

# Lec12 - Simple concepts you need to know about - and Fermi estimation

Peter C. K. Vesborg  
peter.vesborg@fysik.dtu.dk

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## Learning Obejctives

- Fermi Estimates
- CAPEX, OPEX
- WACC, Leverage
- NPV, Time value, Discount rate
- IRR
- LCOE
- Pitfalls of LCOE - aka. economics is not an exact science (or a science at all)

## Fermi Estimates

Take 30 seconds to think of how you would answer the following (classic) question posed by Enrico Fermi:

*How many piano tuners live in Chicago?*

Don't do a web search - think for 30 seconds. How would you begin to answer that question?

As an engineer, you must hone your ability to quickly (often on the fly) perform estimates of various things. How much concrete does the Copenhagen metro tunnel system contain? What is the fuel burn rate for a Boeing 737 in level cruise flight? What must be the flow rate of water through

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the living room radiator to maintain room temperature when it's freezing outside? How much fuel is burnt daily by all the cars in Germany? How much sewage water must the Danish water treatment system handle every day? How much water lands on DTU campus every hour during heavy rain? Which uses more electrical power - the sound system of a stadium concert, or a Tesla plugged into a supercharger? How much TNT does the orbital kinetic energy of an astronaut floating inside the ISS correspond to?

You might think that you don't need the ability to quickly make such estimates without touching your phone, but you would be wrong for a couple of reasons. For example, if you start by searching for an answer - without first doing any thinking for yourself - you have no defense against wrong information. Your search result might be way off (a factor of 1000 is common) and you might not recognize it (!) - but if you've calculated an approximate number, you'd immediately be skeptical if your estimate and the search result are wildly different. This also allows you to recognize many kinds of bogus arguments - *e.g.* in political debates. Secondly, very often you do not require 3 significant digits of precision in order to reach a conclusion to your real question. Most often it's fine to have an estimate within a factor of 2 from the true number, and this is typically much faster to estimate instead of trying to find the true number.

The art of quantitative estimation using limited information (Fermi Estimation) typically comes down to breaking up the question into things you actually know something about - and then multiplying many factors together to arrive at the answer. For example - perhaps you know that the population of Chicago is around  $\sim 3$  million people, so there might be  $\sim 1.5$  million households. We might then proceed to guess that 10% of these households have a piano - and that half of these pianos is actually used regularly (and thus needs occasional tuning) - say 1 tuning/year. Finally we can estimate that a piano tuner might be able to tune up to 2 pianos in a work day, but hardly has a full calendar and perhaps only 220 work-days every year so that's around 300 pianos/year for every tuner. Then the our estimate is  $1.5 \text{ million households} \times 0.05 \text{ active pianos/household} \times 1 \text{ tuning/year}$  divided by  $300 \text{ pianos/tuner-year} = \sim 250 \text{ piano tuners in Chicago}$ . This might be off by a factor of 2 or 3, but it's probably right within an order of magnitude and we didn't have to look anything up to arrive at an estimate within a few seconds. The reason it works well is you don't know what answer to expect and thus do not apply a consistent bias for all the guesstimates you make along the way: Perhaps we overestimated the number of piano tunings, but this error would be counteracted if we also overestimated the number of tunings each tuner can do in a year...

It's not hard to see how the method extends to other things: Population

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of Germany  $\rightarrow$  number of cars  $\times$  average daily drive distance  $\times$  typical fuel efficiency  $\rightarrow$  Fermi estimate for daily fuel burn. *etc.*

Next time you are out for a walk - Fermi-estimate something!

## CAPEX and OPEX

There is a fair amount of “lingo” used in economics which is useful to know and understand. Projects in renewable energy often involve significant up-front investment (*i.e.* construction costs for a wind farm) followed by more modest costs to operate and maintain the facility - and perhaps terminated by some decommissioning cost to restore the site to its prior condition and handle all waste. *CAPEX* is short for CAPital EXpenditure and means the up-front investment cost. This is the cost to build the facility before it starts operating. *OPEX* is short for OPERating EXpenditure and is the schedule of costs to keep the facility running once, it has been commissioned. Of course, while the facility is operating, it will hopefully generate some revenue to (more than?) offset the OPEX, but we’ll get to that later. The OPEX only considers the costs.

## WACC

All projects require money/financing. Money for CAPEX - and perhaps also for OPEX unless the project can cover it’s own OPEX from day 1. Where do we get money from? There are generally two options: 1) We can borrow it - *i.e. Debt Financing*, or 2) we can get investments - *i.e. Equity Financing*. In Debt Financing we get *the principal* (the borrowed amount) and over time we must pay back the principal plus *interest*. Often the loan has fixed interest rate and a deterministic payback schedule in which case we know exactly when we must pay back what. Equity financing is taking investor money (*e.g.* by selling shares) and since investors typically get no certain yield on their invested capital - nor an assurance that they can ever recoup their investment - they tend to demand a higher yield on their capital than lenders. Another way to say this is that investors take greater risk than lenders, and thus demand higher returns. Thus equity financing tends to be more expensive than debt financing. On the flip-side, debt financing is more dangerous because it is merciless - when the bond comes due, payment must happen, while investors can be made to wait for dividends or other returns on their investment. For this reason, it is generally recommended that projects (and whole companies) be funded by a linear combination of debt and equity. Pure debt might be cheaper - but risky since there is no flexibility in case of trouble, while equity provides flexibility, but comes at a higher cost. The *WACC* - short for Weighted Average Cost of Capital -

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is the effective *price of money*:

$$\text{WAAC} = \frac{E \times r_E + D \times r_D}{D + E} \quad (1)$$

where  $E$  and  $D$  is the amount of equity financing and debt financing respectively and  $r_E$  is the return the investors expect while  $r_D$  is the effective interest rate paid on loans:  $r_D = r_c(1 - T)$  where  $r_c$  is the actual interest rate paid to the debt buyers (lenders) and  $T$  is the corporate tax rate.  $r_D$  is reduced by a factor  $(1 - T)$  because interest expenses are tax deductible. As an aside - since selling debt is more risky than selling equity one often measures how much risk a project (or company) is taking by calculating the *financial leverage*  $= D/E$  (sometimes called gearing factor). High leverage means that a large amounts of debt are used to do big things with small investments. Generally speaking larger leverage increases investor returns (or losses) depending on how things go.

Example:

Say you need 100 million to finance your planned wind turbine park. You raise 30 million from investors promising them an average return on 15% *p.a.* on their investment. You then go to the bank and ask to borrow 70 million (using your 30 million as collateral) and they say “Hmmm, your project looks OK, but your gearing is  $70/30 = 2.3$  on the high side, thus we want 12% interest.” You calculate that the WACC is going to be 12.9% (ignoring taxes) which seems high so you decline the loan and go back to your investors and raise an extra 10 million, but this time your investors want 20% yield because you’re decreasing their leverage. Then you go back to the bank and now that your gearing is only 1.5 ( $60/40=1.5$ ) they offer you the loan at 9% interest and you calculate your WACC to be 11.9% so you are much happier because you managed to simultaneously decrease your WACC from 12.9% to 11.9% and your risk (leverage dropped from 2.3 to 1.5).

## NPV and IRR

How did the bank decide that your project looked OK in the previous example? What calculation do they (and your investors) do in order to assess the soundness of the project? Obviously there are several calculations to do, but one of them is surely a *Net Present Value* (or *NPV*) calculation. To motivate the idea of NPV consider the following question: Let’s say I’m going to give you 1000 kr. Would you prefer to get it today - or one year from now? You’ll almost certainly prefer to get it now. One reason you prefer immediate payment is that you know that we have inflation (the phenomenon that the same amount of kr buys you less and less goods and services as

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time goes on, typically said to be around 2% *p.a.*); but even if I were to offer you 1000 kr today - or 1020 kr one year from now (assuming that 20 kr is the correct inflation) you'll still prefer to get it now, because there's no benefit to waiting. It's the same purchasing power, so better to get it in your pocket immediately. The interesting question is: How much *more* you want next year, before you choose to forego getting the money now. 1050 kr? 1100 kr? The (inflation adjusted) difference to the present value reflects the *time value of money*. Time value is subjective and different from person to person. If you are in critical need of money right now your preference is different from someone who doesn't know what to do with it and puts it in a zero interest rate bank account. The equivalent interest rate you demand for waiting to get the money in your pocket is called the *discount rate*. If your discount rate is 5% it means you'd chose 1000 kr today over 1045 kr next year, but also that you would be willing wait a year for 1055 kr payout. This clearly relates to the WACC. In a sense the return "demanded by the lenders" can be thought of as their discount rate (plus some *risk premium*). The above discussion of discount rate refers to an assured payment in year - *i.e.* a zero risk situation. The risk premium corrects for the potential outcome that you never get what you were promised. Let's say your zero-risk discount rate is 5%, and that estimate that there is a 10% risk that I can't or won't pay up next year. In that case, I must offer to give you more than 1150 kr next year to for you to not prefer the 1000 kr here and now.

If I know my discount rate, I can take any future net revenue stream and back-calculate its current value to me: *It's net present value*. Ignoring inflation (*i.e.* calculating in *fixed prices*) and risk let's take a simple example: You invest 1000 kr in something today and the agreement is that one year from now you get 500 kr back and the year after that you get another 600 kr whereupon the contract is over. Your total payout is thus (500 + 600) kr = 1100 kr and since you put in 1000 kr the *net return on investment* is 100 kr. What is the NPV of this investment? If we say that your discount rate is 10% then the present value of the 500 kr payment one year out is  $500/(1 + 0.1)^1 = 454.5$  kr while the present value of 600 kr 2 years out is  $600/(1 + 0.1)^2 = 495.9$  kr so the NPV of that 1100 kr (or 1000 kr investment) is actually only 950.4 kr which is less than the 1000 kr you invest! This investment thus has an overall *negative* NPV and should be avoided. Another way of saying this is that the 1000 kr in your pocket has (49.6 kr) more value to you than 500 kr next year and 600 kr in two years.

In general we have:

$$NPV = \sum_{t=0}^N \frac{R_t}{(1 + i)^t} \quad (2)$$

The sum is over the duration of the investment, which is divided into  $N$

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intervals of time  $t$ <sup>1</sup>,  $R_t$  is the net return (revenue minus expenses) in the time period  $t$  and  $i$  is the discount rate.

In general, all financially worthwhile investments must have a positive NPV, and if we must choose between two projects we would pick the one with the highest NPV. If we go back to our wind turbine park project, we know that (after some massage of our *capital structure*) we could raise funding for the project at a WACC of 11.9% so if we substitute  $i = 11.9\%$  in the NPV calculation we better get a positive result when we sum over the lifetime of the project. Since the denominator grows exponentially in time, for a high-ish discount rate of 11.9% it turns out, that only the first handful of years dominate the sum. In other words, unless we hit positive NPV within the first 6 or 7 years of the sum, we probably shouldn't invest.

Another way to look at the same problem is to ask what is the “break even” discount rate for the project. In other words, for a given set of net returns  $R = \{R_0, R_1, \dots, R_{N-1}, R_N\}$ , which value of  $i$  results in  $\text{NPV} = 0$ ? This special discount rate is called the *Internal Rate of Return (IRR)*. In our example with the 1000 kr investment returning  $R_0 = 0, R_1 = 500, R_2 = 600$  we get  $i = \text{IRR} = 0.0639$  so in that case the internal rate of return was 6.39% and this was not a good investment since we defined our discount rate as 10%. Demanding a positive NPV is the same as demanding  $\text{IRR} > \text{discount rate}$ .

So how did the bank conclude that our wind turbine project looked OK? It seems hard to know which is a reasonable value for the discount rate  $i$  and there is clearly great uncertainty in  $R_t$  (probably more, the higher  $t$  is because  $R_t$  is simply our best guess for the net income derived by the project in at some future time). The answer is that they probably tried to calculate the set of net returns  $R$  and from that they calculated an IRR value. They then compared your project's IRR to those of other projects in their portfolio with a similar *risk profile* and liked what they saw. That's how professional investments are evaluated: You compare the projected IRR to the project risks and invest if the IRR is high compared to the risk, but the basis for the comparison is NPV of the anticipated revenue stream.

## LCOE

There are many problems to consider when trying to answer questions like “Is wind power now cheaper than coal power?”. Most renewable technologies

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<sup>1</sup>Naturally for large  $N$ , it's often convenient to replace the discrete sum by an integral, but since most accounting is in discrete time (reporting periods of quarters, years, *etc.*) NPV is traditionally written as a discrete sum.

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ignore the (significant) problem of intermittancy, while most non-renewable technology ignore the (significant) problem of *externailties* such as pollution<sup>2</sup> or political/military costs.

Ignoring these problems and focusing exclusively on “pure economics” we can compare energy technologies based on **LCOE: Levelized Cost Of Energy/Electricity**. The basis of LCOE is simply to divide (discounted present value of) the total financial cost of an energy project by (the discounted present value of) the total energy “production”:

$$\text{LCOE} = \frac{\sum_{t=0}^N \frac{I_t + M_t + F_t}{(1+i)^t}}{\sum_{t=0}^N \frac{E_t}{(1+i)^t}} \quad (3)$$

where  $I_t, M_t, F_t, E_t$  is the required Investment, operation and Maintenance, Fuel, and produced Energy for time period  $t$  respectively and  $N$  is the lifetime of the project. Clearly LCOE goes down as expenses go down or energy production goes up. Notice that the discounting factors do not cancel out.

A high discount rate,  $i$ , favors projects with low initial cost (CAPEX) even if the consequence is high running costs (OPEX). This is because expenses “far out in the future” are highly discounted if  $i$  is large and thus count for less. Projects such as fossil power which might be cheap to build but expensive to run (due to fuel costs,  $F_t$ , are examples of this. This is also the thinking that (in the early days) led people to ignore the costs associated with decommissioning and clean-up of retired nuclear power stations: It was many decades into the future and the interest rate (and thus the discount rate) was high - so the NPV expense calculated in the 1960’s was very small.

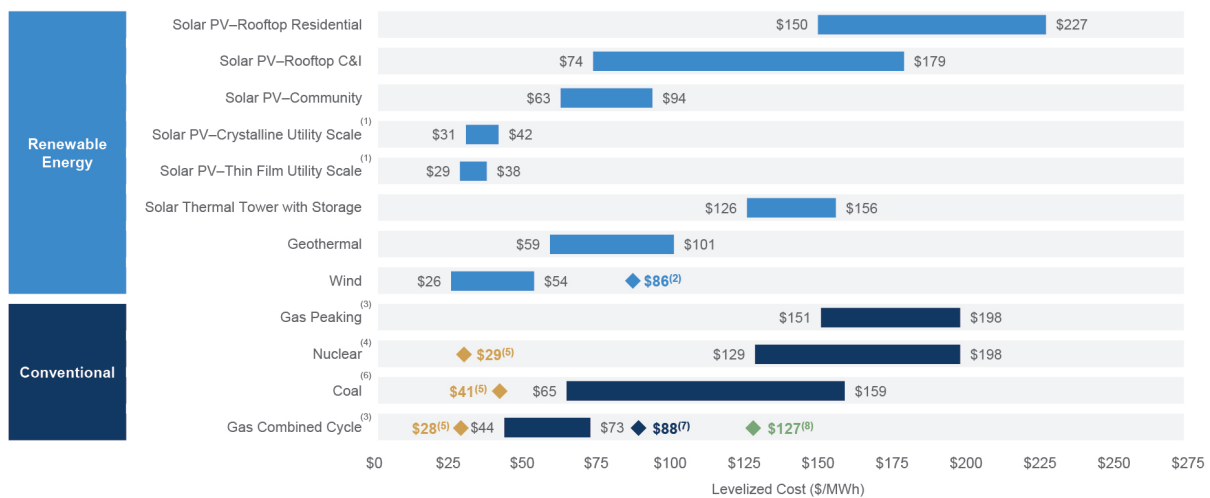
Conversely a low discount rate makes high initial cost less problematic while high OPEX expenses are comparatively more problematic, both of which favors renewable energy which tends to have high CAPEX but very low OPEX (no fuel! - and in the case of photovoltaic energy - essentially no maintenance). This means that the tremendous jump in competitiveness of renewables in the last ten years is not only driven by falling prices for renewables (dropping CAPEX), but also to a significant extent by the precipitous drop in interest rates (and thus also dropping discount rates). Large up-front costs are much easier to bear in a low interest rate environment.

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<sup>2</sup>Pollution, which non-renewable energy rarely pays for, is *not* just CO<sub>2</sub>. For example, besides SO<sub>2</sub> (acid rain) coal power is the main source of mercury (Hg) in the oceanic food chain ending up in your sushi. Or consider the waste problem for nuclear power - or the clean-up or write-off of land contaminated by nuclear accidents. Externailites come in many forms, but they must be factored in for a true apples-to-apples cost-of-energy comparison.

## Levelized Cost of Energy Comparison—Unsubsidized Analysis

Selected renewable energy generation technologies are cost-competitive with conventional generation technologies under certain circumstances



Source: Lazard estimates.

Note: Here and throughout this presentation, unless otherwise indicated, the analysis assumes 60% debt at 8% interest rate and 40% equity at 12% cost. Please see page titled "Levelized Cost of Energy Comparison—Sensitivity to Cost of Capital" for cost of capital sensitivities. These results are not intended to represent any particular geography. Please see page titled "Solar PV versus Gas Peaking and Wind versus CCGT—Global Markets" for regional sensitivities to selected technologies.

(1) Unless otherwise indicated herein, the low case represents a single-axis tracking system and the high case represents a fixed-tilt system.

(2) Represents the estimated implied midpoint of the LCOE of offshore wind, assuming a capital cost range of approximately \$2,600 – \$3,675/kW.

(3) The fuel cost assumption for Lazard's global, unsubsidized analysis for gas-fired generation resources is \$3.45/MMBTU.

(4) Unless otherwise indicated, the analysis herein does not reflect decommissioning costs, ongoing maintenance-related capital expenditures or the potential economic impacts of federal loan guarantees or other subsidies.

(5) Represents the midpoint of the marginal cost of operating fully depreciated gas combined cycle, coal and nuclear facilities, inclusive of decommissioning costs for nuclear facilities. Analysis assumes that the salvage value for a decommissioned gas combined cycle or coal asset is equivalent to its decommissioning and site restoration costs. Inputs are derived from a benchmark of operating gas combined cycle, coal and nuclear assets across the U.S. Capacity factors, fuel, variable and fixed operating expenses are based on upper- and lower-quartile estimates derived from Lazard's research. Please see page titled "Levelized Cost of Energy Comparison—Renewable Energy versus Marginal Cost of Selected Existing Conventional Generation" for additional details.

(6) High end incorporates 90% carbon capture and storage. Does not include cost of transportation and storage.

(7) Represents the LCOE of the observed high case gas combined cycle inputs using a 20% blend of "Blue" hydrogen, (i.e., hydrogen produced from a steam-methane reformer, using natural gas as a feedstock, and sequestering the resulting CO<sub>2</sub> in a nearby saline aquifer). No plant modifications are assumed beyond a 2% adjustment to the plant's heat rate. The corresponding fuel cost is \$5.20/MMBTU.

(8) Represents the LCOE of the observed high case gas combined cycle inputs using a 20% blend of "Green" hydrogen, (i.e., hydrogen produced from an electrolyzer powered by a mix of wind and solar generation and stored in a nearby salt cavern). No plant modifications are assumed beyond a 2% adjustment to the plant's heat rate. The corresponding fuel cost is \$10.05/MMBTU.

Figure 1: LCOE estimates for different electrical power sources without subsidies. In the (small font) notes for the figure the assumptions are given. For example, the capital structure is 60% debt and 40% equity with 8% and 12% *p.a.* cost, respectively. Fuel price estimates and offshore wind plant costs used in the calculation are also given. Notice (note 5) that the gold-colored points for LCOE for nuclear, coal and CCGT power represent fully depreciated plants (*i.e.* pure OPEX); in other words, this is the cost of generating electricity from such a plant if you got the plant for free (!) and despite this, photovoltaics and land-based wind is competitive on a pure LOCE basis.

From: [www.lazard.com/perspective/lcoe2020](http://www.lazard.com/perspective/lcoe2020)



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## Problems with LCOE

LCOE has the potential to be an objective way to compare energy sources from an economic aspect, but since it is boiling a multidimensional picture down to a single point (or single number: \$/kWh for example) much important nuance is lost. If you boil the Mona Lisa down to a single point (probably brown?) you lose some pretty important aspects of the full picture. This is why the preface for the LCOE discussion was pointing out that it's failure to account for intermittancy and externalities. Besides these significant shortcomings of LCOE, you must also beware of the LCOE calculation itself. Remember that the discount rate is quite subjective (it is possible to defend a rather broad range of  $i$  for any given project) likewise there is significant estimation involved in the set  $\{I_t, M_t, F_t, E_t \ \forall t \in [0, \dots, N]\}$ . The result of this is that the *agenda of whoever calculated the LCOE* can affect the result quite a bit. Different people comparing two possible projects (*e.g.* comparing a battery storage facility to a peaker plant - or a wind farm to a solar field) by calculating the LCOE of the projects can come to surprisingly different numbers (and thus conclusion) due to differences in assumptions used in the calculation. This can of course be exacerbated by any personal bias (acknowledged or not). The warning (as usual) then, is that you must never jump to the conclusion. You must dig into the calculation and make sure you agree with the assumptions. If you don't, or if sufficient detail isn't given, you *have to* perform your own calculation to arm you with numbers before reading the given conclusion.