Mathematics 2, course 01034, MC problems, May 2024

Der anvendes en scoringsalgoritme, som er baseret på "One best answer"

Dette betyder følgende:
Der er altid netop ét svar som er mere rigtigt end de andre
Studerende kan kun vælge ét svar per spørgsmål
Hvert rigtigt svar giver 1 point
Hvert forkert svar giver 0 point (der benyttes IKKE negative point)

The following approach to scoring responses is implemented and is based on "One best answer"
There is always only one correct answer – a response that is more correct than the rest
Students are only able to select one answer per question
Every correct answer corresponds to 1 point
Every incorrect answer corresponds to 0 points (incorrect answers do not result in subtraction of points)

The characteristic polynomial for a 4th order homogeneous differential equation with constant coefficients is given as

$$P(\lambda) = (\lambda - 2)^2(\lambda^2 + 1)$$

Find the general real solution to the differential equation. The answer is

Choose one answer

- ○ $y(t) = c_1 e^{2t} + c_2 e^{2t} + c_3 e^{t} + c_4 e^{-t}, \quad c_1, c_2, c_3, c_4 \in \mathbb{R}$
- ○ $y(t) = c_1 + c_2 e^{2t} + c_3 \cos(t) + c_4 \sin(t), \quad c_1, c_2, c_3, c_4 \in \mathbb{R}$
- ○ $y(t) = c_1 e^{2t} + c_2 t e^{2t} + c_3 e^{t} + c_4 t e^{t}, \quad c_1, c_2, c_3, c_4 \in \mathbb{R}$
- ○ $y(t) = c_1 e^{2t} + c_2 t e^{2t} + c_3 e^{t}, \quad c_1, c_2, c_3 \in \mathbb{R}$
- ○ $y(t) = c_1 e^{2t} + c_2 t e^{2t} + c_3 \cos(t) + c_4 \sin(t), \quad c_1, c_2, c_3, c_4 \in \mathbb{R}$

Calculate the transfer function for the differential equation
$y'' + 2y' + y = u'' + 3u$
The answer is

Choose one answer

- ○ $H(s) = s^2 + 2s + 1, \quad s \neq -1$
- ○ $H(s) = \frac{s^2+2s+1}{s^2+3}, \quad s \neq \sqrt{3}$
- ○ $H(s) = \frac{s^2+2s+1}{s^2+3}, \quad s \neq \sqrt{3}i$
- ○ $H(s) = \frac{s^2+3}{s^2+2s+1}, \quad s \neq -1$
- ○ $H(s) = \frac{s^2+3s}{s^2+2s+1}, \quad s \neq \pm 1$

Consider the system of differential equations $\dot{\mathbf{x}} = A\mathbf{x}$, where

$$A = \begin{pmatrix} -2 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 2 & 1 \end{pmatrix}$$

Regarding the stability of the system, the following is true:

Choose one answer

- There is insufficient information to determine stability.
- The system is unstable.
- The system is asymptotically stable.
- The system is stable, but not asymptotically stable.

Find the radius of convergence ρ for the power series

$$\sum_{n=0}^{\infty} \frac{2+n}{2^n} x^n$$

The answer is

Choose one answer

- $\rho = 0$
- $\rho = \infty$
- $\rho = 1/2$
- $\rho = 4$
- $\rho = 2$

Consider the series

$$A = \sum_{n=1}^{\infty} \frac{\cos(n)}{n^2 + n}, \qquad B = \sum_{n=1}^{\infty} \frac{\cos(n\pi)}{\sqrt{n}}$$

For each of the two series, check whether it is divergent, conditionally convergent or absolutely convergent. The answer is

Choose one answer

- A is absolutely convergent, and B is divergent.
- Both A and B are conditional convergent.
- A is conditional convergent, and B is divergent.
- A is absolutely convergent, and B is conditional convergent.
- A is conditional convergent, and B is absolutely convergent.

Two infinite series of real numbers $\sum_{n=0}^{\infty} a_n$ and $\sum_{n=0}^{\infty} b_n$ are given. Assume that for all n, we have $0 < a_n < |b_n|$. If the infinite series $\sum_{n=0}^{\infty} b_n$ is convergent, what can we conclude about the convergence of the series $\sum_{n=0}^{\infty} a_n$?

Choose one answer

- $\sum_{n=0}^{\infty} a_n$ is divergent
- $\sum_{n=0}^{\infty} a_n$ is absolutely convergent
- $\sum_{n=0}^{\infty} a_n$ is conditional convergent
- We can not conclude anything about the convergence of $\sum_{n=0}^{\infty} a_n$

Consider the infinite series of functions

$$R(x) = \sum_{n=1}^{\infty} \frac{\cos(x) + \sin(n^2 x)}{n^2},$$

and the two infinite series of numbers given by

$$A = \sum_{n=1}^{\infty} \frac{3}{n}, \qquad B = \sum_{n=1}^{\infty} \frac{2}{n^2}$$

Which of the following statements is true?

Choose one answer

- A is a majorant series for $R(x)$, and B is a convergent majorant series for $R(x)$
- Both A and B are convergent majorant series for $R(x)$
- A is not a majorant series for $R(x)$, and B is a majorant series for $R(x)$
- $R(x)$ does not have a majorant series
- $R(x)$ has a majorant series, but non of the series A or B are the majorant series for $R(x)$

A 2π-periodic function f is known to have the Fourier series

$$f \sim \sum_{n=1}^{\infty} \left(\frac{1}{n} \cos(nx) + \frac{1}{n^2} \sin(nx) \right)$$

We now consider the Fourier series in complex form,

$$f \sim \sum_{n=-\infty}^{\infty} c_n e^{inx}$$

Determine the Fourier coefficients c_n for $n > 0$

Choose one answer

- $c_n = \frac{1}{2}(\frac{1}{-n} + \frac{i}{n^2}), \quad n > 0$
- $c_n = \frac{1}{2}(\frac{1}{-n^2} + \frac{i}{n}), \quad n > 0$
- $c_n = \frac{1}{2}(\frac{1}{n^2} + \frac{i}{n}), \quad n > 0$
- $c_n = \frac{1}{2}(\frac{1}{n} - \frac{i}{n^2}), \quad n > 0$
- $c_n = \frac{1}{2}(\frac{1}{n^2} - \frac{i}{n}), \quad n > 0$

Consider the differential equation

$$y''(t) + 2ty(t) = -5$$

By inserting the power series $y(t) = \sum_{n=0}^{\infty} c_n t^n$, we obtain

Choose one answer

- $5 + 2c_2 + \sum_{n=1}^{\infty} (c_{n+2}(n+2)(n+1) + 2c_{n-1})t^n = 0$
- $5 + 2c_0 + \sum_{n=1}^{\infty} (c_n(n+2)(n+1) + 2c_{n-1})t^n = 0$
- $5 + 2c_0 + \sum_{n=0}^{\infty} (c_{n+2}(n+2)(n+1) + 2c_{n-1})t^n = 0$
- $5 + \sum_{n=0}^{\infty} (c_{n+2}(n+2)(n+1) + 2c_{n-1})t^n = 0$
- $\sum_{n=0}^{\infty} (5 + c_{n+2}(n+2)(n+1) + 2c_{n-1})t^n = 0$