Hyperbolically speaking

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Submitted to Proceedings of the National Academy of Sciences of the United States of America

Language is not always meant to be interpreted literally. In the real world, people often use imprecise or even exaggerated descriptions to communicate their experiences and opinions. In this paper we focus on the non-literal interpretation of number words, in particular pragmatic halo–the imprecise interpretation of round numbers–and hyperbole–the affective subtext conveyed by exaggerated and unlikely numbers. Building on recent models of pragmatics as rational inference between speaker and listener, we model number interpretation as social inference regarding the communicative goal, meaning, and affective subtext of a numeric utterance. Our model accurately predicts human interpretation of number words with regards to pragmatic halo, hyperbole, and their interaction, suggesting that modeling pragmatics as rational inference allows us to capture a wide range of effects in non-literal language understanding.

Pragmatics | Language understanding | Computational modeling

Abbreviations: IR, Incongruity Resolution

Introduction

Imagine a conversation with a friend about a new restaurant where she recently dined. Your friend says, "It took us 30 minutes to get a table." The number expression 30 can be interpreted to mean somewhere within a range of numbers, and she does not care very much if you know exactly how long she waited. On the other hand, suppose your friend says: "It took us 32 minutes to get a table." You are now more likely to interpret the number expression to mean exactly 32, and believe that she cares for you to know that she waited for exactly that long. Now imagine that your friend says: "It took us a million hours to get a table." You would probably interpret this to mean that she thinks the wait was much too long. In mathematics, the numbers 30, 32, and $1,000,000$ have precise meanings that cannot be confused with each other. In everyday language, however, numbers are treated more flexibly, and people do not always mean what they literally say. In this paper, we examine the pragmatic interpretation of number words using behavioral experiments and computational modeling. In particular, we explore the phenomena of *pragmatic halo*–the imprecise interpretation of round numbers–and *hyperbole*–the affective subtext conveyed by exaggerated and unlikely numbers. Building on recent models of pragmatics as rational inference between speaker and listener, we propose a computational model that captures both halo and hyperbole effects as well as their interaction.

Previous research has shown that people tend to interpret simple number expressions imprecisely and complex number expressions precisely, termed the pragmatic halo effect [**?**]. Krifka [**?**] described how the pragmatic halo can be explained with the assumption that speakers prefer number expressions that are shorter and less costly to utter. Under this assumption, listeners favor approximate interpretations of round numbers even if there is no general bias for approximate interpretations. Bastiaanse [**?**] further argued that interpreting round numbers as approximate is a rational choice that can be formalized via game theory [**?**]. Here, the model we propose captures these arguments within a Bayesian framework for pragmatic inference that takes into account the speaker's communicative goal. Given uncertainty about whether a speaker wishes to communicate precisely or approximately as well as knowledge that some number words are more costly than others, we show that a rational listener will interpret the more costly number words as more precise.

Hyperbolic utterances often express important interpersonal meaning beyond the literal meaning of the statement, and successful interpretation of such expressions hinges on the listener's ability to infer the speaker's intentions [**?**, **?**, **?**]. Previous work has focused on cues for verbal irony and exaggeration, such as a slow speaking rate, heavy stress, nasalization, and interjections [**?**, **?**]. Although lexical and prosodic information has been shown to be important for human and machine detection of hyperbole [**?**, **?**, **?**], we argue that common prior knowledge about the relevant topic also plays an important role in identifying and interpreting hyperbolic statements. That is, part of what makes a statement likely to receive a hyperbolic interpretation is that both speaker and listener know that the literal meaning is very unlikely.

In this paper we describe how these non-literal uses of number terms can be explained as effects of probabilistic inference over recursive social models. We build on a traditional approach within linguistics, which views communication as an interaction between rational, cooperative agents [**?**, **?**]. The speaker in a conversation has a message to communicate, and her goal is for the listener to understand this message. The listener's goal is to infer the intended meaning of the message from the speaker's utterances. The listener performs Bayesian inference to infer the intended meaning, while the speaker is a rational planner who takes into account how the listener will interpret each utterance. Recent work [**?**, **?**] has shown that a simple formal model of this interaction is able to quantitatively explain human pragmatic reasoning. We will be working with extensions of this model, in which the speaker and listener recursively reason about one another [**?**, **?**, **?**]. Here, the listener reasons about a speaker who optimizes informativeness of her utterances; the speaker optimizes assuming that the listener is reasoning in this way about the speaker; and so on. These models of recursive social reasoning are closely related to signaling games [**?**, **?**]. In the case of number words, we will show how this framework can be applied to capture pragmatic halo, hyperbole, and their interactions.

Results

Simulations.

Reserved for Publication Footnotes

Simulation 1

Simulation 2

Real Data.

Discussion

Our behavioral results show that people often interpret numerical expressions in imprecise or non-literal ways, even in the absence of phonological cues. However, there are complex patterns in this non-literal usage: hyperbole depends on prior probability, conveys affective subtext, and interacts with the halo of round numbers. Building upon recent theoretical work, our computational model shows that rational recursive reasoning between speaker and listener can capture these effects of pragmatic halo and hyperbole. Our model introduces an "affect" dimension to language interpretation, such that hyperbolic utterances efficiently convey information regarding the speaker's opinion and affective state. However, the subtext conveyed by language often goes beyond the simple binary state used here. Future work will explore both the rich structure of subtext and the extension of this modeling framework to other cases of non-literal speech: "Bayesian models can explain *everything*."

Materials and Methods

Model.

We begin by describing a version of the framework that is able to capture the basic pragmatic halo and exaggeration effects. Each listener will be associated with a dictionary $\mathcal{D}$, which specifies the literal meaning of each possible utterance: a function assigning a probability to each meaning. (The dictionary can thus be seen as an undirected probabilistic model relating words to meanings.) The listener's dictionary determines a $literal\ interpretation$ of the utterance. We will assume that meanings are integers in a set $\mathcal{I}$, and, with a slight abuse of notation, that utterances come from the same set. For each utterance u, the dictionary entry $\mathcal{D}_u$ will be proportional to a one-dimensional normal distribution $f(x; u, \sigma^2)$ of mean u—that is the word "u" means approximately u. After hearing the utterance u, the listener L_0 updates his prior distribution P over meanings m by conditioning P on the dictionary entry for u:

$$L_0(m|u, \mathcal{D}) \propto \mathcal{D}_u(m)P(m) \quad [1]$$

$$\propto f(m; u, \sigma^2)P(m). \quad [2]$$

This literal listener provides the base case for recursive social reasoning between the speaker and listener. In general, the speaker S_n is assumed to be a rational planner who is optimizing the probability that her intended meaning m will be understood by the listener L_n while minimizing the cost of the utterance. The listener L_n performs Bayesian inference to guess the intended meaning given prior P and his internal model of the speaker S_{n-1}.

The speaker S_n chooses utterances according to a softmax decision rule that describes an approximately rational planner [**?**]:

$$S_n(u|m, \mathcal{D}) \propto e^{\lambda U_n(u|m,\mathcal{D})}, \quad [3]$$

where the constant λ captures the degree of optimality of the speaker. (We used $\lambda = 2$ in the model simulations for this section). The speaker wants to minimize both the cost $c(u)$ of the utterance and the surprisal of the intended meaning m, so the utility function U_n is defined by:

$$U_n(u|m, \mathcal{D}) = \log(L_n(m|u, \mathcal{D})) - c(u), \quad [4]$$

which combined with equation 3 leads to:

$$S_n(u|m, \mathcal{D}) \propto (L_n(m|u, \mathcal{D})e^{-c(u)})^{\lambda}. \quad [5]$$

The speaker S_0 reasons about the literal listener L_0, and assumes that this listener shares her dictionary $\mathcal{D}$. However, in general the listener will be uncertain about the dictionary being used by the speaker, which we call $lexical\ uncertainty$ [**?**]. This lexical uncertainty provides room for context-specific uses of certain words. To determine the speaker's intended meaning, the listener will therefore marginalize over the possible dictionaries being used:

$$L_n(m|u, \mathcal{D}) \propto \sum_{\mathcal{D}_i} P(m)P(\mathcal{D}_i)S_{n-1}(u|m, \mathcal{D}_i). \quad [6]$$

Substituting equation 6 into equation 5 we can see that the dictionary $\mathcal{D}$ plays no role in the reasoning of the listener $L_n(\cdot|u, \mathcal{D})$ or the speaker $S_n(\cdot|m, \mathcal{D})$ for $n > 0$. This leads us to:

$$L_n(m|u) := L_n(m|u, \mathcal{D}) \qquad \text{if } n > 0 \quad [7]$$

$$S_n(u|m) := S_n(u|m, \mathcal{D}) \qquad \text{if } n > 0. \quad [8]$$

We assume in this paper that the prior probability on dictionaries $P(\mathcal{D}_i)$ is uniformly distributed across a set of possible dictionaries. Initially, the dictionary $\mathcal{D}$ simply determines the standard deviation σ_u associated with each number utterance u, therefore specifying how precisely each utterance will be interpreted by the literal listener. Lexical uncertainty thus represents uncertainty about how precisely the speaker believes her utterances will be interpreted.

Pragmatic halo.

This model predicts the pragmatic halo effect. For now we may assume that $P(m)$ is uniform over meanings, but that $c(u)$ varies—some utterances are more costly than others (whether this cost comes from length, frequency, or other factors).

To understand why the model predicts that more complex utterances will be interpreted more precisely, we will look at the simplest possible example of this effect (Figure **??**a). Suppose there are two possible meanings, numbers 1 and 2, and two possible utterances, number words "one" and "two." Suppose that "two" is much more expensive than "one." First suppose the speaker wants to communicate 1. In this case, the speaker will almost never choose to communicate using the utterance "two." The utterance "two" is more expensive than utterance "one," and its literal meaning is strictly farther away from the speaker's intended meaning, implying that it receives a small likelihood from equation 5. Because utterance "one" is cheaper and closer to the intended meaning, it will receive most of the probability mass allocated by this equation.

In contrast, suppose the speaker wants to communicate 2. In this case the two utterances are more evenly balanced, with two factors pulling against each other in equation 5. The literal meaning of utterance "two" is closer to the intended meaning, but utterance "one" is cheaper. The utterance "one" could therefore be used by speakers trying to communicate either meaning, while the utterance "two" will only be used by speakers trying to communicate 2. When the listener uses equation 6 to infer the speaker's intended meaning, he will reason about the likelihood of the utterances being produced by each speaker, and infer that the utterance "two" means 2, while the utterance "one" is ambiguous between the two meanings. It follows that "two" will be assigned a more precise meaning which peaked on 2. Figure **??**a illustrates the pragmatic effect that our model produces. Utterance "two" is assigned a higher cost than utterance "one", and the prior probabilities of the meanings 1 and 2 are identical. After eight levels of recursion between the listener and speaker models, the listener assigns similar probabilities for meanings 1 and 2 to the utterance "one," and a probability of 1 for meaning 2 to the more expensive utterance "two." Our model thus interprets the cheaper utterance loosely and the expensive utterance precisely—this is the pragmatic halo if we assume cost is related to "roundness" of the numbers.

Exaggeration.

We now turn to the effect of the prior distribution over meanings, $P(m)$, and pragmatic exaggeration, i.e. the non-literal interpretation of utterances with extreme meanings. Pragmatic halo results from matched prior probabilities but different utterance costs; we will show that exaggeration is the pragmatic effect that results from matched costs but differing prior probabilities. Hence, we set the cost of all utterances $c(u) = 0$, and set the prior distribution over meanings P to be a unimodal distribution over the numbers $\mathcal{I}$.

Given these assumptions, the model predicts that utterances with unlikely literal meanings will have their interpretations shifted towards the prior, while utterances with likely meanings will be interpreted literally (Figure **??**b). To illustrate this, we again consider an example with two meanings, 1 and 2, and two utterances, "one" and "two." Suppose that the meaning 1 is much more likely $a\ priori$ than 2. If the dictionary entry $\mathcal{D}_{\text{"two"}}$ for the utterance "two" is vague, then the literal listener in equation 1 will revert to his prior and interpret this utterance as likely meaning

1. In contrast, the literal listener will never interpret "one" as meaning *2*: whether the dictionary entry for $\mathcal{D}_{\text{``one''}}$ is vague or precise, the listener's prior will bias him towards interpreting the utterance as meaning *1*. It follows that when the speaker reasons about the listener and chooses an utterance according to equation 5, she will sometimes use the utterance "two" to convey the meaning *1*, but will never use the utterance "one" to convey the meaning *2*. Thus the utterance "two" may be used by speakers intending either meaning, and therefore may be interpreted as exaggerated, while the utterance "one" will only be used by speakers intending meaning *1*, and will be interpreted literally. That is, because 2 is an unlikely meaning, the word "two" is likely to be used loosely. Figure **??**b illustrates the exaggeration effect that our model produces. After eight levels of recursion between the listener and speaker models, the listener assigns a probability of 1 for meaning 1 to utterance "one." However, it assigns a higher probability of meaning 1 to utterance "two" as well, even though the literal meaning of "two" is *2*. Our model thus interprets the utterance whose literal meaning is highly unlikely as exaggerated and as lower than the literal meaning.

Affect and subtext.

How can we capture the subtext of a hyperbolic statement? Hyperbole is similar to exaggeration, except that additional information about the $affect$ of the meaning is conveyed. Affect is a second dimension of meaning, separate from the number value. If A is the set of possible affects, then the set of possible meanings M is given by:

$$M = \mathcal{I} \times A. \tag{9}$$

We extend the model to be compatible with meanings that consist of number-affect pairs. The dictionary entry $\mathcal{D}_u$ for an utterance u now consists of a Gaussian centered around u as before, as well as a truth function $T_u : A \to \{0,1\}$ that determines which affects are compatible with u. The truth function T_u will be determined by the context. This means that in some contexts, an utterance u will convey a negative affect along with its number meaning; in other contexts it will pick out a positive affect; and in still others it will be compatible with any affect. We may modify equation 1 so that the literal listener is now defined by:

$$L_0((k,a)|u,\mathcal{D}) \propto \mathcal{D}_u(k,a)P(k,a) \tag{10}$$

$$\propto f(k;u,\sigma^2)T_u(a)P(k)P(a), \tag{11}$$

where k is the number that the speaker wants to communicate, a is the affect, and we assume for simplicity that number and affect are independent under the prior. The rest of the model is extended in a similar manner. Lexical uncertainty now means that the listener is uncertain about which truth function is associated with each utterance u. This means that while speakers may believe that a specific affect is part of the literal meaning of their utterance, listeners are uncertain about these beliefs. Importantly, the meanings cary no $a\ priori$ content about affect: in expectation, each utterance is uniform over the affect dimension.

We will now illustrate how this model predicts hyperbolic interpretations of extreme utterances (Figure **??**c). We assume that there are two number meanings, *1* and *2*, and two affects, $neutral$ and $negative$, so that there are four pairings of numbers and affects. We assume that *1* is more likely than *2*, and $neutral$ is more likely than $negative$. There are two utterances, "one" and "two." If the speaker wants to communicate *1*-$neutral$, she is likely to succeed by saying "one" whether its dictionary entry $\mathcal{D}_{\text{``one''}}$ is vague or precise: as long as the dictionary entry $\mathcal{D}_{\text{``one''}}$ is compatible with this meaning, the literal listener in equation 10 will be biased towards it. This speaker will therefore assign small probability to the utterance "two," which is less likely to be interpreted correctly. On the other hand, there are two moderately likely meanings that may lead the speaker to say "two": *2*-$neutral$ and *1*-$negative$. It is clear why the speaker would say "two" to communicate the first meaning *2*-$neutral$. For the second meaning, *1*-$negative$, the speaker may use the utterance "two" if she believes that the dictionary entry $\mathcal{D}_{\text{``two''}}$ has a vague number meaning (and is therefore compatible with meaning *1*) and that it uniquely picks out the negative affect. Because the speaker may use "two" to communicate *1*-$negative$, it provides evidence to the listener in equation 6, who is reasoning about the speaker, that this meaning was intended. This is the hyperbolic interpretation of the utterance "two." Figure **??**c illustrates the effect of subtext that our model produces. After eight levels of recursion between the listener and speaker models, the listener assigns no affective information to utterance "one," but assigns a high probability of affect to utterance "two." Our model thus interprets the utterance with the unlikely literal meaning as imprecise and conveying an affective subtext.

Our final model combines the elements of the previous models. It is intended to simultaneously capture three effects: pragmatic halo, the interpretation of extreme utterances as exaggerated, and the interpretation of exaggerated utterances as hyperbolic. This model will allow the costs of utterances to vary, as in the model of pragmatic halo; allow prior probabilities of meanings to vary, as in the model of exaggeration; and introduce affects into the meaning, as in the model of hyperbole. Formally, the model will be identical to the model of hyperbole, except that we allow for utterance costs $c(u) > 0$. In addition to the effects already described, the complete model shows interactions. For instance, pragmatic halo is more extreme for unlikely utterances. We will describe and discuss the behavior of the complete model in comparison with results from our behavioral experiment (Figure 3).

Definition 1.

Theorem 1.

Appendix

An appendix without a title.

Appendix: Appendix title

An appendix with a title.

ACKNOWLEDGMENTS. This work was partially supported by a grant from the Spanish Ministry of Science and Technology.

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