Propriedades envolvendo limites infinitos/no infinito

Continuação (Propriedades)

2-

a) Limites infinitos:

Considere $f, g, h : I \subset \mathbb{R} \to \mathbb{R}$, $p \in \mathbb{R}$, $K \in \mathbb{R}$, de modo que:

$\to \lim_{t \to p} f(t) = \lim_{t \to p} g(t) = +\infty$

$\to \lim_{t \to p} h(t) = K$.

Então:

i) $\lim_{t \to p} f(t) + g(t) = +\infty$, "$\infty + \infty = \infty$"

mas $\lim_{t \to p} f(t) - g(t)$ é indeterminação. "$\infty - \infty$"

ii) $$\lim_{t \to p} K \cdot f(t) = \begin{cases} +\infty, & \text{se } K > 0 \\ -\infty, & \text{se } K < 0 \\ 0, & \text{se } K = 0. \end{cases}$$

iii) $\lim_{t \to p} f(t) \cdot g(t) = +\infty$, "$\infty \cdot \infty = \infty$"

"$\frac{\infty}{\infty}$"

mas ... $f(t)$ ... é indeterminação

$t \to p$

mas $\lim_{t \to p} \dfrac{f(t)}{g(t)}$ é indeterminação. $\xrightarrow{} \text{"}\dfrac{\infty}{\infty}\text{"}$

Obs.: Se tivéssemos $\lim_{t \to p} g(t) = -\infty$:

i) $\lim_{t \to p} f(t) + g(t)$ é indeterminação, $\to$ "$\infty - \infty$"

mas $\lim_{t \to p} g(t) - f(t) = -\infty$. $\to$ "$-\infty - \infty = -\infty$"

ii) $\lim_{t \to p} f(t) \cdot g(t) = -\infty$ $\to$ "$(+\infty) \cdot (-\infty) = -\infty$"

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iv) $\lim_{t \to p} h(t) \cdot f(t) = \begin{cases} +\infty, & \text{se } K > 0 \\ -\infty, & \text{se } K < 0 \\ \text{Indeterminação}, & \text{se } K = 0. \end{cases}$

$\hookrightarrow$ "$K \cdot \infty$" $\hookrightarrow$ "$0 \cdot \infty$"

v) $\lim_{t \to p} f(t) + h(t) = +\infty$

$\hookrightarrow$ "$\infty + c = \infty$"

Ex.:

$$\lim_{t \to +\infty} e^{-t} \cdot t = 0$$

$$\lim_{t \to +\infty} \overset{k=0}{k} \cdot t = \lim_{t \to +\infty} 0 = 0$$

Obs.: As propriedades acima continuam válidas se substituirmos $\underline{p}$ por $+\infty$ ou $-\infty$.

b) Limites finitos no infinito

$$\lim_{t \to +\infty} f(t) = L \,, \quad \lim_{t \to +\infty} g(t) = M$$

$\hookrightarrow$ Análogo às propriedades algébricas já vistas para limites finitos.

Ex.: $\rightarrow \lim_{t \to +\infty} f(t) \underset{\times}{\pm} g(t) = L \underset{\times}{\pm} M$

$$\rightarrow \lim_{t \to +\infty} \frac{f(t)}{g(t)} = \frac{L}{M} \,, \text{ se } M \neq 0.$$

Ex.:

$$\lim t^2 = +\infty$$

Ex.:

$$\lim_{t \to +\infty} t^2 = +\infty$$

$$\lim_{t \to +\infty} 5t^2 + 2t + 1 =$$

$$\underbrace{\lim_{t \to +\infty} 5t^2 + \lim_{t \to +\infty} 2t}_{\infty} + \underbrace{\lim_{t \to +\infty} 1}_{1}$$

$\to +\infty$ $\qquad$ $+\infty$

$= +\infty$

$\to$ "$\infty - \infty$"

$$\lim_{t \to +\infty} t^2 - 5t + 6 = \lim_{t \to +\infty} t^2 \cdot \underbrace{\left(1 - \frac{5}{t} + \frac{6}{t^2}\right)}_{\to 1}$$

$+\infty$ $\qquad$ 0 $\qquad$ 0

$= 1 - 0 + 0 = 1$

~~$h(t)$~~ $\to 1$

$= +\infty$

$$\lim_{t \to -\infty} -3t^5 + 2t^4 + t^2 + 1 =$$

$$= \lim_{t \to -\infty} t^5 \cdot \underbrace{\left(-3 + \frac{2}{t} + \frac{1}{t^3} + \frac{1}{t^5}\right)}_{\to -3}$$

$-\infty$ $\qquad$ 0 $\qquad$ 0 $\qquad$ 0

$= +\infty$

$\to$ função racional.

$$\lim \frac{t^2 + 3}{} \to \text{"}\frac{\infty}{\infty}\text{"}$$

$$t \to +\infty \quad 2t+1$$

$$= \lim_{t\to+\infty} \frac{\overbrace{\cancel{t^2}\cdot\left(1+\cancel{\tfrac{3}{t^2}}^{\to 0}\right)}^{\to 1}}{\underbrace{\cancel{t}\cdot\left(2+\cancel{\tfrac{1}{t}}^{\to 0}\right)}_{\to 2}} = \lim_{t\to+\infty} \cancel{t}^{\to +\infty} \cdot \underbrace{\left[\frac{\left(1+\frac{3}{t^2}\right)}{\left(2+1/t\right)}\right]}_{\to 1/2}$$

$$= +\infty.$$

Obs.: $\displaystyle \lim_{t\to 0} \frac{\sin t}{t} = 1$

$$\parallel$$

$$\lim_{t\to 0} \frac{1}{t} \cdot \cancel{\sin t}^{\to 0} = \lim_{t\to 0} 0 = 0$$

$\to$ $$\lim_{t\to-\infty} \frac{t^3+1}{2t^3+5t+9} = \lim_{t\to-\infty} \frac{\cancel{t^3}\cdot\left(1+\cancel{\frac{1}{t^3}}^{\to 0}\right)}{\cancel{t^3}\cdot\left(2+\underbrace{\cancel{\frac{5}{t^2}}}_{0}+\underbrace{\cancel{\frac{9}{t^3}}}_{0}\right)} \to 1/2$$

$$= \frac{1}{2}$$

$\to$ $$\lim_{t\to+\infty} \frac{t+3}{t^2+5t+1} = \lim_{t\to+} \frac{t\cdot(1+3/t)}{t^{\cancel{2}}\cdot\left(1+\frac{5}{t}+\frac{1}{t^2}\right)}$$

$$= \lim_{t \to +\infty} \left(\frac{1 + \cancel{\frac{3}{t}}^{\to 0}}{1 + \cancel{\frac{5}{t}}^{\to 0} + \cancel{\frac{1}{t^2}}^{\to 0}} \right)_{\to 1} \cdot \cancel{\frac{1}{t}}^{\to 0} = 1 \cdot 0 = 0.$$

$$\to \lim_{t \to \pm\infty} \frac{p(t)}{q(t)} = \begin{cases} \pm\infty, & \text{se } \partial(p) > \partial(q) \\ 0, & \text{se } \partial(p) < \partial(q) \\ \frac{a_n}{b_n}; & \text{se } \partial(p) = \partial(q) = n. \end{cases}$$

$$\to \lim_{t \to +\infty} \sqrt{t^2+2} - t \to \text{“}\infty - \infty\text{”}$$

$$\lim_{t \to +\infty} \sqrt{t^2 \cdot \left(1+\frac{2}{t^2}\right)} - t$$

$$= \lim_{t \to +\infty} t \cdot \sqrt{1+2/t^2} - t$$

$$= \lim_{t \to +\infty} t \cdot \left(\sqrt{1+2/t^2} - 1 \right) \to \text{“}0 \cdot \infty\text{”}$$

X

$$\to \lim_{t \to +\infty} \left(\sqrt{t^2+2} - t \right) \cdot \frac{\sqrt{t^2+2} + t}{\sqrt{t^2+2} + t}$$

$$= \lim_{t \to +\infty} \frac{\cancel{t^2} + 2 - \cancel{t^2}}{\sqrt{t^2+2} + t} = \lim_{t \to +\infty} \frac{\overset{\to 2}{2}}{\underbrace{\left(\sqrt{t^2+2} + t\right)}_{\to +\infty}}$$

$$= 0$$