$|a|-|b| \le |a-b|$

$y \quad x \quad y-x.$

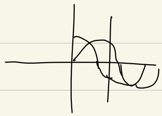
Solve $z^5 = 1+i$

$= \sqrt{2}\left(\frac{1}{\sqrt{2}}+\frac{i}{\sqrt{2}}\right)$

$= \sqrt{2}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right).$

$= \sqrt{2}\, e^{(\frac{\pi}{4}+2\pi k)i}$

$z = 2^{\frac{1}{10}} e^{\frac{1}{5}i(\frac{\pi}{4}+2\pi k)}$ $\quad k=0,1,\dots,4$



Solve $z^4 = 1-\sqrt{3}\,i$

$= 2\left(\frac{1}{2}-\frac{\sqrt{3}}{2}i\right)$

$= 2\left(\cos\frac{5}{3}\pi - i\sin\frac{5}{3}\pi\right)$

$= 2\cdot e^{(\frac{5}{3}\pi+2\pi k)i}$

$z = 2^{\frac{1}{4}}\cdot e^{\frac{1}{4}(\frac{5}{3}\pi+2\pi k)i}$ $\quad k=0,1,2,3.$

$f(z)=z+2z^2 \Rightarrow \lim_{z\to i} f(z) = i-2$

$1 \quad i$

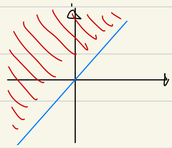
$2 \quad -2i-1$

if $\forall\varepsilon \ge 0,\ \exists\delta$

s.t $|z-i|<\delta \Rightarrow \left|z+2z^2-i+2\right|<\varepsilon$

$|z+i||2z-(2i+1)| \le |z+i|(|z|+5) <$

$A=\{z=x+iy \mid x<y\}$ open 임을 보여라



Let $z\in A$ and $z=x+yi$, $y-x>0$.

$\delta = \frac{1}{\sqrt{2}}(y-x)>0 \rightarrow \sqrt{2}\,\delta = y-x.$ $\quad \sqrt{2}|z-w|<\sqrt{2}\delta$

if $w\in B_\delta(z)$ then $|z-w|<\delta$, $w=a+bi$

$|Im(z-w)|-|Re(z-w)| \le |z-w| \le \sqrt{2}|z-w| < \sqrt{2}\delta$ $=y-x.$

$(y-b)-(x-a) = Im(z-w)-Re(z-w) < y-x.$

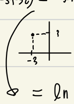
$\therefore b-a>0.$

$\therefore w\in A$

A is open.

* $\log(-3+3i)=?$

$\log(-3+3i) = \ln|-3+3i| + i\arg(-3+3i)$



$= \ln 18 + i\,\frac{3}{4}\pi.$

$i^{i+1} = e^{(i+1)\log i}$

$= e^{(i+1)(\ln|i|+i\arg(i))}$

$= e^{(i+1)\frac{\pi}{2}}.$

$\arg(i) \rightarrow$

$=\frac{\pi}{2}+2n\pi.$

$u_x = v_y$, $u_y = -v_x$

$f : \mathbb{C} \to \mathbb{C}$ $\quad z = x + iy$

$f(z) = u(z) + iv(z)$

"$\Rightarrow$"는 성립
"$\Leftarrow$"는 성립X

THM Suppose f is complex diff at z_0

Then $\frac{\partial u}{\partial x}(z_0) = \frac{\partial v}{\partial y}(z_0)$ $\quad \frac{\partial u}{\partial y}(z_0) = -\frac{\partial v}{\partial x}(z_0)$
($\Rightarrow$)

Cauchy-Riemann equation

Def) $\Omega \subset \mathbb{C}$ is called domain if it is open and connected set

Thm) Let Ω be a domain.

$f : \Omega \to \mathbb{C}$ analytic and $f'(z) = 0 \quad \forall z \in \Omega$

$\Longrightarrow f$: constant.

정의역이 $\mathbb{C}$일때 $f'(z)=0$인 $f(z)$는 상수함수이다.