time. Returns a value congruent to $a \pmod{b}$ in the range $[0, 2b)$. 751a02, 8 lines

```
typedef unsigned long long ull;
struct FastMod {
  ull b, m;
  FastMod(ull b) : b(b), m(-1ULL / b) {}
  ull reduce(ull a) { // a % b + (0 or b)
    return a - (ull)((__uint128_t(m) * a) >> 64) * b;
  }
};
```

FastInput.h

Description: Read an integer from stdin. Usage requires your program to pipe in input from file.

Usage: `./a.out < input.txt`

Time: About 5x as fast as cin/scanf. 7b3c70, 17 lines

```
inline char gc() { // like getchar()
  static char buf[1 << 16];
  static size_t bc, be;
  if (bc >= be) {
    buf[0] = 0, bc = 0;
    be = fread(buf, 1, sizeof(buf), stdin);
  }
  return buf[bc++]; // returns 0 on EOF
}
int readInt() {
  int a, c;
  while ((a = gc()) < 40);
  if (a == '-') return -readInt();
  while ((c = gc()) >= 48) a = a * 10 + c - 480;
  return a - 48;
}
```

BumpAllocator.h

Description: When you need to dynamically allocate many objects and don't care about freeing them. "new X" otherwise has an overhead of something like 0.05us + 16 bytes per allocation. 745db2, 8 lines

```
// Either globally or in a single class:
static char buf[450 << 20];
void* operator new(size_t s) {
  static size_t i = sizeof buf;
  assert(s < i);
  return (void*)&buf[i -= s];
}
void operator delete(void*) {}
```

SmallPtr.h

Description: A 32-bit pointer that points into BumpAllocator memory.

"BumpAllocator.h" 2dd6c9, 10 lines

```
template<class T> struct ptr {
  unsigned ind;
  ptr(T* p = 0) : ind(p ? unsigned((char*)p - buf) : 0)
        {
    assert(ind < sizeof buf);
  }
  T& operator*() const { return *(T*)(buf + ind); }
  T* operator->() const { return &**this; }
  T& operator[](int a) const { return (&**this)[a]; }
  explicit operator bool() const { return ind; }
};
```

BumpAllocatorSTL.h

Description: BumpAllocator for STL containers.

Usage: `vector<vector<int, small<int>>> ed(N);` bb66d4, 14 lines

```
char buf[450 << 20] alignas(16);
size_t buf_ind = sizeof buf;
template<class T> struct small {
  typedef T value_type;
  small() {}
  template<class U> small(const U&) {}
  T* allocate(size_t n) {
    buf_ind -= n * sizeof(T);
    buf_ind &= 0 - alignof(T);
    return (T*)(buf + buf_ind);
  }
  void deallocate(T*, size_t) {}
};
```