Miniproject 2
Robot Control II

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In Miniproject 1 we have set up a system of differential equations that describe the dynamics and control of a robot. In Miniproject 2 we will investigate a test example, where the robot is controlled such that the position of the robot arm follows a given periodic piecewise differentiable trajectory $z(t)$ as closely as possible. At the time t, the current through the robot motor's coils is $j = j(t)$, and the position and velocity of the load respectively $x(t)$ and $v(t) = x'(t)$. The system of differential equations for the robot is given by

$$\frac{d}{dt}\begin{bmatrix} j \\ x \\ v \end{bmatrix} = \begin{bmatrix} -\alpha & 0 & -1 \\ 0 & 0 & 1 \\ 1 & 0 & -\beta \end{bmatrix}\begin{bmatrix} j \\ x \\ v \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} u(t)\,. \tag{1}$$

The voltage $u = u(t)$, across the motor, controls the robot arm's movement. The constant β is a friction term reflecting the friction in the rotor and the movement of the robot arm with load, and α describes the electrical resistance in the motor. We choose a control strategy for the voltage given by

$$u(t) = g_1[z(t) - x(t)]\,. \tag{2}$$

Here g_1 is a real constant.

Problem 1. With the voltage $u(t)$ given by (2), show that the system (1) can be written in the form:

$$\frac{d}{dt}\begin{bmatrix} j \\ x \\ v \end{bmatrix} = \begin{bmatrix} -\alpha & -g_1 & -1 \\ 0 & 0 & 1 \\ 1 & 0 & -\beta \end{bmatrix}\begin{bmatrix} j \\ x \\ v \end{bmatrix} + \begin{bmatrix} g_1 \\ 0 \\ 0 \end{bmatrix} z(t)\,. \tag{3}$$

Problem 2. We choose a trajectory $z(t)$ that is piecewise differentiable, periodic with period T and given on the interval $[0, T]$ by:

$$z(t) = \begin{cases} \frac{2}{T}t & \text{for } 0 \le t < \frac{T}{2}\,, \\ \frac{2}{T}(T-t) & \text{for } \frac{T}{2} \le t < T\,. \end{cases} \tag{4}$$

Plot $z(t)$ on the interval $[0, T]$ for $T = 20\pi$.

It is known that the complex Fourier series for z is given by:

$$z(t) = \frac{1}{2} + \sum_{\substack{n=-\infty \\ n \neq 0}}^{\infty} \frac{(-1)^n - 1}{n^2\pi^2} e^{in\omega t}\,, \tag{5}$$

where $\omega = 2\pi/T$. That means for $n \neq 0$ even, $c_n = 0$ and, for n odd, $c_n = -2/(n^2\pi^2)$.

We remark that in Miniproject 1 we found that the system (3) is asymptotically stable for $g_1 < (\alpha + \beta)(1 + \alpha\beta)$. Introduce

$$y(t) = [0, 1, 0] \begin{bmatrix} j(t) \\ x(t) \\ v(t) \end{bmatrix} = x(t)\,, \tag{6}$$

which corresponds to taking the vector $\mathbf{d}$ in equation (2.35) in the textbook as $\mathbf{d} = [0, 1, 0]^T$.

Problem 3. Determine the transfer function $H(in\omega)$ for the system (3). (It is recommended to use a CAS tool).

Problem 4. Use Fourier's Method to write down the Fourier series for the solution $x(t) = y(t)$.

Problem 5. For $N = 15$, plot the partial sum $S_N^x(t)$ and $z(t)$ for $t \in [0, T]$ in the same figure using the parameter values: $T = 20\pi$, $\alpha = 0.1$, $\beta = 0.1$, $g_1 = 0.2$. Discuss your result. Is the system BIBO stable for these parameter values?