Solve $z^5 = 1+i$

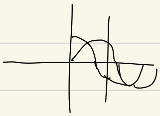
$= \sqrt{2}\left(\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}\right)$

$= \sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right).$

$= \sqrt{2}\, e^{(\frac{\pi}{4}+2\pi k)i}$

$z = 2^{\frac{1}{10}}\, e^{\frac{1}{5}i(\frac{\pi}{4}+2\pi k)}$ $k = 0, 1, \cdots, 4$

Solve $z^4 = 1 - \sqrt{3}\, i$



$= 2\left(\frac{1}{2} - \frac{\sqrt{3}}{2} i\right)$

$= 2\left(\cos\frac{5}{3}\pi - i\sin\frac{5}{3}\pi\right)$

$= 2 \cdot e^{(\frac{5}{3}\pi + 2\pi k)i}$

$z = 2^{\frac{1}{4}} \cdot e^{\frac{1}{4}(\frac{5}{3}\pi + 2\pi k)i}$ $k = 0, 1, 2, 3.$

$f(z) = z + 2z^2 \Rightarrow \lim_{z\to i} f(z) = i - 2$

if $^{\forall}\varepsilon \geq 0$, $\exists\, \delta$

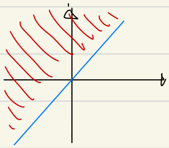
$1 \quad i$

$2 \quad -2i-1$

s.t $|z - i| < \delta \Rightarrow |z + 2z^2 - i + 2| < \varepsilon$

$|z+i||2z - (2i+1)| \leq |z+i|(|z|+5) <$

A = { z = x + iy | x < y } open 임을 보여라



$\delta = \frac{|Re z - Im z|}{\sqrt{2}}$

if $w = B_\delta(z) \Rightarrow |z - w| < \delta$

$z = x + yi$ $w = a + bi$

$(x-a)^2 + (y-b)^2 < \frac{(x-y)^2}{2}.$

$(x-a)^2 < \frac{(x-y)^2}{2}$

$x - a < \frac{x-y}{\sqrt{2}}$

$f: \mathbb{C} \to \mathbb{C}$ $f(z) = u(z) + iv(z)$

Def) $f(z) \to w_0$ as $z \to z_0$ if $^{\forall}\varepsilon \geq 0$, $\exists\, \delta > 0$

s.t $|z - z_0| < \delta \Rightarrow |f(z) - w_0| < \varepsilon$

e.g) $\lim_{z\to i}(z^2 + 2z) = -1 + 2i$ $\left(\lim_{z\to z_0} f(z) = w_0\right)$

pf) Given $\varepsilon > 0$ if $|z - i| < \delta$ and $|z - i| < 1$

then $|z^2 + 2z + 1 - 2i| = |z - i||z + i - 2|$

$\leq |z - i|(|z| + 3) < 5\delta \leq \varepsilon$