(Report 문제)

(1) $(x-2y+5)dx + (2x-y+4)dy = 0$

(2) $(2x-y+2)dx + (4x-2y-1)dy = 0$

(3) $(4x+3y-4)dx + (3x-7y-3)dy = 0$

(4) $(2x+y)dx - (4x+2y-1)dy = 0$

sol)

(1) $(x-2y+5)dx + (2x-y+4)dy = 0$

$x-2y+5=0$

$2x-y+4=0$

$\begin{vmatrix} 1 & -2 \\ 2 & -1 \end{vmatrix} = -1-(-4) \neq 0$

$2x-4y+10=0$

$2x-y+4=0$

$3y=6 \quad y=2 \quad x=-1$

$x = X-1 \quad y = Y+2$.

$dx = dX \quad dy = dY$.

$(X-2Y)dX + (2X-Y)dY = 0$.

$(X-2Y) + \frac{dY}{dX}(2x-Y) = 0$

$\left(1-\frac{2Y}{X}\right) + \frac{dY}{dX}\left(2-\frac{Y}{X}\right) = 0$

$\frac{Y}{X} = t \quad Y = Xt \quad \frac{dY}{dX} = t + \frac{dt}{dX}$

$(1-2t) + \left(t+\frac{dt}{dX}\right)(2-t) = 0$.

$t + \frac{dt}{dX} = -\frac{1-2t}{2-t}$

$\frac{dt}{dX} = -\frac{1-2t+2-t}{2-t} = -\frac{3-3t}{2-t}$

$\frac{dX}{dt} = \frac{2-t}{3t-3}$

$dX = \frac{2-t}{3t-3}dt$

$\int 1 \cdot dX = \int \frac{2-t}{3t-3}dt$

$X = -\frac{1}{3}\int \frac{t-2}{t-1}dt = -\frac{1}{3}\int\left(1-\frac{1}{t-1}\right)dt$

$= -\frac{1}{3}(t - \ln(t-1)) + C$

$X = -\frac{1}{3}\left(\frac{Y}{X} - \ln\left(\frac{Y}{X}-1\right)\right) + C$

$x+1 = -\frac{1}{3}\left(\frac{y-2}{x+1} \ln\left(\frac{y-2}{x+1}-1\right)\right) + C$ 여기서이렇게 고쳐야정답!

(2) $(2x-y+2)dx + (4x-2y-1)dy = 0$

$\begin{cases} 2x-y+2=0 \\ 4x-2y-1=0 \end{cases}$

$\begin{vmatrix} 2 & -1 \\ 4 & -2 \end{vmatrix} = -4-(-4) = 0$

$2x-y = t$.

$2 - \frac{dy}{dx} = \frac{dt}{dx} \qquad \frac{dy}{dx} = 2 - \frac{dt}{dx}$

$(2x-y+2) + (4x-2y-1)\frac{dy}{dx} = 0$

$(t+2) + (2t-1)\left(2-\frac{dt}{dx}\right) = 0$

$2 - \frac{dt}{dx} = -\frac{t+2}{2t-1}$

$\frac{dt}{dx} = \frac{5t}{2t-1}$

$dx = \frac{2t-1}{5t}dt$

$\int 1 \cdot dx = \int \frac{2}{5}\left(\frac{t-\frac{1}{2}}{t}\right)dt$

$x = \int \frac{2}{5}\left(1-\frac{1}{2t}\right)dt = \frac{2}{5}t - \frac{1}{2}\ln t + C$

$x = \frac{2}{5}(2x-y) - \frac{1}{2}\ln(2x-y) + C$.

(3) $(4x+3y-4)dx + (3x-7y-3)dy = 0$

$4x+3y-4=0$

$3x-7y-3=0$

$\begin{vmatrix} 4 & 3 \\ 3 & -7 \end{vmatrix} = -28-9 \neq 0$

$\begin{cases} 4x+3y-4=0 \\ 3x-7y-3=0 \end{cases} \qquad \begin{matrix} 12x+9y-12=0 \\ 12x-28y-12=0 \end{matrix}$.

$x=1 \quad y=0$

$x = a-1 \quad y = b$

$dx = da \quad dy = db$

$(4a+3b)da + (3a-7b)db = 0$

$(4a+3b) + (3a-7b)\frac{db}{da} = 0$

$\left(4+\frac{3b}{a}\right) + \left(3-\frac{7b}{a}\right)\frac{db}{da} = 0$

$\frac{b}{a} = t \quad b = at \quad \frac{db}{da} = t + a\frac{dt}{da}$

$(4+3t) + (3-7t)\left(t + a\frac{dt}{da}\right) = 0$

$t + a\frac{dt}{da} = \frac{4+3t}{3-7t} \qquad a\frac{dt}{da} = \frac{1+10t}{3-7t}$

$a\frac{dt}{da} = \frac{1+10t}{3-7t}$

$\frac{1}{a}\,da = \frac{3-7t}{1+10t}\,dt$ $\qquad 7t = (k-1)\frac{7}{10} - 3$

$\int \frac{1}{a}\,da = \int \frac{3-7t}{1+10t}\,dt \qquad 1+10t = k \qquad 10\,dt = dk.$

$= \int \frac{1}{k}\left((1-k)\frac{7}{10} + 3\right)\frac{1}{10}\,dk.$

$= \int \frac{1}{k}\left(\frac{37}{10} - \frac{7}{10}k\right)\frac{1}{10}\,dk$

$= \frac{37}{100}\ln k - \frac{7}{100}k + C$

$= \frac{37}{100}\ln(1+10t) - \frac{7}{100}(1+10t) + C$

$= \frac{37}{100}\ln\left(1+10\frac{b}{a}\right) - \frac{7}{100}\left(1+10\cdot\frac{b}{a}\right) + C$

$\simeq \frac{37}{100}\ln\left(1+10\frac{y}{x+1}\right) - \frac{7}{100}\left(1+10\cdot\frac{y}{x+1}\right) + C$

(4) $(2x+y)\,dx - (4x+2y-1)\,dy = 0$

$2x+y = 0$

$-4x-2y+1 = 0$

$\begin{vmatrix} 2 & 1 \\ -4 & -2 \end{vmatrix} = -4-(-4) = 0$

$2x+y = t \qquad 2 + \frac{dy}{dx} = \frac{dt}{dx}$

$t\,dx - (2t-1)\,dy = 0$

$t - (2t-1)\frac{dy}{dx} = 0$

$t - (2t-1)\left(\frac{dt}{dx} - 2\right) = 0$

$t + 4t - 2 - (2t-1)\frac{dt}{dx} = 0$

$\frac{dt}{dx} = \frac{5t-2}{2t-1}.$

$dx = \frac{2t-1}{5t-2}\,dt.$

$\int dx = \int \frac{2t-1}{5t-2}\,dt.$

$x = \int \frac{2}{5}\left(\frac{5t-\frac{5}{2}}{5t-2}\right)dt$

$= \frac{2}{5}\int 1 + \frac{-\frac{1}{2}}{5t-2}\,dt$

$= \frac{2}{5}\left(t - \frac{1}{2}\ln|5t-2|\right) + C$

$= \frac{2}{5}(2x+y) - \frac{1}{5}\ln(5(2x+y)-2) + C.$