a)

Let $z_1 = r_1 e^{i\frac{\pi}{3}}$ and $z_2 = r_2 e^{i\frac{\pi}{6}}$, then $z_1 z_2 = r_1 e^{i\frac{\pi}{3}} * r_2 e^{i\frac{\pi}{6}}$.

$$z_1 z_2 = r_1 r_2 e^{i(\frac{\pi}{3}+\frac{\pi}{6})} = r_1 r_2 e^{i\frac{\pi}{2}} = r_1 r_2 i$$

Which is a purely imaginary number.

b)

$$e^z = 1 + i$$

$$z = \ln(1+i) = \ln \sqrt{2} e^{i\frac{\pi}{4}}$$

$$z = \ln \sqrt{2} + i\frac{\pi}{4} \text{ - first solution}$$

$$z = \ln -\sqrt{2} + i\frac{5\pi}{4} \text{ - second solution}$$

c)

$$\begin{array}{r|l} Z^4 + 3Z^3 + 4Z^2 + 6Z + 4 & Z^2 + 3Z + 2 \\ \cline{2-2} \underline{-Z^4 - 3Z^3 - 2Z^2 \qquad\qquad\quad} & Z^2 + 2 \\ 2Z^2 + 6Z + 4 & \\ \underline{-2Z^2 - 6Z - 4} & \\ 0 & \end{array}$$

$$(Z^4 + 3Z^3 + 4Z^2 + 6Z + 4)(Z^2 + 3Z + 2) = (Z^2 + 3Z + 2)(Z^2 + 2)$$

$$Z^2 + 3Z + 2 = 0 \lor Z^2 + 2 = 0$$

$$Z = \frac{-3 \pm \sqrt{9-8}}{2} = -1 \, or \, -2$$

$$Z = \sqrt{-2} = i\sqrt{2} \, or \, -i\sqrt{2}$$

d)

$$\begin{array}{rrr|l}
Z^3 - 3Z^2 & & +4 & Z-2 \\ \cline{4-4}
-Z^3 + 2Z^2 & & & Z^2 - Z - 2 \\ \cline{1-1}
-Z^2 & & & \\
Z^2 & -2Z & & \\ \cline{1-2}
& -2Z & +4 & \\
& 2Z & -4 & \\ \cline{2-3}
& & 0 &
\end{array}$$

That shows that 2 is one of the roots of the polynomial.

$$\begin{array}{rrr|l}
Z^2 & -Z & -2 & Z-2 \\ \cline{4-4}
-Z^2 & +2Z & & Z+1 \\ \cline{1-2}
& Z & -2 & \\
& -Z & +2 & \\ \cline{2-3}
& & 0 &
\end{array}$$

The multiplicity of the root 2 in the polynomial is 2. e)

$$f(1000) \to f(500) \to f(250) \to f(125) \to f(124) \to f(62) \to f(31) \to f(30) \to$$

$$f(15) \to f(14) \to f(7) \to f(6) \to f(3) \to f(2) \to f(1)$$

$$f(1000) = 2*2*2*2*2*2*2*2*2*f(1) = 2^9 * f(1) = 512$$

f)

Base case:

$$n = 0 \Rightarrow 1 \geq 1$$

Induction step:

$$(1+h)^{n+1} \geq 1 + (n+1)h$$

$$(1+h)^{n+1} \geq 1 + hn + h$$

$$1 + (n+1)h + \binom{n+1}{2}h^2 + \binom{n+1}{3}h^3 + ... + h^{n+1} \geq 1 + (n+1)h$$

Cancel out common additives,

$$\binom{n+1}{3}h^3 + ... + h^{n+1} \geq 0$$

Once $\{h \in \mathbb{R} \,|\, h > 0\}$ and $\{n \in \mathbb{Z} \,|\, n \geq 0\}$, $\binom{n+1}{3}h^3 + ... + h^{n+1} \geq 0$ is

True for all .