10/24 #1

Input $x(t)$ $\longrightarrow$ $\boxed{h(t)}$ (given) $\longrightarrow$ $y(t) = x(t) * h(t)$?

② ? ①

last time. →

Input $X(j\omega)$ $\longrightarrow$ $\boxed{H(j\omega)}$ $\longrightarrow$ $Y(j\omega) = X(j\omega) \cdot H(j\omega)$

① $\underline{H(j\omega)} = \int_{-\infty}^{\infty} h(\tau) e^{-j\omega\tau} d\tau$

② $\cancel{X(j\omega) =}$ $x(t) = \sum_k a_k \underline{e^{j\omega_k t}}$

$$e^{j\omega_k t} \longrightarrow \boxed{H(j\omega)} \longrightarrow H(j\omega_k) e^{j\omega_k t} \cdots$$

$$x(t) = \sum_k \underbrace{a_k}_{\text{coeff.}} e^{j\omega_k t} \longrightarrow \boxed{H(j\omega)} \longrightarrow y(t) = \sum_k \underbrace{a_k \overbrace{H(j\omega_k)}^{\text{known from } H(j\omega)}}_{\text{coeff.}} e^{j\omega_k t}$$

Review of Example from last lecture.

$$h(t) = \delta(t) + 5e^{-5t}u(t) \Leftarrow \text{given}$$

$$\Downarrow$$

$$H(j\omega) = \frac{10 + j\omega}{5 + j\omega}$$

Input $e^{j100t} \rightarrow H(j100) = \dfrac{10 + j100}{5 + j100}$ ← when $\omega = 100$.

output $e^{j100t} \cdot H(j100)$

10/24

$x(t) = \Sigma_k a_k e^{j\omega_k t}$

What is a_k ?

periodis signals: $x(t) = x(t+T)$ for all t.

The period is T. frequeny is $f: \frac{1}{T}$

$\omega: \frac{2\pi}{T}$.

harmoics are the frequeny with $\underset{\substack{\uparrow \\ \text{integer}}}{k}\omega$.

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega t} \quad \leftarrow \text{sythesis}$$

$k=0$ a_0 $\leftarrow$ DC gain

$k=\pm 1$ fundamental frequeny

$k=\pm 2$ 2nd harmonic

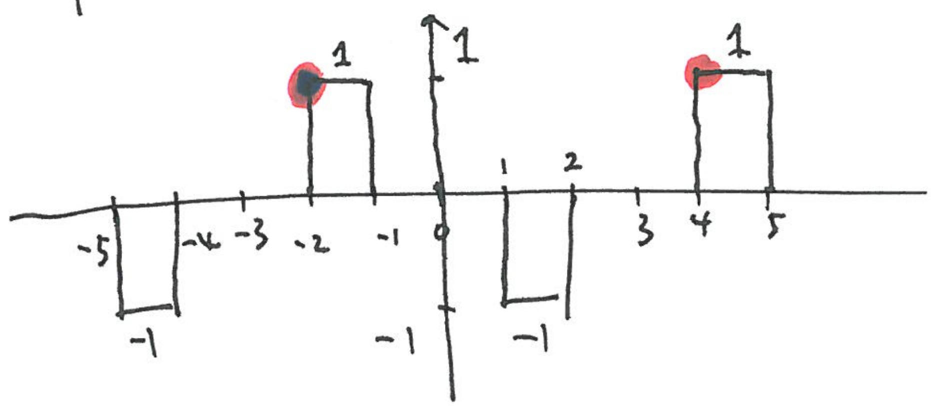
$\vdots$

a_k: Fourier series coeff.

$$a_k = \frac{1}{T}\underbrace{\int_T}_{\text{integral over one period}} x(t) e^{-jk\omega t} dt \quad \leftarrow \text{analysis}.$$

10/24 #3.

Example: Find the Fourier series of



period $= T = 6$ Frequency $= \omega_0 = \frac{2\pi}{6} = \frac{\pi}{3}$

$$x(t) = x(t+T)$$

Calculate a_k.

$$a_k = \frac{1}{T}\int_T x(t)e^{-jk\omega_0 t}dt$$

$$= \frac{1}{6}\int_{-3}^{3} x(t)e^{-jk\omega_0 t}dt$$

$$= \frac{1}{6}\int_{-2}^{-1} 1\, e^{-jk\omega_0 t}dt + \frac{1}{6}\int_{1}^{2}(-1)\,e^{-jk\omega_0 t}dt$$

$$= \frac{1}{6}\int_{-2}^{-1} e^{-jk\frac{\pi}{3}t}dt - \frac{1}{6}\int_{1}^{2} e^{-jk\frac{\pi}{3}t}dt$$

$$= \frac{1}{6}\frac{1}{-jk\frac{\pi}{3}}e^{-jk\frac{\pi}{3}t}\Big|_{-2}^{-1} - \frac{1}{6}\frac{1}{-jk\frac{\pi}{3}}e^{-jk\frac{\pi}{3}t}\Big|_{1}^{2}$$

$$= \frac{1}{-j2k\pi}\left(e^{jk\frac{\pi}{3}} - e^{jk\frac{2\pi}{3}}\right) - \frac{1}{-j2k\pi}\left(e^{-jk\frac{2\pi}{3}} - e^{-jk\frac{\pi}{3}}\right)$$

$$= -\frac{1}{j2k\pi}\left[\underline{e^{jk\frac{\pi}{3}}} - e^{jk\frac{2\pi}{3}} - e^{-jk\frac{2\pi}{3}} + \underline{e^{-jk\frac{\pi}{3}}}\right]$$

10/24

#4

$$= -\frac{1}{j2k\pi}\left[2\cos\left(k\frac{\pi}{3}\right) - 2\cos\left(k\frac{2\pi}{3}\right)\right]$$

$$= -\frac{1}{jk\pi}\left[\cos\left(k\frac{\pi}{3}\right) - \cos\left(k\frac{2\pi}{3}\right)\right]$$

$$= \frac{1}{jk\pi}\left[\cos\left(k\frac{2\pi}{3}\right) - \cos\left(k\frac{\pi}{3}\right)\right]$$

Example: $x(t) = (1+\cos 2\pi t)\sin\left(10\pi t + \frac{\pi}{6}\right)$

Find the Fourier Series Coeff. a_k.

Period $T = 1$ Frequency $\underline{\omega_0 = 2\pi}$

$$x(t) = \left(1 + \frac{e^{j2\pi t} + e^{-j2\pi t}}{2}\right)\frac{e^{j(10\pi t + \frac{\pi}{6})} - e^{-j(10\pi t + \frac{\pi}{6})}}{2j}$$

$$= \left(1 + \frac{1}{2}e^{j2\pi t} + \frac{1}{2}e^{-j2\pi t}\right)\left(\frac{e^{j\frac{\pi}{6}}}{2j}\cdot e^{j10\pi t} - \frac{e^{-j\frac{\pi}{6}}}{2j}e^{-j10\pi t}\right)$$

$$= \underbrace{\frac{e^{j\frac{\pi}{6}}}{2j}}_{a_5}e^{j2\pi t\times 5} \underbrace{- \frac{e^{-j\frac{\pi}{6}}}{2j}}_{a_{-5}}e^{j2\pi t\times(-5)} +$$

$$\underbrace{\frac{e^{j\frac{\pi}{6}}}{4j}}_{a_6}e^{j2\pi t\times 6} \underbrace{- \frac{e^{-j\frac{\pi}{6}}}{4j}}_{a_{-4}}e^{j2\pi t\times(-4)} +$$

$$\underbrace{\frac{e^{j\frac{\pi}{6}}}{4j}}_{a_4}e^{j2\pi t\times 4} \underbrace{- \frac{e^{-j\frac{\pi}{6}}}{4j}}_{a_{-6}}e^{j2\pi t\times(-6)}$$