Technical University of Denmark

Written test, Thursday, 17 August, 2023

Course name Physics 1

Course number 10018

Duration: 4 hours

Aids allowed: All.

Weighting: Submission is assessed as a whole

The exam comprises 18 multiple-choice questions. All questions *must* be answered, even if the answer selected is ”Do not know.” Incorrect answers contribute negatively to the evaluation, “Do not know” is neutral. Each question has at least one correct answer.

Question 1.

A student traveling to Japan gets a little bored on the train ride and measures with an app on his phone that the Japanese high-speed train "Shinkansen" accelerates from rest to a constant speed of 80 m/s in two stages. In the first stage, the acceleration is $2.0\ \mathrm{m/s^2}$ and in the second stage ,the acceleration is $1.0\ \mathrm{m/s^2}$. The two stages take the same amount of time.

How long does it take for the train to reach its constant speed?

A) 30 s
B) 37 s
C) 40 s
D) 50 s
E) 53 s
F) 62 s
G) 76 s
H) 80 s
I) Do not know.

Point system: E) gives 10. I) gives 0. The rest give -3.

Solution:

Let $a = 2\ \mathrm{m/s^2}$ and T is the total time of the acceleration.

$$v = a\frac{T}{2} + \frac{a}{2}\frac{T}{2} = \frac{3}{4}aT$$

$T = \frac{4}{3}\frac{v}{a} = 53.$

Question 2.

A car travels at constant speed on a horizontal road. A small child sitting in the back seat throws a small ball vertically upwards with a velocity relative to the car, v.

A person standing by the side of the road sees that the ball, when it leaves the child's hand, has a speed that forms an angle θ with the horizontal.

What is the speed of the car?

A) v
B) $v \cdot \sin\theta$
C) $v \cdot \cos\theta$
D) $v \cdot \tan\theta$
E) $\frac{v}{\sin\theta}$
F) $\frac{v}{\cos\theta}$
G) $\boldsymbol{\frac{v}{\tan\theta}}$
H) Do not know.

Point system: G) gives 10. H) gives 0. The rest give -3.

Solution:

Seen from the outside, the vertical and horizontal velocity components are v_x and v. So the slope $\tan\theta = \frac{v}{v_{\text{bil}}}$, i.e.,. to $v_{\text{bil}} = \frac{v}{\tan\theta}$.

Question 3.

A car travels 100 km per hour on a flat, straight road. The static coefficient of friction between the wheel and the road is 0.93. The wheels' moment of inertia and air resistance on the car can be neglected. The local gravitational acceleration is 9.82 m/s^2.

With this information, what is the lower limit of the car's braking distance (to a standstill) when braking only via the wheels?

A) 37. m
B) 42. m
C) 63. m
D) 85. m
E) 2.7·10^2 m
F) 5.5·10^2 m
G) There is no lower limit if the brake system works properly.
H) Do not know.

Point system: B) gives 10. H) gives 0. The rest give -3.

Solution:

We call the mass of the car m, but the answer will not depend on it.

The maximum braking force that can work (ideal braking system on all four wheels, no blocking of the wheels) is $f = -0.93 \cdot N = -0.93 \cdot mg$ where g is the acceleration due to gravity, and N the total normal force from the road. This maximum braking force is independent of the speed, and can therefore be assumed to be constant throughout the braking. Thus the acceleration a also becomes constant, , $a = \frac{f}{m}$. We now use the expression for speed as a function of distance traveled L for a constant acceleration:

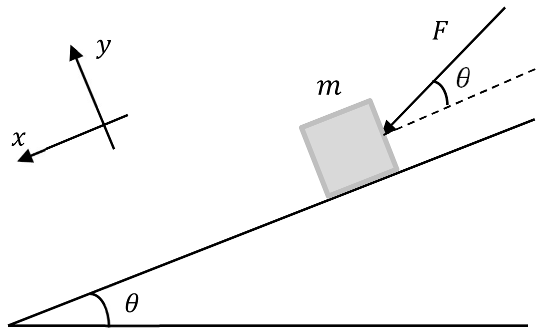
$$v^2 = v_0^2 + 2aL$$

where v_0 is the initial speed of 100 km/h. We count the speed as positive, thus L also becomes positive, while a must be negative as the car brakes. As we brake to a stop, the speed on the left must be set to zero, after which we solve for L:

$$L = -\frac{v_0^2}{2a} = \frac{v_0^2 m}{2 \cdot 0.93 \cdot mg} = \frac{v_0^2}{2 \cdot 0.93g} \approx 42. m$$

Question 4.

A box of mass, m, moves down an inclined plane with a smooth surface. The inclined plane makes an angle θ with the horizontal. A person pushes the box with a constant force, F, whose direction makes an angle θ with the inclined plane.



Which of the following equations corresponds to Newton's second law either down parallel to the inclined plane (x direction in the figure) or perpendicular to the inclined plane (y direction in the figure)?

A) $ma_x = mg\cos\theta - F$
B) $ma_x = mg\sin\theta - F\sin\theta$
C) $ma_x = mg\sin\theta - F\cos\theta$
D) $ma_x = mg\sin\theta + F$
E) $ma_x = mg\sin\theta + F\sin\theta$
F) $\boldsymbol{ma_x = mg\sin\theta + F\cos\theta}$
G) $ma_y = n\cos\theta - mg + F\cos\theta = 0$
H) $ma_y = n - mg\sin\theta + F\cos\theta = 0$
I) $ma_y = n - mg\cos\theta + F\sin\theta = 0$
J) $\boldsymbol{ma_y = n - mg\cos\theta - F\sin\theta = 0}$
K) Do not know.

Point system:

F+J gives 10.

Only F or only J gives 5.

F + either G, H, or I gives 3.

J + either A, B, C , D, E gives 3.

K gives 0.

The rest give -3.

Solution:

N2(x): $ma_x = mg\sin\theta + F\cos\theta$

L) N2(y): $ma_y = n - mg\cos\theta - F\sin\theta = 0$

Question 5.

The human metabolism has an efficiency of about 25%, i.e. around 25% of the energy in a given food you consume can in principle, be converted into mechanical energy. Mayonnaise has a calorie content of 830 kcal per 100 grams (1 kcal = 10^3 cal). In a given fitness machine, a 75 kg weight is lifted vertically 45 cm. The lifts take place slowly.

Approximately how many lifts must be performed on this machine to burn 20 grams of mayonnaise?

A) 53 lifts
B) 88 lifts
C) 106 lifts
D) 125 lifts
E) 131 lifts
F) 525 lifts
G) 1750 lifts
H) 1690 lifts
I) Do not know.

Point system: F) gives 10. I) gives 0. The rest give -3.

Solution:

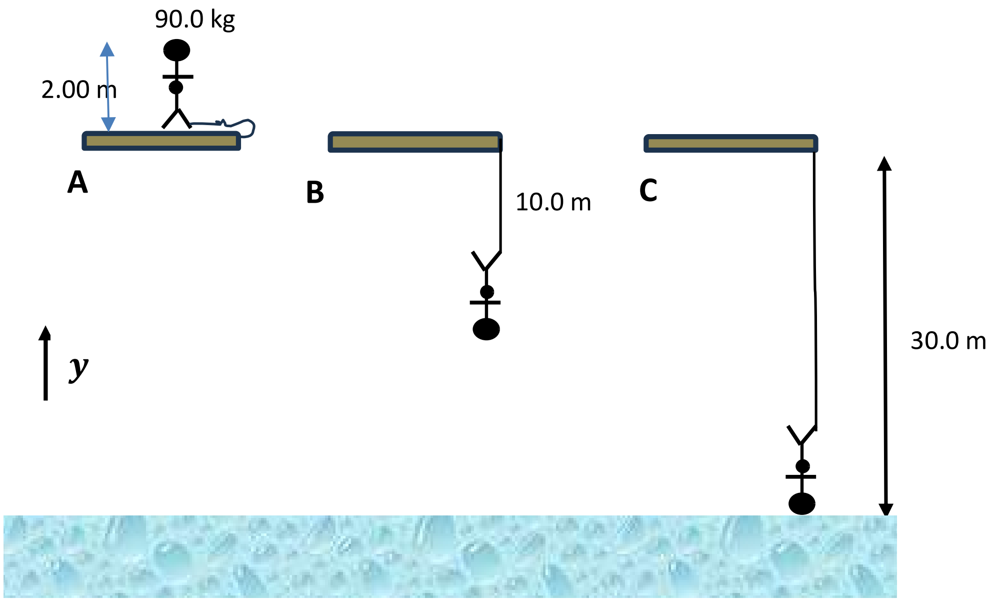
One lift $W = m \cdot g \cdot h = 75 \text{ kg} \cdot 0.45 \text{ m} \cdot 9.82 \text{ m/s}^2 = 331 \text{ J}$

Energy in Mayonnaise $0.02 \text{ kg} \cdot 8300 \text{ kcal/kg} \cdot 4186 \cdot \text{J/kcal} = 695 \text{ kJ}$

Number of lifts $\frac{695 \text{ kJ}}{4 \cdot 331 \text{ J}} = 525$

Question 6.

A student with a height of 2.00 meters and a mass of 90.0 kg is on vacation in the south and uses the time to bungee jump from a bridge located 30.0 meters above a river (figure A). The elastic is 10.0 meters long in the unstretched state and has a spring constant, k. The student's center of mass is in the middle of the body and the gravitational acceleration can be taken as $g = 9.82 \text{ m/s}^2$.



What is the student's speed in figure B, just before the elastic begins to stretch?

A) $v = 11.1 \ \frac{\text{m}}{\text{s}}$
B) $v = -14.0 \ \frac{\text{m}}{\text{s}}$
C) $v = 14.0 \ \frac{\text{m}}{\text{s}}$
D) $\boldsymbol{v = -15.4 \ \frac{\text{m}}{\text{s}}}$
E) $v = -10.2 \ \frac{\text{m}}{\text{s}}$
F) $v = 10.2 \ \frac{\text{m}}{\text{s}}$
G) $v = -11.1 \ \frac{\text{m}}{\text{s}}$
H) $v = 15.4 \ \frac{\text{m}}{\text{s}}$
I) Do not know.

Point system:

D gives 10.

H gives 8.

B gives 5.

C gives 3.

I gives 0.

The rest give -3.

Solution:

$mg\Delta h = \frac{1}{2}mv^2$ (remember the sign when isolating the velocity)

$$v = -\sqrt{2g\Delta h} = -\sqrt{2 * 9.82 * 12} = -15.4\ \frac{m}{s}$$

Question 7. [Continuation from the previous question]

In figure C, the student's head has just touched the water and luckily her speed here is zero. What is the spring constant?

A) $k = 112\ \frac{\mathrm{N}}{\mathrm{m}}$
B) $k = 81.4\ \frac{\mathrm{N}}{\mathrm{m}}$
C) $k = 77.4\ \frac{\mathrm{N}}{\mathrm{m}}$
D) $k = 56.2\ \frac{\mathrm{N}}{\mathrm{m}}$
E) $\boldsymbol{k = 164\ \frac{\mathrm{N}}{\mathrm{m}}}$
F) $k = 1470\ \frac{\mathrm{N}}{\mathrm{m}}$
G) Do not know.

Point system: E) gives 10. G) gives 0. The rest give -3.

Solution:

In C, the decrease in potential energy has been transferred to spring energy. The height difference of the center of mass is Δh=-30 meters while the spring has become x=18 meters longer.

$$mg\Delta h = \frac{1}{2}kx^2$$

$$k = \frac{-mg\Delta h}{x^2} = \frac{2 * 90 * 9.82 * 30}{18^2} = 164\ \frac{N}{m}$$

Question 8.

Two balls collide as shown in the figure below. Before the collision, the light ball is moving with horizontal velocity and the dark ball is at rest. After the impact, the dark sphere moves upwards to the right. The magnitude of the two velocities shown and the masses of the balls are unknown.

Which of the directions shown below for the speed of the light ball after impact are possible? The directions of the arrows are purely horizontal, purely vertical or oblique (the oblique arrows have no specific angle).

A) ←
B) →
C) ↑
D) ↓
E) ↗
F) ↖
G) ↘
H) ↙
I) Do not know.

Point system:

D + G +H gives 10 points

D+G or D+H or G+H 7 points

D + G +H + only another one gives 7 points

D + G +H + two other ones gives 3 points

D+G or D+H or G+H + only another one (not D, G, or H) gives 3 points

Only D or Only G or Only H 3 points

I gives 0 points

Other combinations give -3 points

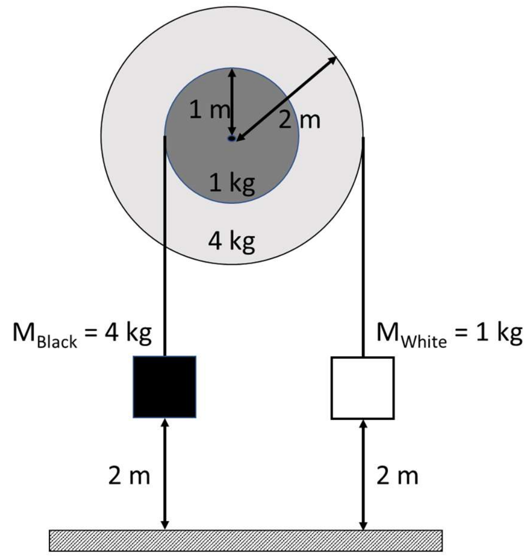
Solution:

Total momentum must be preserved. Before the collision there was no momentum out of the collision direction. After the collision the dark ball has some momentum out of the collision direction (upwards). Thus, to compensate for that, the lighter ball must have some momentum downwards (as in D, G, and H).

Question 9.

The system in the figure consists of two solid disks stuck to each other. They are able to rotate around a fixed, horizontal axis that passes through the center of the disks. The small disk has a radius of R_{small} = 1 m and mass M_{small} = 1 kg. The big disk has a radius of R_{big} = 2 m and mass M_{big} = 4 kg. There is a black box with mass M_{Black} = 4 kg hanging from the small disk. The white box, with mass M_{White} = 1 kg, is hanging from the big disk. Both boxes are 2 m above the floor. The system is freed from rest.

Which of the following statements is true?



A) The white box will move down and the black box will move up.
B) The white box will move up and the black box will move down.
C) Just before the first box touches the floor the speed of the white box will be larger than that of the black box.
D) Just before the first box touches the floor the speed of the white box will be smaller than that of the black box.
E) Just before the first box touches the floor the speed of the white box will be the same as that of the black box.
F) The boxes never move.
G) Do not know.

Point system:

B + C gives 10 points

Only B or Only C gives 5 points

C + A gives 0 points

B + D or B + E gives 0 points

G gives 0 points

Any other combination gives -3 points

Solution

The weight of the black box is producing a larger torque on the stuck disks than the weight of the white box (4kg * g * 1m > 1 kg * g * 2 m). Thus the black box will fall on the floor.

The angular acceleration of both disks is the same (they are stuck). The linear acceleration is the angular acceleration multiplied by the radius of the disk. Thus, the linear acceleration of the white box is larger than that of the black box. Thus, the speed of the black box just before touching the ground will be smaller than that of the white box.

Question 10.

An IC4 train with a total mass of 500 tons travels at 50.0 m/s. The train rolls without slipping on four sets of wheels (i.e. axles) with two wheels each. The individual wheels can be considered as homogeneous disks with a mass of 400 kg and a diameter of 1.00 meters.

What percentage of the train's total kinetic energy exists as rotational energy of the wheels?

A) 0.0399 %
B) 0.0740 %
C) 0.159 %
D) 0.215 %
E) 0.217 %
F) 0.280 %
G) 0.319 %
H) 0.320 %
I) 0.473 %
J) Don't know

Point system: G) gives 10. J) gives 0. The rest give -3.

Solution:

$$\omega = \frac{v}{r} = 100\text{s}^{-1}$$

$$K_{\text{rot, wheel}} = 8 \cdot \frac{1}{2} \cdot \left(\frac{1}{2} \cdot m \cdot r^2\right) \cdot \omega^2 = 2 \cdot m \cdot r^2 \cdot \left(\frac{v}{r}\right)^2 = 2 \cdot m \cdot v^2 = 2\text{MJ}$$

$$K_{\text{trans,gathered}} = \frac{1}{2} M v^2 = 625\text{MJ}$$

$\% = K_{\text{rot, wheel}} / (K_{\text{rot, wheel}} + K_{\text{trans,gathered}}) = 0.319\ \%$

Question 11.

An ice skater with outstretched arms rotates about their longitudinal axis with an angular velocity of 7.50 s^{-1}, and has a moment of inertia of 1.00 kg·m^2. The skater now very quickly pulls their arms in to their body, thereby reducing their moment of inertia to 0.600 kg·m^2. In the subsequent rotations, the ice skater is affected by a constant torque of magnitude 0.230 N·m due to the final extension of the skate tip and friction with the ice. The torque is directed so that it slows down the skater's rotation.

What is the skater's angular velocity after completing 10 full rotations with their arms retracted?

A) $2.84\ s^{-1}$
B) $6.97\ s^{-1}$
C) $8.64\ s^{-1}$
D) $\mathbf{10.4\ s^{-1}}$
E) $12.5\ s^{-1}$
F) $14.3\ s^{-1}$
G) $16.9\ s^{-1}$
H) Don't know

Point system: D) gives 10. A) gives 3. C) gives 3. F) gives 3. J) gives 0. The rest give -3.

Solution:

The momentum is conserved at the moment the skater retracts his arms and reduces his moment of inertia. There applies $L_1 = I_1\omega_1 = I_2\omega_2 \Rightarrow \omega_2 = \omega_1 \frac{I_1}{I_2}$

Subsequently, we have rotation with constant angular acceleration, the magnitude of which is given by:

$$\tau = I_2\alpha \Rightarrow \alpha = \frac{\tau}{I_2}$$

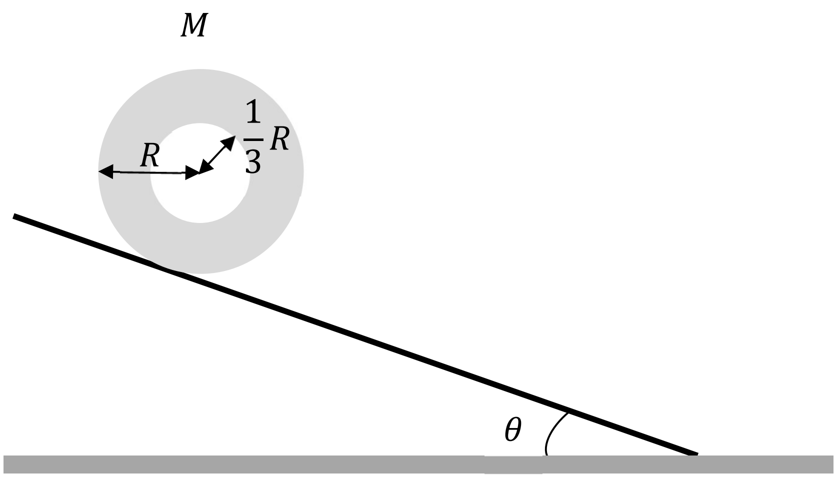
We choose the sign so that the angular velocity is positive and the angular acceleration is negative, as the skater is slowed down. We can now use the equation for angular velocity as a function of rotation angle $\Delta\theta$ with constant angular acceleration α:

$$\omega^2 = \omega_2^2 - 2\alpha\Delta\theta$$

For ten full rotations we have $\Delta\theta=20\pi$. That gives our final expression for the angular angular velocity:

$$\omega = \sqrt{\omega_1^2\left(\frac{I_1}{I_2}\right)^2 - \frac{2\tau}{I_2}20\pi} \approx 10.4\ s^{-1}$$

Question 12.



A hollow cylinder has inner radius, $\frac{1}{3}R$, and outer radius, R. The cylinder has mass, M. The cylinder rolls down an plane inclined with slope θ, with the horizontal.

What is the acceleration of the center of mass of the cylinder, a?

A) $a = \frac{2}{3} g \sin\theta$
B) $a = \frac{1}{2} g \sin\theta$
C) $a = \frac{5}{6} g \sin\theta$
D) $a = \frac{7}{11} g \sin\theta$
E) $a = \frac{3}{7} g \sin\theta$
F) $\boldsymbol{a = \frac{9}{14} g \sin\theta}$
G) $a = \frac{5}{7} g \sin\theta$
H) Don't know

Point system: F) gives 10. H) gives 0. The rest give -3.

Solution:

I(CM): $I = \frac{1}{2}M\left(\left(\frac{1}{3}R\right)^2 + R^2\right) = \frac{1}{2}M\left(\frac{1}{9}R^2 + R^2\right) = \frac{1}{2}M\left(\frac{10}{9}R^2\right) = \frac{5}{9}MR^2$

MMS: $Ma = Mg\sin\theta - f$

IMS: $\frac{5}{9}MR^2\alpha = Rf$

GB: $a = R\alpha$

$\frac{5}{9} Ma = f$

$\frac{14}{9} Ma = Mg \sin \theta$

$a = \frac{9}{14} g \sin \theta$

Question 13. [Continuation of the previous question]

The torque that gives the cylinder its angular acceleration with respect to the center of mass, is due to:

A) Gravity
B) The normal force
C) The kinematic friction force
D) The static friction force
E) Don't know

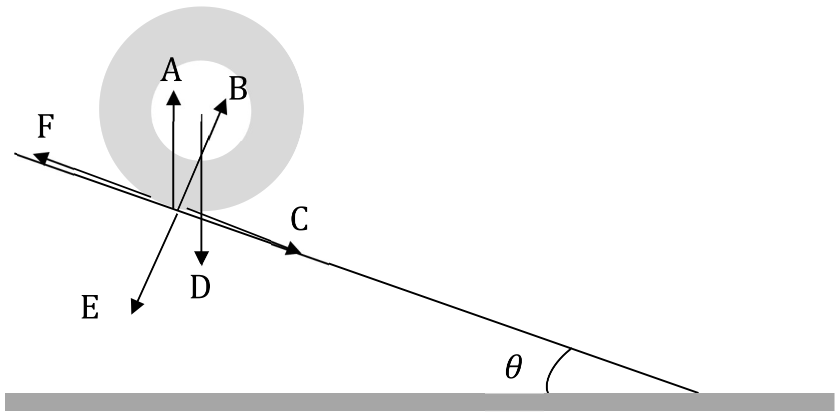
Point system: D) gives 10 E) gives 0. The rest give -3.

Solution:

The static friction force

Question 14. [Continuation of the previous question]

The cylinder now rolls *up* the inclined plane (instead of down).



Which of the vectors shown in the figure must be included in a force diagram for the rolling of the cylinder up the inclined plane?

A) A
B) B
C) C
D) D
E) E
F) F
G) Ved ikke

Point system:

B + D +F gives 10 points

B+D or D+F or B+F gives 7 points

B+D or D+F or B+F + only another one (not B, D, or F) gives 3 points

Only B or Only D or Only F 3 points

I gives 0 points

Other combinations give -3 points

Spørgsmål 15.

En veteranbil der ikke har blokeringsfri bremser kører ad motorvejen med 30. m/s og skal bremses ned til stilstand. Ved korrekt bremsning ruller hjulene under hele nedbremsningen. Hvis der bremses for hårdt 'blokeres' hjulene og skrider hen over vejen. Den statiske og kinematiske friktionskoefficient mellem dæk og vej er henholdsvis 1.0 og 0.7. Hvor stor er forskellen i bremselængde mellem optimal nedbremsning og en nedbremsning hvor hjulene blokeres hele vejen?

Der kan ses bort fra luftmodstand og det kan antages at bremselængderne kun begrænses af friktionen mellem dæk og vej, og ikke af andre komponenter i bremsesystemet.

A) 14 m
B) 20 m (*)
C) 28 m
D) 39 m
E) 45 m
F) Ved ikke

Løsning:

Maksimal acceleration i nedbremsningen er $-g\mu$ hvor μ enten er μ_s (statisk) eller μ_k (kinetisk). Bremselængden kan bestemmes fra:

$$0 - v^2 = -2g\mu L \Rightarrow L = \frac{v^2}{2g\mu}$$

Forskellen i bremselængde bliver så $\Delta L = \frac{v^2}{2g}(\frac{1}{\mu_k} - \frac{1}{\mu_s})$.

Spørgsmål 16. [Fortsættelse af foregående spørgsmål]

Bilen antages nu at køre op ad en skrånende vej, der hælder 5.00 grader i forhold til vandret. Idet fart og friktionskoefficienter antages at være som før, hvad er den mindst mulige bremselængde nu ved korrekt bremsning? Sæt tyngdeaccelerationen til 9.80 m/s^2.

A) 42.2 m
B) 42.4 m (*)
C) 45.9 m
D) 46.1 m
E) 84.5 m
F) 84.8 m
G) Ved ikke

Løsning:

Maksimal bremsekraft er givet ved $f = N\mu_s$ hvor normalkraften fra vejen er givet ved $N = Mg\cos(\theta)$ og M er bilens masse. Dertil har vi komposanten af tyngdekraften langs vejen, $F_{g,x} = Mg\sin(\theta)$. Begge kræfter er rettet mod bevægelsesretningen. Størrelsen af accelerationen bliver således $a_{max} = g(\sin(\theta) + \mu_s\cos(\theta))$ og bremselængden bliver dermed

$$L = \frac{v^2}{2g(\sin(\theta)+\mu_s\cos(\theta))}.$$

Spørgsmål 17.
Tre togvogne, A, B og C, er på samme spor med B imellem A og C. De har alle samme masse, m, og B er koblet sammen med C. Sammenkoblingen kan beskrives som en masseløs fjeder med fjederkonstanten k. B og C står stille på sporet, mens A bevæger sig imod B med en konstant hastighed v_0.

A støder ind i B og undergår en fuldstændig elastisk kollision. Hvad er hastigheden af A (v_A), B (v_B), og C (v_C) i øjeblikket umiddelbart efter kollisionen?

A) $v_A = -\frac{1}{2}v_0,\ v_B = \frac{1}{2}v_0,\ v_C = \frac{1}{2}v_0$

B) $v_A = -\frac{1}{3}v_0,\ v_B = \frac{1}{3}v_0,\ v_C = \frac{1}{3}v_0$

C) $v_A = 0,\ v_B = \frac{1}{2}v_0,\ v_C = \frac{1}{2}v_0$

D) $v_A = 0,\ v_B = v_0,\ v_C = 0$ (*)

E) $v_A = -\frac{1}{2}v_0,\ v_B = \frac{1}{2}v_0,\ v_C = 0$

F) $v_A = \frac{1}{2}v_0,\ v_B = \frac{1}{2}v_0,\ v_C = 0$

G) ved ikke

Løsning: Se nedenfor (efter næste spørgsmål).

Spørgsmål 18. [Fortsættelse af foregående spørgsmål]
Tre togvogne, A, B og C, er på samme spor med B imellem A og C. De har alle samme masse, m, og B er koblet sammen med C. Sammenkoblingen kan beskrives som en masseløs fjeder med fjederkonstanten k. B og C står stille på sporet, mens A bevæger sig imod B med en konstant hastighed v_0.

A støder ind i B og undergår en fuldstændig elastisk kollision. Hvad er den maksimale sammentrykning af fjederen i koblingen?

A) $\sqrt{\frac{mv_0}{k}}$
B) $\sqrt{\frac{mv_0}{2k}}$
C) $\sqrt{\frac{m}{v_0 k}}$
D) $v_0\sqrt{\frac{m}{k}}$
E) $v_0\sqrt{\frac{m}{2k}}$ (*)

F) Ved ikke

Der er impulsbevarelse i kollisionen.
Under stødet er det kun A og B, som vekselvirker idet fjederkraften er meget mindre end kræfterne involveret i stødet.

Da A og B har samme masse ligger A stille efter stødet og impuls og energi overføres til B.
B bevæger sig nu mod C og trykker fjederen sammen.

Når fjederens sammentrykning er maksimal har B og C ingen relativ hastighed ⊛

⊛ Deres relative hastighed er $\frac{dx}{dt}$, hvor x er længden af fjederen. Når fjederens sammentrykning er maksimal må $\frac{dx}{dt} = 0$ (da ændringen i x ændrer fortegn) og B og C må have samme hastighed.

Da impulsen er bevaret må der gælde

$$\underset{p_A}{mv_0} = \underset{p_B}{mv} + \underset{p_C}{mv},$$

hvor v er hastigheden af B og C når fjederen er maksimalt sammentrykket.

$\Rightarrow v = \frac{1}{2} v_0$

Da stødet er elastisk overføres al den kinetiske energi fra A til B og C og fjederen.
Dvs.

$$\cancel{\tfrac{1}{2}} m v_0^2 = \cancel{\tfrac{1}{2}}(m+m)\left(\frac{v_0}{2}\right)^2 + \underbrace{\cancel{\tfrac{1}{2}} k x^2}_{\text{sammentrykning af fjederen}}$$

$$\Rightarrow m v_0^2 = \frac{2}{4} m v_0^2 + k x^2$$

$$\Rightarrow x = \sqrt{\frac{1}{2} m v_0^2 \cdot \frac{1}{k}} = v_0 \sqrt{\frac{m}{2k}}$$