analytic ; talyor 급수한 함수와 원함수가 같을 때.

Todo ; 연습문제 풀어보기.

1장 ; 복소수와 복소평면.

1.1. 복소수계

$i = \sqrt{-1}$ $(i^2 = -1)$.

$z = x + iy$ $(x, y \in \mathbb{R})$

$x = \mathrm{Re}\, z$ $y = \mathrm{Im}\, z$

ex) $z = 2 + 3i$, $\mathrm{Re}\, z = 2$, $\mathrm{Im}\, z = 3$.

이러한 $z \in \mathbb{C}$ 이다.

Thm 1.1. $\mathbb{C}$는 결합, 분배, 교환법칙이 성립.

절대값 ; $|z| = |x + yi| = \sqrt{(x+iy)(x-iy)} = \sqrt{z\bar{z}}$

켤레복소수 : $z = x + iy$, $\bar{z} = x - iy$.

$z^{-1} = \frac{1}{z} = \frac{\bar{z}}{z\bar{z}} = \frac{x-iy}{x^2+y^2}$

연습문제 4.

(1) $z^4 = -1$

$z^2 = \pm i$

$z = \pm\sqrt{i}$ or $\pm\sqrt{-i}$

(3) $z^2 = i$

$z = \pm\sqrt{i}$

$z = x + iy$

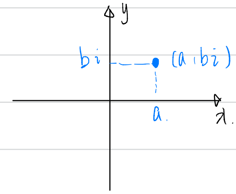
$z^2 = x^2 - y^2 + 2i\,xy = i$

$x^2 - y^2 = 0$ $xy = \frac{1}{2}$

$x^2 = y^2$ $x = \frac{1}{2y}$

$x = \pm y$

1.2 복소평면과 극형식.



$z_1 = x_1 + iy_1$, $z_2 = x_2 + iy_2$.

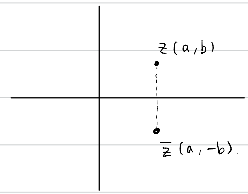
$z_1 + z_2 = (x_1 + x_2) + i(y_1 + y_2)$.

이는 두 벡터의 합의 계산과 같다.

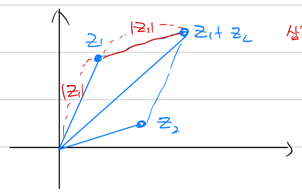
$\left(\vec{a} = (x_1, y_1) \quad \vec{b} = (x_2, y_2) \quad \vec{a} + \vec{b} = (x_1 + x_2, y_1 + y_2) \right)$.

$|z|$ 은 원점으로부터의 거리.

$\|\vec{a}\|$ 는 원점으로부터의 거리.



정리 1.2 절대값의 기본성질.



삼각부등식 ; $|z_1 + z_2| \le |z_1| + |z_2|$

*

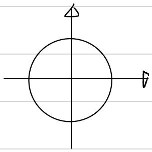
복소평면에서의 직선.

$Re((k_1+i)z+k_2)=0$ or $Re((k_1+i)z)=-k_2$

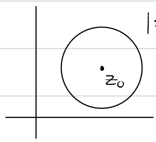
$z=x+iy$, $k_1, k_2 \in \mathbb{R}$

$Re(az+b)=0$. or $Re(az)=k$

복소평면에서의 원.



$|z|=r$



$|z-z_0|=r$.

예제. 1.2.

$|z+i| = \frac{1}{2}|z-1|$ 을 복소평면에 나타내라.

sol1) $z=x+iy$.

$|x+iy+i| = \frac{1}{2}|x+iy-1|$.

$|x+i(y+1)| = \frac{1}{2}|(x-1)+iy|$

$\sqrt{(x+i(y+1))(x-i(y+1))} = \frac{1}{2}\sqrt{((x-1)+iy)((x-1)-iy)}$

$x^2+(y+1)^2 = \frac{1}{4}((x-1)^2+y^2)$

$4x^2+4y^2+8y+4 = x^2-2x+1+y^2$.

$3x^2+3y^2+2x+8y+3=0$.

$(x+\frac{1}{3})^2+(y+\frac{4}{3})^2 = \frac{8}{9}$

Sol 2) 내분점의 원리 이용.

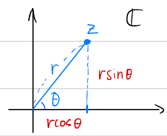
복소수의 극형식.

$z=x+iy$ $x,y \in \mathbb{R}$

$|z| = \sqrt{x^2+y^2} = r$

$z=|z|\left(\frac{x+iy}{|z|}\right) = |z|(\cos\theta+i\sin\theta)$

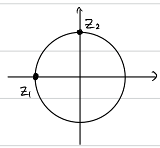
$\frac{x}{|z|}=\cos\theta$ $\frac{y}{|z|}=\sin\theta$. $\underbrace{(-\pi\le\theta\le\pi)}_{\text{주편각}}$.



Arg z

$\arg z = Arg + 2k\pi$.

예제 1.3.



$Arg(z_1z_2) = Arg z_1 + Arg z_2$

정리 1.3. Euler's Identity.

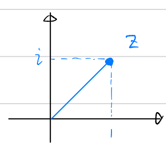
$e^{i\theta} = \cos\theta + i\sin\theta$.

$\theta=\pi \to e^{i\pi}=-1$

$z=r(\cos\theta+i\sin\theta)$.

$=re^{i\theta} = re^{i\arg z}$

예제 1.4. $z=1+i$ $Arg(z)=\frac{\pi}{4}$ $arg(z)=\frac{\pi}{4}+2k\pi$.



곱셈. $z_1 = r_1 e^{i\theta_1}$ $z_2 = r_2 e^{i\theta_2}$

$z_1 \cdot z_2 = r_1 r_2 e^{i\theta_1 + i\theta_2}$

나눗셈 $\frac{z_1}{z_2} = \frac{r_1}{r_2} e^{i(\theta_1 - \theta_2)}$

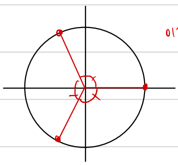
$z^n = r^n(\cos n\theta + i\sin n\theta)$

복소수의 제곱근 .

$z^n = z_0$ 를 만족하는 z를 z_0의 "n-제곱근"

$z = \sqrt[n]{z_0}$ or $z_0^{\frac{1}{n}}$

ex) $z^3 = 1$



이점들이 z^3의 해이다 .

무한점(∞)과 (확장 복소수계).