TECHNICAL UNIVERSITY OF DENMARK

3-hour written exam, May 13, 2024
Course: Advanced Engineering Mathematics 2 01034 / 01035

Allowed aids: All aids allowed by DTU.
Weighting: Multiple-choice(administrated electronically): 55%, Problem 1: 15%, Problem 2: 10%, Problem 3: 20%

The weighting is only a guide. The exam is evaluated as a whole. In order to receive full points in Part B, all answers must be substantiated, if necessary with reference to the text book, and a reasonable number of intermediate steps must be included.

The exam consists of 2 parts: an electronic multiple choice section (**Part A**) and this (**Part B**).

- **Part A is administered and answered electronically.**.
- **Part B follows below**, and can be handed in either electronically or on paper.

Part B

Problem 1

1. Show that the series $\sum_{n=1}^{\infty}(-1)^n \frac{100+n}{2+n^3}$ is absolutely convergent.
2. For $k \in \mathbb{N}$, consider the series
$$S = \sum_{n=1}^{\infty}(-1)^n \frac{100+n}{2+n^k}$$
   (a) Find the smallest value for $k \in \mathbb{N}$, such that the series S is absolutely convergent.
   (b) Find the smallest value for $k \in \mathbb{N}$, such that the series S is convergent.

The problem set continues!

Problem 2

Consider the differential equation

$$y^{(4)}(t) + y^{(3)}(t) - 2y^{(2)}(t) = u(t), \tag{1}$$

where $u : \mathbb{R} \to \mathbb{R}$ is a continuous function.

1. Write down the characteristic polynomial of the equation (1) AND find the general real solution to the associated homogeneous system.
2. The function $y(t) = t^3$ is a solution of (1). Write down the general real solution to the inhomogeneous differential equation (1).
3. Use the information that $y(t) = t^3$ is a solution to the inhomogeneous differential equation to determine the function $u(t)$.

Problem 3

A function $f : \mathbb{R} \to \mathbb{R}$ has the following properties: f is an even function, f is 2π-periodic, and on the interval $[0, \pi]$ the function is given by

$$f(x) = \begin{cases} 1 & \text{for } x \in [0, \pi/2], \\ 2 & \text{for } x \in \,]\pi/2, \pi]. \end{cases}$$

1. Sketch the function f on the interval $[-\pi, \pi]$.
2. Determine all the real Fourier coefficients a_n and b_n of the Fourier series

$$f \sim \frac{1}{2}a_0 + \sum_{n=1}^{\infty}(a_n \cos(nx) + b_n \sin(nx)),$$

3. What is the sum of the Fourier series at the points $x = -\pi, -\frac{\pi}{2}, 0, \frac{\pi}{2}$ and π?
4. Does the Fourier series for f converge uniformly? Explain your answer.

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The problem set continues!