

Blockholder Influence

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Abstract

Blockholder influence has attracted recent interest, not only in the context of corporate boards but also in the context of decentralized autonomous organizations (DAOs). I analyze a model of project choice with dispersed information. I focus on the question whether a blockholder should delegate control to a set of inside delegates. I assume that agents' preferences are not aligned, in the sense that inside delegates derive private benefits from the acceptance of the proposal while the blockholder derives benefits from the rejection of the proposal. The blockholder chooses composition of the board between inside delegates and direct representatives of her own interests. I find that when private interests are low, or signals are imprecise there can be multiple equilibria. The most preferred equilibrium for the blockholder is the one with minimum meaningful delegation. This equilibrium also turns out to be the second best and in particular, the equilibrium value of the firm is higher compared to full delegation. This gives rise to a blockholder premium. Furthermore, I solve the information acquisition problem of the committee in closed form.

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1 Introduction

Boards of independent directors are often proposed as an effective means to protect shareholder interests, addressing the agency problem that arises from the separation of ownership and control. However, the significant influence of blockholders —shareholders with significant stakes in a firm— over board composition has raised questions about the alignment of these so-called independent boards with the interests of small shareholders. Similarly, the rise of decentralized autonomous organizations (DAOs), initially praised for their democratic and decentralized governance structures, has raised similar concerns. In DAOs, large tokenholders, such as venture capital firms, have significant voting power, and recent cases suggest that their objectives may not always align with those of smaller tokenholders². Despite the significant implications of blockholder influence, particularly regarding the protection of small stakeholders, the theoretical literature has paid relatively little attention to the influence of blockholders on boards.

In this paper, I analyze a model of project choice with dispersed information. I focus on the question of whether and under which conditions a blockholder would voluntarily delegate control to a set of delegates. Each delegate observes a signal about the quality of the project. In addition, he can exert costly effort to improve the quality of its signal. The blockholder on the other hand is not able to obtain any information about the quality of the project. Furthermore, I assume that the blockholder and the delegates preferences are not aligned: delegates derive private benefits from the acceptance of the project, while the blockholder derives private benefits from its rejection. The blockholder strategically chooses the composition of the board between delegates and her own representatives weighing the value of information provided by delegates against the risk of them being too favorable towards the proposal, leading some but not full delegation. Interestingly, in both corporate boards and DAOs, blockholders often indeed refrain from exercising their full control, opting instead to delegate some authority to a set of delegates ex-ante, even though they anticipate potential disagreements ex-post.

Building on the strategic voting literature, which explores how informed committee mem-

²see for example <https://tokeninsight.com/en/news/why-did-a16z-vote-against-uniswap-s-latest-expansion>

bers make decisions based on their pivotality and the information contained in such event, this paper extends the analysis to settings where the number of informed voters is endogenously determined by the blockholder. Unlike previous studies that fix the proportion of informed to uninformed voters or assume the distribution of heterogeneous preferences among voters ex-ante, this model allows the blockholder to choose the number of informed delegates versus direct representatives of the blockholders, thereby endogenously determining the proportion of informed voters and distribution of preferences.

In this environment, I derive conditions under which the blockholder voluntarily limits her influence on the board by delegating control to a meaningful number of delegates. The intuition is that she trades off the benefits of better information aggregation against the loss of direct control. Delegates, in turn, adjust their voting strategies based on anticipated delegation levels. The more board seats they expect the blockholder to fill with her direct representatives, the more strongly they vote in the opposite direction to offset her biased decision making. This strategic interplay can lead to multiple equilibria where the blockholder and delegates' decisions counterbalance each other. The blockholder-preferred equilibrium, which in some cases is also the unique equilibrium, involves minimal meaningful delegation where the blockholder delegates just enough seats to lose her majority. This equilibrium is also the second best, as the blockholder internalizes the information aggregation decision. Consequently, the equilibrium value of the firm is higher under minimal delegation compared to full delegation from the perspective of all shareholders, including small investors who do not derive private benefits. This results in a control premium.

In equilibrium, the blockholder always rejects proposals and effectively determines the voting rule by selecting the number of delegates. Her decision to delegate to the minimal meaningful number of informed voters leads to a scenario where all informed committee members must agree to pass a proposal - effectively implementing a unanimity requirement. This finding that unanimity among informed board members leads to optimal information aggregation differs sharply from previous studies where the number of informed board members is fixed. In those settings, unanimity rules tend to result in poor information aggregation because an individual informed voter becomes pivotal only if all other informed voters accept the proposal. The informational content of this event is so overwhelming that the voter may

disregard their private signal entirely and simply follow others. In contrast, in my setting, the blockholder's rejection motivates informed voters to accept proposals more frequently to counterbalance her influence on the vote. This behavior offsets the tendency to rely solely on being pivotal, thereby enhancing information aggregation compared to scenarios where all informed voters vote and the blockholder has no voice.

Furthermore, I solve the committee's information acquisition problem in closed form, finding that lower levels of delegation lead to higher precision, as fewer informed delegates increase the probability of an individual delegate being pivotal and the investment thus having effect. As a consequence, scenarios with blockholder representation on the committee are superior to full delegation to inside board members also in terms of providing the right incentives for delegates to become better informed.

Related Literature

This work is rooted in property rights theory, as established by Grossman and Hart (1986) and Hart and Moore (1990), which provides a framework for understanding control as the residual right of owners to make decisions in contingencies not covered by contracts due to contractual incompleteness. When control is delegated from owners to managers, agency problems arise due to the separation of ownership and control. These issues are exacerbated in firms with dispersed ownership, where collective action problems among shareholders hinder effective monitoring of management Jensen and Meckling (1976).

Blockholders can mitigate these problems. Their substantial holdings provide them with the right incentives to monitor management, thereby overcoming the collective action problem inherent in dispersed ownership (e.g. Shleifer and Vishny (1986), Admati, Pfleiderer, and Zechner (1994), Bolton and Von Thadden (1998), Maug (1998)), for a comprehensive review of the literature on blockholders, see Edmans and Holderness (2017).

However, the impact of blockholders is not always entirely positive. Empirical studies by Zingales (1994), Bianco and Casavola (1999), and La Porta et al. (1998) provide evidence that blockholders may pursue private interests not aligned with those of minority shareholders, extracting benefits at their expense—a phenomenon known as entrenchment. From a theoretical perspective, Burkart, Gromb, and Panunzi (1997) argue that while blockholders

can reduce agency costs through monitoring, excessive oversight can undermine managerial initiative and deter firm-specific investments.

Despite these concerns, blockholder-controlled firms remain prevalent, particularly in Europe. Studies by La Porta, Lopez-de-Silanes, and Shleifer (1999), Claessens, Djankov, and Lang (2000), and Barca and Becht (2001) document the widespread existence of large shareholders in European firms. Similarly, in the world of Decentralized Autonomous Organizations (DAOs)—blockchain-based entities governed collectively by their members through smart contracts—blockholders are omnipresent (Rossello (2024); Fritsch, Müller, and Wattenhofer (2022); Barbereau et al. (2023)). These observations suggest to reevaluate the role of the blockholder, suggesting that it may not always be detrimental and can, in fact, play a positive role in corporate governance even for minority shareholders.

While much of the literature focuses on the relationships between blockholders and managers, less attention has been paid to the interaction between blockholders and boards of directors. One exception is Krüger, Limbach, and Voss (2022), who analyze the conditions under which blockholders choose to have board representation. They find that blockholders weigh the benefits of enhanced monitoring against the potential market impact of perceived agency problems.

The present work explores how blockholders may voluntarily cede control over the board of directors to capitalize on the superior information held by other board members. By stepping back, blockholders can allow for better information aggregation within the board, leading to more informed decision-making that benefits the firm as a whole. This voluntary delegation contrasts with the traditional view of blockholders primarily exerting control through board representation.

This work also contributes to the understanding of information aggregation in strategic voting. Seminal works by Austen-Smith and Banks (1996) and Feddersen and Pesendorfer (1997) explore how private information is aggregated in voting settings and the implications for rational decision-making. These studies focus on strategic voting behavior, where informed committee members consider their pivotal role in the outcome. In these models, voters decide whether to support a proposal based not only on their private information but also by anticipating that they will influence the outcome only if they are pivotal. This leads

them to consider what information other voters may have received. Such strategic behavior can result in voters disregarding their private signals in favor of inferred collective information, potentially hindering optimal information aggregation. For a review of the literature on voting in committees, see Gerling et al. (2005).

Contrasting with traditional models, the current work examines a scenario where the number of informed voters is endogenously determined by a blockholder who balances private interests and information aggregation. In equilibrium, the blockholder always rejects proposals. By choosing the number of informed delegates, the blockholder implicitly sets the proportion of informed delegates required to accept a proposal—effectively determining the voting rule.

The blockholder’s privately and socially optimal decision is to delegate to the minimal meaningful number of informed voters so that they no longer hold a majority. Consequently, all informed committee members must agree to pass a proposal, implementing a unanimity requirement. This mechanism aligns the blockholder’s incentives with optimal information aggregation, ensuring that only proposals supported by all informed members are accepted.

The finding that unanimity among informed board members leads to optimal information aggregation deviates from previous studies where the number of informed board members is fixed. In traditional models, such as Feddersen and Pesendorfer (1998), the unanimity rule results in poor information aggregation because an individual informed voter becomes pivotal only if all other informed voters accept the proposal. The informational content of this event is so significant that the voter may disregard their own private signal and follow the expected collective decision.

In the current setting, however, the blockholder’s consistent rejection incentivizes informed voters to accept proposals more frequently to counterbalance their influence. This dynamic offsets the tendency of a voter to accept based solely on information inferred from being pivotal. As a result, the inferiority of the unanimity rule observed in earlier studies no longer holds, and information aggregation improves compared to scenarios where all informed voters participate and the blockholder has no voice.

Another strand of the voting literature examines how committee size influences information acquisition. Persico (2004) analyzes how the size of a committee affects members’

incentives to acquire information. When committee members perceive that they are less likely to be pivotal and that their individual vote is unlikely to decide the outcome, they have less motivation to incur the costs associated with obtaining accurate information. While larger committees have the potential to aggregate more information due to the higher number of members, the decreased individual incentives can lead to less overall information being acquired. In smaller groups, each member’s vote carries more weight, increasing the likelihood that they will invest in acquiring information. Similar themes are explored by Malenko and Malenko (2019) and Meirowitz and Pi (2021).

Most closely related to this work is the literature on shareholder voting where shareholders and/or blockholders vote directly on a proposal without delegating to a board of directors. Bar-Isaac and Shapiro (2020) find that a blockholder may choose not to vote with all their shares to prevent crowding out of information from smaller shareholders, thus enhancing informational efficiency. Maug (1999) studies the impact of trading and the information contained in prices on the voting process and consequently information aggregation.

This paper proceeds as follows: First the model is described. Then a numerical example is provided to highlight the intuition of the model. Next, the equilibrium is derived for the case of no information acquisition. Finally, the information acquisition problem is solved.

2 Model

An organization has the opportunity to adopt a valuable project proposal.

The project can either increase or decrease the value of the organization and is represented by a random variable $\tilde{\theta}$, which is distributed uniformly over an interval symmetric around zero.

$$\tilde{\theta} \sim \mathcal{U}[-\omega, \omega]$$

where $\omega > 0$. Whether the project proposal is accepted or not is determined by simple majority voting of the committee. We say $d = 1$ if the outcome of the vote is for the project to be accepted and $d = 0$ if it is to be rejected.

A committee is appointed to decide on whether or not the proposed project is to be implemented. The committee can be thought of as the board of directors of a corporation

or as the set of delegates of a DAO. For simplicity, I will use the board terminology in what follows; however, the mechanisms are equally applicable to delegation by blockholders in the context of DAOs. A blockholder holds the absolute majority of voting power and can thus determine the composition of the board. There are $2n + 1$ board members in total, who can be of two different types: Blockholder representatives vote as instructed by the blockholder and have no own decision power. I assume that the blockholder instructs all of his delegates to vote in the same direction. Inside board members, on the other hand vote independently.

Preferences I assume that the agents' preferences are not aligned: the blockholder derives private benefits from the rejection of the proposal. She values the outcome of the project as $\theta - b$ if it is accepted and 0 if it is rejected, where b is a positive constant representing her bias. Her preferences can be represented by the utility function:

$$u_b(d, \theta) = \mathbb{1}_{\{d=1\}}(\theta - b)$$

where $\mathbb{1}_{\{d=1\}}$ is an indicator function that equals 1 if the decision is to accept the project ($d = 1$) and 0 if it is rejected ($d = 0$). This formulation ensures that, without delegation, the blockholder always votes against the project as the expected utility of accepting the proposal is negative.

Inside board members on the other hand derive private benefits from the acceptance of the proposal. They can be thought of as employees of the company such as members of the management team in the context of corporations or developers in the context of DAO. As they are closely related to who proposes the project, they are more favorable towards its acceptance: They value the outcome of the project by $\theta + l$ if it is accepted and 0 if it is rejected where l is a positive constant. Their preferences can be represented by

$$u_i(d, \theta) = \mathbb{1}_{\{d=1\}}(\theta + l)$$

The project choice is, however, not contractible. Otherwise, the blockholder could just compensate all inside board members by paying a side payment of $b + l$ in case of rejection to perfectly align incentives - if feasible and worthwhile. The blockholder can choose the number m of inside board members, the remaining $2n + 1 - m$ board members are consequently

blockholder representatives.

Information Structure Because of their proximity to the operations and expert knowledge inside board members also possess superior knowledge. Each inside board member i privately observes a noisy signal about the realization of θ of $\tilde{\theta}$

$$s_i = \theta + \sigma_i$$

where the noise term σ_i is uniformly distributed over the interval $[-\epsilon, \epsilon]$

$$\sigma_i \sim \mathcal{U}[-\epsilon, \epsilon]$$

and ϵ represents the precision of the signal: the smaller the value of ϵ , the more precise the signal. The blockholder does not receive any signals about the project and nor do its delegates.

Each inside board member can reduce the noise of their signal by incurring a linear cost. However, inside board members are not allowed to communicate the signal - an assumption that at first may seem demanding. As sharing information undermines the motive for acquiring it, one could reinterpret this assumption as the ability to commit not to share information ex-post.

Parametric Assumptions It is assumed that the noise in the signal is not too large, compared to the noise in the underlying project value, specifically $2\epsilon < \omega$. This assumption is necessary for the existence of an equilibrium in the voting sub-game. Furthermore, the potential upside of the project for the blockholder, represented as $\omega - b$, needs to be large relative to the difference in private benefits $b + l$ in order to allow for a positive surplus of delegation: $b + l < \omega - b$.

Timing is as follows: (i) the blockholder chooses the composition of the committee, (ii) inside board members decide on the signal quality, (iii) signals realize, and (iv) inside board members observe their signal vote. Even though the delegation decision of the blockholder happens chronologically before voting and effort decisions of the committee, it is assumed that committee members cannot observe the blockholders choice of board composition. Those actions are thus strategically simultaneous. I focus on symmetric equilibria in monotone

strategies where the likelihood of voting in favor of the project is increasing both in the received signal as well as in the private benefits.

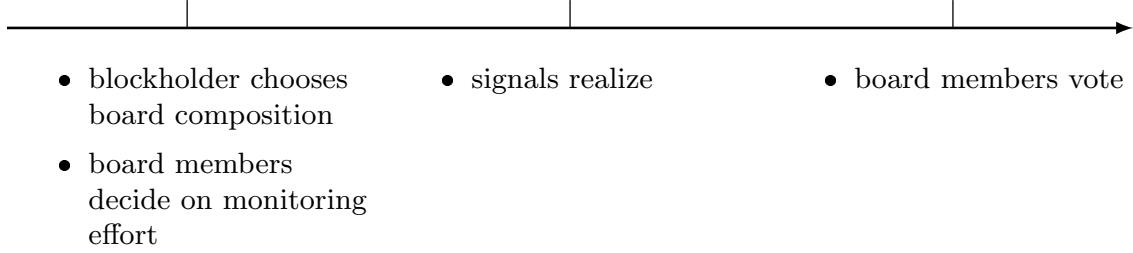


Figure 1: Timeline

The equilibrium concept is that of a Bayes Nash Equilibrium. An equilibrium consists of a set of inside board members' voting decision rules, the blockholder's board composition m^* , and the inside board members' signal precision choice ϵ^* such that

1. Board Composition: Given the noise ϵ_i^* and voting strategy s^* of inside board members, the blockholder chooses the number of inside board members

$$m^* \in \arg \max_m \mathbb{E}_\theta [P(d = 1 | \theta) (\theta - b)]$$

2. Signal Precision: Given the board composition m^* , the noise of other inside board members $\epsilon^{*, -i}$, and the voting strategy of other inside board members, a board member i chooses ϵ_i^* such that

$$\epsilon_i^* \in \arg \max_{\epsilon} \mathbb{E}_{s_i} [\mathbb{E}_{\theta | s_i, piv} [(\theta + l)] P(piv_i | s_i; \epsilon^{*, -i}) | d = 1] + c\epsilon$$

3. Voting Decision: Given the board composition m^* , the monitoring effort ϵ^* , and the voting strategy of other board members, a board member i accepts the project whenever

$$\mathbb{E}[\theta | piv_i, s_i] + l \geq 0$$

where piv_i stands for the event of being pivotal

4. beliefs are consistent.

3 Numerical Example

Before proceeding to a detailed analysis of the previously described model, I first present a numerical example to intuitively explain the mechanisms at play. This example deviates from the general setup only in that the state space is assumed to be discrete rather than continuous: I assume that θ can take two values, 1 if the project succeeds and -1 if it fails, with each outcome being equally likely. Furthermore, I assume $n = 1$, which implies that the board has $2n + 1 = 3$ members. The blockholder, as well as her direct representatives, have a bias of $b = \frac{1}{8}$ so that the utility in case of acceptance of the proposal is given by $\theta - \frac{1}{8}$. For illustrative purposes and for arithmetic simplicity, I assume that inside board members have no private benefits ($l = 0$) and therefore evaluate the proposal purely based on its fundamental value, θ . The noise in the signal is assumed to be $\epsilon = 2$. In the good state, signals are then uniformly distributed over the interval $[-1, 3]$ and in the bad state over $[-3, 1]$.

The blockholder decides on the composition of the board, considering three possible scenarios:

1. **No delegation.** The blockholder could fill all board seats with her representatives, resulting in decisions that fully reflect the blockholder's preferences. In this scenario, no inside information can affect the decision. As the expected value of θ equals zero, the blockholder has a negative expected utility of $-\frac{1}{8}$ in case the proposal is accepted and therefore instructs her representatives to reject the proposal.
2. **Partial delegation.** The blockholder could fill two of the board seats with inside board members and one with a blockholder representative. This arrangement allows the inside board members some degree of control but does not eliminate the blockholder's influence.
3. **Full delegation.** The blockholder could choose to fill all board seats with inside board members, thereby completely abstaining from influence. In this case, the outcome mirrors that of a firm without a blockholder, as all decisions reflect the views of the inside board members.

The latter two scenarios require a more detailed analysis, which I will address in the remainder of this example.

Partial Delegation

In the partial delegation scenario, the board is composed of two inside board members and one blockholder representative. An equilibrium exists where the blockholder always directs her representative to reject the proposal. Each inside board member accepts the proposal only if his observed signal exceeds -1 .

An inside board member only considers the scenario in which his vote is pivotal, meaning it can swing the outcome of the vote. This is the case when exactly one other board member votes in favor and one votes against. Given the conjectured strategies, the blockholder representative always votes against the proposal, so an inside board member is pivotal only if the other inside board member votes in favor. In such cases, the pivotal board member infers that the other inside board member's signal must have been greater than -1 , based on their conjectured voting strategy..

Given this information as well as his own signal, the inside board member updates his beliefs about the project's quality. If his signal is less than -1 , he concludes that the project must be bad (since signals for a good project are uniformly distributed over $[-1, 3]$) and rejects the proposal. Conversely, if his signal is greater than 1 , he concludes that the project is good (since signals for a bad project are uniformly distributed over $[-3, 1]$) and accepts the proposal. If his signal lies in the interval $[-1, 1]$, the signal provides no information as it is equally likely under both project states.

However, in this scenario, being pivotal implies that the other inside board member has accepted the proposal. This implies that the other inside board member must have received a signal greater than -1 . For a bad project, where signals are uniformly distributed over the interval $[-3, 1]$, the probability of receiving a signal greater than -1 is $\frac{1}{2}$ (see figure 2). For a good project on the other hand, where signals are uniformly distributed over $[-1, 3]$, the probability of a signal exceeding -1 , leading to acceptance, is 1 (see figure 3).

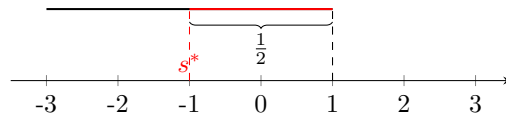


Figure 2: individual acceptance probability in low state

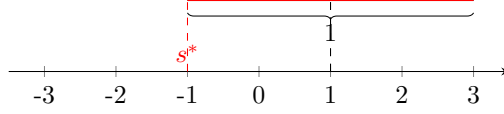


Figure 3: individual acceptance probability in high state

An inside board member can now use these observations to revise his beliefs about the likelihood of a good project in the scenario that he is pivotal. By Bayes' law we get

$$\begin{aligned}
 P(\theta = 1 \mid piv) &= \frac{P(piv \mid \theta = 1) P(\theta = 1)}{P(piv \mid \theta = 1) P(\theta = 1) + P(piv \mid \theta = 0) P(\theta = 0)} \\
 &= \frac{1 \times \frac{1}{2}}{1 \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2}} = \frac{2}{3}
 \end{aligned}$$

Given these revised beliefs, it is optimal for the inside board member to accept the proposal for any signal within the interval $[-1, 1]$. This confirms that -1 is his optimal cutoff level.

The blockholder does not receive a signal but can still infer information from being pivotal. Being pivotal from the view of the blockholder means that one inside board member accepted the proposal and the other rejected it. Given the voting strategy of the inside board members, this can only occur if the project is bad, as for a good project, both inside board members would always accept. Thus, the blockholder representative concludes that the proposal is bad and rejects it.

We have thus confirmed that the proposed strategies form an equilibrium. Similarly, it can be shown that an alternative equilibrium exists in which the blockholder always accepts, while inside board members use a signal cutoff of 1 (accept only when they are sure that the project is good). However, an equilibrium where the blockholder with negative private benefits always accepts and an inside board member with positive private benefits rejects with high probability seems less plausible. Such equilibria are not considered further by assuming monotonicity with respect to private benefit, meaning that a stronger private benefit leads to a higher likelihood of acceptance.

A proposal is accepted by a majority if at least two of the three board members vote in favor. Since the blockholder representative always rejects, the proposal can be accepted only when both inside board members vote in favor.

For a good project, both inside board members always accept the proposal (see figure 3), so the probability of approval is 1. For a bad project, each inside board member accepts with a probability of $\frac{1}{2}$ (see figure 2), which implies that the probability of both voting in favor and approving the proposal is $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$. The ex-ante utility of inside board members and minority shareholders is then:

$$\begin{aligned}
& P(\text{acceptance} \mid \theta = 1) P(\theta = 1) u_i(1, 1) + P(\text{acceptance} \mid \theta = -1) P(\theta = -1) u_i(1, -1) \\
&= 1 \times \frac{1}{2} \times 1 + \frac{1}{4} \times \frac{1}{2} \times (-1) \\
&= 0.375
\end{aligned}$$

while the ex-ante utility of the blockholder is given by

$$\begin{aligned}
& P(\text{acceptance} \mid \theta = 1) P(\theta = 1) u_d(1, 1) + P(\text{acceptance} \mid \theta = -1) P(\theta = -1) u_d(1, -1) \\
&= 1 \times \frac{1}{2} \times \left(1 - \frac{1}{8}\right) + \frac{1}{4} \times \frac{1}{2} \times \left(-1 - \frac{1}{8}\right) \\
&\simeq 0.297
\end{aligned}$$

Full Delegation

In the full delegation scenario, only the inside board members vote on the proposal. When deciding how to vote, each inside board member considers only scenarios where their vote is pivotal, so that it can determine the outcome.

The inside board members' voting strategy of accepting the proposal if their signal is greater than 0 constitutes an equilibrium. Under this strategy, the probability that an inside board member receives a signal greater than 0—and therefore votes to accept the proposal—is $\frac{3}{4}$ in case of a good project (see figure 4),

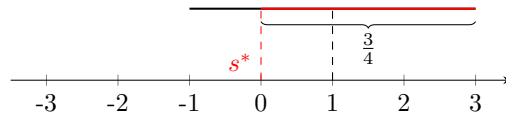


Figure 4: individual acceptance probability in high state

while it is $\frac{1}{4}$ in case of a bad project (see figure 5).

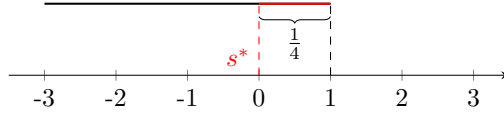


Figure 5: individual acceptance probability in low state

An inside board member is pivotal when exactly one of the other two inside board members accepts and one rejects the proposal. Due to the symmetry of this example with a cutoff value of 0, the probability of being pivotal is the same in both the high and low state.

$$P(piv \mid \theta = 1) = 2 \times \frac{1}{4} \times \frac{3}{4} = P(piv \mid \theta = -1)$$

Thus, no additional information can be inferred from being pivotal, and each inside board member bases their decision entirely on their own signal.

A signal greater than 1 can only occur if the project is good and is therefore fully revealing, leading the inside board member to always accept. A signal less than -1 can only occur if the project is bad and is also fully revealing, leading to rejection. For signals in the range of $[-1, 1]$, each realization is equally likely under both outcomes, providing no additional information. No updating takes place for signals in the interval $[-1, 1]$

$$P(\theta = 1 \mid s_i, piv) = P(\theta = 1) = \frac{1}{2}$$

At the cutoff signal of 0, the expected utility of an inside board member is 0, making them indifferent as required for this to be the equilibrium cutoff.

A proposal is accepted if at least two of the three inside board members vote in favor. Each inside board member votes in favor with a probability of $\frac{3}{4}$ for a good project and $\frac{1}{4}$ for a bad project.

For a good project, the probability that all three vote in favor is $(\frac{3}{4})^3$, and the probability that exactly two vote in favor is $3(\frac{3}{4})^2(\frac{1}{4})$, where the factor 3 accounts for the different combinations of two members voting in favor. Adding these two expressions, the probability of acceptance in the good state is given as approximately 0.844.

For a bad project, the probability that all three vote in favor is $\left(\frac{1}{4}\right)^3$, and the probability that exactly two vote in favor is $3\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right)$. Adding these two expressions again, the probability of acceptance in the bad state is given as approximately 0.156.

This leads to an ex-ante utility for inside board members and minority shareholders of:

$$\begin{aligned} & P(\text{acceptance} \mid \theta = 1) P(\theta = 1) u_i(1, 1) + P(\text{acceptance} \mid \theta = -1) P(\theta = -1) u_i(1, -1) \\ &= 0.844 \times \frac{1}{2} \times 1 + 0.156 \times \frac{1}{2} \times (-1) \\ &= 0.344 \end{aligned}$$

(compared to 0.375 in the partial delegation scenario) while the ex-ante utility of the blockholder is given by

$$\begin{aligned} & P(\text{acceptance} \mid \theta = 1) P(\theta = 1) u_d(1, 1) + P(\text{acceptance} \mid \theta = -1) P(\theta = -1) u_d(1, -1) \\ &= 0.844 \times \frac{1}{2} \times \left(1 - \frac{1}{8}\right) + 0.156 \times \frac{1}{2} \times \left(-1 - \frac{1}{8}\right) \\ &\simeq 0.282 \end{aligned}$$

(compared to 0.297 in the partial delegation scenario).

In this example, all agents are better off in the partial delegation scenario. The presence of the blockholder has increased welfare by better aggregating information, even though fewer signals have contributed to the decision.

4 No Information Acquisition

This section derives and analyzes the equilibrium given a specific level of noise, ϵ , in the inside board members' signals. The next section will endogenize the level of noise. First, the voting behavior of the blockholder's representatives and inside board members is examined individually, before determining the blockholder's optimal delegation decision. From there, the full equilibrium is derived. This approach allows me to obtain the equilibrium value of

delegation and, consequently, the control premium that a blockholder gains by determining the board's composition. Furthermore, the impact of the blockholder's control on small shareholders is analyzed.

4.1 Blockholder's Voting Decision

Although the blockholder does not receive any private information about the project's true value (θ), she votes strategically by extracting information from the event of being pivotal. When her decision can determine the outcome of the vote, this event provides her with indirect information about the private signals received by the insider board members. She uses this insight to update her belief about θ .

Since the insider board members derive positive private benefits from accepting the proposal, they vote in favor even when their private signals are slightly negative.³

As a result, the blockholder is more likely to be pivotal when the project's value θ is slightly negative. Recognizing this, she revises her beliefs about θ downward when conditioning on the event of being pivotal. This intuition is formalized in the following proposition.

Proposition 1. *A blockholder revises her expectation downwards in the event of being pivotal and thus always rejects the proposal.*

Note that the blockholder could also instruct a small number of his delegates to vote yes, as long as this does not affect the overall rejection of the proposal. Such kind of equilibria are however ruled out by assumption.

4.2 Inside board members' Voting Decision

Inside board members decide whether to vote in favor or against the proposal based on their private signal and the event of being pivotal. The information they infer from being pivotal depends on their conjectured presence of blockholder representatives. Specifically, the number of blockholder representatives influences how likely it is for an inside board member to be pivotal for a project of quality θ , which in turn affects the information they extract

³In general, there might also be equilibria where the opposite is true; these type of equilibria are ruled out due to the monotonicity assumption.

from this event. In what follows, we analyze how the number of blockholder representatives influences the information extracted.

A single inside board member rejects the project after observing signal s_i whenever is lower than the threshold value s^* ,

$$s_i \leq s^*$$

. Larger private benefits in case of acceptance result in a lower threshold value s^* , thereby increasing the likelihood of accepting the proposal. The probability of a single inside board member rejecting the proposal in the state θ is given by

$$\begin{aligned} P(v_i = 0 \mid \theta) &= P(s_i \leq s^* \mid \theta) \\ &= \begin{cases} 0 & s^* \leq \theta - \epsilon \\ \frac{1}{2} - \frac{1}{2\epsilon} (\theta - s^*) & s^* \in (\theta - \epsilon, \theta + \epsilon) \\ 1 & s^* \geq \theta + \epsilon \end{cases} \end{aligned} \quad (1)$$

and likewise the probability of acceptance by

$$P(v_i = 1 \mid \theta) = \begin{cases} 1 & s^* \leq \theta - \epsilon \\ \frac{1}{2} + \frac{1}{2\epsilon} (\theta - s^*) & s^* \in (\theta - \epsilon, \theta + \epsilon) \\ 0 & s^* \geq \theta + \epsilon \end{cases} \quad (2)$$

The number of blockholder representatives influences the information extracted from the event of being pivotal as it affects the proportion of yes and no votes that are required for a split vote. An inside board member is pivotal when the other board members' votes are split such that exactly n vote yes and n vote no. Given that the blockholder always votes no with all her votes, exactly n yes votes are required from the other $m - 1$ inside board members in order to obtain a total of n yes votes. The probability of being pivotal in state θ in the case of m inside board members is thus:

$$P(piv_i | \theta) = \binom{m-1}{n} \left(\frac{1}{2} + \frac{1}{2\epsilon} (\theta - s^*) \right)^n \left(\frac{1}{2} - \frac{1}{2\epsilon} (\theta - s^*) \right)^{m-n-1} \mathbb{1}_{\{\theta \in [s^* - \epsilon, s^* + \epsilon]\}}$$

The event of being pivotal implies proportionally fewer no votes compared to yes votes in the presence of a blockholder. An inside board member thus revises his beliefs stronger upwards in the event of being pivotal and is more willing to accept the project for lower values of his signal s_i the more blockholder representatives there are on the board. This leads to the inside board member voting as if they would counterbalance the blockholder's opposition by accepting the project more often than they would without the presence of the blockholder.

This adjustment is stronger when the signals are less precise (i.e., when ϵ is larger), as the inside board members pay less attention to the signal and rely more on their original bias due to the higher uncertainty. To illustrate this intuition, consider the following two scenarios:

In the case of full delegation ($m = 2n + 1$), all delegates are inside board members. Since all inside board members have the same bias and use the same cutoff, being pivotal does not provide additional information, and the inside board member bases his decision solely on his own signal and bias.

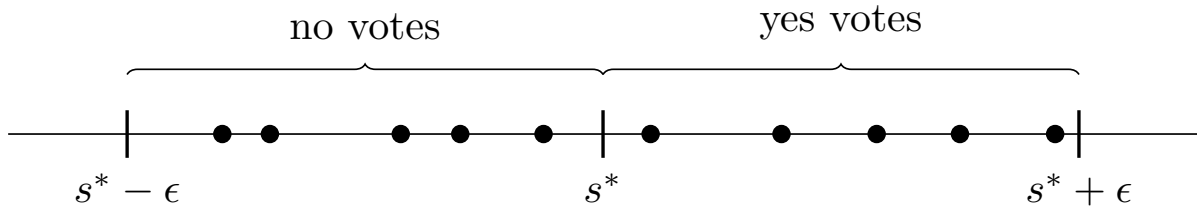


Figure 6: Full delegation: $n = 5$, $m = 11$

In contrast, with partial delegation ($2n + 1 > m > n + 1$), some delegates are blockholder representatives who always vote in opposition. When an inside board member is pivotal, he infers that the votes against the project include both blockholder representatives and some inside board members. This implies that fewer inside board members voted against and more voted in favor of the project, suggesting that the signals among inside board members are

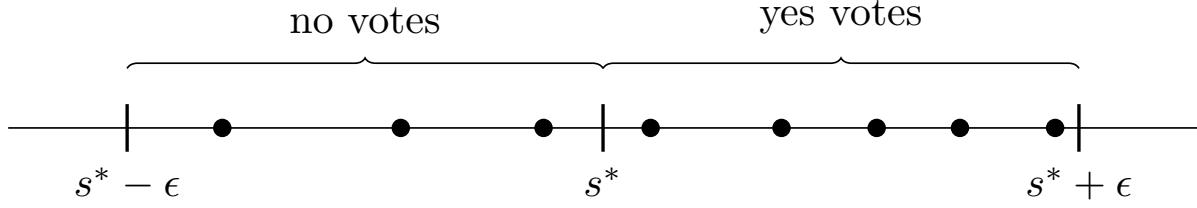


Figure 7: Partial delegation: $n = 5, m = 9$

higher on average. A larger value of ϵ suggests a higher potential upside on the possible values of the signals. This leads the pivotal inside board member to revise his belief about θ upwards and become more inclined to accept the project by lowering his acceptance threshold further. More noise or a larger presence of blockholder representatives strengthens this effect.

The following proposition formalizes this intuition and shows that there is a unique equilibrium in the voting sub-game in cutoff strategies.

Proposition 2. *For*

$$-l \in [-\omega + 2\epsilon, \omega - 2\epsilon]$$

there exists a unique equilibrium in cutoff strategies of the voting sub-game where an inside board member accepts the project whenever

$$s_i \geq s^*$$

with

$$s^* = -\frac{2n+1-m}{m+1}\epsilon - l.$$

Proof. See Appendix. □

The sub-game is unique because the best response functions of the inside board members are strictly increasing in the cutoff s^* . An increase in the cutoff used by other inside board members leads each inside board member to increase their own cutoff, as their posterior belief about θ upon being pivotal improves. This strategic complementarity ensures that there is a unique intersection point where all inside board members' best responses coincide.

4.3 The Blockholder's Delegation Decision

The blockholder aims to choose the optimal number of inside board members m^* to maximize her expected utility. This involves balancing the benefit of improved decision-making through better information aggregation against the cost of the inside board members' bias toward accepting the project. The blockholder's expected utility is given by:

$$v(m) = \frac{1}{2\omega} \int_{-\omega}^{\omega} (\theta - b) P_{n,m}(d = 1 \mid \theta) d\theta$$

where $P_{n,m}(d = 1 \mid \theta)$ is the probability that the project is accepted given θ , with n and m specifying the number of blockholder and inside board members, respectively. This can be written as the probability of at most $m - n - 1$ inside board members rejecting, plus the $2n + 1 - m$ blockholder representatives, so in total at most n inside board members rejecting.

$$P_{n,m}(d = 1 \mid \theta) = \sum_{i=0}^{m-n-1} \binom{m}{i} \left[\underbrace{\frac{1}{2} + \frac{1}{2\epsilon}(\theta - s^*)}_{P(v_i=1|\theta)} \right]^{m-i} \left[\underbrace{\frac{1}{2} - \frac{1}{2\epsilon}(\theta - s^*)}_{P(v_i=0|\theta)} \right]^i$$

By integrating over θ , we immediately obtain the value of delegation for the blockholder.

Lemma 3. *The blockholder's value of delegation is given by*

$$v_b(m) = \begin{cases} \frac{1}{2\omega} \left\{ \frac{2\epsilon(m-n)}{(m+1)(m+2)} [(n+1)\epsilon - (m+2)(b-s)] + \frac{1}{2} ((\omega - s - \epsilon)(\omega + s + \epsilon - 2b)) \right\} & \text{for } m \geq n+1 \\ 0 & \text{otherwise} \end{cases}$$

Proof. See Appendix. □

From here, the benefit of adding an additional inside board member is readily derived as

$$v_b(m+1) - v_b(m) = \frac{\epsilon}{\omega} \frac{(n+1)}{(m+1)(m+2)} \left[\frac{2n+1-m}{m+3} \epsilon - (b - s^*) \right]$$

This expression has the following interpretation. If the difference between the blockholder's bias and the inside board members' threshold value is relatively large compared to the noise

term, adding an additional inside board member increases the chance that the project is accepted in states where the blockholder has negative utility. The loss of control is thus relatively large compared to the gain in precision due to an additional signal. If, on the other hand, the signal is less precise (large ϵ), the benefit of adding an additional inside board member is considerable because the existing inside board members' aggregated information is relatively noisy. This argument becomes clearer when considering the limiting case of almost perfect information. In this case, the m inside board members already know the state very precisely, and adding an additional inside board member disproportionately increases the risk of unfavorable decisions from the blockholder's perspective. The factor $\frac{2n+1-m}{m+3}$ in front of the noise implies that the benefit of information aggregation is decreasing with the number of inside board members.

The blockholder thus increases the number of inside board members as long as $v_b(m+1) - v_b(m) > 0$ or

$$m < \frac{\epsilon(2n+1) - 3(b-s^*)}{\epsilon + b - s^*}.$$

The optimal m is thus when, for the first time,

$$m \geq \frac{\epsilon(2n+1) - 3(b-s^*)}{\epsilon + b - s^*}.$$

If, on the other hand,

$$n+1 \geq \frac{\epsilon(2n+1) - 3(b-s^*)}{\epsilon + b - s^*},$$

the blockholder does not delegate above the minimal meaningful level of delegation and might or might not prefer even no delegation at all. It then depends on the surplus of delegation whether minimal delegation is attractive or not. These findings are formalized in the following proposition.

Proposition 4. *If either*

$$(\omega - b)^2 < \frac{4(n+1)}{(n+2)^2(n+3)}\epsilon^2$$

or

$$b > s^* + \frac{n}{n+1}\epsilon + \sqrt{(\omega - b)^2 - \frac{4(n+1)}{(n+2)^2(n+3)}\epsilon^2},$$

delegation is never optimal ($m^* = 0$). Otherwise, the optimal level of delegation is given by

$$m^* = \begin{cases} n + 1 & \text{if } b - s^* \geq \frac{n}{n+4}\epsilon \\ \left\lceil \frac{(2n+1)\epsilon - 3(b-s^*)}{\epsilon + b - s^*} \right\rceil & \text{if } b - s^* \in \left(\frac{n}{n+4}\epsilon, \frac{1}{2n+3}\epsilon \right) \\ 2n + 1 & \text{if } b - s^* \leq \frac{1}{2n+3}\epsilon \end{cases}$$

The proposition states that if the maximal upside on the project value ω is low relative to the blockholder's bias b , or her bias is large with respect to the cutoff value of the inside board members s^* , then the blockholder prefers a board only composed of her own representatives. If none of these conditions are satisfied, there is value in delegation, and the number of inside board members decreases with the difference in bias. Furthermore, the optimal m^* is increasing in noise or decreasing in precision. This is because if information is relatively precise (small ϵ), a lower number of inside board members aggregates information comparatively well, so that adding an extra inside board member brings relatively little advantage compared to the cost of shifting the bias against in the direction of acceptance.

4.4 Equilibrium

From the individual optimization problems, we have seen that the inside board members revise their threshold value s^* downwards the more blockholder representatives there are, while the blockholder increases delegation to her own representatives the lower s^* . If the signal noise is relatively large (but within the limits of a positive surplus of delegation), this counterbalancing behavior of both strategies can lead to a situation of multiple equilibria where any level of delegation can be supported as an equilibrium. This is because, at high levels of noise, the benefit of adding an additional inside board member is large as we have just seen in the previous section, which in turn allows to sustain high levels of delegation. If, on the other hand, signals are relatively precise, this benefit is too small so that large numbers of inside board members are not sustainable. Either scenario is, of course, subject to delegation having a positive surplus. These observations are formalized in the following proposition.

Proposition 5. *For a large bias of the blockholder in respect to ω*

$$(\omega - b)^2 < \frac{4(n+1)}{(n+2)^2(n+3)}\epsilon^2$$

or a large difference in biases

$$(b + l)^2 > (\omega - b)^2 - \frac{4(n+1)}{(n+2)^2(n+3)}\epsilon^2,$$

no delegation can be supported as an equilibrium. Otherwise, if also

$$b + l \geq \frac{1}{2n+3}\epsilon,$$

the only equilibrium that can be supported is that of minimal delegation

$$m^* = n + 1,$$

while if

$$b + l < \frac{1}{2n+3}\epsilon,$$

there is a multiplicity of equilibria, and any

$$m^* \in \{n + 1, \dots, 2n + 1\}$$

can be supported as an equilibrium.

From the expressions above, we can also derive the equilibrium value of the firm.

Proposition 6. *The equilibrium value for the blockholder is given by*

$$v_b^*(m) = \frac{(b - \omega)^2 - (b + l)^2 - 4\frac{(n+1)(m-n)}{(m+1)^2(m+2)}\epsilon^2}{2(2\omega)}$$

We can see that the optimal level of delegation is not dependent on either the private benefits or the distribution of the project or the level of noise.

Corollary 7. *For $n \geq 3$, the blockholder prefers the equilibrium with minimal delegation ($m^* = n + 1$).*

This alignment occurs because the blockholder internalizes the effects of delegation on information aggregation and takes into account the impact on the project's acceptance probability.

4.5 Second Best

Suppose that a social planner would choose the optimal number of inside board members with the objective of minimizing the error of falsely accepting a project with negative value or falsely rejecting a project with positive value. The social planner does not have any bias in favor of or against the project but takes into account the inside board members' voting behavior. His objective function is thus

$$v_0(m) = \frac{1}{2\omega} \int_{-\omega}^{\omega} \theta P_{n,m}(d = 1 \mid \theta) d\theta$$

As the following proposition will show, the presence of a blockholder improves the welfare of small shareholders. This improvement is not caused by enhanced monitoring by either the inside board members or the blockholder, as is usually argued in the blockholder literature. Instead, it results from a more balanced decision-making body. The blockholder voluntarily restrains her power because she internalizes the benefits of aggregating information.

The equilibrium with the least possible delegation provides the best expected outcome for small shareholders because it requires that for a project to be accepted, all inside board members vote in favor of the project using the lowest possible threshold level and thus a very high probability of voting yes. This is the most efficient way of aggregating information in this biased environment.

Proposition 8. *The value of the firm for a small shareholder under blockholder influence is given by*

$$v_0^*(m) = \frac{\omega^2 - l^2 - 4 \frac{(n+1)(m-n)}{(m+1)^2(m+2)} \epsilon^2}{2(2\omega)}$$

For $n \geq 3$, the equilibrium with minimal delegation $m = n+1$ delivers the highest expected utility, and the blockholder equilibrium is thus constrained efficient.

4.6 Blockholder Premium

The benefit the blockholder adds to the value of the firm can be derived by comparing the value of the firm at the socially optimal level of delegation ($m^* = n + 1$)

$$v_0(n+1) = \frac{\omega^2 - l^2 - 4 \frac{(n+1)}{(n+2)^2(n+3)} \epsilon^2}{4\omega}$$

to the value that would prevail if there was no blockholder, i.e., at full delegation

$$v_0(2n+1) = \frac{\omega^2 - l^2 - 4 \frac{(n+1)^2}{(2n+2)^2(2n+3)} \epsilon^2}{4\omega}.$$

The difference is given by

$$\frac{n((n-1)n-4)\epsilon^2}{4(n+2)^2(n+3)(2n+3)\omega}$$

This expression is increasing in the noise term, does not depend on any bias term, and is in fact the same from the perspective of the blockholder. This confirms that the welfare improvement is purely due to an improvement in information aggregation.

5 Information Acquisition

This section extends the previous analysis by giving inside board members the opportunity to choose the signal precision at a cost. Each inside board member decides how much to invest in his signal, considering only the contingency in which he can use his signal, which is the case when he is pivotal. Suppose an inside board member acquires a signal with precision ϵ while all other inside board members acquire signals with precision $\epsilon^{*, -i}$. His expected profit from acquiring the signal is then

$$\max_{\epsilon} \int_{s^*}^{\omega+\epsilon} \mathbb{E}[\theta + l \mid s_i, piv_i] P(piv \mid s_i) f(s_i) ds_i - c\epsilon$$

Proposition 9. *The inside board members' optimal level of noise is given by*

$$\epsilon^* = \frac{m+2}{(2n+1-m)^2 + m+1} \left[\binom{m+1}{n} c - (2n+1-m)(l+s^*) \right]$$

Proof. See Appendix. □

The optimal noise is increasing in the cost and decreasing in the difference in biases.

5.1 Equilibrium with Information Acquisition

Inserting the equilibrium value of s^* , we can derive the equilibrium level of noise

$$\epsilon = \frac{(m+2)(m+1)^2 \binom{m}{n+1}}{4(m-n)} c$$

which is increasing in m since

$$\begin{aligned} \epsilon(m+1) - \epsilon(m) &= c \frac{(m(n+3) + n+5) \Gamma(m+3)}{4\Gamma(n+2)\Gamma(m-n+2)} \\ &> 0 \end{aligned}$$

An equilibrium with more delegation to inside board members leads to more noise, while fewer inside board members improve signal precision. This further increases the advantage of minimal delegation and thus blockholder delegation.

6 Conclusion

We have seen that blockholder influence on boards is not necessarily a bad thing, and in cases of opposing private benefits of the boards, blockholder influence can lead to a welfare improvement over pure board decision-making. However, there are also situations where the blockholder has private benefits of the same sign as the boards', and one can expect that the same is no longer true in this scenario. Furthermore, we have left out the issue of communication, which might also be impacted by the presence of blockholder representatives

on the board. The issue of communication or deliberation in such scenarios would be an interesting topic for future research.

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Appendix

Proof of Proposition 1

The proof proceeds by showing that the blockholder always instructs her own representatives to reject the project independently of the number of inside board members she has chosen. This is because she is already biased against the project through her private benefits, and the more she delegates to blockholder representatives, the more she becomes pessimistic about the project by extracting information from the event of being pivotal.

In order to understand the blockholder's voting behavior, the concept of being pivotal first needs to be extended to a voter who controls multiple votes: Suppose there are $2n+1$ delegates in total, consisting of m inside board members and $2n+1-m$ blockholder representatives. For delegation to be meaningful, the blockholder must delegate at least $n+1$ inside board members ($m \geq n+1$); otherwise, her own representatives retain the majority, and she can never be pivotal. As a consequence, in all cases where $m < n+1$, the blockholder votes according to her prior and thus rejects the project with a majority, resulting in all of those choices being outcome-equivalent.

In all that follows, we thus mean by delegation that at least $m \geq n+1$ of the board members are inside board members. Assume now that, out of the m inside board members, i vote in favor of the project, and $m-i$ vote against. If $i \geq n+1$ inside board members vote in favor of the project, there is a majority in favor of the project among the inside board members alone, and the blockholder cannot be pivotal, so that the project will pass regardless of her representatives' votes. Conversely, if there are $m-i \geq n+1$ inside board members rejecting the proposal, there is a majority against the proposal among the inside board members, and again the blockholder cannot be pivotal as the project will be rejected regardless of her votes. Therefore, the blockholder can be pivotal only when the number of votes in favor of the project among inside board members satisfies $i \in \{m-n, \dots, n\}$.

The probability that the blockholder is pivotal, given the state θ , can then be written as:

$$P(piv_b \mid \theta) = \sum_{i=m-n}^n \binom{m}{i} [P(v_i = 1 \mid \theta)]^i [P(v_i = 0 \mid \theta)]^{m-i} \mathbb{1}_{\{\theta \in [s^* - \epsilon, s^* + \epsilon]\}} \quad (3)$$

Substituting the individual inside board members acceptance and rejection probabilities (1) and (2) into equation (3) The probability of being pivotal in state θ can then be expressed as

$$P(piv_b | \theta) = \sum_{i=m-n}^n \binom{m}{i} \left[\frac{1}{2} + \frac{1}{2\epsilon} (\theta - s^*) \right]^i \left[\frac{1}{2} - \frac{1}{2\epsilon} (\theta - s^*) \right]^{m-i} \mathbb{1}_{\{\theta \in [s^* - \epsilon, s^* + \epsilon]\}}$$

. The posterior probability of θ in the event of being pivotal can then be derived as

$$\begin{aligned} f(\theta | piv_b) &= \frac{P(piv_b | \theta) f(\theta)}{\int_{-\infty}^{\infty} P(piv_b | \theta) f(\theta) d\theta} \\ &= \frac{\sum_{i=m-n}^n \binom{m}{i} \left[\frac{1}{2} + \frac{1}{2\epsilon} (\theta - s^*) \right]^i \left[\frac{1}{2} - \frac{1}{2\epsilon} (\theta - s^*) \right]^{m-i} \mathbb{1}_{\{\theta \in [s^* - \epsilon, s^* + \epsilon]\}}}{\int_{s^* - \epsilon}^{s^* + \epsilon} \sum_{i=m-n}^n \binom{m}{i} \left[\frac{1}{2} + \frac{1}{2\epsilon} (\theta - s^*) \right]^i \left[\frac{1}{2} - \frac{1}{2\epsilon} (\theta - s^*) \right]^{m-i} \mathbb{1}_{\{\theta \in [s^* - \epsilon, s^* + \epsilon]\}} d\theta} \\ &= \frac{1}{2\epsilon} \frac{m+1}{2n+1-m} \sum_{i=m-n}^n \binom{m}{i} \left[\frac{1}{2} + \frac{1}{2\epsilon} (\theta - s^*) \right]^i \left[\frac{1}{2} - \frac{1}{2\epsilon} (\theta - s^*) \right]^{m-i} \mathbb{1}_{\{\theta \in [s^* - \epsilon, s^* + \epsilon]\}} \end{aligned}$$

where the integrals in the denominator are evaluated by change of variable

$$p = \frac{1}{2} + \frac{1}{2\epsilon} (\theta - s^*)$$

$$\frac{dp}{d\theta} = \frac{1}{2\epsilon}$$

and noting that $\int_0^1 (p)^n (1-p)^{m-n} dp$ is the The Euler integral of the first kind with the value $\frac{n!(m-n)!}{(m+1)!}$ so that

$$\int_{s^* - \epsilon}^{s^* + \epsilon} \binom{m}{n} \left(\frac{1}{2} + \frac{1}{2\epsilon} (\theta - s^*) \right)^n \left(\frac{1}{2} - \frac{1}{2\epsilon} (\theta - s^*) \right)^{m-n} d\theta$$

the expected conditional mean can now be derived byso that

$$\begin{aligned}
\mathbb{E}[\theta \mid piv_b] &= \int_{-\infty}^{\infty} \theta f(\theta \mid piv_i) d\theta \\
&= \frac{1}{2\epsilon} \frac{m+1}{2n+1-m} \sum_{i=m-n}^n \int_{s^*-\epsilon}^{s^*+\epsilon} \theta \binom{m}{i} \left[\frac{1}{2} + \frac{1}{2\epsilon} (\theta - s^*) \right]^i \left[\frac{1}{2} - \frac{1}{2\epsilon} (\theta - s^*) \right]^{m-i} d\theta \\
&= \frac{1}{2\epsilon} \frac{m+1}{2n+1-m} \sum_{i=m-n}^n 2\epsilon \frac{((2i-m)\epsilon + (m+2)s^*)}{(m+2)(m+1)} \\
&= s^* \\
&< 0
\end{aligned}$$

Proof of Proposition 2

The first step of the proof proceeds by confirming that the proposed voting strategy is an equilibrium in the voting sub-game. Suppose all inside board members vote according to the threshold strategy s^* . The likelihood of an inside board member being pivotal in state θ is then the likelihood that n of the $m-1$ other inside board members are voting in favor of the proposal.

$$P(piv_i \mid \theta) = \binom{m-1}{n} \left(\frac{1}{2} + \frac{1}{2\epsilon} (\theta - s^*) \right)^n \left(\frac{1}{2} - \frac{1}{2\epsilon} (\theta - s^*) \right)^{m-n-1} \mathbb{1}_{\{\theta \in [s^*-\epsilon, s^*+\epsilon]\}}$$

The posterior probability of θ in the event of being pivotal and having observed s^* can then be derived as

$$g(\theta \mid s^*, piv_i) = \frac{f(s^* \mid \theta) P(piv_i \mid \theta) f(\theta)}{\int_{-\infty}^{\infty} f(s^* \mid \theta) P(piv_i \mid \theta) f(\theta) d\theta} \quad (4)$$

$$= \frac{\frac{1}{2\epsilon} \mathbb{1}_{s^* \in \{\theta-\epsilon, \theta+\epsilon\}} P(piv_i \mid \theta) \frac{1}{2\omega} \mathbb{1}_{\{\theta \in [-\omega, \omega]\}}}{\int_{-\infty}^{\infty} \frac{1}{2\epsilon} \mathbb{1}_{s^* \in \{\theta-\epsilon, \theta+\epsilon\}} P(piv_i \mid \theta) \frac{1}{2\omega} \mathbb{1}_{\{\theta \in [-\omega, \omega]\}} d\theta} \quad (5)$$

$$= \frac{\left(\frac{1}{2} + \frac{1}{2\epsilon} (\theta - s^*) \right)^n \left(\frac{1}{2} - \frac{1}{2\epsilon} (\theta - s^*) \right)^{m-n-1} \mathbb{1}_{\{\theta \in [s^*-\epsilon, s^*+\epsilon]\}}}{\int_{s^*-\epsilon}^{s^*+\epsilon} \left(\frac{1}{2} + \frac{1}{2\epsilon} (\theta - s^*) \right)^n \left(\frac{1}{2} - \frac{1}{2\epsilon} (\theta - s^*) \right)^{m-n-1} d\theta} \quad (6)$$

The integral in the denominator can be evaluated by change of variable

$$p = \frac{1}{2} + \frac{1}{2\epsilon} (\theta - s^*)$$

$$\frac{dp}{d\theta} = \frac{1}{2\epsilon}$$

and noting that $\int_0^1 p^n (1-p)^{m-n-1} dp$ is the Euler integral of the first kind with the value $\frac{n!(m-n-1)!}{m!}$, we have

$$\begin{aligned} & \int_{s^*-\epsilon}^{s^*+\epsilon} \binom{m-1}{n} \left(\frac{1}{2} + \frac{1}{2\epsilon} (\theta - s^*) \right)^n \left(\frac{1}{2} - \frac{1}{2\epsilon} (\theta - s^*) \right)^{m-n-1} d\theta \\ &= \binom{m-1}{n} 2\epsilon \int_0^1 p^n (1-p)^{m-n-1} dp \\ &= \binom{m-1}{n} 2\epsilon \frac{n!(m-n-1)!}{m!} \\ &= \frac{2\epsilon}{m} \end{aligned}$$

So, the posterior density is thus given by

$$g(\theta \mid s^*, piv_i) = \frac{1}{2\epsilon} \frac{m!}{n!(m-n-1)!} \left(\frac{1}{2} + \frac{1}{2\epsilon} (\theta - s^*) \right)^n \left(\frac{1}{2} - \frac{1}{2\epsilon} (\theta - s^*) \right)^{m-n-1} \mathbb{1}_{\theta \in [s^*-\epsilon, s^*+\epsilon]}$$

The expected conditional mean can now be derived using the fact that

$$\begin{aligned} & \frac{1}{2\epsilon} \int_{s^*-\epsilon}^{s^*+\epsilon} \theta \left(\frac{1}{2} + \frac{1}{2\epsilon} (\theta - s^*) \right)^n \left(\frac{1}{2} - \frac{1}{2\epsilon} (\theta - s^*) \right)^{m-n-1} d\theta \\ &= 2\epsilon \frac{(n+1)!(m-n-1)!}{(m+1)!} - \left(\frac{1}{2} - \frac{s^*}{2\epsilon} \right) 2\epsilon \frac{n!(m-n-1)!}{m!} \end{aligned}$$

Thus

$$\begin{aligned} \mathbb{E}[\theta \mid piv_i] &= \int_{-\infty}^{\infty} \theta g(\theta \mid s^*, piv_i) d\theta \\ &= \frac{1}{2\epsilon} \frac{m!}{n!(m-n-1)!} \int_{s^*-\epsilon}^{s^*+\epsilon} \theta \left(\frac{1}{2} + \frac{1}{2\epsilon} (\theta - s^*) \right)^n \left(\frac{1}{2} - \frac{1}{2\epsilon} (\theta - s^*) \right)^{m-n-1} d\theta \\ &= \frac{m!}{n!(m-n-1)!} \left[2\epsilon \frac{(n+1)!(m-n-1)!}{(m+1)!} - \left(\frac{1}{2} - \frac{s^*}{2\epsilon} \right) 2\epsilon \frac{n!(m-n-1)!}{m!} \right] \\ &= 2\epsilon \frac{n+1}{m+1} - (\epsilon - s^*) \\ &= \frac{2n+1-m}{m+1} \epsilon + s^* \end{aligned}$$

An inside board member who observes exactly s^* has to be indifferent between accepting and rejecting, so

$$\frac{2n+1-m}{m+1}\epsilon + s^* + l = 0$$

or

$$s^* = -\frac{2n+1-m}{m+1}\epsilon - l$$

To show uniqueness, I proceed to show that the best response is strictly increasing. Let $s^{*,-i}$ denote the cutoff signal of all other inside board members. Then, the conditional expectation of an inside board member who observes a signal s_i in the event of being pivotal is given by

$$\mathbb{E}[\theta \mid s_i, \text{piv}; s^{*,-i}] = \int_{\max\{s_i, s^*\} - \epsilon}^{\min\{s_i, s^*\} + \epsilon} \theta \left(\frac{1}{2} + \frac{1}{2\epsilon}(\theta - s^*) \right)^n \left(\frac{1}{2} - \frac{1}{2\epsilon}(\theta - s^*) \right)^{m-n-1} d\theta$$

Suppose the other inside board members increase their cutoff from s^* to \hat{s}^* , then

$$\min\{s^*, \hat{s}^*\} + \epsilon = s^* + \epsilon$$

$$\max\{s^*, \hat{s}^*\} - \epsilon = \hat{s}^* - \epsilon$$

Thus,

$$\mathbb{E}[\theta \mid s^*, \text{piv}; \hat{s}^*] = \int_{\hat{s}^* - \epsilon}^{s^* + \epsilon} \theta \left(\frac{1}{2} + \frac{1}{2\epsilon}(\theta - \hat{s}^*) \right)^n \left(\frac{1}{2} - \frac{1}{2\epsilon}(\theta - \hat{s}^*) \right)^{m-n-1} d\theta$$

Using the Leibniz rule, we get the derivative with respect to \hat{s}^*

$$\begin{aligned} & \frac{\partial}{\partial \hat{s}^*} \mathbb{E}[\theta \mid s^*, \text{piv}; \hat{s}^*] \\ &= \int_{\hat{s}^* - \epsilon}^{s^* + \epsilon} \frac{1}{2\epsilon} \theta \left(\frac{1}{2} + \frac{1}{2\epsilon}(\theta - s^*) \right)^{n-1} \left(\frac{1}{2} - \frac{1}{2\epsilon}(\theta - s^*) \right)^{m-n-2} \left(\frac{1}{2} + \frac{1}{2\epsilon}(\theta - s^*) \right) \left(\underbrace{m-n-1}_{\geq 0} \right) d\theta \\ &> 0 \end{aligned}$$

Thus, after an increase in \hat{s}^* ,

$$\mathbb{E}[\theta \mid s^*, \text{piv}; \hat{s}^*] + l > 0$$

and the inside board member will strictly increase her cutoff point where she is indifferent. The best response is thus strictly increasing.

Proof of Lemma 3

The proof consists of integrating over each summand and then adding up the sum. The proof uses the result that

$$\int_{s^*-\epsilon}^{s^*+\epsilon} (\theta - b) \binom{m}{i} \left[\frac{1}{2} + \frac{1}{2\epsilon} (\theta - s^*) \right]^{m-i} \left[\frac{1}{2} - \frac{1}{2\epsilon} (\theta - s^*) \right]^i d\theta = \frac{\epsilon(m-2i) + (s-b)(m+2)}{(m+1)(m+2)} 2\epsilon$$

$$\begin{aligned} v(m) &= \frac{1}{2\omega} \int_{-\omega}^{\omega} (\theta - b) P_{n,m}(d=1 | \theta) d\theta \\ &= \frac{1}{2\omega} \left\{ \sum_{i=0}^{m-n-1} \int_{s^*-\epsilon}^{s^*+\epsilon} (\theta - b) \binom{m}{i} \left[\frac{1}{2} + \frac{1}{2\epsilon} (\theta - s^*) \right]^{m-i} \left[\frac{1}{2} - \frac{1}{2\epsilon} (\theta - s^*) \right]^i d\theta + \int_{s^*+\epsilon}^{\omega} (\theta - b) d\theta \right\} \\ &= \frac{1}{2\omega} \left\{ \sum_{i=0}^{m-n-1} \frac{\epsilon(m-2i) + (s^*-b)(m+2)}{(m+1)(m+2)} 2\epsilon + \frac{1}{2} (\omega^2 - (s^*+\epsilon)^2) + b((s^*+\epsilon) - \omega) \right\} \\ &= \frac{1}{2\omega} \left[\frac{(m-n)[(n+1)\epsilon + (m+2)(s-b)]}{(m+1)(m+2)} 2\epsilon + \frac{1}{2} (\omega^2 - (s^*+\epsilon)^2) + b((s^*+\epsilon) - \omega) \right] \\ &= \frac{1}{2\omega} \left\{ \frac{2\epsilon(m-n)}{(m+1)(m+2)} [(n+1)\epsilon - (m+2)(b-s)] + \frac{1}{2} (\omega^2 - (s^*+\epsilon)^2) + b((s^*+\epsilon) - \omega) \right\} \\ &= \frac{1}{2\omega} \left[\frac{2\epsilon(m-n)}{(m+1)(m+2)} [(n+1)\epsilon - (m+2)(b-s)] + \frac{1}{2} ((\omega - s - \epsilon)(\omega + s + \epsilon - 2b)) \right] \end{aligned}$$

Proof of Proposition 4

•

$$\frac{1}{2\omega} \left\{ \frac{2\epsilon(m-n)}{(m+1)(m+2)} [(n+1)\epsilon - (m+2)(b-s)] + \frac{1}{2} ((\omega - s - \epsilon)(\omega + s + \epsilon - 2b)) \right\}$$

just above

• FOC wrt s^*

$$\frac{1}{2\omega} \left(b - s + \frac{m-2n-1}{m+1} \epsilon \right)$$

SOC wrt s^*

$$-\frac{1}{2\omega}$$

thus there is a max at

$$s^* = b - \frac{2n+1-m}{m+1}\epsilon$$

in equilibrium

$$-l = \frac{2n+1-m}{m+1}\epsilon + s^* = b$$

however for

$$b > -l$$

$$s^* < b - \frac{2n+1-m}{m+1}\epsilon$$

so that in the FOC

$$\frac{1}{2\omega} \left(b - s + \frac{m-2n-1}{m+1}\epsilon \right) > \frac{2}{2\omega} \left(\frac{2n+1-m}{m+1}\epsilon \right) > 0$$

thus in total we have that for a bias $b > -l$ or equivalently $s^* < b - \frac{2n+1-m}{m+1}\epsilon$ the value fct is increasing in s^*

- at maximum

$$v \left(s^* = b - \frac{2n+1-m}{m+1}\epsilon \right) = \frac{1}{2\omega} \left[\frac{1}{2} (\omega - b)^2 - 2\epsilon^2 \frac{(n+1)(m-n)}{(m+1)^2(m+2)} \right]$$

so that if

$$(\omega - b)^2 - 4\epsilon^2 \frac{(n+1)(m-n)}{(m+1)^2(m+2)} < 0$$

the value function is always negative

- positivity constraint is thus

$$\frac{1}{2\omega} \left\{ \frac{2\epsilon(m-n)}{(m+1)(m+2)} [(n+1)\epsilon - (m+2)(b-s)] + \frac{1}{2} ((\omega - s - \epsilon)(\omega + s + \epsilon - 2b)) \right\} = 0$$

(need positive solution for $b > -l$, but how about out of equilibrium values?)

- need that at optimum value fct is positive otherwise no delegation is possible
- or for

$$b - \sqrt{(\omega - b)^2 - \underbrace{\frac{4(n+1)(m-n)}{(m+1)^2(m+2)}}_{<1} \epsilon^2} \leq s^* + \frac{2n+1-m}{m+1} \epsilon \leq b + \sqrt{(\omega - b)^2 - \underbrace{\frac{4(n+1)(m-n)}{(m+1)^2(m+2)}}_{<1} \epsilon^2}$$

(RHS values of b below -1 (not considered here), LHS values of b above -1) alternatively the value fct is positive

- condition for VF to be negative at $m = n + 1$

$$\begin{aligned} s^* &\leq b - \frac{n}{n+2} \epsilon - \sqrt{(\omega - b)^2 - \frac{4(n+1)}{(n+2)^2(n+3)} \epsilon^2} \\ &< b - \frac{n}{n+4} \epsilon \end{aligned}$$

which is equivalent to

$$n+1 > \frac{\epsilon(2n+1) - 3(b - s^*)}{\epsilon + b - s^*}$$

so that the value function being negative at $m = n + 1$ implies that it is even lower for all other m and thus negative everywhere.

Proof of Proposition 9

First note that Bayes law gives

$$f_{\theta|s_i,piv}(\theta) = \frac{f_{s_i|\theta}(s_i) P(piv | \theta) f_{\theta}(\theta)}{P(piv_i | s_i) f_{s_i}(s_i)} \mathbb{1}_{\theta \in [\max\{s_i - \epsilon^*, -i, s_i - \epsilon\}, \min\{s^* + \epsilon^*, -i, s_i + \epsilon\}]}$$

Using this result we can write

$$\begin{aligned}
& \int_{s^*}^{\omega+\epsilon} \mathbb{E}[(\theta + l) \mid s_i, piv_i] P(piv \mid s_i) f(s_i) ds_i \\
&= \int_{s^*}^{\omega+\epsilon} \int_{\max\{s_i-\epsilon, s^*-\epsilon^*, -i\}}^{\min\{s_i+\epsilon, s^*+\epsilon^*, -i\}} (\theta + l) f_{\theta|s_i, piv}(\theta) d\theta P(piv \mid s_i) f(s_i) ds_i \\
&= \int_{s^*}^{\omega+\epsilon} \int_{\max\{s_i-\epsilon, s^*-\epsilon^*, -i\}}^{\min\{s_i+\epsilon, s^*+\epsilon^*, -i\}} (\theta + l) \frac{f_{s_i|\theta}(s_i) P(piv \mid \theta) f_{\theta}(\theta)}{P(piv_i \mid s_i) f_{s_i}(s_i)} d\theta P(piv \mid s_i) f(s_i) ds_i \\
&= \frac{1}{2\omega} \left[\int_{s^*-\epsilon^*, -i}^{s^*+\epsilon^*, -i} P(piv \mid \theta) (\theta + l) \int_{\theta-\epsilon}^{\theta+\epsilon} \mathbb{1}_{s_i \geq s^*} \frac{1}{2\epsilon} ds_i d\theta \right] \\
&= \frac{1}{2\omega} \left[\int_{s^*-\min\{\epsilon^*, -i, \epsilon\}}^{s^*+\min\{\epsilon^*, -i, \epsilon\}} P(piv \mid \theta) (\theta + l) \frac{\theta + \epsilon - s^*}{2\epsilon} d\theta + \int_{s^*+\min\{\epsilon^*, -i, \epsilon\}}^{s^*+\epsilon^*, -i} P(piv \mid \theta) (\theta + l) d\theta \right]
\end{aligned}$$

upward deviation (less info acq then others)

$$\epsilon > \epsilon^{*, -i}$$

$$\frac{1}{2\omega} \left[\int_{s^*-\epsilon^*, -i}^{s^*+\epsilon^*, -i} \frac{\partial}{\partial \epsilon} P(piv \mid \theta) (\theta + l) \frac{\theta + \epsilon - s^*}{2\epsilon} d\theta \right]$$

- downward deviation (more info acq then others)

$$\epsilon < \epsilon^{*, -i}$$

$$\frac{1}{2\omega} \left[\int_{s^*-\epsilon}^{s^*+\epsilon} P(piv \mid \theta) (\theta + l) \frac{\theta + \epsilon - s^*}{2\epsilon} d\theta + \int_{s^*+\epsilon}^{s^*+\epsilon^*, -i} P(piv \mid \theta) (\theta + l) d\theta \right]$$

$$\begin{aligned}
& \frac{\partial}{\partial \epsilon} \frac{1}{2\omega} \left[\int_{s^*-\epsilon}^{s^*+\epsilon} \frac{1}{2\epsilon} (\theta + \epsilon - s^*) P(piv \mid \theta) (\theta + l) d\theta + \int_{s^*+\epsilon}^{s^*+\epsilon^*, -i} P(piv \mid \theta) (\theta + l) d\theta \right] \\
&= \frac{1}{2\omega} \left[P(piv \mid s^* + \epsilon) (s^* + \epsilon + l) + \frac{1}{2\epsilon} (s^* - \epsilon + \epsilon - s^*) P(piv \mid s^* - \epsilon) (s^* - \epsilon + l) \right] \\
&\quad + \frac{1}{2\omega} \int_{s^*-\epsilon}^{s^*+\epsilon} \frac{\partial}{\partial \epsilon} \frac{1}{2\epsilon} (\theta + \epsilon - s^*) P(piv \mid \theta) (\theta + l) d\theta \\
&\quad - \frac{1}{2\omega} P(piv \mid s^* + \epsilon) (s^* + \epsilon + l) \\
&= \frac{1}{4\epsilon\omega} \int_{s^*-\epsilon}^{s^*+\epsilon} \frac{\partial}{\partial \epsilon} P(piv \mid \theta) (\theta + l) (\theta + \epsilon - s^*) d\theta
\end{aligned}$$

so derivatives are the same

$$\begin{aligned}
& \frac{\partial}{\partial \epsilon} \int_{s^* - \epsilon}^{s^* + \epsilon} (\theta + l) \underbrace{\left(\frac{1}{2} + \frac{1}{2\epsilon} (\theta - s^*) \right)}_{P(v=1|piv, \theta; \epsilon)} P(piv | \theta) f(\theta) d\theta \\
&= -\frac{1}{2\epsilon^2} \int_{s^* - \epsilon}^{s^* + \epsilon} (\theta + l) (\theta - s^*) P(piv | \theta) f(\theta) d\theta \\
&= -\frac{1}{\epsilon^2} \int_{s^* - \epsilon}^{s^* + \epsilon} \frac{1}{2} (\theta + l) (\theta - s^*) \left(\frac{1}{2} + \frac{1}{2\epsilon} (\theta - s^*) \right)^n \left(\frac{1}{2} - \frac{1}{2\epsilon} (\theta - s^*) \right)^{m-n-1} d\theta \\
&= -\frac{1}{\epsilon^2} \left[\frac{\epsilon^2 n! (m-n-1)! \{ (m+2)(l+s)(2n+1-m) + \epsilon [(2n+1-m)^2 + m+1] \}}{(m+2)!} \right] \\
&= -\frac{n! (m-n-1)! \{ (m+2)(l+s)(2n+1-m) + \epsilon [(2n+1-m)^2 + m+1] \}}{(m+2)!}
\end{aligned}$$

The FOC then becomes

$$-\frac{n! (m-n-1)!}{(m+2)!} ((m+2)(l+s)(2n+1-m) + \epsilon ((2n+1-m)^2 + m+1)) = -c$$

$$\epsilon ((2n+1-m)^2 + m+1) = \frac{(m+2)!}{n! (m-n-1)!} c - (m+2)(l+s)(2n+1-m)$$

$$\begin{aligned}
\epsilon &= \frac{c - \frac{n!(m-n-1)!}{(m+2)!} (m+2)(l+s)(2n+1-m)}{\frac{n!(m-n-1)!}{(m+2)!} ((2n+1-m)^2 + m+1)} \\
&= \frac{c}{\frac{n!(m-n-1)!}{(m+2)!} ((2n+1-m)^2 + m+1)} - \frac{(m+2)(2n+1-m)}{((2n+1-m)^2 + m+1)} (l+s) \\
&= \frac{m+2}{(2n+1-m)^2 + m+1} \left[\binom{m+1}{n} c - (2n+1-m)(l+s) \right]
\end{aligned}$$