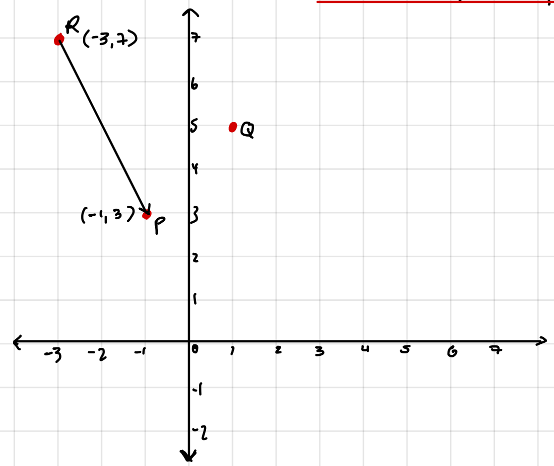
chapter 2.1

HW #1 4, 12, 15, 25, 46

JEWEL MALDONADO

consider points



Determine the requested vectors and express each of them a) in component form and b) by using standard unit vectors

Given

4) $\overrightarrow{RP}$ P(-1,3) Q(1,5) R(-3,7)

(-3,7) (-1,3)

-1-(-3), 3-7

$\boxed{\overrightarrow{RP} = \langle 2, -4 \rangle}$

$\boxed{\overrightarrow{QP} = 2\hat{i} - 4\hat{j}}$

---

12) A vector v has initial point (-2,5) and terminal point (3,-1) find the unit vector in the direction of v Express the answer in component form.

Given

V_i (-2,5) V_t (3,-1)

find $\overrightarrow{V_i V_t}$

in $(\overset{x}{-2}, \overset{y}{5}) \rightarrow$ term$(\overset{x}{3}, \overset{y}{-1})$

$\|v\| = (3-(-2), -1-5)$

$\|v\| = \langle 5, -6 \rangle$

$\|v\| = \sqrt{5^2 + (-6)^2} = \sqrt{61}$

$\frac{v}{\|v\|} = \frac{1}{\sqrt{61}} \langle 5, -6 \rangle = \boxed{\left\langle \frac{5}{\sqrt{61}}, -\frac{6}{\sqrt{61}} \right\rangle}$

---

15) Given:

$a = 2i + j$, $b = i + 3j$

Step a) determine a+b in component & standard form: $\boxed{\overrightarrow{ab} = \langle 3, 4 \rangle}$

$2\hat{i} + \hat{j}$

$\hat{i} + 3\hat{j}$

$a + b = \boxed{3\hat{i} + 4\hat{j}}$

Step b) determine a-b in component & standard form: $\boxed{\overrightarrow{ab} = \langle 1, -3 \rangle}$

$2\hat{i} + \hat{j}$

$a - b = -\ \hat{i} + 3\hat{j}$

$\boxed{\hat{i} - 2\hat{j}}$

Step c) verify that the vectors a, b, & a+b & respectively a, b, & a-b satisfy the triangle inequality

$\|a+b\| \leq \|a\| + \|b\|$

$\|a\| = \sqrt{2^2 + 1^2} = \sqrt{5}$

$\|b\| = \sqrt{\underset{1+9}{1^2 + 3^2}} = \sqrt{10}$

$\|a\| + \|b\| = \sqrt{5} + \sqrt{10}$

$\|a+b\| = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$

$\|a-b\| = \sqrt{1^2 + (-2)^2} = \sqrt{5}$

$\boxed{\begin{array}{l} 5 < \sqrt{5} + \sqrt{10} \\ \sqrt{5} < \sqrt{5} + \sqrt{10} \end{array}}$

d) $a = 2\hat{i} + \hat{j}$

$b = \hat{i} + 3\hat{j}$

$2a = 2(2\hat{i} + \hat{j})$

$-b = -(\hat{i} + 3\hat{j})$

$\boxed{\begin{array}{l} 4\hat{i} + 2\hat{j} \\ -\hat{i} - 3\hat{j} \end{array}}$ $\boxed{\begin{array}{l} \langle 4, 2 \rangle \\ \langle -1, -3 \rangle \end{array}}$

$4\hat{i} + 2\hat{j}$

$-\hat{i} - 3\hat{j}$

$-3\hat{i} - \hat{j}$

$\cancel{4\hat{i} + 2\hat{j}}$

$\cancel{-(-\hat{i} - 3\hat{j})}$

$\cancel{5\hat{i} + 5\hat{j}}$

$\boxed{\langle -3, -1 \rangle}$

Given

25) $\|v\| = 7$, $u = \langle 3,4 \rangle$ find vector v w/ the given magnitude & in the same direction of u.

$v = \langle 3c, 4c \rangle$

$(3c)^2 + (4c)^2 = \|v\| = 7$

$25c^2 = 7$

$5c = 7$

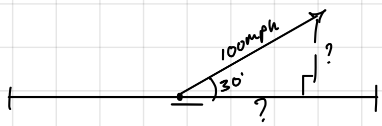
$c = \frac{7}{5}$ $\left\langle \frac{7*3}{5} \quad \frac{7*4}{5} \right\rangle$

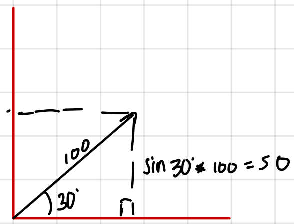
$\boxed{\left\langle \frac{21}{5}, \frac{28}{5} \right\rangle}$

---

46) A Baseball player throws a baseball at an angle of 30° with the horizontal. If the initial Speed of the Ball is 100 mph find the horizontal and vertical components of the initial velocity vector of the ball. (Round two decimal places.)

30·60·90△





cos 30°*100 = 86.6

$\boxed{\begin{array}{c} \langle 86.6, 50 \rangle \\ \langle 86.60, 50.00 \rangle \end{array}}$

$s = \frac{o}{h}$ $c = \frac{a}{h}$ $t = \frac{o}{a}$

$\cos 30° = \frac{a}{100} = 86.6$

$\sin 30 = \frac{o}{100} = 50$

#77, 81, 89, 99, 115 (forces Newtons)

Given

P = initial Q = terminal

P = (3, 0, 2)

Q = (-1, -1, 4)

a) component form

standard unit vector

77)

$\vec{PQ} = \langle 3-(-1), 0-(-1), 2-4 \rangle$

$\boxed{(4, 1, -2) \quad 4\hat{i} + \hat{j} - 2\hat{k}}$

81) Given

$\vec{PQ} = \langle 7, -1, 3 \rangle$ $P_i = (-2, 3, 5)$

$(Q_1 - (-2) = 7, Q_2 - 3 = -1, Q_3 - 5 = 3)$

$Q_1 = 5 \quad Q_2 = 2 \quad Q_3 = 8$

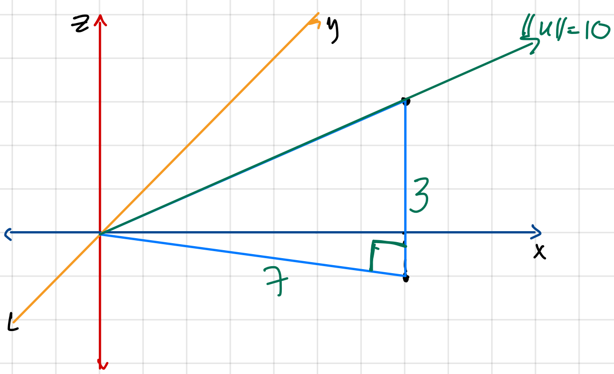
$\boxed{Q\ (5, 2, 8) \quad 5\hat{i} + 2\hat{j} + 8\hat{k}}$

99) Given

$v = \langle 7, -1, 3 \rangle$, $\|u\| = 10$

u & v are in the same direction

find $\vec{u}$



$u = \hat{k} \cdot v$

$= \hat{k} \langle 7, -1, 3 \rangle$

$\|u\| = \| \langle 7\hat{k}, -\hat{k}, 3\hat{k} \rangle \|$

$= \sqrt{49\hat{k}^2 + \hat{k}^2 + 9\hat{k}^2} = \sqrt{59\hat{k}^2} = \sqrt{59}\,\hat{k}$

$\|u\| = 10$

$\frac{10}{\sqrt{59}} = \hat{k}$

$\boxed{u = \left(\frac{70}{\sqrt{59}}, -\frac{10}{\sqrt{59}}, \frac{30}{\sqrt{59}} \right)}$

115) Given

$F_1 = \langle 10, 6, 3 \rangle$

$F_2 = \langle 0, 4, 9 \rangle$

$F_3 = \langle 10, -3, -9 \rangle$

$F_1 + F_2 + F_3 + F_4 = \emptyset$

find F_4 so that

$\Sigma F_1 F_2 F_3 F_4 = \emptyset$

$\langle 10, 6, 3 \rangle + \langle 0, 4, 9 \rangle + \langle 10, -3, -9 \rangle + \langle x, y, z \rangle = \langle 0, 0, 0 \rangle$

$\langle 20 + x, 7 + y, 3 + z \rangle = \langle 0, 0, 0 \rangle$

$x = -20 \quad y = -7 \quad z = -3$

$\boxed{F_4 = \langle -20, -7, -3 \rangle \text{ N}}$

$= -20\hat{i} - 7\hat{j} - 3\hat{k}$

#125, 131, 145, 171, 176

#125) Given

$u = \langle 2, 2, -1 \rangle \quad v = \langle -1, 2, 2 \rangle$ find $u \cdot v$

$2(-1) + 2(2) + -1(2)$

$-2 + 4 + -2 = \emptyset$

$\boxed{u \cdot v = \langle 0, 0, 0 \rangle}$

#131) find θ btwn a & b

express in radians

is θ acute?

Given: $a = \langle 3, -1 \rangle$

$b = \langle -4, 0 \rangle$

$$\cos\theta = \frac{a \cdot b}{\|a\| \, \|b\|}$$

$$\frac{3(-4) + (-1)(0)}{\sqrt{(3)^2 + (-1)^2}\sqrt{(-4)^2 + (0)^2}}$$

$$\frac{-12}{\sqrt{10}\quad\sqrt{16}} = \frac{-12}{4\sqrt{10}} = -\frac{3}{\sqrt{10}}$$

$$\boxed{\arccos\left(-\tfrac{3}{\sqrt{10}}\right) = 2.82 \text{ rad}}$$

not acute

#145 Given

$b = \langle 3, 4 \rangle$

find all 2-D vectors a

orthogonal to vector b

express in component form

$a \cdot b = \langle a_1\ a_2 \rangle \cdot \langle 3, 4 \rangle$

$3a_1 + 4a_2 = 0$

$3a_1 = -\dfrac{4a_2}{3}$

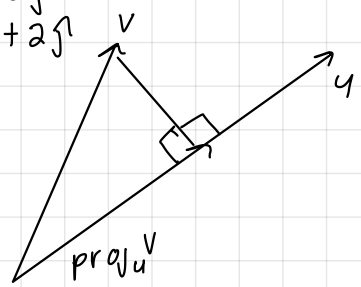
$a_1 = -\dfrac{4a_2}{3}$

$$\boxed{a = \left\langle -\frac{4a_2}{3}, a_2 \right\rangle = \left\langle -\frac{4a}{3}, a \right\rangle \qquad a \neq 0,\ a = \mathbb{R}}$$

#171 Given

$u = 4\hat{i} - 3\hat{j}$

$v = 3\hat{i} + 2\hat{j}$



$$\text{proj}_u v = \frac{u \cdot v}{\|u\|^2} u$$

$$= \frac{\langle 4, -3 \rangle \cdot \langle 3, 2 \rangle}{4^2 + (-3)^2} (4\hat{i} - 3\hat{j})$$

$$= \frac{12 - 6}{25} (4\hat{i} - 3\hat{j})$$

$$= \frac{6}{25} (4\hat{i} - 3\hat{j}) = \boxed{\text{proj}_u v = \left\langle \frac{24}{25} - \frac{18}{25} \right\rangle}$$

$w = \text{proj}_u v$

$$q = v - w = \langle 3, 2 \rangle - \left\langle \frac{24}{25}, \frac{-18}{25} \right\rangle$$

$$\left\langle \frac{51}{25}, \frac{68}{25} \right\rangle$$

w + q

$$\boxed{v = \left\langle \frac{24}{25}, -\frac{18}{25} \right\rangle + \left\langle \frac{51}{25}, \frac{68}{25} \right\rangle}$$

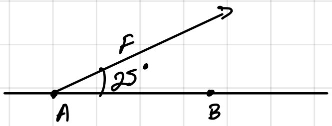
#176

Given

$F = 100$

$\alpha = 25°$

$AB = 40$



$W = \|F\| \cdot \|\overrightarrow{AB}\| \cos\alpha$

$100 \cdot 40 \cos 25$

$\boxed{\approx 3625.2 \text{ N}}$