

(Report 문제)

$$(1) (x-2y+5)dx + (2x-y+4)dy = 0$$

$$(2) (2x-y+2)dx + (4x-2y-1)dy = 0$$

$$(3) (4x+3y-4)dx + (3x-7y-3)dy = 0$$

$$(4) (2x+y)dx - (4x+2y-1)dy = 0$$

sol)

$$(1) (x-2y+5)dx + (2x-y+4)dy = 0$$

$$x-2y+5=0$$

$$2x-y+4=0$$

$$\begin{vmatrix} 1 & -2 \\ 2 & -1 \end{vmatrix} = -1 - (-4) \neq 0$$

$$2x-4y+10=0$$

$$2x-y+4=0$$

$$3y = 6 \quad y=2 \quad x = -1$$

$$x = X - 1 \quad y = Y + 2$$

$$dx = dX \quad dy = dY$$

$$(X-2Y)dX + (2X-Y)dY = 0$$

$$(X-2Y) + \frac{dY}{dX}(2X-Y) = 0$$

$$(1 - \frac{2Y}{X}) + \frac{dY}{dX}(2 - \frac{Y}{X}) = 0$$

$$\frac{Y}{X} = t \quad Y = Xt \quad \frac{dY}{dX} = t + \frac{dt}{dX}$$

$$(1-2t) + (t + \frac{dt}{dX})(2-t) = 0$$

$$t + \frac{dt}{dX} = -\frac{1-2t}{2-t}$$

$$\frac{dt}{dX} = -\frac{1-2t+2-t}{2-t} = -\frac{3-3t}{2-t}$$

$$\frac{dX}{dt} = \frac{2-t}{3t-3}$$

$$dX = \frac{2-t}{3t-3} dt$$

$$\int 1 \cdot dX = \int \frac{2-t}{3t-3} dt$$

$$X = -\frac{1}{3} \int \frac{t-2}{t-1} dt = -\frac{1}{3} \int (1 - \frac{1}{t-1}) dt$$

$$= -\frac{1}{3} (t - \ln(t-1)) + C$$

$$X = -\frac{1}{3} (\frac{Y}{X} - \ln(\frac{Y}{X}-1)) + C$$

$$X+1 = -\frac{1}{3} (\frac{Y-2}{X+1} \ln(\frac{Y-2}{X+1}-1)) + C$$

(여기서부터가
2차분할때)

$$(2) (2x-y+2)dx + (4x-2y-1)dy = 0$$

$$\begin{cases} 2x-y+2=0 \\ 4x-2y-1=0 \end{cases}$$

$$\begin{vmatrix} 2 & -1 \\ 4 & -2 \end{vmatrix} = -4 - (-4) = 0$$

$$2x-y = t$$

$$2 - \frac{dy}{dx} = \frac{dt}{dx} \quad \frac{dy}{dx} = 2 - \frac{dt}{dx}$$

$$(2x-y+2) + (4x-2y-1) \frac{dy}{dx} = 0$$

$$(t+2) + (2t-1)(2 - \frac{dt}{dx}) = 0$$

$$2 - \frac{dt}{dx} = -\frac{t+2}{2t-1}$$

$$\frac{dt}{dx} = \frac{5t}{2t-1}$$

$$dx = \frac{2t-1}{5t} dt$$

$$\int 1 \cdot dx = \int \frac{2}{5} (\frac{t-1}{t}) dt$$

$$x = \int \frac{2}{5} (1 - \frac{1}{2t}) dt = \frac{2}{5} t - \frac{1}{2} \ln t + C$$

$$x = \frac{2}{5} (2x-y) - \frac{1}{2} \ln(2x-y) + C$$

$$(3) (4x+3y-4)dx + (3x-7y-3)dy = 0$$

$$4x+3y-4=0$$

$$3x-7y-3=0$$

$$\begin{vmatrix} 4 & 3 \\ 3 & -7 \end{vmatrix} = -28-9 \neq 0$$

$$\begin{cases} 4x+3y-4=0 \\ 3x-7y-3=0 \end{cases} \quad \begin{cases} 12x+9y-12=0 \\ 12x-28y-12=0 \end{cases}$$

$$3x-7y-3=0$$

$$12x-28y-12=0$$

$$x=1 \quad y=0$$

$$x = a-1 \quad y = b$$

$$dx = da \quad dy = db$$

$$(4a+3b)da + (3a-7b)db = 0$$

$$(4a+3b) + (3a-7b) \frac{db}{da} = 0$$

$$(4 + \frac{3b}{a}) + (3 - \frac{7b}{a}) \frac{db}{da} = 0$$

$$\frac{b}{a} = t \quad b = at \quad \frac{db}{da} = t + a \frac{dt}{da}$$

$$(4+3t) + (3-7t)(t + a \frac{dt}{da}) = 0$$

$$t + a \frac{dt}{da} = \frac{4+3t}{3-7t} \quad a \frac{dt}{da} = \frac{1+10t}{3-7t}$$

$$a \frac{dt}{da} = \frac{1+10t}{3-10t}$$

$$\frac{1}{a} da = \frac{3-10t}{1+10t} dt \quad 10t = (k-1) \frac{1}{10} - 3$$

$$\int \frac{1}{a} da = \int \frac{3-10t}{1+10t} dt \quad 1+10t = k \quad 10dt = dk.$$

$$= \int \frac{1}{k} \left((1-k) \frac{1}{10} + 3 \right) \frac{1}{10} dk.$$

$$= \int \frac{1}{k} \left(\frac{3}{10} - \frac{1}{10} k \right) \frac{1}{10} dk$$

$$= \frac{3}{100} \ln k - \frac{1}{100} k + C$$

$$= \frac{3}{100} \ln(1+10t) - \frac{1}{100}(1+10t) + C$$

$$= \frac{3}{100} \ln\left(1+10\left(\frac{k}{a}\right)\right) - \frac{1}{100}\left(1+10\left(\frac{k}{a}\right)\right) + C$$

$$= \frac{3}{100} \ln\left(1+10\left(\frac{y}{x+1}\right)\right) - \frac{1}{100}\left(1+10\left(\frac{y}{x+1}\right)\right) + C$$

$$(4) (2x+y) dx - (4x+2y-1) dy = 0$$

$$2x+y=0$$

$$-4x-2y+1=0$$

$$\begin{vmatrix} 2 & 1 \\ -4 & -2 \end{vmatrix} = -4 - (-4) = 0$$

$$2x+y=t \quad 2 + \frac{dy}{dx} = \frac{dt}{dx}$$

$$t dx - (2t-1) dy = 0$$

$$t - (2t-1) \frac{dy}{dx} = 0$$

$$t - (2t-1) \left(\frac{dt}{dx} - 2 \right) = 0$$

$$t + 4t - 2 - (2t-1) \frac{dt}{dx} = 0$$

$$\frac{dt}{dx} = \frac{5t-2}{2t-1}$$

$$dx = \frac{2t-1}{5t-2} dt$$

$$\int dx = \int \frac{2t-1}{5t-2} dt$$

$$x = \int \frac{2}{5} \left(\frac{5t-2}{5t-2} \right) dt$$

$$= \frac{2}{5} \int \left(1 + \frac{-1}{5t-2} \right) dt$$

$$= \frac{2}{5} \left(t - \frac{1}{5} \ln|5t-2| \right) + C$$

$$= \frac{2}{5} (2x+y) - \frac{1}{5} \ln|5(2x+y)-2| + C.$$