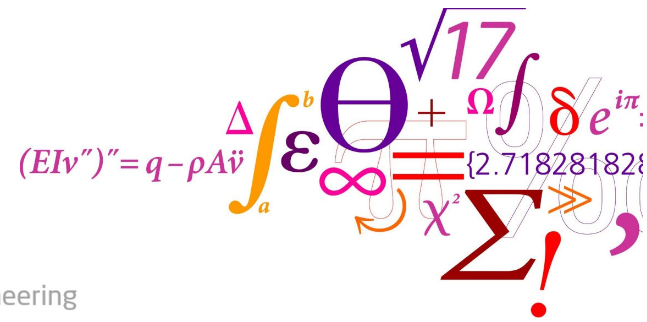
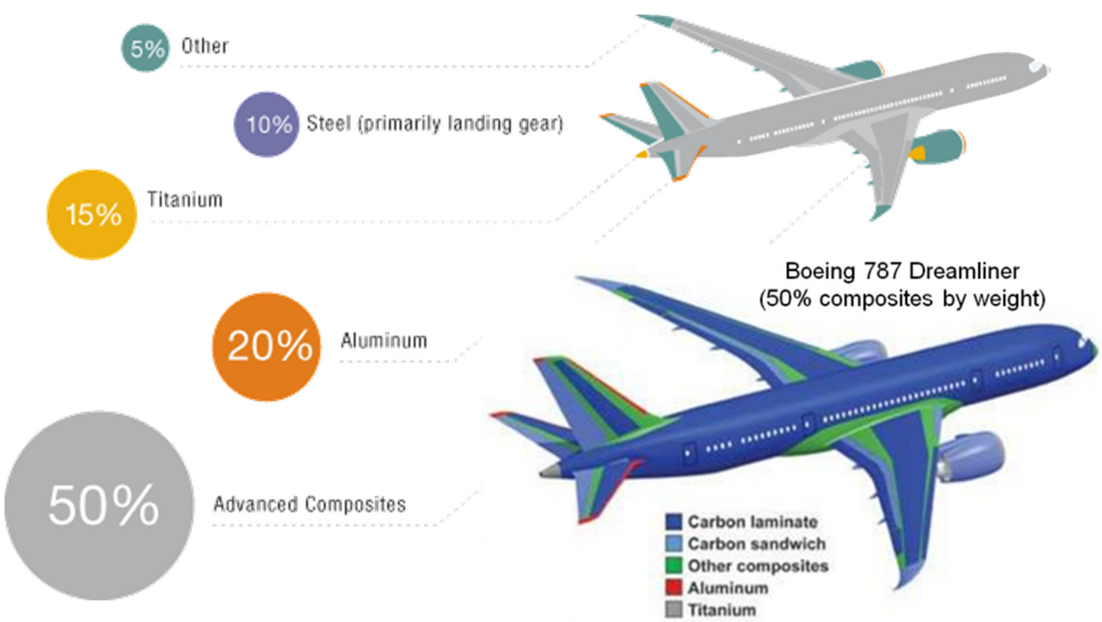
Composites and materials indices

41680 Introduction to advanced materials



DTU Construct
Department of Civil and Mechanical Engineering

Composites



Composites

- Combination of two or more materials
- Aim: achievement of desired properties

- Artificial multiphase materials

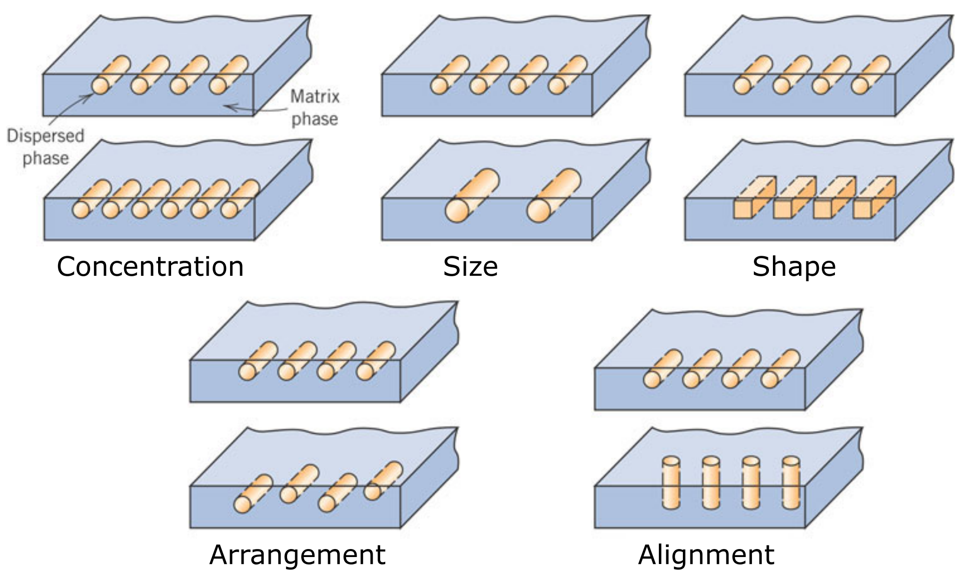
- Phase 1: continuous matrix
- Phase 2: discontinuously distributed in matrix

- Types

MMC	PMC	CMC
Metal	Polymer	Ceramic
Matrix Composite		

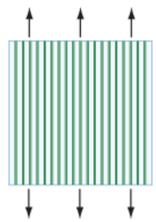
- Natural multiphase material, e.g. eutectics

Composites



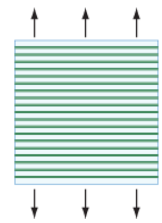
Elastic properties of laminar composites

- Longitudinal lamellae (iso-strain)



$$\varepsilon_c = \varepsilon_m = \varepsilon_f$$
$$\sigma_{c\|} = f_m \sigma_m + f_f \sigma_f$$

- Transversal lamellae (iso-stress)



$$\sigma_c = \sigma_m = \sigma_f$$
$$\varepsilon_{c\perp} = f_m \varepsilon_m + f_f \varepsilon_f$$

- Rule of mixture for stress

$$E_{c\|} = f_m E_m + f_f E_f$$

- Rule of mixture for strain

$$\frac{1}{E_{c\perp}} = f_m \frac{1}{E_m} + f_f \frac{1}{E_f}$$

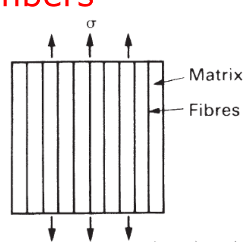
$$\boxed{E_{c\|} > E_{c\perp}}$$

- Voigt model
- Upper bond
- Volume fractions f_m, f_f

- Reuss model
- Lower bond
- Volume fractions f_m, f_f

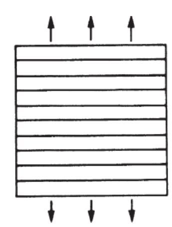
Elastic properties of fiber composites (formulae same as for laminar composites)

- Longitudinal fibers (iso-strain)



$$\varepsilon_c = \varepsilon_m = \varepsilon_f$$
$$\sigma_{c\|} = f_m \sigma_m + f_f \sigma_f$$

- Transversal fibers (iso-stress)



$$\sigma_c = \sigma_m = \sigma_f$$
$$\varepsilon_{c\perp} = f_m \varepsilon_m + f_f \varepsilon_f$$

- Rule of mixture for stress

$$E_{c\|} = f_m E_m + f_f E_f$$

- Rule of mixture for strain

$$\frac{1}{E_{c\perp}} = f_m \frac{1}{E_m} + f_f \frac{1}{E_f}$$

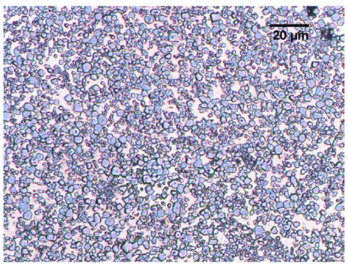
$$\boxed{E_{c\|} > E_{c\perp}}$$

- Voigt model
- Upper bond
- Volume fractions f_m, f_f

- Reuss model
- Lower bond
- Volume fractions f_m, f_f

Properties of particulate composites

- Particles (p)
  dispersed in matrix (m)
- Example
  W particles (blue)
  in Cu matrix (red)



- Volume fractions

$$f_m + f_p = 1$$

- Rule of mixture

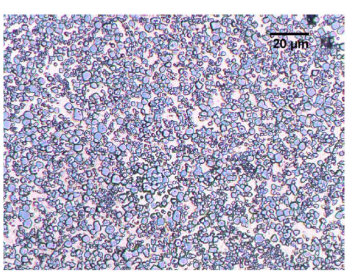
$$x_c = f_m x_m + f_p x_p$$

- Mass density

$$\rho_c = f_m \rho_m + f_p \rho_p$$

Elastic properties of particulate composites

- Particles (p)
  dispersed in matrix (m)
- Example
  W particles (blue)
  in Cu matrix (red)

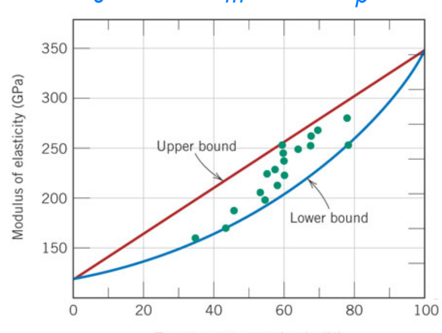


- Young's modulus
- Upper limit (Voigt)

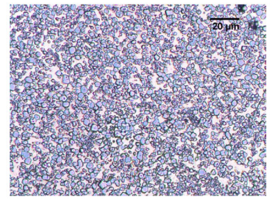
$$E_c = f_m E_m + f_p E_p$$

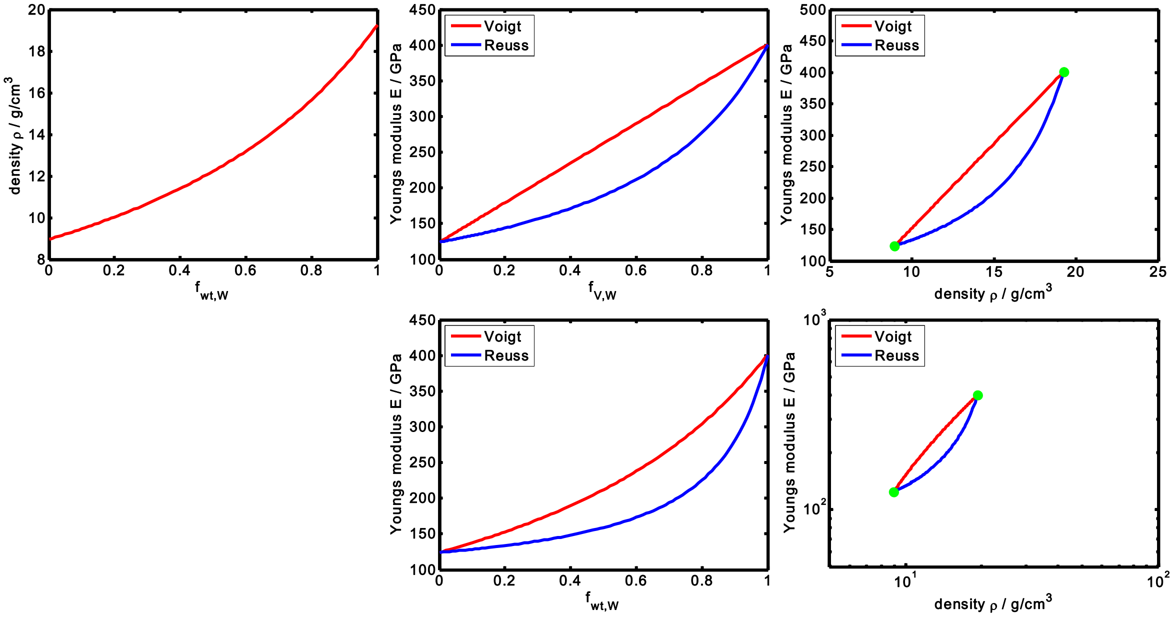
- Lower limit (Reuss)

$$\frac{1}{E_c} = f_m \frac{1}{E_m} + f_p \frac{1}{E_p}$$



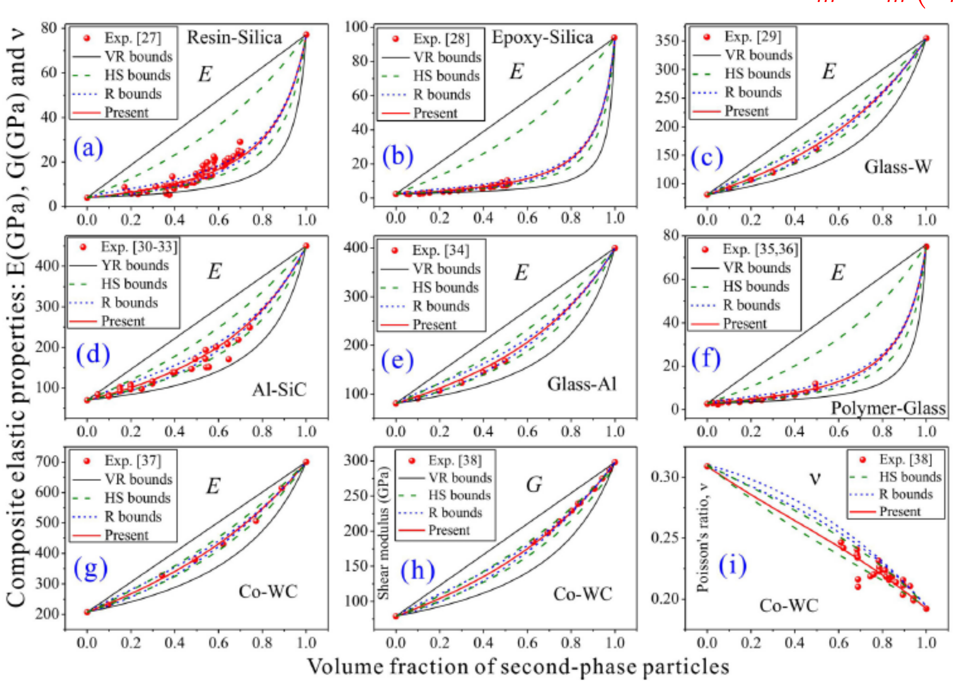
Tungsten copper composites





Elastic properties of particulate composites
Analytical approximation
(Zhang et al., Phys. Rev. B (2017))

$$E_{aa} = E_m \frac{3E_p + 2f_m\left(E_m - E_p\right)}{3E_m + f_m\left(E_m - E_p\right)}$$

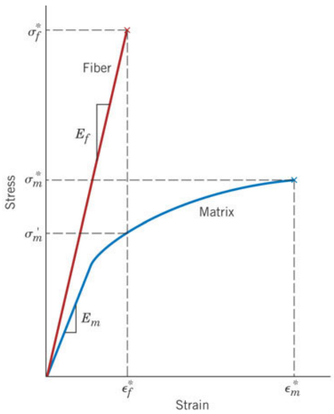


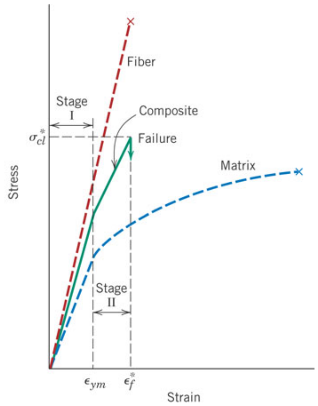
VR

Voigt
Reuss
boonds

Mechanical properties of fiber composites Continuous aligned fibers

- Stress strain curve





- Tensile stress (iso-strain)

$$\sigma(\varepsilon) = f_m \sigma_m(\varepsilon) + f_f \sigma_f(\varepsilon)$$
$$= f_m \sigma_m(\varepsilon) + f_f E_f \varepsilon$$

- Yield strength

$$\sigma_{y,c} = f_m \sigma_{y,m} + f_f E_f \varepsilon_{y,m}$$

- Tensile strength

$$\sigma^* = f_m \sigma_m\left(\varepsilon_f^*\right) + f_f \sigma_f^*$$
$$= f_m \sigma_m' + f_f \sigma_f^*$$

- Fracture strength of fibers

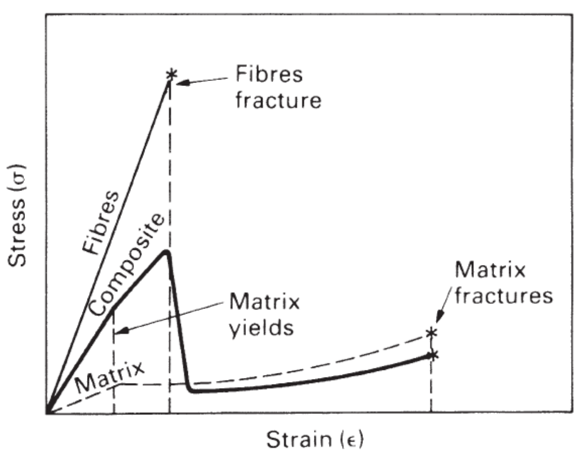
$$\sigma_f^*$$

- Strain to failure of fibers

$$\varepsilon_f^* < \varepsilon_m^*$$

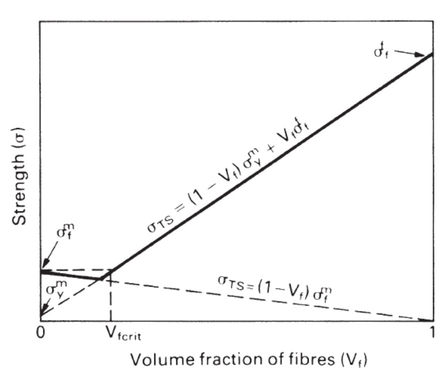
Mechanical properties of fiber composites Continuous aligned fibers

- What happens after the fracture of the fibers?



- Fibers are not under stress
- Deformation continues in matrix
- Strain at matrix fracture ε_m^*
- Reduced tensile strength

$$\sigma = f_m \sigma_m\left(\varepsilon_m^*\right) = f_m \sigma_m^*$$



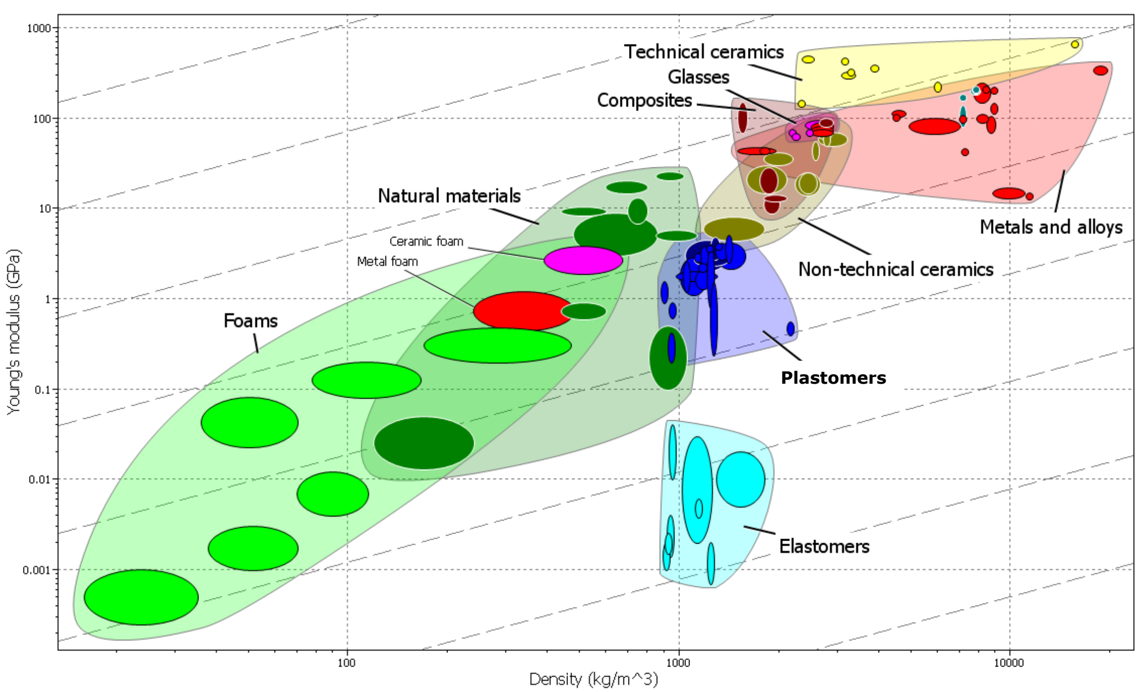
Applications of ANSYS Granta EduPack

- Selection for specific purposes
  - Materials indices

- Design of new materials
  - Composites (synthesizer tool)

Ansys
GRANTA EDUPACK

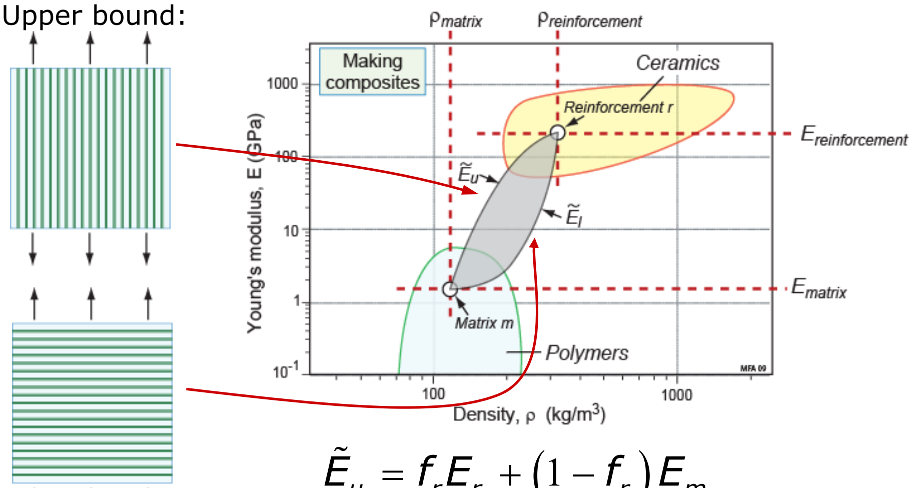
Material property chart
Young’s modulus vs. mass density



General composites: reinforcement in matrix

$$\rho_c = f_r \rho_r + \left(1 - f_r\right) \rho_m$$

Upper bound:



$$\tilde{E}_u = f_r E_r + \left(1 - f_r\right) E_m$$

Lower bound: E_l

$$\tilde{E}_l = \left[\frac{f_r}{E_r} + \frac{1 - f_r}{E_m} \right]^{-1}$$

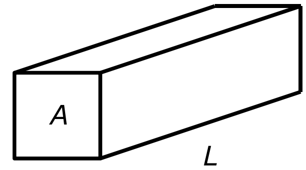
Materials indices

41680 Introduction to advanced materials

$(EIv'')'' = q - \rho A \ddot{v}$

DTU Construct
Department of Civil and Mechanical Engineering

Design: minimize weight of a **light, stiff** member



- Force

$$F = \sigma A$$

- Elongation

$$\Delta L = \varepsilon L$$

- Elastic behavior

$$\sigma = E\varepsilon$$

- Stiffness of member

$$S = \frac{F}{\Delta L} = \frac{\sigma A}{\varepsilon L} = \frac{EA}{L}$$

- Requirements:
  specified stiffness S
  fixed length L

- Mass

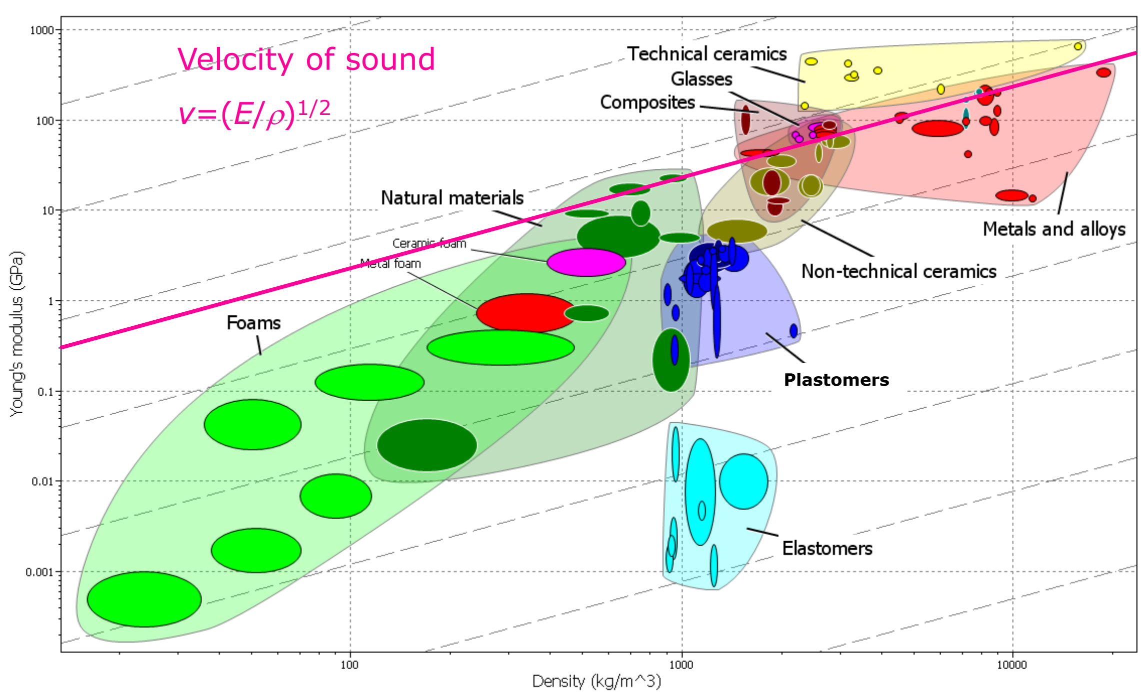
$$m = \rho A L = \frac{\rho}{E} S L^2$$

- Minimizing mass means maximizing ratio

$$E/\rho$$

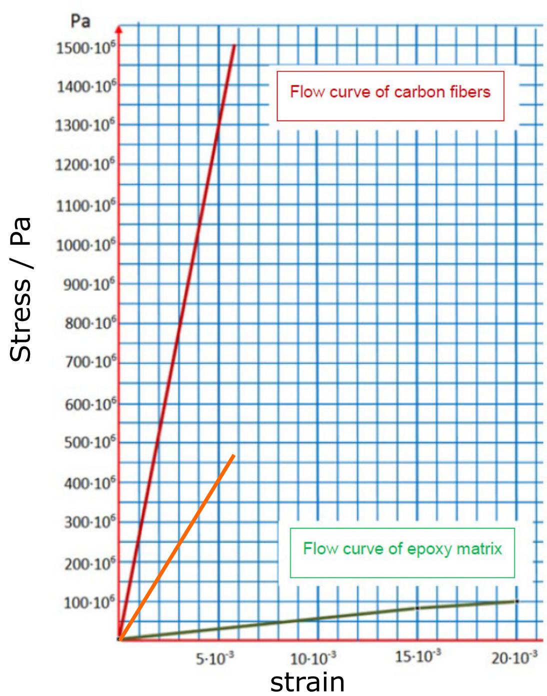
- *Selection line in ANSYS GRANTA EduPack*

Material property chart
Young's modulus vs. mass density



Group exercises

Carbon fiber-reinforced PMC



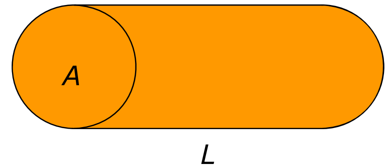
- Young's modulus of the composite (Voigt)

$$E_c = f_f E_f + f_m E_m = 80 \text{ GPa}$$

- Fracture strength of epoxy $\sigma_m{}^*$ = 100 MPa
- Stress in matrix, when carbon fibers crack σ'_m = 30 MPa
- fracture strength of composite

$$\sigma_c^* = f_f \sigma_f^* + f_m \sigma'_m = 471 \text{ MPa}$$

Design: minimize weight of a **light, strong** rod



- Requirements:
  fixed length L
  specified load F before yield

- Force at yield

$$F = \sigma_y A$$

- Mass

$$m = \rho A L = F L \frac{\rho}{\sigma_y}$$

- Minimizing mass means maximizing ratio

$$\sigma_y / \rho$$

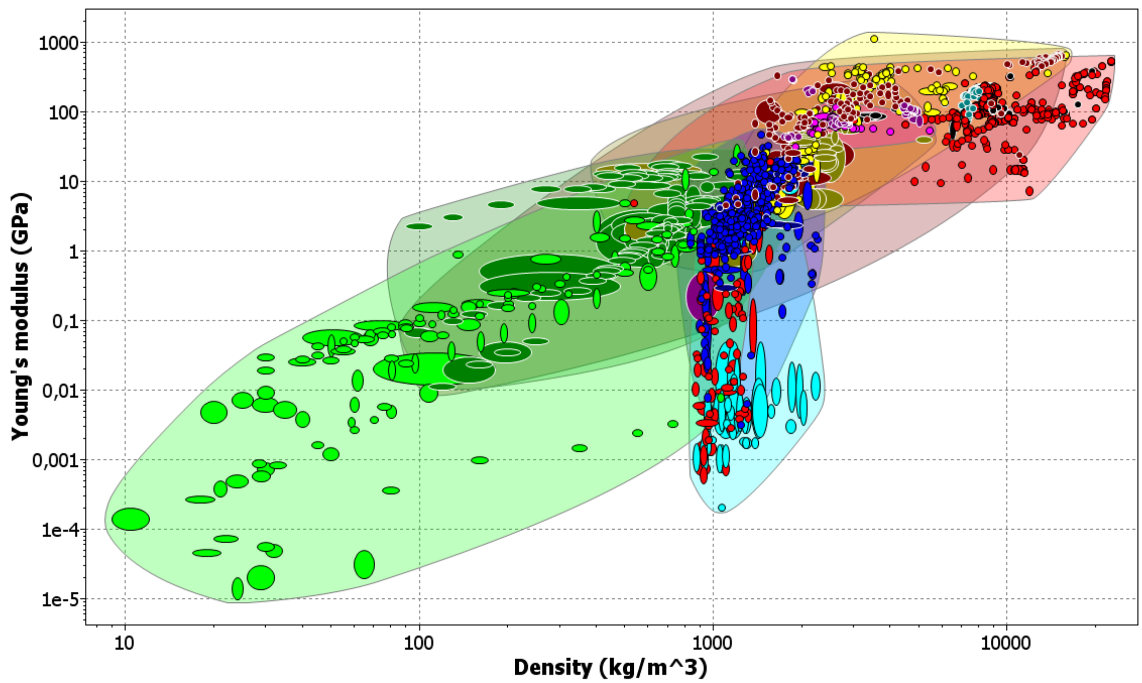
- *Selection line in ANSYS GRANTA EduPack*

Material indices (light weight structures)

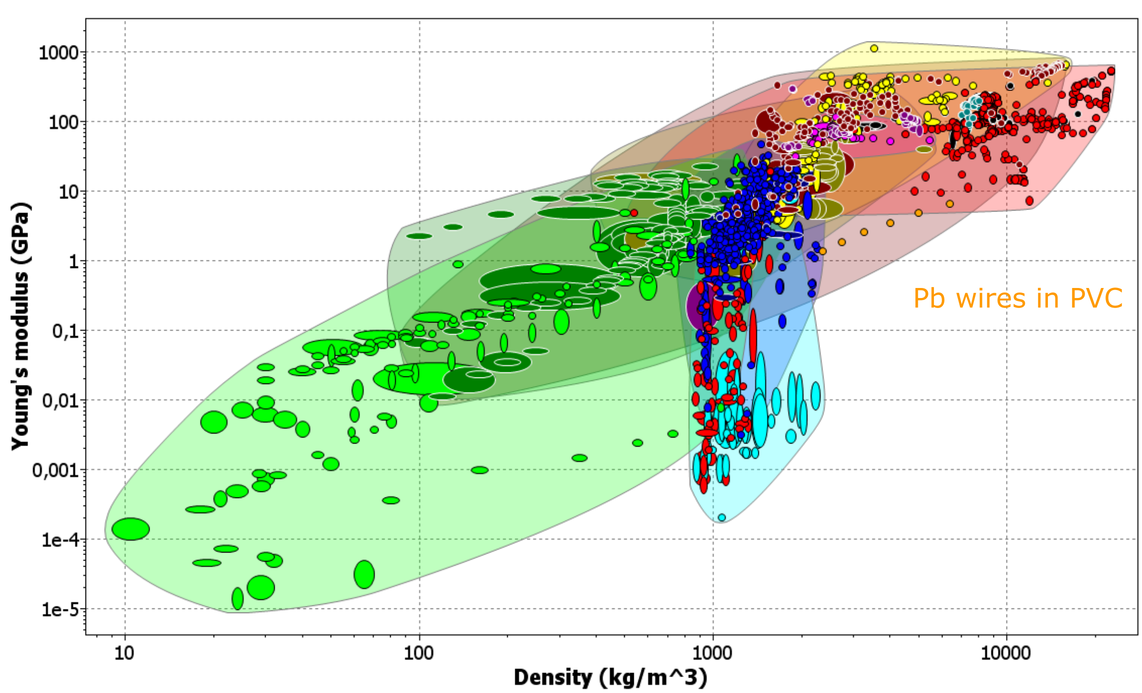
Member		Loading	Index
Beam	Stiffness	Tension / compression	E/ρ
		Torsion	G/ρ
		Bending	$E^{1/3}/\rho$ $E^{1/2}/\rho$
	Buckling	Compression	$E^{1/2}/\rho$
Panel	Stiffness	Bending	$E^{1/3}/\rho$
Beam	Strength*	Tension / compression	σ_y/ρ
	Strength*	Bending	$\sigma_y^{2/3}/\rho$
Panel	Strength*	Bending	$\sigma_y^{1/2}/\rho$
Spring	Resilience		$\sigma_y^2/E\rho$

* Either yield strength or failure strength

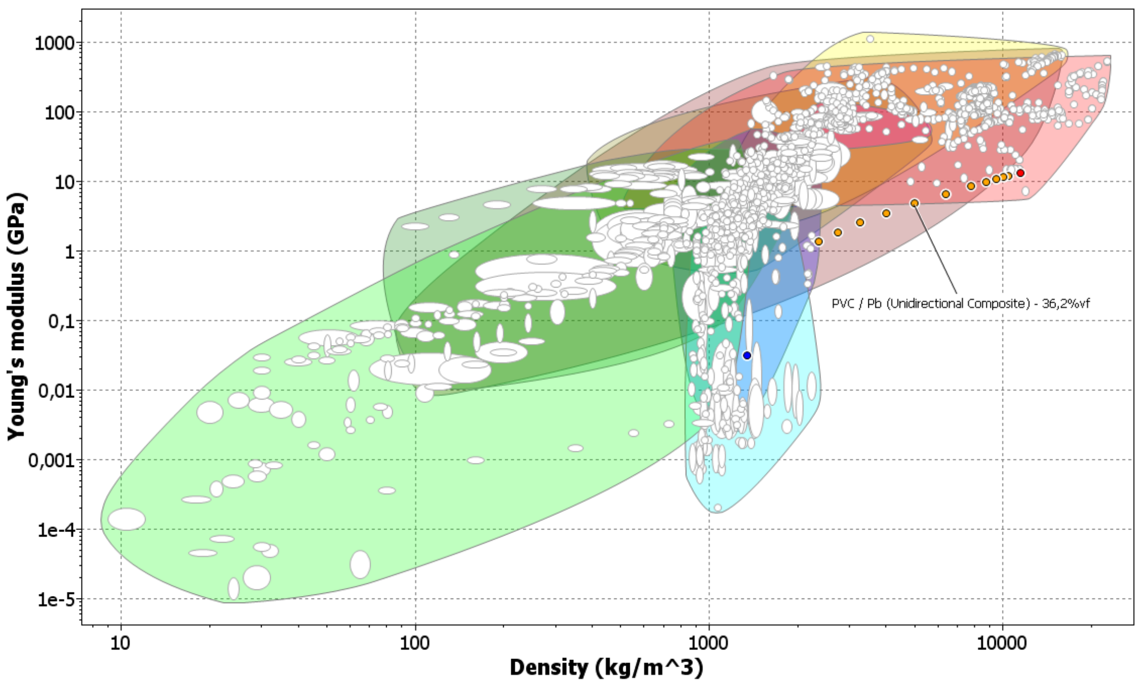
Material property chart
Young’s modulus vs. mass density



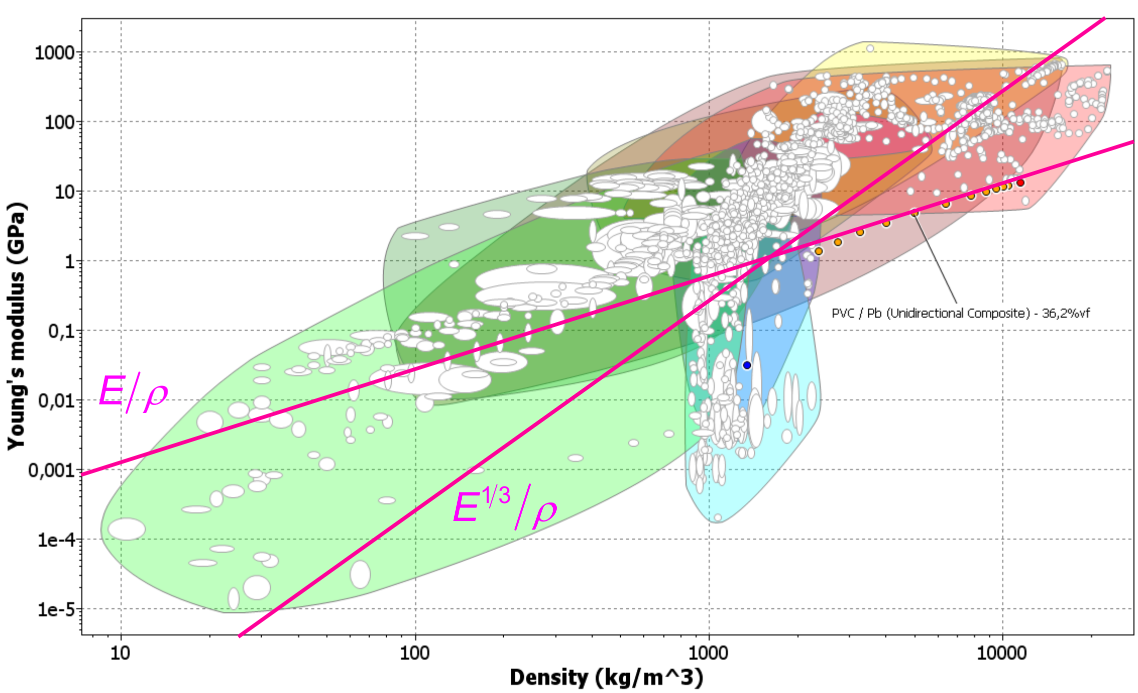
Material property chart
Young’s modulus vs. mass density



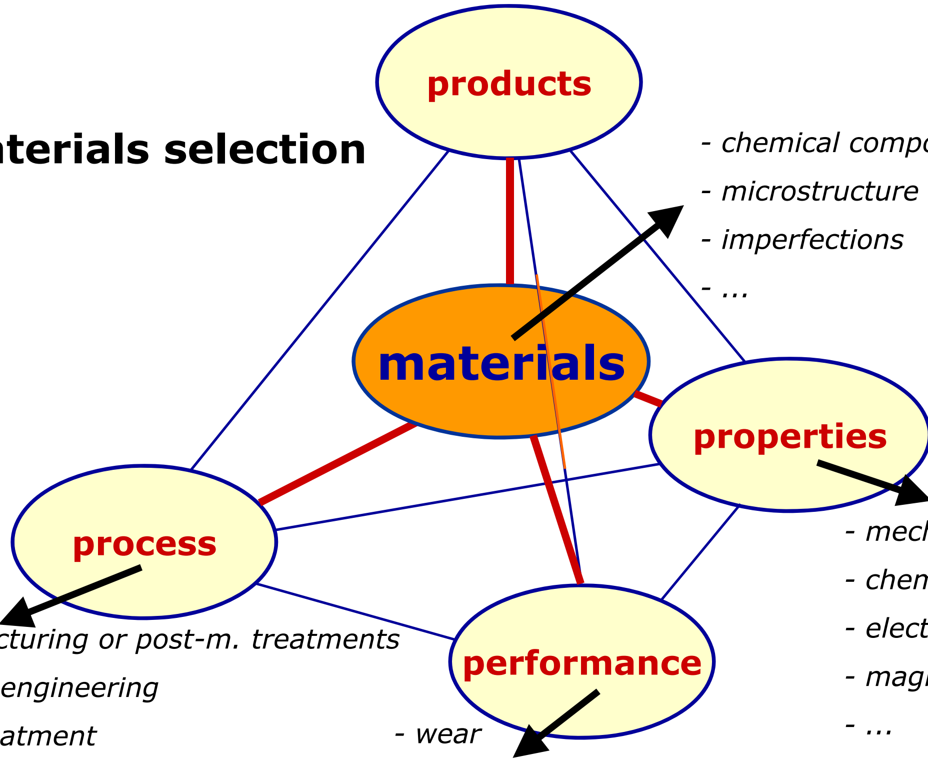
Material property chart
Young's modulus vs. mass density



Material property chart
Young's modulus vs. mass density



Materials selection



- chemical composition
- microstructure
- imperfections
- …

- mechanical
- chemical
- electrical
- magnetical
- …

- manufacturing or post-m. treatments
- surface engineering
- heat treatment
- plastic deformation
- …

- wear
- corrosion
- …

Materials and cyber materials
Atoms - Microstructure - Properties