Triangular Body-Cover Model: Description and Main Equations

Gabriel A. Alzamendi

June 4, 2020

Abstract

This document describes the basics concepts in the Triangular Body-Cover model (TBCM) and the most important equations involved, as introduced in [1]. This document pays special attention to some improved/revisited rules.

1 Dynamic equations in the TBCM

TBCM is a lumped-element biomechanical model of the vocal folds introduced in [1]. It is composed of three masses representing the cover-body structure of the human vocal folds, where upper m_u and lower m_l masses constitute the cover layer, and the body mass m_b emulates the internal body in the vocal fold. The dynamic of the TBCM is described through a second order differential equations:

$$\begin{aligned} m_u\,\ddot{x}_u &= F_{k\,u} + F_{d\,u} - F_{kc} + F_{e\,u} + F_{col\,u}, \\ m_l\,\ddot{x}_l &= F_{k\,l} + F_{d\,l} + F_{kc} + F_{e\,l} + F_{col\,l}, \\ m_b\,\ddot{x}_b &= F_{k\,b} + F_{d\,b} - (F_{k\,u} + F_{d\,u} + F_{k\,l} + F_{d\,l}), \end{aligned} \tag{1}$$

where for $* \in \{u, l, b\}$ the variables x_* describe the displacement of the masses with respect to the rest (equilibrium) position $x_{0\,*}$. The force terms correspond to the (elastic) spring response ($F_{k\,*}$ and F_{kc}), the damping ($F_{d\,*}$), the reaction during left-right vocal folds collision ($F_{col\,*}$), and aerodynamics force due to glottal airflow ($F_{e\,*}$). The masses parameters (m_u, m_l, m_b) are obtained as a function of the activation of laryngeal muscles [4].

According to the TBCM, the phonatory (triangular) glottal configuration is a function of arytenoid cartilage posturing. Let ξ_u and ξ_l be the position of the posterior ends of the upper and lower masses, respectively, due to medial-lateral vocal process displacement. The equilibrium positions for the upper and lower masses are equal to the average horizontal glottal position:

$$x_{0\,u} = \frac{\xi_u}{2}, \qquad \text{and} \qquad x_{0\,l} = \frac{\xi_l}{2}. \tag{2}$$

For the body mass, the equilibrium position $x_{0\,b} = 0.3$ cm.

1.1 Elastic spring forces

The spring forces are computed as follows:

$$\begin{aligned}
F_{k\,u} &= -k_u \left[\left[(\delta_u - x_{0\,u}) - (x_b - x_{0\,b}) \right] + \eta_u \left[(\delta_u - x_{0\,u}) - (x_b - x_{0\,b}) \right]^3 \right], \\
F_{k\,l} &= -k_l \left[\left[(\delta_l - x_{0\,l}) - (x_b - x_{0\,b}) \right] + \eta_l \left[(\delta_l - x_{0\,l}) - (x_b - x_{0\,b}) \right]^3 \right], \\
F_{k\,b} &= -k_b \left[(x_b - x_{0\,b}) + \eta_b \left(x_b - x_{0\,b} \right)^3 \right], \\
F_{kc} &= -k_c \left[(\delta_u - x_{0\,u}) - (\delta_l - x_{0\,l}) \right],
\end{aligned} \tag{3}$$

where $\delta_u = x_u + \frac{\xi_u}{2}$ and $\delta_l = x_l + \frac{\xi_l}{2}$.

The expression can be written as:

$$\begin{aligned}
F_{k\,u} &= -k_u \left[(x_u - x_b) + \eta_u \left(x_u - x_b \right)^3 \right], \\
F_{k\,l} &= -k_l \left[(x_l - x_b) + \eta_l \left(x_l - x_b \right)^3 \right], \\
F_{k\,b} &= -k_b \left[(x_b - x_{0\,b}) + \eta_b \left(x_b - x_{0\,b} \right)^3 \right], \\
F_{kc} &= -k_c \left[x_u - x_l \right].
\end{aligned} \tag{4}$$

The elastic parameters (k_u, k_l, k_b, k_c) are obtained as a function of the activation of laryngeal muscles [4].

1.2 Damping forces

Damping forces are [1, 2]:

$$\begin{aligned}
F_{d\,u} &= -2\zeta_u \sqrt{m_u k_u} \left(1 + 150|\delta_u| \right) \left(\dot{x}_u - \dot{x}_b \right), \\
F_{d\,l} &= -2\zeta_l \sqrt{m_l k_l} \left(1 + 150|\delta_l| \right) \left(\dot{x}_l - \dot{x}_b \right), \\
F_{d\,b} &= -2\zeta_b \sqrt{m_b k_b} \left(\dot{x}_b \right).
\end{aligned} \tag{5}$$

where $\delta_u = x_u + \frac{\xi_u}{2}$ and $\delta_l = x_l + \frac{\xi_l}{2}$.

1.3 Collision forces

In order to compute the reaction forces due to the collision of left/right vocal folds, glottis membranous area and contact area during collision are first described.

According to the TBCM, the parametric representations of the rest configuration for the cover masses are $r_*(z) = \xi_* \left(1 - \frac{z}{L_g} \right)$, $0 \le z \le L_g$, and $* \in \{u, l\}$. Then, the phonatory configuration can be computed as follows:

$$w_*(z) = x_* + \xi_* \left(1 - \frac{z}{L_g} \right). \tag{6}$$

Thus, vocal fold collision occurs for all z given that $w_*(z) \le 0$.

1.3.1 Region under collision

In the vertical axis, the region under collision is delimited by two extreme points. Solving for $w_*(z) = 0$, one of these extreme points is computed:

$$\tilde{z} = \min\left(0, \max\left(\frac{x_* + \xi_*}{\xi_*}, 1\right)\right) L_g. \tag{7}$$

The other extreme points depends on ξ_*. For the case $\xi_* > 0$, the vertical region under collision is $[\tilde{z}, L_g]$, whereas if $\xi_* < 0$ the region is $[0, \tilde{z}]$. In the case $\xi_* = 0$, the TCBM tends to the original *body-cover* model [3].

Let α_* be the portion of length of the cover masses under collision, with $0 \leq \alpha_* \leq 1$ and $* \in \{u, l\}$. Following the definition of the collision region introduced above, the formulation of α_* depends on x_* and ξ_*:

Case $\xi_* = 0$: parallel bars as in the body-cover model,

$$\alpha_* = \frac{1 - \operatorname{sgn}(x_*)}{2}. \tag{8}$$

Case $\xi_* > 0$: triangular configuration with the lateral displacement in the posterior end,

$$\alpha_* = \min\left(\max\left(0, -\frac{x_*}{\xi_*}\right), 1\right). \tag{9}$$

Case $\xi_* < 0$: triangular configuration with the lateral displacement in the anterior end,

$$\alpha_* = \min\left(\max\left(0, \frac{x_* + \xi_*}{\xi_*}\right), 1\right). \tag{10}$$

1.3.2 Rule for collision forces

Considering the definitions introduced in the previous section, the resulting collision forces $F_{col\,*}$, $* \in \{u, l\}$, along the region under collision is:

$$F_{col\,*} = \frac{1}{L_g} \int_{z_1}^{z_2} -\hat{h}_{col\,*} \left[w_*(z) + \hat{\eta}_{col\,*} w_*(z)^3\right] dz, \tag{11}$$

where z_1 and z_2 are the inferior and superior extremes of the collision region. Applying integration by substitution, Eq. (11) is integrated out:

$$F_{col\,*} = \frac{\hat{h}_{col\,*}}{\xi_*} \left[\frac{1}{2} w_*(z)^2 + \frac{\hat{\eta}_{col\,*}}{4} w_*(z)^4\right]_{z_1}^{z_2}. \tag{12}$$

Next, the different solutions are obtained:

Case $\xi_* \geq 0$: Correspond to a triangular configuration with the lateral displacement in the posterior end. For the case $x_* \leq 0$, the inferior and superior extremes are $z_1 = \tilde{z}$ and $z_2 = L_g$, and $w_*(z_1) = 0$ and $w_*(z_2) = x_*$. The resulting collision forces is:

$$
\begin{aligned}
F_{col\,*} &= \frac{\hat{h}_{col\,*}}{\xi_*}\left[\frac{1}{2}x_*^2 + \frac{\hat{\eta}_{col\,*}}{4}x_*^4\right] = -\alpha_*\hat{h}_{col\,*}\left[\frac{1}{2}x_* + \frac{\hat{\eta}_{col\,*}}{4}x_*^3\right] \\
&= -\alpha_*\hat{h}_{col\,*}\left\{\left(\delta_* - (1-\alpha_*)\frac{\xi_*}{2}\right) + \hat{\eta}_{col\,*}\left(\delta_* - (1-\alpha_*)\frac{\xi_*}{2}\right)\right. \\
&\qquad \left.\left[\left(\delta_* - (1-\alpha_*)\frac{\xi_*}{2}\right)^2 + \left(\alpha_*\frac{\xi_*}{2}\right)^2\right]\right\},
\end{aligned} \tag{13}
$$

where $\delta_* = x_* + \frac{\xi_*}{2}$.

Case $\xi_* < 0$: Correspond to a triangular configuration with the lateral displacement in the anterior end. For the case $x_* > 0$, the inferior and superior extremes are $z_1 = 0$ and $z_2 = \tilde{z}$, and $w_*(z_1) = x_* + \xi_*$ and $w_*(z_2) = 0$. The resulting collision forces is:

$$
\begin{aligned}
F_{col\,*} &= \frac{\hat{h}_{col\,*}}{\xi_*}\left[\frac{-1}{2}(x_* + \xi_*)^2 - \frac{\hat{\eta}_{col\,*}}{4}(x_* + \xi_*)^4\right] \\
&= -\alpha_*\hat{h}_{col\,*}\left[\frac{1}{2}(x_* + \xi_*) + \frac{\hat{\eta}_{col\,*}}{4}(x_* + \xi_*)^3\right] \\
&= -\alpha_*\hat{h}_{col\,*}\left\{\left(\delta_* - (1-\alpha_*)\frac{(-\xi_*)}{2}\right) + \hat{\eta}_{col\,*}\left(\delta_* - (1-\alpha_*)\frac{(-\xi_*)}{2}\right)\right. \\
&\qquad \left.\left[\left(\delta_* - (1-\alpha_*)\frac{(-\xi_*)}{2}\right)^2 + \left(\alpha_*\frac{\xi_*}{2}\right)^2\right]\right\},
\end{aligned} \tag{14}
$$

where $\delta_* = x_* + \frac{\xi_*}{2}$.

2 Membranous glottal area

The membranous glottal area A_m is delimited for the triangular posture of the glottal folds, from the posterior vocal processes to the anterior ends. This parameter can be computed as follows:

$$
A_m = 2\begin{cases}(1-\alpha_*)L_g\left(\delta_* + \alpha_*\frac{\xi_*}{2}\right), & 0 \le \xi_*, \\ (1-\alpha_*)L_g\left(\delta_* + \alpha_*\frac{(-\xi_*)}{2}\right), & \xi_* < 0,\end{cases} \tag{15}
$$

where $\delta_* = x_* + \frac{\xi_*}{2}$. Taking into account the definition of α_*, membranous area formulation for the original body-cover model is obtained through Eq. (15) in the case $\xi_* = 0$.

References

[1] G. E. Galindo, S. D. Peterson, B. D. Erath, C. Castro, R. E. Hillman, and M. Zañartu. Modeling the Pathophysiology of Phonotraumatic Vocal Hyperfunction With a Triangular Glottal Model of the Vocal Folds. *Journal of Speech, Language, and Hearing Research*, 60(9):2452–2471, sep 2017.

[2] J. C. Lucero and L. L. Koenig. Simulations of temporal patterns of oral airflow in men and women using a two-mass model of the vocal folds under dynamic control. *The Journal of the Acoustical Society of America*, 117(3):1362–1372, mar 2005.

[3] B. H. Story and I. R. Titze. Voice simulation with a body-cover model of the vocal folds. *The Journal of the Acoustical Society of America*, 97(2):1249–1260, feb 1995.

[4] I. R. Titze and B. H. Story. Rules for controlling low-dimensional vocal fold models with muscle activation. *The Journal of the Acoustical Society of America*, 112(3):1064–1076, sep 2002.