Electrical and thermal properties

Henning Friis Poulsen, hfpo@dtu.dk

Curriculum:

Callister edition 10: 18.1-18.13, 18.17, 19

Exercises:

Callister edition 9: 19.2, 19.5, 19.11, 19.21, 19.25, 19.29,
20.2, 20.4, 20.7, 20.23, 20.14, 20.26.

For more physical background try as extras: 19.19, 20.22

Transport into Copenhagen

Stochastic process:
net flow

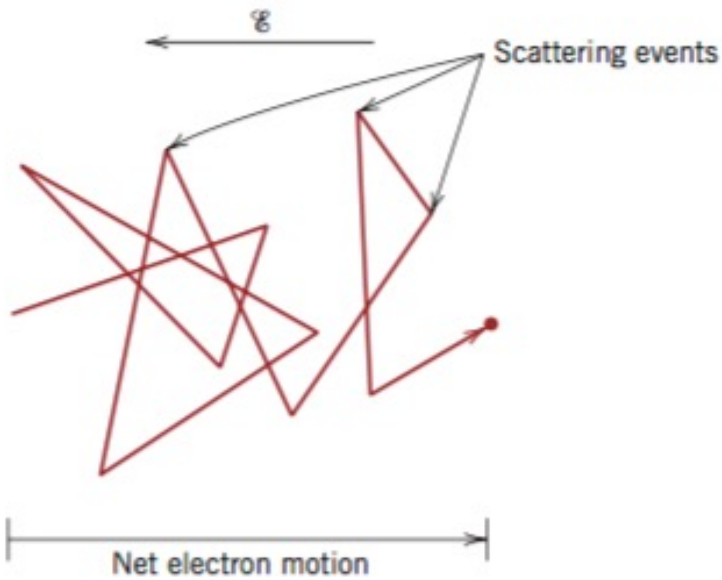
Several carriers:
car, train, bikes…

Capacity depends on
defintion:
per area....

Flow depends on
definition:
per area, per line …

Traffic: heterogeneous and non-linear flow

Transport of electrons

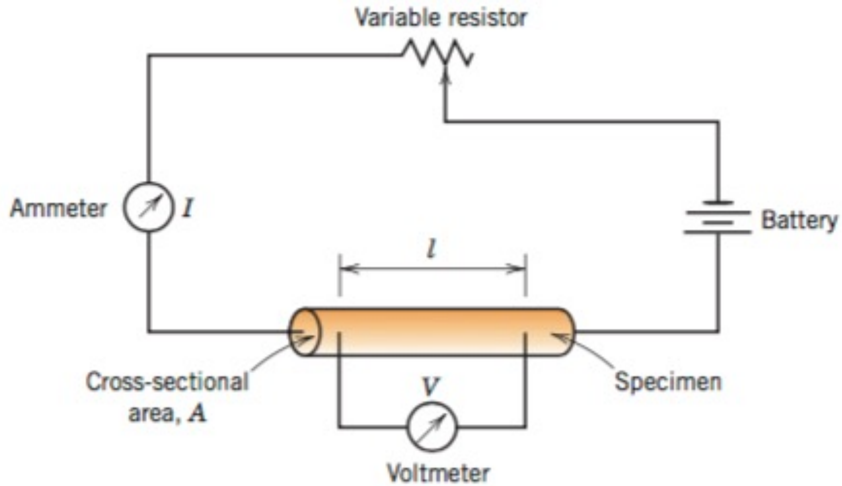


Stochastic process with a net current, but homogeneous and linear behaviour

We can understand and quantify this using SOLID STATE PHYSICS

Today: Borhs model that allow a qualitative understanding: conductors, semiconductors, insulators.

Definitions



Resistance, R $\quad$ V = R I $\quad$ Units: Ω (V/A), Volt (J/C), Amperes (C/s)

Conductance, C $\quad$ C = 1/R $\quad$ Unit: Ω^{-1}

Current density: $\quad$ J = I/A $\quad$ Unit: A/m^2

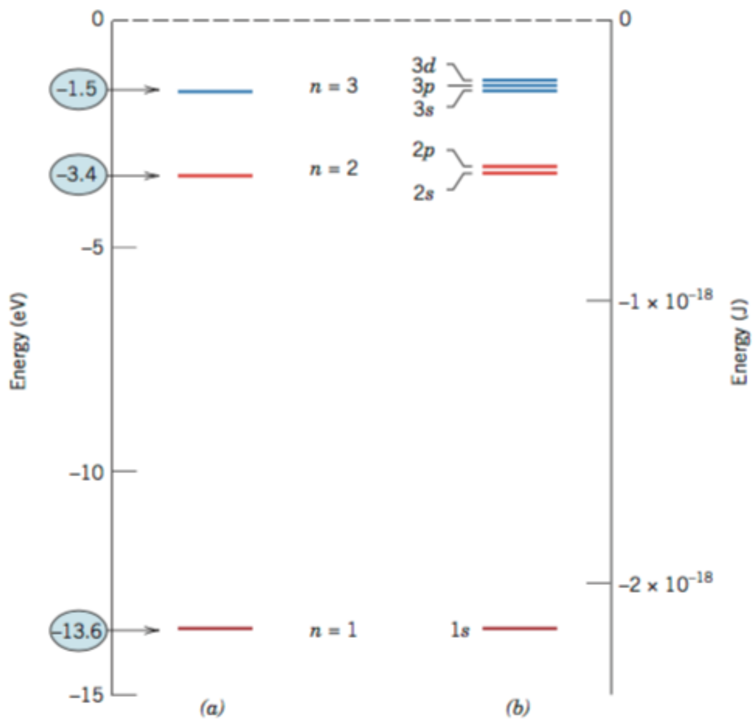
Electric field intensity: $\mathcal{E} = V/L$ $\quad$ Unit: V/m

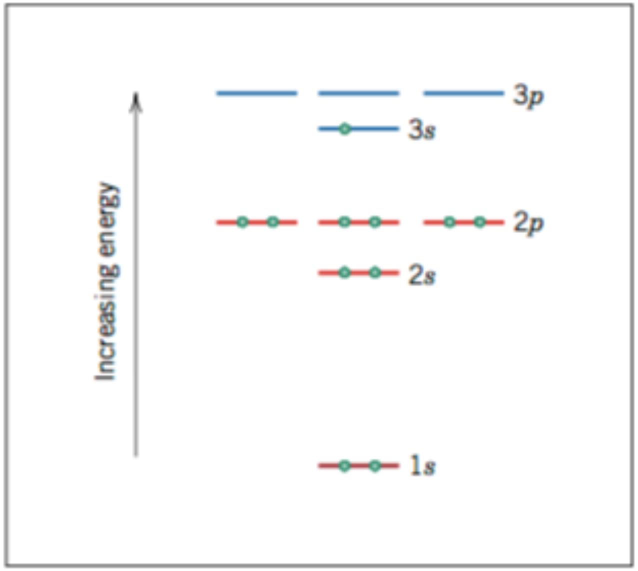
Ohms law: $\boxed{J = 1/\rho\, \mathcal{E} = \sigma\, \mathcal{E}}$ $\quad$ NB: Linear relationship

Resistivity, ρ $\quad$ ρ = RA/L $\quad$ Unit: Ωm

Conductivity, σ $\quad$ $\sigma = 1/\rho$ $\quad$ Unit: $\Omega^{-1}\ m^{-1}$

Bohrs model for one atom





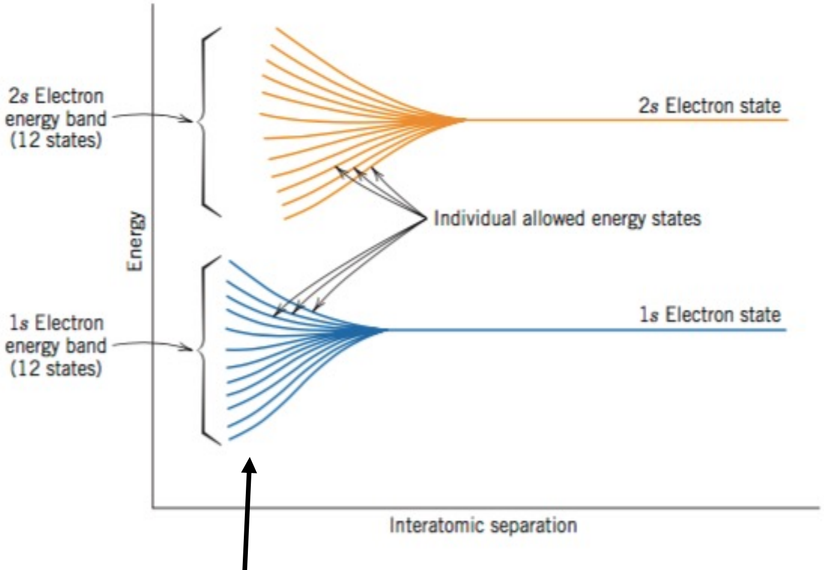
Hydrogen

Sodium

Shell	Subshell	Total Number of Electrons in Shell
1st Shell	1s	2
2nd Shell	2s, 2p	2 + 6 = 8
3rd Shell	3s, 3p, 3d	2 + 6 + 10 = 18
4th Shell	4s, 4p, 4d, 4f	2 + 6 + 10 + 14 = 32

Energy bands

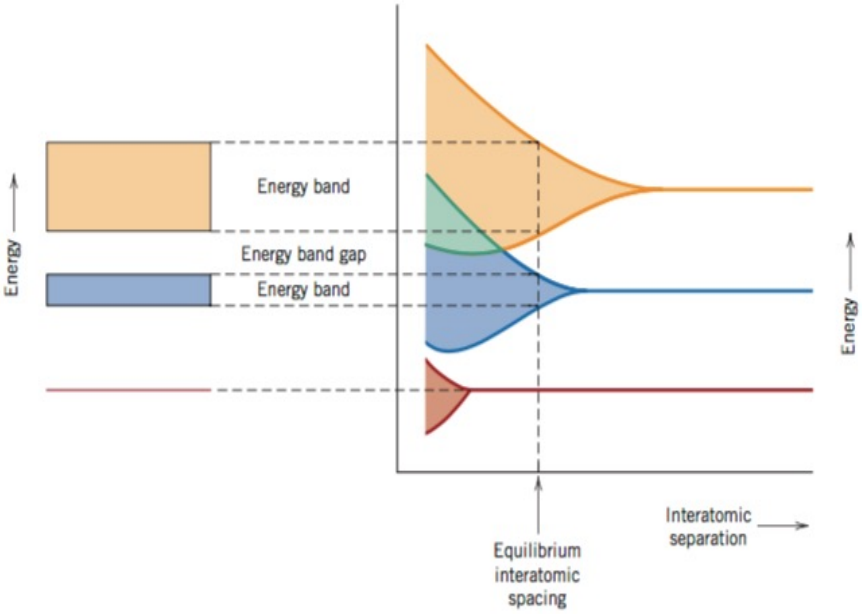
Borhs model for 12 atoms



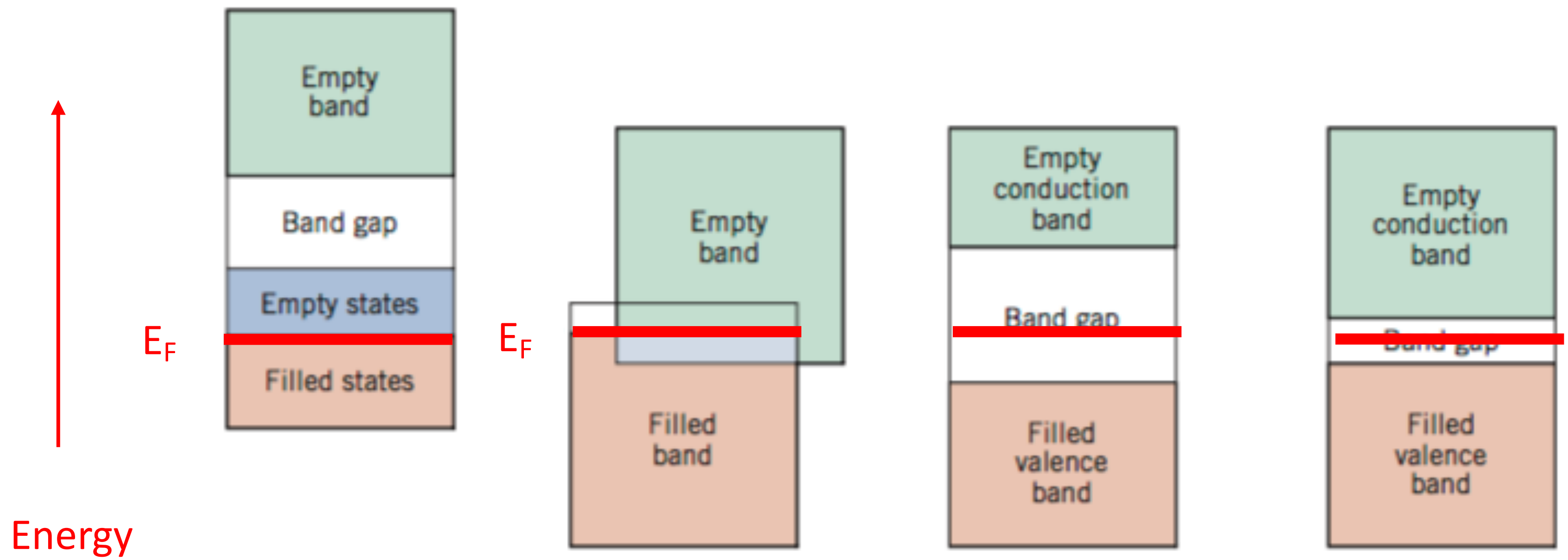
Splitting af states

Bohrs model for N_A atoms

=> Energy bands



Fermi energy and thermal exitation



Metal, Cu | Metal, Mg | Insulator | Semiconductor

Band gap > 2 eV | Band gap < 2 eV

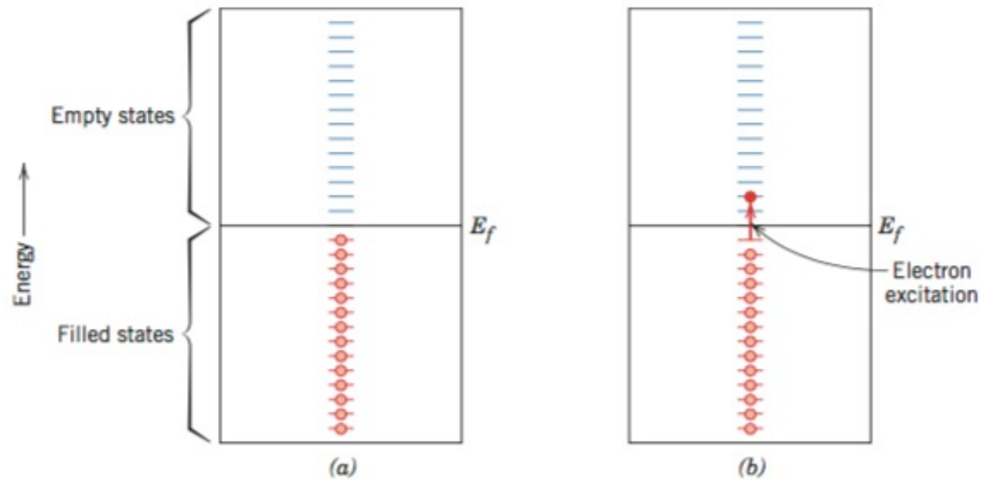
Thermal variation: probability goes as $\exp(-(E-E_f)/kT)$

$kT = 25$ meV at 300 K

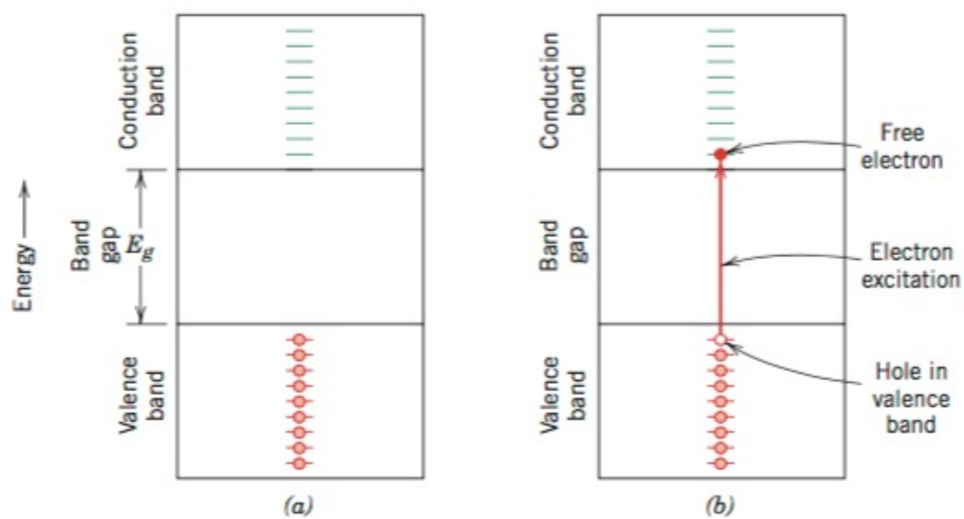
$\sigma \sim 10^{7}\ (\Omega m)^{-1}$ | $\sigma \sim 10^{-20} - 10^{-10}\ (\Omega m)^{-1}$ | $\sigma \sim 10^{-6} - 10^{4}\ (\Omega m)^{-1}$

Same but with figures from book

Metal

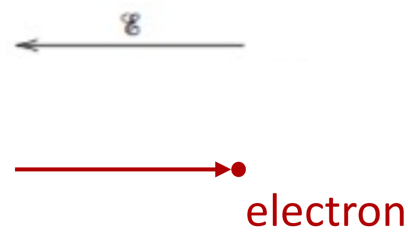


Insulator



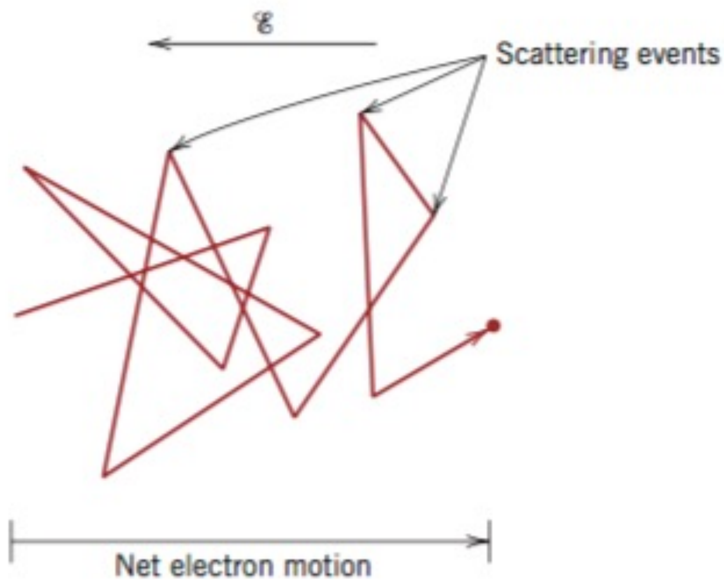
What causes the resistance

Perfect crystal lattice:



Imperfect crystal lattice:

- impurities
- vacancies
- dislocations
- vibrations (heat)



Electron mobility

Current density $\boxed{J = n\,|e|\,v_d}$ NB: SIGN

Electron density: n Unit: m^{-3}
Drift (net) velocity v_d Unit: m/s

$|e| = 1.6\ 10^{-19}$ C

$$\boxed{v_d = \mu_e\,\mathcal{E}}$$

Electron mobility, μ_e Unit: $m^2/(Vs)$ NB: depends on scattering processes

Ohms law: $\sigma = J/\mathcal{E} = n\,|e|\,\mu_e$ NB: Linear relationship

Electrical resistivity of metals

Metal	*Electrical Conductivity* $[(\Omega\text{-}m)^{-1}]$
Silver	6.8×10^7
Copper	6.0×10^7
Gold	4.3×10^7
Aluminum	3.8×10^7
Brass (70Cu–30Zn)	1.6×10^7
Iron	1.0×10^7
Platinum	0.94×10^7
Plain carbon steel	0.6×10^7
Stainless steel	0.2×10^7

Several obstacles: $\boxed{\rho_{total} = \rho_{temp} + \rho_{imp} + \rho_{disloc}}$

Temp. Variation: $\rho_{temp} = \rho_0 + aT$

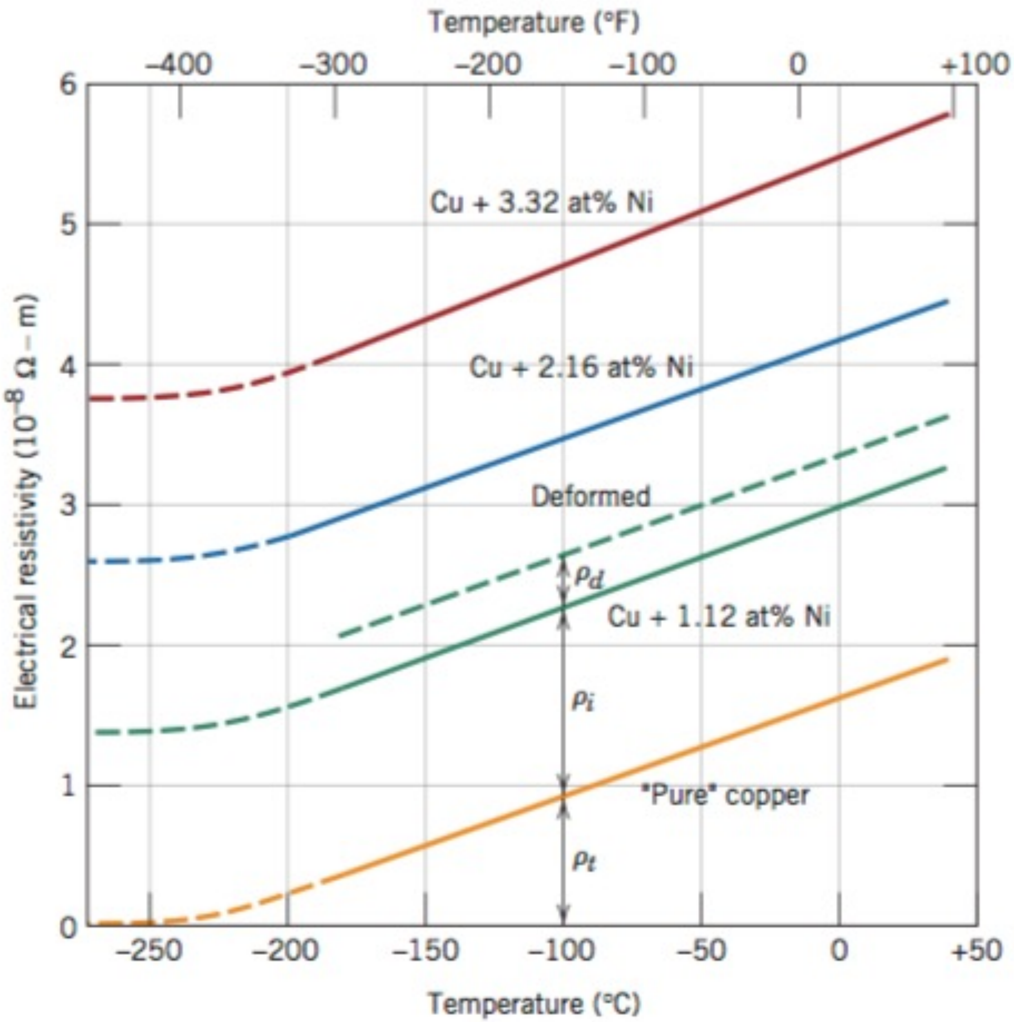
Impurity variation,

solid state $\rho_{imp} = A\, c_i(1-c_i)$; c_i: concentration of impurities

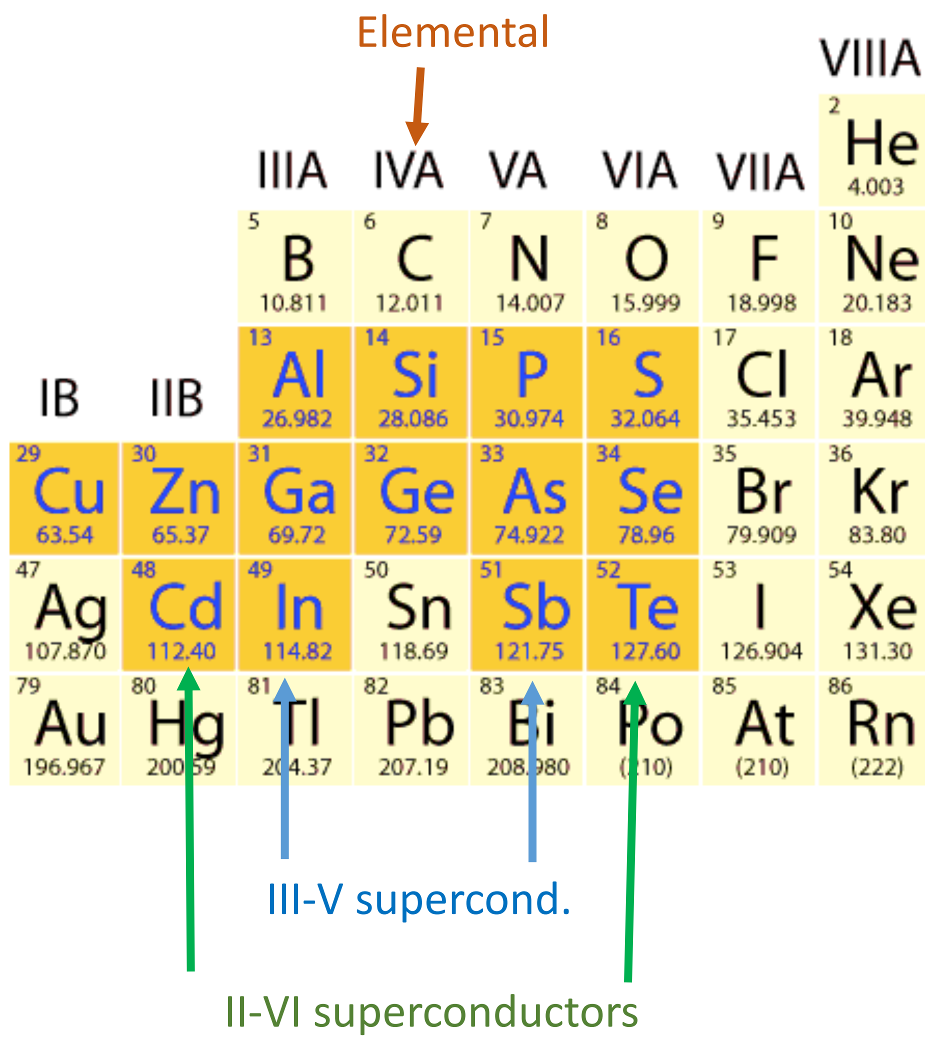
two phase $\rho_{imp} = V_1\rho_1 + V_2\rho_2$ V_1 and V_2: volume fractions

ρ_1, ρ_2: resistivity of the two phases

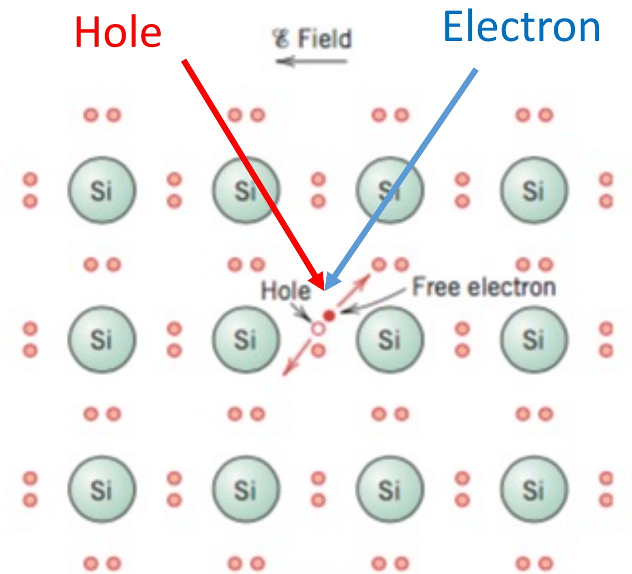
Electrical resistivity of Cu and Cu alloys



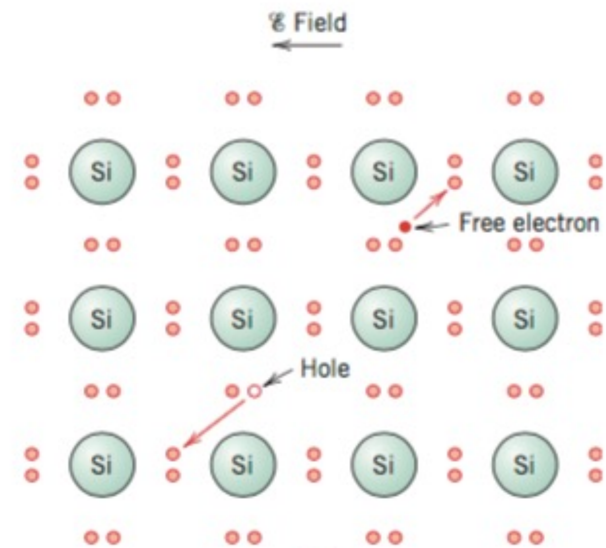
Semiconductors



Intrinsic (pure) semiconductors



(b)



(c)

Intrinsic conductivity:

$$\sigma = - n\,(-e)\,\mu_e + p\,e\,\mu_h$$

$$= n\,|e|\,\mu_e + p\,|e|\,\mu_h$$

$$= n_i\,|e|\,(\mu_e + \mu_h)$$

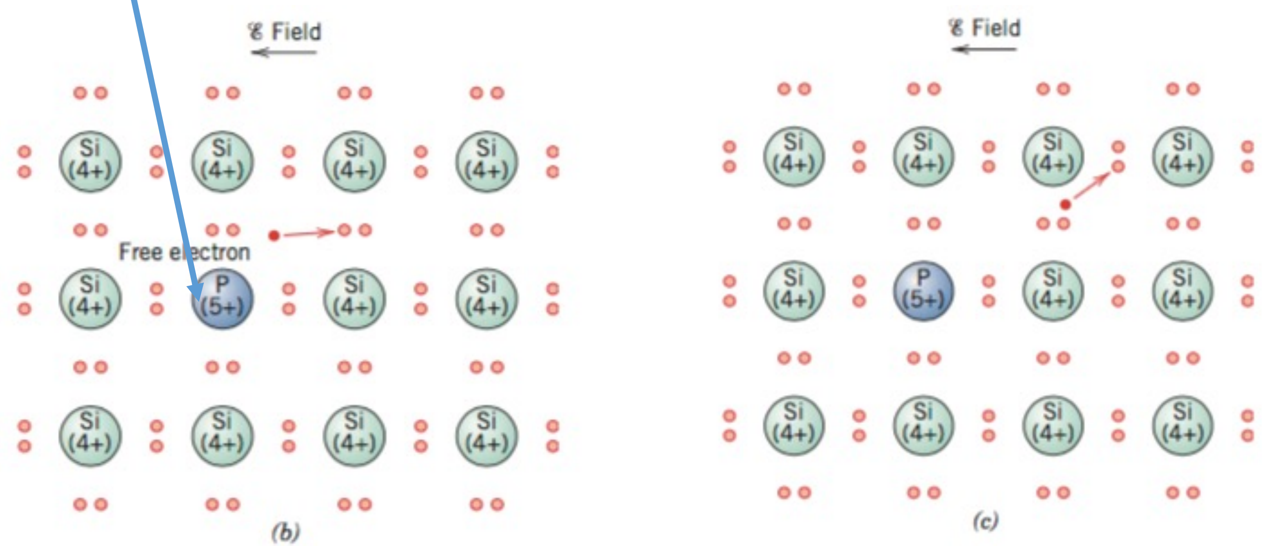
with intrinsic carrier concentration n_i defined as $n_i = n = p$

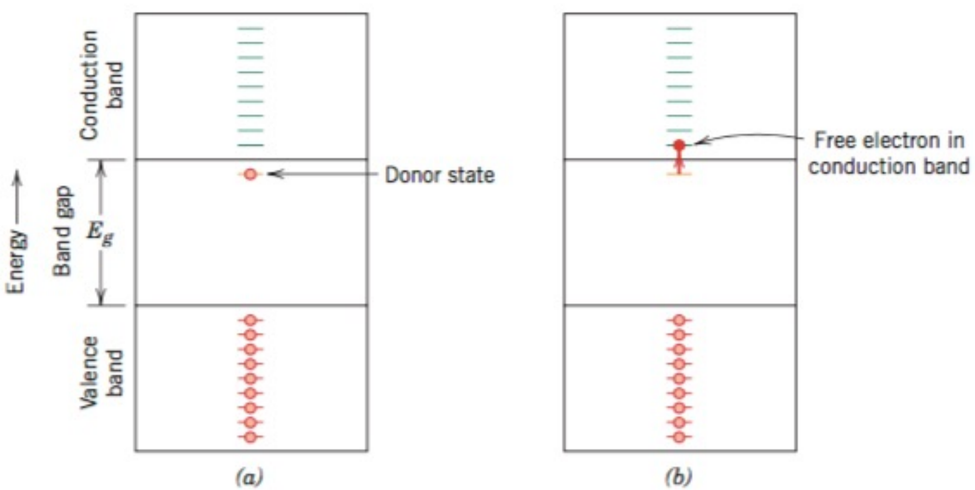
Intrinsic (pure) semiconductors

Material	*Band Gap (eV)*	*Electrical Conductivity* $[(\Omega\text{-}m)^{-1}]$	*Electron Mobility* $(m^2/V\text{-}s)$	*Hole Mobility* $(m^2/V\text{-}s)$
		Elemental		
Si	1.11	4×10^{-4}	0.14	0.05
Ge	0.67	2.2	0.38	0.18
		III–V Compounds		
GaP	2.25	—	0.03	0.015
GaAs	1.42	10^{-6}	0.85	0.04
InSb	0.17	2×10^{4}	7.7	0.07
		II–VI Compounds		
CdS	2.40	—	0.03	—
ZnTe	2.26	—	0.03	0.01

Extrinsic (doped) semiconductors: n-type

Donor

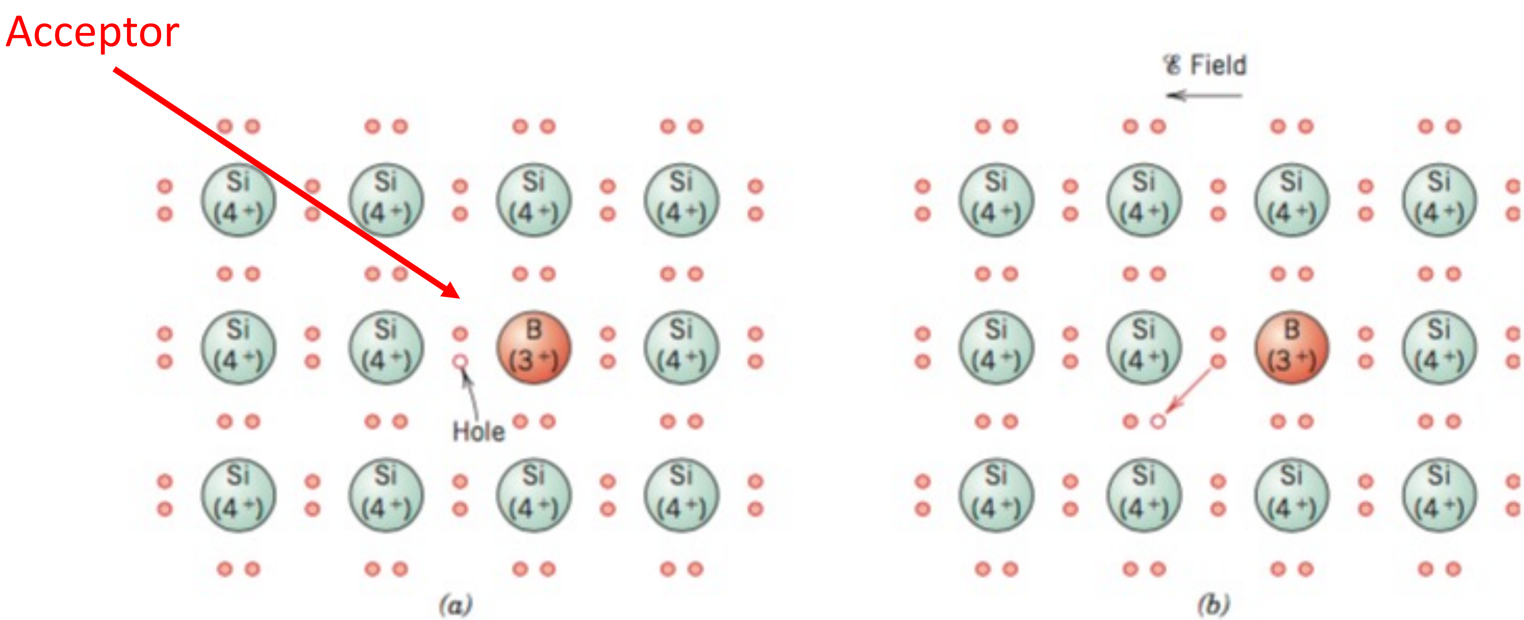


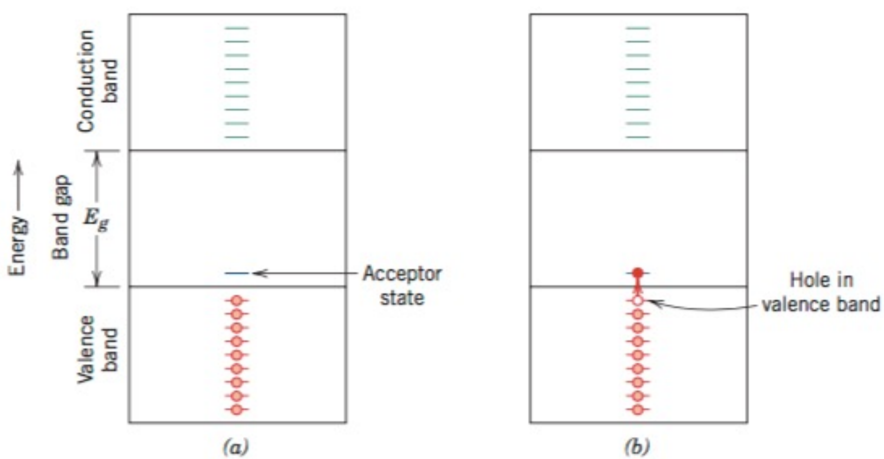


Extrinsic conductivity:

$$\sigma = n\,|e|\,\mu_e$$

Extrinsic (doped) semiconductors: p-type

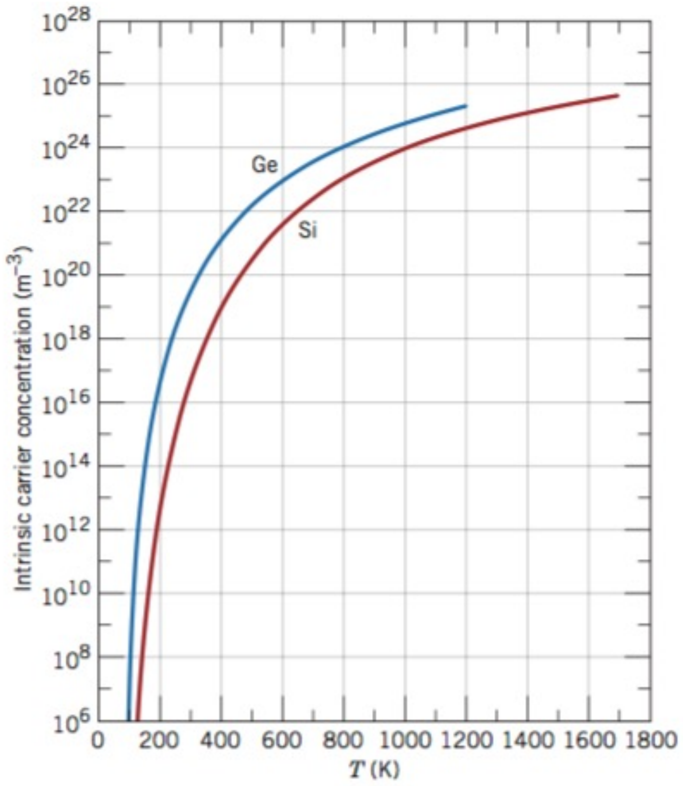




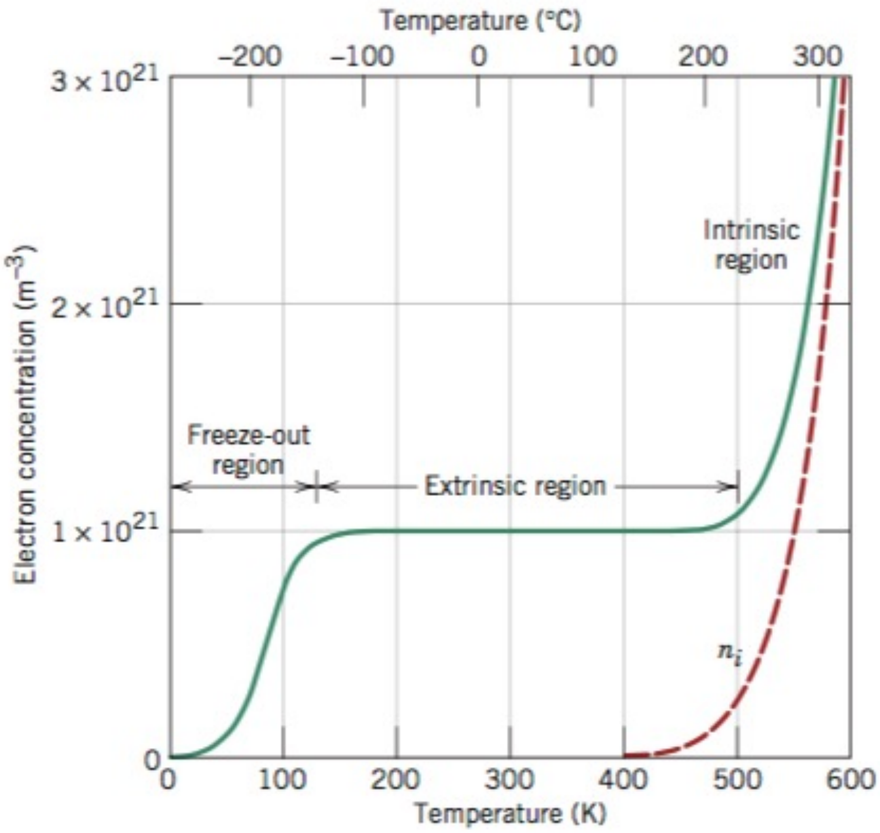
Extrinsic conductivity:

$$\sigma = p\,|e|\,\mu_h$$

Temperature variation of carrier concentration



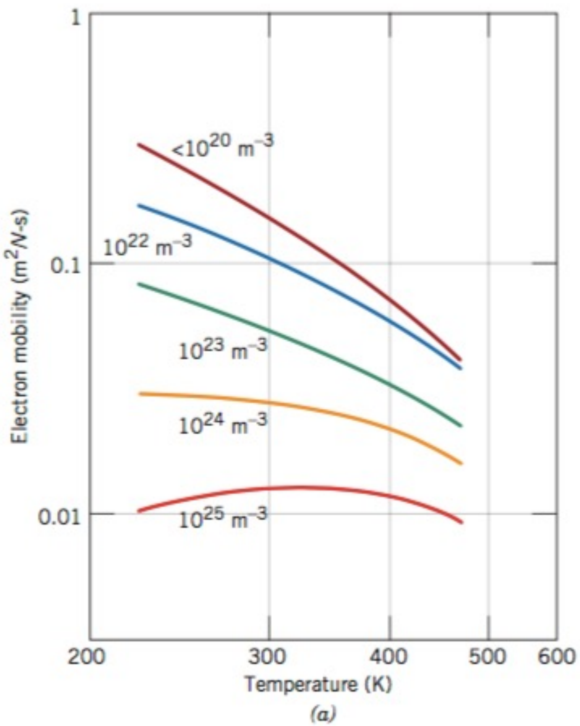
Intrinsic



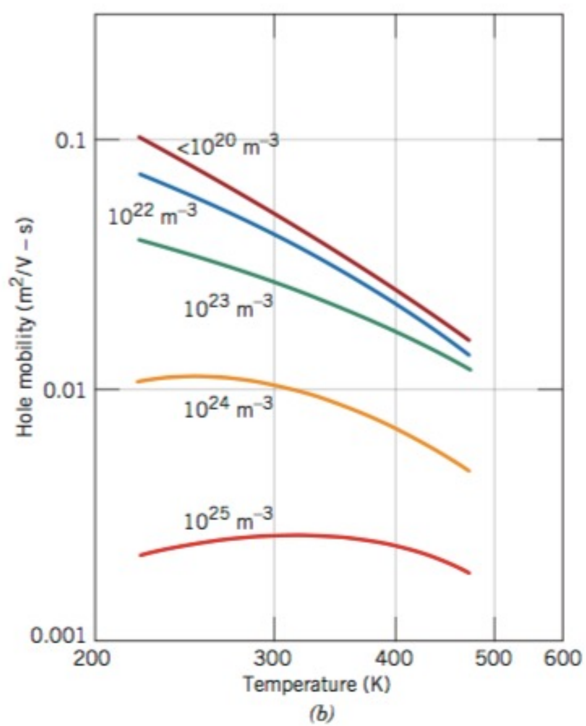
Extrinsic

Temperature variation of carrier mobility in Si

Electrons

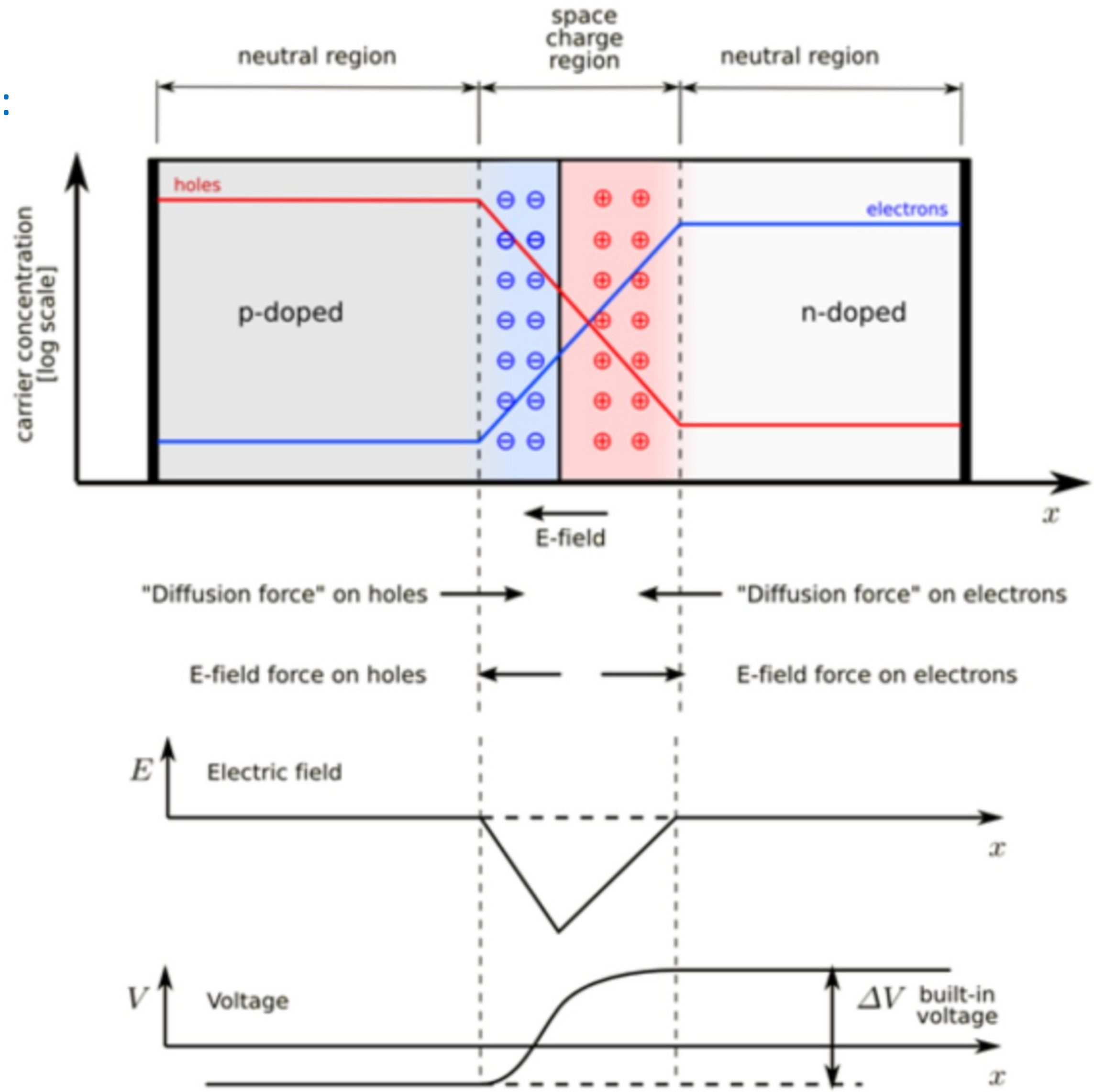


Holes



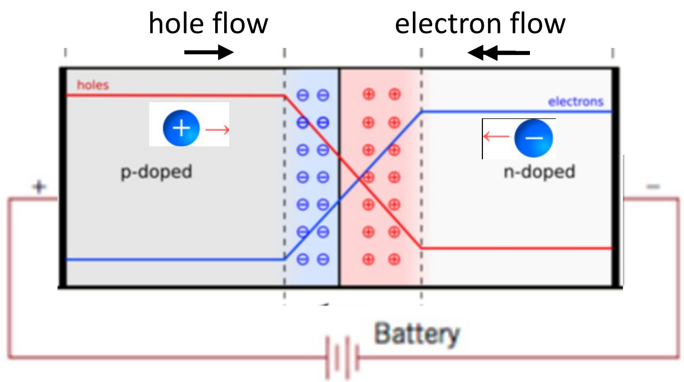
Diode: no circuit

Equilibrium:

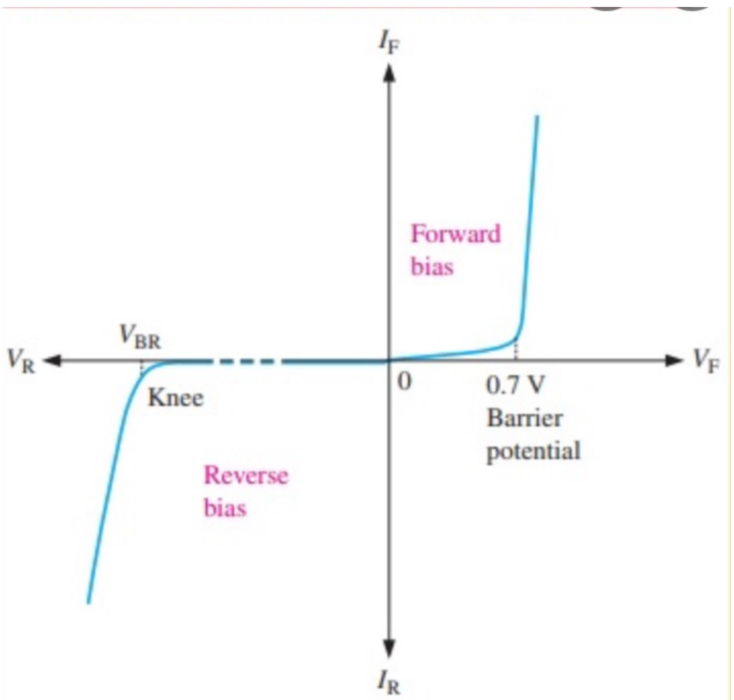


Diode: circuit

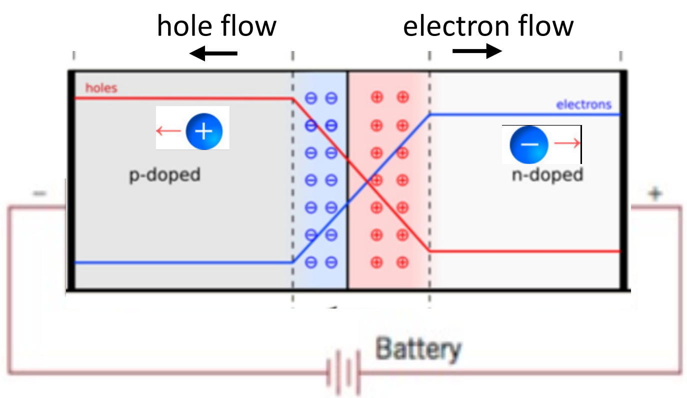
Forward bias



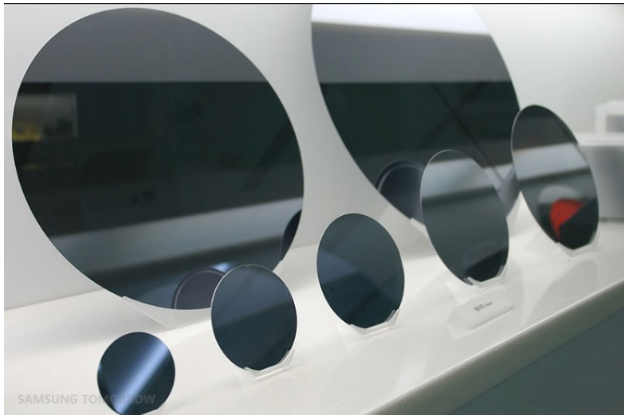
I/V curve



Reverse bias

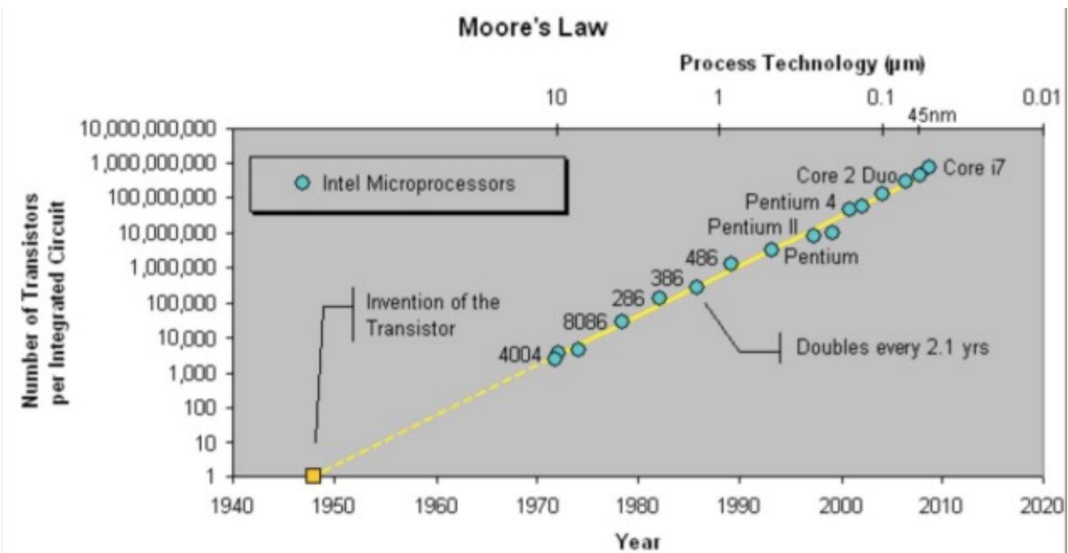


IT revolution



Purify large crystals

Solid state physics



Nanomanufacturing

Integration

Transport of heat

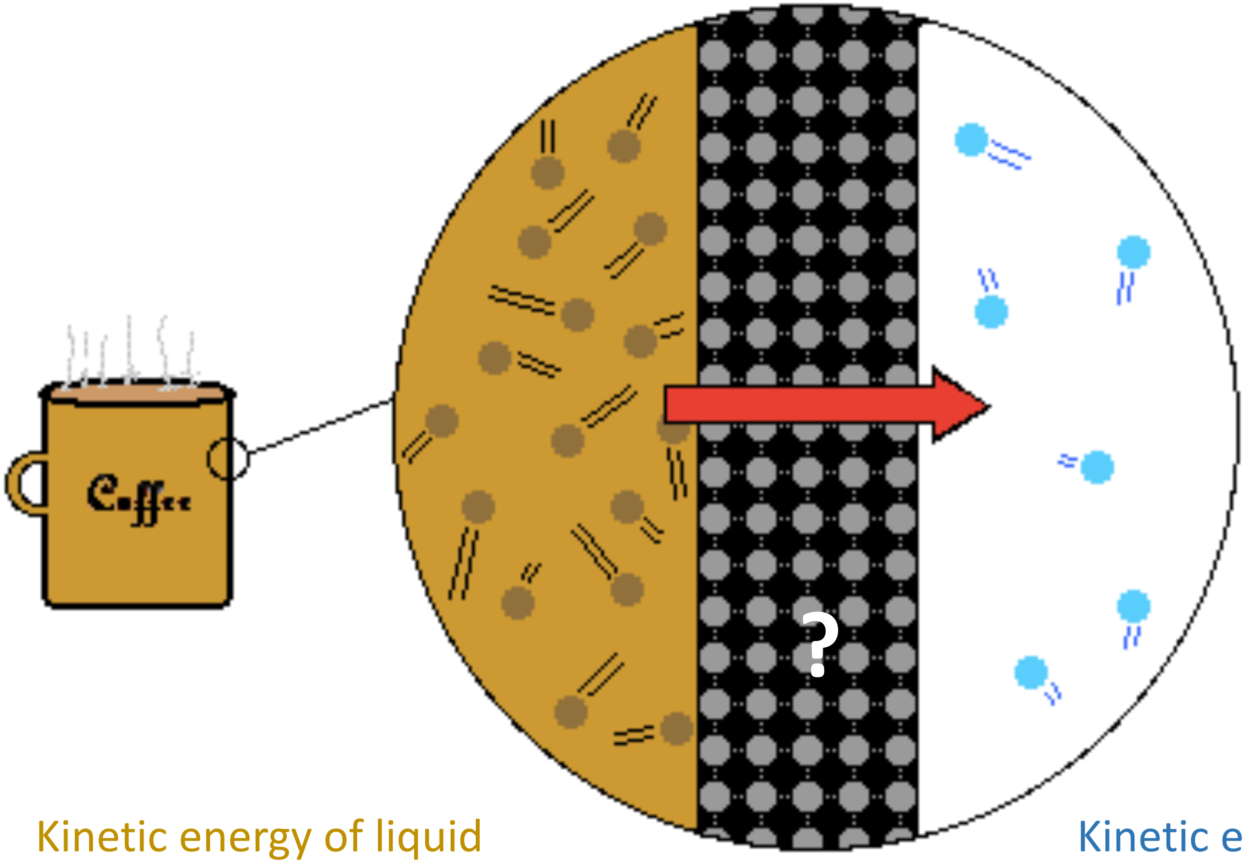
Consider for 2.5 minutes:

What is heat?

What is the unit of heat?

What is the carrier of heat transport?

Transport of heat through coffee mug



Kinetic energy of liquid

$$E = \frac{1}{2}\sum_{molecules} mv^2$$

Kinetic energy of gas

$$E = \frac{1}{2}\sum_{molecules} mv^2$$

Heat

Heat — Q — Unit: J

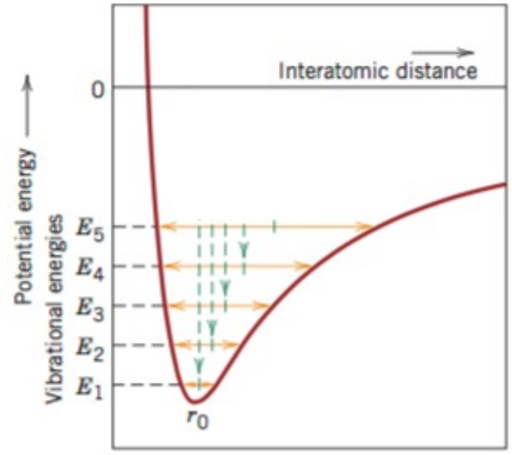
Heat capacity: — $C = dQ/dT$ — Unit: J/(mol K)

Specific heat — c_v , c_p — Unit: J/(kg K) – for fixed volume or pressure

$\boxed{\Delta E = m\, c_p\, \Delta T}$ — m is mass

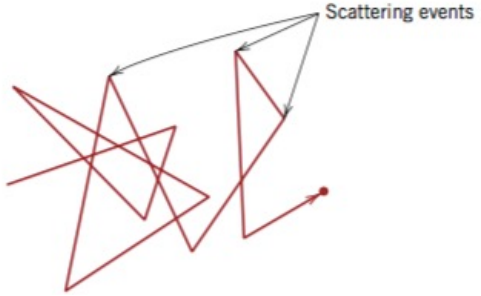
Capacitors:

Vibrational heat capacity



(a)

Conduction electron heat capacity
(only metals and only minor contribution)



Vibrational Heat

Debye

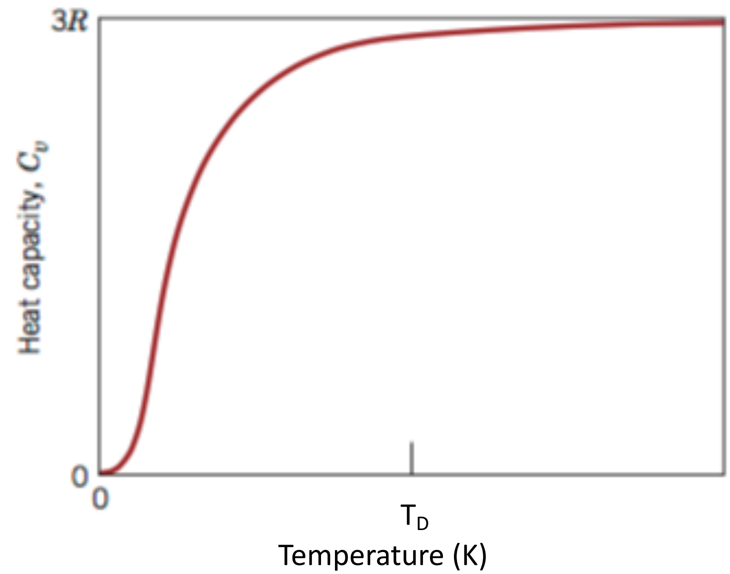
Debye: Nr of occupied vibrational states times energy of states

General

$$C_V = 9NkT(T/T_D)^3 \int_0^{T_D/T} \frac{x^3}{e^x - 1}\, dx\,,$$

$T \ll T_D$
$$\frac{C_V}{Nk} \sim \frac{12\pi^4}{5}\left(\frac{T}{T_D}\right)^3$$

$T \gg T_D$
$$\frac{C_V}{Nk} \sim 3\,.$$



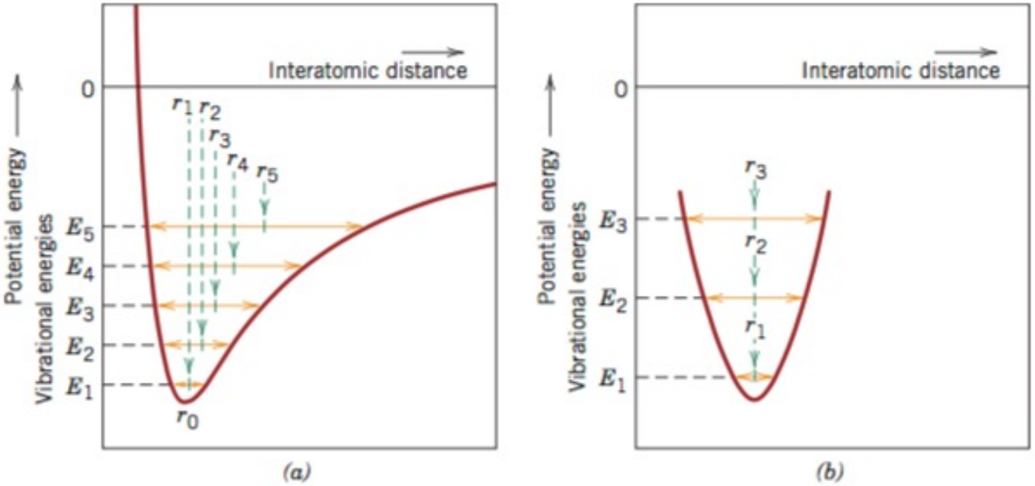
So dependence is defined by one parameter the Debye temperature: T_D

Aluminium	428 K	Manganese	410 K
Beryllium	1440 K	Nickel	450 K
Cadmium	209 K	Platinum	240 K
Caesium	38 K	Sapphire	1047 K
Carbon	2230 K	Silicon	645 K
Chromium	630 K	Silver	215 K
Copper	343.5 K	Tantalum	240 K
Gold	170 K	Tin (white)	200 K
Iron	470 K	Titanium	420 K
Lead	105 K	Tungsten	400 K

Thermal expansion

Linear expansion coefficient, α_l: $\Delta L/L = \alpha_l \, \Delta T$ Unit: K^{-1}

Volume expansion coefficient, α_v: $\Delta V/V = \alpha_v \, \Delta T$ Unit: K^{-1}



Loose bond

Harder bond

Metals

α_l = 5-25 10^{-6} K^{-1}

Ceramics

α_l = 0.5-15 10^{-6} K^{-1}

Polymers

α_l = 50-400 10^{-6} K^{-1}

Thermal stress

Stress: $\sigma = E\, \alpha_l\, \Delta T$

Transport of heat

<u>Steady state heat flow</u>

$q = -k \, \Delta T/\Delta x$ Ficks first law

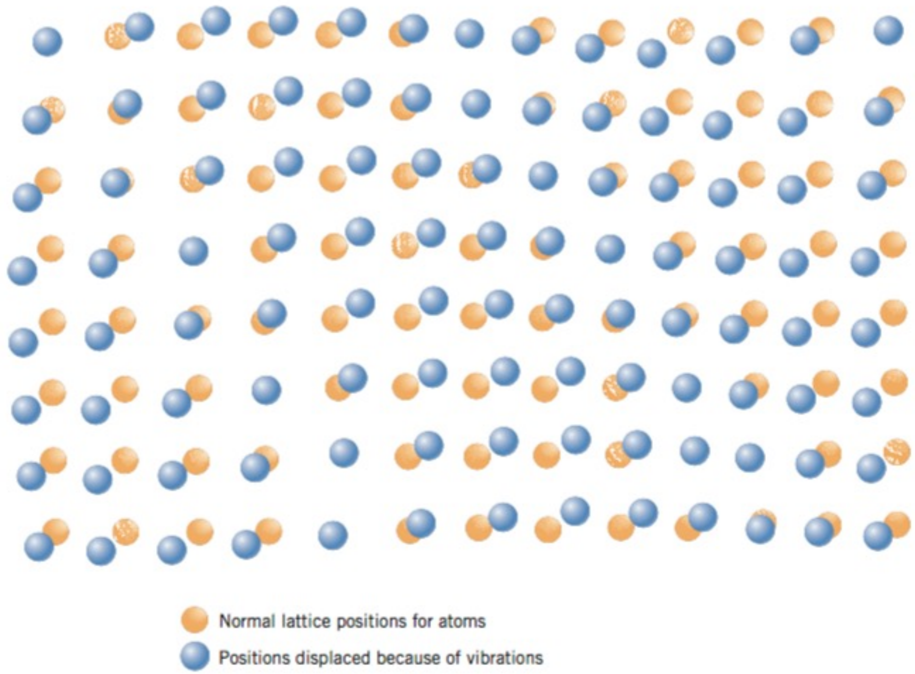
Heat flow	$q = Q/(At)$	Unit: Wm^{-2}
Thermal conductivity	k	Unit: $W/(mK)$

Several carriers: $k = k_{lattice} + k_{electron}$ (plus convection and radiation)

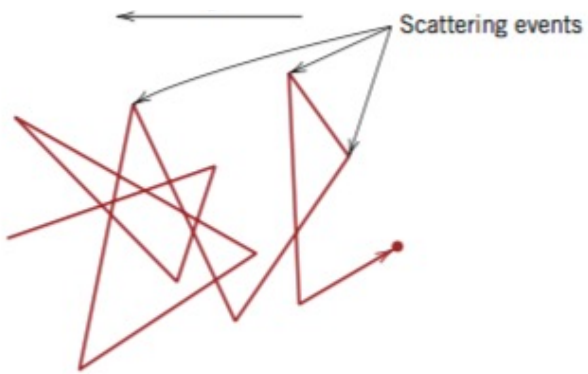
How can atoms dissipate heat when they are fixed in position within a few Å?

Transport in solids

Phonons = waves in lattice:



Free electrons



Thermal properties of metals and ceramics

Material	c_p (J/kg-K)[a]	α_l $[(°C)^{-1} \times 10^{-6}]$[b]	k (W/m-K)[c]	L $[\Omega\text{-W/(K)}^2 \times 10^{-8}]$
		Metals		
Aluminum	900	23.6	247	2.20
Copper	386	17.0	398	2.25
Gold	128	14.2	315	2.50
Iron	448	11.8	80	2.71
Nickel	443	13.3	90	2.08
Silver	235	19.7	428	2.13
Tungsten	138	4.5	178	3.20
1025 Steel	486	12.0	51.9	—
316 Stainless steel	502	16.0	15.9	—
Brass (70Cu–30Zn)	375	20.0	120	—
Kovar (54Fe–29Ni–17Co)	460	5.1	17	2.80
Invar (64Fe–36Ni)	500	1.6	10	2.75
Super Invar (63Fe–32Ni–5Co)	500	0.72	10	2.68
		Ceramics		
Alumina (Al_2O_3)	775	7.6	39	—
Magnesia (MgO)	940	13.5[d]	37.7	—
Spinel ($MgAl_2O_4$)	790	7.6[d]	15.0[e]	—
Fused silica (SiO_2)	740	0.4	1.4	—
Soda–lime glass	840	9.0	1.7	—
Borosilicate (Pyrex™) glass	850	3.3	1.4	—

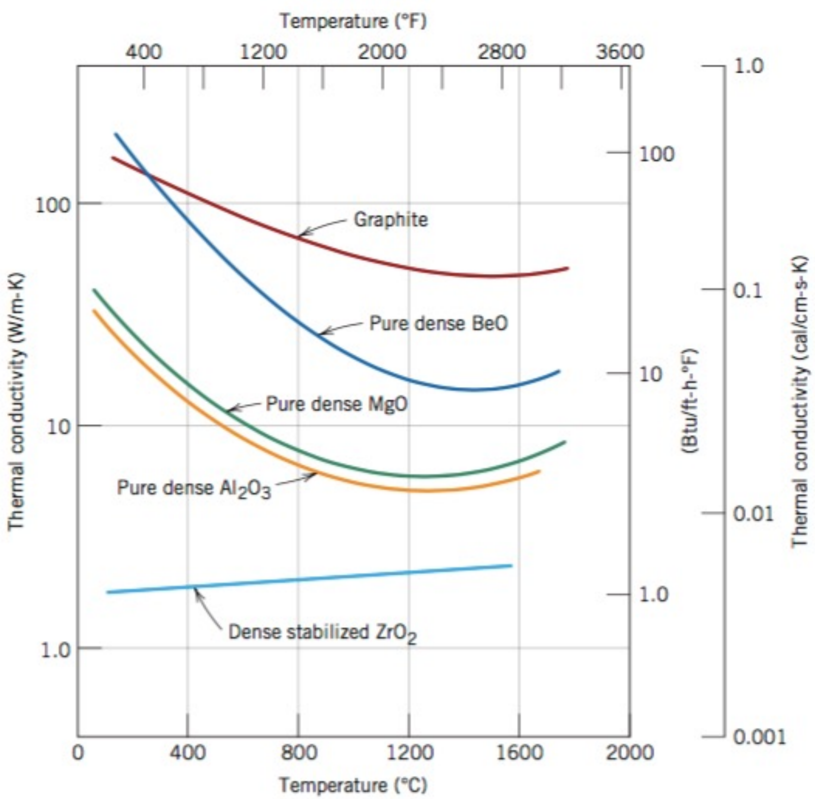
Metals: same carrier for both heat and electricity

The ratio $L = k/\sigma T$ is a constant

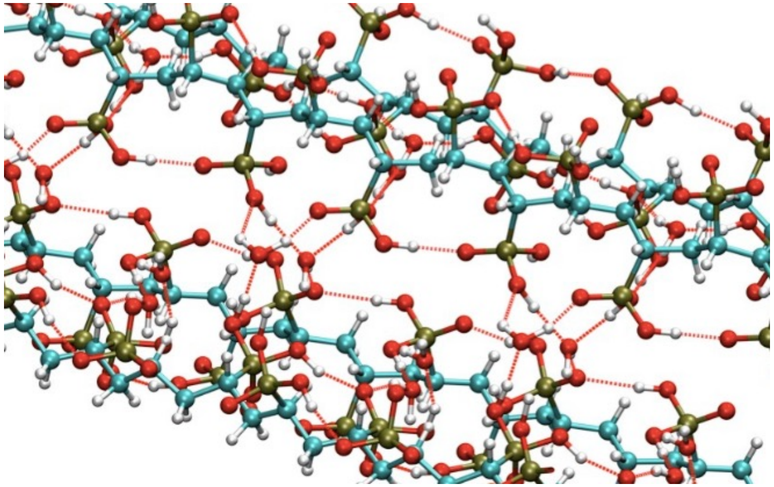
Thermal conductivity, ceramics

Only phonons as carriers!

Drastic effect of porosity



Polymers



Electrical conductivity, σ	$10^{-10} – 10^{-15}$ $(\Omega m)^{-1}$	$10^{-15} – 10^{-20}$ times metals
Specific heat, C_p	1000-2000 J/(kgK)	2 times metals
Thermal expansion , α_l	$100 – 200$ 10^{-6} K^{-1}	10 times metals
Thermal conductivity, k	0.1-0.2 W/(mK)	10^{-4} times metals

Properties

	ELECTRICAL CONDUCTIVITY $\Omega^{-1}\ m^{-1}$	SPECIFIC HEAT J/(kg K))	THERMAL CONDUCTIVITY W/(mK)
METALS	$2\ 10^{6} - 10^{8}$	$10^{2} - 10^{3}$	10-400
CERAMICS	$10^{-9} – 10^{-15}$	$6\ 10^{2} – 10^{3}$	1-50
SEMICONDUCTORS	$10^{-4} - 10^{4}$	$10^{2} - 10^{3}$	100
POLYMERS	$10^{-10} – 10^{-15}$	$10^{3} – 2\ 10^{3}$	0.1-0.2