$|a|-|b| \le |a-b|$

$y \qquad x \qquad y-x.$

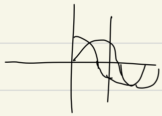
Solve $z^5 = 1+i$

$= \sqrt{2}\left(\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}\right)$

$= \sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right).$

$= \sqrt{2}\, e^{(\frac{\pi}{4}+2\pi k)i}$

$z = 2^{\frac{1}{10}} e^{\frac{1}{5}i(\frac{\pi}{4}+2\pi k)} \quad k = 0, 1, \cdots, 4$



Solve $z^4 = 1-\sqrt{3}\, i$

$= 2\left(\frac{1}{2} - \frac{\sqrt{3}}{2} i\right)$

$= 2\left(\cos\frac{5}{3}\pi - i\sin\frac{5}{3}\pi\right)$

$= 2\cdot e^{(\frac{5}{3}\pi + 2\pi k)i}$

$z = 2^{\frac{1}{4}}\cdot e^{\frac{1}{4}(\frac{5}{3}\pi + 2\pi k)i} \quad k = 0, 1, 2, 3.$

$f(z) = z + 2z^2 \Rightarrow \lim_{z\to i} f(z) = i-2$

$\qquad 1 \qquad i$

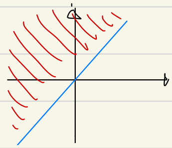
$\qquad 2 \qquad -2i-1$

if $^\forall \varepsilon \ge 0,\ \exists\, \delta$

s.t $|z-i| < \delta \Rightarrow \left|z+2z^2 - i + 2\right| < \varepsilon$

$|z+i|\,|2z-(2i+1)| \le |z+i|\left(|z|+5\right) <$

$A = \{ z = x+iy \mid x<y \}$ open 임을 보여라



Let $z \in A$ and $z = x+yi$, $y-x>0$.

$\delta = \frac{1}{\sqrt{2}}(y-x) > 0 \to \sqrt{2}\,\delta = y-x.$

$\sqrt{2}|z-w| < \sqrt{2}\delta$

if $w \in B_\delta(z)$ then $|z-w| < \delta$, $w = a+bi$

$|Im(z-w)| - |Re(z-w)| \le |z-w| \le \sqrt{2}|z-w| < \sqrt{2}\delta$

$= y-x.$

$(y-b)-(x-a) = Im(z-w) - Re(z-w) < y-x.$

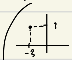
$\therefore b-a>0.$

$\therefore w \in A$

A is open.

$\log(-3+3i) = ?$

$\log(-3+3i) = \ln|-3+3i| + i\arg(-3+3i)$



$= \ln\sqrt{18} + i\,\frac{3}{4}\pi.$

$f: \mathbb{C} \to \mathbb{C} \quad f(z) = u(z) + iv(z)$

Def) $f(z) \to w_0$ as $z \to z_0$ if $^\forall \varepsilon \ge 0,\ \exists\, \delta > 0$

s.t $|z-z_0| < \delta \Rightarrow |f(z) - w_0| < \varepsilon$

e.g) $\lim_{z\to i}(z^2+2z) = -1+2i \quad \left(\lim_{z\to z_0} f(z) = w_0\right)$

pf) Given $\varepsilon > 0$ if $|z-i| < \delta$ and $|z-i| < 1$

then $\left|z^2+2z+1-2i\right| = |z-i|\,|z+i-2|$

$\le |z-i|\left(|z|+3\right) < 5\delta \le \varepsilon$