HW4

November 29, 2024

[1]:
```python
import numpy as np
import matplotlib.pyplot as plt
from Newtonsys import Newtonsys
from time import time
import scipy as sp
```

1 1) Section

1.1 A - 2

1.2 B - 4

1.3 C - 2

1.4 D - 4

2) 1.

$$F(X) = 0, \quad X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \quad F(X) = \begin{bmatrix} x_1^2 + 2x_2 - 2 \\ x_1 + 4x_2^2 - 4 \end{bmatrix}$$

$$J_F(X) = \begin{bmatrix} 2x_1 & 2 \\ 1 & 8x_2 \end{bmatrix}$$

2.

$$J_F(X_k) \cdot H_k = -F(X_k)$$

$$H_k = -\left(J_F(X_k)\right)^{-1} F(X_k)$$

$$X_{k+1} = X_k + H_k$$

[41]:
```python
def funFdF(X):
    x1, x2 = X
    F = np.array([x1**2 + 2*x2 - 2, x1 + 4*x2**2-4])
    dF = np.array([[2*x1, 2], [1,8*x2]])
    return F, dF
```

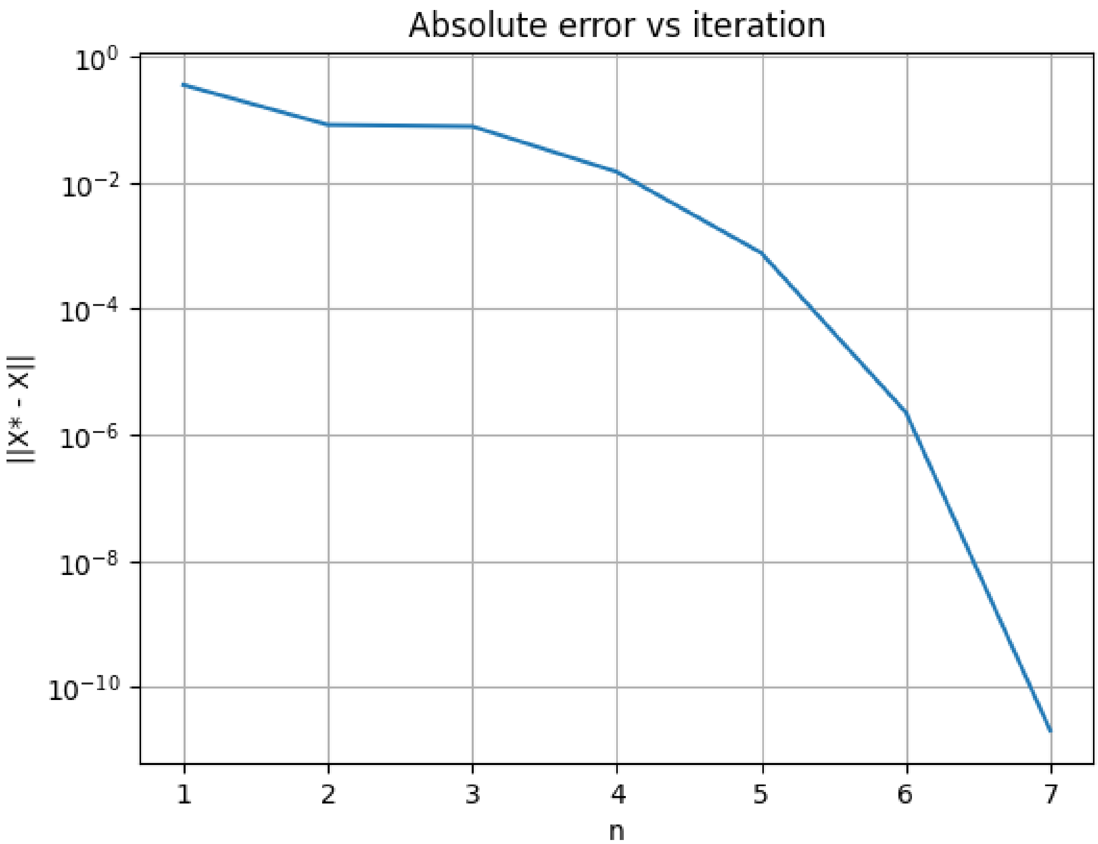
[44]:
```python
def Newton_Sys(FdF, X, n):
    F, dF = FdF(X)
    X_iter = []
    for i in range(n):
        H = np.linalg.solve(dF,F)
        X = X - H
        X_iter.append(X)
        F, dF = FdF(X)
    return X_iter
```

[48]:
```python
X0 = np.array([1,2])
X = Newton_Sys(funFdF, X0, 8)
X[-1]
```

[48]:
```
array([2.5471828e-16, 1.0000000e+00])
```

[62]:
```python
X_last = X[-1]
err = []
for x in X[:-1]:
    norm = np.linalg.norm(X_last - x)
    err.append(norm)
```

[64]:
```python
n = np.linspace(1, 7, 7)
plt.semilogy(n, err)
plt.xlabel('n')
plt.ylabel('||X* - X||')
plt.title('Absolute error vs iteration')
plt.grid(True)
```


Absolute error vs iteration
||X* - X||
10^{0}
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^{-10}
1
2
3
4
5
6
7
n

3. Matrix factorization

1) $A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 3 & \\ 1 & 1 & 2 \end{bmatrix} \begin{matrix} R_2 = R_2 + \frac{1}{3} R_1 \\ R_3 = R_3 - \frac{1}{3} R_1 \end{matrix} \Rightarrow \begin{bmatrix} 3 & -1 & 1 \\ 0 & 8/3 & 4/3 \\ 0 & 4/3 & 5/3 \end{bmatrix} R_3 = R_3 - \frac{1}{2} R_2 \Rightarrow$

$\Rightarrow \begin{bmatrix} 3 & -1 & 1 \\ 0 & 8/3 & 4/3 \\ 0 & 0 & 1 \end{bmatrix} = U \qquad L = \begin{bmatrix} 1 & 0 & 0 \\ -\frac{1}{3} & 1 & 0 \\ \frac{1}{3} & \frac{1}{2} & 1 \end{bmatrix}$

2) LDL^T factorization

The matrix A has to be symmetric. ✓

$D = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 8/3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, diagonal of U.

L is the same as in LU.

3) The matrix A is symmetric and positive defined $\Rightarrow$ there is a Cholesky factorization.

$\tilde{L} = LD^{\frac{1}{2}} \Rightarrow \tilde{L} = \begin{bmatrix} 1 & 0 & 0 \\ -1/3 & 1 & 0 \\ 1/3 & 1/2 & 1 \end{bmatrix} \begin{bmatrix} \sqrt{3} & 0 & 0 \\ 0 & \sqrt{8/3} & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \sqrt{3} & 0 & 0 \\ -\sqrt{3}/3 & \sqrt{8/3} & 0 \\ \frac{\sqrt{3}}{3} & \sqrt{2/3} & 1 \end{bmatrix}$

[ ]: