Forces 2

Problem 1

A block with mass is located on a horizontal table. The coefficient of kinematic friction between the block and the table is . The block is pulled with constant force angled with respect to the  
horizon.

Et billede, der indeholder linje/række, skærmbillede, diagram, design

Automatisk genereret beskrivelse

1. Draw a force diagram for the block accelerating towards right.

Et billede, der indeholder linje/række, diagram, Parallel, skærmbillede

Automatisk genereret beskrivelse

1. Find an expression for the acceleration of the block, as it accelerates towards the right.

We first compute the horizontal and vertical components of the force :

Then, the normal force is gravity minus upwards force:

Thus, the friction force is:

Now, the net force forward is the vertical component minus friction force:

And since :

The block is now polled with a force , so it travels with constant velocity

1. Find an expression for the force

If the velocity is constant, , thus (solving for ):

*Ligningen løses for F vha. WordMat.*

1. Determine the angle , so the force is as small as possible.

The minimum of occurs when the denominator attains its maximum value. This value is found by finding the stationary point, differentiating to and setting this expression equal to zero, solving for :

Finding the stationary point, solving for :

Problem 2

Two different masses () are connected by a rope which passes over a frictionless, massless pulley. When the system is released from rest, the masses begin to accelerate. The system is shown in the figure to the right. Shown below are four different pairs of force diagrams for the two masses. Which pair of force diagrams best describes the moving masses?

Et billede, der indeholder skitse, tegning, linje/række, clipart

Automatisk genereret beskrivelseEt billede, der indeholder diagram, skitse, Plan, linje/række

Automatisk genereret beskrivelse

Tension should be the same in whole of the string, therefore, the magnitude of the directions upwards should be the same for and . The block should of course have a downwards arrow of bigger magnitude than block because force of gravity is greater (). Therefore, it can only be .

1. A
2. B
3. **C**
4. D
5. Do not know

Problem 3

A curling stone is affected by a slanted, downwards force from a player as shown in the figure below. The curling stone slides and accelerates to the right over a rough surface with .

Et billede, der indeholder diagram, skærmbillede, design

Automatisk genereret beskrivelse

Which of the above forces should be included in a force diagram for the curling stone in the situation described? The direction and magnitude of the forces should be considered.

Curling stone is affected by gravity , normal force , friction force . Also push force is included. Length of normal force is sum of lengths of gravitational force and vertical push force. So it is quite large. Must be larger than vertical component of push force. This goes for . looks like it fits as difference in lengths between and vertical component of . Also the friction force, must not be bigger than horizontal comp. of push force, so it must be .

1. A
2. **B**
3. **C**
4. D
5. **E**
6. **F**
7. G
8. Do not know

Problem 4

A block is transported upwards on an escalator (see the figure to the right). The acceleration of the blocks and the escalator both have the magnitude and angle 45° above the horizontal. The block does not move relative to the escalator.

Et billede, der indeholder diagram, skitse, linje/række, design

Automatisk genereret beskrivelse

Which of the force diagrams of the block below are most correct regarding the direction and magnitude of the forces? The numbers in the figure indicate the magnitude of the forces in Newton.

Et billede, der indeholder diagram, linje/række, typografi

Automatisk genereret beskrivelse

As the direction of the force is , the sum of all forces should have vertical and horizontal components that are equal:

This is only the case for D:

1. A
2. B
3. C
4. **D**
5. E
6. F
7. Do not know

Problem 5

A block with mass is placed on a smooth slope. The slope angle is above the horizontal. A rope goes from the block and over a massless, frictionless pulley. The other end of the rope is fixed to the second block with mass . The system is released from rest, and the block on the slope begins to accelerate downwards.

Et billede, der indeholder linje/række, skærmbillede, Rektangel, design

Automatisk genereret beskrivelse

1. Determine the acceleration of the blocks and the magnitude of the tension.

Force on left block leftwards:

Force on right block rightwards

Tension is sum of opposite forces connected through string:

Net force considering down the slope as axis:

Total acceleration of the system:

1. Determine the acceleration of the blocks if the slope has a rough surface. The kinematic coefficient of friction between the block and the slope is .

Now we calculate friction force and subtract it from :

Net force considering down the slope as axis:

Total acceleration of the system:

Problem 6

A particle with mass , is hooked onto one end of a massless spring. The other end of the spring can freely rotate around an axis. Gravity and friction are neglected. The particle moves in a uniform circular motion with period of rotation . The radius of the movement is . The spring constant is . The situation is shown in the figure below, from the top.

Et billede, der indeholder cirkel, ur

Automatisk genereret beskrivelse

Which of the following distances is closest to the un-stretched length of the spring?

Compressed length is and length of the spring is .

When spring is un-stretched, :

Hooke’s law:

Velocity of particle:

Particle’s acceleration is given by (because constant velocity):

And:

We set up the two expression we have for the force , and solve for :

*Ligningen løses for x\_0 vha. WordMat.*

1. 0.16 m
2. 0.20 m
3. **0.84 m**
4. 0.92 m
5. 1.16 m
6. Don’t know

Problem 7

A block with mass is dragged over a rough horizontal surface. When the force pulling the block has the magnitude , the block moves with constant velocity.

Et billede, der indeholder skærmbillede, Rektangel, design, linje/række

Automatisk genereret beskrivelse

1. Which of the following statements regarding the friction force is correct?

The friction force and forward force must be equal, as the acceleration is zero because of constant velocity.

1. The friction force is greater than
2. The friction force is smaller than 𝐹
3. **The friction force has magnitude .**
4. The friction force cannot be determined as the friction coefficient is unknown.
5. Don’t know
6. The block is now pulled with a force of magnitude in the same direction as the previous problem.

What is the acceleration of the block?

New computation of net force:

New computation of acceleration:

1. Don’t know

Problem 8

A homogeneous cylindrical disc with mass and radius rotates with angular velocity with respect to the center of mass as shown in the figure. A small coin with mass is situated in the point in a distance away from . At small angular velocities, the coin does not move with respect to the disk due to static friction. The gravitational acceleration is , and the coefficient of static friction is . The period of rotation for the disk is .

Et billede, der indeholder tegning, skitse, diagram, cirkel

Automatisk genereret beskrivelse

What is the period of rotation, , when the coin begins to slide with respect to the disk?

Velocity of coin, since it is constant (acceleration given by centripetal acceleration):

Acceleration is thus,

Maximum static friction force:

Force of static friction is mass times acceleration:

*Ligningen løses for T vha. WordMat.*

1. Don’t know

Work and Energy

Problem 1

A block is situated on a horizontal smooth table. At time the block is at rest. A constant horizontal force is pushing the block forward. Which of the following figures represents the power as a function of time, as the force is acting on the block?

Et billede, der indeholder diagram, linje/række, Kurve

Automatisk genereret beskrivelse

Since, is constant (and is constant), must also be constant:

Then, we have that:

That is, the power increases linearly with time , (since acceleration is constant)!

1. **A**
2. B
3. C
4. D
5. E
6. Don’t know

Problem 2

A block with mass is placed on a smooth horizontal surface. The block is pulled with a constant force, , forming the angle with horizontal. The block starts from rest.

Et billede, der indeholder linje/række, skærmbillede, diagram, Rektangel

Automatisk genereret beskrivelse

1. Determine the speed of the block when it has moved the horizontal distance .

We first determine horizontal component of applied force , factor must be 1 when is 0 degrees and 0 when is 90 degrees:

Then, we determine the work done by on the distance :

At last, we see that due to conservation of energy, the following applies (solving for ):

*Ligningen løses for v vha. WordMat.*

1. The block is now placed at a horizontal, rough surface. The coefficient of kinematic friction between the block and the surface is . Else, the situation is as described before.

First, we compute the vertical component of applied force , factor must be 0 when is 0 degrees and 1 when is 90 degrees:

Then, we determine normal force , by gravitational force subtracted by :

The work done by the friction force is therefore:

At last, we have the following expression due to conservation of energy (there is now also some energy spent on , not just as before:

*Ligningen løses for v vha. WordMat.*

Problem 3

A block is placed on a horizontal table. The surface is rough. Another block is hanging in a string attached to the block on the table through a massless pully. The blocks have the masses and .

Et billede, der indeholder design, Rektangel, kunst

Automatisk genereret beskrivelse

The system is set in motion and moves with a constant speed. The system is observed from when it is set in motion, until it has moved the distance .

1. Determine the work done on the block placed on the table by the friction force and the tension .

Since SPEED is CONSTANT:

Since SPEED is CONSTANT:

We then compute the work, noting that works in negative direction and in positive :

1. Determine the work of the tension and gravity done on the hanging block.

Exactly the same as before, but now tension is negative direction and gravity is positive:

Problem 4

A block with mass moves in a hilly terrain, from one horizontal surface to another. The difference in height between the two horizontal surfaces are . The block leaves position 1, the first horizontal surface, with speed . The block arrives in position 2, the beginning of the second horizontal surface, with speed . From here, the block travels a distance before it, in situation 3, hits a spring which is compressed a distance . The spring constant is and the coefficient of kinematic friction between the block and the surface is .

Et billede, der indeholder diagram, linje/række, Kurve, tekst

Automatisk genereret beskrivelse

1. Determine the work done by the friction force from 1 to 2.

Conservation of mechanical energy (some is converted to work by friction at 2):

*Ligningen løses for W\_f vha. WordMat.*

This work is of course in the negative direction, so .

1. Determine the speed of the block, , immediately before the block hits the spring

First, we calculate work done by second friction force, knowing that :

Then, due to conservation of energy, kinetic energy at 2 is equal to the sum of the kinetic energy at 3 and the energy spent on work done by friction (solving for ):

*Ligningen løses for v\_3 vha. WordMat.*

1. Determine the max compression of the spring .

Force applied on spring (Hooke’s law):

Energy of compression on spring (Integral of Hooke’s law):

Spring constant unit conversion:

Work done by friction while going into spring:

Conservation of energy (kinetic energy before equals sum of compression energy and friction work):

*Ligningen løses for d vha. WordMat.*

Problem 5

A person is holding a block at rest attached to a string (situation 1). The block has mass. The person now lowers the block vertically at a constant acceleration . In total, the block is lowered a distance . Notice, the speed of the block in situation 2 is not zero.

Et billede, der indeholder diagram, linje/række, Kurve

Automatisk genereret beskrivelse

1. What is the magnitude of the tension ?

We calculate the downwards force:

We compute the tension by the gravitational force subtracted by the downwards force:

1. Don’t know
2. What is the magnitude of the work , the person has done on the block from 1 to 2?

The work the person is doing goes to the tension in the string, (the falling is done by the gravity, which the tension works by resisting):

The direction of the work done by the tension is opposite to that of the direction in which the mass is moving (thus, ):

1. Don’t know
2. What is the speed of the block when it is in situation 2?

We calculate the work performed by gravity:

And the total work performed:

Another expression for total work is total energy converted:

So, we set these two expressions equal each other and solve for :

*Ligningen løses for v vha. WordMat.*

Problem 6

A block with mass is moving over a horizontal surface. The coefficient of kinematic friction between the block and the surface is . The initial speed of the block is .

1. What is the average power the friction force is delivering to the block?

We first calculate the deceleration caused by the friction:

Then we calculate time before velocity is zero (block stops):

*Ligningen løses for t vha. WordMat.*

Then the distance before it stops (braking distance formula):

Then, we calculate the work performed by the friction over distance :

At last,we calculate the average power of the friction force over time :

Problem 7

A ball with mass is at rest on the ground. A shot putter picks up the ball and shoots it. The ball lands away from where it is shot. The point from where the ball is shot is above ground. The ball is shot at an angle with respect to horizontal.

1. Determine the speed of the ball in the moment it is shot.

Formula for projectile motion:

We replace landing coordinates with as origin, :

*Ligningen løses for v vha. WordMat.*

1. Determine the magnitude of the work done on the ball by the shot putter.

Conservation of mechanical energy, including work on left side, solving for :

Problem 8

A block is situated on a horizontal table. The surface is rough. The coefficient of kinematic friction between the block and the table is . Another block hangs in a string, which goes over a massless frictionless pully and is attached to the block on the table. The blocks have masses and .

Et billede, der indeholder design

Automatisk genereret beskrivelse

The system is released from rest and the blocks starts accelerating.

1. Determine the speed of the blocks when they have traveled the distance

Force of gravity on :

Friction force on :

Net force:

Change in kinetic energy:

Total work done by net force over distance :

Conservation of energy, work done by net force equals change in kinetic energy (solve for ):

*Ligningen løses for v vha. WordMat.*

Energy Conservation

Problem 1

A snowboarder first slides down a hill and then across a ramp, where the ramp’s angle with the hill gives her an upward velocity component. The snowboarder starts from rest at time and reaches a maximum speed of down the hill, jumping 20 meters and finishing at rest at time .

Which of the following graphs is the best representation of her jump?

Et billede, der indeholder tekst, diagram, linje/række, Kurve

Automatisk genereret beskrivelse

Snowboarder starts and ends at rest,

So A and D are disqualified.

Since, the kinetic energy starts at zero, the potential energy can NOT exceed its initial value

So B is disqualified

It can therefore only be A

1. **Graph A**
2. Graph B
3. Graph C
4. Graph D
5. Do not know

Problem 2

Et billede, der indeholder skitse, linje/række, diagram, hvid

Automatisk genereret beskrivelse

A block is initially at rest in position (1) at height above the ground. It slides down a slippery hill to the beginning of a horizontal segment (2). The horizontal segment is long and has a rough surface. At the end of the horizontal segment, the block is in position (3). After the horizontal segment, the block goes up a second slippery hill. The block reaches a height ℎ before it comes to a halt.

Consider the motion from (3) to (4) and determine the speed of the block in position (3).

1. The speed of the block at (3) is

Conservation of mechanical energy from (3) to (4), solving for :

*Ligningen løses for v\_3 vha. WordMat.*

1. Do not know
2. The friction force’s work on the horizontal segment of length is

Conservation of mechanical energy from (1) to (3), solving for :

*Ligningen løses for W vha. WordMat.*

1. Do not know

Problem 3

You are riding a sled down an icy hill. You reach a speed of at the bottom of the hill. Your friends have built a tall slippery ramp, which you launch yourself from. The ramp is horizontal just before the jump.

Et billede, der indeholder diagram, linje/række, skitse, hvid

Automatisk genereret beskrivelse

How far are you airborne in the horizontal direction from the edge of the ramp (x on the drawing)?

Conservation of mechanical energy between (1) and (2), we solve for :

*Ligningen løses for v\_1 vha. WordMat.*

This velocity is entirely horizontal because the ramp turns horizontal.

Then, we calculate amount of time airborne :

*Ligningen løses for t vha. WordMat.*

To calculate the distance, we simply take the product of time and horizontal velocity:

1. **5.0 m**
2. 7.8 m
3. 3.5 m
4. 4.2 m
5. Do not know

Problem 4

A small block, whose length can be ignored, is initially at rest in the shown position. The block is released from rest and slides down a smooth track. The center of the track follows a circular trajectory with radius .

Et billede, der indeholder skitse, cirkel, diagram, design

Automatisk genereret beskrivelse

In the following, we consider the motion of the block in the circular part of the track.

1. Make a force-diagram for the block when it is at a point .

Et billede, der indeholder ur, cirkel

Automatisk genereret beskrivelse

1. Determine the speed of the block as a function of the angle 𝜃 when it moves along the circular part of the track.

Conservation of mechanical energy from (1) to (P):

*Ligningen løses for v\_P vha. WordMat.*

1. Determine the angle when the block loses contact with the track.

Radial direction

Net radial force forward

Gravity radial component

Net radial force second definition:

The net radial force (Block loses contact when normal force is zero):

*Ligningen løses for θ vha. WordMat.*

Problem 5

A weight with mass is dropped from a height over a vertical spring with spring constant .

1. Determine the maximum compression, , of the spring.

Force applied on spring (Hooke’s law):

Energy of compression on spring (Integral of Hooke’s law):

Conservation of energy (remember to count as addition in potential energy):

*Ligningen løses for d vha. WordMat.*

Problem 6

A block is initially at rest and slides in a valley. The figure below illustrates the valley’s shape and the block’s initial position. The two horizontal segments have a kinematic coefficient of friction of . The rest of the valley’s segments are smooth.

Et billede, der indeholder diagram, linje/række, design

Automatisk genereret beskrivelse

At what height, above the short horizontal segment is the block stationary for the first time?

From (0) to (4):

We calculate the work lost to the friction (by normal force and ):

Then, due to conservation of energy:

*Ligningen løses for h vha. WordMat.*

1. (the block stops before it begins to move upwards)
2. Do not know

Problem 7

A pendulum consists of a string with length and a small weight with mass . The weight is dropped from rest at a position where the string is at angle with the vertical.

1. What is the speed of the weight when the string is parallel to the vertical for the first time?

Expression for , as factor should be 1, when pendulum is released (90 degrees) and 0 when pendulum is parallel to vertical (0 degrees). It’s NOT sinus, because it would SHRINK if the angle went above 90 degrees.

Conservation of mechanical energy:

*Ligningen løses for v vha. WordMat.*

1. How large is the tension when the string is parallel to the vertical for the first time?

We use radial direction, since the weight forms a circular motion :

Tension is as big as directional component of gravitational force:

We set the two expressions for equal, substituting and solve for :

*Ligningen løses for T vha. WordMat.*

Problem 8

A pendulum consists of a particle with mass 𝑚𝑚 at the end of a string of length . The pendulum is released from rest with and during the motion, the string hits a stationary nail. The nail is fixed a distance below the point of attachment of the string. When the string hits the nail, the now shorter pendulum rotates with the nail as the rotation center.

Et billede, der indeholder linje/række, diagram

Automatisk genereret beskrivelse

1. Show that if the pendulum is released, and is to make a full rotation, the smallest value of is equal to .

At the end of full rotation, the tension has to be zero, , thus:

The pendulum forms a circular motion around the nail with radius .

The speed at end is determined by conservation of energy (reference point for at nail):

*Ligningen løses for v vha. WordMat.*

Setting the two expressions equal and solving for :

*Ligningen løses for d vha. WordMat.*

Collisions

Problem 1

A block of mass is on a horizontal table. There is a hole in the table, through which a bullet is shot into the block. The bullet is stopped completely and becomes embedded in the block. The bullet has mass and speed . You can assume that the penetration occurs quickly.

Et billede, der indeholder skitse, linje/række, diagram, tegning

Automatisk genereret beskrivelse

1. Determine the speed of the block immediately after the collision.

Conservation of momentum:

*Ligningen løses for v\_M2 vha. WordMat.*

1. How far above the table does the block-and-bullet system travel?

Using the braking distance formula :

Using conservation of energy:

*Ligningen løses for h vha. WordMat.*

Problem 2

A particle with mass has at time a velocity of . The particle is affected by a time-dependent force 𝐹, shown in the figure below.

Et billede, der indeholder linje/række, diagram, Kurve

Automatisk genereret beskrivelse

What is the particle’s speed at time

First, we calculate the change in momentum/impulse by the area below the curve:

Then, the change in velocity:

Then the sum of initial and positive change in velocity:

1. The speed is
2. The speed is
3. The speed is
4. **The speed is**
5. The speed is
6. The speed is
7. Do not know

Problem 3

Two blocks with masses (small block) and (large block) are at rest on a smooth, horizontal table. The two blocks are connected to a compressed spring. Now, the spring expands and accelerates the two blocks.

Which of the following statements are correct?



The force applied on one is the same as that applied on the other. And, since :

Then, we have that:

And we see that the momentum gained is the same:

Also, we see that the kinetic energy is NOT the same

Thus, the kinetic energy gained for the small block is larger than for the big block:

1. **The two blocks receive the same amount of momentum from the spring**
2. The large block receives a larger amount of momentum from the spring than the small block.
3. The small block receives a larger amount of momentum from the spring than the large block.
4. The two blocks receive the same kinetic energy from the spring.
5. **The small block receives more kinetic energy from the spring than the large block.**
6. The large block receives more kinetic energy from the spring than the small block
7. Do not know

Problem 4

A ball collides with a stationary wall as shown in the figure. The ball’s mass is , and it arrives with a speed of . The collision is elastic, and there is no force of friction between the ball and the wall. A high-speed-camera measures the contact between the ball and the wall to last .

Et billede, der indeholder cirkel, skærmbillede, ur, diagram

Automatisk genereret beskrivelse

What is the magnitude of the average force, on the ball from the wall in this time-frame?

We are told that the collision is elastic (conservation of kinetic energy):

Now we calculate horizontal component of velocity:

Then, the momentum before and after collision:

Then, the change in momentum:

At last, the average force exerted by the wall for the time of contact:

1. Do not know

Problem 5

Two blocks A and B move towards each other in a straight line on a smooth table. The blocks have the same speed . Block A has mass m and moves to the right (positive direction), while block B has mass 4m and moves to the left. There is Velcro at the front of each block, ensuring that the blocks undergo a completely inelastic collision.

1. Determine the speed of the blocks after the collision.

Conservation of momentum - same velocity at last due to inelastic collision:

Solve for

*Ligningen løses for v\_AB vha. WordMat.*

1. The total kinetic energy of the blocks after the collision is the following fraction of the total kinetic energy of the blocks before the collision:

Energy before collision:

Energy after collision:

Energy fraction:

1. 0
2. Do not know

Problem 6

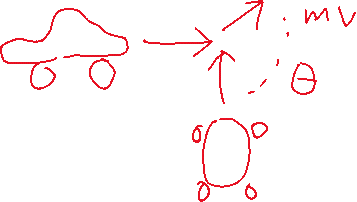
Two blocks with masses and lie on a horizontal, smooth table. A bullet with speed and mass passes completely through block . Immediately after, the bullet impacts block and thereafter travels with . After perforating , the bullet has a speed of . After the collisions, and the body consisting of and both have a speed of .

1. First collision, conservation of momentum
2. The speed of condition
3. Second collision, end up with same speed

*Ligningssystemet løses for v\_M1,v\_1,v\_2 vha. WordMat's 'Løs Ligninger' funktion,*

Problem 7

Two cars crash into each other on a road. After the collision, they stick together and move as a single body. The first car has mass 1500 kg and drives to the east with speed 25.0 m/s before the collision. The second car has mass 2500 kg and drives northwards with speed 20.0 m/s before the collision.



Conservation of momentum in :

Solving for and :

*Ligningssystemet løses for v\_x,v\_y vha. WordMat's 'Løs Ligninger' funktion,*

At last, we find find the magnitude of the velocity vector:

The angle is :

Problem 8

Two blocks A and B move towards each other in a straight line on a smooth table. The blocks have the same speed . Block A has mass and moves to the right (positive direction), while block B has mass and moves to the left. On the front of each block is a spring, which ensures that the blocks undergo an elastic collision.

The velocities of the blocks and after the collision are

Conservation of momentum (remember different signs)

Conservation of kinetic energy (remember same signs no matter what)

Solving for :

*Ligningssystemet løses for v\_A1,v\_B1 vha. WordMat's 'Løs Ligninger' funktion,*

1. **and**
2. and
3. and
4. and
5. Do not know

Problem 9

Three blocks, A, B and C can slide along a horizontal, smooth surface. The blocks have mass , and , respectively. Block A moves towards block B with a speed of and undergoes a central, elastic collision with B. Afterwards, B undergoes a central, completely inelastic collision with C.

Et billede, der indeholder linje/række, diagram, Rektangel, design

Automatisk genereret beskrivelse

What is the final speed of block C?

For the first, central elastic collision

1. Conservation of momentum (remember different signs)
2. Conservation of kinetic energy (remember same signs no matter what)
   1. Same signs, because of squaring

For the inelastic collision

1. Conservation of momentum (B and C end up with same velocity)

Solving for and :

*Ligningssystemet løses for v\_A,v\_B,v\_BC vha. WordMat's 'Løs Ligninger' funktion,*

1. Do not know

Rotation 1

Problem 1

A revolving door consists of three vertical, thin, homogeneous wings, each with mass . The length of the wings is . The revolving door is shown from above in the figure. The three wings are attached to each other and can rotate freely about a vertical rotation axis through the center of the revolving door.

Et billede, der indeholder cirkel, ur, diagram, linje/række

Automatisk genereret beskrivelse

The revolving door’s moment of inertia with respect to the rotation axis is

For a thin rectangular plate, axis along edge:

There are three doors, thus:

1. Do not know

Problem 2

A thin, homogeneous cylinder has mass 𝑀𝑀 and radius 6𝑅𝑅. A hole with radius 𝑅𝑅 is drilled into the right side of the cylinder.

Et billede, der indeholder cirkel, tegneserie

Automatisk genereret beskrivelse

What is the moment of inertia of the cylinder with the hole with respect to an axis perpendicular to the paper’s plane and going through the center of the cylinder?

For a solid cylinder, the big green circle:

We calculate the mass that would have been, by assuming same density, computing the fraction of area

For a solid cylinder, the small hole:

The moment of inertia of the hole with respect to CM of green cylinder:

At last, subtract the inertia of small hole from green cylinder:

1. Do not know

Problem 2

A thin, homogeneous rod with mass and length can rotate freely about a horizontal rotation axis, which goes through one end of the rod. The rod is released from rest in a horizontal position (0). During its rotation, the rod is at vertical position (1) at one point in time, and sometime later, it is at an angle with the vertical (2).

Et billede, der indeholder linje/række

Automatisk genereret beskrivelse

1. Determine the angular velocity, , when the rod is vertical (1).

For a thin rectangular plate, axis along edge:

Conservation of energy (solving for :

*Ligningen løses for ω vha. WordMat.*

1. Determine the speed of the center of mass, , of the rod when it is at to the vertical (2).

The potential energy left at :

The rotational energy:

Conservation of energy (solving for ):

*Ligningen løses for ω vha. WordMat.*

At last, calculating :

Problem 4

A body, which looks circular from the side, starts from rest (1) and rolls down a slope to point (2). At position (2), the body’s speed is determined to be between and . The mass of the body is and its radius is . The difference in height between (1) and (2) is .

Et billede, der indeholder diagram

Automatisk genereret beskrivelse

Which of the following shapes can the body have?

Conservation of energy:

For :

*Ligningen løses for I vha. WordMat.*

*Ligningen blev løst med GeoGebra:*

For :

*Ligningen løses for I vha. WordMat.*

Thus, we get that

1. A thin ring
2. **A thick ring with inner radius**
3. A cylinder
4. A solid sphere
5. **A hollow, thin-walled sphere**
6. Do not know

Problem 5

A homogeneous cylinder with radius and mass is on a horizontal, rough surface. The cylinder is pulled by a constant, horizontal force , acting on the cylinder’s center of mass. The cylinder starts from rest and rolls on the substrate when the force is applied.

Et billede, der indeholder cirkel, skærmbillede, diagram, linje/række

Automatisk genereret beskrivelse

1. Determine the cylinder’s angular velocity when its center of mass has been displaced by .

Inertia for solid cylinder:

Definition of work:

Calculate change in kinetic energy (geometric rolling relation ):

Work-energy theorem (solve for ):

*Ligningen løses for ω vha. WordMat.*

The same constant force is now applied on the cylinder’s vertex.

Et billede, der indeholder skærmbillede, ur, cirkel, diagram

Automatisk genereret beskrivelse

1. Determine the cylinder’s angular velocity when its center of mass has been displaced by .

Et billede, der indeholder tekst, diagram, skærmbillede, cirkel

Automatisk genereret beskrivelse

Now, this is the same as displacing by instead of as before:

*Ligningen løses for ω vha. WordMat.*

Problem 6

A homogeneous cylinder with radius and mass can rotate freely about a horizontal axis of rotation through its center. A rope is coiled around the cylinder, which does not move relative to the cylinder. A block with mass hangs from the other end of the rope. The rope passes over a nail (no friction). The system is released from rest.

Et billede, der indeholder skærmbillede, ur, cirkel, diagram

Automatisk genereret beskrivelse

1. Determine the angular velocity of the cylinder when the small block has moved a distance ℎ.

Inertia for solid cylinder:

Conservation of mechanical energy

Due to the geometrical constraint between velocity and angular velocity :

*Ligningen løses for ω vha. WordMat.*

Work-energy theorem (solve for ):

*Ligningen løses for ω vha. WordMat.*

Rotation 2

Exercise 2

A cylinder with mass , radius and moment of inertia with respect to a rotational axis through its center of mass lies on a rough slope with inclination to the horizontal. The cylinder is connected to a block with mass via a cord running over a massless pulley. The block lies on the right slope, which is smooth and inclined with to the horizontal.

Et billede, der indeholder linje/række, trekant, diagram

Automatisk genereret beskrivelse

Now, the cylinder is released, and it begins to roll down the left slope without sliding.

1. Make a force-body diagram for the cylinder and the block, respectively.

Et billede, der indeholder diagram, tegning, skitse, linje/række

Automatisk genereret beskrivelse

1. Determine the acceleration of the cylinder.

Inertia

Equation of torque:

Parallel gravitational forces:

Net force:

Expression 1:

Expression 2:

Solve the two expressions of net force for :

*Ligningen løses for a vha. WordMat.*

1. Determine the speed of the block when it reaches the top of the slope.

There exist two formulas for the relation between the height of one of the two entities and length traveled (equal length of cord moving over the pulley - is the same).

In this scenario, is at the top, so:

Now, we express as a function of :

Thus, the potential energy in the system becomes

Conservation of energy (solve for ):

*Ligningen løses for v vha. WordMat. med følgende antagelser/definitioner:*

Exercise 4

A solid sphere with radius and mass lies at rest on a table. A bullet with mass and velocity in the horizontal direction pierces the sphere in its center. When the bullet leaves the sphere, its velocity has been halved. The sphere is left intact by bullet, i.e. it can still be modelled as solid. The moment of inertia for a homogeneous sphere with mass and radius with respect to a horizontal rotational axis through its center of mass is .

1. Determine the sphere’s velocity,, right after the bullet leaves it.

Momentum is preserved.

*Ligningen løses for v\_M2 vha. WordMat.*

There is friction between the table and the sphere; the coefficients of friction are

1. Show that the sphere’s angular velocity (at a time after the penetration) is given by .

We calculate friction force with given coefficient and normal force (opposite to gravitational):

Then, we calculate angular acceleration with the found friction force:

At last, we calculate angular velocity:

1. Find how much time passes before the sphere rolls without slipping.

The sphere begins to roll without slipping when .

*Ligningen løses for t vha. WordMat.*