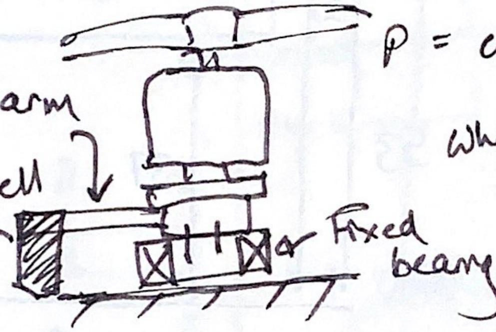
-> Check motor power/voltage

-> Power draw ~ Each motor? ~ESC?

-> Motor rpm ~ via ESC? or Auto pilot?

-> Calibrate efficiency of each motor with static torque, ω, η test.



$P = \omega T = f(VI, \omega)$

where VI is measured upstream of ESC

BUY

-> Load cell, get sensitive.

Look at megs, more sensitive than that Amazon?

-> Lightweight pressure scanners

-> Evolution Measurements.

-> postage stamp sized 8ch?

- -> Own branding

-> store data?

-> serial? Output?

-> Arduino maybe?

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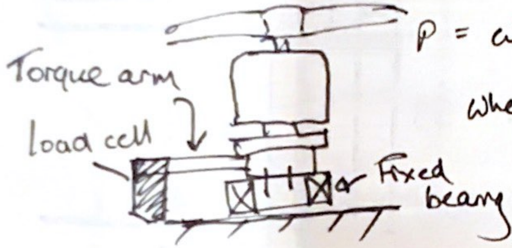

BUY

→ Load cell, get sensitive.

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→ Lightweight pressure scanners

→ Evolutron Measurements.

→ postage stamp sized 8ch?

↳ Own branding

↳ Store data?

↳ serial? Output?

↳ Arduino maybe?

→ Consider pressure sensing wrt tappings

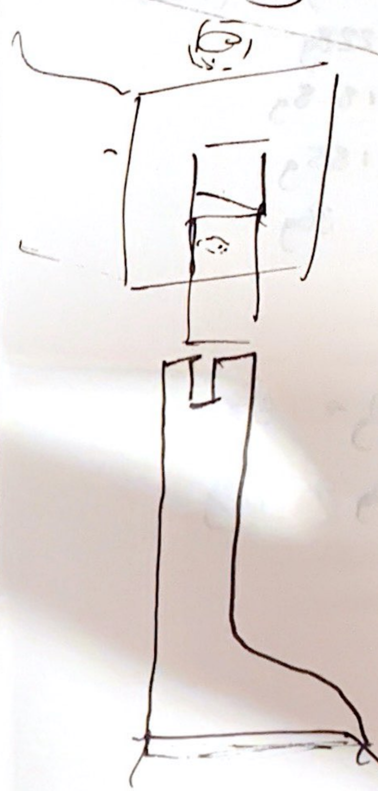
→ Make test mating joint. Use assembly

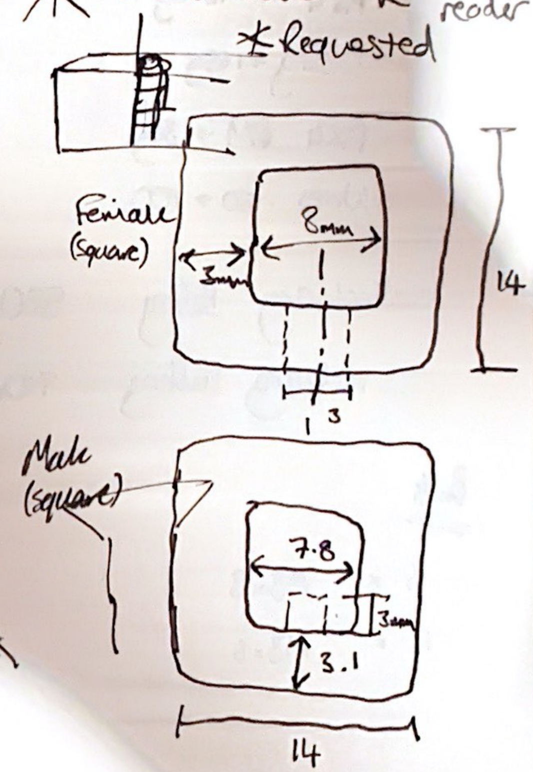
→ Bingo. Asst Megatron.

→ Assembly in CURA to change infills.

→ Load cells & SD reader

*Requested





Weights

Adaptor →	~5g ×4	20g	
Arm →	15g ×4	60g	
Base →	~30g	30g	
Electronics →			
ESC →	57g ×4	228g	(4×20g = 80)
Motor →	57g ×4	228g	
Pix4 →	15.8g	15.8g	
Battery →	188g	188g	
PX4 PM →	36g	36g	
Wires	50 → 100		

∴ Excluding Battery 520g → 570g

including battery 710g → 760g

Bolts

16 × M3×8

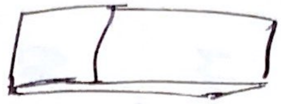
12 × M3×6

Charger

→ Torsion Load cell

→ Load cell on arm

Inventory

3x3cm 

WOOD

① ~~2.5m~~ → 2.4m × 6 ✓
② 2.18m × 4 ✓
③ 1.90m ~~× 6~~ × 7 ✓
④ 0.90m × 3

14 × 2.4m
3 × 3m

SCREWS

~5cm wood ~ ×80
Hinges × 3.

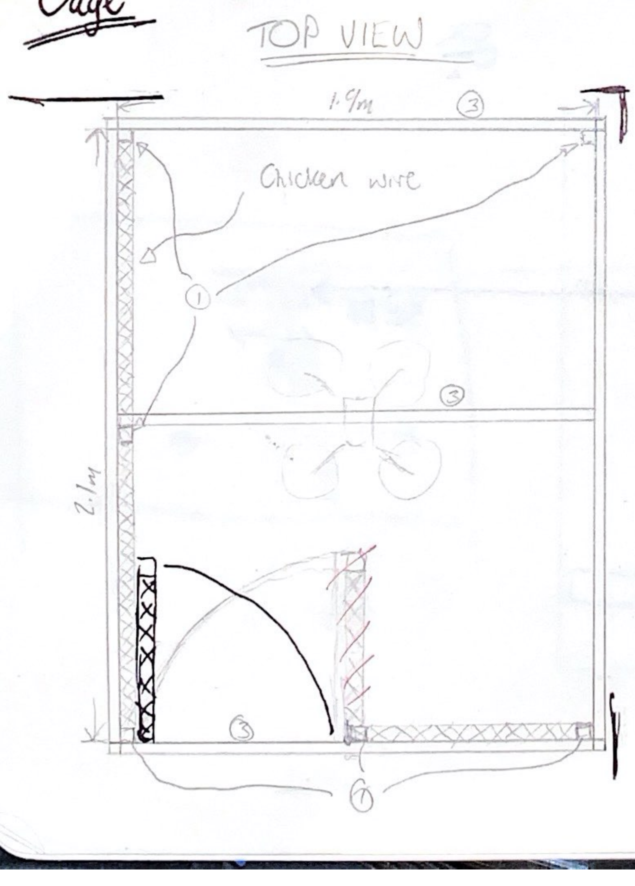
CHICKEN WIRE ~> ~~1cm~~ grid

Ⓐ 2.16m × 2.5m ×1 (1.08 × 2.5 × 2)
Ⓑ 0.96m × 0.5m ×1
Ⓒ 0.96m × 2.5m ×1
Ⓓ 0.96m × 1.96m ×1
Ⓔ ~1.1m × 1.96m ×2

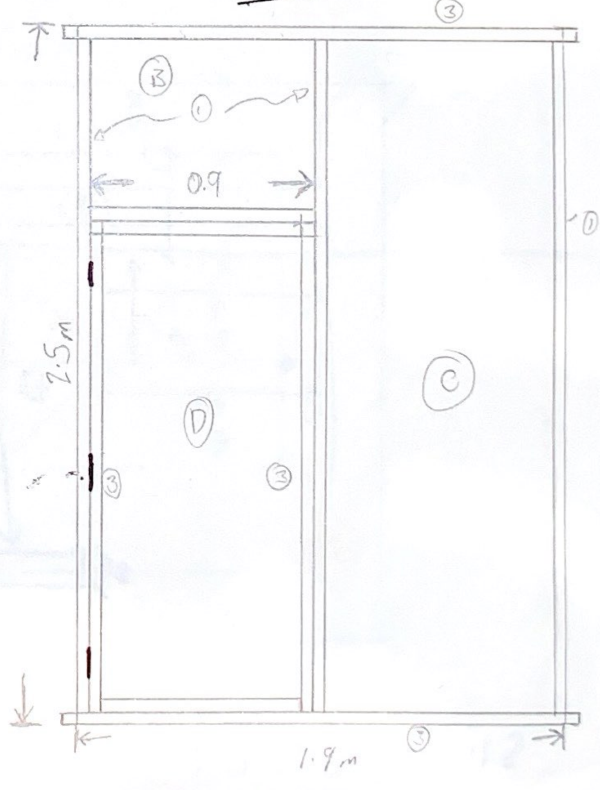
6x 8ft × 4ft
1" holes

~£177 Amazon

Cage



FRONT



Chat with Pawel @ RR

GCS → Q Ground Control

~~Sensor~~

RPM

→ optical sensors on ali tape stuck on half of motor

Power

→ Use on board V & A measurements

→ How accurate?

→ How to get end power out, shaft power?

Pressure

→ Mini pressure sensor, talk to RPi

Position Hold

→ Use Maxbotix I²C XL Max Sonar ultrasonic sensor

→ How do I incorporate the sensors into the FC?

→ Tuning PID?

RPi

→ Sensible to use RPi for sensor management?

→ Sychronizing data?

- Log rate on FC vs Log rate on RPi?

Pawel

- EWTOL
- Sensor integration
- Plug in from Simulink

→ Simulink
  - Long time to debug
  - Need to develop
  - Contact Matlab to get help.
  - Can use

→ Sensors
  - Use PWM, 6 analogue output sensors on PM4

→ Write control software on RP:
Can incorporate with simulink.

→ Carmen filters?

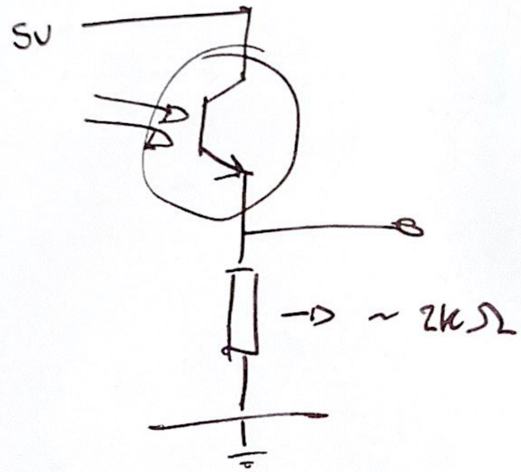
Power

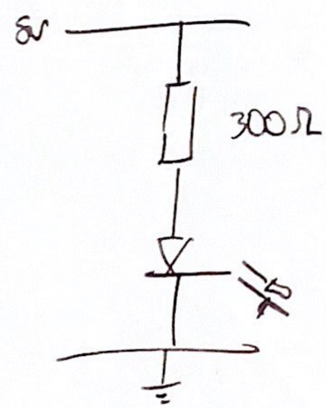
- Calibrated on rig

PWM/input to speed
to power.

// 3-4 MHz dongle for serial comms.

Photo transistor





$r = f \times 60$

$\frac{1}{T} = f$

time → rpm

time → f

FIXED QUANTITIES

ϕ, φ $\qquad \phi = \dfrac{V_x}{u}$

$$\varphi = \frac{1}{2}\frac{V_x^2}{u^2}$$

T_T $\qquad T_T = \dfrac{mg}{4}$

$$T_T = T_{in} + T_{fan} + T_{diff}$$

Others

σ $\qquad$ Diffusion ratio

$$= \frac{A_e}{A_2} = \frac{A_4}{A_3}$$

RPM/POWER

$$\text{Weight} = fn\left(\sigma, \overset{\sim ?\ \therefore L}{r_c}, r_h\right)$$

$(T-W)$ fixed at ≥ 5

A_2/A_1 vs L/h_1

$h_1 = r_c - r_n$

L = length of diffuser

Data Points

0.9, 1.33

1, 1.35

2, 1.48

3, 1.59

4, 1.68

5, 1.76

8, 2.0

10, 2.1

30, 3

$$\sigma = A\left(L/h_i\right)^B + C$$

L/h_i = AR

$AR = L/h_i$

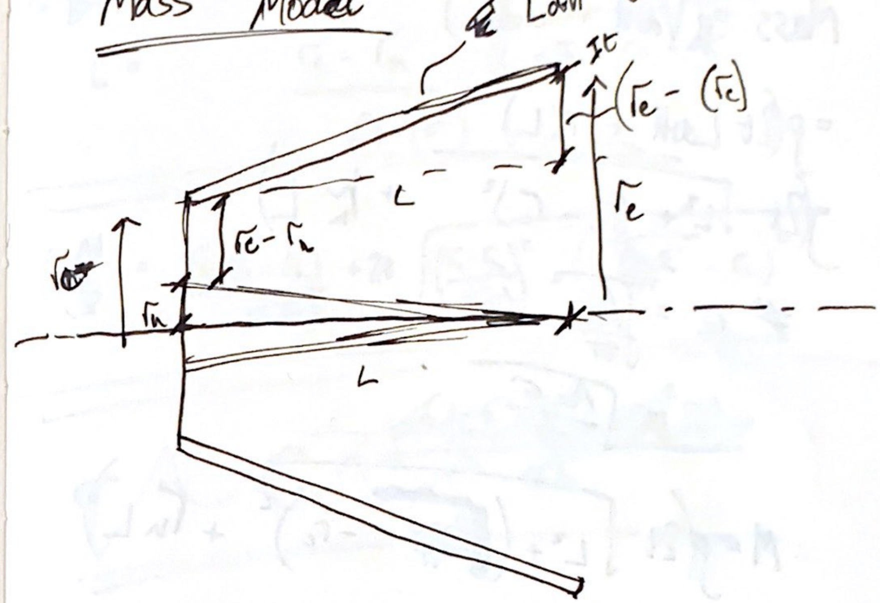
$$\sigma = A_2/A_1 = A_e / \pi(r_c^2 - r_n^2)$$

$$\left(\frac{\sigma - C}{A}\right)^{1/B} L = (r_c - r_n)$$

$h_i = r_c - r_n$

Mass Model

$L_{diff} = \sqrt{L^2 + (r_e - r_c)^2}$



$\therefore$ Volume of end cone $\sim \frac{1}{3} h \pi r^2$

$$V_{cone} = 2 \times \frac{r_n L}{2} = r_n L$$

$\therefore$ Volume of diffuser

$V_{diff} = 2 \times \ldots$

$A_e = \pi r_e^2$

$\therefore V_{diff} = 2 \times L_{diff} \times t$

$\therefore$ <u>Diffuser weight</u>

$$Mass = \rho(V_{diff} + V_{cone})$$

$$= \rho(2t\, L_{diff} + r_n L)$$

$$= \rho\left(2t\sqrt{L^2 + (r_e - r_c)^2} + r_n L\right)$$

$$\# \; r_e = \sqrt{\frac{A_e}{\pi}}$$

$$= \sqrt{\frac{A_x \sigma}{\pi}}$$

$$\therefore M = \rho\left(2t\sqrt{L^2 + \left(\sqrt{\frac{A_x\sigma}{\pi}} - r_c\right)^2} + r_n L\right)$$

$$\frac{M}{\rho} = 2t\sqrt{L^2 + \left(\sqrt{\frac{A_x\sigma}{\pi}} - r_c\right)^2} + r_n L$$

$$\# \; \underbrace{\left(\frac{\sigma - c}{A}\right)^{-1/3}}_{\to DF} \cdot L + r_n = r_c$$

$$\therefore \frac{M}{\rho} = 2t\sqrt{L^2 + \left(\sqrt{\frac{A_x\sigma}{\pi}} - r_n - L\cdot DF\right)^2} + r_n L$$

$$\frac{M}{\rho} = r_n L + 2t\sqrt{L^2 + \left(\sqrt{\frac{A_x\sigma}{\pi}} - r_n - L\cdot DF(\sigma)\right)^2}$$

\# BUT

$$L = \frac{r_c - r_n}{DF(\sigma)} \quad \Bigg\| \; \begin{array}{l} - r_n - (r_c - r_n) \\ = \cancel{-r_n} - r_c + \cancel{r_n} \end{array}$$

$$\therefore \frac{M}{\rho} = \frac{r_n(r_c - r_n)}{DF(\sigma)} + 2t\sqrt{\left(\frac{r_c - r_n}{DF(\sigma)}\right)^2 + \left(\sqrt{\frac{A_x\sigma}{\pi}} - r_c\right)^2}$$

$$\sigma = \phi^2 \; \cancel{\frac{\rho\pi(r_c - r_n)(r_c + r_n)^2\Omega^2}{4\pi}}$$

$$\frac{\phi^2}{\varphi} = 2\sigma^2 \Rightarrow \sigma = \frac{\phi}{\sqrt{2\varphi}}$$

In the unit of seperation, diffuser mass can be written wrt ϕ, φ, r_c where $DF = fn(\phi, \varphi)$

$$\left(\frac{M}{\rho_\rho}\right) = \frac{r_n(r_c - r_n)}{DF()} + 2t\sqrt{\left(\frac{r_c - r_n}{DF}\right)^2 + \left(\sqrt{\frac{A_x\sigma}{\pi}} - r_c\right)^2}$$

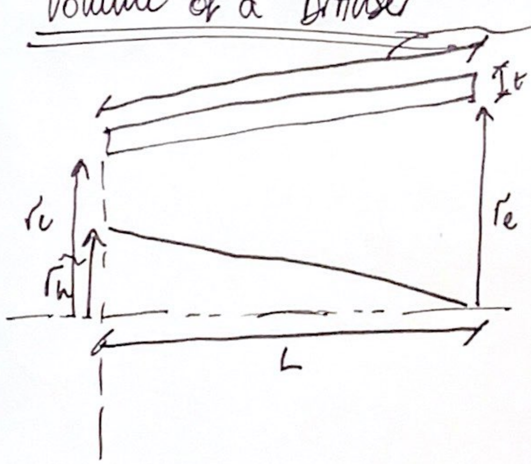
$$\frac{M}{\rho} = \frac{r_h(r_c - r_h)}{DF} + 2t\sqrt{\left(\frac{r_c - r_h}{DF}\right)^2 + \left(\frac{(r_c^2 - r_h^2)\phi}{\sqrt{2\varphi}} - r_c\right)^2}$$

$$A(r_c - r_h)^2 + (B r_c^2 - r_h^2 B - r_c)^2$$

$$= A(r_c - r_h)^2 + r_c^4$$

~~Area~~

Volume of a Diffuser



$$L_{diff}^2 = L^2 + (r_e - r_c)^2$$

$$L_{diff} = L^2 + (r$$

$$= \sqrt{L^2 + \left(\sqrt{\frac{A_x \sigma}{\pi}} - r_c\right)^2}$$

$$= \sqrt{L^2 + \left(\sqrt{\frac{\phi^2(r_c^2 - r_h^2)}{\sqrt{2\varphi}}} - r_c\right)^2}$$

$$A_e = \pi r_e^2$$

$$A_x = \frac{\pi r_e^2}{\sigma}$$

$$\therefore r_e = \sqrt{\frac{A_x \sigma}{\pi}}$$

$$A_x = \pi(r_c^2 - r_h^2)$$

$$\sigma = \frac{\phi^2}{\sqrt{2\varphi}}$$

$$\therefore V_{diff} \approx L_{diff} \cdot t \cdot 2\pi r_e$$

$$\approx L_{diff} \cdot t \cdot \cancel{2\pi\sqrt{\frac{A_x\sigma}{\pi}}} \quad 2\pi\sqrt{\frac{\phi^2(r_c^2 - r_h^2)}{\sqrt{2\varphi}}}$$

$$= L_{diff} \cdot t \cdot \cancel{\sqrt{4\pi A_x \sigma}}$$

$$= 2\pi t\sqrt{\frac{\phi^2(r_c^2 - r_h^2)}{\sqrt{2\varphi}}} \cdot \sqrt{\left(\frac{r_c - r_h}{DF}\right)^2 + \left(\frac{\phi^2(r_c^2 - r_h^2)}{\sqrt{2\varphi}} - r_c\right)^2}$$

$$V_{cone} = \frac{1}{3} h \pi r^2$$

$$= \frac{1}{3} L \pi r_h^2$$

$$= \frac{\pi r_h^2 (r_c - r_h)}{3DF}$$

RPI 3 A+ Header Board

	+3.3V	+5V	RPM +Ve (Red)
	SDA	+5V	Telem 2 +Ve (Red)
	SCL	⏚	Telem 2 Gnd (Black)
	GPIO 4	TXD	Telem 2 TXD (Green)
(Gnd) RPM	⏚	RXD	Telem 2 RXD (Blue)
	GPIO 17	18	RPM (Yellow)
	GPIO 27	⏚	RPM GND (Purple)
	GPIO 22	23	LC (Yellow)
	+3.3V	24	LC (Orange)
	10	⏚	LC (Purple)
	9	25	
	11	8	
	⏚	7	
	ID_SD	ID_SC	
	5	⏚	
	6	12	
	13	⏚	
RPM Motor 1	19	16	RPM Motor 3
RPM Motor 4	26	20	RPM Motor 2
	⏚	21	

RPi Function index

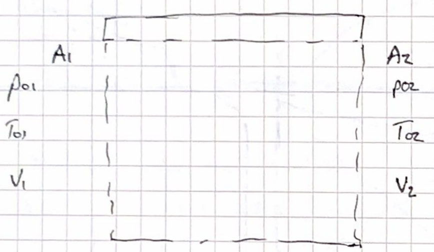
~/drone/

rpm-countlow.py	Use rpm sensors, output either rpm at each motor & time stamp, or print sensor states
rpm-sensor.py	Use gap ~~measureme~~ to determine rev #. Replaced by rpm-countlow.py.
poshold.py	test script for MAVProxy module mavproxy_cmdlong.py

$\phi = \frac{V_x}{U}$ $\psi = \frac{\Delta h_0}{U^2}$ $\Lambda = 0.5$

$\dot{m} = \rho A V_x$

Assume 1:1 diffuser $\therefore A_1 = A_2 = A$



Assume $T_{01} = T_{02}$ ie isentropic expansion

$\therefore \Delta h_0 \sim \frac{\Delta P_0}{\rho}$

Due to:

$\cancel{Tds}^{\,0} = dh - \frac{dp}{\rho}$

$dh = \frac{dp}{\rho}$

$\Delta h_0 = U \Delta V_\theta = \frac{W_x}{\dot{m}}$

$\Delta h_0 \dot{m} = W_x$ $W_x =$

SFME p_1 far field $= p_{01}$

$F = \dot{m}_2 V_2 - \dot{m}_1 V_1 + (p_2 - p_1) A$

$= \dot{m}(V_2 - \cancel{V_1}^{\,0}) + (p_2 - p_{01}) A$

$= \dot{m} V_2 + (p_2 - p_{01}) A$

$T = \frac{Mg}{4} \approx \frac{2 \cdot 9.81}{4}$

$= 4.905$

$T = \rho A V_2^2 + (\overset{p_a}{p_2} - p_{01}) A$

$= A(\rho V_2^2 + \ldots)$

Assume incompressible

$p_2 = p_a + \frac{1}{2}\rho V_2^2$

$\therefore T = A\rho V_2^2 + \frac{1}{2} A\rho V_2^2$

$= \frac{3}{2} A\rho V_2^2$

$\therefore V_2^2 = \frac{2T}{3A\rho}$

$V_2 = \sqrt{\frac{2T}{3A\rho}}$

WRONG

$\phi = \frac{V_x}{U}$

$\psi = \frac{\Delta h_0}{U^2}$

$\Lambda = \frac{\Delta h\ \text{rotor}}{\Delta h\ \text{stage}} = 0.5$

$\Omega = 4000$ rpm

$= \frac{4000}{60} \cdot 2\pi$

$\Delta h_0 \sim \frac{\Delta p_0}{\rho}$

$\therefore \Delta h_0 \simeq \frac{1}{2} V_2^2$

$\therefore \psi \sim \frac{\frac{1}{2} V_2^2}{U^2}$

PROJECT MEETING

→ Blade count

→ Flow coeff. ? Lower?

Hows the efficiency? → UROP.

→ Jonny's report
  Fritz
  - Check thrust.

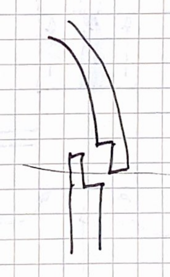
BL write IBL Jonny

THIS WEEK

→ Turbo Comp. ~

→ Latex compressor design ✓

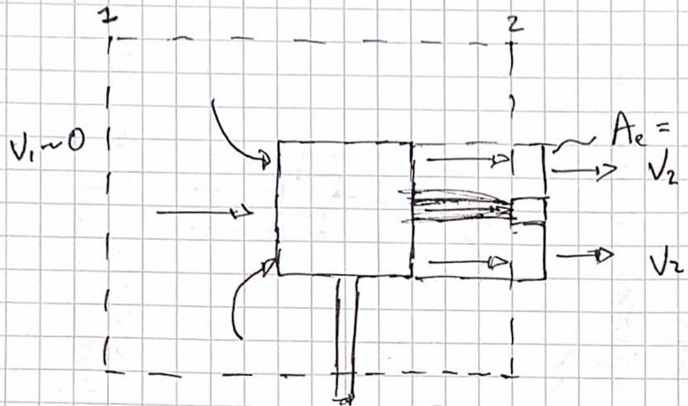
→ Scripts. ~



// Free Vortex const. rV_θ at exit
~~See~~ See where in code & change.

CONTROL VOLUME ANALYSIS

Upstream velocity & pressure ~ zero, atmospheric respectively.



$A_e = \pi(r_c^2 - r_n^2)$

$\dot{m} = \rho A_e V_2$

T_T ~ on Arm ∴ equal opposite on fluid

ρ = const (Low V, $M \ll 1$)

$\therefore P_0 = P_2 + \frac{1}{2}\rho V^2$

CV large enough such that $\dot{m}_n V_n \simeq 0$ for all circumference excluding A_e.

$$\therefore T_T = \dot{m}(V_2 - \cancel{V_1}^{\to 0}) + (P_2 - P_{01})A_e \qquad P_2 + \tfrac{1}{2}\rho V_2^2 = P_1 + \tfrac{1}{2}\rho V_1^2$$

$$T_T = \dot{m}(V_2 - V_1) + A_e(P_2 - P_1) \qquad P_{02} - \tfrac{1}{2}\rho V_2^2 - (P_{01} - \tfrac{1}{2}\rho V_1^2)$$

$$= \Delta h_0 \rho - \tfrac{1}{2}\rho(V_2^2 - V_1^2)$$

$$= \tfrac{1}{2}V_2^2\rho - \tfrac{1}{2}\rho V_2^2 + \tfrac{1}{2}\rho V_1^2$$

$V_1 \approx 0$; $P_1 \simeq P_{01} \simeq P_a$; $\cancel{P_2 = P_{02} - \frac{1}{2}\rho V^2}$

$$P_2 + \tfrac{1}{2}\rho V_2^2 = P_1 + \cancel{\tfrac{1}{2}\rho V_1^2}^{\,0}$$

$$\Delta h_0 = \frac{\Delta P_0}{\rho} \qquad \Delta P_0 = P_{02} - P_{01} = P_2 + \tfrac{1}{2}\rho V_2^2 - \cancel{P_{01}} \qquad P_{02} = P_2 + \tfrac{1}{2}\rho V_2^2$$

$$P_2 = P_{01} = P_a$$

$$\Delta h_0 = \tfrac{1}{2}V_2^2$$

If assume exit static = inlet static

$P_2 - P_1 = 0$

$P_{02} - \frac{1}{2}\rho V_2^2$

$$T_p = \dot{m}(V_2 - V_1) + A_e(p_2 - p_1)$$

$$T\cancel{ds}^{\,0} = dh - \frac{dp}{\rho}$$

$$dh = \frac{dp}{\rho}$$

$$\therefore \Delta h_0 = \frac{\Delta p_0}{\rho} \qquad \Delta p_0 = p_{02} - \cancel{p_{01}}^{\,p_1}$$

$$= p_2 + \tfrac{1}{2}\rho V_2^2 - p_1$$

$$= \tfrac{1}{2}\rho V_2^2 + (p_2 - p_1)$$

$\therefore$ To maximise thrust, maximise $p_2 - p_1$ and V_2 ie diffuse flow downstream of fan. Most efficient when flow leaves $=$ ie parallel and pressure matched

and accelerate $\therefore$ Max p_{02} + max V_2

$$\Downarrow$$

$$p_2 = p_1$$

$$\therefore \Delta p_0 = \tfrac{1}{2}\rho V_2^2 \Rightarrow \Delta h_0 = \tfrac{1}{2}V_2^2$$

$$\therefore T_p = \dot{m}(V_2 - \cancel{V_1}^{\,0})$$

$$= \rho A_e V_2^2$$

$$\therefore V_2 = \sqrt{\frac{T_p}{\rho A_e}}$$

$$\phi = \frac{V_2}{U}$$

$$\psi = \frac{\tfrac{1}{2}V_2^2}{U^2}$$

$$\phi = \frac{V_2}{U} \qquad V_2 = \sqrt{\frac{T_p}{\rho A_2}} = \sqrt{\frac{T_p}{\rho\pi(r_c^2 - r_h^2)}}$$

$$U = \Omega\left(\frac{r_c + r_h}{2}\right)$$

$$\phi = \sqrt{\frac{T_p}{\rho\pi(r_c^2 - r_h^2)}} \cdot \frac{2}{\Omega(r_c + r_h)}$$

$$\psi = \frac{T_p}{\rho\pi(r_c^2 - r_h^2)} \cdot \frac{2}{\Omega^2(r_c + r_h)^2}$$

Mam

r_c
r_h
r_m
span
r.radius
HTR
AR_rotor
AR-stator
tip_gap-percentage
r.11
s.12
ac_r

$$DF \approx 1 - \frac{V_2}{V_1} + \frac{1}{2} \frac{\Delta V_\theta}{V_1} \frac{s}{c}$$

$$\Delta V_{\theta\_m} = |V_{\theta_2} - V_{\theta_1}|$$

Jonny's CV Analysis:

$$p_0 = p_{00} - \tfrac{1}{2}\rho V_0^2$$

$$p_4 = p_{04} - \tfrac{1}{2}\rho V_4^2$$

$$p_4 - p_1 = \tfrac{1}{2}\rho(V_1^2 - V_4^2)$$

CV Analysis $\rightarrow$ fan design

$p_2 = p_{atm}$

work in' $\therefore p_{02} \neq p_{01}$

$p_{atm} = p_{02} = p_2 \because V = 0$

2.1 "M< ..."

Vectorized graphics drawing program
Coral draw
Illustrator (mac OS

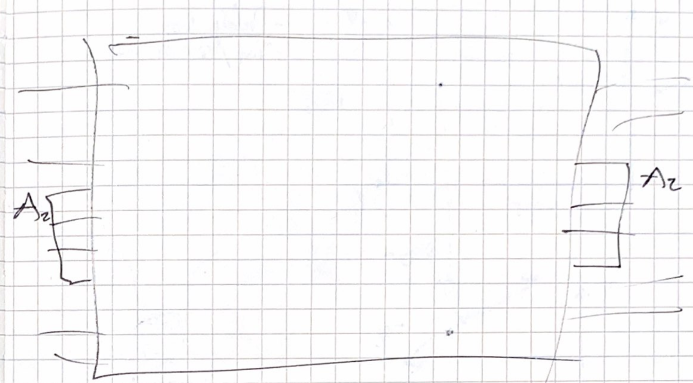
(11) & (12) Mean line

Define how (12)

$$\Delta h_0 = \frac{\Delta P_0}{\rho} \Rightarrow \frac{\frac{1}{2}\rho V^2}{\rho}$$

$$\Delta h_0 = \frac{\frac{1}{2}\rho V^2}{\rho}$$

$$\varphi = \frac{\frac{1}{2}V^2}{U^2}$$

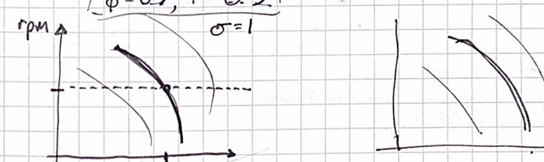


$$\psi = \frac{\Delta P_0}{\frac{1}{2}\rho U_{mid}^2}$$

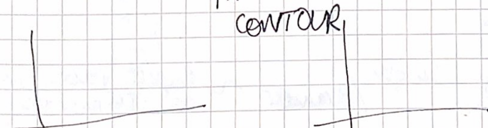
- Flip dependent variables so φ + ϕ are pre-set
- Get contours of the below.

$\boxed{\phi = 0.7,\ \psi = 0.3}$

$\sigma = 1$

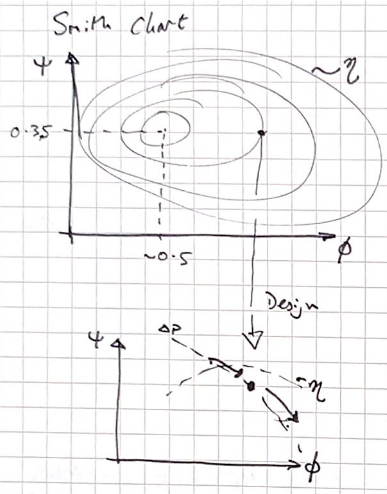


THRUST CONTOUR



+ Power to analysis. SFEE

Smith Chart



- Profile loss
- Tip/corner loss

$\sigma = 1.5 \Rightarrow P_{02} - P_{01}$ across turbomachine

- Get σ into equations. ✓
- Do power analysis as well as thrust ✓
- Contour maps of design. ✓

Simple mass model of diffuser

Diffusers

L/D length/diameter ~ Aspect ratio increases as DF increases

How does that trade off with weight?

Optimising weight vs pressure force from diff.

$$T_p = \rho A V_2^2 = \frac{\rho A V_\infty^2}{\sigma}$$

$V_2 =$

How does σ effect triangles?

Simple mass model relates σ to weight of diffuser. ⇝ AR limited by BL separation.

σ related to thrust by pressure change

$H = 1.8 + 3.75B$ $\qquad$ H = shape factor δ^*/θ

$B = \frac{2\delta^*}{R}$

Journies

Ref.: [14]

[16]

~~Fixed~~

RPM (or Power...) → Torque vs rpm
→ Work eqn.
(Matlab)

ϕ ~
ψ

} Linked as fn (σ) once it is included.

T → { Intake
Fan
Exit

See Sam's email

r_h

~~Vary~~

~~σ~~

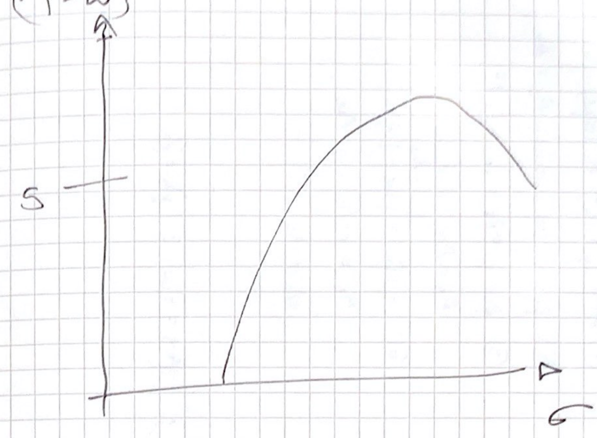
Put in σ, spits out r_c (via area)

Then from ESDU (expressed analytically)

σ → length from r_c & r_h as above.

→ length modelled to weight

(T-W)

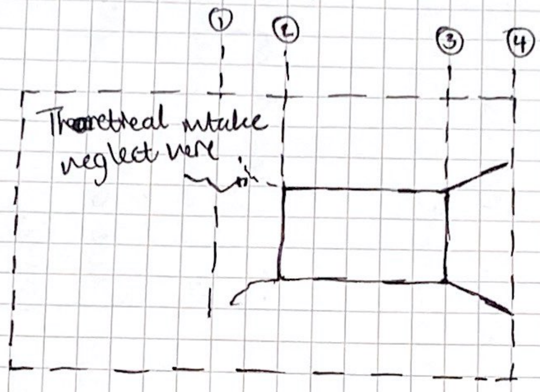


→ Want POWER optimised with constraint on (T-W)

→ ~~Then fix~~
L & r ∴ σ limited by mass model

∴ Vary r (hence σ) to optimise power.

CV Analyses with σ, OPTIMISE FOR MIN POWER wrt $r_c, (r_h)$



① ~~Atm~~ $p_1 = p_a = p_{01}$ A_1 large, $V_1 = 0$

② $p_2 = p_a$ (no intake) $A_2 = A_x$

③ $p_3 = p_3 \cancel{\ne p_a} = p_{03} - \frac{1}{2}\rho V_3^2$

$A_3 = A_x$, $V_3 = V_x$

④ $p_4 = p_a = \overset{= p_{03}}{p_{04}} - \frac{1}{2}\rho V_4^2$

$A_4 = \sigma A_3 = \sigma A_x$

$V_4 \ne V_x$

Relating ③ & ④

$p_3 = p_{03} - \frac{1}{2}\rho V_3^2$; $p_4 = p_{04} - \frac{1}{2}\rho V_4^2$; $p_{03} = p_{04}$; $p_4 = p_a$

$p_3 + \frac{1}{2}\rho V_3^2 = p_4 + \frac{1}{2}\rho V_4^2$

By continuity

$A_3 V_3 = A_4 V_4 \cancel{\rho}$

$\therefore \left(\frac{V_3}{V_4}\right)^2 = \sigma = \left(\frac{V_x}{V_4}\right)^2$ $\therefore V_x^2 = \sigma V_4^2$

SFME

$T_T = \dot{m}_4 V_4 \overset{-\dot{m}_1 V_1}{+} A_4 p_4 - A_1 p_1$ $\qquad p_4 = p_1$ $\therefore p_4 - p_1 = 0$

$= \dot{m}_4 V_4$

$= \rho A_4 V_4^2$

$= \rho A_4 \frac{V_x^2}{\sigma}$ $\qquad A_x = \frac{A_4}{\sigma}$

$= \frac{\rho A_x V_x^2}{\sigma}$

$\rho A_4 V_4^2 = \frac{\rho A_x V_x^2}{\sigma}$

$\dot{m} V_4 = \rho A_x V_x V_4$

$= \frac{\rho A_x V_x^2}{\sqrt{\sigma}}$

$\therefore V_x = \sqrt{\frac{T_T \sigma}{\rho A_x}}$ ✓

Flow coefficients; stage loading

$\phi = \frac{V_x}{U}$; $\psi = \frac{\Delta h_0}{U^2}$

Near isentropic pressure rise

$\therefore T ds^{\to 0} = dh - \frac{dp}{\rho}$

$\therefore dh_0 = \frac{dp}{\rho}$

$\Delta h_0 = \frac{\Delta P_0}{\rho} \quad \therefore \quad \psi = \frac{\Delta P_0}{\rho U^2}$

$\Delta P_0 = P_{03} - P_{02}$

$\Delta P_0 = P_3 + \frac{1}{2}\rho V_3^2 - P_a$

$$\begin{cases} P_{02} = P_a \\ P_{03} = P_3 + \frac{1}{2}\rho V_3^2 \end{cases}$$

$P_3 = P_4 + \frac{1}{2}\rho(V_4^2 - V_3^2)$ // from bernoulli

$= P_a + \frac{1}{2}\rho\left(\frac{V_3^2}{\sigma^2} - V_3^2\right) \qquad V_3 = V_x$

$\therefore \Delta P_0 = \frac{1}{2}\rho V_x^2\left(\frac{1}{\sigma^2} - 1\right) + \frac{1}{2}\rho V_x^2$

$= \cancel{\ldots} = \frac{1}{2}\rho V_x^2\left(\frac{1}{\sigma^2}\right)$

$\therefore \psi = \cancel{\ldots} = \frac{\frac{1}{2}\rho V_x^2}{\rho U^2 \sigma^2}$

$P_3 = P_a + \frac{1}{2}\rho\left(\frac{V_x^2}{\sigma} - V_x^2\right)$

$\Delta P_0 = \frac{1}{2}\rho\left(\frac{V_x^2}{\sigma} - V_x^2\right) + \frac{1}{2}\rho V_x^2$

$= \frac{1}{2}\rho\frac{V_x^2}{\sigma}$

$\therefore \psi = \frac{V_x^2}{2\sigma \cdot U^2}$

$\therefore \psi = \frac{V_x^2}{2U^2\sigma^2}; \quad \phi = \frac{V_x}{U}; \quad V_x = \sqrt{\frac{T_T}{\rho}\frac{\sigma}{A_x}}$

~~Redefine in terms of A_x & T_T~~

$\frac{\phi^2}{2\sigma} = \psi$

$\frac{\phi^2}{\psi} = 2\sigma$

$\phi = \sqrt{\frac{T_T\sigma}{\rho A_x U^2}}; \quad \psi = \frac{T_T}{2\rho A_x U^2 \sigma}$

Rearrange for σ

$$\left.\begin{aligned} \phi^2 \cdot \frac{\rho A_x U^2}{T_T} &= \sigma \\ \frac{T_T}{2\rho A_x U^2 \psi} &= \sigma \end{aligned}\right\} \quad \cancel{\frac{\phi^2}{\psi} = 2\sigma^2}$$

$A_x = \pi(r_c^2 - r_h^2)$

$\therefore$ Limit of diffusion factor can be determined by flow coeff & stage loading

$\sigma = \frac{\phi}{\sqrt{2\psi}}; \quad M = \rho\left[\frac{r_h(r_c - r_h)}{DF} + 2t\sqrt{\cancel{\left(\frac{r_c - r_h}{DF}\right)^2 + \left(\frac{A_x\phi}{\pi\sqrt{2\psi}} - r_c\right)^2}}\right]$

Optimise for mass wrt r_c Assume fixed: r_h, φ, ϕ

$$DF = \left(\frac{\sigma - C}{A}\right)^{-1/B} = \left(\frac{\phi}{2A\sqrt{2\varphi}} - \frac{C}{A}\right)^{-1/B}$$

$A = 0.2863$
$B = 0.5627$
$C = 1.06$
} Empirically from ESDU 75026

$$\cancel{\therefore M = \rho_{PLA}\left[\frac{r_h(r_c-r_h)}{DF} + 2t\sqrt{\left(\frac{r_c-r_h}{DF}\right)^2 + \left(\frac{(r_c^2-r_h^2)\phi}{\sqrt{2\varphi}} - r_c\right)^2}\right]}$$

Total Diffuser Volume

$$\left(V_{cone} + \underbrace{V_{thin\ shell}}_{\Downarrow\ t \times 2\pi r \times L}\right)$$

$$V_{total} = \frac{\pi r_h^2 (r_c^2 - r_h^2)}{3DF} + 2\pi t \sqrt{\frac{\phi(r_c^2-r_h^2)}{\sqrt{2\varphi}}} \cdot \sqrt{\left(\frac{r_c-r_h}{DF}\right)^2 + \left(\frac{\phi(r_c^2-r_h^2)}{\sqrt{2\varphi}} - r_c\right)^2}$$

$\therefore$ Mass = $V_{total} * \rho_{PLA}$

$$\cancel{\phi}\ \varphi = \frac{T_T}{2\rho A_x U^2 \sigma} \qquad \sigma = \frac{\phi}{\sqrt{2\varphi}}$$

$$= \frac{T_T\sqrt{2\varphi}}{2\rho A_x U^2 \phi}$$

$$A_x = \pi(r_c^2 - r_h^2)$$
$$U = \Omega\frac{(r_c+r_h)}{2}$$
$$U^2 = \frac{\Omega^2(r_c+r_h)^2}{4}$$

$$\phi\sqrt{\varphi} \cdot \sqrt{2}\rho A_x U^2 = T_T$$

$$\frac{\phi\rho\pi(r_c^2-r_h^2)\Omega^2(r_h+r_c)^2\sqrt{\varphi}}{\sqrt{2}} = T_T$$

$$\phi\sqrt{\varphi}\rho\pi\Omega\frac{(r_c^2-r_h^2)(r_c+r_h)}{\sqrt{2}} = T_T$$

$$\phi\sqrt{\varphi}\rho\pi\Omega\frac{(r_c-r_h)(r_c+r_h)^3}{\sqrt{2}} = T_T$$

$$\therefore \varphi = \frac{T_T \cdot \sqrt{2\varphi} \cdot 4}{2\rho\pi(r_c^2-r_h^2)\Omega^2(r_c+r_h)^2\phi}$$

$$\therefore T_T = \frac{\varphi\, 2\rho\pi(r_c^2-r_h^2)\Omega^2(r_c+r_h)^2\phi}{4\sqrt{2\varphi}}$$

$$= \frac{\sqrt{\varphi}\,\phi\,\rho\pi(r_c^2-r_h^2)\Omega^2(r_c+r_h)^2}{2\sqrt{2}}$$

$$L = \frac{r_c - r_h}{DF} \qquad \frac{\frac{\phi}{\sqrt{2\varphi}} - C}{A}$$

SPEE

$$\dot m\left(h_0 + \cancel{gz_0}^{\,0} + \frac{V_0^2}{2}\right) + \dot Q = \dot m\left(h_4 + \cancel{gz_4}^{\,0} + \frac{V_4^2}{2}\right) + \dot w \qquad \dot m = 1$$

$$h_4 - h_0 + \tfrac{1}{2}(V_4^2 - V_0^2) = \cancel{\dot Q}^{\,0} - \dot w \qquad \text{Assume } \dot Q = 0$$

$$\Delta h_0 + \tfrac{1}{2}V_4^2 = -\dot w$$

$$+ve \quad +ve \quad = -\dot w$$

$\dot w = -ve$ ie work done on fluid

$$\frac{\Delta P_0}{\rho} = \frac{1}{2}\frac{V_x^2}{\sigma} \qquad V_4^2 = \frac{V_x^2}{\sigma}$$

$$P = \frac{\tfrac{1}{2}\rho A V_x^3}{\sigma^2} = \tfrac{1}{2}\rho A T \qquad P_{sp} = \frac{\tfrac{1}{2}V_x^2}{\sigma^2} = \frac{\tfrac{1}{2}T\sigma}{\rho A \sigma^2}$$

$$P = \frac{T^{3/2}}{\sqrt{4\sigma\rho A}} \quad W \qquad W/kg$$

$$\therefore \Delta h_0 + \tfrac{1}{2}V_4^2 = -\dot w$$

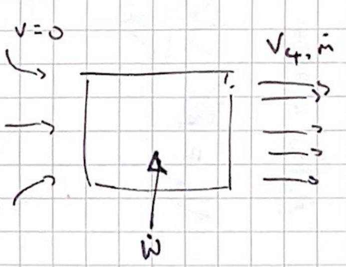
$$-\dot w = \frac{V_x^2}{2\sigma} + \frac{V_x^2}{2\sigma} = \frac{V_x^2}{\sigma} = \frac{T_T}{\rho A_x \sigma} = \frac{T_T}{2\rho A_x \sigma}$$

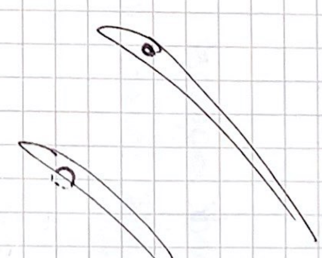
$$\therefore -\dot W = \frac{\sqrt{2\varphi}\,\varphi\, U^2}{\cancel{2\sigma}} = 2\varphi U^2 = \frac{2\varphi\Omega^2(r_c + r_h)^2}{4\,2}$$

$$\therefore |\dot W| = \frac{\varphi\Omega^2(r_c + r_h)^2}{2}$$

Omega vs r_c ✓

Fix omega, r_c vs r_h ?





$$\dot w = \tfrac{1}{2}\dot m V_4^2 = \tfrac{1}{2}\dot m \frac{V_x^2}{\sigma}$$

HTR ~ 0.4 as a maximum

effects ?

119.4
59.57

MT5W

Using loss models

produce S&S of ϕ vs ψ for loss.

Lo Smith chart.

A. Dicksen → Check out appendix as well.
Ch 9

→ well written

A bit beyond 4A3

→ Check against notes Sam sent!!

3 control volumes.

α ⟹ efficiency.

2. ΔP_0 rotor
3. ΔP_0 stator ✓ $A_3 = A_{x}$

5 $V_4 = \frac{V_3}{\sqrt{6}}$

$\sigma \rightarrow \sigma^2$

2.1 Mach # < ... ✓

2.2 (1) P_{atm}

2.3

2.4 (18)/(17) replace constants with a or b

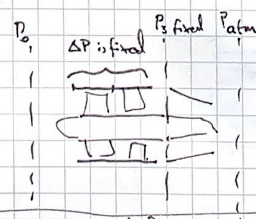
2.4.1 (21) Model rotors & stators as flat plates & relate to N

∴ relates to P/c - pitch chord.

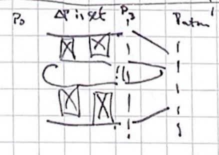
Trade off between weight + N

(22) Want thickness of diffuser also defined as predetermined variable.

(21) ~~Compare~~ Combine motor weight with rest of drone instead of propulsion



⟹ Gives σ

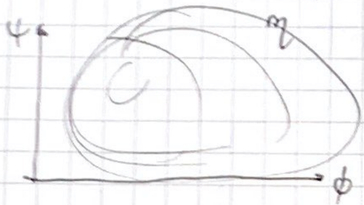


Ω $\Rightarrow$ r_c

r_h, r_c $\quad \sigma$

t

θ



See if combining M_f with η gives any optimum ~~ateams~~ in OP.

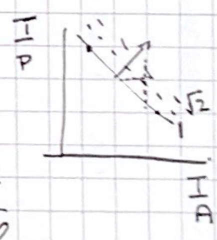
Use existing data for η, too prove concept before using Loss model.

$$M_f = \frac{M}{P}\sqrt{\frac{M}{2\rho A}}$$

$$\frac{1}{2}\,\frac{\rho A V_x^3}{\sigma^2} = \frac{1\,\rho A\, T^{3/2}\,\cancel{\sigma^{3/2}}}{4^{1/2}\rho^{3/2}A_x^{3/2}\,\cancel{\sigma^{1/2}}}$$

$$= \frac{T^{3/2}}{\sqrt{4\rho A_x \sigma}} = \frac{M^{3/2}}{\sqrt{4\rho A_x \sigma}}$$

$$\omega = \frac{T_T}{2\rho A_x \sigma} \qquad T_T = \sqrt{2\psi}\,\phi\,\rho A_x U^2$$



$$= \frac{\sqrt{2\psi}\,\phi\,\cancel{\rho A_x}U^2}{\sqrt{2}\,\cancel{\rho A_x}\,\sigma} = \frac{\sqrt{\psi}\,\phi\,U^2}{\sqrt{2}\,\sigma} \qquad \sigma = \frac{\phi^2}{2\psi}$$

$$= \frac{\sqrt{2}\,\psi^{3/2}\,\theta\,U^2}{\phi}$$

$$P = \eta\,\frac{\sqrt{2}\,\psi^{3/2}U^2}{\phi}\cdot \rho A_x V_x$$

$$M_f = \frac{T}{P}\sqrt{\frac{T}{2\rho A}}$$

σ

$\phi\ \psi\ r_c\ r_h\ \Omega\ \theta$

σ

$$P = \frac{T^{3/2}}{\sqrt{4\rho A_x \sigma}} \qquad T = \sqrt{2\psi}\,\phi\,\rho A_x U^2$$

$$\sigma^{1/2} = \frac{\phi}{\sqrt{2\psi}} \qquad \frac{\phi^2}{2\psi}$$

$$= \frac{(2\psi)^{3/4}\,\phi^{3/2}\,\rho^{3/2}\,A_x^{3/2}\,U^3}{2\,\rho^{1/2}\,A_x^{1/2}\,\sigma^{1/2}}$$

$$= \frac{\sqrt{2\psi}\,\psi^{3/4}\,\phi^{3/2}\,\rho A_x U^3}{2^{1/4}\,\phi}$$

$$= \frac{2^{3/4}\,\psi^{5/4}\,\rho A_x U^3\,\phi^{1/2}}{\cancel{\phi}}$$

$$= \sqrt{\sqrt{2^3\psi^5}\cdot\phi}\cdot\rho A_x U^3$$

$$= \sqrt{\sqrt{2^3\psi^5}\cdot\phi}\cdot\rho A_x U^3 = \sqrt{\sqrt{2^3\psi^5}\,\phi}\cdot\rho\pi(r_c^2 - r_h^2)\frac{\Omega^3}{8}(r_c + r_h)^3$$

$$P = \frac{\sqrt{2\psi\cdot T^3}}{\sqrt{4\rho A_x\,\phi}}$$

$$= \sqrt{\frac{\psi\cdot T^3}{\rho\,\cancel{2}\phi\,\cancel{(r_c+r_h)}\,\pi(r_c^2 - r_h^2)}}$$

$$= \sqrt{\frac{\psi T^3}{2\rho\phi\pi(r_c^2 - r_h^2)}}$$

4A3 Turbomachinery - Turbine Cascade

1.1 Free-stream Δp

$$p_2 - p_1 = (p_{01} - p_1) - (p_{01} - p_{02}) - (p_{02} - p_2)$$

$$\approx -200 \text{ mm } H_2O$$

$$\frac{p_1}{\rho_1^\gamma} = \frac{p_2}{\rho_2^\gamma}$$

$$\frac{p_1}{p_2} = \left(\frac{\rho_1}{\rho_2}\right)^\gamma \qquad p_2 = p_{atm} = \cancel{100126}\ 100130$$

$$|p_2 - p_1| = 200\times10^{-3}(1000)(9.81)$$

$$= 200(9.81)\ \text{Pa}$$

$$\therefore p_1 = 100130 + 200(9.81)$$

$$= 102092$$

$$\therefore \frac{p_1}{p_2} = 1.0196$$

$\gamma = 1.4$ # ~~as~~ Air, uncombusted

$$\therefore \frac{\rho_1}{\rho_2} = 1.0140$$

$\therefore$ change in $\rho \sim 1\%$

Within context of turbomachinery 1% changes should NOT be neglected.

$\rho A_x V_x = \rho A_4 V_4$ $\qquad A_x = \frac{A_4}{\sigma}$ $\therefore$

σA_x

$$\frac{A_x}{A_4} = \frac{V_4}{V_x} = \frac{1}{\sigma} \quad \therefore \quad \frac{V_x}{V_4} = \sigma$$

$$T_T = \frac{\rho A_x V_x^2}{\sigma^2}$$

$$\therefore V_x^2 = \sqrt{\frac{T_T \sigma^2}{\rho A_x}}$$

$$p_3 + \tfrac{1}{2}\rho V_x^2 = p_4 + \tfrac{1}{2}\rho V_4^2$$

$p_4 \to p_a$

$$\cancel{p_a + \tfrac{1}{2}\rho\left(V_x^2 - \frac{V_x^2}{\sigma^2}\right)} \quad p_3 = p_4 + \tfrac{1}{2}\rho\left(\frac{V_x^2}{\sigma^2} - V_x^2\right)$$

$$\Delta p_0 = p_3 + \tfrac{1}{2}\rho V_x^2 - p_a$$

$$= \left[\cancel{p_a} + \tfrac{1}{2}\rho\left(\frac{V_x^2}{\sigma^2} - \cancel{V_x^2}\right)\right] + \tfrac{1}{2}\rho \cancel{V_x^2} - \cancel{p_a}$$

$$= \frac{\rho V_x^2}{2\sigma^2}$$

$$\varphi = \frac{\Delta p_0}{\rho U_0^2} = \frac{V_x^2}{2U^2\sigma^2}$$

$$\sigma^2 = \frac{\phi^2 \rho A_x U^2}{T_T} \qquad \sigma^{-2} = \frac{T_T}{2\rho A_x U^2 \varphi}$$

$$2\rho^2 A_x^2 U^4 \phi^2 \varphi = T_T^2$$

$$\rho A_x \phi U^2 \sqrt{2\varphi} = T_T$$

$$\phi^2 = \frac{T_T \sigma^{-2}}{\rho A_x U^2} \qquad \varphi = \frac{T_T}{2\rho A_x U^2 \sigma^{-2}} \qquad 2\varphi\sigma^{-4} = \phi^2$$

$$\sigma^{-4} = \left(\frac{\phi^2}{2\varphi}\right)$$

$$\frac{\varphi}{\phi^2} = \frac{1}{\sigma^4}$$

$$\sigma^4 = \frac{\phi^2}{\varphi} \qquad \sigma = \sqrt{\frac{\phi}{\sqrt{\varphi}}} = \left(\frac{\phi^2}{\varphi}\right)^{1/4}$$

$$\phi^2 = \frac{T_T \sigma^2}{\rho A_x U^2} \qquad \psi = \frac{T_T}{2\rho A_x U^2}$$

$$\phi^2 = \frac{2\psi}{\cancel{m}} \frac{\sigma^2}{U^2} U^2 \qquad \frac{T_T}{\rho A_x} = \psi 2\rho U^2 = \frac{\phi^2 U^2}{\sigma^2}$$

$$\sigma^2 = \frac{\phi^2 \cancel{m}}{2\psi}$$

$$T_T = \frac{\phi^2 \rho A_x U^2}{\sigma^2}$$

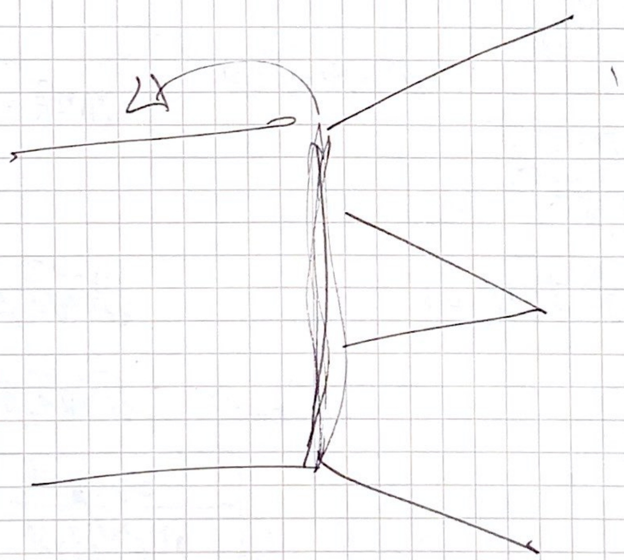
$$\phi^2 = \frac{T_T}{\rho A_x U^2} \cdot \frac{\phi^2}{2\psi}$$

$$T_T = 2\psi \rho A_x U^2 \qquad \phi \quad \sigma$$

$$T_T = \frac{\phi^2 \rho A_x U^2}{\sigma^2}$$

sigh

feigh



SAM'S EQUATIONS

$$P_i = \frac{T_{TOT}^{3/2}}{\sqrt{4\sigma}\,\rho A_x}$$

$$= \left[\frac{\phi^6 \rho^3 A_x^3 U^6}{4\sigma \rho A_x} \right]^{1/2}$$

$$= \frac{\phi^3 \rho A_x U^3}{2\sqrt{\sigma}} \cdot \sigma^{-3}$$

$$= \frac{\phi^3 \rho A_x U^2}{2\sigma^{3/2}}$$

$$T_T = \frac{\phi^2 \rho A_x U^2}{\sigma^2} = \underbrace{\phi^2 \rho A_x U^2}\,.\,\sigma^{-2}$$

kg m^{-3} m^2 m^{2}s^{-2} = kg m s^{-2}

$$\frac{(\text{kg m s}^{-2})^{3/2}}{(\text{kg m}^{-3}\text{ m}^2)^{1/2}} = \text{kg m}^2\text{s}^{-3}$$

$$\therefore P = \frac{T_T \cdot \phi \cancel{\rho A_x}}{2\sigma^{3/2}}$$

$$= \frac{T_T \cdot \phi U}{2\sigma^{3/2}}$$

$$P_4 = \tfrac{1}{2}\dot m V_4^2$$

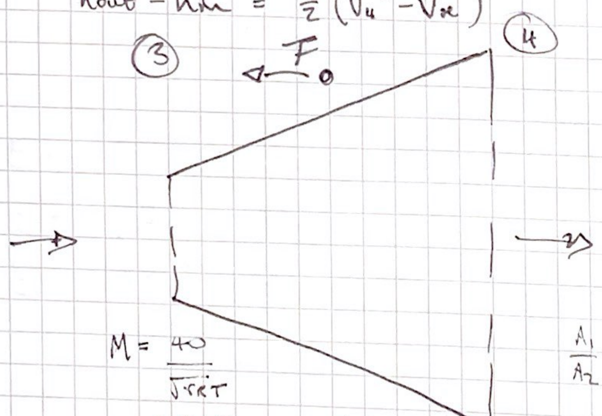
$$\phi^3 \rho^{3/2} A_x^{3/2} U^3$$

$$M_c = \frac{T}{P}\sqrt{\frac{T}{2\rho A_x}}$$

$$= \frac{2\sigma^{3/2}}{\phi U}\sqrt{\frac{\phi^2 A_x U^2}{2\sigma^{-2}}} = \sqrt{2}\,A_x \sigma^{1/2}$$

$$M_f = \cancel{\sigma^{3/2}} \sqrt{\frac{2A_x\sigma}{A_x}} = \sqrt{2\sigma}$$

$$h_{out} - h_{in} = \tfrac{1}{2}(V_4^2 - V_x^2)$$



~~Energy:~~

Force/mom

$$F + \tfrac{1}{2}\dot m V_4 = \dot m V_3$$

Mass

$$\rho A_3 V_3 = \rho A_4 V_4$$

Energy

$$\tfrac{1}{2}\dot m V_3^2 = \tfrac{1}{2}\dot m V_4^2 + \int F V \, dx$$

$$M = \frac{40}{\sqrt{\gamma R T}} = 0.12$$

$$\frac{A_1}{A_2} = 2$$

adiabatic ⇒ T_0 = const

isentropic ⇒ p_0 = const

$$\frac{\dot m\sqrt{c_p T_0}}{A p_0} = 0.2633$$

$$T = 293 \qquad \frac{T}{T_0} = 0.9971$$

$$T_0 = 293.85$$

$$\frac{\dot m\sqrt{c_p T_{02}}}{A_2 p_{02}} = \frac{\dot m\sqrt{c_p T_{01}}}{A_1 p_{01}} \cdot \frac{A_1}{A_2} = 0.316$$

$M = 0.06$

$$\frac{T}{T_0} = 0.9993$$

$$= \frac{293.49}{293.65}$$

$$V(x) = V_3 - \frac{(V_3 - V_4)}{L}x$$

$$= V_3 - (V_3 - V_4)\cdot\frac{x}{L}$$

$$V_3\left(1 - \left(1 - \frac{1}{\sigma}\right)\frac{x}{L}\right)$$

$$\frac{L}{V_3 F} \therefore \frac{1}{2}\dot m (V_3^2 - V_4^2) = \left[x - \left(1 - \frac{1}{\sigma}\right)\frac{x^2}{2L}\right]_0^L$$

$$= L - \left(1 - \frac{1}{\sigma}\right)\frac{L}{2}$$

$$= L\left(\frac{1}{2} + \frac{1}{2\sigma}\right) = \frac{L}{2\sigma}(\sigma + 1)$$

$$\therefore \frac{\dot m (V_3^2 - V_4^2)}{2V_3 F} = \left(\frac{\sigma + 1}{2\sigma}\right)$$

$$\frac{\dot{m}\,(V_3^2 - V_4^2)}{2V_3 F} = \frac{\sigma + 1}{2\sigma}$$

$$\frac{\dot{m}}{2F}\left(V_3 - \frac{V_3}{\sigma^2}\right) = \frac{\sigma + 1}{2\sigma}$$

$$\frac{\dot{m} V_3}{2\sigma F}(\sigma - 1) = \frac{\sigma + 1}{2\sigma}$$

$$\frac{\dot{m} V_3}{F} = \frac{\sigma + 1}{\sigma - 1} \qquad \therefore \sigma > 1$$

$$F + \dot{m} V_4 = \dot{m} V_3$$

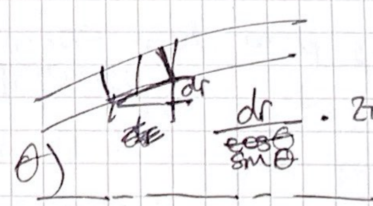
$$F + \dot{m}\left(\frac{V_3}{\sigma} - V_3\right) = 0$$

$$F + \frac{\dot{m} V_3}{\sigma}(1 - \sigma) = 0$$

$$F = \frac{\dot{m} V_3}{\sigma}(\sigma - 1)$$

$\therefore$ More diffusion, more resistive force on propulsor from flow deceleration

Now assuming $p_4 = p_a$



$$\frac{dr}{\sin\theta} \cdot 2\pi r = dA$$

$$\text{Total Pressure force} = \int p\,dA = \int_{r_h}^{r_e} p(r) \cdot \frac{2\pi r}{\sin\theta} \cdot dr$$

$$p_4 + \tfrac{1}{2}\rho V_4^2 = p_r + \tfrac{1}{2}\rho V_r^2$$

$$\rho A_r V_r = \rho A_4 V_4$$

$$V_r = \frac{A_4 V_4}{A_r}$$

$$A_r =$$

QUESTIONS FOR SAM

- Having corrected the σ issue I've obtained an expression for thrust. which has correct dimensions.

$$\phi^3 = (2\varphi\sigma^2)^{3/2}$$
$$= (2\varphi)^{3/2}\sigma^3$$
$$P = \frac{(2\varphi)^{3/2}\sigma^3 \rho A x U^3}{2\sigma^{7/2}}$$
$$= \frac{(2\varphi)^{3/2}\rho A x U^3}{2\sqrt{\sigma}}$$
$$= \frac{\sqrt{2\varphi^3}\cdot\sqrt{2\varphi}}{\phi}\rho A x U^3 = \frac{2\varphi^3 \rho A x}{\phi}$$

$$\sigma^2 = \frac{\phi^2}{2\varphi}$$

- → Tip velocity limit wrt Stress →
- → What geometry & features drive figure of merit.

"Reasons its happening is A B C

Don't want to go there because

→ Efficiency changes across map and I've assumed $\eta = 0.9$.

- → Flip to Smith chart explain why not in bottom rig
- → Ideal prop $M_f = 1$
- → Ideal fan $M_f = \sqrt{2}$ // FOR $\sigma = 1$

we are $\sigma > 1$ ∴ M_f can be greater.

- → Weight UROP chat towards 4th Yr & UROP / Proj.
- → Presentation

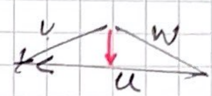
Build drone - cage	- Instrument - Get flying - Get data.

- → Mass model, outline EXACTLY what the set-up looks like.

Replace V_0 with V_m

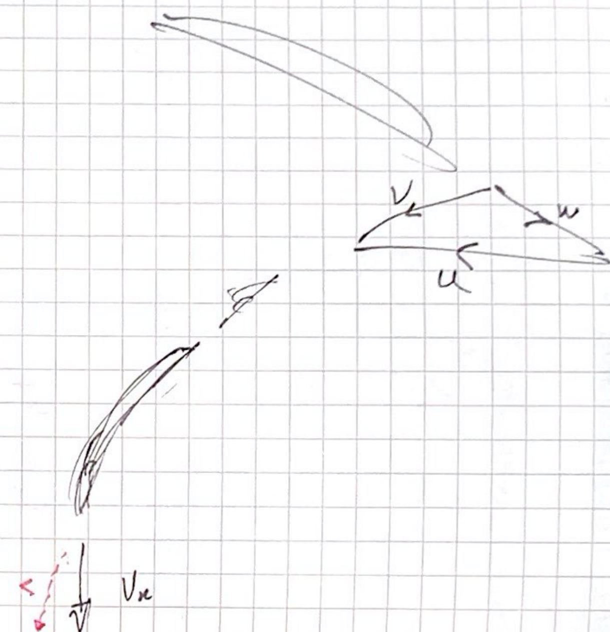
2. Leave out $(\cdots)$

3. Consider exit swirl



$$V_3 \cos\alpha_3 = V_x$$

Maybe talk later about how exit deviation effects thrust re how well are stators working.



Mass Model

→ Solid cone ?

(19) look at how payload changes $\tilde{T}_E$ Me on M_f.

$$\sigma^2 = \frac{\phi^2}{2\psi}$$

$$\sigma^{\frac{1}{2}} = \frac{\phi^{\frac{1}{2}}}{(2\psi)^{\frac{1}{4}}}$$

$$\phi^3 = (\sigma^2\, 2\psi)^{3/2}$$
$$= \sigma^3 (2\psi)^{3/2}$$

$$\frac{\sigma^3}{\sigma^{7/2}} = \sigma^{\frac{6}{2}-\frac{7}{2}} = \sigma^{-\frac{1}{2}} = \frac{1}{\sqrt{\sigma}}$$

$$P = \frac{\rho A V_x^3}{2\sigma^2} \qquad \sigma^2 = \frac{\phi^2}{2\psi}$$

$$= \frac{\rho A V_x^3 \cdot 2\psi}{2\phi^2}$$

$$= \frac{\rho A V_x^3 \psi}{\frac{V_x^2}{U^2}} = \rho A V_x \psi U^2$$

$$= \rho A \phi \psi U^3$$

$$P = \frac{\phi^3 \rho A U^3}{2\sigma^{7/2}} = \frac{\sigma^3 (2\psi)^{3/2} \rho A U^3}{2\sigma^{7/2}}$$

$$= \frac{(2\psi)^{3/2} \rho A U^3}{2\sqrt{\sigma}}$$

$$= \frac{\sqrt{2}\,\psi^{3/2} \rho A U^3}{\sqrt{\sigma}}$$

$$= \frac{2\psi^2 \rho A U^3}{2\phi}$$

$$\dot{m} = \rho A_x^e V_x$$
$$= \rho A_x \sigma \frac{V}{\sqrt{\sigma}}$$

$$\dot{m}'$$

$$\rho A_x V_x V_x$$
$$\rho A_x \frac{V_x^2}{\sqrt{\sigma}}$$

$$V_x = \sqrt{\frac{T_T\,\sigma}{\rho A_x}}$$

$$\phi = \sqrt{\frac{T_T\,\sigma}{\rho A U^2}} \qquad \varphi = \frac{T_T}{2\rho A_x U^2 \sigma} \qquad \frac{\phi^2}{\sigma} = \frac{\phi^2\sqrt{2\varphi}}{\phi} = \phi\sqrt{2\varphi}$$

$$T_T = \frac{\phi^2 \rho A U^2}{\sigma} \cdot (12)$$

$$\frac{T_T}{\rho A U^2} = \frac{\phi^2}{\sigma} \qquad 2\varphi\sigma = \frac{T_T}{\rho A_x U^2}$$

$$2\varphi\sigma = \frac{\phi^2}{\sigma}$$

$$\sigma^2 = \frac{\phi^2}{2\varphi} \qquad M_f = \sqrt{2\sigma}$$

$$P = \frac{\rho A_x V_x^3}{2\sigma^2} \qquad V_x^3 = \frac{T_T^{3/2}\,\sigma^{3/2}}{\rho^{3/2}\,A_x^{3/2}}$$

$$P = \frac{\rho A_x}{2\sigma^2} \cdot \frac{T_T^{3/2}\,\sigma^{3/2}}{\rho^{3/2} A_x^{3/2}}$$

$$= \frac{T_T^{3/2}}{\sqrt{4\sigma\rho A_x}} \qquad T_T = \frac{\phi^2 \rho A_x U^2}{\sigma} \qquad 0.4 = \frac{\Delta P_0}{\frac{1}{2}\rho U^2}$$

$$= \frac{\phi^3 \rho^{3/2} A_x^{3/2} U^3}{2\sigma^{3/2}\sigma^{1/2}\rho^{1/2} A_x^{1/2}} \qquad \frac{1}{2}0.4 = \frac{\Delta P_0}{\rho U^2}$$

$$= \frac{\phi^3 \rho A_x U^3}{2\sigma^2}$$

$$\rho\,\frac{U^3}{U^3} = \text{same}$$

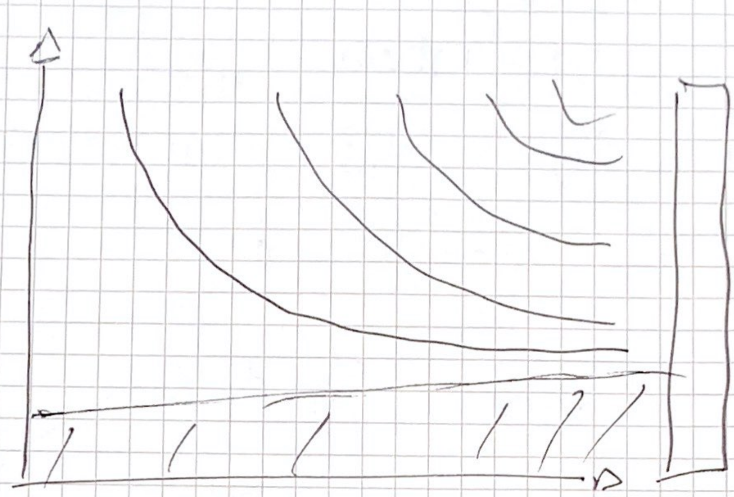
$$T_T = \phi\sqrt{2\varphi}\,\rho A_x U^2$$

$$P = \rho A U^3 \phi\varphi$$

Ask about $\varphi = \frac{\Delta P_0}{\frac{1}{2}\rho U^2}$ vs $\frac{\Delta P_0}{\rho U^2}$ → Have 3 plots each at a different speed. How does it effect payload

Plots range y 0:0.5

x 0:1



$P =$ $T_T \sim M$

$T_J = M$

M_p

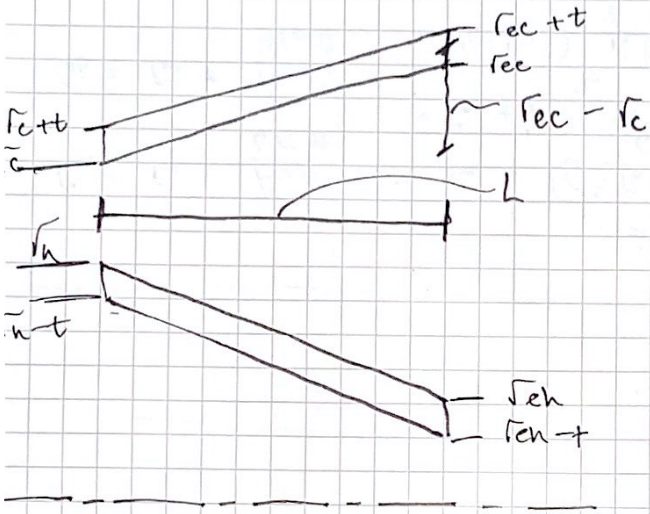
- Make rotor lighter
- Reduce AR a bit.
- Be more generous with fillets.
- Change connector to motor.

- Get flying, get measurements.

~~F1~~ Jobs !
Apply?

- What does it look like? ✓
- Bar chart of component weights ✓
- Thrust vs RPM at design ✓
- RPM required for Thrust = Weight? ✓
- Max payload for theoretical max RPM?
- Thrust vs Power at design ✓
- Power required for Thrust = weight ✓
- Max payload at max power?
- Sensitivity with efficiency.

DIFFUSER VOLUME



$(r_c+r_n)(r_c-r_n)$

$r_c^2 - r_c r_n + r_n r_c - r_n^2$

$$L = \frac{r_c - r_n}{DF}$$

$$r_{ec} = r_c(1+\sigma) + r_n(1-\sigma)$$

$$= r_n + \frac{A_e}{4 r_m \pi} \qquad A_e = \pi(r_c^2 - r_n^2)\sigma$$

$$= \frac{r_c + r_n}{2} + \frac{\cancel{2\pi}(r_c^2 - r_n^2)\sigma}{2\cancel{4}(r_c+r_n)\cancel{\pi}}$$

$$= \frac{r_c + r_n}{2} + \frac{\cancel{(r_c+r_n)}(r_c - r_n)\cdot\sigma}{2\cancel{(r_c+r_n)}}$$

$$= \frac{r_c + r_n + (r_c - r_n)\sigma}{2}$$

$$= \frac{r_c + r_c\sigma + r_n - r_n\sigma}{2}$$

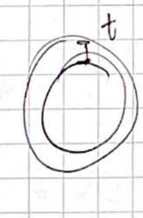
$$= \frac{r_c(1+\sigma) + r_n(1-\sigma)}{2}$$

$$r_{ec} - r_c = \frac{r_c(1+\sigma) + r_n(1-\sigma)}{2} - r_c$$

$$= \frac{r_c(1+\sigma) - 2r_c + r_n(1-\sigma)}{2}$$

$$= \frac{r_c(\sigma - 1) + r_n(1-\sigma)}{2}$$

$$\therefore L_{diff} = \mathrm{sqrt}\left[\left(\frac{r_c - r_n}{Df}\right)^2 + \left(\frac{r_c(\sigma-1) + r_n(1-\sigma)}{2}\right)^2\right] \times t \times 2\pi$$



$$dA = t\, dl \cdot \sqrt{L_{diff}^2 - (r_{ec} - r_c)^2}$$

$$dl^2 = dl\, d_{diff}^2 - (r_{ec} - r_c)^2$$

$$\frac{1}{3} h\pi r^2$$

$$\frac{L}{r_{ec} - r_c} \cdot r_{ec} = h \qquad r_{ec} - t$$

$$r^2 = r_{ec}^2 \sim (r_{ec} - t)^2$$

$$\therefore A = \frac{L r_{ec}^3 \pi}{3(r_{ec} - r_c)} - \frac{L(r_{ec} - t)^3 \pi}{3(r_{ec} - r_c)} - \left[\frac{L}{(r_{ec} - r_c)} \cdot r_{ec} - h\right]\pi$$

$$h_1^* = \frac{L}{r_{ec} - r_c} \cdot (r_{ec} + t) \qquad h_2^* = \frac{L}{r_{ec} - r_c} \cdot (r_{ec})$$

$$\therefore V = \frac{h_1 \pi (r_{ec} + t)^2}{3} - \frac{h_2 \pi (r_{ec})^2}{3} - (h_1 - L)\pi(r_c + t)$$

$$= \frac{h_1 \pi (r_{ec} + t)^2}{3} - \frac{(h_1 - L) \pi (r_c + t)^2}{3} - \left[\frac{h_2 \pi r_{ec}^2}{3} - \frac{(h_2 - L) \pi r_c^2}{3} \right]$$

$$h_1 = \frac{L}{r_{ec} - r_c} \cdot (r_{ec} + t)$$

$$h_2 = \frac{L}{r_{ec} - r_c} \cdot (r_{ec})$$

$$V = \cancel{K \pi (r_{ec} + t)^2}$$

$$V_{case} = \frac{\pi}{3} \left[h_1 (r_{ec} + t)^2 - (h_1 - L)(r_c + t)^2 - h_2 r_{ec}^2 + (h_2 - L) r_c^2 \right]$$

$$V_{hub} = \frac{\pi}{3} \left[h_1 r_h^2 - (h_1 - L)(r_{eh})^2 - h_2 (r_h - t)^2 + (h_2 - L)(r_{eh} - t)^2 \right]$$

COVER LETTER

UPDATE CV

$$K_v V = rpm \qquad V = 14.8 \text{ V}$$

$$V = \frac{rpm}{K_v}$$

$$P = IV$$

$$\therefore I = \frac{P}{V} = \frac{P K_v}{rpm}$$

Battery $\rightarrow$ Control $\rightarrow$ Motor $\rightarrow$ Propulsor

$$P_E = VI \quad \frac{\text{rpm}}{kv}$$

$= \text{inlet} \times \text{drift} \times \text{blades}$

P_E

$$\therefore P = \eta_c \, \eta_m \eta_p P_E$$

$$P = \eta_T \, VI = \eta_T \frac{\text{rpm}}{kv} I = P$$

$$\frac{\text{rpm}}{kv} I = \eta_T P_E$$

$$I = \frac{kv P}{\eta_T \, \text{rpm}}$$

$$\cancel{I = \eta_T \frac{P_E \, kv}{\text{rpm}}}$$

$$P_T = \eta_T P_E$$

$$= \eta_T \frac{\text{rpm}}{kv} I$$

$$\therefore I = \frac{P_T \, kv}{\eta_T \, \text{rpm}}$$

Prop - RPM Power characteristics

APC props with known calibration

- Same prop diameter
-

Sigma limit due to geometric constraints of diffuser becoming cone.

$\sigma \Rightarrow \quad r_{eh} = 0$

$$\therefore 0.5\left[r_c(1-\sigma) + r_h(1+\sigma)\right] = 0 \qquad M_t = \sqrt{2\sigma}$$

$$r_c(\sigma - 1) = r_h(1+\sigma)$$

$$\sigma(r_c - r_h) = r_h + r_c$$

$$\sigma_{max} = \frac{r_h + r_c}{r_c - r_h} = \frac{\phi^2}{2\psi} \qquad \therefore \psi = \frac{\phi^2}{2\sigma_{max}} =$$

C_p

$\rightarrow$ Update MESH SECTION.

$\rightarrow$ Need to relate ϕ, ψ, r_h, r_c, u

$$\phi^2 = \frac{T_T \sigma}{\rho A_x U^2} \qquad \varphi = \frac{T_T}{2\rho A_x U^2 \sigma}$$

// Equate σ $\cancel{\frac{\phi^2 \rho A_x U^2}{\sigma}} = \cancel{\varphi 2 \rho A_x}$

$$\frac{\phi^2 \rho A_x U^2}{T_T} = \frac{T_T}{2\rho A_x U^2 \varphi}$$

$$T_T^2 = 2\varphi\phi^2\rho^2 A_x^2 U^4$$

$$\underline{T_T = \phi\sqrt{2\varphi}\,\rho A_x U^2} \qquad T_T = M$$

$$= \phi\sqrt{2\varphi}\,\rho_a \pi (r_t^2 - r_h^2) . \Omega^2 \left(\frac{r_t + r_h}{4}\right)^2$$

$$M = \rho_p (V_D + V_1 + V_c)$$

$$fn(r_t, r_h, t, \phi, \varphi).$$

4A2 Computational Fluid Dynamics

flow_guess

Exit velocity & ρ ~~a~~ given isentropic flow with known inlet & exit area.

$p_{0in} = p_{0out}$ $\qquad A_e = a_{flow}(ni)$

$T_{0in} = T_{0out}$

$p_e = p_{down}$

$\therefore$ At exit $\left(\frac{p_e}{p_{0out}}\right) = \left(1 + \frac{\gamma-1}{2} M^2\right)^{-\gamma/\gamma-1}$

$$\therefore \quad M^2 = \frac{2}{\gamma-1}\left(\left(\frac{p_{0out}}{p_e}\right)^{\gamma-1/\gamma} - 1\right)$$

$$M = \text{sqrt(ans)}$$

Further $\frac{T}{T_0} = \left(1 + \frac{\gamma-1}{2} M^2\right)^{-1}$

$$T = T_0\left(1 + \frac{\gamma-1}{2} M^2\right)^{-1}$$

Assuming uniform flow

$$\dot{m} = \rho A V$$

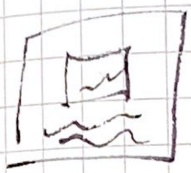
$$\therefore \quad V = M\sqrt{\gamma R T}$$

Density (Ideal gas)

$$\frac{p_e}{\rho} = RT_e \qquad \therefore \rho = \frac{p_e}{RT_e}$$

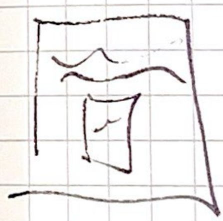
$$\dot{m} = \frac{p_e}{RT} . A . M\sqrt{\gamma R T_e} = p_e A M \sqrt{\frac{\gamma}{RT}}$$

Presentation Chat



~~Line~~ In Physics

Line In	Diagram	Physics	Line Out
Introduce	Explain whats on it	Explain why	Sum-up.



① Analytical model. Slide 11
- Have details on back up slide only

Intro to section
- Model to static thrust
- Slides 19/20 become bullet points in an introduction.

② Slide 12 is intro slide to 2^{nd} Q.
- More like earlier work.

- Slide 14, more interested in outputs as in poster

Combine 13 + 14

~~4~~ - Animation to replace drone ~~pic~~ with schematic
+ explain where M_F comes from.

Modular

- Made to compare engines

14 - 2 problems

Control I've used ....

Measurement I've developed...

$T = k_1 \sin\phi$

$\Omega = k_v V$

(2) . 4 slides

12, 13, 14 & results of payload & power.

(1) ~~Does~~ 11 -> 4

?

No control over M_F for ideal prop.

With ducted fan, diffuser adds DoF hence M_F = fn(σ).

How do I design it to give good M_F?.

Then prelim slide with ONLY

New Fixed ....

To close problem need to match weight to thrust. To do this mass model.