Solutions to exercises for lesson 4

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1 Problems from the textbook

Problem 4.61
The lattice spacing is $d = a/\sqrt{1^2 + 1^2 + 3^2}$ with $a = 2\sqrt{2}R$, where $R = 0.1387 nm$ is the atomic radius. Hence $d = 0.118$ nm.

Braggs law: $n\lambda = 2d\sin(\theta)$ with wavelength $\lambda = 0.154$ nm and $n = 1$ implies the diffraction angle is $2\theta = 2asin(\frac{2d}{\lambda}) = 1.42\, rad = 81.46\,\text{deg}$.

Problem 4.62
With an atomic radius $R = 0.1431$ nm, acc. to table, the lattice constant is $a = 2\sqrt{2}R = 0.404$ nm.

The d-spacing is $d = a/\sqrt{h^2 + k^2 + \ell^2}$. Hence for (hk$\ell$) = (110): $d = 2.856$ nm and for (hkℓ) = (221): $d = 1.347$ nm.

Problem 4.63
Braggs law: $n\lambda = 2d\sin(\theta)$ with wavelength $\lambda = 0.154$ nm, diffraction angle $2\theta = 29.2\,\text{deg}$ and $n = 1$ implies $d = 0.135$ nm.

The d-spacing is $d = a/\sqrt{h^2 + k^2 + \ell^2}$. Hence with (hk$\ell$) = (220) we have $a = 2.856$ nm. From this follows $R = a/(2\sqrt{2}) = 0.135 nm$.

Problem 4.64
Braggs law: $n\lambda = 2d\sin(\theta)$ with wavelength $\lambda = 0.071$ nm, diffraction angle $2\theta = 27.0\,\text{deg}$ and $n = 1$ implies $d = 0.152$ nm.

The d-spacing is $d = a/\sqrt{h^2 + k^2 + \ell^2}$. Hence with (hk$\ell$) = (321) we have $a = 0.569$ nm. From this follows $R = a * \sqrt{3}/4 = 0.247 nm$.

Problem 4.66
By inspection we find the peak positions at $2\theta = 44.8\,\text{deg}, 65.3\,\text{deg}$ and $82.7\,\text{deg}$, respectively.

From Braggs law: $n\lambda = 2d\sin(\theta)$ with wavelength $\lambda = 0.154$ nm and $n = 1$ we have $d = 2.019$ nm, 1.43 nm and 1.17 nm, respectively. The corresponding values for lattice constant $a = d\sqrt{h^2 + k^2 + \ell^2}$. Inserting we have $a = 0.286$ nm, 0.284 nm and 0.285 nm, respectively. As anticipated these values are identical within the error of reading out the peak positions.

Problem 4.68
This exercise illustrates the concept of indexing. We shall we don't know the properties of the crystalline lattice of Cu. By inspection we find the peak positions at $2\theta = 43\,\text{deg}, 50\,\text{deg}$ and $74\,\text{deg}$, respectively.

Using Braggs law: $n\lambda = 2d\sin(\theta)$ with wavelength $\lambda = 0.154$ nm and $n = 1$ we have $d = 0.210$ nm, 0.182 nm and 0.128 nm, respectively.

There is a limited number of crystalline symmetries and we can in principle test them one by one. Here we make the inspired guess that Cu has an cubic lattice. Then $d^2 = (h^2 + k^2 + \ell^2)a^2$ should be an integer. Let us make the ratios $d_1^2/d_2^2 = 1.33 \approx 4/3$. Likewise $d_1^2/d_3^2 = 2.69 \approx 8/3$. This is consistent with the three peaks being (111) $h^2 + k^2 + \ell^2 = 3$, (200) $h^2 + k^2 + \ell^2 = 4$ and (220) $h^2 + k^2 + \ell^2 = 8$. These are the three first indecies for an ffc lattice. Going through other crystal symmetries the consistency is worse. In conclusion the lattice is fcc and $a \approx 0.364$ nm. In comparison the tabulated value on the internet is $a = 0.361$ nm.

2 Microscopy

Problem 1
The focal length is $f = 10\mu$m and $d_0 = 100\mu$m. From the lensmakers equation:

$$1/d_i = 1/f - 1/d_0 \tag{1}$$

Inserting $1/d_i = 1/10 - 1/100 \implies d_i = 11.11\mu$m.

Moreover, the magnification is

$$M = d_i/d_0 \tag{2}$$

Inserting $M = 11.11/100 = 0.111$.

What does it mean that M is smaller than 1?

Problem 2
We have two equations with two unknowns, Eqs. 1,2. Solving these gives

$$d_0 = f(1/M + 1) \tag{3}$$
$$d_i = Md_0 = f(1 + M) \tag{4}$$

Inserting $f = 0.2$ m and $M = 100$ we have $d_0 = 0.202$ m and $d_i = 20.2$ m.

Problem 3
The Rayleigh criterion is

$$Res = 0.6\frac{\lambda}{n\sin(\theta)}, \tag{5}$$

while geometry implies (with D being the physical aperture of the lens)

$$\sin(\theta) = \frac{D/2}{f}. \tag{6}$$

By inserting $n = 1$ (vacuum) and $f = 12$ mm, $\lambda = 550$ nm and $Res = 900$ nm, we have $D = 8.8$ mm. If $f = 24$mm instead, then $D = 17.6$ mm.