MECE 5397

Project B – Diffusion Equation

Write a computer code to solve the two-dimensional diffusion equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{\partial u}{\partial t} \tag{1}$$

The domain of interest is the rectangle

$$a_x < x < b_x\,, \qquad a_y < y < b_y \tag{2}$$

and the boundary conditions

$$u(x, y = b_y) = (1 - e^{-\lambda t}) f_a(x)\,, \qquad u(x, y = a_y) = (1 - e^{-\lambda t}) g_a(x)\,, \tag{3}$$

$$\left.\frac{\partial u}{\partial x}\right|_{x=a_x} = 0\,, \qquad u(x = b_x, y) = (1 - e^{-\lambda t}) \left[g_a(b_x) + \frac{y - a_y}{b_y - a_y} [f_a(b_x) - g_a(b_x)] \right] \tag{4}$$

$$a_x = a_y = 0\,, \qquad b_x = b_y = 2\pi \tag{5}$$

$$g_a(x) = (x - a_x)^2 \cos x\,, \qquad f_a(x) = x(x - a_x)^2 \tag{6}$$

Use a value between 0.05 and 0.5 for λ. The initial condition is

$$u(x, y, t = 0) = 0\,. \tag{7}$$

Use ghost node(s) for Neumann condition(s).
Carry out the time integration to steady state, i.e., until the result becomes independent of time.