Three Laws of Behavior: Allocation, Induction, and Covariance

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At the beginning of the 20th century, research on behavior depended on the concept of stimulus-response bonds. With the invention of operant behavior, Skinner (1938) pointed the way to a new formulation that transcends discrete events and contiguity—to what has been called a "molar" view. Three developments in the 1960s moved behavior analysis further in that direction: (a) the matching law (Herrnstein, 1961); (b) the "misbehavior" of organisms (Breland & Breland, 1961) or induction (Segal, 1972); and (c) contingency defined as covariance (Rescorla, 1967). These developments lead to three laws of behavior: (a) the Law of Allocation; (b) the Law of Induction; and (c) the Law of Covariance. Time allocation results from competition among activities for the limited time available. Induction of activities by phylogenetically important events (PIE) results from natural selection operating during phylogeny. Like stimulus control, induction modulates the rates of activities. Covariance replaces contiguity and explains acquisition of inducing signals and induced operant activities—that is, conditional inducers and conditionally induced activities. These 3 laws explain virtually all we know about behavior. Some examples illustrate the superiority of this framework over the older molecular view.

Keywords: Allocation, induction, covariance, contingency, reinforcement

At the beginning of the 20th century, scientists studying behavior had available to them only two concepts: reflexes and associative bonds. Both entailed discrete events and contiguity between the events. Pavlov's (1960) conditional reflexes (called "conditioned" due to a translating error) entailed contiguity between a conditional stimulus (CS) and an unconditional stimulus (which he also called a "reinforcer"), and the result was supposed to be a bond between the CS and a conditional response. Nineteenth-century associationists considered associative bonds to connect ideas, but the associative bond, when combined with the reflex, became a bond between stimulus and response, or an S-R bond. In ethology, a similar concept emerged, in which a sign stimulus was said to "release" a fixed action pattern. Thus was born the vocabulary that behavior analysts live with today: stimulus, response, and reinforcer.

The early behaviorists Watson (1930) and Thorndike (1911/2000) wrote of S-R bonds. Watson considered S-R bonds sufficient for a science, but Thorndike added to the associative laws, such as the law of contiguity, another law, which he called the "law of effect." According to the law of effect, an S-R bond could be strengthened by a satisfying event that closely followed the S-R sequence.

With Skinner's (1938) invention of operant behavior, a new concept emerged. In 1938, he tied it to the reflex, but he soon recognized that operant behavior could not be characterized by S-R bonds, because no stimulus could be identified before each occurrence of the response. Two inventions followed: (a) response rate as a valid measure of behavior and (b) stimulus control. With these two new concepts, Skinner left S-R bonds behind. Instead, he thought of response rate as an indicator of response probability and a discriminative stimulus as exerting "control" by modulating response rate via response probability. Skinner's innovations pointed in a direction away from discrete responses and contiguity, but he never made a

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further move in that direction because he never went beyond the "operant" as a class of discrete responses or the theory that an immediately following reinforcer "strengthens" an operant response.

The legacies left by Pavlov, Watson, Thorndike, and Skinner forced scientists studying behavior to rely, from the 1930s to the 1960s, on a jumble of poorly related concepts, such as classical conditioning, operant conditioning, reinforcement, discrimination, and stimulus control. As the phenomena of interest expanded with research, applying these concepts became awkward. For example, textbooks stated that classical conditioning and operant conditioning "interact," but only for the reason that unconditional stimuli are the same events as reinforcers and punishers. For theory during this period, behavioral scientists appealed to hypothetical constructs such as response strength and S-R bonds.

The science of behavior moved along in the direction Skinner had pointed out, toward a "molar" view of behavior with three discoveries in the 1960s: a) the matching law (Herrnstein, 1961); (b) the "misbehavior of organisms," or induction (Breland & Breland, 1961; Segal, 1972); and (c) contingency defined as covariance (Rescorla, 1967, 1968, 1988). These three discoveries lead to three laws of behavior: (a) the Law of Allocation, (b) the Law of Induction, and (c) the Law of Covariance. Together, these laws allow behavior analysts to organize most, if not all, of what we know about behavior. Let us examine each discovery and law in turn.

The Law of Allocation

As Herrnstein (1961) originally presented it, the matching law stated that the proportion of behavior attracted by an alternative tended to match the proportion of reinforcers obtained by that alternative:

$$\frac{B_1}{B_1 + B_2} = \frac{r_1}{r_1 + r_2} \tag{1}$$

where B_1 and B_2 are rates of behavior allocated to Alternatives 1 and 2, such as two response keys, and r_1 and r_2 are the rates at which reinforcers, such as bits of food, were obtained. Herrnstein (1970) generalized Equation 1 to any number, N, of alternatives:

$$\frac{B_j}{\sum_{i=1}^{N} B_i} = \frac{r_j}{\sum_{i=1}^{N} r_i} \tag{2}$$

Though not fully appreciated at the time, the matching law was revolutionary, because it introduced reinforcer rate as a valid independent variable for understanding response rate. Just as Skinner had recognized an extended measure, response rate, as a dependent variable, the matching law introduced an extended measure, reinforcer rate, as an independent variable, and together they indicated that behavior and its controlling relations should be seen as extended in time.

From the recognition that the matching law implies temporally extended variables and relations, only a short step was required to write matching more generally in terms of time (Baum & Rachlin, 1969):

$$\frac{T_j}{\sum_{i=1}^{N} T_i} = \frac{V_j}{\sum_{i=1}^{N} V_i}$$
 (3)

which states that the proportion of time taken up by one activity j matches V_j relative to the total of V_i across all alternatives, and each V_i is a composite measure of reinforcer variables, such as rate, amount, and immediacy, that determine the relative time.

Equation 3 may be rewritten in a variety of ways (Baum, 2012a), but it is general enough for present purposes to be called the Law of Allocation. It has been used to explain impulsive choice (Aparicio, Elcoro, & Alonso-Alvarez, 2015) and resurgence—the reappearance of extinguished responding when an alternative activity is extinguished (Shahan & Craig, 2017). Like any scientific law, it embodies and depends upon a number of assumptions or axioms. These were discussed less formally in an earlier article (Baum, 2013). Let us spell them out here.

Axiom 1: Only Whole Organisms Behave

Axiom 1 partially defines what we mean by "behavior." It rules out behavior by parts of an organism. In particular, it denies that the brain behaves (Bennett & Hacker, 2003). The brain does not perceive, choose, or sense, any more than the brain can walk or talk; these are activities of whole organisms.

Axiom 2: To Be Alive Is to Behave

Axiom 2 says that so long as an organism is alive, it behaves continuously. It immediately implies that behavior takes up all the time available. If one observes an organism for an hour, a day, or a year, one observes an hour's worth, a day's worth, or a year's worth of behavior. If behavior is allocated among various activities, those activities each take up some of the time, and together take up all of the time.

Axiom 3: Every Activity Is Composed of Parts That Are Themselves Activities

Axiom 3 could also be considered a corollary of Axiom 2, but for clarity it is better stated as a separate axiom. It introduces scale into Equation 3. It says that the time taken up by any one activity may be subdivided into the lessextended, smaller-scale activities of which it is composed, and that the time taken up by those parts adds up to the time taken up by the moreextended, longer-scale activity of which they are parts. If I play tennis for an hour, during that hour I am serving shots, returning shots, keeping score, exchanging remarks with my opponent, and so on. Together these activities constitute playing tennis, and together they take up the whole hour of my playing tennis. Thus, Equation 3 may apply at any time scale, to the parts of playing tennis or to the activities of a day, one of which is playing tennis. It may apply even at time scales of fractional seconds, to the parts of a pigeon's key peck or a rat's lever press.

Axioms 1, 2, and 3 support the left side of Equation 3. The numerator equals the time taken up by one activity, and the denominator equals the total time taken up by the *N* activities. The right side of Equation 3 derives from the Law of Induction.

The Law of Induction

In an article published in 1961, Breland and Breland reported on what they called the "misbehavior" of organisms and ushered in a period of questioning of assumptions in the study of behavior. The Brelands had been training animals to perform tricks at venues like state fairs for profit, requiring visitors to donate coins that induced the animals to perform. They observed

that food reinforcers would usually work for a time to shape the animals' operant behavior, but that the performance would then be disrupted by the intrusion of activities that were parts of the organism's natural feeding activity. A pig required to drop a wooden coin into a slot to receive food would perform the trick for a while, but then would begin to push the coin around with its snout and toss it about (the activity called "rooting"), an activity that pigs exhibit when they encounter a food item that needs to be broken up before it can be ingested. A raccoon that was required to deposit a real coin in a slot similarly performed well for a while, but then began to hold on to the coin and rub it with its paws, as it would do naturally with a food item to clean it before eating it.

Critics of behavior analysis pointed to more examples of such deviations from a simple-minded notion of reinforcers and their effects. Seligman (1970) wrote that the limitations to which the Brelands' article pointed undermined the "generality" of the laws of learning, particularly the law of effect. He criticized the assumption of "equipotentiality" on which he supposed the law of effect rested. Hinde and Stevenson-Hinde (1973) put together a whole book on "Constraints on Learning."

In response to the criticism, behavior analysts acknowledged that such constraints exist, but tended to consider them as peripheral boundary conditions that needed sometimes to be noted. In an article that came out in 1972, Segal went beyond this position. She catalogued the many constrained relations that behavior analysts and others had uncovered. She went on then to suggest that they could all be understood with a single concept: *induction*. Induced behavior has been called "adjunctive," "interim," and "terminal" (Staddon, 1977). Here it will just be called "induced."

Induction operates exactly like stimulus control. It modulates the time spent in several activities related to the presented environmental event—say, food or electric shock. Like stimulus control, induction too is extended in time, not momentary, and not limited to a contiguous S-R relation. Presenting food to a hungry pigeon induces a lot of locomotor activity and particularly induces pecking—at the floor, the wall, in the air, or at a shiny disk (Staddon, 1977). In a rat, food induces locomotor activity and sniffing, licking, biting, and chewing in the

hopper where food appears. Thus the inducing event (*inducer* hereafter) increases time spent in some activities, and, because activities compete for time, decreases time spent in some other activities. The word *constraint* captures both of these effects, because the very presence of the inducer ensures that some activities are likely and others unlikely.

Inducers like food and electric shock induce activities that may be understood in the light of evolutionary history. They are phylogenetically important events (PIE). To call events "phylogenetically important" is to say that the events affect fitness. Food, water, a potential mate, and shelter exemplify good PIEs, in the sense that their presence tends to enhance reproductive success. Predators, injury, illness, and excessive cold or heat exemplify bad PIEs, in the sense that their presence tends to reduce reproductive success. Traditionally, PIEs have been called "reinforcers," "punishers," "aversive stimuli," or "unconditional stimuli," but their tie to phylogeny makes these labels unnecessary (Baum, 2012b).

Because PIEs impact fitness, they facilitate natural selection of the activities they induce. Individuals in a population that behave in ways that enhance the effects of good PIEs when they are present tend to leave more offspring, and thus are selected over those that behave less effectively. Individuals in a population that behave in ways that mitigate the effects of bad PIEs when they are present also tend to leave more offspring and are selected over those that behave less effectively. Thus, when grass seeds are present in an area, pigeons peck at them, and when a hawk flies overhead, quail on the ground freeze or hide.

Not all the events that we call reinforcers or punishers—that is, inducers—are due to natural selection; some arise during ontogeny, as a result of an organism's exposure to correlations or contingencies in the environment between "neutral" events—ones that have no direct effect on fitness initially—and PIEs. These tokens of PIEs have effects similar to the PIEs themselves. Money is a classic example of a token of good PIEs, and warning labels are a similar example of tokens of bad PIEs.

Not only do tokens of PIEs become effective during ontogeny, but also operant activities become PIE-induced as a result of their correlations with PIEs. When lever pressing produces food, food and lever pressing are correlated. When lever pressing is correlated with food and other activities not correlated with food, lever pressing is selected to occur whenever food is present. A feedback cycle occurs in which the lever pressing produces the food and the food induces lever pressing, producing food again. When lever pressing reduces the rate of electric shocks and other activities do not, lever pressing is selected to occur whenever electric shock occurs. In free-operant avoidance, lever pressing is induced by shocks, particularly by shocks during a "warm-up" period at the beginning of a session (Hineline, 1977).

Although Segal (1972) stopped short of integrating induction with traditional concepts about operant behavior, she pointed in that direction, and acknowledged (personal communication) that my reasoning (Baum, 2012b) had taken her work to its "logical conclusion." Once operant behavior is established, it is induced by the PIE maintaining it, even after the operant activity has been extinguished—the "reinstatement" effect (Reid, 1958). Because the PIEs feed back on the time spent in the operant activity, we may ask what the quantitative role of the PIE is in the feedback cycle between operant activity and resulting PIEs.

Some evidence supports a power function relating food rate and pigeons' rate of key pecking (e.g., Baum, 2015):

$$T_j = b_j r_j^{s_j} \tag{4}$$

where T_j is time spent pecking, r_j is rate of food, s_j is sensitivity of T_j to r_j , and b_j is a coefficient. Equation 4 states that r_j induces time spent pecking according to the power s_j and in proportion to b_j . In principle, r_j need not be only rate of food; for example r_j could represent amount of food or immediacy of food (Baum & Rachlin, 1969).

The Law of Covariance

In 1967, a review came out by Rescorla of appropriate controls for Pavlovian conditioning. He argued that the correct control is a procedure in which no relation exists between occurrences of the putative CS and the unconditional stimulus. He described what such a nonrelation would be like. In doing so, he pointed out that a

nonrelation would be a procedure in which the probability of the unconditional stimulus is the same regardless of the occurrence of the CS. By implication, however, the argument means that a contingent relation between two events entails that the probability of the unconditional stimulus differs when the CS occurs from when it is absent.

Rescorla's insight meant that responsereinforcer contiguity could no longer suffice to define the controlling relation. Instead, a contingency controlling behavior had to include a difference between two conditions: likelihood of the signaled event (PIE) when the signal (S) is present and likelihood of the signaled event (PIE) when the signal (S) is absent. Table 1 diagrams the requirements for a contingency to exist.

For S to be correlated to PIE, a difference must exist between two co-occurrences: (a) Cell B—S absent and PIE present—must occur relatively rarely; and (b) cell D—S present and PIE present—must occur relatively often. If these co-occurrences occur equally often, no relation exists. Cells A and C differ also, but the difference in their rate is determined by the difference between cells B and D, because PIE can only be present or absent in the view of Table 1; if the rate of cell B is low, then the rate of cell A must be high, and the same for cells C and D.

Table 1, being couched in terms of absence and presence, is not the most general way to characterize what it means for S and a PIE to be related. When the rates of the four co-occurrences differ as in Table 1, the two events (signal and signaled in Table 1) covary—that is, when the rate of one event changes, the rate of the other event changes too.

Figure 1 illustrates the covariance implied by Table 1. The shape of the curve will vary from one relationship to another. The dashed line

Table 1 Relative Rates of Co-Occurrences in a Positive Contingency or Positive Covariance Between a Signal and a Phylogenetically Important Event (PIE)

	PIE absent	PIE present
Signal absent	A high rate	B low rate
Signal present	C low rate	D high rate

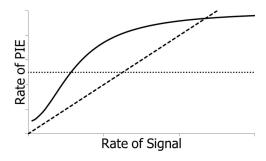


Figure 1. Covariance and noncovariance between rate of a signal and rate of a phylogenetically important event (PIE). The dashed line and solid curve show examples of covariance. The dotted horizontal line shows noncovariance.

shows a perfect relationship, in which the rate of the PIE exactly follows the rate of the signal. The solid curve shown suggests that some PIEs occur in the absence of signals, when the rate of S is zero, and that the rate of PIEs approaches an upper limit, above which signals cease to be related to the PIEs, when the curve flattens. The point of Figure 1, however, is just to illustrate that when S and a PIE covary, some relation between their rates is implied. Were no relation to exist—shown by the dotted flat line in Figure 1—S and the PIE would be unrelated, because the rate of the PIE is independent of the rate of S.

The line of reasoning that Rescorla (1967, 1968) followed applies, not only to relations between signals and PIEs, but also to relations between activities and PIEs. Contiguity between an activity and a PIE cannot suffice to determine the contingency or correlation between the activity and the PIE. Table 2 repeats Table 1, but for an activity instead of a signal.

As in Table 1, for a contingency to exist between the activity and the PIE, a difference in rate must exist between cells B and D. When the activity is absent, the rate of the PIE is relatively

Table 2 Relative Rates of Co-Occurrences in a Positive Contingency or Positive Covariance Between an Operant Activity and a Phylogenetically Important Event (PIE)

	PIE absent	PIE present
Activity absent	A high rate	B low rate
Activity present	C low rate	D high rate

low, and when the activity is present, the rate of the PIE is relatively high. Were the rates of cells B and D to be equal, no relation would exist.

Tables 1 and 2 indicate that contiguity between a signal or an activity and a PIE is insufficient to define a contingency, but contiguity is not entirely irrelevant, either. The co-occurrences in the tables imply some degree of contiguity in cell D, but also imply a contrast with cell B. Thus, if a PIE has been absent for a substantial time, and an accidental coincidence between an action and a PIE occurs, some repetition of the activity might follow. The activity will be short-lived, however, if the PIE occurs just as often when the activity is absent as when the activity is present.

As Table 1 implies Figure 1, so Table 2 implies Figure 2. The relations shown in Figure 2 are known as feedback functions. They specify the dependence of PIE rate or reinforcer rate on activity rate or response rate. Many different relations are possible. Figure 2 illustrates two common relations. The straight line shows the relation imposed by a variable-ratio (VR) schedule, which specifies that PIE rate is directly proportional to activity rate. The two curves show feedback functions imposed by variable-interval (VI) schedules. Each curve ap-

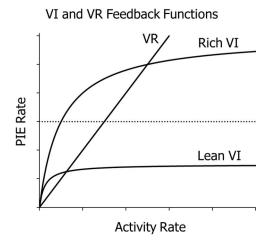


Figure 2. Covariance and noncovariance between rate of an activity and rate of a phylogenetically important event (PIE). The solid curves and line, also known as feedback functions, show examples of covariance. The curves show the covariance imposed by two different variable interval schedules. The straight line illustrates the covariance imposed by a ratio schedule. The dotted horizontal line shows noncovariance.

proaches an upper limit determined by the programming of the schedule. The lower curve shows the relation for a relatively lean VI, and the higher curve shows the relation for a relatively rich VI schedule. As activity rate increases, a VI schedule results in less and less covariance between PIE rate and activity rate. In contrast, the covariance imposed by a VR schedule remains strong throughout the range of activity rate. Both schedules' feedback functions contrast with the dotted horizontal line, which specifies the absence of any covariance between the PIE rate and activity rate.

Our discussion of covariance so far has focused only on positive covariance. Tables 1 and 2 and Figures 1 and 2 show only relations in which the rate of the PIE and the rate of the signal or activity increase together. In negative relations, PIE rate decreases as signal rate or activity rate increases. Table 3 illustrates the co-occurrences in a negative relation. As before, we focus on cells B and D. Now, however, Table 3 shows that when the signal or activity is absent, the PIE is present at a high rate, and when the signal or activity is present, the PIE is present at a low rate. In other words, in negative covariance, the signal indicates the PIE is unlikely, and the activity avoids or prevents the PIE from occurring. If the PIE is fitnessreducing (e.g., a predator), the signal indicates "safety," and the activity is avoidance or escape.

Figure 3 shows the sort of relation that constitutes negative covariance. The curve shown indicates that, as signal or activity rate increases, PIE rate decreases ever more slowly and approaches zero. At high signal or activity rates, covariance drops toward zero, but at lower rates, covariance is substantial. A horizontal line would indicate lack of any covariance. The curve shown might apply to free-operant avoidance (Sidman, 1966). As with positive covariance, the curve could take many different shapes, depending on circumstances in the environment.

The Law of Covariance states that (a) when covariance exists between a signal and a PIE, the signal becomes a proxy for the PIE and induces the same activities as the PIE, and (b) when covariance exists between an activity and a PIE, the activity becomes a PIE-induced activity. The first part replaces the concepts of CS, discriminative stimulus, and secondary reinforcer with the single concept of conditional

Table 3
Relative Rates of Co-Occurrences in a Negative
Contingency or Negative Covariance Between a
Signal or Operant Activity and a Phylogenetically
Important Event (PIE)

	PIE absent	PIE present
Signal or activity absent	A low rate	B high rate
Signal or activity present	C high rate	D low rate

inducer. The second part defines a conditionally induced activity, which is often called "operant" or "instrumental" behavior.

As noted earlier, many critics in the 1960s and 1970s emphasized "constraints" on signals and operant behavior, meaning that not all events readily become signals (conditional inducers) and not all activities readily become operant activities (conditionally induced activities). Indeed, such warnings apply sometimes in the artificial situations of the laboratory and therapeutic intervention. For example, the "misbehavior" reported by the Brelands resulted from the success of having created covariance between operant behavior and food—when the food became regularly forthcoming, the activities induced by food during phylogeny began to appear. Similarly, in a therapeutic setting, covariance between desirable behavior and earned PIEs may end up increasing the PIE rate to a level at which the PIEs actually induce disruptive behavior.

In the natural world outside of the laboratory. issues about constraint have little force, because the constraints themselves are part of creatures' present and past environment. Garbage trucks and fishing boats induce approach in seagulls for the simple reason that they dump food; whether a constraint operates or not is of little concern. Snakes come to induce avoidance in human children with ease (Öhman & Mineka, 2003). Both of these examples—and many more—possess a social component: seagulls approach places where others of their kind feed, and human children fear snakes because they see adults fearing snakes. In 1960s England, birds (e.g., blue tits) opened milk bottles left at one's door by pecking through the aluminum foil top. They were able this way to eat the cream inside. This operant activity spread rapidly through the bird population, indicating that individuals were imitating others pecking at the

shiny milk bottle tops (Aplin, Sheldon, & Morand-Ferron, 2013).

When covariance between rate of an operant activity (e.g., lever pressing or shopping at a market) and rate of a PIE (e.g., food or other goods) is established, the PIE induces the activity. As noted earlier, the activity produces the PIE, and the PIE induces the activity, creating a feedback loop that maintains the activity. As lever pressing produces food, which in turn induces further lever pressing, so, for example, shopping produces goods, which in turn induce further shopping. The chief simplification underlying this statement is the omission of motivating factors—food deprivation for lever pressing or a discriminative stimulus (e.g., empty refrigerator) inducing shopping.

The Behavior–Environment Feedback System

Some earlier work suggested that the interaction of behavior with the environment may be compared to a feedback system (Baum, 1973, 1981, 1989, 2016). Figure 4 shows a diagram of the feedback system for one activity. (A more detailed presentation may be found in Baum, 1981). The set-point of the system ("competition") accords with the Law of Allocation; one may think of it as Equation 3 or Equation 5. It is compared with the current rate of the activity, B, and the difference or "error" equals ΔB , which is input to the third law, the Law of Covariance. The function g represents a feedback function—a property of the environment

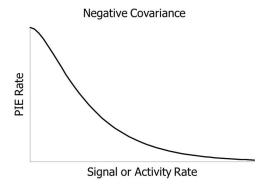


Figure 3. Negative covariance between a signal or an activity and a phylogenetically important event (PIE). The curve could characterize a "safety" signal or avoidance of a bad (fitness-reducing) PIE.

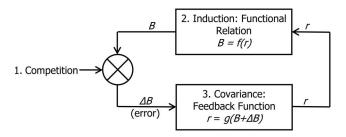


Figure 4. Behavior and environment as a feedback system. The Law of Allocation ("competition"; Equation 3 or 5) determines the set-point. The Law of Covariance translates error (ΔB) into a PIE rate (r). The Law of Induction translates r into rate of the operant activity (B).

(see Figure 2). The output of the feedback function, r, is a rate of consequences, PIE rate (e.g., food rate). The rate r is input to the second law, the Law of Induction. The function f may be thought of as Equation 4, with B equal to T_j . The system stabilizes when ΔB equals zero. That equilibrium is often called "stable performance." Although local variation never completely ceases, allocation may be considered stable when it ceases to exhibit a trend across time.

Applications

The Generalized Matching Law

When Equation 4 (the Law of Induction) is combined with Equation 3 (the Law of Allocation), we may substitute for V_i and V_i as follows:

$$\frac{T_j}{\sum_{i=1}^{N} T_i} = \frac{b_j r_j^{s_j}}{\sum_{i=1}^{N} b_i r_i^{s_i}}$$
 (5)

Equation 5 applies to all activities induced by the variable r, which has often been studied as rate of food. If N, the number of activities measured, equals 2, Equation 5 reduces to a familiar equation, called the "generalized matching law." If one transforms Equation 5 with N equal to 2 to ratios, assumes that s_1 equals s_2 , and takes the logarithm of the equation, the result is:

$$log\left(\frac{T_1}{T_2}\right) = s \, log\left(\frac{r_1}{r_2}\right) + log\left(\frac{b_1}{b_2}\right) \tag{6}$$

where s is the common sensitivity, and the ratio b_1/b_2 is treated as bias.

The Law of Allocation applies to more than just the activities induced by a single PIE, however. It applies to all activities and all PIEs. Thus, V in Equation 3 could stand for induction by food, but also induction by sounds, lights, or itches. When Herrnstein (1970) proposed an equation for describing a single activity as choice between that activity and all others, he included a term in the equivalent of Equation 3 that he labeled r_o , which he thought of as reinforcers from other activities than the operant activity measured and considered to reduce the equation to choice between just two activities (or choice between operant activity and a conglomerate of other "background" activities). Evidence indicates now that, contrary to Herrnstein's theory, r_o actually depends on food rate r (Baum, 2015; Baum & Davison, 2014). However, because Herrnstein's original reasoning about r_o was correct, two other terms, r_e and T_e , must be added. Thus, Herrnstein's equation becomes:

$$\frac{T}{T+T_o+T_e} = \frac{br^s}{br^s+b_or^{s_o}+r_e}$$
 (7)

where r_e is assumed to depend directly on T_e (Herrnstein, 1979). In the more familiar terms of response rate, Equation 7 would be rewritten:

$$B = K \frac{br^s}{br^s + b_o r^{s_o} + r_e} \tag{8}$$

where *B* is the rate of the operant activity, and *K* is the tempo of the operant activity, or the total of behavior measured in the units of *B*. When the activities induced by the food, including the

operant activity (B or T) and the other activities induced by the food (B_o or T_o), take up all the time available, T_e in Equation 7 equals zero, and r_e in Equation 8 equals zero. The resulting equation fits extant data well (Baum, 2015).

Equations 7 and 8 embody all three rules. The Law of Covariance explains that the operant activity B is induced by the PIE (e.g., food) because B covaries with the PIE. The Law of Induction explains that both B and B_o are induced by the PIE. The Law of Allocation explains that B and B_o compete for time, as expressed in the proportions on the right of the equations.

When B and B_o take up all the time available, we may rewrite Equation 8 by dividing through by br^s , with this result:

$$B = \frac{K}{1 + b'r^{s'}} \tag{9}$$

where b' equals $\frac{b_o}{b}$ and s' equals s_o -s. This equation fits response rates maintained by variable interval schedules as well as Herrnstein's (1970) proposed equation with r_o (Baum, 2015).

Avoidance

Avoidance responding challenges any theory of reinforcement that requires an immediate consequence to follow responses, because avoidance results in nonoccurrence of a punisher or at least a reduction in its probability. Typically, avoidance responses are followed by nothing. To try to meet this challenge, some researchers proposed "two-factor" theory, which appealed to unobserved "fear" that avoidance responses supposedly reduced in a fashion also unobserved (Anger, 1963; Kimble, 1961). Sidman (1966) proposed instead that lever pressing that avoided electric shocks was maintained by shock-rate reduction. Herrnstein (1969) expanded on this theory of avoidance and criticized two-factor theory as implausible and contrary to experimental evidence.

An experiment by de Villiers (1974) tested the theory that the reinforcement for avoidance is shock-rate reduction. He arranged variable interval schedules of shock-avoidance. If a rat's lever press occurred anywhere within a scheduled interval, the shock that would have occurred at the end of the interval was cancelled. He varied the schedule across conditions, waiting for response rate to stabilize before changing conditions. Lever pressing occurred at steady rates within conditions and varied across conditions, and de Villiers attributed the press rates to shock-rate reduction.

The theory that shock-rate reduction reinforces avoidance contains a weakness, however. Behavior can be maintained only by events and covariances with which it actually comes into contact, and how behavior can come into contact with shock-rate reduction remains unclear. In de Villiers's experiment, the programmed shock rate would have occurred rarely, as long as the rate of pressing exceeded zero, and would thus have been rarely available for comparison with the shock rate received.

The Law of Induction suggests instead that the shocks with which lever pressing came into contact—the shocks received—ought to be the PIEs inducing the lever pressing. Since deVilliers published the data for the four rats, I was able to reanalyze the press rates in relation to the received shock rates according to Equation 9, assuming that the shocks induced both lever pressing and other activities too—that is, B and B_o . Figure 5 shows the results of applying Equation 9 with r equal to the received shock rate. The dashed curves show the fits of Equation 9 produced by Solver in MS Excel. The parameter K could not be estimated for R 8 and R 13, so it was set to 50 as a reasonable estimate. Equation 9 fits the response rates well (variance accounted for ranged from .61 to .85). The exponent s' ranged from -1.19 to -0.48 across rats.

The success of Equation 9 with shocks received represents a theoretical superiority of the present framework over a view of reinforcement as strengthening. Operant activity induced by food might seem to be equally well attributed to strengthening as to induction, but operant avoidance induced by received shocks would contradict strengthening by reinforcement, because electric shocks are considered aversive. If anything, the shocks would be thought to weaken the lever pressing. Although shock-rate reduction also fits de Villiers's data well, the rationale for setting r equal to shock-rate reduction seems weak, whereas the present three laws afford a strong rationale for induction by received shocks.

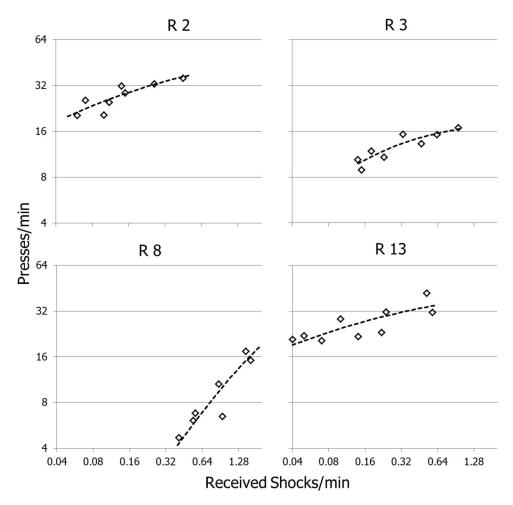


Figure 5. Avoidance as induced by received shock rate. The data are from de Villiers (1974). The dashed curves show Equation 9 fitted to the avoidance press rates with r equal to received shock rate. Note logarithmic axes.

The Differential-Outcomes Effect

When an organism is trained in a successive discrimination between two stimuli (e.g., two different tones), with two response alternatives (e.g., levers)—one response reinforced following one stimulus and the other response reinforced following the other stimulus—discrimination is enhanced if the reinforcers following the responses differ (e.g., food pellets vs. sucrose solution). The improvement in discrimination is known as the differential-outcomes effect (see Urcuioli, 2005, for a review). The phenomenon presents a challenge for any simple contiguity-based concept of reinforcement,

because the different outcomes follow the responses; they could differentially strengthen the responses if one reinforcer was stronger than the other, but they could not strengthen both responses in the presence of their appropriate stimuli. For example, suppose that sucrose was a more powerful reinforcer than a food pellet, and that the responses were pressing two levers (R1 and R2). If R1 produced sucrose, and R2 produced a food pellet, then sucrose might differentially strengthen R1, but similar reasoning could not explain why R2 was differentially strengthened in the presence of S2. Besides their roles as reinforcers, the two different outcomes play a discriminative role. But

how could they? The problem is that at the moment when R1 or R2 occurs, the outcomes are in the future, and future events are not supposed to affect present behavior.

The supposed "solution" to this problem of control by future events has been to posit "expectancies" that act as cues in addition to the stimuli, S1 and S2 (Urcuioli, 2005). Expectancies, however, constitute a form of mentalism that cannot stand up to scrutiny. Positing an unobservable event with no understandable connection to behavior may be tempting, but it violates basic scientific principles. It is an example of what Skinner (1969) called "mentalism." In science, causes must be observable.

Instead of unobservable expectancies, we may escape from the limitations of contiguity-based theory with the Law of Induction and the Law of Covariance. If S1 and R1 covary with sucrose, then S1 and sucrose will induce R1 rather than R2. Similarly, if S2 and R2 covary with food pellets, then S2 and food pellets will induce R2 rather than R1. The different covariances with the different PIEs across experimental trials explain the increase in R1 relative to R2 in the presence of S1 and the increase in R2 relative to R1 in the presence of S2. Discrimination improves because the differential outcomes induce the two responses differentially.

Conclusion

The three laws-allocation, induction, and covariance—may be used to explain virtually all we know about behavior. For example, they illuminate self-control and rule-governed behavior, both of which entail a combination of short-term and long-term covariance. Selfcontrol is a choice alternative that competes with impulsiveness, another choice alternative. In the short-term, impulsiveness has good consequences, but in the long-term it has bad consequences; it constitutes a bad habit (Baum, 2016). Drunkenness has enjoyable consequences in a short timeframe, but covaries with disastrous events in a long timeframe. The activity known as self-control also participates in two conflicting covariances; it has bad consequences in the short-term and good consequences in the long-term and constitutes a good habit (Baum, 2016). Sobriety entails giving up enjoyment in a short timeframe, but avoids the disastrous consequences of drunkenness. (See

Baum, 2017, for further discussion). Solving a problem in self-control requires bringing the activity of self-control into contact with its long-term covariance with good PIEs. Rule-governance often facilitates the solution by inducing self-control with a rule (a verbal conditional inducer) that signals good (usually social) PIEs. Friends and family may exhort an alcoholic to sober-up and attend AA meetings and give approval when the alcoholic does so. The rule thus drives time allocation toward self-control and away from impulsiveness. If self-control comes into contact with the long-term covariance with good PIEs, rule-governance may become unnecessary.

The laws of allocation, induction, and covariance cover the phenomena that have been called classical conditioning, operant conditioning, and adjunctive behavior. Through the concept of phylogenetically important event (PIE)—that is, events important in phylogeny that result in natural selection of activities that enhance good PIEs and mitigate bad PIEs—the laws make explicit connection with evolutionary theory. This connection allows us to understand why some events function as reinforcers and others as punishers, because the Law of Covariance applies both to behavioral selection in phylogeny and behavioral selection in ontogeny. Exceptions may exist, but only in the artificial circumstances of the laboratory and other controlled settings. In the world of nature and everyday human affairs, these three laws illuminate the interaction between organism and environment that we call behavior (Baum, 2013).

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Received July 18, 2017
Revision received October 20, 2017
Accepted October 23, 2017