01017/01019 Discrete Mathematics E24
Home assignment 1
To be handed in no later than Sunday, September 22 at 11:59pm

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Important regarding the home assignments: Please note that the home assignments must be handed in in groups of two or three students.

Exercise A

In the following exercise it is very important to explain your solution and argue for its correctness. The task is not trivial, so start thinking about it in due time!

On a remote island there are two types of people:

- Truth sayers, that always tell the truth (everything they say is true).

- Liars, that always lie (everything they say is false).

It is not possible to distinguish between them by the way they look, but only by the truth of falsehood of the things they say.

1. Once a foreigner visited the island. There, he met two of the island's inhabitants, Peter and Signe. He asked them: "Are any of you liars?". "At least one of us is a liar", Peter replied. What are Peter and Signe (truth sayers or liars)?

   Hint: Use a propositional variable p to denote the proposition "Peter is a truth sayer" and a propositional variable s to denote "Signe is a truth sayer". We can then e.g. express the proposition "Peter and Signe are both truth sayers" by the formula $p \wedge s$. How do you then express "at least one of them is a liar"? Now find a formula with propositional variables p and s, that must be true for the described situation. By looking at which truth values of p and s that make this formula true (e.g. by truth table), you are able to determine what Peter and Signe are.

2. While the the foreigner wandered around the island, he met two other inhabitants, Anne and Bob. He asked Anne: "Are any of you truth sayers?". "If Bob is a liar, then I am also a liar", Anne replied. What are Anne and Bob?

Hint: Use the same strategy as above. Let a denote "Anne is a truth sayer" and b denote "Bob is a truth sayer". How do you then express "If Bob is a liar, then Anne is also a liar"? Now find a formula with propositional variables a and b, that must be true for the described situation, and use this formula to determine the truth values of a and b.

3. The foreigner kept wandering. At night he started searching for shelter for the night, but was very careful, as he knew some of the island's inhabitants to be cannibals, and it was not possible to distinguish between them by their looks. He then met three of the island's inhabitants, Carsten, Diana and Erika. He asked Carsten: "How many of you are truth sayers?". "Flam flim", Carsten replied in his own language. "What did he say?", the stranger asked Dianna. "He said 'Only one' ", Diana replied. "Don't trust Diana, she is a liar. Come with me, I am not a cannibal", Erika said. "No, come with me, I am not a cannibal", Diana said. What should the stranger do?

Hint: Like in the previous questions we can let c denote that Carsten is a truth sayer, d for Dianna being a truth sayer and e for Erika being a truth sayer. Then start by writing the formula that expresses "only one of us is a truth sayer", where "us" is Carsten, Diana and Erika. Carsten's proposition is "flim flam". Diana claims that "flim flam" means "only one of us is a truth sayer". So if Diana tells the truth (is a truth sayer), then Carsten's claim is that "only one of us is a truth sayer", and this claim must then be true exactly when Carsten is a truth sayer. Try to write down a formula that captures all of this, and determine which truth assignments that satisfy this formula.

Exercise B

Translate the following English sentences into formulas in predicate logic, and determine their truth value (justify your answer). Make sure to specify the domain for each formula.

1. Every real number is smaller than its square.

2. There exists a smallest positive real number.

3. For every odd integer, there is an even integer greater than it.

Also translate each of the following formulas into English and determine their truth value and justify your answer.

4. $\exists x \in \mathbb{R} \ (x^5 = -1)$

5. $\forall x \in \mathbb{N} \ \exists y \in \mathbb{N} \ (y^2 = x)$

6. $\forall x \in \mathbb{R} \ \forall z \in \mathbb{R} \ (x < z \rightarrow (\exists y \in \mathbb{R} \ (x < y < z)))$

Exercise C

Do Exercise 32 in Section 2.2. Justify your answers.

Exercise D

Argue whether each of the following functions are surjective, injective, both, neither, or not actually a properly defined function.

1. $f : \mathbb{N} \to \mathbb{N}$ defined as $f(x) = 2x + 3$

2. $f : \mathbb{N} \to \mathbb{Z}$ defined as $f(x) = \begin{cases} x/2 & \text{if } x \text{ is even} \\ -(x+1)/2 & \text{if } x \text{ is odd} \end{cases}$

3. $f : \mathbb{R} \to \mathbb{R}$ defined as $f(x) = \sqrt{x}$

4. $f : \mathbb{R} \to \mathbb{R}$ defined as $f(x) = \sin(x)$

5. $f : \mathbb{Z}^+ \to \mathbb{R}$ defined as $f(x) = \lfloor \log_2(x) \rfloor$

Exercise E

Do Exercise 44 in Section 2.3. You must read the definition given above the exercise in the book to do this exercise.

Exercise F (Only for groups of size 3)

Do Exercise 74 in Section 2.3.