Jewel Maldonado Week 2 WHW1 184, 189, 199, 209, 235, 237
WHW2 247, 251, 257, 268, 271 (b,c), 280 a

Section 2.4 excercises

184)

a) Find the cross product $u \times v$ of vectors u and v express in component form

b) Sketch the vectors u, v, and $u \times v$

given: $u = \langle 3, 2, -1 \rangle$, $v = \langle 1, 1, 0 \rangle$

a)

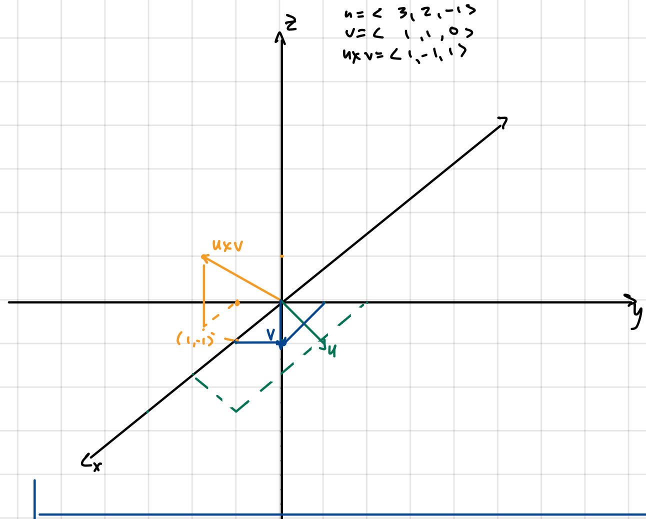
$$u \times v = \begin{vmatrix} i & j & k \\ 3 & 2 & -1 \\ 1 & 1 & 0 \end{vmatrix} = \hat{i}((2 \cdot 0) - (-1) \cdot 1) - \hat{j}((3 \cdot 0) - (-1) \cdot 1) + \hat{k}(3 - 2)$$

$$\hat{i} \quad - \hat{j} \quad + \quad k$$

$$\boxed{u \times v = 1\hat{i} - 1\hat{j} + 1k = \langle 1, -1, 1 \rangle}$$

b)

$u = \langle 3, 2, -1 \rangle$
$v = \langle 1, 1, 0 \rangle$
$u \times v = \langle 1, -1, 1 \rangle$



189) In the following exercises, vectors u and v are given. Find unit vector w in the direction of the cross product vector u × v. Express your answer using standard unit vectors.

$u = \langle 3, -1, 2 \rangle \qquad v = \langle -2, 0, 1 \rangle$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & 2 \\ -2 & 0 & 1 \end{vmatrix} : \hat{i}((-1)1 - (2) \cdot 0) - \hat{j}(3 \cdot 1 - 2(-2)) + k(3(0) - (-1)(-2))$$

$$\hat{i}(-1 - 0) \quad - \hat{j} \quad 3 - (-4) \quad + k(0 - (2))$$

$$\hat{i}(-1) - \hat{j}(7) + k(-2)$$

$$\boxed{w = -\hat{i} - 7\hat{j} - 2\hat{k}}$$

199) find w orthogonal to u & v

$u = \langle -1, 0, e^t \rangle$
$v = \langle 1, e^{-t}, \emptyset \rangle$

$$\begin{vmatrix} \hat{i} & \hat{j} & k \\ -1 & 0 & e^t \\ 1 & e^{-t} & 0 \end{vmatrix} = \hat{i}(0 - 1) - \hat{j}(0 - e^t) + k(-e^{-t})$$

$$-\hat{i} + e^t\hat{j} - ke^{-t}$$

$$u \cdot (u \times v) = \langle -1, 0, e^t \rangle \circ \langle -1, e^t, -e^{-t} \rangle$$

$$-1, e^t, -e^{-t}$$

$$1 - 1$$

$$\boxed{w = u \times v = -\hat{i} + e^t\hat{j} - e^{-t}k}$$

209) find the area of the parallelogram with adjacent sides $\|u \times v\|$

Given:

$u = \langle 3, 2, 0 \rangle \quad v = \langle 0, 2, 1 \rangle$

$(2 - 0)i - (3 - 0)j + (6 - 0)k$

$2i \quad -3j \quad 6k$

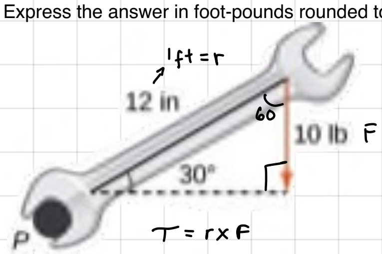
$\|2i - 3j + 6k\|$

$= \sqrt{2^2 + (-3)^2 + 6^2}$

$4 + 9 + 36$

$\sqrt{49} = \boxed{7 \text{ units}^2}$

235. [T] A mechanic uses a 12-in. wrench to turn a bolt. The wrench makes a 30° angle with the horizontal. If the mechanic applies a vertical force of 10 lb on the wrench handle, what is the magnitude of the torque at point P (see the following figure)? Express the answer in foot-pounds rounded to two decimal places.



$T = r \times F$

$\tau = \|r\| \|F\| \sin\theta$

$= (1 ft)(10 lb) \sin 60$

$= \frac{10\sqrt{3}}{2}$ ft lb

$5\sqrt{3}$ ft lb

$\boxed{\approx 8.66 \text{ ft lb}}$

237. [T] Find the magnitude of the force that needs to be applied to the end of a 20-cm wrench located on the positive direction of the y-axis if the force is applied in the direction $\langle 0, 1, -2 \rangle$ and it produces a 100 N·m torque to the bolt located at the origin.

$\tau = r \times F$

$\|\tau\| = \|r \times F\| = \|r\|\|F\| \sin\theta$

$\|r\| = 20cm = .20m$ $\quad r = .2j$ $\quad r \cdot \langle 0,1,-2\rangle = \langle 0,.2,0\rangle \cdot \langle 0,1,-2\rangle$

$= (0)0 + (.2)(1) + (0)$

$= .2$

$r = .2j$

F lies along $\langle 0,1,-2\rangle$

$r \cdot \langle 0,1,-2\rangle = \|r\|\left(\sqrt{(1)^2 + (-2)^2}\right)\cos\theta$

$.20 = \; = \sqrt{(.20)^2}\,($

$.20 = .2(\sqrt{5})\cos\theta$

$\frac{1}{\sqrt{5}} = \cos\theta$

$\cos\theta^2 + \sin^2\theta = 1$

$\sin\theta = \sqrt{1-\cos^2\theta}$

$\sin\theta = \sqrt{1 - \frac{1}{5}}$

$\sqrt{4/5}$

$\sin\theta = \frac{2}{\sqrt{5}}$

$100Nm = (.2m)\|F\|\frac{2}{\sqrt{5}}$

$\|F\| = \frac{100}{.2} \times \frac{\sqrt{5}}{2} N$

$= 559.0169943749474$

≈ 559.017

$\boxed{559.02N}$

WHW2 247, 251, 257, 268, 271 (b,c), 280a

For the following exercises, point P and vector $\mathbf{v}$ are given. Let L be the line passing through point P with direction $\mathbf{v}$.

A)Find parametric equations of line L.

B)Find symmetric equations of line L.

C)Find the intersection of the line with the xy-plane.

Given

247) $P(1,-2,3),\ V = \langle 1,2,3\rangle$

$\vec{PQ} = tv$

$\langle x-1, y+2, z-3\rangle = t\langle 1,2,3\rangle$, t = Real numbers

$r = \langle x,y,z\rangle$

$r_0 = \langle 1,-2,3\rangle$

$r = r_0 + tv$

$\langle x,y,z\rangle = \langle 1,-2,3\rangle + t\langle 1,2,3\rangle$

$\boxed{x = 1+t \quad y = -2+2t \quad z = 3+3t}$

$x = 1+t$ $\quad t = \frac{x-1}{1}$

$-1 \quad -1$

$\frac{x-1}{1} = \frac{t}{1}$

$\boxed{\frac{x-1}{1} = \frac{y+2}{2} = \frac{z-3}{3}}$

$y = -2+2t$ $\quad$ $-2-2$ $\quad$ $y = -4$

$\boxed{\langle 0,-4,-1\rangle}$

$z = 3+3t = 0 \quad t = -1$

$y = -2+2t$ $\quad t = \frac{y+2}{2}$

$\frac{y+2}{2} = \frac{2t}{2}$

$\frac{x-1}{1} = \frac{y+2}{2} = \frac{0-3}{3}$

$x = 1+t = \emptyset$

$t = \frac{z-3}{3}$

$\frac{x-1}{1} = \frac{y+2}{2} = -1$

$z = 3+3t$

$\frac{z-3}{3} = t$

Find point P that belongs to the line and direction vector $\mathbf{v}$ of the line.

a. Express $\mathbf{v}$ in component form.

b. Find the distance from the origin to line L

251) $x = 1+t,\ y = 3+t,\ z = 5+4t,\ t \in \mathbb{R}$

$x = 1+t$

$y = 3+t$

$z = 5+4t$

$\frac{x-1}{1} = \frac{y-3}{1} = \frac{z-5}{4}$

$\boxed{\begin{array}{l} P = P(1,3,5) \\ v = \langle 1,1,4 \rangle \end{array}}$

$\begin{vmatrix} i & j & k \\ -1 & -3 & -5 \\ 1 & 1 & 4 \end{vmatrix} = -12-(-5),\ -(-4)-(-5),\ -1+3$

$-7, -1, 2\rangle$

$d = \frac{\|\overrightarrow{PM} \times v\|}{v}$

$= \frac{\|\langle -1,-3,-5\rangle \times \langle 1,1,4\rangle\|}{\sqrt{2+16}}$

$\sqrt{18}$

$\frac{\|\langle -7,-1,2\rangle\|}{3\sqrt{2}}$

$-7^2 + 1 + 4 = \frac{\sqrt{54}}{3\sqrt{2}}$

$49 \quad 54$

$\frac{\sqrt{9}\sqrt{6}}{3\sqrt{2}} = \frac{3\sqrt{6}}{3\sqrt{2}}\ \boxed{= \sqrt{3}}$

257) 257. Show that the line passing through points $P(3, 1, 0)$ and $Q(1, 4, -3)$ is perpendicular to the line with equation $x=3t, y=3+8t, z=-7+6t,\ t \in$ R.

Given $L = \begin{array}{l} x = 3t \\ y = 3+8t \\ z = -7+6t \end{array}\ t \in \mathbb{R}$

$P(3,1,0)\quad Q(1,4,-3)$

$\frac{x}{3} = \frac{y-3}{8} = \frac{z+7}{6}$

$(0,3,-7)\quad v = \langle 3,8,6\rangle$

$\overrightarrow{PQ} = \langle 1-3,\ 4-1,\ -3-0\rangle$

$\langle -2, 3, -3\rangle$

$v \cdot \overrightarrow{PQ} = \langle 3,8,6\rangle \circ \langle -2,3,-3\rangle$

$3(-2) + 8(3) + 6(-3$

$-6 + 24 + -18 = \emptyset$

$\boxed{v \circ \overrightarrow{PQ} = 0 \quad \text{so the two lines are orthogonal}}$

251, 257, 268, 271 (b,c), 280 a

For the following exercises, point P and vector $\mathbf{n}$ are given.

a. Find the scalar equation of the plane that passes through P and has normal vector $\mathbf{n}$.

b. Find the general form of the equation of the plane that passes through P and has normal vector $\mathbf{n}$.

268) $P(3,2,2)$ $n = 2i + 3j - k$

a) $P: a(x-x_0)+b(y-y_0)+c(z-z_0)=0$

where $n=\langle a\ b\ c\rangle$ $P(x_0\ y_0\ z_0)$

$2(x-3)+3(y-2)+(z-2)=0$

$2x-6+3y-6+z+2=0$

$2x+3y+z-10=0$

271) For the following exercises, the equation of a plane is given.

a. Find normal vector $\mathbf{n}$ to the plane. Express $\mathbf{n}$ using standard unit vectors.

b. Find the intersections of the plane with the axes of coordinates.

c. Sketch the plane.

Given : $4x+5y+10z-20=0$

$n=\langle 4,5,10\rangle$

$n = 4\hat{i}+5\hat{j}+10\hat{k}$

$\langle a\ b\ c\rangle$

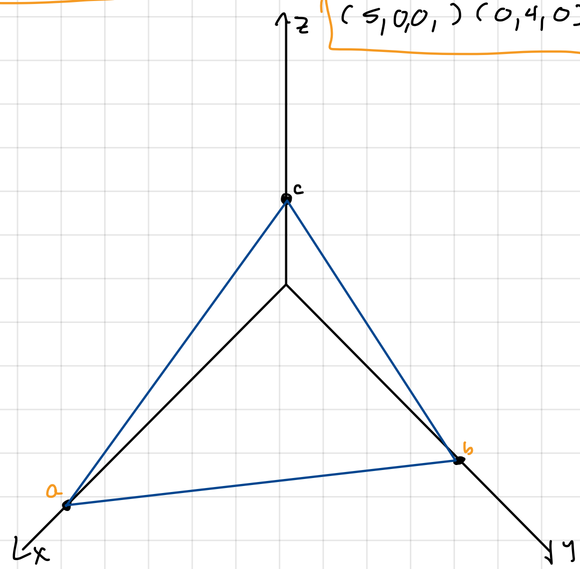
$\langle a,0,0\rangle$ $\langle 0b0\rangle$ $\langle 00c\rangle$

$4a-20=0$ $a=5$

$5b-20=0$ $b=4$

$10c-20=0$ $c=2$

$(5,0,0)(0,4,0)(0,0,2)$



280) find the real number α such that the line of parametric equations $x=t,\ y=2-t,\ z=3+t,\ t\in\mathbb{R}$ is parallel to the plane of equation $\alpha x+5y+z-10=0$

Given $ax+5y+z-10=0$

$x=t$ $y=2-t$ $z=3+t$

$\frac{x}{1}=\frac{y-2}{-1}=\frac{z-3}{1}$ $v=\langle 1,-1,1\rangle$

$v\circ n=0$

$\langle 1,-1,1\rangle\circ\langle \alpha,5,1\rangle=0$

$\alpha-5+1=0$ (4)

$\boxed{\alpha=4}$