Indeterminações do tipo "$\frac{0}{0}$" no limite

$\frac{0}{0}$

$\longrightarrow \frac{1}{0} \to$ Não existe!!

$$\lim_{t\to 0} \frac{\cos t}{t} = \frac{\lim_{t\to 0}\cos t}{\lim_{t\to 0} t} = \frac{1}{0} \quad X$$

"$\frac{1}{0}$"

X

**Caso 1:** $\lim_{t\to\lambda} \frac{p(t)}{q(t)}$ → Estratégia: Fatorar os polinômios!

Ex.: $\lim_{t\to 1} \frac{t^2-1}{t-1}$ → "$\frac{0}{0}$"

$$= \lim_{t\to 1} \frac{\cancel{(t-1)}\cdot(t+1)}{\cancel{t-1}}$$

$$= \lim_{t\to 1} t+1 = 2$$

Ex.: $\lim_{t\to -1} \frac{t^2-t-2}{t+1}$ → "$\frac{0}{0}$"

Obs.: $t^2-t-2 = (t+1)\cdot(t-2)$

$$= \lim \frac{\cancel{(t+1)}\cdot(t-2)}{}$$

$$= \lim_{t \to -1} \frac{\cancel{(t+1)} \cdot (t-2)}{\cancel{t+1}}$$

$$t^2 - t - 2 = (t+1) \cdot (t-2)$$

$$= \lim_{t \to -1} t-2 = -3$$

Ex:
$$\lim_{t \to 3} \frac{t^2 - 5t + 6}{t^2 - 11t + 24}$$

$$= \lim_{t \to 3} \frac{(t-2) \cdot \cancel{(t-3)}}{\cancel{(t-3)} \cdot (t-8)}$$

$$= \lim_{t \to 3} \frac{t-2}{t-8} = \frac{\lim_{t \to 3} t-2}{\lim_{t \to 3} t-8} = \frac{1}{-5} = -\frac{1}{5}.$$

Ex:
$$\lim_{t \to -1} \frac{t^3 + 2t^2 + t}{t^2 + 2t + 1}$$

$$= \lim_{t \to -1} \frac{t \cdot \cancel{(t+1)} \cdot \cancel{(t+1)}}{\cancel{(t+1)} \cdot \cancel{(t+1)}} = \lim_{t \to -1} t = -1$$

Ex:
$$\lim_{t \to 1} \frac{t - 1}{t^3 - 1} = \lim_{t \to 1} \frac{\cancel{t-1}}{\cancel{(t-1)} \cdot (t^2 + t + 1)}$$

$$= \lim \frac{1}{t^2+t+1} = 1/3$$

$$= \lim_{t \to 1} \frac{1}{t^2+t+1} = 1/3$$

Obs.: Escrevam os detalhes de como encontrar as raízes dos polinômios!!

## Caso 2: Funções envolvendo radicais

Estratégia: "Conjugado".

$$z = a + bi, \quad a, b \in \mathbb{R}, \quad i = \sqrt{-1}.$$

$$\bar{z} = a - bi$$

Ex.: $$\lim_{t \to 1} \left( \frac{\sqrt{t} - 1}{t - 1} \right) \cdot \left( \frac{\sqrt{t} + 1}{\sqrt{t} + 1} \right)$$

$$= \lim_{t \to 1} \frac{\cancel{t - 1}}{\cancel{(t-1)} \cdot (\sqrt{t} + 1)} = \lim_{t \to 1} \frac{1}{\sqrt{t} + 1}$$

$$= \frac{\lim_{t \to 1} 1}{\lim_{t \to 1} \sqrt{t} + 1} = \frac{1}{\sqrt{1} + 1} = \frac{1}{2}$$

"0"

0

Ex.: $$\lim \left( \frac{\sqrt{t^2+9} - 3}{t^2} \right) \cdot \left( \frac{\sqrt{t^2+9} + 3}{\quad} \right)$$

Ex: $\lim\limits_{t \to 0} \left( \dfrac{\sqrt{t^2+9} - 3}{t^2} \right) \cdot \left( \dfrac{\sqrt{t^2+9} + 3}{\sqrt{t^2+9} + 3} \right)$

$= \lim\limits_{t \to 0} \dfrac{(\cancel{t^2} + \cancel{9} - \cancel{9})}{\cancel{t^2} \cdot \left(\sqrt{t^2+9} + 3\right)}$

$= \lim\limits_{t \to 0} \dfrac{1}{\sqrt{t^2+9} + 3} = \dfrac{1}{6}$

Ex: $\lim\limits_{t \to 1} \dfrac{\sqrt[3]{t} - 1}{t - 1}$ $\rightarrow$ Exercício!!!

$a^2 - b^2 = (a-b) \cdot (a+b)$

$a^3 - b^3 = (a-b) \cdot (a^2 + ab + b^2)$

$a = \sqrt{u}$ , $b = \sqrt{w}$

$u - w = \left(\sqrt{u} - \sqrt{w}\right) \cdot \left(\sqrt{u} + \sqrt{w}\right)$

$\sqrt{u} - \sqrt{w} = \dfrac{u - w}{\sqrt{u} + \sqrt{w}}$

$a = \sqrt[3]{u}$ $\quad$ $b = \sqrt[3]{w}$

# Caso 3: Funções trigonométricas

<u>Afirmação:</u> $\lim\limits_{t \to 0} \dfrac{\sin t}{t} = 1 \rightarrow$ "0/0"

↳ Limite trigonométrico fundamental (L.T.F.)

$$\lim_{t \to 0} \left( \frac{\cos t - 1}{t^2} \right) \cdot \left( \frac{\cos t + 1}{\cos t + 1} \right) \rightarrow \text{"}\frac{0}{0}\text{"}$$

$$(\sin y)^2 + (\cos y)^2 = 1$$

$$= \lim_{t \to 0} \frac{(\cos t)^2 - 1}{t^2 \cdot (\cos t + 1)}$$

$$= \lim_{t \to 0} \frac{-(\sin t)^2}{t^2 \cdot (\cos t + 1)} = \lim_{t \to 0} - \frac{\sin t}{t} \cdot \frac{\sin t}{t} \cdot \frac{1}{(\cos t + 1)}$$

$$= - \underbrace{\lim_{t \to 0} \frac{\sin t}{t}}_{\text{L.T.F. } 1} \cdot \underbrace{\lim_{t \to 0} \frac{\sin t}{t}}_{\text{L.T.F. } 1} \cdot \underbrace{\lim_{t \to 0} \frac{1}{\cos t + 1}}_{\frac{1}{2}}$$

$$= -1 \cdot 1 \cdot \frac{1}{2} = -\frac{1}{2}$$

Ex: $\lim \quad \cos t \quad - 1$

Ex: $\lim_{t \to 0} \dfrac{\cos t - 1}{t}$ ... Exercício!

## Teorema do confronto

Considere funções $f, g, h : I \subset \mathbb{R} \to \mathbb{R}$.

Suponha que $\exists\, M \in \mathbb{R}$ t.q.

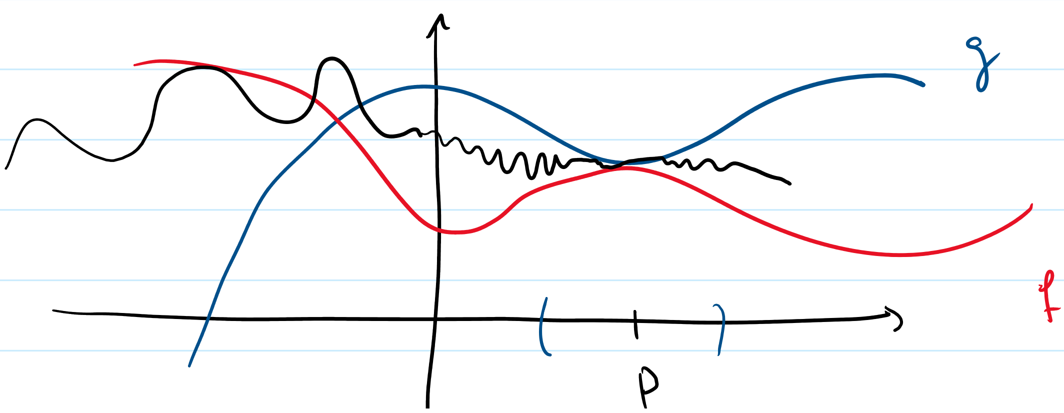
$$\lim_{t \to p} f(t) = \lim_{t \to p} g(t) = M.$$

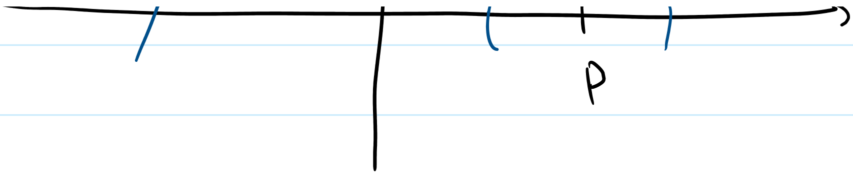
Suponha ainda que

$$f(t) \leq h(t) \leq g(t), \quad \forall t \text{ numa vizinhança de } p.$$

Então:

$$\exists \lim_{t \to p} h(t) = M.$$






Obs.: Teor. do confronto $\Rightarrow$ L.T.F.