2)

1.

$$F(X)=0\ ,\quad X=\begin{bmatrix}x_1\\x_2\end{bmatrix},\quad F(X)=\begin{bmatrix}x_1^2+2x_2-2\\x_1+4x_2^2-4\end{bmatrix}$$

$$J_F(X)=\begin{bmatrix}2x_1 & 2\\ 1 & 8x_2\end{bmatrix}$$

2.

$$J_F(X_k)\cdot H_k=-F(X_k)$$

$$H_k=-\left(J_F(X_k)\right)^{-1}F(X_k)$$

$$X_{k+1}=X_k+H_k$$

3. Matrix factorization

1) $A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 3 & \\ 1 & 1 & 2 \end{bmatrix} \begin{matrix} R_2 = R_2 + \frac{1}{3}R_1 \\ R_3 = R_3 - \frac{1}{3}R_1 \end{matrix} \Rightarrow \begin{bmatrix} 3 & -1 & 1 \\ 0 & 8/3 & 4/3 \\ 0 & 4/3 & 5/3 \end{bmatrix} R_3 = R_3 - \frac{1}{2}R_2 \Rightarrow$

$\Rightarrow \begin{bmatrix} 3 & -1 & 1 \\ 0 & 8/3 & 4/3 \\ 0 & 0 & 1 \end{bmatrix} = U \qquad L = \begin{bmatrix} 1 & 0 & 0 \\ -\frac{1}{3} & 1 & 0 \\ \frac{1}{3} & \frac{1}{2} & 1 \end{bmatrix}$

2) LDL^T factorization

The matrix A has to be symmetric. ✓

$D = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 8/3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, diagonal of U.

L is the same as in LU.

3) The matrix A is symmetric and positive defined $\Rightarrow$ there is a Cholesky factorization.

$\tilde{L} = LD^{\frac{1}{2}} \Rightarrow \tilde{L} = \begin{bmatrix} 1 & 0 & 0 \\ -1/3 & 1 & 0 \\ 1/3 & 1/2 & 1 \end{bmatrix} \begin{bmatrix} \sqrt{3} & 0 & 0 \\ 0 & \sqrt{8/3} & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \sqrt{3} & 0 & 0 \\ -\sqrt{3}/3 & \sqrt{8/3} & 0 \\ \frac{\sqrt{3}}{3} & \sqrt{2/3} & 1 \end{bmatrix}$