

Homework Assignment 2



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a)

Let $z_1 = r_1 e^{i\frac{\pi}{3}}$ and $z_2 = r_2 e^{i\frac{\pi}{6}}$, then $z_1 z_2 = r_1 e^{i\frac{\pi}{3}} * r_2 e^{i\frac{\pi}{6}}$.

$$z_1 z_2 = r_1 r_2 e^{i(\frac{\pi}{3} + \frac{\pi}{6})} = r_1 r_2 e^{i\frac{\pi}{2}} = r_1 r_2 i$$

Which is a purely imaginary number.

b)

$$e^z = 1 + i$$

$$z = \ln(1 + i) = \ln \sqrt{2} e^{i\frac{\pi}{4}}$$

$$z = \ln \sqrt{2} + i\frac{\pi}{4} \text{ - first solution}$$

$$z = \ln -\sqrt{2} + i\frac{5\pi}{4} \text{ - second solution}$$

c)

$$\begin{array}{r|l} Z^4 + 3Z^3 + 4Z^2 + 6Z + 4 & Z^2 + 3Z + 2 \\ \hline - Z^4 - 3Z^3 - 2Z^2 & Z^2 + 2 \\ \hline & 2Z^2 + 6Z + 4 \\ & - 2Z^2 - 6Z - 4 \\ \hline & 0 \end{array}$$

$$(Z^4 + 3Z^3 + 4Z^2 + 6Z + 4)(Z^2 + 3Z + 2) = (Z^2 + 3Z + 2)(Z^2 + 2)$$

$$Z = \frac{-3 \pm \sqrt{9-8}}{2} = -1 \text{ or } -2$$

d)

That shows that 2 is one of the roots of the polynomial.

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The multiplicity of the root 2 in the polynomial is 2. e)

$$f(1000) \rightarrow f(500) \rightarrow f(250) \rightarrow f(125) \rightarrow f(124) \rightarrow f(62) \rightarrow f(31) \rightarrow f(30) \rightarrow$$

$$f(15) \rightarrow f(14) \rightarrow f(7) \rightarrow f(6) \rightarrow f(3) \rightarrow f(2) \rightarrow f(1)$$

$$f(1000) = 2 * 2 * 2 * 2 * 2 * 2 * 2 * 2 * 2 * f(1) = 2^9 * f(1) = 512$$

f)

Base case:

$$n = 0 \Rightarrow 1 \geq 1$$

Induction step:

$$(1 + h)^{n+1} \geq 1 + (n + 1)h$$

$$(1 + h)^{n+1} \geq 1 + hn + h$$

$$1 + (n + 1)h + \binom{n+1}{2}h^2 + \binom{n+1}{3}h^3 + \dots + h^{n+1} \geq 1 + (n + 1)h$$

Cancel out common additives,

$$\binom{n+1}{3}h^3 + \dots + h^{n+1} \geq 0$$

Once $\{h \in \mathbb{R} \mid h > 0\}$ and $\{n \in \mathbb{Z} \mid n \geq 0\}$, $\binom{n+1}{3}h^3 + \dots + h^{n+1} \geq 0$ is

True for all .