[PRINT]

01034 Matematik 2 E24,
Fedir Vasyliev, 9/11/24 at 11:25:04 AM CEST

Question1: *Score 0.66/1*

Consider the system consisting of 3 first-order differential equations of the form:

$x_1{}'(t) = 2\ x_1(t)\ + 4\ x_2(t)\ - 4\ x_3(t)$

$x_2{}'(t) = 16\ x_1(t)\ + 2\ x_2(t)\ - 2\ x_3(t)$

$x_3{}'(t) = 0$

Let $\lambda_1,\ \lambda_2$ and λ_3 be the eigenvalues of the system matrix.
a) It is assumed $\lambda_1 < \lambda_2 < \lambda_3$, enter the three eigenvalues below:

$\lambda_1 =$

Your response	Correct response
-6	**-6**

Auto graded Grade: 1/1.0

$\lambda_2 =$

Your response	Correct response
0	**0**

Auto graded Grade: 1/1.0

$\lambda_3 =$

Your response	Correct response
10	**10**

Auto graded Grade: 1/1.0

b) We consider the solution which has the following initial value conditions:

$x_1(0) = \ 4\ \ ,\quad x_2(0) = -3\ \ \text{and}\ \ x_3(0) = 1.$

Enter an expression for each of the three solutions below:

$x_1(t) =$

Your response	Correct response
-355/60*exp(6*t)*(-0.5) + 1/15 + 117/60*exp(10*t)*0.5	**exp(10*t)+3*exp(-6*t)**

Auto graded Grade: 0/1.0

$x_2\,(t) =$

Your response	Correct response
-355/60*exp(6*t) + 29/30 + 117/60*exp(10*t)	**2*exp(10*t)-6*exp(-6*t)+1**

Auto graded Grade: 0/1.0

$x_3\,(t) =$

Your response	Correct response
1	**1**

Auto graded Grade: 1/1.0

Total grade: 1.0×1/6 + 1.0×1/6 + 1.0×1/6 + 0.0×1/6 + 0.0×1/6 + 1.0×1/6 = 17% + 17% + 17% + 0% + 0% + 17%

Question2: *Score 0.6/1*

Consider the system consisting of 2 first-order differential equations of the form:

$\frac{\mathrm{d}}{\mathrm{d}\,t}\,x = A\;x \qquad (1)$

where $x\,(t) = \begin{bmatrix} x_1\,(t) \\ x_2\,(t) \end{bmatrix}$ is a to dimensional vectorfunction.

$A = \begin{bmatrix} 0 & 1 \\ -34 & -10 \end{bmatrix} \qquad (2)$

Question 2a

Determine the complete complex solution of the system (1) and show it can be written on the form.

$$x\,(t) = c_1\,e^{-mt}\;e^{int}\begin{bmatrix} 1 \\ -m+in \end{bmatrix} \;+\; c_2\,e^{-mt}\;e^{-int}\;\begin{bmatrix} 1 \\ -m-in \end{bmatrix}$$

and state the values of m and n below

$m =$

Your response	Correct response
5	**5**

Auto graded Grade: 6/6.0

$n =$

Your response	Correct response
3	**3**

Auto graded Grade: 6/6.0

Question 2b

Specify the type of numbers c_1 and c_2 by checking the correct answer below:

Your response	Correct response
Choice 1: c_1 and c_2 are arbitrary real numbers.	**c_1 and c_2 are arbitrary complex numbers.**

Auto graded Grade: 0/2.0

Question 2c

The complete real solution can be determined from the complete complex solution. Show that the complete real solution can be written in the form:

$$x(t) = c_1 e^{-mt} \begin{bmatrix} \cos(nt) \\ f(t) \end{bmatrix} + c_2 e^{-mt} \begin{bmatrix} \sin(nt) \\ g(t) \end{bmatrix}$$

and determine the two functions:

$f(t)$ =

Your response	Correct response
m*cos(n*t) - n*sin(n*t)	**-5*cos(3*t)-3*sin(3*t)**

Auto graded Grade: 0/3.0

$g(t)$ =

Your response	Correct response
m*sin(n*t) + n*cos(n*t)	**3*cos(3*t)-5*sin(3*t)**

Auto graded Grade: 0/3.0

Total grade: 1.0×6/20 + 1.0×6/20 + 0.0×2/20 + 0.0×3/20 + 0.0×3/20 = 30% + 30% + 0% + 0% + 0%

Question3: *Score 0/1*

Let $\Phi(t)$ denote the fundamental matrix of the homogeneous system $\dot{x} = A \cdot x \quad , \quad A \in \mathbb{R}^{2\times 2}$.

$$\Phi(t) = \begin{bmatrix} e^{4\ t} & 2e^{3\ t} \\ 0 & e^{3\ t} \end{bmatrix}$$

We consider now the inhomogeneous system:

$$\dot{x} = A \cdot x\ + \begin{bmatrix} 7 \\ -3 \end{bmatrix}$$

State below the solution $x_p\ (t) = \begin{bmatrix} x_{1\ p}\ (t) \\ x_{2\ p}\ (t) \end{bmatrix}$ to the inhomogeneous system which satisfies.

$$x_p\ (0) = \begin{bmatrix} x_{1\ p}\ (0) \\ x_{2\ p}\ (0) \end{bmatrix} = \begin{bmatrix} 7 \\ 5 \end{bmatrix}$$

$x_{1\ p}\ (t) =$

Your response	Correct response
-3*exp(4*t) + 10*exp(3*t) - 13*exp(4*t)+6*exp(3*t) + 13 - 6*exp(-t)	**1/4*exp(4*t)+8*exp(3*t)-5/4**

Auto graded Grade: 0/1.0

$x_{2\ p}\ (t) =$

Your response	Correct response
5*exp(3*t) + 3*exp(3*t) - 3*exp(-t)	**4*exp(3*t)+1**

Auto graded Grade: 0/1.0

Hint: Use Theorem 2.20

Total grade: 0.0×1/2 + 0.0×1/2 = 0% + 0%