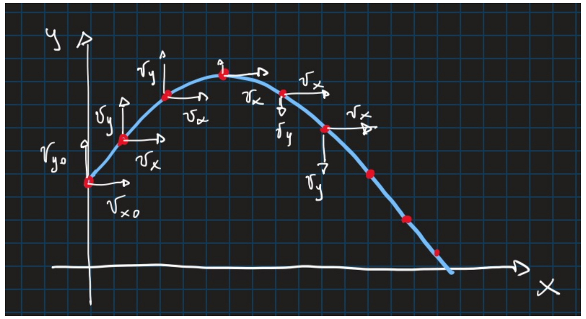
10063 Physics (Polyteknisk grundlag)

Recap of Kinematics in more than1D

The two types of motion we explored in 2D

- Projectile Motion

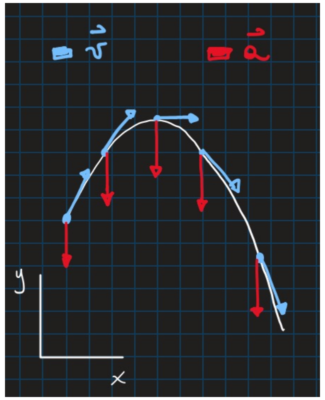


x-direction

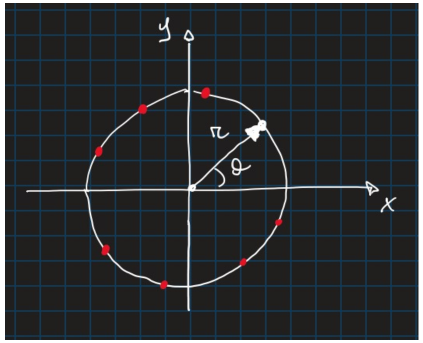
$$x(t) = x_0 + v_{0,x}t$$

y-direction

$$y(t) = y_0 + v_{0,y}t - \frac{1}{2}gt^2$$



- Uniform Circular Motion



Polar representation

$$\vec{r}(t) = r\cos(\omega t)\hat{i} + r\sin(\omega t)\hat{j}$$

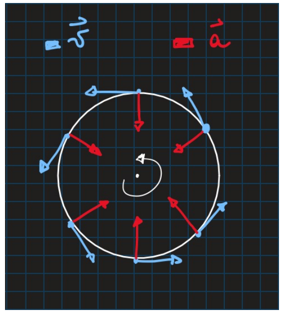
Velocity and centripetal acceleration

$$\vec{v}(t) = \frac{d\vec{r}(t)}{dt} = -r\omega\sin(\omega t)\hat{i} + r\omega\cos(\omega t)\hat{j}$$

$$\vec{a}(t) = \frac{d\vec{v}(t)}{dt} = r\omega^2\sin(\omega t)\hat{i} + r\omega^2\cos(\omega t)\hat{j}$$

Their magnitude

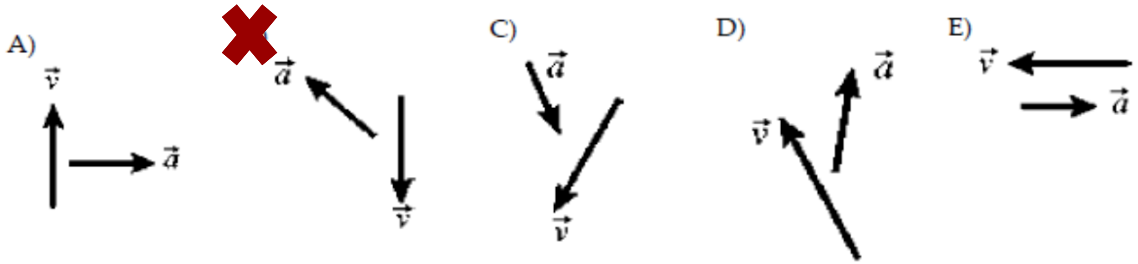
$$|\vec{v}(t)| = r\omega \qquad |\vec{a}(t)| = r^2\omega^2 = \frac{v^2}{r}$$



Quizzes

Question 1

What situation corresponds to an object that reduces its speed and at the same time turns towards right?



Question	Average Grade	Standard Deviation
Question 1	47.13 %	49.92 %
Question 2	69.89 %	29.38 %
Question 3	20.69 %	40.51 %
Question 4	14.94 %	35.65 %

Question 3

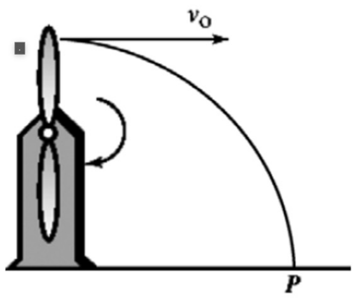
In class we have launched a projectile with a projectile launcher like in the picture:

$$v_0 = R\sqrt{\frac{g}{h+R}}$$

⇨ **3.3 m/s**

Question 4

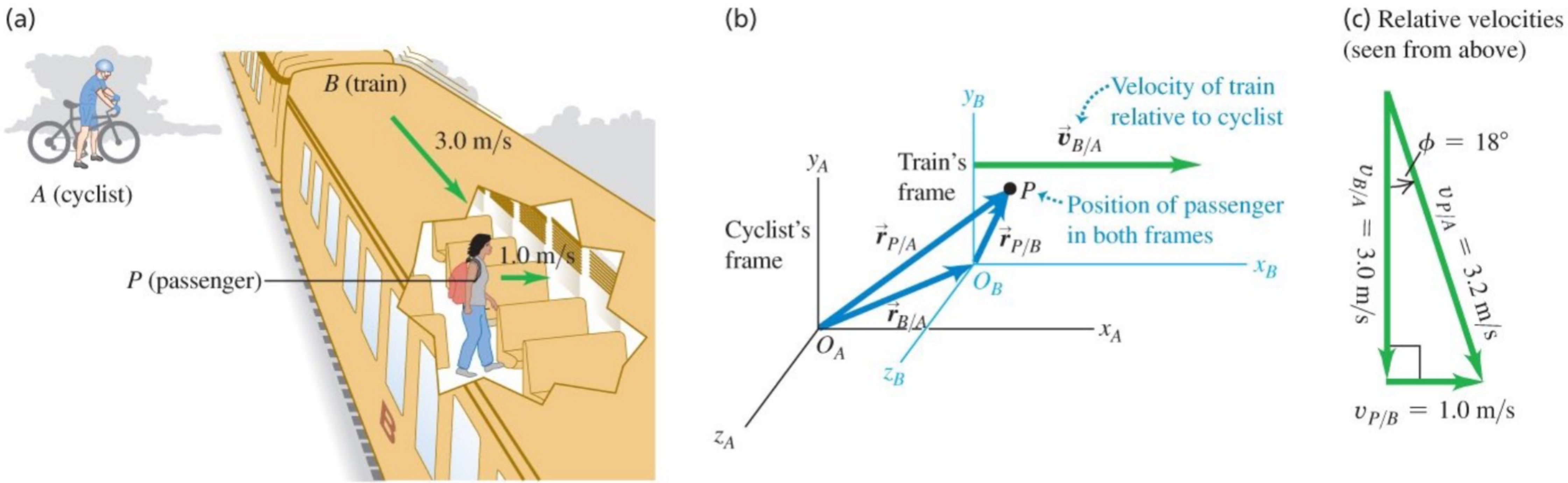
The wings of a wind turbine has radius 14 m. The wings are mounted at a height of 27 m. The period of revolution for the wings is 1.5 s. A fragment of the wing in the top position leaves the wing with a horizontal velocity. The acceleration of gravity is g=9.81 m/s^2.



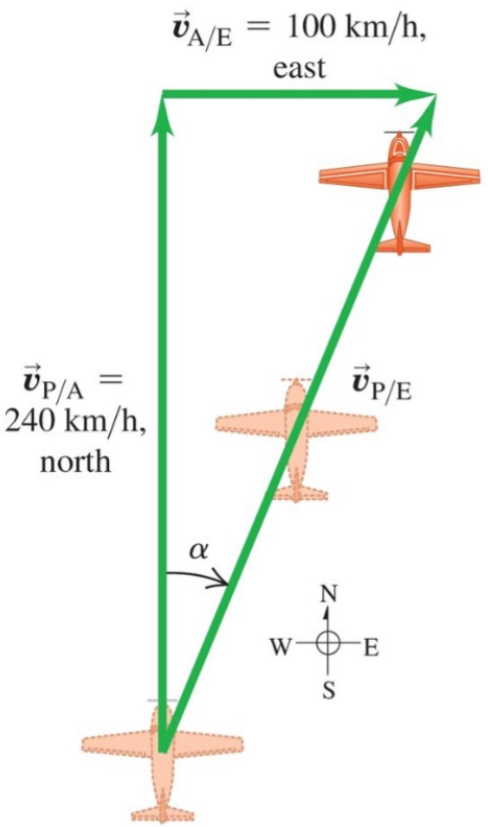
$$P = \frac{2\pi r}{t}\sqrt{\frac{2h}{g}}$$

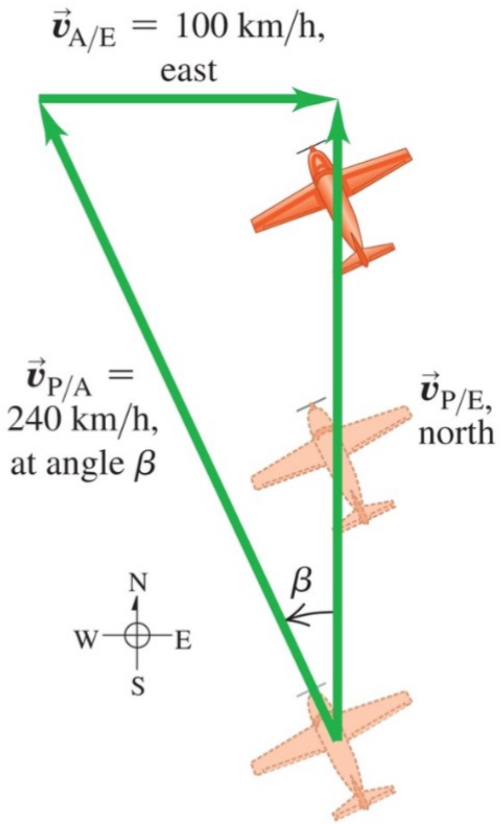
⇨ **138 m**

Relative Motion



Example: Flying in a crosswind





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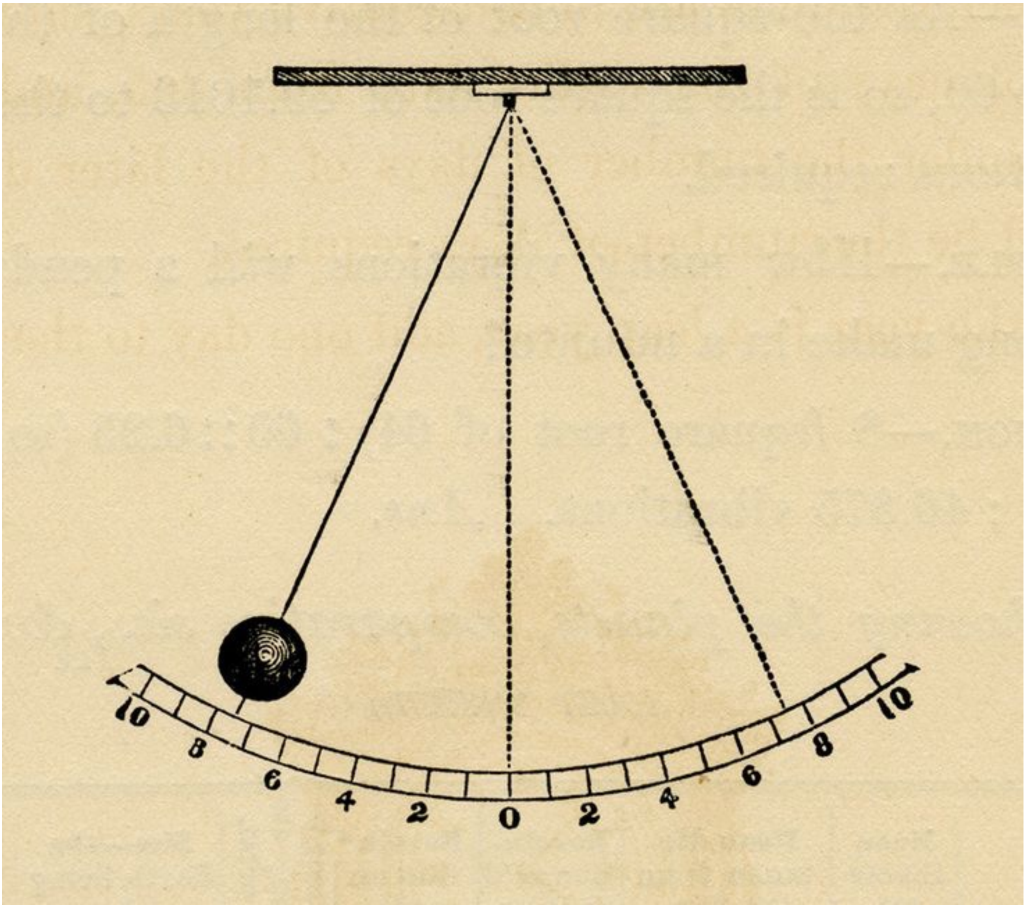
Uncertanties

Let’s measure something…



Source: Pinterest

Why is this relevant to you?

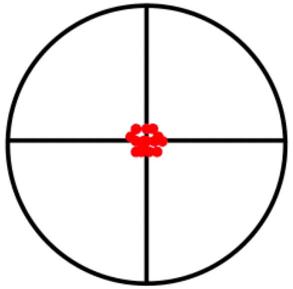
Errors are a fact of life!

- Quality control of data; Is data trustworthy and useful?
- Comparison of measured sizes with each other and against standards.
- Testing predictions.
- Help improve measurements and calculations.

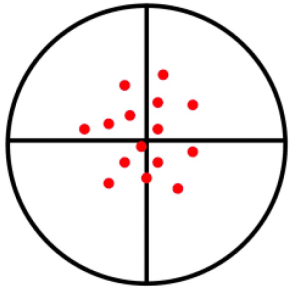
Random vs Systematic Errors

Precision: Is it repeatable?

Very precise:



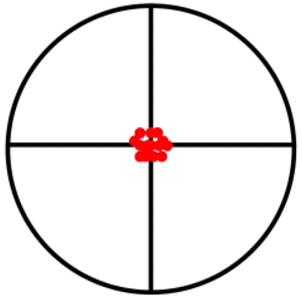
Not very precise:



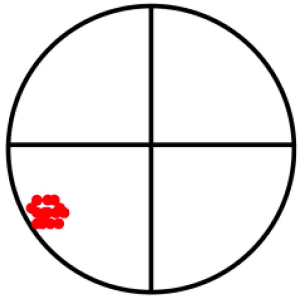
1. The deviations from the bullseye are random (i.e. **random error**)
2. The *average* of those deviations is zero

Accuracy: Is it correct?

Very accurate:



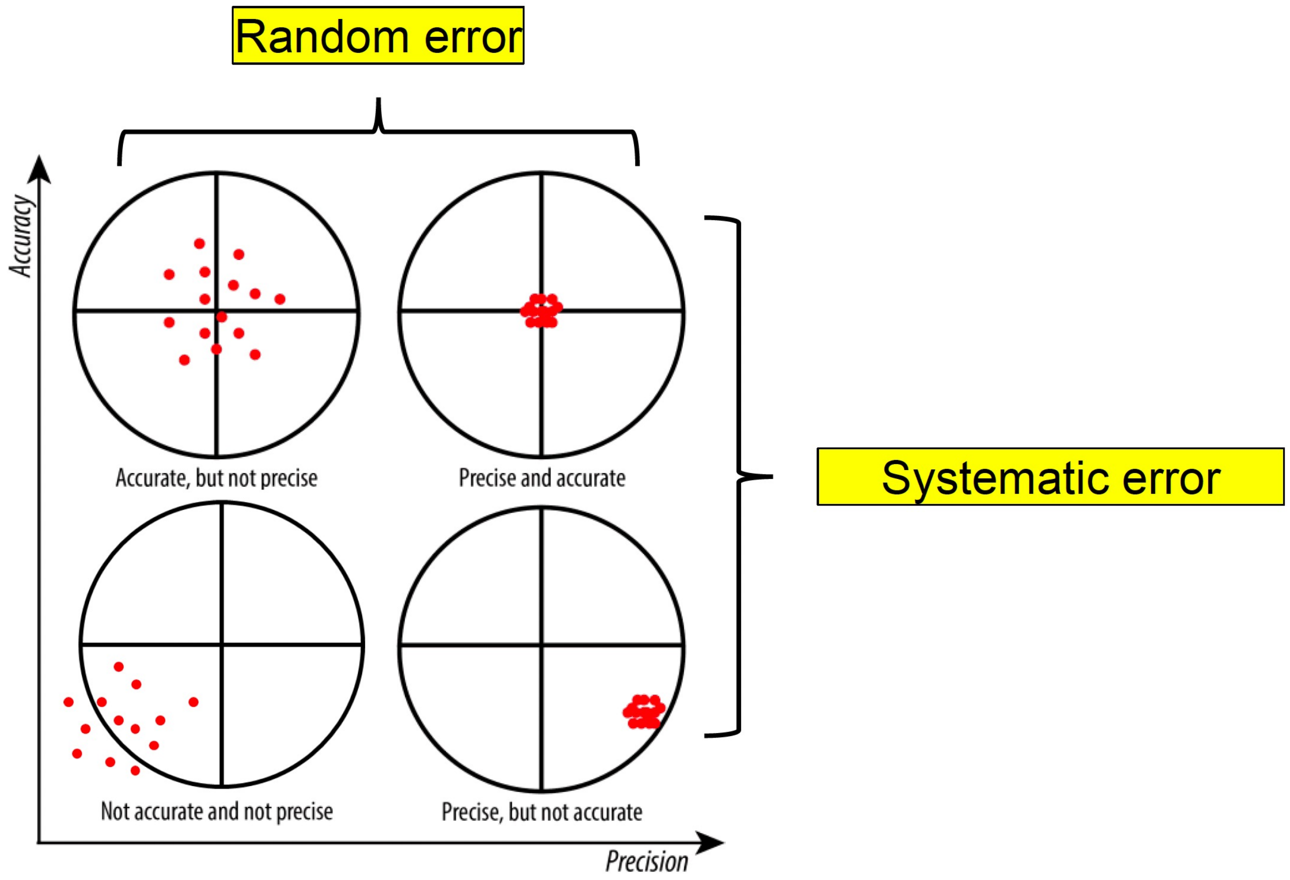
Not very accurate:



1. The deviations from the bullseye are similar (i.e. **systematic error**)
2. The *average* of those deviations is <u>not</u> zero

Precision and Accuracy

Summary:

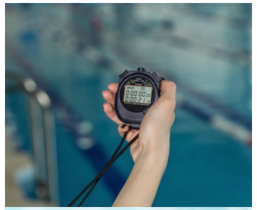


Dependent vs Independent errors

Dependent errors arise from the **same source**

E.g. measuring in paces by the same person

E.g. using the same (inaccurate) instrument



Independent errors arises from **different sources**

E.g. measuring in paces by different people

E.g. measuring different quantities like time and distance

Error/Uncertainty (used interchangeably)

Definitions:

x	**Best estimate,** i.e. what you think the value is
δx	**Uncertainty/error,** i.e. how much it could vary in each +/- direction
$x \pm \delta x$	**Absolute error,** i.e. how we write uncertainty in practise
$\frac{\delta x}{\lvert x \rvert}$	**Relative error**, i.e. how important is the error

Rounding errors to significant figures

Normally, the uncertainty is rounded to one significant digit. If this digit is 1, 2, or 3, an additional digit is included to avoid excessive rounding errors.

$$\delta x = 0.44566 \rightarrow \delta x = 0.4$$
$$\delta x = 0.14327 \rightarrow \delta x = 0.14$$

```python
from uncertainties import *
x = ufloat(3.0,0.15)
print(x)
x = ufloat(3.0,0.25)
print(x)
x = ufloat(3.0,0.35)
print(x)
x = ufloat(3.0,0.42)
print(x)
x = ufloat(3.0,0.45)
print(x)
x = ufloat(3.0,0.55)
print(x)
```

```
3.00+/-0.15
3.00+/-0.25
3.00+/-0.35
3.0+/-0.4
3.0+/-0.5
3.0+/-0.6
```

Rounding errors to significant figures

Rounding errors to significant figures:

Rule #1: Know and use significant figures (sig figs) properly

- E.g. 4732.188 to: 2 sig figs → 4700, 4 sig figs → 4732, 6 sig figs → 4732.19
- E.g. 0.00752 to 1 sig fig → 0.008 = 8E-3 (scientific notation)

Rule #2: Experimental uncertainties should be stated to one sig fig.

- E.g. 4732.188 ± 0.0392 → 4732.188 ± 0.04

Rule #3: The last sig fig should be in the same place as the uncertainty.

- E.g. 1261.29 ± 200 → 1300 ± 200

Rule #4: Uncertainties cannot be more precise than the measured value.

- (self explanatory from above examples)

Rule #5: When using sig figs in calculations, the number of sig figs in the answer should be the same as the smallest number of sig figs in any of any terms.

- E.g. 2.21 X 1.2 → 2.652 = 2.7

Pro tip: Using scientific notation makes life easier!

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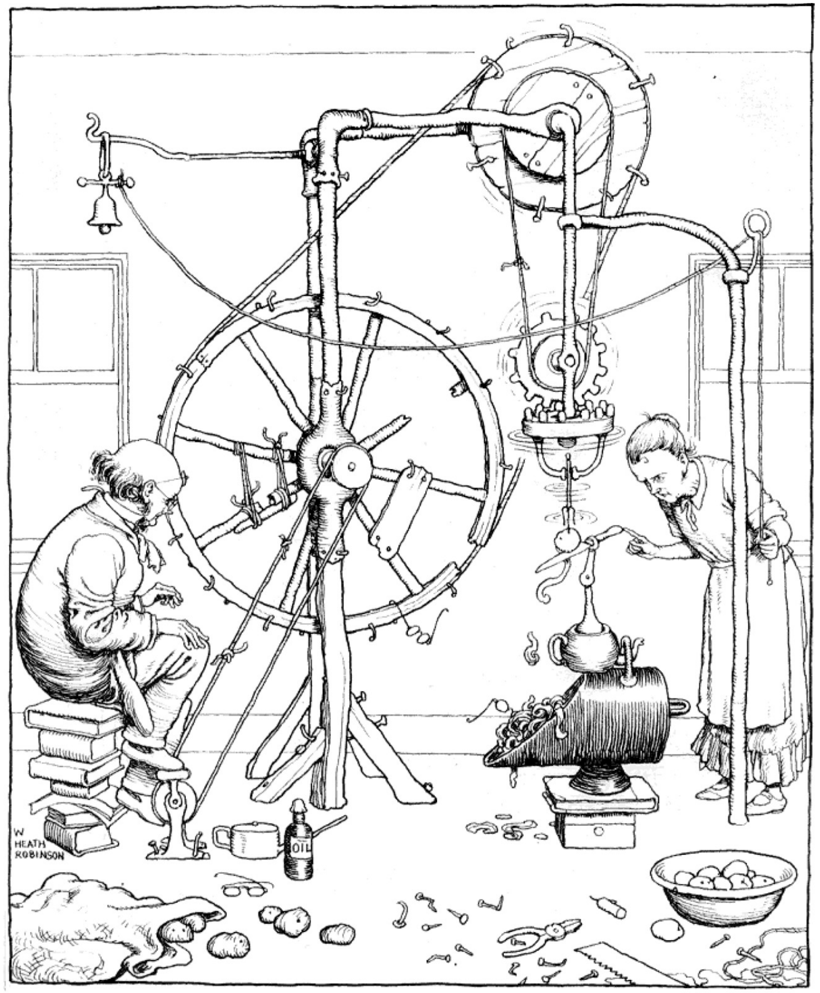
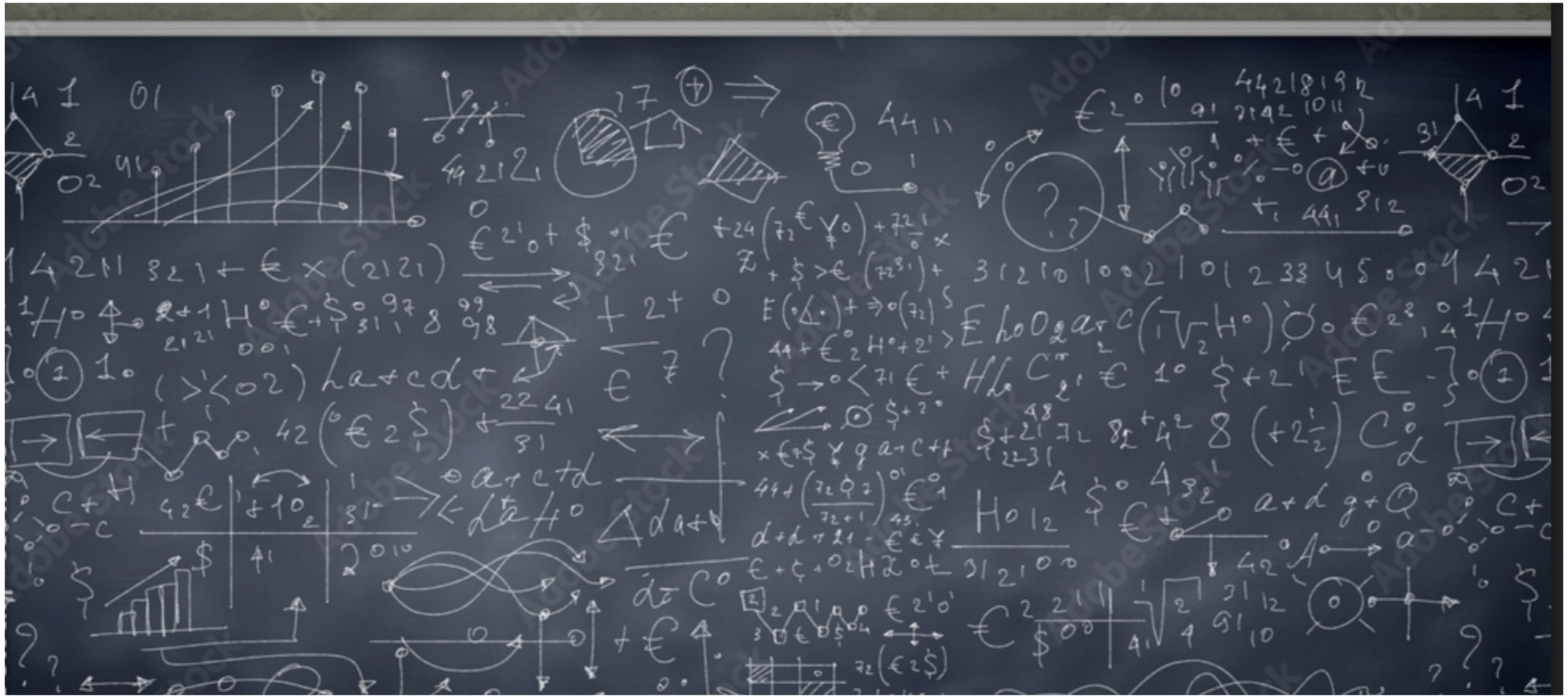
Error Propagation

The error propagates



The Professor's invention for peeling potatoes.

Error propagation for a single variable function



Black board time…

Adding and substracting errors

Let's say A is a combination of two measurements, x and y:

$$A = x + y$$

If x and y are dependent: the error in A is the sum of the errors in x and y:

$$\delta A = \delta x + \delta y$$

If x and y are independent: the square of the error in A is the sum of the square of the errors in x and y:

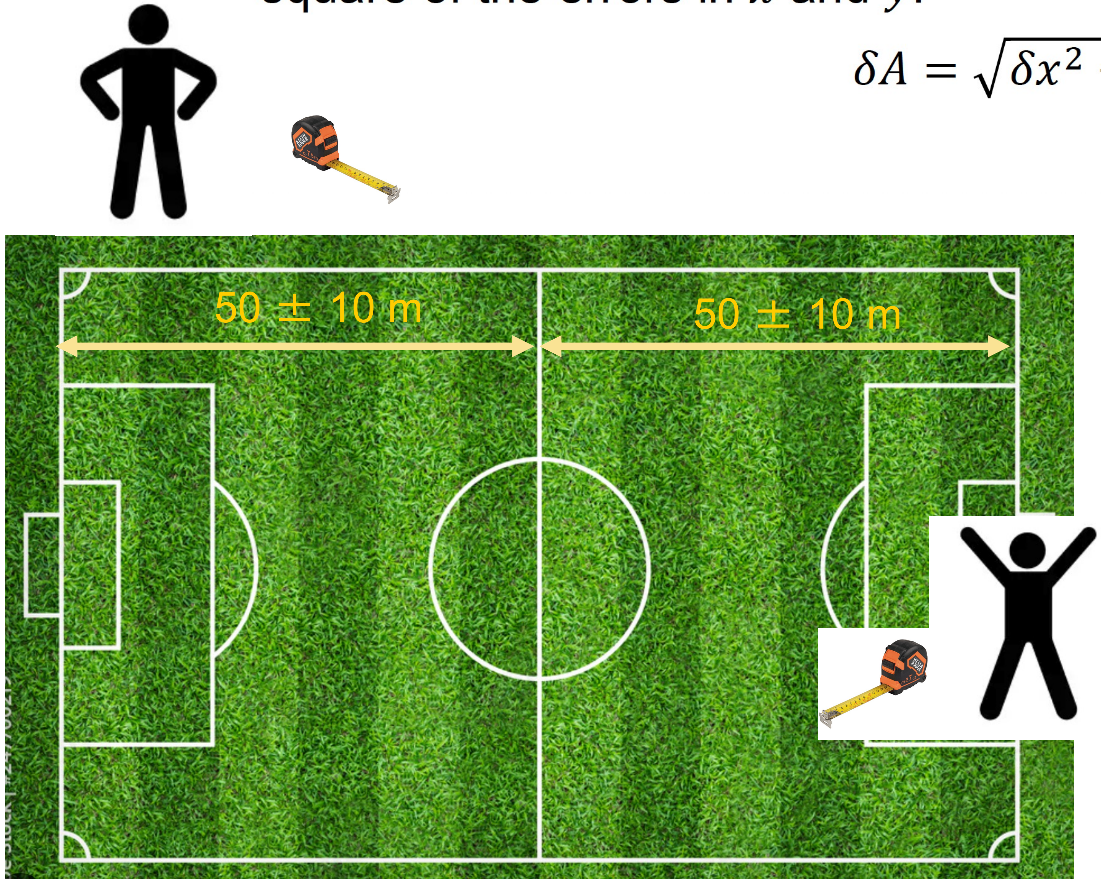
$$\delta A = \sqrt{\delta x^2 + \delta y^2}$$

$$\delta x = \sqrt{\left(\delta x_{\text{random}}\right)^2 + \left(\delta x_{\text{systematic}}\right)^2}$$

Adding and substracting errors

If x and y are independent: the square of the error in A is the sum of the square of the errors in x and y:

$$\delta A = \sqrt{\delta x^2 + \delta y^2}$$



Your Discrepancy	Friend's Discrepancy	Total Discrepancy
+10	+10	+20
+10	0	+10
+10	-10	0
0	+10	+10
0	0	0
0	-10	-10
-10	+10	0
-10	0	-10
-10	-10	-20

Multiplying and dividing errors

Multiplying/dividing uncertainties/errors:

Let's say A is a product of two measurements, x and y:

$$A = xy$$

If x and y are dependent: the error in A is the sum of the *relative* errors in x and y:

$$\frac{\delta A}{|A|} = \frac{\delta x}{|x|} + \frac{\delta y}{|y|}$$

If x and y are independent: the square of the error in A is the sum of the square of the *relative* errors in x and y:

$$\frac{\delta A}{|A|} = \sqrt{\left(\frac{\delta x}{|x|}\right)^2 + \left(\frac{\delta y}{|y|}\right)^2}$$

Error propagation in a known function

Uncertainty for known functions

Let's say A is a function of just x :

$$A = f(x)$$

Use calculus to approximate the error in A from x (first order):

$$\delta A = \left|\frac{\partial f}{\partial x}\right| \delta x$$

Now let's extent it such that A is a function of x and y :

$$A = f(x, y)$$

If x and y are dependent: we again just add the errors to find the error in A:

$$\delta A = \left|\frac{\partial f}{\partial x}\right| \delta x + \left|\frac{\partial f}{\partial y}\right| \delta y$$

If x and y are independent: we again add their squares to find the error in A:

$$\delta A = \sqrt{\left(\frac{\partial f}{\partial x}\delta x\right)^2 + \left(\frac{\partial f}{\partial y}\delta y\right)^2}$$

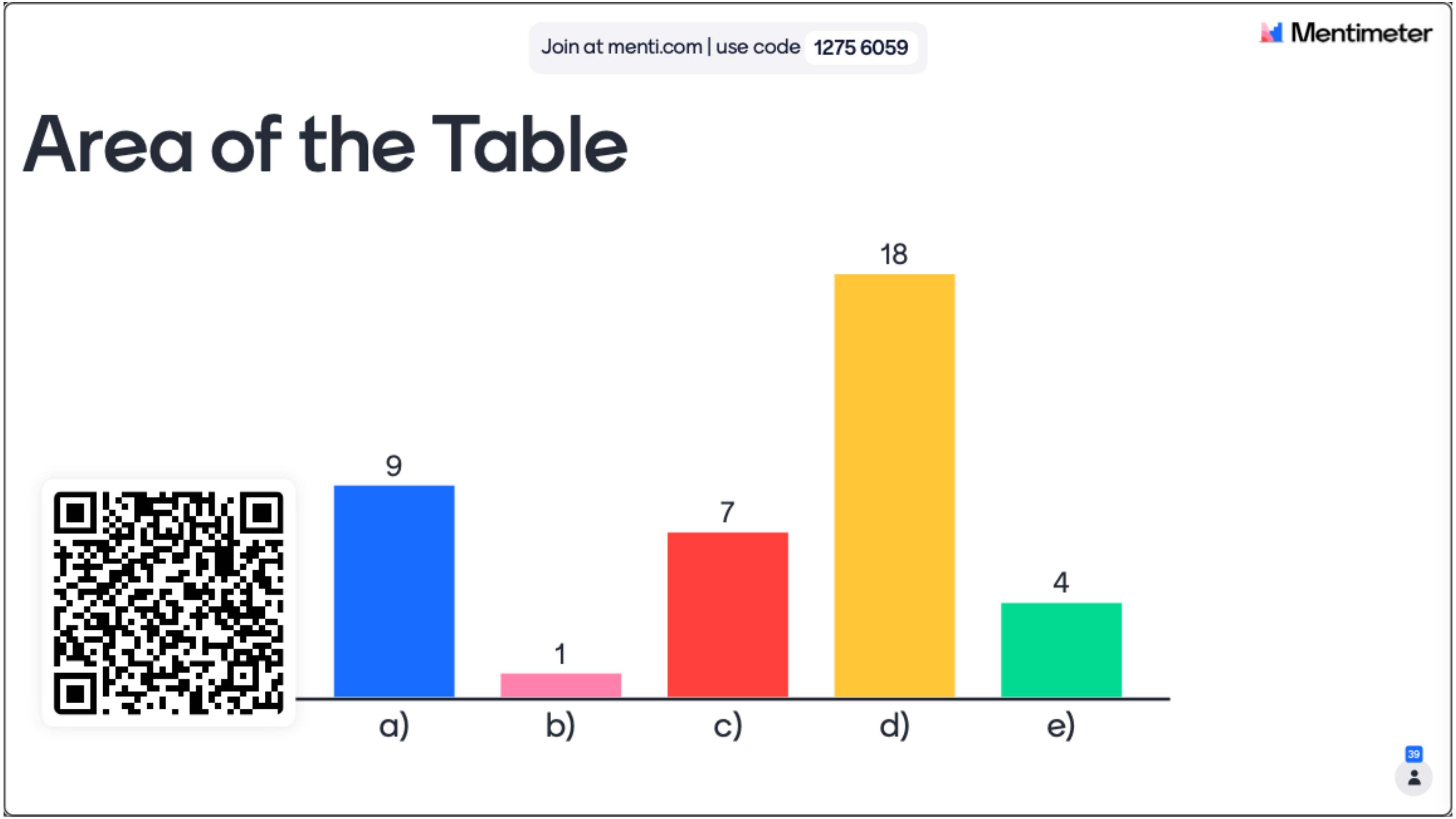
Summary

Situation:	Dependent:	Independent:
Adding/subtracting: $A = x + y$	Add the absolute errors: $\delta A = \delta x + \delta y$	Sum-square the absolute errors: $\delta A = \sqrt{\delta x^2 + \delta y^2}$
Multiplying/dividing: $A = xy$	Add the relative errors: $\frac{\delta A}{\lvert A\rvert} = \frac{\delta x}{\lvert x\rvert} + \frac{\delta y}{\lvert y\rvert}$	Sum-square the relative errors: $\frac{\delta A}{\lvert A\rvert} = \sqrt{\left(\frac{\delta x}{\lvert x\rvert}\right)^2 + \left(\frac{\delta y}{\lvert y\rvert}\right)^2}$
Functions: $A = f(x,y)$	Add the gradient-error products $\delta A = \left\lvert\frac{\partial f}{\partial x}\right\rvert \delta x + \left\lvert\frac{\partial f}{\partial y}\right\rvert \delta y$	Sum-square the gradient-error products: $\delta A = \sqrt{\left(\frac{\partial f}{\partial x}\delta x\right)^2 + \left(\frac{\partial f}{\partial y}\delta y\right)^2}$

Area of table

What is the area?

a) $A = 0.27 \pm 0.03\ \mathrm{m}^2$
b) $A = 0.265 \pm 0.03\ \mathrm{m}^2$
c) $A = 0.265 \pm 0.06\ \mathrm{m}^2$
d) $A = 0.265 \pm 0.026\ \mathrm{m}^2$
e) $A = 0.2652 \pm 0.0260\ \mathrm{m}^2$


Join at menti.com | use code 1275 6059
Mentimeter
Area of the Table
9
1
7
18
4
a)
b)
c)
d)
e)
39

Area of table

Best bet:

$$A = b \cdot d = 0.51 \cdot 0.52 = 0.2652 \text{ m}^2$$

Relative uncertainty due to product form:

$$\frac{\delta A}{A} = \sqrt{\left(\frac{\delta b}{b}\right)^2 + \left(\frac{\delta d}{d}\right)^2} = \sqrt{\left(\frac{1}{51}\right)^2 + \left(\frac{5}{52}\right)^2} = 0.0981$$

$\delta A = 0.0981 \cdot A = 0.26 \text{ m}^2$

$A = 0.265 \pm 0.026 \text{ m}^2$

Let's try it out with python

Area of a table

We want to calculate the area of a table for which we know that $l_1 = 52 \pm 5$ cm and $l_2 = 51 \pm 1$ cm

```python
from uncertainties import ufloat

l1 = ufloat(52/100,5/100,"l1")
l2 = ufloat(51/100, 1/100, "l2")

display(l1)
display(l2)

# let's calculate the area
A = l1 * l2

display(A)
```

[7] ✓ 0.0s

```
< l1 = 0.52+/-0.05 >

< l2 = 0.51+/-0.01 >

0.2652+/-0.026024795868555582
```

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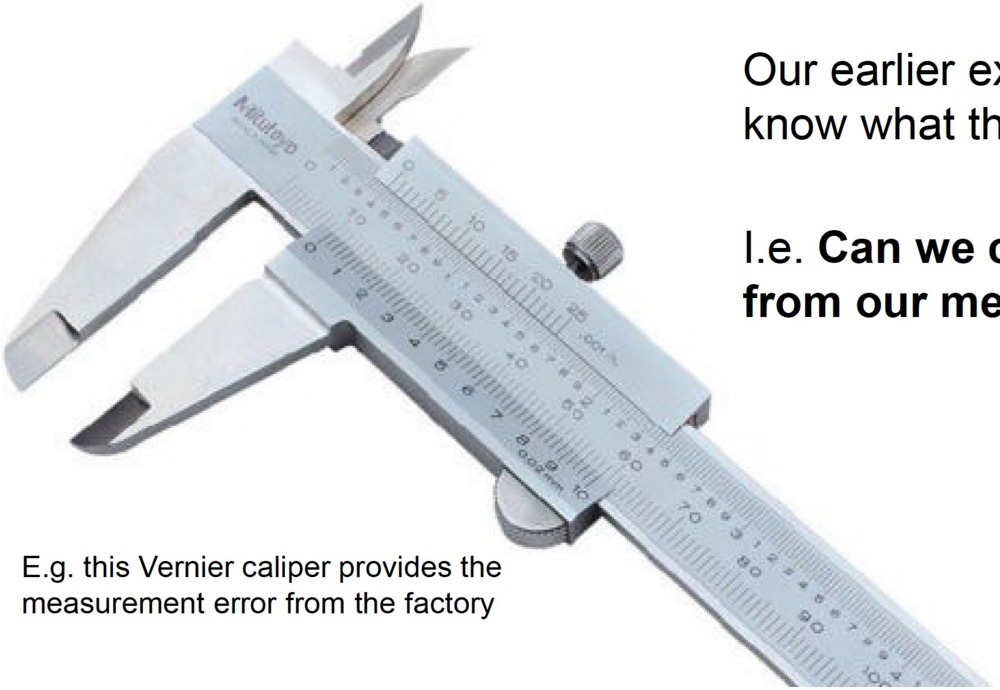
Uncertainty Statistics

What if we do not know our error source?

The problem with our approach thus far:

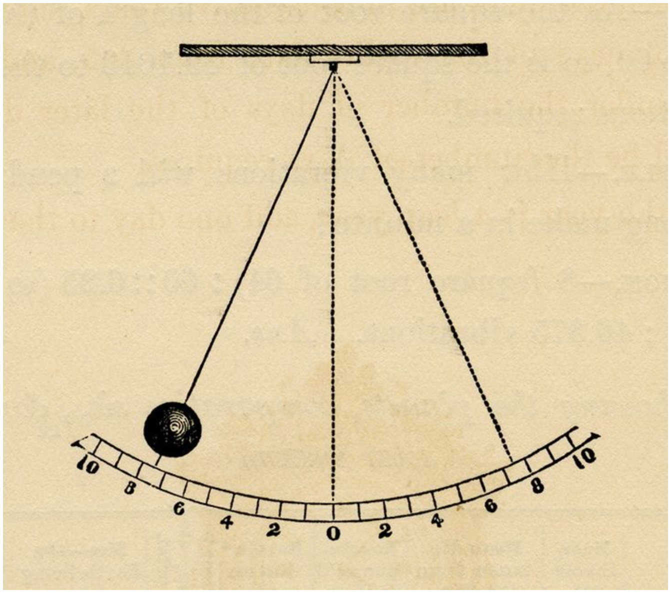


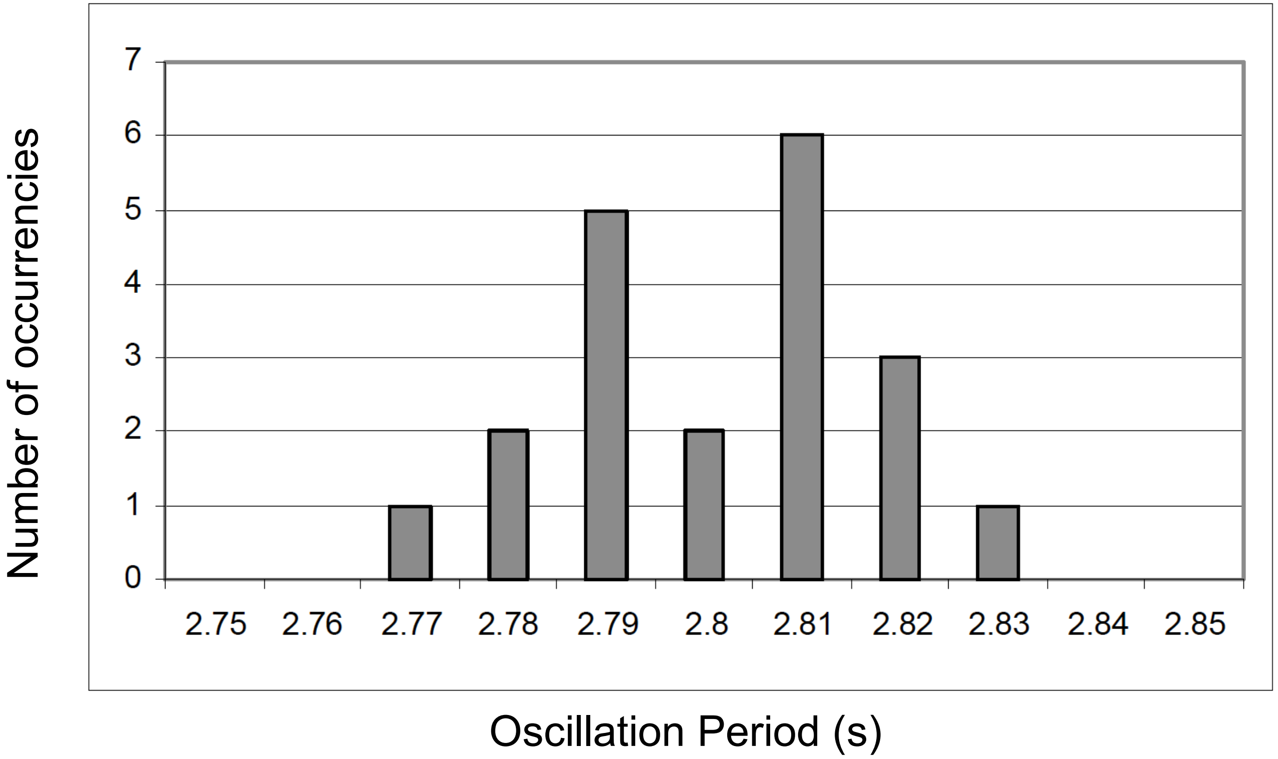
E.g. this Vernier caliper provides the measurement error from the factory

Our earlier examples assume we already know what the error is! **What if we don't?**

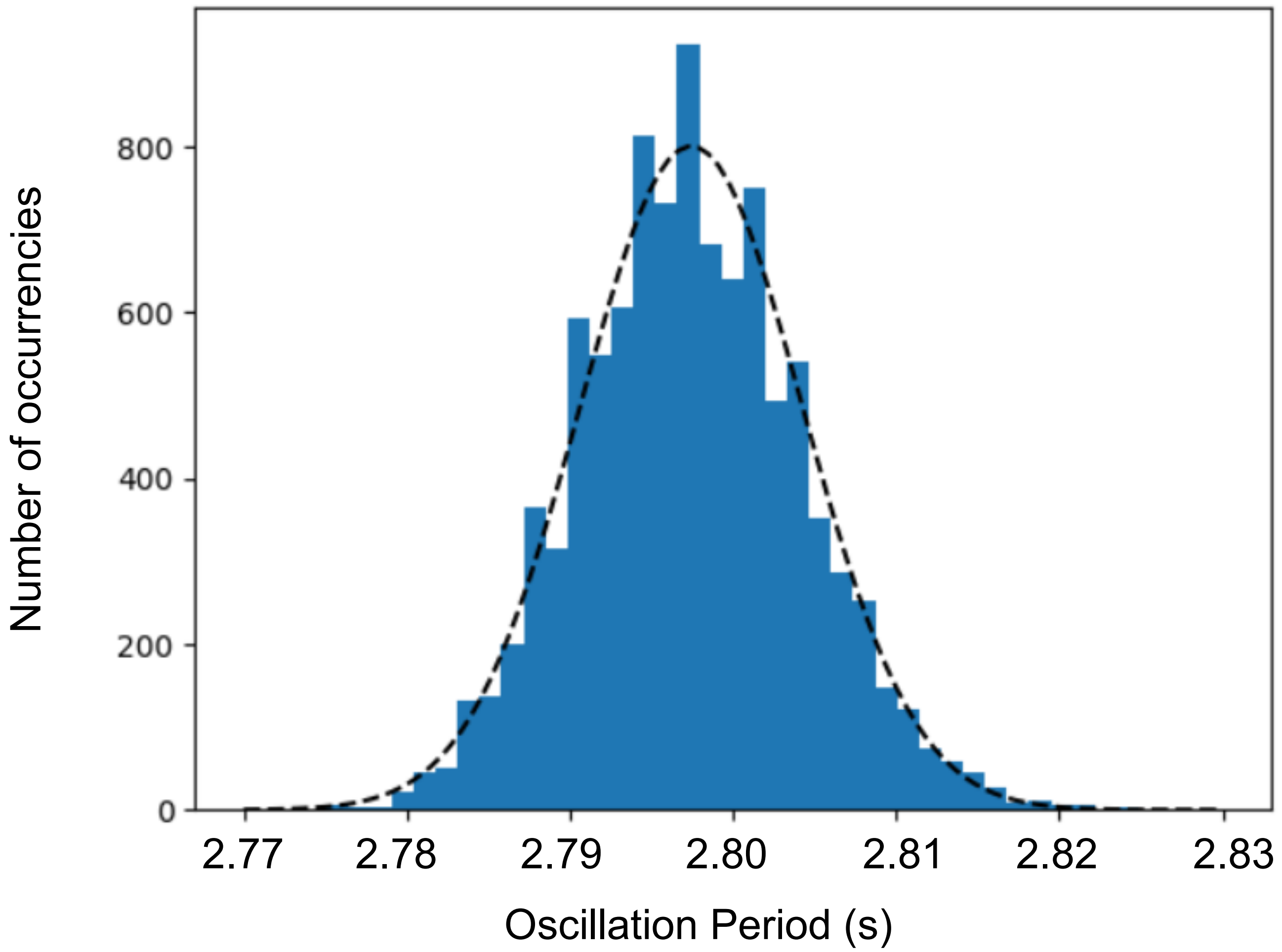
I.e. **Can we calculate the uncertainty from our measurements alone?**

Example of repeated measurement





Distribution of time measurements for large number of measurements



The Gaussian distribution

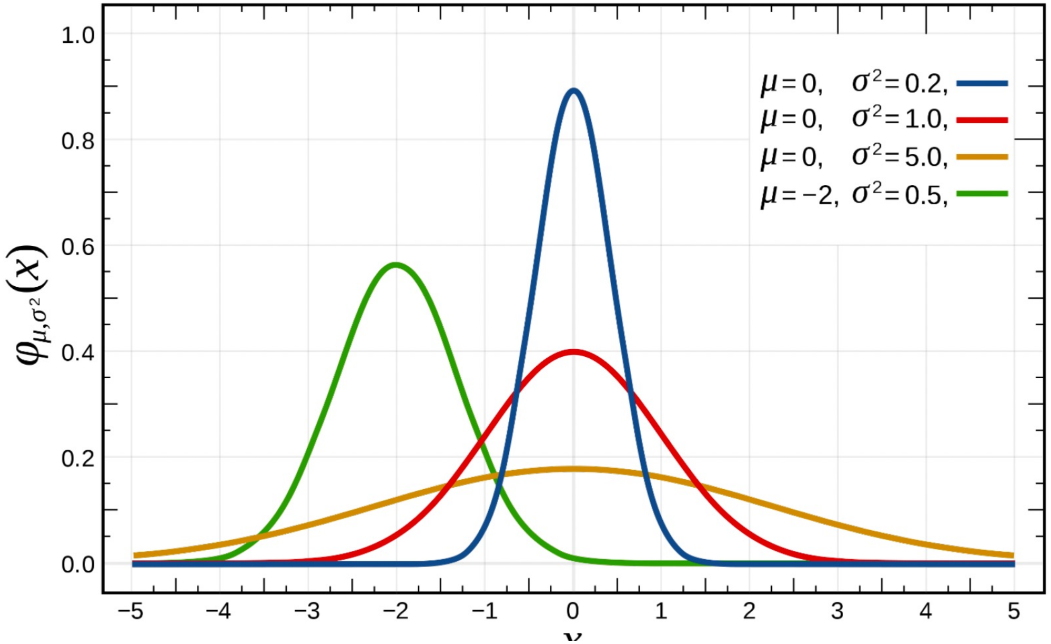
The normal (Gaussian) distribution:

(Central Limit Theorem states that independent random variables sum to give a normal distribution)

Relevant quantities of a Gaussian Distribution

The normal (Gaussian) distribution:

$$\boxed{\varphi(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right]}$$



Think of it as a probability distribution:

- Width of distribution given by σ
- Position of distribution given by μ

σ is the standard deviation

- Proportional to random error
- Related to $FWHM = 2\sqrt{2\ln 2}\,\sigma$

μ is the offset

- Equivalent to systematic error (assuming target is zero)

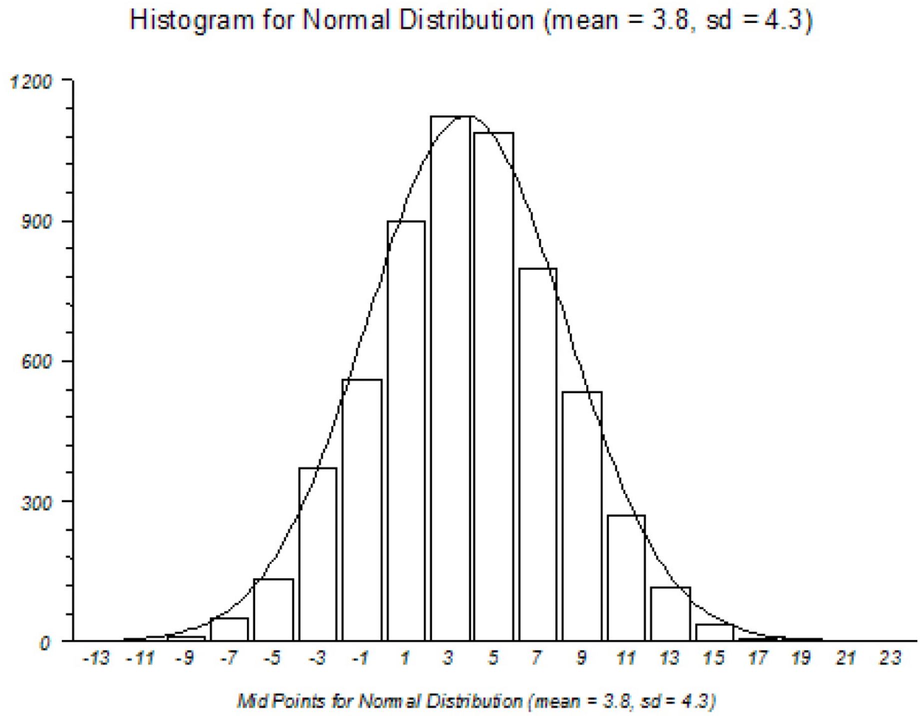
Relation to measurements

The normal (Gaussian) distribution:

Mean value: $$\bar{x} = \frac{1}{N}\sum_{n=1}^{N} x_n$$

Std. deviation: $$\sigma_x = \sqrt{\frac{1}{N}\sum_{n=1}^{N}(x_n - \bar{x})^2}$$

Std. error: $$\sigma_{\bar{x}} = \frac{\sigma_x}{\sqrt{N}} = \frac{1}{N}\sqrt{\sum_{n=1}^{N}(x_n - \bar{x})^2}$$



Standard deviation vs standard error

Standard deviation vs. standard error:

Or: what a difference $\sqrt{N}$ makes…

Think about it: Making more measurements should make us more “sure” of our average result, but the standard deviation (i.e. the distribution width) doesn’t change…

The concept of the “standard error” corrects for this by normalizing with $\sqrt{N}$.

Then, we write uncertainties from statistical data as: $\bar{x} \pm \sigma_{\bar{x}}$