Hyperbolically speaking

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One of the biggest challenges in language understanding is that language is not always meant to be interpreted literally. In everyday situations, people often use imprecise, exaggerated, or otherwise literally false descriptions to communicate their experiences and opinions. In this paper we focus on the non-literal interpretation of number words, in particular the effects of pragmatic halo (the imprecise interpretation of round numbers) and hyperbole (the affective subtext conveyed by exaggerated and unlikely numbers). Building on recent models of pragmatics as rational inference between speaker and listener, we model number interpretation as social inference regarding the communicative goal, meaning, and affective subtext of an utterance. Our model accurately predicts humans' pragmatic interpretation of number words, and is one of the first computational models to quantitatively capture a range of effects in non-literal language understanding.

Pragmatics | Language understanding | Computational modeling

Abbreviations: IR, Incongruity Resolution

Introduction

Imagine a conversation with a friend about a new restaurant where she recently dined. Your friend says, "It took 30 minutes to get a table." You are likely to interpret this to mean she waited approximately 30 minutes. Now suppose your friend says: "It took 32 minutes to get a table." You are more likely to interpret the number expression to mean exactly 32, and believe that she cares to communicate the exact wait time. Suppose she says: "It took a million hours to get a table." You will probably interpret her to mean that the wait was shorter than a million hours, but importantly that she thinks it took much too long.

Given the incredible flexibility of language, a crucial part of a listener's job is to understand an utterance even when its literally meaning is extremely unlikely. Non-literal language understanding is one of the biggest challenges in language research, and it has been difficult to build formal models or design empirical studies that capture effects in non-literal language understanding quantitatively. In this paper, we present a computational model that predicts people's non-literal interpretation of number words. We build on a traditional approach in language understanding that views communication as an interaction between rational, cooperative agents [**?**, **?**], and show how non-literal interpretation of number terms can be explained as effects of probabilistic inference over recursive social models.

Recent work has shown that modeling communication as recursive social inference is able to quantitatively explain rich phenomena in human pragmatic reasoning [**?**, **?**, **?**, **?**]. However, a limitation of these models is that they are unable to handle utterances where the intended meaning directly contradicts the literal meaning, as is the case in metaphor ("Juliet is the sun") and hyperbole ("It took a million hours to get a table.") Here we extend the model by introducing uncertainty over the speaker's communicative goal, and propose that non-literal language understanding relies on considering the possibility of communicative goals that are distinct from what is conveyed by the literal meaning of an utterance. More concretely, a listener reasons about a speaker who optimizes informativeness of her utterances given a particular communicative goal; the speaker choses the optimal utterance assuming that the listener is reasoning in this way about the speaker; and so on. In this paper, we show how this framework of recursive social inference can be applied to capture non-literal hyperbolic interpretation of number words.

We focus on number words for three reasons: first, despite their flexible and non-literal usages in everyday language, numbers have precise literal meanings that can be easily formalized, unlike more complex concepts such as "Juliet" or "the sun." Second, number words can be systematically manipulated on a continuous scale to yield quantitative predictions. Third, there are two particular well-known phenomena regarding number interpretation: *pragmatic halo*–the imprecise interpretation of round numbers, and *hyperbole*–the affective subtext conveyed by exaggerated and unlikely numbers.

Pragmatic halo describes the phenomenon in which people tend to interpret simple number expressions imprecisely and complex number expressions precisely [**?**]. This effect has been formalized via game theory as a rational choice given different costs of utterances [**?**, **?**]. The model we propose captures these arguments within a Bayesian framework for pragmatic inference. Given uncertainty about whether a speaker wishes to communicate precisely or approximately in addition to differential utterance costs, we show that a rational listener will interpret costlier number words as more precise.

While hyperbolic utterances are literally false, listeners usually successfully infer the speaker's intended meaning and often regard hyperbole as a source of humor or signal of interpersonal closeness [**?**, **?**, **?**]. Previous work on human and machine identification of irony and hyperbole has focused on cues such as a slow speaking rate, heavy stress, nasalization, and interjections [**?**, **?**, **?**, **?**, **?**]. Here we show that common prior knowledge about the relevant topic also plays an important role in identifying and interpreting hyperbolic statements. That is, part of what makes an utterance likely to receive a hyperbolic interpretation is that both speaker and listener know that the literal meaning is extremely unlikely. Furthermore, given the possibility that the speaker wishes to convey a subjective opinion instead of simply the objective state of the world, the listener can infer the affective subtext that goes beyond the literal meaning of the utterance.

By modeling language understanding as social inference regarding the communicative goal, meaning, and affective subtext of an utterance, we show that our model captures the non-literal effects of halo and hyperbole as well as their interaction.

Reserved for Publication Footnotes

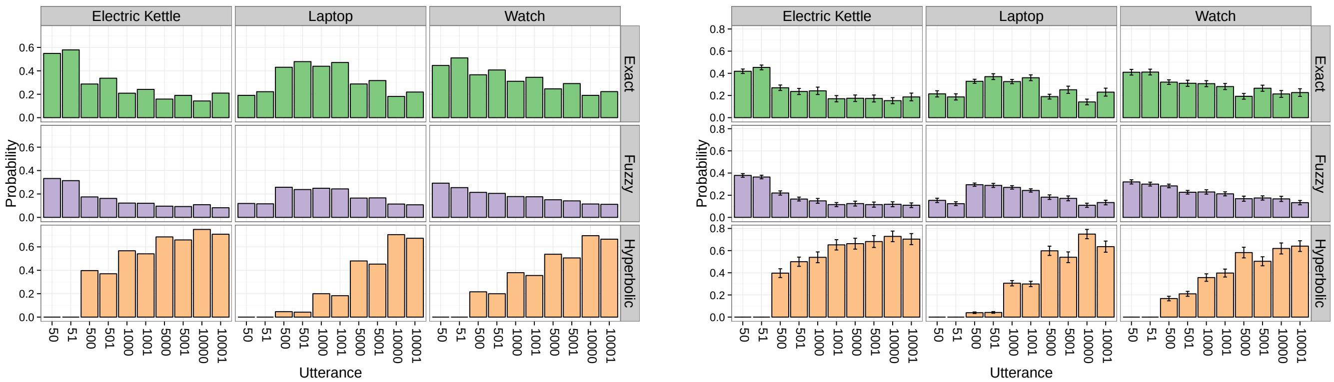
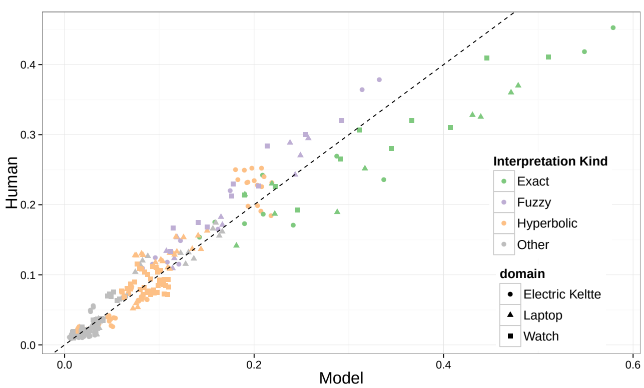


Fig. 1. Vertical panels show results for the three item kinds. Horizontal panels show the three kinds of interpretations, *exact*, *fuzzy*, and *hyperbolic*. The x axis in each panel represents the utterance. The y axis shows the probability of a kind of interpretation given the utterance.



Model	R^2
Full	0.92
Uniform prior	bad
Uniform cost	bad
Uniform affect	bad

Fig. 2. We show the model and human interpretation probabilities for all 300 items (10 utterances crossed with 10 price states and 3 item kinds. The model significantly predicts human interpretations with very high correlation ($r = 0.96$, $p < 0.0001$.)

Results

We test our model's interpretation of number expressions regarding the prices of three kinds of everyday items: *electric kettles*, *watches*, and *laptops*. We chose to focus on price because it is a common and natural topic of conversation in which people use number expressions frequently. We chose ten numbers as the set of possible price states for the three kinds of items: $S = \{50, 50', 500, 500', 1000, 1000', 5000, 5000', 10000, 10000'\}$. The ten numbers can be seen as five pairs of numbers in which each pair consists of a "round" number state s (which is a number divisable by 10) and its "sharp" counterpart s' (which we define as $s \pm k$, $k \in \{1, 2, 3\}$). We assume that the space of possible utterances U is the same as the space of possible price states. For example, a speaker can say, "That electric kettle cost u dollars," for $u \in U$.

Model. Our model considers the following five communicative goals: whether the speaker wants to convey the price precisely, imprecisely, or not at all, crossed with whether the speaker wants to convey her opinion about the price or not, minus the vacuous goal in which the speaker wants to convey nothing. We assume a uniform prior over goals. The model considers the possible price state meanings S and the prior probability $P(s)$ of each price state for a given kind of item (see Experiment 2(a) in materials). It also considers the prior conditional probability $E(s)$ of a given price state being considered expensive (see Experiment 2(b) in materials). Finally, it considers the cost of each utterance $C(u)$ and assumes that sharp numbers are costlier to utter than round numbers.

Given the formal setup of our model described in the methods section, we obtained posterior meaning distributions for each of the ten numerical utterances for each of the three item kinds. Figure 3 in the Appendix shows the full meaning distribution for each utterance. We broke down the interpretations into three separate kinds: *exact interpretations*, which are interpretations that are identical to the utterance (e.g. when"1000" is interpreted as meaning 1000); *fuzzy interpretations*, which are interpretations that are counterpart to the utterance (e.g. "1000" interpreted as meaning 1001); and *hyperbolic interpretations*, which are interpretations that are smaller than the utterance besides fuzzy interpretations (e.g. "1000" interpreted as meaning 50). The model also returns the probability of an utterance being interpreted as conveying *affect* about the price being too expensive. Figure **??** summarizes these effects. We first see a basic effect of the prior, in which utterances that are more likely given the price prior of the item are more likely to be interpreted exactly or fuzzily (e.g. "1000" is more likely to be interpreted exactly or fuzzily for laptops than for electric kettles). The model demonstrates the pragmatic halo effect, in which round utterances such as "500" and "1000" are interpreted less exactly and more fuzzily than their sharp counterparts "501" and "1001." It also demonstrates the hyperbole effect, in which utterances that are less likely given the price prior are more likely to be interpreted hyperbolically (e.g. "1000" is more likely to be interpreted as $50, 51, 500$, or 501 for electric kettles than for laptops). We also see an interaction between halo and hyperbole, where round utterances such as "5000" and "10000" are more likely to be interpreted hyperbolically than their sharp counterparts.

Experiment. We conducted an experiment on humans' interpretation of number words using the same set of items, price states S, and utterances U as provided to the model. Subjects read scenarios in which a buyer makes an utterance u about the price of an item he just bought. Subjects rate the likelihood of the buyer thinking that the item was expensive, as well as the likelihood that the item had actually cost s dollars for $s \in S$ (see Experiment 1 in materials). Figure 3 in the Appendix shows the full meaning distribution for each utterance. We broke down the interpretations into *exact*, *fuzzy*, and *hyperbolic*. Figure 1 summarizes these effects. We see that round utterances tend to be interpreted less exactly and more fuzzily than their sharp counterparts, and utterances that are less likely given the price prior are more likely to be interpreted hyperbolically. We also see that round utterances are more likely to be interpreted hyperbolically.

Comparison. Our model results correlate significantly with human interpretations of number words ($r = 0.96$, $p < 0.0001$) (Figure 2). We lesion stuff and fit gets much worse.

Discussion

Our behavioral results show that people often interpret numerical expressions in imprecise or non-literal ways, even in the absence of phonological cues. However, there are complex patterns in this non-literal usage: hyperbole depends on prior probability, conveys affective subtext, and interacts with the halo of round numbers. Building upon recent theoretical work, our computational model shows that rational recursive reasoning between speaker and listener can capture these effects of pragmatic halo and hyperbole. Our model introduces an "affect" dimension to language interpretation, such that hyperbolic utterances efficiently convey information regarding the speaker's opinion and affective state. However, the subtext conveyed by language often goes beyond the simple binary state used here. Future work will explore both the rich structure of subtext and the extension of this modeling framework to other cases of non-literal speech: "Bayesian models can explain *everything*."

Materials and Methods

Model. Here we describe our model in detail. A literal listener L_0 provides the base case for recursive social reasoning between the speaker and listener. L_0 interprets an utterance literally without taking into account the speaker's communicative goals.

The literal listener L_0's interpretation of an utterance u can be given as follows:

$$L_0(m|u) \propto \sum_{<i \in \{s,s',0\}, j \in \{a,0\}>} P(m|g_{<i,j>}, u). \qquad [1]$$

L_0 interprets an utterance "n" with some uncertainty about the speaker's communicative goal regarding the price state S and the affective subtext A. We represent this goal as $g_{<S,A>}$, where $g_{<s',a>}$ represent the goal to communicate both the precise price state and affective subtext, $g_{<s,0>}$ represent the goal to communicate the imprecise price state and no affective subtext, and $g_{<0,a>}$ to communicate the affective subtext only, and so on.

L_0's interpretation m of an utterance "n" has two parts: the price state and the affect. We represent this interpretation as $m_{<S,A>}$. Given $g_{<s',\{a,0\}>}$ L_0 interprets "n" as meaning n with probability 1 given $g_{<s',\{a,0\}>}$, as meaning $n-1$, n, or $n+1$ with uniform probability given $g_{<s,\{a,0\}>}$, and as meaning the price is expensive with probability $E(n)$ when $g_{<\{s',s\},e>}$.

The speaker S_n is assumed to be a rational planner who optimizes the probability that her intended meaning m will be understood by the listener L_n and thus satisfy her communicative goal while minimizing the cost of the utterance. The speaker S_n chooses utterances according to a softmax decision rule that describes an approximately rational planner [?]:

$$S_n(u|m,g) \propto e^{\lambda U_n(u|m,g)}, \qquad [2]$$

where the constant λ captures the degree of optimality of the speaker. (We used $\lambda = 2$ in the model simulations for this section). The speaker wants to minimize both the cost $c(u)$ of the utterance and the surprisal of the intended meaning m, so the utility function U_n is defined by:

$$U_n(u|m,g) = \log(L_n(m|u,g)) - c(u), \qquad [3]$$

which combined with equation 2 leads to:

$$S_n(u|m,g) \propto (L_n(m|u,g)e^{-c(u)})^{\lambda}. \qquad [4]$$

The listener L_n performs Bayesian inference to guess the intended meaning given the prior P and his internal model of the speaker. To determine the speaker's intended meaning, the listener will marginalize over the possible goals under consideration. The listener L_n performs Bayesian inference to guess the intended meaning given the prior P and his internal model of the speakerS_{n-1}.

$$L_n(m|u) \propto \sum_i P(m)P(g_i)S_{n-1}(u|m,g_i). \qquad [5]$$

Experiment 1a: Halo and hyperbole. 120 subjects were recruited from Amazon's Mechanical Turk to participate in our experiment on number word interpretation. Each subject read 15 scenarios in which a person (e.g. Bob) buys an item (e.g. a watch) and is asked by a friend whether the item is expensive. Bob responds by saying "It cost u dollars," where $u \in \{50, 50 \pm k, 500, 500 \pm k, 1000, 1000 \pm k, 5000, 5000 \pm k, 10000, 10000 \pm k\}$. k was randomly selected from the set $\{1, 2, 3\}$ for each trial. From now on this set of prices will be referred to as U. Given this utterance, subjects rated the probability of Bob thinking that the item was expensive. They then rated the probability of the item costing the following amounts of money: 50, $50 \pm k$, 500, $500 \pm k$, 1000, $1000 \pm k$, 5000, $5000 \pm k$, 10000, $10000 \pm k$, where k was randomly selected from the set $\{1, 2, 3\}$ for each trial. From now on this set of prices will be referred to as S. Ratings for each price state were on a continuous scale from "impossible" to "Extremely likely", represented as real values between 0 and 1. We normalized subjects' ratings across price points for each trial to sum up to 1. The average normalized ratings across subjects for each item/utterance pair is shown in Figure 3.

Experiment 1a: Affective subtext.

Experiment 2a: Price prior. To obtain people's prior knowledge of the price distributions for electric kettles, laptops, and watches, 30 subjects were recruited from Amazon's Mechanical Turk. Each subject rated the probability of an electric kettle, laptop, and watch costing s dollars, where $s \in S$. Ratings for each price state were on a continuous scale from "impossible" to "Extremely likely", represented as real values between 0 and 1. We normalized subjects' ratings across price points for each trial to sum up to 1. The average normalized ratings across subjects for each item were taken as the prior probability distribution of item prices. These price distributions were used in the model to determine the prior probability of each price state.

Experiment 2b: Affect prior. To obtain people's prior knowledge of the affect likelihood given a price state, 30 subjects were recruited from Amazon's Mechanical Turk. Each subject read 15 scenarios where someone had just bought an item that cost s dollars ($s \in S$). They then rated how likely the buyer would think the item was expensive on a continuous scale ranging from "impossible" to "absolutely certain," represented as real values between 0 and 1. The average ratings for each item/price state pair were taken as the prior probability of an affect given a price state. This was used in the model to determine the prior probability of an affect given each price state.

Appendix

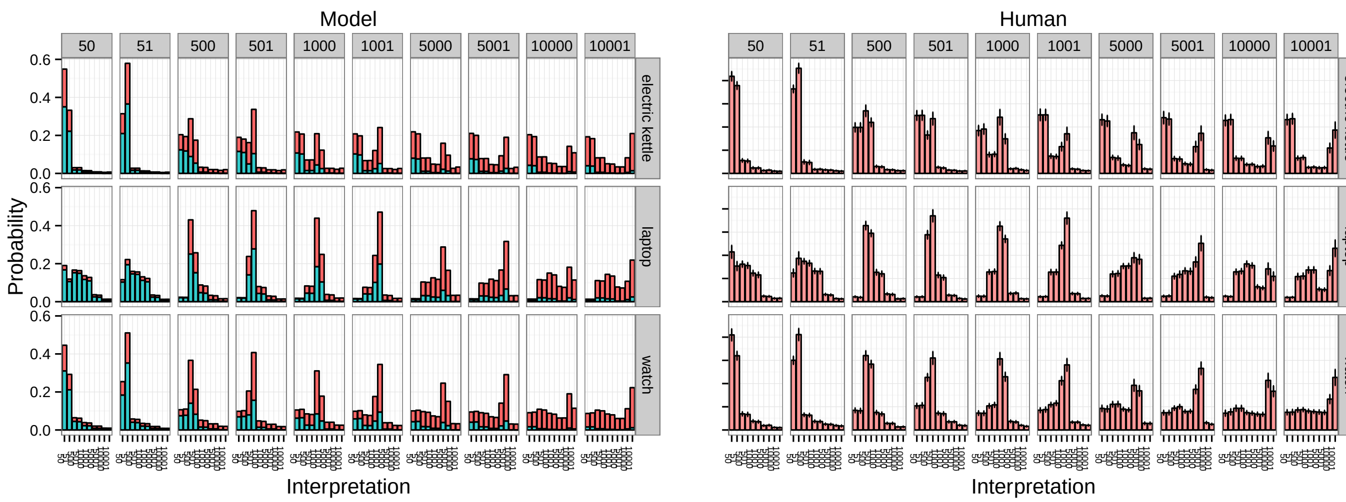


Fig. 3. Hello

Appendix: Appendix title

An appendix with a title.

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