

Home work:

1. Find β_0, β_1 :

$$f(\beta_0, \beta_1) = \sum_{i=1}^n (y_i - \beta_0 - \beta_1 x_i)^2$$

To find β_0, β_1 , we'll minimize the function f . In other words, we'll need to evaluate the following partial derivatives and set them to zero:

$$\left\{ \begin{array}{l} \frac{\partial f}{\partial \beta_0} = -2 \sum (y_i - \beta_0 - \beta_1 x_i) = 0 \quad (1) \\ \frac{\partial f}{\partial \beta_1} = -2 \sum x_i (y_i - \beta_0 - \beta_1 x_i) = 0 \quad (2) \end{array} \right.$$

(1) is equivalent to:

$$\sum y_i - n\beta_0 - \beta_1 \sum x_i = 0$$

$$\Leftrightarrow \bar{y} - \beta_0 - \beta_1 \bar{x} = 0 \quad (\text{both sides divided by } n)$$

$$\Leftrightarrow \boxed{\beta_0 = \bar{y} - \beta_1 \bar{x}} \quad (3)$$

(2) can be expanded as:

$$\sum x_i y_i - \beta_0 \sum x_i - \beta_1 \sum x_i^2 = 0$$

Divide both sides by n , we have

$$\frac{1}{n} \sum x_i y_i - \beta_0 \bar{x} - \beta_1 \frac{1}{n} \sum x_i^2 = 0$$

Replace $\beta_0 = \bar{y} - \beta_1 \bar{x}$ into the above eqn, it becomes:

$$\frac{1}{n} \sum x_i y_i - \bar{x} (\bar{y} - \beta_1 \bar{x}) - \beta_1 \frac{1}{n} \sum x_i^2 = 0$$

$$\Leftrightarrow \beta_1 \left(\bar{x}^2 - \frac{1}{n} \sum x_i^2 \right) = \bar{x} \bar{y} - \frac{1}{n} \sum x_i y_i$$

$$\Leftrightarrow \beta_1 = \frac{\frac{1}{n} \sum x_i y_i - \bar{x} \bar{y}}{\frac{1}{n} \sum x_i^2 - \bar{x}^2}$$

$$\Leftrightarrow \beta_1 = \frac{\frac{1}{n} \sum x_i y_i - \bar{x} \bar{y}}{\frac{1}{n} \sum x_i^2 - \bar{x}^2} \quad (\text{divided by } \left(\bar{x}^2 - \frac{1}{n} \sum x_i^2 \right) \text{ and multiplied by } -1).$$

$$\beta_1 \text{'s numerator} = \frac{1}{n} \sum x_i y_i - \bar{x} \bar{y} - \bar{x} \bar{y} + \bar{x} \bar{y}$$

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$$= \frac{1}{n} \sum x_i y_i - \bar{x} \frac{1}{n} \sum y_i - \bar{y} \frac{1}{n} \sum x_i + \bar{x} \bar{y}$$

Multiply both sides with n :

$$n\beta_1 =$$

$$\begin{aligned} n\beta_1 \text{'s num} &= \sum x_i y_i - \bar{x} \sum y_i - \bar{y} \sum x_i + n\bar{x}\bar{y} \\ &= \sum (x_i y_i - \bar{x} y_i - \bar{y} x_i + \bar{x}\bar{y}) \\ &= \sum [x_i (y_i - \bar{y}) - \bar{x} (y_i - \bar{y})] \\ &= \sum (x_i - \bar{x}) (y_i - \bar{y}). \end{aligned}$$

Similarly: $n\beta_1 \text{'s denominator} = \sum (x_i - \bar{x})^2$

$$\Rightarrow \boxed{\beta_1 = \frac{\sum (x_i - \bar{x}) (y_i - \bar{y})}{\sum (x_i - \bar{x})^2}}$$

Ex 4.2 | $\hat{w} = -79.24 + 4.16h$, $R^2 = 0.72$, $SE_R = 12.6$

a) $\hat{w}(64) = 187(16)$

$$\bar{w}(68) = 203.64(16)$$

$$\bar{w}(72) = 220.24(16)$$

b) $\Delta w = \beta_1 \Delta h = 4.16 \times 2 = 8.32(16)$


```
Lecture_3_1_Chapter4.R x probability-theory.r* x linear-reg-with-one-regressor.r* x e42-a_through_d.r* x
Source on Save Run Source
2 setwd("/Users/joal/Downloads/Econometrics - Course material/Data/")
3
4 getwd()
5
6 library(foreign) # to read dta files
7 library(tidyverse) # to apply filters
8
9 data <- read.dta("Earnings_and_Height.dta")
10
11 head(data)
12 summary(data)
13
14
15
16 ### START OF QUESTION A ###
17
18 # What is the median value of the height in the sample?
19
20 heights <- data$height # set a variable for reuse
21
22 median_height <- median(heights)
23 median_height # 67
24
25 ### END OF QUESTION A ###
26
13:1 (Top Level) R Script
```

```
Console Terminal Background Jobs x
R 4.5.1 · ~/Downloads/Econometrics - Course material/Data/
Error: unexpected symbol in:
"t.test(heights_greater_than_67$earnings, heights_at_most_67$earnings
t.test"

> t.test(heights_greater_than_67$earnings, heights_at_most_67$earnings)

Welch Two Sample t-test

data: heights_greater_than_67$earnings and heights_at_most_67$earnings
t = 13.59, df = 16624, p-value < 2.2e-16
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 4706.237 6292.643
sample estimates:
mean of x mean of y
 49987.88  44488.44

> |
```

Environment History Connections Tutorial

Import Dataset 528 MiB

R Global Environment

difference	5499.43990178219
ES	7
estimated_slope	Named num 708
EX	1.5
heights	num [1:17870] 65 65 60 67 68 63 67 65 67 66 ...
intercept	Named num -513
k	int [1:11] 0 1 2 3 4 5 6 7 8 9 ...
mean_of_heights_at_mos...	64.0738580185881
median_height	67
n	420L
predictions	Named num [1:3] 46901 49024 45486
prob	num [1:11] 0.000977 0.010742 0.054687 0.171875 0.376953 ...

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