TECHNICAL UNIVERSITY OF DENMARK

3-hour written exam, May 13, 2024

Course: Advanced Engineering Mathematics 2

01034 / 01035

Allowed aids: All aids allowed by DTU.

Weighting: Multiple-choice(administrated electronically): 55%, Problem 1: 15%, Problem 2: 10%,

Problem 3: 20%

The weighting is only a guide. The exam is evaluated as a whole. In order to receive full points in Part B, all answers must be substantiated, if necessary with reference to the text book, and a reasonable number of intermediate steps must be included.

The exam consists of 2 parts: an electronic multiple choice section (**Part A**) and this (**Part B**).

- Part A is administered and answered electronically..
- Part B follows below, and can be handed in either electronically or on paper.

Part B

Problem 1

- 1. Show that the series $\sum_{n=1}^{\infty} (-1)^n \frac{100+n}{2+n^3}$ is absolutely convergent.
- 2. For $k \in \mathbb{N}$, consider the series

$$S = \sum_{n=1}^{\infty} (-1)^n \frac{100 + n}{2 + n^k}$$

- (a) Find the smallest value for $k \in \mathbb{N}$, such that the series S is absolutely convergent.
- (b) Find the smallest value for $k \in \mathbb{N}$, such that the series S is convergent.

Problem 2

Consider the differential equation

$$y^{(4)}(t) + y^{(3)}(t) - 2y^{(2)}(t) = u(t), (1)$$

where $u: \mathbb{R} \to \mathbb{R}$ is a continuous function.

- 1. Write down the characteristic polynomial of the equation (1) AND find the general real solution to the associated homogeneous system.
- 2. The function $y(t) = t^3$ is a solution of (1). Write down the general real solution to the inhomogeneous differential equation (1).
- 3. Use the information that $y(t) = t^3$ is a solution to the inhomogeneous differential equation to determine the function u(t).

Problem 3

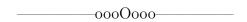
A function $f: \mathbb{R} \to \mathbb{R}$ has the following properties: f is an even function, f is 2π -periodic, and on the interval $[0, \pi]$ the function is given by

$$f(x) = \begin{cases} 1 & \text{for } x \in [0, \pi/2], \\ 2 & \text{for } x \in]\pi/2, \pi]. \end{cases}$$

- 1. Sketch the function f on the interval $[-\pi, \pi]$.
- 2. Determine all the real Fourier coefficients a_n and b_n of the Fourier series

$$f \sim \frac{1}{2}a_0 + \sum_{n=1}^{\infty} (a_n \cos(nx) + b_n \sin(nx)),$$

- 3. What is the sum of the Fourier series at the points $x=-\pi,-\frac{\pi}{2},0,\frac{\pi}{2}$ and π ?
- 4. Does the Fourier series for f converge uniformly? Explain your answer.



The problem set continues!