Technical University of Denmark

Multiple-choice test exam, November 2023

Course name: Mathematics 1a (Polytechnical foundation) Course no. 01001 and 01003
Exam duration: 2 hours

Aid: All by DTU permitted aid.

"Weighting": All questions in this test exam are weighted equally. This part of the test exam constitutes 50% of the entire test exam.

Additional information: The questions are posed first in English, after that in Danish. All questions are multiple choice and no explanatory text or calculations will be taken into account. *Note: This test exam is a transcipt; the test exam will be digital and is to be answered on the DTU Digital Exam platform.*

Question 1

Given is the following system of linear equations over $\mathbb{R}$ in the three unknowns x_1, x_2, and x_3:

$$\begin{cases} x_2 + x_3 &= 1 \\ x_1 - 2x_2 + x_3 &= 4 \end{cases}.$$

Which of the below sets denotes the complete solution to the system?

(1) $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 6 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} 3 \\ 1 \\ -1 \end{bmatrix}, t \in \mathbb{R}$

(2) $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 6 \\ 1 \\ 0 \end{bmatrix}$

(3) $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 6 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix}, t \in \mathbb{R}$

(4) $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 6 \\ 1 \\ 1 \end{bmatrix} + t \begin{bmatrix} 3 \\ 1 \\ -1 \end{bmatrix}, t \in \mathbb{R}$

(5) $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 6 \\ 1 \\ 1 \end{bmatrix} + t \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}, t \in \mathbb{R}$

Question 2

We are given:

$$\mathbf{A} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 0 \end{bmatrix}, \text{ and } \mathbf{v} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$$

Which of the following expressions is identical to the expression $(\mathbf{A} \cdot \mathbf{B})^T \cdot \mathbf{v}$?

(1) $\begin{bmatrix} 5 \\ 2 \\ -1 \end{bmatrix}$

(2) $\begin{bmatrix} -1 \\ 2 \\ 5 \end{bmatrix}$

(3) $\begin{bmatrix} 5 & 11 \\ 2 & 4 \\ -1 & -3 \end{bmatrix}$

(4) $\begin{bmatrix} 5 \\ 11 \end{bmatrix}$

(5) $\begin{bmatrix} 1 \\ 6 \end{bmatrix}$

Question 3

We are given

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 9 \\ 1 & a & 1 \\ 0 & -1 & a \end{bmatrix} \in \mathbb{R}^{3\times 3}.$$

For which values of $a \in \mathbb{R}$ is matrix $\mathbf{A}$ invertible?

(1) $a \in \mathbb{R} \setminus \{-2, 4\}$

(2) $a \in \{-2, 4\}$

(3) $a \in \mathbb{R}$

(4) $a \in \mathbb{R} \setminus \{0\}$

(5) $a \in \mathbb{R} \setminus \{0, 1\}$

Question 4

We are informed that $Q(z)$ is a polynomial of degree 2 with the roots 1 and 3. We are given

$$P(z) = Q(z) \cdot (z^3 - 5z^2 - z + 5),$$

and we are informed that $P(z)$ has the root $z = -1$. What are all roots and corresponding multiplicities of P?

(1) $z_1 = -1$ with multiplicity 1, $z_2 = 3$ with multiplicity 1, $z_3 = 5$ with multiplicity 1, and $z_4 = 1$ with multiplicity 2.

(2) $z_1 = -1$ with multiplicity 1, $z_2 = 1$ with multiplicity 1, and $z_3 = 3$ with multiplicity 1.

(3) $z_1 = -1$ with multiplicity 2, $z_2 = 1$ with multiplicity 2, and $z_3 = 3$ with multiplicity 1.

(4) $z_1 = -1$ with multiplicity 1, $z_2 = 3$ with multiplicity 1, $z_3 = 5$ with multiplicity 1, and $z_4 = 1$ with multiplicity 1.

(5) $z_1 = 0$ with multiplicity 1, $z_2 = 1$ with multiplicity 1, $z_3 = 3$ with multiplicity 1, and $z_4 = 5$ with multiplicity 1.

Question 5

Let

$$\mathbf{B} = \begin{bmatrix} 4 & -1 & -1 \\ 6 & -3 & -1 \\ -6 & 5 & 3 \end{bmatrix} \in \mathbb{R}^{3\times 3}.$$

Choose matrices Λ and $\mathbf{V}$ such that $\mathbf{V}^{-1}\,\mathbf{B}\,\mathbf{V} = \Lambda$.

(1) $\mathbf{V} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ -1 & 1 & -1 \end{bmatrix}, \Lambda = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & -2 \end{bmatrix}$

(2) $\mathbf{V} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ -1 & 1 & -1 \end{bmatrix}, \Lambda = \begin{bmatrix} -2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \end{bmatrix}$

(3) $\mathbf{V} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ -1 & 1 & -1 \end{bmatrix}, \Lambda = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(4) $\mathbf{V} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ -1 & 1 & -1 \end{bmatrix}, \Lambda = \begin{bmatrix} 2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -4 \end{bmatrix}$

(5) No matrices $\mathbf{B}$ and Λ that fulfill the above relation exist.

Question 6

Let V be a subspace of $\mathbb{R}[Z]$, and let V be equipped with the ordered basis $\alpha = (1, Z, Z^2)$. We are given the following information about a linear map $f : V \to V$:

$$f(1) = 1 + Z\,,\ f(Z) = 1 - Z\,,\ \text{and}\ f(Z^2) = Z + Z^2.$$

What are the eigenvalues of this map?

(1) $1, \sqrt{2}, -\sqrt{2}$

(2) $1, -1, 1$

(3) $-1, -\sqrt{2}, \sqrt{2}$

(4) $\sqrt{2}, -1, 1$

(5) $-\sqrt{2}, 0, 1$

Question 7

A real system of differential equations is given by

$$\begin{bmatrix} f_1'(t) \\ f_2'(t) \end{bmatrix} = \mathbf{A} \begin{bmatrix} f_1(t) \\ f_2(t) \end{bmatrix}, \text{ where } \mathbf{A} = \begin{bmatrix} -1 & 3 \\ 3 & -1 \end{bmatrix}.$$

We are informed that $\begin{bmatrix} f_1(0) \\ f_2(0) \end{bmatrix} = \begin{bmatrix} 6 \\ 0 \end{bmatrix}$. What is the solution?

(1) $f_1(t) = 3e^{2t} + 3e^{-4t}$, $f_2(t) = 3e^{2t} - 3e^{-4t}$

(2) $f_1(t) = e^{2t} + 5e^{-4t}$, $f_2(t) = e^{2t} - e^{-4t}$

(3) $f_1(t) = 5e^{2t} + e^{-4t}$, $f_2(t) = 5e^{2t} - 5e^{-4t}$

(4) $f_1(t) = 6e^{2t}$, $f_2(t) = 0$

(5) $f_1(t) = e^{2t}$, $f_2(t) = e^{-4t}$

Question 8

A polynomial is given by

$$P(Z) = Z^2 + aZ + b \ , \ a,b \in \mathbb{C}.$$

We are informed that $z_1 = 1+i$ and $z_2 = 2i$ are roots in P. What are the values of a and b?

(1) $a = -3i - 1 \ , \ b = -2 + 2i$

(2) $a = 1 + i \ , \ b = 2i$

(3) $a = 3i + 1 \ , \ b = 2 - 2i$

(4) $a = 3i - 1 \ , \ b = -2 + 2i$

(5) $a = -3i - 1 \ , \ b = -2 - 2i$

END OF THE EXAM

Spørgsmål 1

Givet er følgende lineære ligningssystem over $\mathbb{R}$ i de tre ubekendte x_1, x_2, og x_3:

$$\begin{cases} x_2 + x_3 &= 1 \\ x_1 - 2x_2 + x_3 &= 4 \end{cases} .$$

Hvilke af nedenstående mængder angiver den fuldstændige løsning til systemet?

(1) $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 6 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} 3 \\ 1 \\ -1 \end{bmatrix}, t \in \mathbb{R}$

(2) $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 6 \\ 1 \\ 0 \end{bmatrix}$

(3) $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 6 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix}, t \in \mathbb{R}$

(4) $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 6 \\ 1 \\ 1 \end{bmatrix} + t \begin{bmatrix} 3 \\ 1 \\ -1 \end{bmatrix}, t \in \mathbb{R}$

(5) $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 6 \\ 1 \\ 1 \end{bmatrix} + t \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}, t \in \mathbb{R}$

Spørgsmål 2

Givet er:

$$\mathbf{A} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 0 \end{bmatrix}, \text{ og } \mathbf{v} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$$

Hvilke af følgende udtryk er identisk med udtrykket $(\mathbf{A} \cdot \mathbf{B})^T \cdot \mathbf{v}$?

(1) $\begin{bmatrix} 5 \\ 2 \\ -1 \end{bmatrix}$

(2) $\begin{bmatrix} -1 \\ 2 \\ 5 \end{bmatrix}$

(3) $\begin{bmatrix} 5 & 11 \\ 2 & 4 \\ -1 & -3 \end{bmatrix}$

(4) $\begin{bmatrix} 5 \\ 11 \end{bmatrix}$

(5) $\begin{bmatrix} 1 \\ 6 \end{bmatrix}$

Spørgsmål 3

Givet er

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 9 \\ 1 & a & 1 \\ 0 & -1 & a \end{bmatrix} \in \mathbb{R}^{3\times 3}.$$

For hvilken værdi af $a \in \mathbb{R}$ er matricen $\mathbf{A}$ invertibel?

(1) $a \in \mathbb{R} \setminus \{-2, 4\}$

(2) $a \in \{-2, 4\}$

(3) $a \in \mathbb{R}$

(4) $a \in \mathbb{R} \setminus \{0\}$

(5) $a \in \mathbb{R} \setminus \{0, 1\}$

Spørgsmål 4

Det er givet, at $Q(z)$ er et polynomium af grad 2 med rødderne 1 og 3. Givet er

$$P(z) = Q(z) \cdot (z^3 - 5z^2 - z + 5),$$

og det oplyses, at $P(z)$ har roden $z = -1$. Angiv alle rødder i P samt deres multipliciteter?

(1) $z_1 = -1$ med multiplicitet 1, $z_2 = 3$ med multiplicitet 1, $z_3 = 5$ med multiplicitet 1, og $z_4 = 1$ med multiplicitet 2.

(2) $z_1 = -1$ med multiplicitet 1, $z_2 = 1$ med multiplicitet 1, og $z_3 = 3$ med multiplicitet 1.

(3) $z_1 = -1$ med multiplicitet 2, $z_2 = 1$ med multiplicitet 2, og $z_3 = 3$ med multiplicitet 1.

(4) $z_1 = -1$ med multiplicitet 1, $z_2 = 3$ med multiplicitet 1, $z_3 = 5$ med multiplicitet 1, og $z_4 = 1$ med multiplicitet 1.

(5) $z_1 = 0$ med multiplicitet 1, $z_2 = 1$ med multiplicitet 1, $z_3 = 3$ med multiplicitet 1, og $z_4 = 5$ med multiplicitet 1.

Spørgsmål 5

Lad

$$\mathbf{B} = \begin{bmatrix} 4 & -1 & -1 \\ 6 & -3 & -1 \\ -6 & 5 & 3 \end{bmatrix} \in \mathbb{R}^{3\times 3}.$$

Vælg matricer Λ og $\mathbf{V}$ således at $\mathbf{V}^{-1}\,\mathbf{B}\,\mathbf{V} = \Lambda$.

(1) $\mathbf{V} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ -1 & 1 & -1 \end{bmatrix}, \Lambda = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & -2 \end{bmatrix}$

(2) $\mathbf{V} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ -1 & 1 & -1 \end{bmatrix}, \Lambda = \begin{bmatrix} -2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \end{bmatrix}$

(3) $\mathbf{V} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ -1 & 1 & -1 \end{bmatrix}, \Lambda = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(4) $\mathbf{V} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ -1 & 1 & -1 \end{bmatrix}, \Lambda = \begin{bmatrix} 2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -4 \end{bmatrix}$

(5) Der findes ingen matricer $\mathbf{B}$ og Λ, som opfylder den givne sammenhæng.

Spørgsmål 6

Lad V være et underrum i $\mathbb{R}[Z]$, og lad V være udstyret med den ordnede basis $\alpha = (1, Z, Z^2)$. Om en lineær afbildning $f : V \to V$ oplyses følgende:

$$f(1) = 1 + Z\,,\ f(Z) = 1 - Z\,,\ \text{og } f(Z^2) = Z + Z^2.$$

Hvad er egenværdierne af denne afbildning?

(1) $1, \sqrt{2}, -\sqrt{2}$

(2) $1, -1, 1$

(3) $-1, -\sqrt{2}, \sqrt{2}$

(4) $\sqrt{2}, -1, 1$

(5) $-\sqrt{2}, 0, 1$

Spørgsmål 7

Et reelt differentialligningssystem er givet ved

$$\begin{bmatrix} f_1'(t) \\ f_2'(t) \end{bmatrix} = \mathbf{A} \begin{bmatrix} f_1(t) \\ f_2(t) \end{bmatrix}, \text{ hvor } \mathbf{A} = \begin{bmatrix} -1 & 3 \\ 3 & -1 \end{bmatrix}.$$

Det oplyses at $\begin{bmatrix} f_1(0) \\ f_2(0) \end{bmatrix} = \begin{bmatrix} 6 \\ 0 \end{bmatrix}$. Hvad er løsningen?

(1) $f_1(t) = 3e^{2t} + 3e^{-4t}$, $f_2(t) = 3e^{2t} - 3e^{-4t}$

(2) $f_1(t) = e^{2t} + 5e^{-4t}$, $f_2(t) = e^{2t} - e^{-4t}$

(3) $f_1(t) = 5e^{2t} + e^{-4t}$, $f_2(t) = 5e^{2t} - 5e^{-4t}$

(4) $f_1(t) = 6e^{2t}$, $f_2(t) = 0$

(5) $f_1(t) = e^{2t}$, $f_2(t) = e^{-4t}$

Spørgsmål 8

Et polynomium er givet ved

$$P(Z) = Z^2 + aZ + b \ , \ a,b \in \mathbb{C}.$$

Det oplyses at $z_1 = 1 + i$ og $z_2 = 2i$ er rødder i P. Hvad er værdierne af a og b?

(1) $a = -3i - 1 \ , \ b = -2 + 2i$

(2) $a = 1 + i \ , \ b = 2i$

(3) $a = 3i + 1 \ , \ b = 2 - 2i$

(4) $a = 3i - 1 \ , \ b = -2 + 2i$

(5) $a = -3i - 1 \ , \ b = -2 - 2i$

EKSAMEN SLUT