Stellenbosch University

Department of Industrial Engineering

Optimisation (Eng) 774/874

Pre-block Assignment — Due on 1 August 2021 at 23:55
(On optimisation problems)

Instructions

Kindly complete this assignment and submit it as a single electronic submission in PDF format on SUNLearn by the above date.

Optimisation (Eng) 774 students should attempt Questions 1–3 only, while all four questions should be attempted by Optimisation (Eng) 874 students.

Late submission (*i.e.* later than the required submission time mentioned above) will be penalised by 20% per day or part thereof. A final cut-off per assignment will apply after four days, and no assignments will be accepted after this cut-off (*i.e.* a 100% penalty will be applied). Please work independently and ensure that the assignment you submit contains your own work.

Question 1 [36]

(a) Within the context of an optimisation problem, explain the meaning of each of the following problem components: [6]

i. An objective function,

ii. The decision variables,

iii. The constraints.

(b) A variety of different classes of optimisation problems exist and it is important to be able classify a given problem as belonging to one of these classes before attempting to solve the problem, because algorithms for solving optimisation problems are typically tailored for certain classes of problems. Provide a concise descriptions for each of the following classes of optimisation problems. [16]

i. Combinatorial optimisation problems,

ii. Integer programming problems,

iii. Linear programming problems,

iv. Nonlinear programming problems,

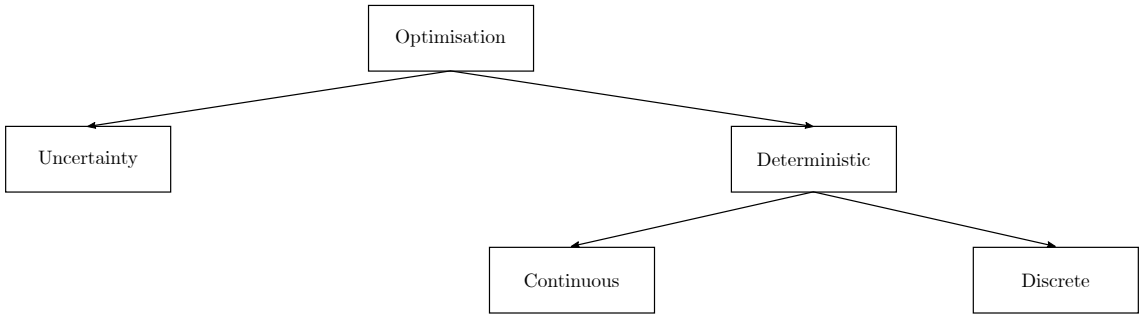
v. Quadratic programming problems,

vi. Mixed integer programming problems,

vii. Robust optimisation problems,

viii. Stochastic programming problems.

(c) Complete the following diagram by associating each of the problem classes in (b) above with one of the (leaf) categories in the following classification tree: [8]



(d) Describe the differences (in terms of solution quality returned and execution speed) between the following three types of algorithms for solving optimisation problems: [6]

i. An exact algorithm,

i. An approximate algorithm (such as a heuristic or metaheuristic),

i. An approximation algorithm.

Question 2 [26]

Consider the following examples of real-world optimisation problems. For each problem, define appropriate decision variables, and formulate the the objective function and constraints mathematically. Finally, classify the optimisation problem according as belonging to one of the classes mentioned in Question 1(b). You are *not* required to solve the problems.

(a) WooDécor manufactures two types of wooden décor items: Floating shelves and rails. A floating shelf sells for R500 and is manufactured from R200 worth of raw materials. Each floating shelf manufactured increases WooDécor's variable labour and overhead costs by R250. A rail sells for R450 and is manufactured from R220 worth of raw materials. Each rail manufactured increases WooDécor's variable labour and overhead costs by R200. The manufacturing of wooden floating shelves and rails requires two types of skilled labour: Carpentry labour and finishing labour. A floating shelve requires 3 hours of finishing labour and 1.5 hours of carpentry labour, while a rail requires 1.5 hours of finishing labour and 1.5 hours of carpentry labour. Each week, WooDécor can obtain all the raw materials it requires, but only 150 finishing hours and 120 carpentry hours are available each week. Demand for rails is unlimited, but at most 30 floating shelves are bought each week. WooDécor wishes to maximise its weekly profit (revenues less costs). Formulate an optimisation problem that will inform of WooDécor how to go about maximising its weekly profit. [10]

(b) Investo is considering four investment options. Investment 1 is expected to yield a *net present value* (NPV) of R 12 000; investment 2, an NPV of R 16 500; investment 3, an NPV of R 9 000; and investment 4, an NPV of R 6 000. Investing in an investment requires a certain cash outflow at the present time: Investment 1, R 3 750; investment 2, R 5 250; investment 3, R 3 000; and investment 4, R 2 250. Investo can either invest fully in an investment, or not at

all (fractions of investments are not possible). Currently, R 10 500 is available for investment and Investo cannot invest in more than two options. Formulate an optimisation problem whose solution will inform Investo how to maximize the NPV obtained from the investments selected. [10]

(c) Suppose you wish to design a custom fish tank with a square bottom and an open top of largest possible enclosed volume, subject to the constraint of using at most four square metres of glass. Formulate an optimisation problem whose solution will inform you as to optimal dimensions for the fish tank. [6]

Question 3 [10]

The complexity of an optimisation problem relates to the computational effort expended by the best algorithm available for solving the problem. In order to formalise this notion, each optimisation problem is associated with underlying decision problem (posed as a yes-no question), which can be categorised into different complexity classes, of which the most prominent are P and NP.

(a) Provide a concise definition for each of the following complexity classes for decision problems associated with optimisation problems: [8]

i. The complexity class P,

ii. The complexity class NP,

ii. The complexity class NP-complete,

iv. The complexity class NP-hard.

(b) Draw a venn-diagram illustrating the inclusion and exclusion relationships between the complexity classes in (a) above. [2]

Question 4 [8]

(This question is only applicable to Optimisation (Eng) 874 students.)

The question of whether or not P = NP is one the so-called *millennium problems* posed by the Clay Institute (with a prize of one million USD for a first-round resolution). Discuss your understanding of what this question entails. [8]