01017/01019 Discrete Mathematics E24
Home assignment 4

Fedir Vasyliev s234542
William Carlsen s223818

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1 Exercise A

$$|A_1 \cup A_2 \cup A_3 \cup A_4| = \sum_{i=1}^{k} |A_i| - \sum_{i_1<i_2}^{k} |A_{i_1} \cap A_{i_2}| +$$
$$\sum_{i_1<i_2<i_3}^{k} |A_{i_1} \cap A_{i_2} \cap A_{i_3}| + (-1)^{k+1}|A_1 \cap A_2 \cap A_3 \cap A_4|$$
$$|A_1 \cup A_2 \cup A_3 \cup A_4| = 4 \cdot 100 - \binom{4}{2} \cdot 50 + \binom{4}{3} \cdot 25 - 5 =$$
$$= 400 - 6 \cdot 50 + 4 \cdot 25 - 5 = 195$$

2 Exercise B

We can use the same formula for the union of 4 sets from the previous exercise.

$$P(A \cup B \cup C \cup D) = P(A) + P(B) + P(C) + P(D)$$
$$-\big(P(A \cap B) + P(A \cap C) + P(A \cap D) + P(B \cap C) + P(B \cap D) + P(C \cap D)\big)$$
$$+\big(P(A \cap B \cap C) + P(A \cap B \cap D) + P(A \cap C \cap D) + P(B \cap C \cap D)\big)$$
$$-P(A \cap B \cap C \cap D).$$

As the probability of having 3 events at the time is 0, we can exclude them from the expression.

$$P(A \cup B \cup C \cup D) = P(A) + P(B) + P(C) + P(D)$$
$$-\big(P(A \cap B) + P(A \cap C) + P(A \cap D) + P(B \cap C) + P(B \cap D) + P(C \cap D)\big)$$
$$-P(A \cap B \cap C \cap D).$$

3 Exercise C

To count the derangements of {1,2,3,4,5,6} that ends with the integers 1, 2 and 3 in some order, I will first calculate the number of arrangements of the last three digits. This is simply the number of permutations of three objets, which is equal to $3! = 6$. Now the number of derangements of 4, 5 and 6 is calculated as

$$D(n) = n! \sum_{k=0}^{n} \frac{(-1)^k}{k!}$$
$$D(3) = 3! \cdot \big(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!}\big) = 2$$

This is the number of derangements of the list {4,5,6}, where the original list does not count. So the derangements of the list {1,2,3,4,5,6} with number 4, 5, and 6 as the first three digits is actually 3, because 4, 5, and 6 is not in the first three digits in the list.

So the total amount of derangements is the way of arranging the first three numbers times the way of arranging the last three, which is equal to $(2+1) \cdot 6 = 18$.

4 Exercise D

To prove that there exist a function with A as its domain such that $(x, y) \in R \iff f(x) = f(y)$, where A is a nonempty set and R is an equivalence class, I will prove the relation is reflexive, symmetric and transitive.
Because $f(x) = f(x)$ it follows that (x, x) is in R and it is reflective.
Because $f(x) = f(y)$ is the same as $f(y) = f(x)$ it follows that both (x, y) and (y, x) is in R and it is therefore symmetric.
If $f(x) = f(y)$ and $f(y) = f(z)$ then it follows that $f(x) = f(z)$, and therefore (x, y), (y, z) and (x, z) is in R and it is transitive.

5 Exercise E

Because the string x is equal, the relation xRx holds.(reflexive)
As we compare the nth latter in 2 strings it is evident that there is no difference in which order we pick, from the string x or y.(symmetric)
Suppose we have 3 strings x, y, and z. We know that relations xRy and yRz hold. $\Rightarrow$ xRz clearly holds as it is easy to see that all 3 strings are the same.(transitive)
Thus, R is an equivalence relation.

6 Exercise F

a) (S, R) is a poset.
aRa holds because $a = a$. $(aRb) \wedge (bRc) \Rightarrow aRc$ holds because if $a \geq b \wedge b \geq c \Rightarrow a \geq c$. And the symmetric relation only holds when $a = b$ because if one person is higher than the other it can't be in the opposite way.
b) (S, R) is not a poset.
The given relation doesn't allow for (a, a).
c) (S, R) is a poset.
The reflexive relation obviously holds. The transitive relation holds because If a is a descendant of b and b is a descendant of c, then a is a descendant of c. Similarly, if a=b or b=c, the property still holds. The relation is antisymmetric because If a is a descendant of b and b is also a descendant of a, the only possibility is a=b (since no person can "descend" from themselves).
d) (S, R) is not a poset.
The reflexive relation doesn't hold because the same person must have common friends with itself.

7 Exercise G

Since there are $2n$ players and every player will play excatly one match against every other player, each player will play $2n - 1$ games. Since the tournament is played over $2n - 1$ games, it is possible to select a winning player from each day without picking the same player twice.
This is due to the fact there is just as many games every day, as there is days in total, and that every game is unique and only played once. Now thinking of this as the pigeonhole principle, there can be picked excatly one winner everyday without picking the same person twice. If some of the players played more than on game per day, then it would not be possible because there would come days with fewer than $2n-1$ games.