Physics 344 Assignment 5

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1 Introduction

We would like to investigate the bond percolation phase transition in the inspected model further by considering the correlation length. We will also investigate the critical exponents related to the correlation length.

2 Methods

For a given (μ, β) combination, we generate s number of graphs typical for the chosen parameters. For each we then choose a random vertex not in the largest cluster (to ensure that we are not in an 'infinite' cluster) and then we calculate the distance to each vertex in the same cluster as the chosen vertex. We then calculate the average probability of a vertex at distance r (in the graph with all possible edges) to be in the same cluster as the chosen vertex. This function of r we then take as an approximation of the correlation function. We then calculate the correlation length ξ by fitting the correlation function to $e^{-\frac{r}{\xi}}$ using Least Squares. All the data in this report is generated in this manner.

3 Correlation Length

We would first like to get an idea of what the correlation length varies like in the $\mu - \beta$ plane. The result of sampling 40000 (μ, β) combinations is shown in the figure below. Each point is calculated using 256 graphs generated from 100×100 lattices.

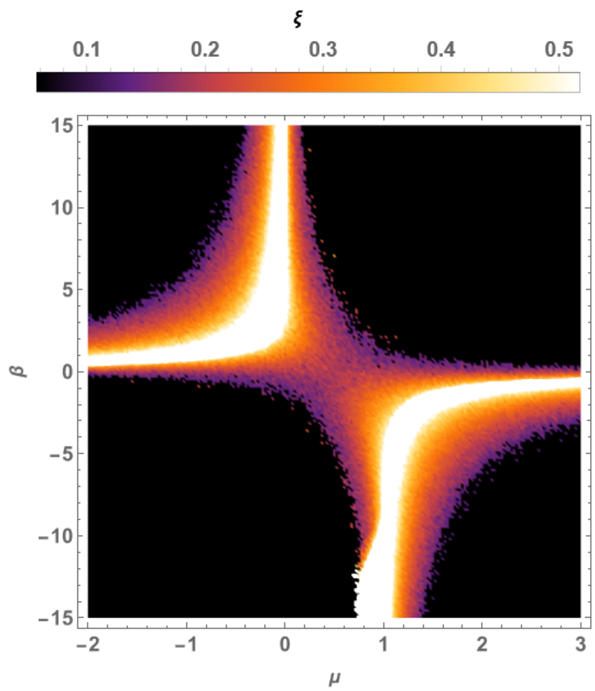

Figure 1: Correlation length in the $\mu - \beta$ plane, where the scale has been cut off at $\xi = 0.5$, such that the interesting structure near the origin is visible.

The results are shown in Figure 1 above. We can see some non-trivial behavior happens around the origin, again hinting at an interesting regime in the amorphous phase of the model lying in this region.

4 Scaling Exponents

We would now like to investigate the scaling exponents of the correlation length. We will do this by sampling the correlation length at (μ, β) combinations that lie on some straight line and crosses the divergence seen in Figure 1 above. We will then take the data seeming to follow a power-law (by identifying them lying in straight lines in the appropriate log-log plot) and fit them to the relevant scaling relation, in order to estimate the scaling exponents.

4.1 Constant β

If we take $\beta = 6$ and vary μ we would like to fit our data to the scaling relations $\xi \sim \frac{a}{(\mu_c - \mu)^{\nu_\beta}}$ below μ_c and $\xi \sim \frac{a'}{(\mu - \mu_c)^{\nu'_\beta}}$ above μ_c. We find the approximate parameters in Table 1 below.

	$\mu < \mu_c$	$\mu > \mu_c$
ν_β	0.916	0.323
μ_c	0.004	0.005
a	0.137	0.172

Table 1: Fitted parameters above and below transition.

The fits with these paramters are shown on the right in Figure 2 below.

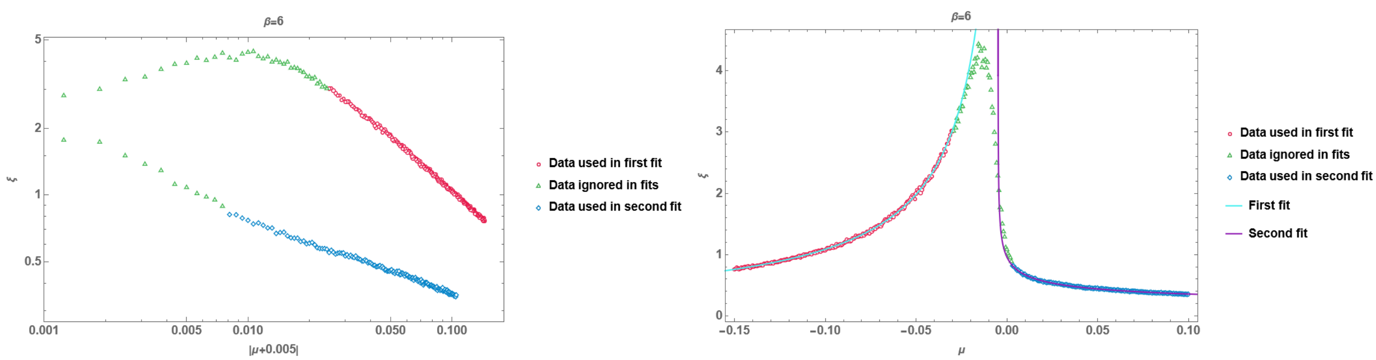

Figure 2: (Left) The correlation length ξ as a function of μ for $\beta = 6$ on a log-log plot, in order to identify the part of the data acting like a power-law. (Right) The fit of the chosen data to a power-law.

We shockingly find complete disagreement in the scaling exponent on the two sides of the transition, even though the other parameters are very similar. This is a very interesting result, and warrants further thought and investigation. The fits seem very reasonable, and it seems like no reasonable fits exist when one tries to force equality of the exponents.

4.2 Constant μ

If we take $\mu = -2$ and vary β we would like to fit our data to the scaling relations $\xi \sim \frac{b'}{(\beta_c - \beta)^{\nu'_\mu}}$ below β_c and $\xi \sim \frac{b}{(\beta - \beta_c)^{\nu_\mu}}$ above β_c. We find the approximate parameters in Table 2 below.

	$\beta < \beta_c$	$\beta > \beta_c$
ν_μ	0.316	0.788
β_c	0.440	0.434
b	0.191	0.291

Table 2: Fitted parameters above and below transition.

The fits with these parameters are shown on the right in Figure 3 below.

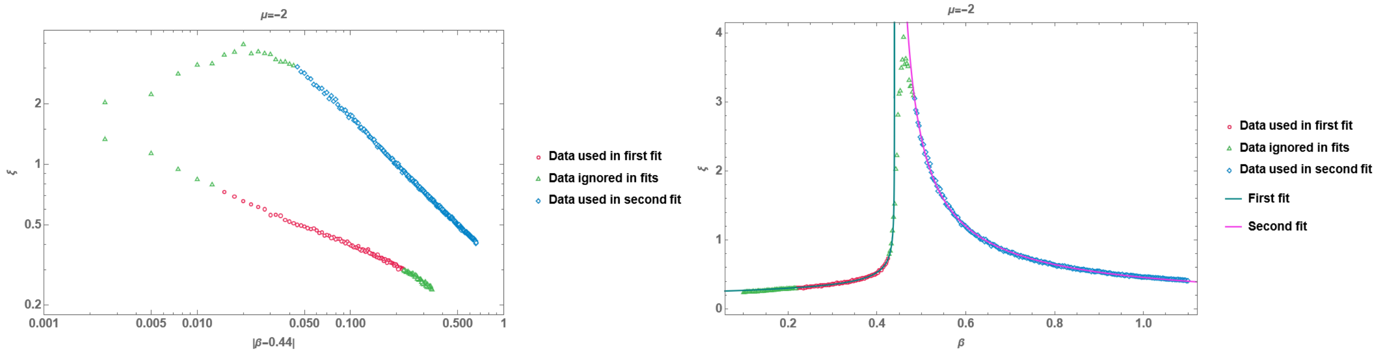

Figure 3: (Left) The correlation length ξ as a function of β for $\mu = -2$ on a log-log plot, in order to identify the part of the data acting like a power-law. (Right) The fit of the chosen data to a power-law.

We again find similar disagreement in the scaling exponent on the two sides of the transition, even though the other parameters are very similar. One might then wonder about the possibility of systematic or statistical errors in the data. In order to check for statistical errors, we can look at the 95% confidence intervals of the fitted exponents.

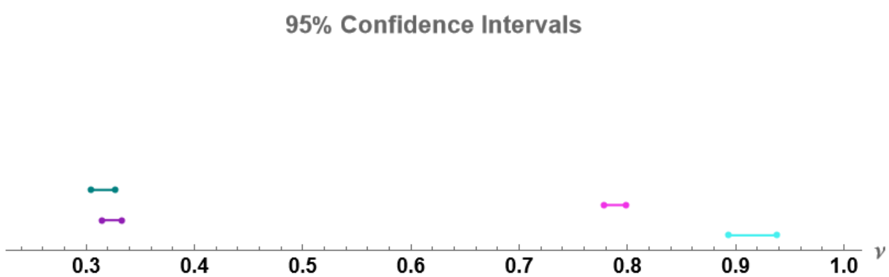

Figure 4: Confidence intervals of the fitted exponents.

4.3 $\beta = -4\mu$

Although we have so far steered clear of going near the origin, if we consider the $\beta = -4\mu$ line, we would go through the origin as well as two transitions. We then need to fit four different power-laws, ordered from smallest to largest μ. We find the approximate parameters in Table 3 below.

	$\mu < \mu_{c1}$	$\mu > \mu_{c1}$	$\mu < \mu_{c2}$	$\mu > \mu_{c2}$
ν	0.900	0.320	0.227	0.675
μ_c	-0.341	-0.350	1.05	1.04
a	0.163	0.171	0.240	0.167

Table 3: Fitted parameters on $\beta = -4\mu$ line.

The fits with these parameters are shown on the bottom of Figure 5 below.

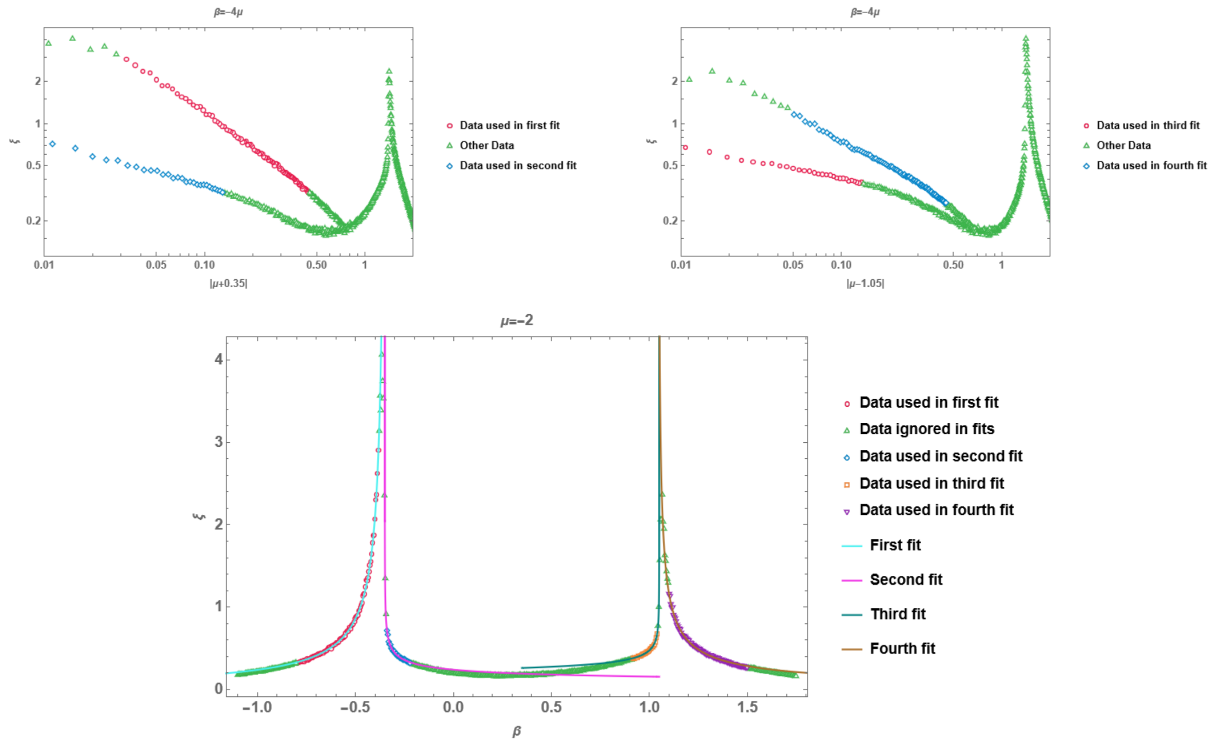

Figure 5: (Top) The correlation length ξ as a function of μ for $\beta = -4\mu$ on a log-log plot around the two points of divergence respectively, in order to identify the part of the data acting like a power-law. (Bottom) The fit of the chosen data to a power-law.

We again find a very similar result where the exponents differ significantly on either side of the transitions. We can again check for statistical errors as can be seen in Figure 6 below.

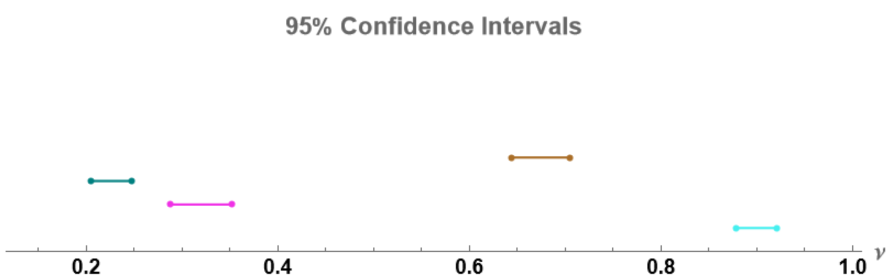

Figure 6: Confidence intervals of the fitted exponents.

5 Conclusion

We have found the correlation length and how it varies over the $\mu-\beta$ plane, as well as how it diverges close to the transition line of the bond-percolation transition at $\beta = 6$, $\mu = -2$ and $\beta = -4\mu$. For all three these cases, we find that the correlation length diverges in a power-law, but the exponents on either side of the transition differ significantly. This is quite unusual, as one expects the exponents to be the same on either side of the transition. These results seem to be robust against different choices of fitting intervals and lattice sizes, and the data seems otherwise reasonable and well-behaved. The validity and ubiquity of these disagreements in critical exponents warrants further and thorough study. If the disagreement persists, then investigating possible violations of hyper-scaling relations can also give more insight into possible reasons for these disagreements.