

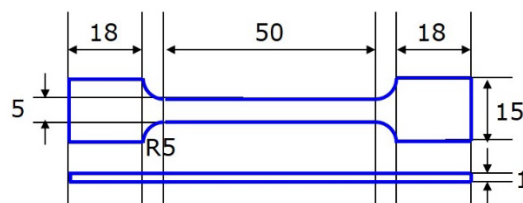
Lesson 05: Mechanical testing

Exercise 05.1: Tensile testing

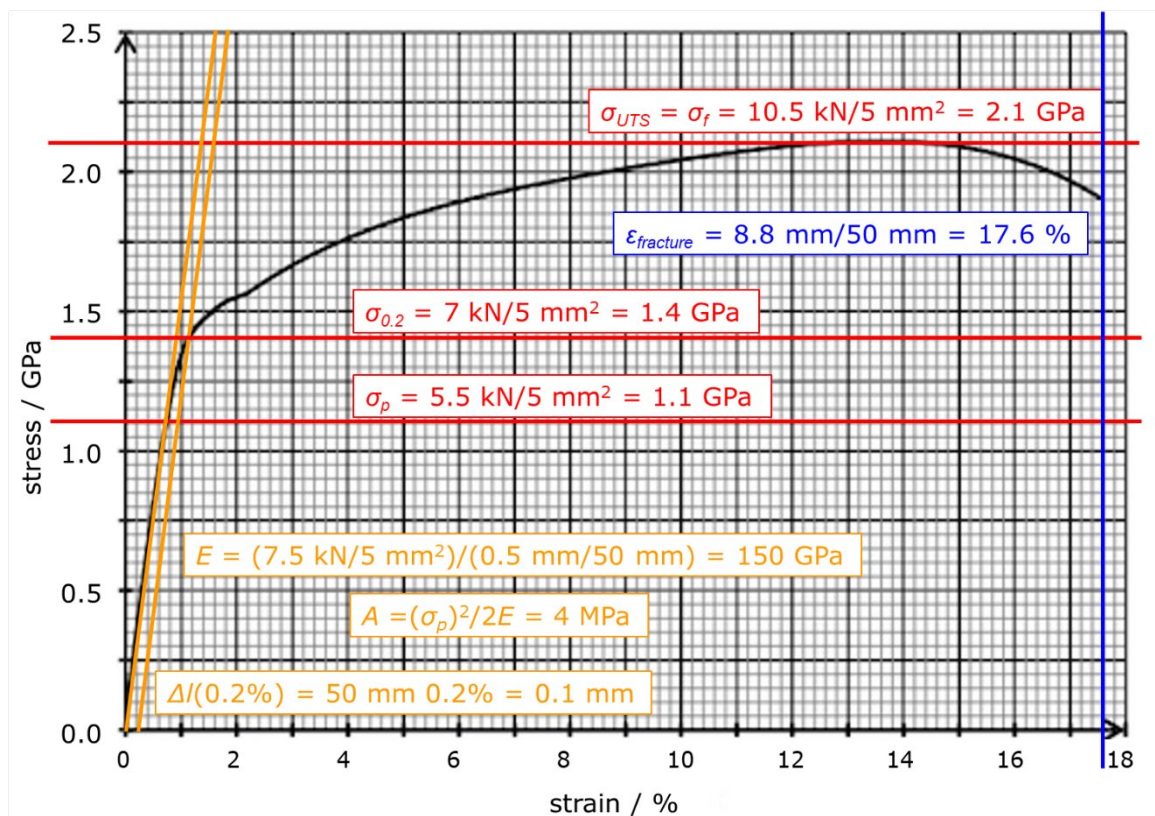
The relevant formulae for stress and strain are

$$\sigma = \frac{F}{A_0} \quad \varepsilon = \frac{\Delta l}{l_0}$$

The force elongation diagram can easily be transformed into a stress strain diagram by changing the axes: the force must be divided by the initial cross sectional area ($A_0 = 5 \text{ mm}^2$) and the elongation by the initial length of the gauge section ($l_0 = 50 \text{ mm}$) as deformation will occur only (mainly) there. The relevant sizes are obtained from the drawing.



The stress strain curve looks like the force elongation curve, just the axes are changed.



As seen from the graph:

1) Young's modulus

(Choose good points to read stress and strain for obtaining the slope)

$$E = \sigma / \epsilon = (7.5 \text{ kN/5 mm}^2) / (0.5 \text{ mm/50 mm}) = 150 \text{ GPa}$$

2) Proportional limit $\sigma_p = 5.5 \text{ kN/5 mm}^2 = 1.1 \text{ GPa}$

3) Stress at 0.2-limit $\sigma_{0.2} = 7 \text{ kN/5 mm}^2 = 1.4 \text{ GPa}$

4) Tensile strength $\sigma_{UTS} = 10.5 \text{ kN/5 mm}^2 = 2.1 \text{ GPa}$

5) Fracture strength (just another term for tensile strength) $\sigma_{UTS} = 10.5 \text{ kN/5 mm}^2 = 2.1 \text{ GPa}$

6) Largest elastic strain before plastic yielding $\epsilon_{el,p} = \sigma_p / E = 1.1 \text{ GPa} / 150 \text{ GPa} = 0.7\%$

7) Largest elastic strain during entire test $\epsilon_{el,UTS} = \sigma_{UTS} / E = 2.1 \text{ GPa} / 150 \text{ GPa} = 1.4\%$

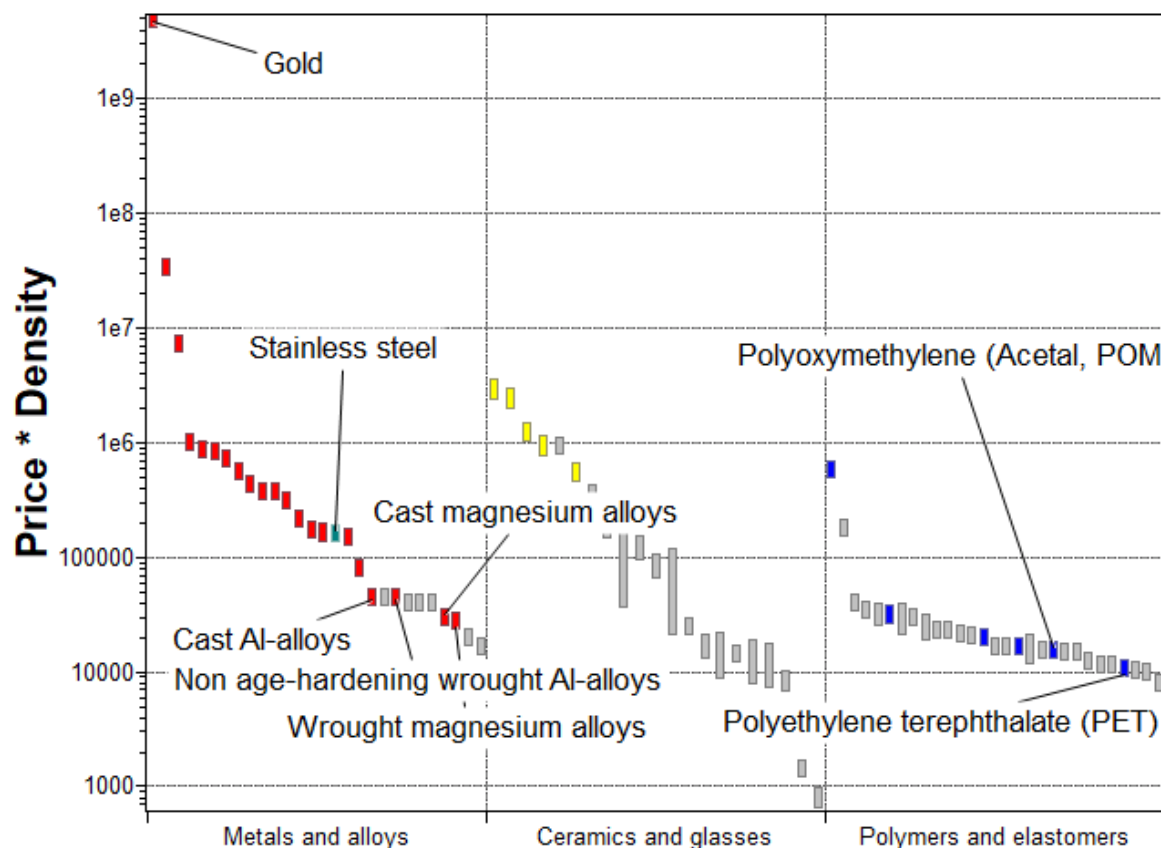
8) Maximal strain before fracture $\epsilon_{fracture} = 8.8 \text{ mm/50 mm} = 17.6\%$ (also called ductility)

9) Resilience, i.e. area under flow curve in elastic region (up to plastic yielding)

$$A = (\sigma_p \epsilon_{el,p}) / 2 = (\sigma_p)^2 / 2E = (1.1 \text{ GPa})^2 / 2 \cdot 150 \text{ GPa} = 4 \text{ MPa} = 4 \cdot 10^6 \text{ N/m}^2 = 4 \cdot 10^6 \text{ J/m}^3 = 4 \text{ mJ/mm}^3$$

Exercise 05.2: Beverage can

The solution of the exercise will be presented during lecture while using ANSYS GRANTA EduPack.



Exercise 05.3: Stiff and lightweight rod

A round rod with a length of 2 m should be manufactured from a metal (Fe, W, Ni, Al, Mg or Ti). It must not extend more than 2 mm when loaded with a mass of 2 t.

- (a) Which is the minimum diameter required for each metal?
- (b) Which metal should be selected for a rod as light as possible? Which is the relevant parameter?
- (c) Which metal should be selected for a rod as cheap as possible? Which is the relevant parameter?

The relations between the different quantities are the same as in exercise 06.2,

$$\sigma = \frac{F}{A_0} \quad \varepsilon = \frac{\Delta l}{l_0} \quad \sigma = E\varepsilon$$

but this time the area has to be found

$$A_0 = \frac{Fl_0}{E\Delta l}$$

- (a) The force $F = m_L g$ is determined from the loading mass and m_L the gravitational acceleration $g = 9.82 \text{ m/s}^2$; for the diameter follows

$$d_0 = \sqrt{\frac{4A_0}{\pi}} = \sqrt{\frac{4}{\pi} \frac{Fl_0}{E\Delta l}} = \sqrt{\frac{4}{\pi} \frac{mgl_0}{E\Delta l}}$$

	Fe	W	Ni	Al	Mg	Ti
d_0/mm	10.9	7.8	11.0	18.9	23.5	15.3
$\rho/\text{g cm}^{-3}$	7.9	19.3	8.9	2.7	1.73	4.5
m_0/kg	1.48	1.86	1.68	1.51	1.51	1.65

- (b) When taking into account the different mass density ρ of the metals follows a total mass of the rod

$$m_0 = \rho V_0 = \rho A_0 l_0 = \rho \frac{Fl_0}{E\Delta l} l_0 = \frac{\rho}{E} \frac{F}{\Delta l} l_0^2$$

The initial length l_0 is demanded by the design, the load (force F) and maximum elongation required by the construction, hence, the relevant materials parameter which determines the mass of the rod is ρ/E . For choosing a material for a light and stiff member it is not sufficient to check mass density or Young's modulus, it is the ratio ρ/E which should be as small as possible.

- (c) When the price

$$P_0 = m_0 p$$

is relevant instead of the rod, the price per mass p has to be considered and the relevant parameter becomes $p\rho/E$.