Problem 1

Project.

a. $T_0 = 4$, $\omega_0 = \frac{2\pi}{T_0} = \frac{\pi}{2}$

The Fourier Series for the signal.

$$f(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}.$$

where

$$a_k = \frac{1}{T_0} \int_{T_0} f(t) \cdot e^{-jk\omega_0 t} dt.$$

$$= \frac{1}{4} \left[\int_{-2}^{0} (-1) e^{-jk\frac{\pi}{2}t} dt + \int_{0}^{2} e^{-jk\frac{\pi}{2}t} dt \right]$$

$$= \frac{1}{4 \cdot (-jk\frac{\pi}{2})} \left[-e^{-jk\frac{\pi}{2}t} \Big|_{-2}^{0} + e^{-jk\frac{\pi}{2}t} \Big|_{0}^{2} \right]$$

$$= \frac{j}{2k\pi} \cdot \left[-(1 - e^{jk\pi}) + (e^{-jk\pi} - 1) \right]$$

$$= \frac{j}{2k\pi} \left[e^{jk\pi} + e^{-jk\pi} - 2 \right]$$

$$= \frac{j}{k\pi} \left[\cos(k\pi - 1) \right], \text{ and } a_0 = 0$$

Problem 2.

The differential equation for the circuit is:

$$f(t) = y(t) + RC \cdot \frac{d\,y(t)}{dt} \qquad (1)$$

In order to determine its frequency response $H(j\omega)$, we note that, by definition, with input voltage $f(t) = e^{j\omega t}$

thus, $y(t) = H(j\omega) \cdot e^{j\omega t}$.

$$(1) \Rightarrow e^{j\omega t} = H(j\omega) e^{j\omega t} + R \cdot C \cdot j\omega H(j\omega) \cdot e^{j\omega t}$$

$$H(j\omega) = \frac{1}{1 + RC \cdot j\omega}$$