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$$|a| - |b| \leq |a - b|$$

$$y \quad x \quad y - x.$$

Solve  $z^5 = 1 + i$

$$= \sqrt{2} \left( \frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}} \right)$$

$$= \sqrt{2} \left( \cos \frac{\pi}{4} + i \sin \frac{\pi}{4} \right).$$

$$= \sqrt{2} e^{i(\frac{\pi}{4} + 2\pi k)}$$

$$z = 2^{\frac{1}{5}} e^{i(\frac{\pi}{4} + 2\pi k)} \quad k = 0, 1, \dots, 4$$

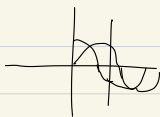
Solve  $z^4 = 1 - \sqrt{3}i$

$$= 2 \left( \frac{1}{2} - \frac{\sqrt{3}}{2}i \right)$$

$$= 2 \left( \cos \frac{5}{3}\pi - i \sin \frac{5}{3}\pi \right)$$

$$= 2 \cdot e^{i(\frac{5}{3}\pi + 2\pi k)}$$

$$z = 2^{\frac{1}{4}} \cdot e^{i(\frac{5}{3}\pi + 2\pi k)} \quad k = 0, 1, 2, 3.$$



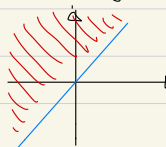
$$f(z) = z + 2z^2 \Rightarrow \lim_{z \rightarrow i} f(z) = i - 2$$

$$\text{if } \forall \epsilon \geq 0, \exists \delta \quad \begin{matrix} 1 & i \\ z & -2i - 1 \end{matrix}$$

$$\text{s.t. } |z - i| < \delta \Rightarrow |z + 2z^2 - i + 2| < \epsilon$$

$$|z + i| |2z - (2i + 1)| \leq |z + i| (|z| + 5) <$$

$$A = \{ z = x + iy \mid x < y \} \text{ open in } \mathbb{C}$$



$$\text{Let } z \in A \text{ and } z = x + yi, \quad y - x > 0.$$

$$\delta = \frac{1}{\sqrt{2}} (y - x) > 0 \Rightarrow \sqrt{2} \delta = y - x. \quad \sqrt{2} |z - w| < \sqrt{2} \delta$$

$$\text{if } w \in B_\delta(z) \text{ then } |z - w| < \delta, \quad w = a + bi$$

$$| \operatorname{Im}(z - w) | - | \operatorname{Re}(z - w) | \leq |z - w| \leq \sqrt{2} |z - w| < \sqrt{2} \delta$$

$$(y - b) - (x - a) = \operatorname{Im}(z - w) - \operatorname{Re}(z - w) < y - x.$$

$$\therefore b - a > 0.$$

$$\therefore w \in A$$

$$A \text{ is open.}$$

$$\star \log(-3 + 3i) = ?$$

$$\log(-3 + 3i) = \ln|-3 + 3i| + i \arg(-3 + 3i)$$

$$\left( \begin{matrix} 1 \\ -3 \end{matrix} \right) = \ln 18 + i \frac{3}{4}\pi.$$

$$i^{i+1} = e^{(i+1)\log i}$$

$$= e^{(i+1)(\ln|i| + i \arg(i))}$$

$$= e^{(i+1)\frac{\pi}{2}}$$

