Blockholder Influence

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1 February 2025

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Abstract

Blockholder influence has attracted recent interest, not only in the context of corporate boards but also in the context of decentralized autonomous organizations (DAOs). I analyze a model of project choice with dispersed information. I focus on the question of whether a blockholder should delegate control to a set of inside delegates. I assume that agents' preferences are not aligned, in the sense that inside delegates derive private benefits from the acceptance of the proposal while the blockholder derives benefits from the rejection of the proposal. The blockholder chooses composition of the board between inside delegates and direct representatives of her own interests. I find that when private interests are low, or signals are imprecise there can be multiple equilibria. The most preferred equilibrium for the blockholder is the one with minimum meaningful delegation. This equilibrium also turns out to be the second best and in particular, the equilibrium value of the firm is higher compared to full delegation. This gives rise to a blockholder premium. Furthermore, I solve the information acquisition problem of the committee in closed form.

[1]I would like to thank Franklin Allen, Patrick Bolton, Gilles Chemla, Ramana Nanda, Ailsa Röell, Magda Rola-Janicka, Savitar Sundaresan and Ansgar Walther for valuable comments and discussions. I gratefully acknowledge support from the Brevan Howard Centre for Financial Analysis.

1 Introduction

Boards of independent directors are often proposed as an effective measure to safeguard shareholder interests, addressing the agency problem that arises from the separation of ownership and control. Boards are intended to reduce conflicts of interest by ensuring that managers act in the best interests of shareholders. However, in practice, shareholders with significant stakes in a company - known as blockholders - often exert substantial influence over the choice of board members. This raises concerns about whether these supposedly independent boards genuinely represent the interests of smaller shareholders, or if they primarily serve the interests of the blockholders as their priorities may diverge.

Similar concerns have been raised in the context of decentralized autonomous organizations (DAOs)—blockchain-based entities governed collectively by their members through smart contracts—which were initially praised for their democratic and decentralized governance structures. In practice, however, large tokenholders, such as venture capital firms, frequently hold significant voting power, potentially marginalizing smaller tokenholders as recent cases suggest that blockholder objectives may not always align with those of smaller tokenholders. [2]. Despite the importance of understanding blockholder influence, particularly regarding the protection of small shareholders, the theoretical literature has paid relatively little attention to the mechanisms by which blockholders influence board decisions.

In this paper I analyze a model to understand the conditions under which a blockholder may choose to delegate decision-making to a group of independent board members and the welfare implications thereof. In the model, independent board members possess superior knowledge about the quality of the project, as they receive a signal of it. They can also exert effort to improve the precision of their signal. The blockholder on the other hand cannot directly obtain information about the project quality. Furthermore, I assume that the preferences of blockholder and independent board members are not aligned: independent board members are more inclined to accept the proposal because of private benefits they derive from its acceptance, while the blockholder leans towards rejecting the proposal because of private benefits he derives from its rejection. The blockholder must therefore decide

[2] see for example https://tokeninsight.com/en/news/why-did-a16z-vote-against-uniswap-s-latest-expansion

how much authority to delegate, considering the trade-off between achieving more accurate decisions and the risk that delegates may approve the proposal overly optimistically, contrary to his preferences. In the blockholder-preferred equilibrium this leads to some but not full delegation. Interestingly, in both corporate boards and DAOs, blockholders often indeed refrain from exercising their full control, preferring instead to delegate some authority to a set of delegates ex-ante, even though they anticipate potential disagreements ex-post.

This model builds on the strategic voting literature, which explores how informed voters make decisions considering the event of being pivotal and the information contained in such event. This paper extends the analysis to settings where the number of informed voters is endogenously determined by the blockholder. Unlike previous studies which fix the proportion of informed to uninformed voters or assume a distribution of heterogeneous preferences among voters ex-ante, this model allows the blockholder to choose the number of informed inside board members versus direct representatives of the blockholders, thereby endogenously determining the proportion of informed voters and distribution of preferences.

In this environment, I derive conditions under which the blockholder voluntarily limits her influence on the board by delegating control to a meaningful number of inside board members. The intuition behind this result is that she trades off the benefits of better information aggregation against the loss of direct control. Inside board members, in turn, also adjust their voting strategies based on their anticipated presence of the blockholder on the board. The more board seats they expect the blockholder to fill with her direct representatives, the more strongly they vote in the opposite direction to offset the representatives' bias on the decision making process. This strategic interplay can lead to multiple equilibria where the blockholder and delegates' decisions counterbalance each other. The blockholder-preferred equilibrium, which under certain conditions also results as the unique equilibrium, is the one where the blockholder delegates just enough seats to inside board members to lose her majority. This equilibrium is also the second best, as the blockholder internalizes the information aggregation decision. Consequently, the equilibrium value of the firm is higher under this minimal delegation compared to full delegation from the perspective of all shareholders, including small investors who are not assumed to derive any private benefits. This results in a control premium of the value of the firm.

I show that in equilibrium, the blockholder instructs her delegates to always reject the proposal. As the total committee size is fixed, her choice of the number inside board members effectively determines the voting rule. For example, when she finds it optimal to delegate to the minimal meaningful number of inside board members (i.e. so that her direct representatives just loose absolute majority), this leads to a scenario where all informed committee members must agree to pass a proposal - effectively implementing a unanimity requirement. The finding that unanimity among informed voters leads to optimal information aggregation differs sharply from previous studies where the number of informed voters is fixed. In those settings, unanimity rules tend to result in poor information aggregation because an individual informed voter becomes pivotal only if all other informed voters accept the proposal. The informational content of this event is so overwhelming that the voter may disregard their private signal entirely and simply follow the crowd. In contrast, in this setting, the blockholder delegates' outright rejection decision leads informed voters to accept the proposal more frequently to offset the blockholder delegates' bias impact on the voting outcome. This behavior offsets the tendency to disregard their own signal, thereby enhancing information aggregation compared to scenarios where only informed inside board members vote without the influence of a blockholder.

Furthermore, I solve the committee's information acquisition problem in closed form, finding that lower levels of delegation lead to higher precision, as fewer informed delegates increase the probability of an individual delegate being pivotal and the investment thus having effect. As a consequence, scenarios with blockholder representation on the committee can be superior to full delegation to inside board members also in terms of providing the right incentives for delegates to become better informed.

Related Literature

This work is rooted in the property rights theory, as established by Grossman and Hart (1986) and Hart and Moore (1990), which provides a framework for understanding control as the residual right of owners to make decisions in contingencies not covered by contracts due to contractual incompleteness. When control is delegated from owners to managers, agency problems arise due to the separation of ownership and control. These issues are exacerbated in firms with dispersed ownership, where collective action problems among shareholders hinder effective monitoring of management (Jensen and Meckling (1976)).

Blockholders can mitigate these problems. Their substantial holdings provide them with the right incentives to monitor management, thereby overcoming the collective action problem inherent in dispersed ownership (e.g. Shleifer and Vishny (1986), Admati, Pfleiderer, and Zechner (1994), Bolton and Von Thadden (1998), Maug (1998)), for a comprehensive review of the literature on blockholders, see Edmans and Holderness (2017).

However, the impact of blockholders is not always entirely positive. Empirical studies by Zingales (1994), Bianco and Casavola (1999), and La Porta et al. (1998) provide evidence that blockholders may pursue private interests not aligned with those of minority shareholders, extracting benefits at their expense—a phenomenon known as entrenchment. From a theoretical perspective, Burkart, Gromb, and Panunzi (1997) argue that while blockholders can reduce agency costs through monitoring, excessive oversight can undermine managerial initiative and deter firm-specific investments.

Despite these concerns, blockholder-controlled firms remain prevalent, particularly in Europe. Studies by La Porta, Lopez-de-Silanes, and Shleifer (1999), Claessens, Djankov, and Lang (2000), and Barca and Becht (2001) document the widespread existence of large shareholders in European firms. See for an excellent comparison of institutional arrangements and governance structures across countries and their impact on firm outcomes, see Allen and Gale.

Similarly, in the world of Decentralized Autonomous Organizations (DAOs) blockholders are omnipresent (Rossello (2024); Fritsch, Müller, and Wattenhofer (2022); Barbereau et al. (2023)). These observations suggest to reevaluate the role of the blockholder, suggesting that it may not always be detrimental and can, in fact, play a positive role in corporate governance even for minority shareholders.

While much of the literature focuses on the relationships between blockholders and managers, less attention has been paid to the interaction between blockholders and boards of directors. One exception is Krüger, Limbach, and Voss (2022), who analyze the conditions under which blockholders choose to have board representation. They find that blockholders weigh the benefits of enhanced monitoring against the potential market impact of perceived agency problems.

The present work explores how blockholders may voluntarily cede control over the board of directors to capitalize on the superior information held by other board members. By stepping back, blockholders can allow for better information aggregation within the board, leading to more informed decision-making that benefits the firm as a whole. This voluntary delegation contrasts with the traditional view of blockholders primarily exerting control through board representation.

This work also contributes to the understanding of information aggregation in strategic voting. Seminal works by Austen-Smith and Banks (1996) and Feddersen and Pesendorfer (1997) explore how private information is aggregated in voting settings and the implications for rational decision-making. These studies focus on strategic voting behavior, where informed committee members consider their pivotal role in the outcome. In these models, voters decide whether to support a proposal based not only on their private information but also by anticipating that they will influence the outcome only if they are pivotal. This leads them to consider what information other voters may have received. Such strategic behavior can result in voters disregarding their private signals in favor of inferred collective information, potentially hindering optimal information aggregation. For a review of the literature on voting in committees, see Gerling et al. (2005).

Contrasting with traditional models, the current work examines a scenario where the number of informed voters is endogenously determined by a blockholder who balances private interests and information aggregation. In equilibrium, the blockholder always rejects proposals. By choosing the number of informed delegates, the blockholder implicitly sets the proportion of informed delegates required to accept a proposal—effectively determining the voting rule.

The blockholder's privately and socially optimal decision is to delegate to the minimal meaningful number of informed voters so that they no longer hold a majority. Consequently, all informed committee members must agree to pass a proposal, implementing a unanimity requirement. This mechanism aligns the blockholder's incentives with optimal information aggregation, ensuring that only proposals supported by all informed members are accepted.

The finding that unanimity among informed board members leads to optimal information aggregation deviates from previous studies where the number of informed board members is fixed. In traditional models, such as Feddersen and Pesendorfer (1998), the unanimity rule results in poor information aggregation because an individual informed voter becomes pivotal only if all other informed voters accept the proposal. The informational content of this event is so significant that the voter may disregard their own private signal and follow the expected collective decision.

In the current setting, however, the blockholder's consistent rejection incentivizes informed voters to accept proposals more frequently to counterbalance their influence. This dynamic offsets the tendency of a voter to accept based solely on information inferred from being pivotal. As a result, the inferiority of the unanimity rule observed in earlier studies no longer holds, and information aggregation improves compared to scenarios where all informed voters participate and the blockholder has no voice.

Another strand of the voting literature examines how committee size influences information acquisition. Persico (2004) analyzes how the size of a committee affects members' incentives to acquire information. When committee members perceive that they are less likely to be pivotal and that their individual vote is unlikely to decide the outcome, they have less motivation to incur the costs associated with obtaining accurate information. While larger committees have the potential to aggregate more information due to the higher number of members, the decreased individual incentives can lead to less overall information being acquired. In smaller groups, each member's vote carries more weight, increasing the likelihood that they will invest in acquiring information. Similar themes are explored by Malenko and Malenko (2019) and Meirowitz and Pi (2021).

Most closely related to this work is the literature on shareholder voting where shareholders and/or blockholders vote directly on a proposal without delegating to a board of directors. Bar-Isaac and Shapiro (2020) find that a blockholder may choose not to vote with all their shares to prevent crowding out of information from smaller shareholders, thus enhancing informational efficiency. Maug (1999) studies the impact of trading and the information contained in prices on the voting process and consequently information aggregation.

This paper proceeds as follows: First the model is described. Then a numerical example is provided to highlight the intuition of the model. Next, the equilibrium is derived for the case of no information acquisition. Finally, the information acquisition problem is solved.

2 Model

An organization is presented with a proposal to implement a new project.

The project can either increase or decrease the value of the organization and is represented by a random variable $\tilde{\theta}$, which is distributed uniformly over an interval symmetric around zero.

$$\tilde{\theta} \sim \mathcal{U}[-\omega, \omega]$$

where $\omega > 0$.

A committee is appointed to decide on whether or not to implement the proposed project. The committee can be thought of as the board of directors of a corporation or as the set of delegates of a DAO. For simplicity, I will use the board terminology in what follows; however, the mechanisms are equally applicable to delegation by blockholders in the context of DAOs. There are in total 2n+1 board members, who can be of two different types: blockholder representatives, who vote as instructed by the blockholder and have no independent decision power, and inside board members, who vote independently. A blockholder holds the absolute majority of shares and can thus determine the composition of the board. The project proposal is accepted if a simple majority of the committee members vote in favor. We say $d = 1$ if more than n out of the committee members vote in favor and the proposal is thus accepted, and $d = 0$ otherwise.

Preferences I assume that the agents' preferences are not aligned: Inside board members derive private benefits from the acceptance of the proposal. They can be thought of as employees of the company such as members of the management team in the context of corporations or developers in the context of DAO. As they are closely related to who proposes the project, they are more favorable towards its acceptance: They value the outcome of the project by $\theta + l$ if it is accepted and 0 if it is rejected, where l is a positive constant. Their preferences can be represented by

$$u_i(d, \theta) = \mathbb{1}_{\{d=1\}}(\theta + l)$$

. The blockholder on the other hand, derives private benefits from the rejection of the proposal. She values the outcome of the project as $\theta - b$ if it is accepted and 0 if it is rejected, where b is a positive constant representing her bias. Her preferences can be represented by the utility function:

$$u_b(d, \theta) = \mathbb{1}_{\{d=1\}} (\theta - b)$$

where $\mathbb{1}_{\{d=1\}}$ is an indicator function that equals 1 if the decision is to accept the project ($d = 1$) and 0 if it is rejected ($d = 0$). This formulation ensures that, without delegation, the blockholder always votes against the project as the expected utility of accepting the proposal is negative. The project choice is, however, not contractible. Otherwise, the blockholder could just compensate all inside board members by paying a side payment of $b + l$ in case of rejection to perfectly align incentives - if feasible and worthwhile.

The blockholder can choose the number m of inside board members and the remaining $2n + 1 - m$ board members are consequently blockholder representatives.

Information Structure Because of their proximity to the operations and expert knowledge inside board members also possess superior knowledge. Each inside board member i privately observes a noisy signal about the realization of θ of $\tilde{\theta}$

$$s_i = \theta + \sigma_i$$

where the noise term σ_i is uniformly distributed over the interval $[-\epsilon, \epsilon]$

$$\sigma_i \sim \mathcal{U}[-\epsilon, \epsilon]$$

and ϵ represents the precision of the signal: the smaller the value of ϵ, the more precise the signal. The blockholder does not receive any signals about the project and nor do its delegates.

Each inside board member can reduce the noise of their signal by incurring a linear cost. However, inside board members are not allowed to communicate the signal - an assumption that at first may seem demanding. As sharing information undermines the motive for acquiring it, one could reinterpret this assumption as the ability to commit not to share information ex-post.

Parametric Assumptions It is assumed that the noise in the signal is not too large, compared to the noise in the underlying project value, specifically $2\epsilon < \omega$. This assumption is necessary for the existence of an equilibrium in the voting sub-game. Furthermore, the potential upside of the project for the blockholder, represented as $\omega - b$, needs to be large relative to the difference in private benefits $b + l$ in order to allow for a positive surplus of delegation: $b + l < \omega - b$.

Timing is as follows: (i) the blockholder chooses the composition of the committee, (ii) inside board members decide on the signal quality, (iii) signals realize, and (iv) inside board members observe their signal vote. Even though the delegation decision of the blockholder and information happens chronologically before voting and effort decisions of the committee, it is assumed that committee members cannot observe the blockholders choice of board composition. Furthermore it is assumed that each inside board members' information acquisition decision is not observed by other inside board members. Those actions are thus strategically simultaneous. I focus on symmetric equilibria in strictly monotone strategies where the likelihood of voting in favor of the project is strictly increasing both in the received signal as well as in the private benefits obtained in case of acceptance.

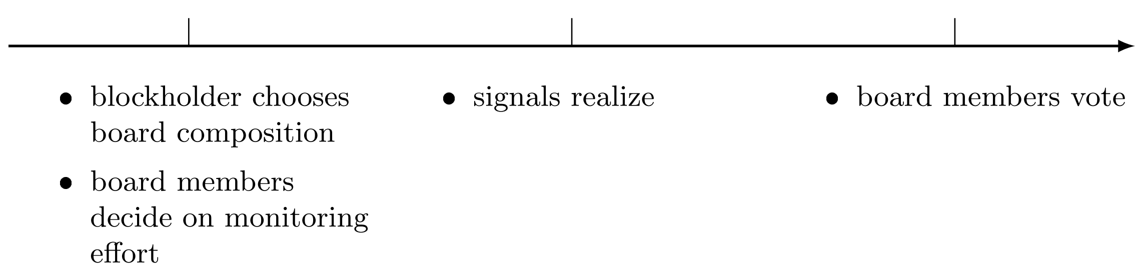

Figure 1: Timeline

The equilibrium concept is that of a Bayes Nash Equilibrium. An equilibrium consists of a set of inside board members' voting decision rules, the blockholder's board composition m^*, and the inside board members' signal precision choice ϵ^* such that

1. Board Composition: Given the noise ϵ_i^* and voting strategy s^* of inside board members,

the blockholder chooses the number of inside board members

$$m^* \in \arg\max_{m} \mathbb{E}_\theta \left[P\left(d = 1 \mid \theta\right) (\theta - b) \right]$$

2. Signal Precision: Given the board composition m^*, the noise of other inside board members $\epsilon^{*,-i}$, and the voting strategy of other inside board members, a board member i chooses ϵ_i^* such that

$$\epsilon_i^* \in \arg\max_{\epsilon} \mathbb{E}_{s_i} \left[\mathbb{E}_{\theta|s_i,piv} \left[(\theta + l) \right] P\left(piv_i \mid s_i; \epsilon^{*,-i}\right) \mid d = 1 \right] + c\epsilon$$

taking into account the effect of his choice of ϵ on his own cutoff value $s^*(\epsilon)$

3. Inside Board Member Voting Decision: Given the board composition m^*, the monitoring effort ϵ^*, and the voting strategy of other board members, an inside board member i accepts the project whenever

$$\mathbb{E}\left[\theta \mid piv_i, s_i\right] + l \geq 0$$

where piv_i stands for the event of being pivotal

4. Blockholder Voting Decision: The blockholder instructs her representatives to vote in favor or against the proposal.

5. beliefs are consistent.

3 Numerical Example

Before proceeding to a detailed analysis of the previously described model, I first present a numerical example to intuitively explain the mechanisms at play. This example deviates from the general setup only in that the state space is assumed to be discrete rather than continuous: I assume that θ can take two values, 1 if the project succeeds and -1 if it fails, with each outcome being equally likely. Furthermore, I assume $n = 1$, which implies that the board has $2n + 1 = 3$ members. The blockholder, as well as her direct representatives, have a bias of $b = \frac{1}{8}$ so that the utility in case of acceptance of the proposal is given by

$\theta - \frac{1}{8}$. For illustrative purposes and for arithmetic simplicity, I assume that inside board members have no private benefits ($l = 0$) and therefore evaluate the proposal purely based on its fundamental value, θ. The noise in the signal is assumed to be $\epsilon = 2$. Signals are then uniformly distributed over the interval $[-1, 3]$ in the good state and over $[-3, 1]$ in the bad state.

The blockholder decides on the composition of the board, considering three possible scenarios:

1. **No delegation.** The blockholder retains control by assigning at most one board seat to an inside member and filling the remaining seats with her own representatives. This configuration leads to her always being pivotal, and as a consequence no information can be extracted from this event. Since the expected value of θ is zero, accepting the proposal would yield a negative expected utility of $-\frac{1}{8}$ and she thus instructs her representatives to reject the proposal.

2. **Partial delegation.** The blockholder could fill two of the board seats with inside board members and one with a blockholder representative. This arrangement allows the inside board members some degree of control but does not eliminate the blockholder's influence.

3. **Full delegation.** The blockholder could choose to fill all board seats with inside board members, thereby completely abstaining from influence. In this case, the outcome mirrors that of a firm without a blockholder, as all decisions reflect the views of the inside board members.

The latter two scenarios require a more detailed analysis, which I will address in the remainder of this example.

Partial Delegation

In the partial delegation scenario, the board consists of two inside board members and one blockholder representative. This section confirms that an equilibrium exists where the blockholder always instructs her representative to vote against the proposal. Each inside board member accepts the proposal only if the signal he observes exceeds -1.

An inside board member only considers the scenario in which his vote is pivotal, i.e. it can swing the outcome of the vote. This is the case when exactly one other board member votes in favor and one votes against. Given the conjectured strategies, the blockholder representative always votes against the proposal, so an inside board member is pivotal only if the other inside board member votes in favor. Given the conjectured voting strategy, the inside board member thus infers from the event of being pivotal that the signal of the other inside board member must have been greater than -1.

From this information as well as his own signal, the inside board member updates his beliefs about the quality of the project. If his own signal is less than -1, he concludes that the project must be bad (since signals for a good project are uniformly distributed over $[-1, 3]$) and rejects the proposal. Conversely, if his signal is greater than 1, he concludes that the project is good (since signals for a bad project are uniformly distributed over $[-3, 1]$) and accepts the proposal.

If his signal lies in the interval $[-1, 1]$, the signal alone provides no information as it is equally likely under both project states. However, in this latter scenario, being pivotal implies that the other inside board member has accepted the proposal. This implies that the other inside board member must have received a signal greater than -1. For a bad project, where signals are uniformly distributed over the interval $[-3, 1]$, the probability of receiving a signal greater than -1 is $\frac{1}{2}$ (see Figure 2). For a good project on the other hand, where signals are uniformly distributed over$[-1, 3]$, the probability of a signal exceeding -1 is 1 (see Figure 3).

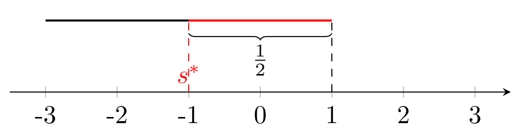

Figure 2: individual acceptance probability in low state

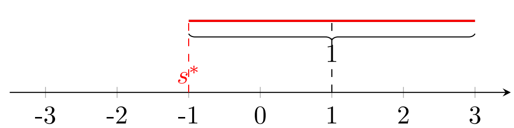

Figure 3: individual acceptance probability in high state

An inside board member can now use these observations to revise his beliefs about the likelihood of a good project in the event that he is pivotal. By Bayes' law we get

$$
\begin{aligned}
P(\theta = 1 \mid piv) &= \frac{P(piv \mid \theta = 1)\, P(\theta = 1)}{P(piv \mid \theta = 1)\, P(\theta = 1) + P(piv \mid \theta = 0)\, P(\theta = 0)} \\
&= \frac{1 \times \frac{1}{2}}{1 \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2}} = \frac{2}{3}
\end{aligned}
$$

The inside board member thus revises the likelihood of a good project from $\frac{1}{2}$ to $\frac{2}{3}$. Given these revised beliefs, the expected utility in case of acceptance is $\frac{2}{3} + \frac{1}{3}(-1) = \frac{1}{3}$ and it is thus optimal for the inside board member to accept the proposal for any signal within the interval $[-1, 1]$. This confirms that -1 is the optimal signal cutoff for accepting the proposal.

The blockholder does not receive a signal but can nonetheless infer information from being pivotal. Being pivotal from the view of the blockholder means that one inside board member accepted the proposal and the other rejected it. Given the voting strategy of the inside board members, this can only occur if the project is bad, as for a good project, both inside board members would always accept. Thus, the blockholder representative concludes that the proposal must be bad whenever he is pivotal and thus rejects it.

We have thus confirmed that the proposed strategies form an equilibrium. Similarly, it can be shown that an alternative equilibrium exists in which the blockholder always accepts, while inside board members use a signal cutoff of 1 (or equivalently accept only when they are sure that the project is good). However, an equilibrium where the blockholder with negative private benefits always accepts and an inside board member with positive private benefits rejects with high probability seems not plausible. Such equilibria are thus ruled out by assuming strict monotonicity with respect to private benefits, meaning that larger positive private benefits lead to a higher likelihood of acceptance.

In order to assess the efficiency of this board structure not that the proposal is accepted if at least two of the three board members vote in favor. Since the blockholder representative always rejects, the proposal can be accepted only when both inside board members vote in favor.

For a good project, both inside board members always accept the proposal (see Figure 3), so the probability of approval is 1 in this case. For a bad project on the other hand, each inside board member accepts with a probability of $\frac{1}{2}$ (see Figure 2), which implies that the probability of both voting in favor and approving the proposal in case of a bad project is $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$. The ex-ante utility of inside board members and minority shareholders is then:

$$
\begin{aligned}
&P\left(acceptance \mid \theta = 1\right) P\left(\theta = 1\right) u_i\left(1,1\right) + P\left(acceptance \mid \theta = -1\right) P\left(\theta = -1\right) u_i\left(1,-1\right) \\
=&1 \times \frac{1}{2} \times 1 + \frac{1}{4} \times \frac{1}{2} \times (-1) \\
=&0.375
\end{aligned}
$$

while the ex-ante utility of the blockholder is taking into account of her private benefits of $-\frac{1}{8}$ and is given by

$$
\begin{aligned}
&P\left(acceptance \mid \theta = 1\right) P\left(\theta = 1\right) u_d\left(1,1\right) + P\left(acceptance \mid \theta = -1\right) P\left(\theta = -1\right) u_d\left(1,-1\right) \\
=&1 \times \frac{1}{2} \times \left(1 - \frac{1}{8}\right) + \frac{1}{4} \times \frac{1}{2} \times \left(-1 - \frac{1}{8}\right) \\
\simeq&0.297
\end{aligned}
$$

Full Delegation

We now turn to the full delegation scenario, where only inside board members vote on the proposal. This section demonstrates that an equilibrium exists in which inside board members accept the proposal if their signal is greater than 0. The efficiency of this equilibrium is then compared to that of the partial delegation case discussed in the previous section. Given the signal distribution, the probability that an inside board member receives a signal greater than 0—and therefore votes to accept the proposal—is $\frac{3}{4}$ in case of a good project (see Figure 4),

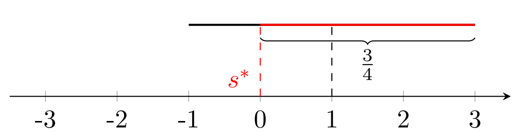

Figure 4: individual acceptance probability in high state

while it is $\frac{1}{4}$ in case of a bad project (see Figure 5).

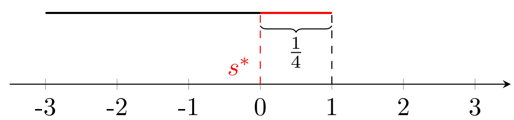

Figure 5: individual acceptance probability in low state

An inside board member is pivotal when exactly one of the other two inside board members accepts and one rejects the proposal. Due to the symmetry of this example with a cutoff value of 0, the probability of being pivotal is the same in both the high and low state.

$$P\left(piv \mid \theta=1\right)=2\times\frac{1}{4}\times\frac{3}{4}=P\left(piv \mid \theta=-1\right)$$

Thus, no additional information can be inferred from the event of being pivotal, and each inside board member bases their decision solely on their own signal.

As in the previous section, a signal larger than 1 (smaller than -1) is fully revealing of a good (bad) project and thus leads to the acceptance (rejection) by the inside board members. For signals in the range of $[-1,1]$, each realization is equally likely under both types of projects, providing no additional information. As a consequence no updating takes place for signals in the interval $[-1,1]$

$$P\left(\theta=1 \mid s_i, piv\right)=P\left(\theta=1\right)=\frac{1}{2}$$

At the cutoff signal of 0, the expected utility of an inside board member is 0, making him indifferent. The cutoff level of 0 is thus indeed a optimal response. However, as is common in voting games, also the equilibrium under full delegation is not unique. For example, always rejecting the proposal regardless of the signal is also an equilibrium. In this case, no inside board member is ever pivotal, so that each weakly prefers to reject as their vote has no impact on the outcome. Similarly, always accepting the proposal is another equilibrium strategy. Note however, that both of these alternative strategies are ruled out by the strict monotonicity requirement with respect to the signal. In both cases, inside board members cast the same vote regardless of the signal they receive.

In case of three inside board members proposal is accepted if at least two of the three inside board members vote in favor. Each inside board member votes in favor with a probability of

$\frac{3}{4}$ for a good project and $\frac{1}{4}$ for a bad project.

For a good project, the probability that all three inside board members vote in favor is given by $\left(\frac{3}{4}\right)^3$, while the probability that exactly two of the three vote in favor is given by $3\left(\frac{3}{4}\right)^2\left(\frac{1}{4}\right)$, where the factor 3 accounts for the different combinations of two members voting in favor. Adding these two expressions, the probability of acceptance in the good state is given as approximately 0.844.

For a bad project, the probability that all three vote in favor is $\left(\frac{1}{4}\right)^3$, and the probability that exactly two vote in favor is $3\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right)$. Adding these two expressions again, results in the probability of acceptance in the bad state as given by approximately 0.156.

This leads to an ex-ante utility for inside board members and minority shareholders of:

$$
\begin{aligned}
&P\left(acceptance \mid \theta=1\right)P\left(\theta=1\right)u_i\left(1,1\right)+P\left(acceptance \mid \theta=-1\right)P\left(\theta=-1\right)u_i\left(1,-1\right)\\
=&0.844\times\frac{1}{2}\times 1+0.156\times\frac{1}{2}\times(-1)\\
=&0.344
\end{aligned}
$$

(compared to 0.375 in the partial delegation scenario) while the ex-ante utility of the blockholder is given by

$$
\begin{aligned}
&P\left(acceptance \mid \theta=1\right)P\left(\theta=1\right)u_d\left(1,1\right)+P\left(acceptance \mid \theta=-1\right)P\left(\theta=-1\right)u_d\left(1,-1\right)\\
=&0.844\times\frac{1}{2}\times\left(1-\frac{1}{8}\right)+0.156\times\frac{1}{2}\times\left(-1-\frac{1}{8}\right)\\
\simeq&0.282
\end{aligned}
$$

(compared to 0.297 in the partial delegation scenario).

We have thus seen in this example, that all agents are better off in the partial delegation scenario. The presence of the blockholder has increased welfare by improving how information is aggregated, even though fewer signals contribute to the decision. This improvement arises because, in the case of partial delegation, both inside board members must vote in favor for the proposal to be accepted, whereas in the case of full delegation, only two out of three inside board members need to approve it. This stricter voting requirement better screens out bad projects, reducing the likelihood of approving a bad project.

Interestingly, good projects are also more likely to be accepted in the partial delegation scenario. This is because the blockholder's strategy of always rejecting the proposal leads the inside board members to counterbalance this voting behavior by voting more strongly in favor - in this example to the extent that they always accept a good project. As a result, the blockholder's presence not only reduces the likelihood of approving bad projects but also guarantees that good projects are more frequently accepted, thereby enhancing welfare for all agents.

4 Equilibrium with Fixed Information Precision

This section derives and analyzes the equilibrium taking the level of noise, ϵ, in the inside board members' signals as given. The next section will endogenize the level of noise. First, the voting behavior of the blockholder and inside board members is each examined individually, before determining the blockholder's optimal delegation decision. From there, the full equilibrium is derived. This leads to the equilibrium value of delegation and, consequently, the control premium which is obtained by letting the blockholder determine the board's composition. Furthermore, the welfare impact of the blockholder's control on small shareholders is analyzed.

4.1 Blockholder's Voting Decision

Although the blockholder does not receive any private information about the project's true value θ, she votes strategically by extracting information from the event of being pivotal. When her decision can determine the outcome of the vote, the blockholder can infer the number of positive and negative votes of inside delegates that led to such a scenario. From there, she can make some inferences about the private signals that inside board members likely received and which underlying project quality may have led to such signals. She uses this insight to update her belief about θ.

Since the inside board members derive positive private benefits from accepting the proposal, they tend to vote in favor even when their private signals are slightly negative.[3] Given the uniform distribution of signals, the blockholder is more likely to be pivotal when the project's value θ is slightly negative. Recognizing this, she revises her belief about θ downward, conditioning on the event of being pivotal. This intuition is formally derived in the proof of the following proposition.

Proposition 1. *A blockholder revises her expectation downwards in the event of being pivotal and instructs all of her representatives to reject the proposal.*

The blockholder revises his expectations of the project's quality downward and because of his extra private benefit if the project is rejected, so he is even more convinced that the proposal should be rejected. To ensure this outcome, she instructs all her representatives to vote against the proposal. It must to be all of them, as one vote configuration that leads the blockholder to be pivotal is the one where n of the inside board members have voted "yes". The blockholder needs to instruct all representatives to vote "no" to ensure the proposal is defeated.

4.2 Inside board members' Voting Decision

Inside board members decide whether to vote in favor or against the proposal based on their private signal and the information they extract from the event of being pivotal. This information depends on their conjectured presence of blockholder representatives as the number of blockholder representatives influences how likely it is that an inside board member will be pivotal for a project of quality θ. This in turn affects the information inside board members extract from this event. The following section provides a detailed analysis how the number of blockholder representatives influences the inside board member beliefs.

A single inside board member i accepts the proposal after observing his private signal s_i whenever it is higher than the threshold value s^*, i.e. $s_i > s^*$. Larger private benefits result in a lower threshold value s^*, thereby increasing the likelihood of accepting the proposal.

[3] In general, there might also be equilibria where the opposite is true; these type of equilibria are ruled out due to the strict monotonicity assumption.

The probability of a single inside board member voting in favor of the proposal in the state θ is given by

$$P\left(s_i > s^* \mid \theta\right) = \begin{cases} 1 & s^* \leq \theta - \epsilon \\ \frac{1}{2} + \frac{1}{2\epsilon}\left(\theta - s^*\right) & s^* \in \left(\theta - \epsilon, \theta + \epsilon\right) \\ 0 & s^* \geq \theta - \epsilon \end{cases} \tag{1}$$

The number of blockholder representatives influences the information extracted from the event of being pivotal, as it affects the proportion of “yes” and “no” votes required for a split vote. An inside board member is pivotal when the other board members’ votes are split such that exactly n vote “yes” and n vote “no”. Given that the blockholder instructs all of her representatives to vote “no” , all n “yes” votes are required to come from the remaining $m-1$ inside board members in order for inside board member i to be pivotal. The probability of being pivotal for an inside board member in state θ, given a total of m inside board members, is then the probability that exactly n of the other $m-1$ inside board vote “yes”. This probability can be expressed by the binomial distribution, with the success probability given by the likelihood that an individual inside board member votes "yes", as expressed in (1). This leads to the following expression:

$$P\left(piv_i \mid \theta\right) = \binom{m-1}{n}\left(\frac{1}{2} + \frac{1}{2\epsilon}\left(\theta - s^*\right)\right)^n \left(\frac{1}{2} - \frac{1}{2\epsilon}\left(\theta - s^*\right)\right)^{m-n-1} \mathbb{1}_{\{\theta \in [s^* - \epsilon, s^* + \epsilon]\}}$$

The larger the presence of the blockholder on the board, the fewer “no” votes compared to “yes” votes of inside board members lead to a pivotal event. An inside board member thus revises his beliefs upwards in the event of being pivotal and is more willing to accept the project for lower values of his signal s_i the more blockholder representatives there are on the board. This leads to the inside board member voting as if they would counterbalance the blockholder’s opposition by accepting the project more often than they would without the presence of the blockholder. This adjustment is stronger when the signals are less precise (i.e., when ϵ is larger), as the inside board members pay less attention to the signal and rely more on their original bias due to the higher uncertainty. To illustrate this intuition, consider the following two scenarios:

Full Delegation $(m = 2n + 1)$

In the case of full delegation, all board members are inside board members. In this scenario, being pivotal does not provide additional information for an inside board member who receives a signal exactly at the cutoff value, since all inside board members share the same bias and use the same cutoff. Therefore, the inside board member bases his decision solely on his own signal and bias. This is illustrated in Figure 6 for the case $n = 5$, which implies a board size of 11 consisting entirely of inside board members $(m = 11)$. In this case, for an inside board board member to be pivotal means that 5 of the other inside board members have voted "yes" since they have received a signal larger than s^* and 5 have voted "no" since they received a signal smaller than s^*.

An inside board member who has received a signal $s_i = s^*$ thus does not obtain any further information from the event of being pivotal. His revised expected value of the project is exactly equal to s^*. For s^* to be his equilibrium cutoff value, he must be indifferent between accepting and rejecting the proposal. Therefore, the equilibrium voting strategy is to accept the proposal for values larger than $s^* = -l$ and reject otherwise.

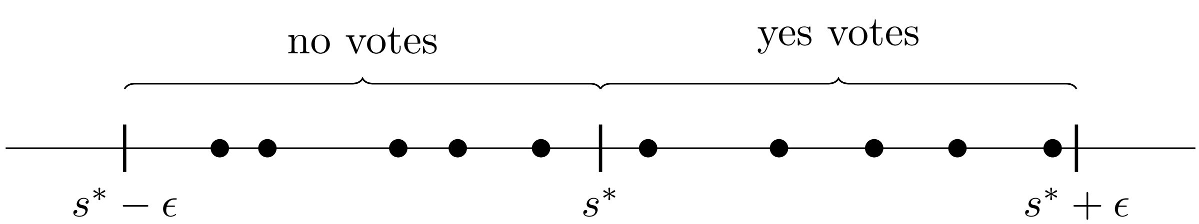

Figure 6: Full delegation: $n = 5$, $m = 11$

Partial Delegation $(2n + 1 > m > n + 1)$

In contrast, with partial delegation, some delegates are blockholder representatives who always vote against the proposal. This scenario is depicted in Figure 7 for the case $n = 5$ which means a board size of 11 with $m = 9$ inside board members and the remaining two board members blockholder delegates.

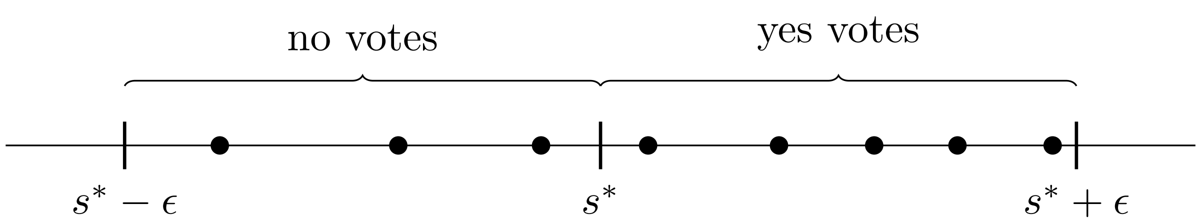

Figure 7: Partial delegation: $n = 5$, $m = 9$

For an inside board board member to be pivotal is its required that 5 of the other inside board members have voted "yes" and 5 have voted "no". However, among the "no" votes are now the two blockholder delegates who are always instructed to vote "no," thus not contributing any (negative) information. This implies that out of the now $m - 1 = 8$ other inside board members, only 3 have received a signal being below s^* while 5 have received a signal larger than s^*. An inside board member which has received exactly a signal s^*, concludes that the signals among other inside board members are higher on average. A larger value of ϵ suggests a higher potential upside on the possible values of the signals.

As a result, the pivotal inside board member revises his belief about θ upwards and becomes more inclined to accept the project by lowering his acceptance threshold further. This effect is amplified by more noise (larger ϵ), as the range of more positive signals becomes larger.

The following proposition formalizes this intuition and shows that there is a unique equilibrium in the voting sub-game in cutoff strategies.

Proposition 2. *For*

$$-l \in [-\omega + 2\epsilon, \omega - 2\epsilon]$$

there exists a unique equilibrium in strictly monotone strategies where an inside board member accepts the proposal whenever

$$s_i \geq s^*$$

with

$$s^* = -\frac{2n + 1 - m}{m + 1}\epsilon - l$$

Proof. See Appendix. $\square$

The uniqueness results from the best response functions of the inside board members being strictly increasing in the cutoff s^*. An increase in the cutoff used by other inside board members leads each inside board member to increase their own cutoff, as their posterior belief about θ in the event of being pivotal improves. This strategic complementarity ensures that there is a unique intersection point where all inside board members' best responses coincide.

4.3 The Blockholder's Delegation Decision

The blockholder to chooses the optimal number of inside board members m^* to maximize her expected utility. This involves balancing the benefit of improved decision-making through better information aggregation against the cost of the inside board members being overly optimistic in approving the proposal. By including more inside board members, the blockholder can potentially benefit from their informational advantage, as more signals are included in the decision making. This better information aggregation can lead to improved decision-making. On the other hand inside board members may vote to accept the too readily due to their private benefits. Increasing the number of inside board members thus raises the risk of approving projects that bring negative utility to the blockholder.

The blockholder's expected utility given m inside board members can be written as

$$v_b(m) = \frac{1}{2\omega} \int_{-\omega}^{\omega} (\theta - b) P_{n,m}(d = 1 \mid \theta) \, d\theta \tag{2}$$

where $P_{n,m}(d = 1 \mid \theta)$ is the probability a project of quality θ is approved by a board of size $2n+1$ with m inside board members. Intuitively, $v_b(m)$ represents the blockholder's expected utility, once we account for the committee's voting outcomes across all possible values of θ. The blockholder will choose m^* to maximize $v_b(m)$.

The the acceptance probability $P_{n,m}(d = 1 \mid \theta)$ depends on the number of blockholder representatives and how likely inside board members are to vote "yes" given their private signals about θ. Blockholder representatives always vote against the proposal as we have seen in proposition (1). Inside board members on the other hand vote in favor of the proposal whenever their signal is above the threshold s^*. The proposal is accepted when at least $n+1$ of the m inside board members approve on it. The probability of acceptance can thus be written as

$$P_{n,m}\left(d=1 \mid \theta\right)=\sum_{i=n+1}^{m}\binom{m}{i}\left[\underbrace{\frac{1}{2}+\frac{1}{2\epsilon}\left(\theta-s^{*}\right)}_{P(v_i=1|\theta)}\right]^{i}\left[\underbrace{\frac{1}{2}-\frac{1}{2\epsilon}\left(\theta-s^{*}\right)}_{P(v_i=0|\theta)}\right]^{m-i}$$

The blockholder's value of delegation can then be derived by substituting this expression into (2) and evaluating the integral. This leads to the following closed-form expression:

Lemma 3. *The blockholder's value of delegation is given by*

$$\begin{aligned}
& v_b\left(m\right) \\
=& \frac{1}{2\omega}\left\{\underbrace{\sum_{i=n+1}^{m}\int_{s^*-\epsilon}^{s^*+\epsilon}\left(\theta-b\right)\binom{m}{i}\left[\frac{1}{2}+\frac{1}{2\epsilon}\left(\theta-s^{*}\right)\right]^{i}\left[\frac{1}{2}-\frac{1}{2\epsilon}\left(\theta-s^{*}\right)\right]^{m-i}d\theta}_{\textit{probabilistic acceptance}}+\underbrace{\int_{s^*+\epsilon}^{\omega}\left(\theta-b\right)d\theta}_{\textit{certain acceptance}}\right\} \\
=& \begin{cases}\frac{1}{2\omega}\left\{\frac{2\epsilon(m-n)}{(m+1)(m+2)}\left[\left(n+1\right)\epsilon-\left(m+2\right)\left(b-s\right)\right]+\frac{1}{2}\left[\left(\omega-b\right)^{2}-\left(b-s-\epsilon\right)^{2}\right]\right\} & \textit{for } m\geq n+1 \\ 0 & \textit{otherwise}\end{cases}
\end{aligned}$$

Proof. See Appendix. □

The right summand stems from the region of θ where the proposal is always accepted as $\theta \geq s^* + \epsilon$, so that all inside board members vote to approve according to (1). When inside board members hold the majority ($m \geq n+1$), the outcome of the vote is unaffected by the actual number of inside board members because the proposal is approved already for $m = n + 1$. An increase in the noise ϵ makes the interval $[s^* + \epsilon, \omega]$ shrink, thereby reducing the range of θ in which the project is accepted for sure. If at the lower bound $\theta = s^* + \epsilon$ accepting the proposal would bring positive utility to the blockholder ($\theta - b > 0$ since $b - s^* - \epsilon < 0$), then increasing ϵ reduces the value of the blockholder because projects that carry positive value are forgone. If, on the other hand, the blockholder obtains negative utility at the cutoff ($\theta - b < 0$ since $b - s^* - \epsilon > 0$), then increasing ϵ has a beneficial effect from the blockholder's perspective because fewer projects with negative utility are accepted.

The left summand represents the region of project qualities $\theta \in [s^* - \epsilon, s^* + \epsilon]$ that are approved with some positive probability. In this "probabilistic acceptance" zone, at least $n+1$ of the m inside board members receive signals above s^*. A larger ϵ widens this intermediate region but also changes the acceptance probabilities, thus affecting the blockholder's value in a more complex way compared to the "certain acceptance" region. To gain better intuition into the role of inside board members in this intermediate range, consider the increase from m to $m+1$ inside board members (where $m \geq n+1$, so the blockholder does not hold a majority to begin with). This difference can be written as

$$v_b\,(m+1) - v_b\,(m) = \frac{\epsilon}{\omega}\frac{(n+1)}{(m+1)\,(m+2)}\left[\frac{2n+1-m}{m+3}\epsilon - (b - s^*)\right]$$

This expression can be interpreted as follows: If the difference between the blockholder's bias and the inside board members' threshold value $b - s^*$, is relatively large compared to the noise term ϵ, the likelihood increases that the additional delegate accepts the proposal in states where the blockholder has negative utility. Consequently, the loss of control exceeds the gain in precision and the increase in the blockholder's utility from adding an additional inside board member $v_b\,(m+1) - v_b\,(m)$, is negative under these circumstances.

If, on the other hand, the signal is less precise (i.e., ϵ is large), the marginal benefit of adding an additional inside board member becomes significant because the information aggregated by the board is relatively imprecise. In such a setting, each additional inside board member provides a stronger incremental effect in reducing the noise of the decision process. As the number of inside board members m increases, however, the diminishing returns from information aggregation become evident, which is captured by the factor $\frac{2n+1-m}{m+3}$ that decreases as m increases: When m is small, each new delegate considerably improves the precision of the boards's decision. As m becomes larger, the board already possesses more precise information, and the incremental value of adding an additional member is reduced. The value of additional members depends critically on the precision of the signals (ϵ), the the blockholder's private benefits and thus sensitivity to potential unfavorable outcomes.

The blockholder increases the number of inside board members as long as $v_b\,(m+1) -$

$v_b(m) > 0$, or equivalently, as long as

$$m < \frac{\epsilon(2n+1) - 3(b - s^*)}{\epsilon + b - s^*}.$$

The optimal number of inside board members, m^*, is reached when this condition for the first time no longer holds so that m just satisfies

$$m^* \geq \frac{\epsilon(2n+1) - 3(b - s^*)}{\epsilon + b - s^*}.$$

and

$$m^* - 1 < \frac{\epsilon(2n+1) - 3(b - s^*)}{\epsilon + b - s^*}.$$

In cases where

$$n + 1 \geq \frac{\epsilon(2n+1) - 3(b - s^*)}{\epsilon + b - s^*},$$

the blockholder does not delegate beyond the minimal meaningful level of delegation ($m = n + 1$) and might or might not prefer delegation to full control: Whether minimal delegation is more attractive to no delegation depends on the net surplus it provides. If the surplus from delegation is insufficient - either because the signals are already very precise (low ϵ) or because the private benefits of the blockholder are large in comparison to the inside board members cutoff level - the blockholder may prefer to retain full control rather than delegate to any inside board members.

These findings are formalized in the following proposition, which summarizes the conditions under which the blockholder optimally delegates authority to inside board members and the circumstances in which minimal or no delegation is preferred. .

Proposition 4. *If either*

$$(\omega - b)^2 < \frac{4(n+1)}{(n+2)^2(n+3)}\epsilon^2$$

or

$$b > s^* + \frac{n}{n+1}\epsilon + \sqrt{(\omega - b)^2 - \frac{4(n+1)}{(n+2)^2(n+3)}\epsilon^2},$$

delegation is never optimal ($m^* = 0$). Otherwise, the optimal level of delegation is given by

$$m^* = \begin{cases} n+1 & \text{if } b - s^* \geq \frac{n}{n+4}\epsilon \\ \left\lceil \frac{(2n+1)\epsilon - 3(b-s^*)}{\epsilon + b - s^*} \right\rceil & \text{if } b - s^* \in \left(\frac{n}{n+4}\epsilon, \frac{1}{2n+3}\epsilon\right) \\ 2n+1 & \text{if } b - s^* \leq \frac{1}{2n+3}\epsilon \end{cases}$$

The proposition highlights the blockholder's trade-off between control and the benefits of information aggregation. If the maximum upside of the project, ω, is low relative to the blockholder's private benefits, b, or if b are large relative to the cutoff value of the inside board members, s^*, the blockholder prefers to maintain complete control by relying solely on her direct representatives. This minimizes the risk of decisions being made against her preferences.

When these conditions are not met, delegation becomes valuable. The optimal number of inside board members, m^*, increases with the noise in the signals (i.e., larger ϵ): when information is precise (small ϵ), adding more inside members brings only marginal benefits, whereas in the presence of large noise (large ϵ), a larger m^* substantially improves decision quality through better information aggregation.

Thus, the optimal number of inside board members reflects a balance between the cost of reduced control due to delegation and the benefit of improved information aggregation. This balance depends on the relative magnitude of the blockholder's bias, the cutoff value s^*, the noise level ϵ, and the potential upside of the project ω.

4.4 Equilibrium

From the individual optimization problems, we have seen that the inside board members revise their threshold value s^* downwards the more blockholder representatives there are, while the blockholder increases delegation to her own representatives the lower (more negative) s^*. If the signal noise is relatively large (but within the limits of a positive surplus of delegation), this counterbalancing behavior of both strategies can lead to a situation of multiple equilibria where any level of delegation m^* can be supported as an equilibrium. At high levels of noise, the benefit of adding an additional inside board member is large as we have just seen in the previous section, which in turn allows to sustain high levels of delegation. If, on the other hand, signals are relatively precise, this benefit is too small so that large numbers of inside board members are not sustainable. Either scenario is, of course, subject to delegation having a positive surplus. These observations are formalized in the following proposition.

Proposition 5. *For a large private benefits of the blockholder in respect to* ω

$$(\omega - b)^2 - \frac{4(n+1)}{(n+2)^2(n+3)}\epsilon^2 < 0$$

or a large difference in private benefits

$$(b+l)^2 > (\omega - b)^2 - \frac{4(n+1)}{(n+2)^2(n+3)}\epsilon^2,$$

no delegation can be supported as an equilibrium. Otherwise, if also

$$b + l \geq \frac{1}{2n+3}\epsilon,$$

the only equilibrium that can be supported is that of minimal delegation

$$m^* = n + 1,$$

while if

$$b + l < \frac{1}{2n+3}\epsilon,$$

there is a multiplicity of equilibria, and any

$$m^* \in \{n+1, \ldots, 2n+1\}$$

can be supported as an equilibrium.

If the blockholder could delegate to a board with perfectly aligned interests and perfect information about the project quality θ she would be able to obtain an expected value proportional to $(\omega - b)^2$. The term $\frac{4(n+1)}{(n+2)^2(n+3)}\epsilon^2$ adjust for the noise in the signals of the inside board members. The larger the noise, the lower the adjusted surplus of delegation in case of perfectly aligned preferences. If the precision of the inside board members' signals is very low, this results in the surplus being negative and delegation can never be viable. The next inequality accounts for the difference in private benefits. The inside board members gain a private benefit l whenever the project is implemented, so $b + l$ measures how misaligned the blockholder's preference for rejecting is compared to the inside members' preference for accepting. If this misalignment is large, delegation is again not viable.

If these conditions are satisfied, no delegation emerges as an equilibrium because the blockholder wants to avoid a situation where inside delegates' lack of precise signals or tendency to approve projects erodes her payoff. Otherwise, when the misalignment is relatively large compared to the noise ϵ (specifically, $b + l \geq \frac{1}{2n+3}\epsilon$), the only sustainable outcome is minimal delegation, where the blockholder assigns exactly $n + 1$ inside board members so that she just loses majority but still retains influence on decisions. If the misalignment $b + l$ is smaller than that threshold, there is a multiplicity of equilibria where any number of inside seats from $n + 1$ up to $2n + 1$ is possible.

From the expressions in the previous proposition, we can derive the equilibrium value of the firm for a blockholder with private benefits b and m inside board members.

Proposition 6. *The equilibrium value for the blockholder is given by*

$$v_b^*(m) = \frac{(\omega - b)^2 - (b + l)^2 - 4\frac{(n+1)(m-n)}{(m+1)^2(m+2)}\epsilon^2}{4\omega}$$

The first term stands for the maximum surplus obtainable from accepting the project when signals are perfect and there is no misalignment of preferences. The second term reflects the loss incurred due to the misalignment of preferences. The third term results from the information loss due to less than perfect signals and also describes the equilibrium effect of the number of inside board members on information aggregation and thus the value of delegation. These last two terms enter into the value in an additive way. As a consequence the optimal level of delegation is not dependent on level private benefits as long as they are small enough to make delegation viable (see previous proposition). Furthermore we can see from this last term that the equilibrium value of delegation is decreasing in the number of inside board members (for a sufficiently large board size) and the highest value is obtained in case of $m^* = n + 1$.

Corollary 7. *For $n \geq 3$, the blockholder prefers the equilibrium with minimal delegation ($m^* = n + 1$).*

4.5 Small Investor Welfare

Suppose that a social planner would choose the optimal number of inside board members with the objective of minimizing the error of falsely accepting a project with negative value or falsely rejecting a project with positive value. The social planner does not have any private benefits in favor of or against the project but takes into account the inside board members' voting behavior. His objective function is thus

$$v_0(m) = \frac{1}{2\omega} \int_{-\omega}^{\omega} \theta P_{n,m}(d = 1 \mid \theta)\, d\theta$$

As the following proposition will show, the presence of a blockholder improves the welfare of small shareholders. This improvement is not caused by enhanced monitoring by either the inside board members or the blockholder, as is usually argued in the blockholder literature (e.g. Jensen and Meckling (1976), Shleifer and Vishny (1986)). Instead, it results from a more balanced decision-making body. The blockholder voluntarily restrains her power because she internalizes the benefits of aggregating information.

The equilibrium with the least possible delegation provides the best expected outcome for small shareholders because it requires that for a project to be accepted, all inside board members vote in favor of the project using the lowest possible threshold level and thus a very high probability of voting yes. This is the most efficient way of aggregating information in this biased environment.

Proposition 8. *The value of the firm for a small shareholder under blockholder influence is given by*

$$v_0^*(m) = \frac{\omega^2 - l^2 - 4\frac{(n+1)(m-n)}{(m+1)^2(m+2)}\epsilon^2}{4\omega}$$

For $n \geq 3$, the equilibrium with minimal delegation $m = n+1$ delivers the highest expected utility

4.6 Blockholder Premium

The benefit the blockholder adds to the value of the firm (from the perspective of small shareholders) can be derived by comparing the value of the firm at the socially optimal level of delegation ($m^* = n + 1$)

$$v_0\left(n+1\right) = \frac{\omega^2 - l^2 - 4\frac{(n+1)}{(n+2)^2(n+3)}\epsilon^2}{4\omega}$$

to the value that would prevail if there was no blockholder, i.e., at full delegation

$$v_0\left(2n+1\right) = \frac{\omega^2 - l^2 - 4\frac{(n+1)^2}{(2n+2)^2(2n+3)}\epsilon^2}{4\omega}.$$

The difference is given by

$$\frac{n\left((n-1)n-4\right)\epsilon^2}{4\left(n+2\right)^2\left(n+3\right)\left(2n+3\right)\omega}$$

This expression is increasing in the noise term, but does not depend on any bias term. Therefore it is in fact the same from the perspective of the blockholder as from the perspective of a small shareholder. A key insight from the model is thus that having a blockholder who strategically delegates board seats—but retains enough power to vote "no" unilaterally—can improve the firm's overall decision quality for all shareholders. Inside board members internalize the blockholder delegates' consistent rejection when deciding how to vote. As a result, borderline or questionable projects must clear a higher "effective threshold," thereby reducing acceptance of value-destroying proposals. Conversely, when the proposal is truly beneficial, all informed members agree, ensuring that good projects are approved despite the blockholder's negative bias. Because this mechanism raises ex-ante expected payoffs for all shareholders (minority included), the firm's market valuation incorporates a "blockholder premium"—it is worth more, ex ante, under a blockholder's partial-delegation structure than it would be under full delegation to insiders (i.e. a diffuse ownership structure).

5 Information Acquisition

This section extends the previous analysis by letting inside board members choose the signal precision at a cost. Each inside board member decides how much to invest in his signal, considering only the contingency in which he can use his signal, which is the case when he is pivotal. Suppose an inside board member acquires a signal with precision ϵ while all other inside board members acquire signals with precision $\epsilon^{*,-i}$. His expected profit from acquiring the signal is then

$$\max_{\epsilon} \int_{s^*(\epsilon)}^{\omega+\epsilon} \mathbb{E}\left[\theta + l \mid s_i, piv_i\right] P\left(piv \mid s_i\right) f\left(s_i\right) ds_i - c\epsilon$$

Proposition 9. *The inside board members' optimal level of noise is given by*

$$\epsilon^* = \frac{m+2}{\left(2n+1-m\right)^2 + m + 1}\left[\binom{m+1}{n} c - \left(2n+1-m\right)\left(l+s^*\right)\right]$$

Proof. See Appendix. □

An inside board member chooses a more noisy signal the larger the cost of information acquisition. A larger (less negative) cutoff value s^* moves the individual acceptance probabilities towards $\frac{1}{2}$Larger private benefits or a higher cutoff value lead to a more precise signals on the other hand. When the inside board members anticipate that a higher approval threshold s^* is adopted, they realize that borderline (moderately good) projects could fail if their information is too noisy. Missing out on such borderline-good projects is especially costly if each member's private benefit l (gained only when the project is accepted) is large. Hence, to avoid mistakenly rejecting profitable proposals, inside members invest more in improving their signal precision—i.e., they choose a larger ϵ in your setup if ϵ corresponds to spending more resources so that the effective noise variance shrinks. Intuitively, the higher either s^* or l is, the greater the payoff from correctly identifying (and accepting) good projects, which increases each insider's incentive to refine their information and separate good from mediocre proposals. This is why, in equilibrium, both a stricter voting cutoff and a larger private gain from acceptance push each inside director to devote more effort or expenditure toward signal precision, thereby raising ϵ when ϵ denotes higher precision.

5.1 Equilibrium with Information Acquisition

Inserting the equilibrium value of s^*, we can derive the equilibrium level of noise

$$\epsilon = \frac{(m+2)(m+1)^2 \binom{m}{n+1}}{4(m-n)} c$$

which is increasing in m since

$$\begin{aligned} \epsilon(m+1) - \epsilon(m) &= c\binom{m+2}{n+1}\frac{(m(n+3)+n+5)}{4} \\ &> 0 \end{aligned}$$

An equilibrium with more delegation to inside board members leads to more noise, while fewer inside board members improve signal precision. This further increases the advantage of minimal delegation and thus blockholder delegation.

6 Conclusion

We have seen that blockholder influence on boards is not necessarily a bad thing, and in cases of opposing private benefits of the boards, blockholder influence can lead to a welfare improvement over pure board decision-making. However, there are also situations where the blockholder has private benefits of the same sign as the boards', and one can expect that the same is no longer true in this scenario. Furthermore, we have left out the issue of communication, which might also be impacted by the presence of blockholder representatives on the board. The issue of communication or deliberation in such scenarios would be an interesting topic for future research.

7 References

References

Admati, A. R., P. Pfleiderer, and J. Zechner. 1994. Large Shareholder Activism, Risk Sharing, and Financial Market Equilibrium. *Journal of Political Economy* 102:1097–130.

Allen, F., and D. Gale. ???? *Comparing financial systems*. MIT Press. ISBN 978-0-262-01177-8.

Austen-Smith, D., and J. S. Banks. 1996. Information Aggregation, Rationality, and the Condorcet Jury Theorem. *The American Political Science Review* 90:34–45.

Bar-Isaac, H., and J. Shapiro. 2020. Blockholder voting. *Journal of Financial Economics* 136:695–717.

Barbereau, T., R. Smethurst, O. Papageorgiou, J. Sedlmeir, and G. Fridgen. 2023. Decentralised finance's timocratic governance: The distribution and exercise of tokenised voting rights. *Technology in Society* 73:102251–.

Barca, F., and M. Becht. 2001. *The Control of Corporate Europe*. Oxford University Press. ISBN 978-0-19-924742-4.

Bianco, M., and P. Casavola. 1999. Italian corporate governance:: Effects on financial structure and firm performance. *European Economic Review* 43:1057–69.

Bolton, P., and E.-L. Von Thadden. 1998. Blocks, Liquidity, and Corporate Control. *The Journal of Finance* 53:1–25.

Burkart, M., D. Gromb, and F. Panunzi. 1997. Large Shareholders, Monitoring, and the Value of the Firm. *The Quarterly Journal of Economics* 112:693–728.

Claessens, S., S. Djankov, and L. H. P. Lang. 2000. The separation of ownership and control in East Asian Corporations. *Journal of Financial Economics* 58:81–112.

Edmans, A., and C. G. Holderness. 2017. Blockholders: A Survey of Theory and Evidence. In B. E. Hermalin and M. S. Weisbach, eds., *The Handbook of the Economics of Corporate Governance*, vol. 1 of *The Handbook of the Economics of Corporate Governance*, 541–636. North-Holland. doi:10.1016/bs.hecg.2017.11.002.

Feddersen, T., and W. Pesendorfer. 1997. Voting Behavior and Information Aggregation in Elections With Private Information. *Econometrica* 65:1029–58.

———. 1998. Convicting the Innocent: The Inferiority of Unanimous Jury Verdicts under Strategic Voting. *The American Political Science Review* 92:23–35.

Fritsch, R., M. Müller, and R. Wattenhofer. 2022. Analyzing Voting Power in Decentralized Governance: Who controls DAOs? doi:10.48550/arXiv.2204.01176.

Gerling, K., H. P. Grüner, A. Kiel, and E. Schulte. 2005. Information acquisition and decision making in committees: A survey. *European Journal of Political Economy* 21:563–97.

Grossman, S. J., and O. D. Hart. 1986. The Costs and Benefits of Ownership: A Theory of Vertical and Lateral Integration. *Journal of Political Economy* 94:691–719.

Hart, O., and J. Moore. 1990. Property Rights and the Nature of the Firm. *Journal of Political Economy* 98:1119–58.

Jensen, M. C., and W. H. Meckling. 1976. Theory of the firm: Managerial behavior, agency costs and ownership structure. *Journal of Financial Economics* 3:305–60.

Krüger, S., P. Limbach, and P. Voss. 2022. Blockholder Representation on the Board: Theory and Evidence. doi:10.2139/ssrn.4270536.

La Porta, R., F. Lopez-de-Silanes, and A. Shleifer. 1999. Corporate Ownership around the World. *The Journal of Finance* 54:471–517.

La Porta, R., F. Lopez-de Silanes, A. Shleifer, and r. w. vishny. 1998. Law and Finance. *Journal of Political Economy* 106:1113–55.

Malenko, A., and N. Malenko. 2019. Proxy Advisory Firms: The Economics of Selling Information to Voters. *The Journal of Finance* 74:2441–90.

Maug, E. 1998. Large Shareholders as Monitors: Is There a Trade-Off between Liquidity and Control? *The Journal of Finance* 53:65–98.

Maug, E. G. 1999. How Effective is Proxy Voting? Information Aggregation and Conflict Resolution in Corporate Voting Contests. doi:10.2139/ssrn.157693.

Meirowitz, A., and S. Pi. 2021. Information Acquisition, Voting and Trading. doi:10.2139/ssrn.4890065.

Persico, N. 2004. Committee Design with Endogenous Information. *The Review of Economic Studies* 71:165–91.

Rossello, R. 2024. Blockholders and strategic voting in DAOs' governance. doi:10.2139/ssrn.4706759.

Shleifer, A., and R. W. Vishny. 1986. Large Shareholders and Corporate Control. *Journal of Political Economy* 94:461–88.

Zingales, L. 1994. The Value of the Voting Right: A Study of the Milan Stock Exchange Experience. *The Review of Financial Studies* 7:125–48.

Appendix

Proof of Proposition 1

The proof proceeds by showing that the blockholder always instructs her own representatives to reject the project independently of the number of inside board members she has chosen. This is because she is already biased against the project through her private benefits, and the more she delegates to blockholder representatives, the more she becomes pessimistic about the project by extracting information from the event of being pivotal.

In order to understand the blockholder's voting behavior, the concept of being pivotal first needs to be extended to a voter who controls multiple votes: Suppose there are $2n+1$ delegates in total, consisting of m inside board members and $2n+1-m$ blockholder representatives. For delegation to be meaningful, the blockholder must delegate at least $n+1$ inside board members ($m \geq n+1$) - otherwise, her own representatives retain the majority, and she can never be pivotal. As a consequence, in all cases where $m < n+1$, the blockholder cannot extract any information from the event of being pivotal and thus votes according to her prior to reject the project with a majority, resulting in all of those choices ($m < n+1$) being outcome-equivalent.

In all that follows, we thus mean by delegation that at least $m \geq n+1$ of the board members are inside board members. Furthermore it is clear that it must be that $m \leq 2n$ as otherwise only inside board members vote and the blockholder has no representatives at all. Assume now that, out of the m inside board members, i vote in favor of the project, and $m-i$ vote against. If $i \geq n+1$ inside board members vote in favor of the project, there is a majority in favor of the project among the inside board members alone, and the blockholder cannot be pivotal, so that the project will pass regardless of the blockholder representatives' votes. Conversely, if there are $m-i \geq n+1$ inside board members rejecting the proposal, there is a majority against the proposal among the inside board members, and again the blockholder cannot be pivotal as the project will be rejected regardless of her representatives' votes. Therefore, the blockholder can be pivotal only when the number of votes in favor of the project among inside board members satisfies $i \in \{m-n, \ldots, n\}$.

The probability that the blockholder is pivotal, given the state θ, can then be written as:

$$P\left(piv_b \mid \theta\right) = \sum_{i=m-n}^{n} \binom{m}{i} \left[P\left(v_i = 1 \mid \theta\right)\right]^i \left[P\left(v_i = 0 \mid \theta\right)\right]^{m-i} \mathbb{1}_{\{\theta \in [s^* - \epsilon, s^* + \epsilon]\}} \tag{3}$$

.

Substituting the individual inside board members acceptance probability (1) into equation (3) The probability of being pivotal in state θ can then be expressed as

$$P\left(piv_b \mid \theta\right) = \sum_{i=m-n}^{n} \binom{m}{i} \left[\frac{1}{2} + \frac{1}{2\epsilon}\left(\theta - s^*\right)\right]^i \left[\frac{1}{2} - \frac{1}{2\epsilon}\left(\theta - s^*\right)\right]^{m-i} \mathbb{1}_{\{\theta \in [s^* - \epsilon, s^* + \epsilon]\}}$$

. The posterior probability of θ in the event of being pivotal can then be derived as

$$\begin{aligned} f\left(\theta \mid piv_b\right) &= \frac{P\left(piv_b \mid \theta\right) f\left(\theta\right)}{\int_{-\infty}^{\infty} P\left(piv_b \mid \theta\right) f\left(\theta\right) d\theta} \\ &= \frac{\sum_{i=m-n}^{n} \begin{pmatrix} m \\ \\ i \end{pmatrix} \left[\frac{1}{2} + \frac{1}{2\epsilon}\left(\theta - s^*\right)\right]^i \left[\frac{1}{2} - \frac{1}{2\epsilon}\left(\theta - s^*\right)\right]^{m-i} \mathbb{1}_{\{\theta \in [s^* - \epsilon, s^* + \epsilon]\}}}{\int_{s^*-\epsilon}^{s^*+\epsilon} \sum_{i=m-n}^{n} \begin{pmatrix} m \\ \\ i \end{pmatrix} \left[\frac{1}{2} + \frac{1}{2\epsilon}\left(\theta - s^*\right)\right]^i \left[\frac{1}{2} - \frac{1}{2\epsilon}\left(\theta - s^*\right)\right]^{m-i} \mathbb{1}_{\{\theta \in [s^* - \epsilon, s^* + \epsilon]\}} d\theta} \\ &= \frac{1}{2\epsilon} \frac{m+1}{2n+1-m} \sum_{i=m-n}^{n} \begin{pmatrix} m \\ \\ i \end{pmatrix} \left[\frac{1}{2} + \frac{1}{2\epsilon}\left(\theta - s^*\right)\right]^i \left[\frac{1}{2} - \frac{1}{2\epsilon}\left(\theta - s^*\right)\right]^{m-i} \mathbb{1}_{\{\theta \in [s^* - \epsilon, s^* + \epsilon]\}} \end{aligned}$$

where the integrals in the denominator are evaluated by change of variable

$$p = \frac{1}{2} + \frac{1}{2\epsilon}\left(\theta - s^*\right)$$

$$\frac{dp}{d\theta} = \frac{1}{2\epsilon}$$

and noting that $\int_0^1 \left(p\right)^n \left(1-p\right)^{m-n} dp$ is the The Euler integral of the first kind with the value $\frac{n!(m-n)!}{(m+1)!}$ so that

$$
\begin{aligned}
&\int_{s^*-\epsilon}^{s^*+\epsilon} \left(\frac{1}{2} + \frac{1}{2\epsilon}\left(\theta - s^*\right)\right)^{n} \left(\frac{1}{2} - \frac{1}{2\epsilon}\left(\theta - s^*\right)\right)^{m-n} d\theta \\
=&2\epsilon\frac{n!\,(m-n)!}{(m+1)!}
\end{aligned}
$$

using this formula iteratively for n and $n+1$ the expected conditional mean can now be derived by

$$
\begin{aligned}
&2\epsilon\frac{(n+1)!\,(m-n)!}{(m+2)!} \\
=&\int_{s^*-\epsilon}^{s^*+\epsilon} \left(\frac{1}{2} + \frac{1}{2\epsilon}\left(\theta - s^*\right)\right)^{n+1} \left(\frac{1}{2} - \frac{1}{2\epsilon}\left(\theta - s^*\right)\right)^{m-n} d\theta \\
=&\left(\frac{1}{2} - \frac{s^*}{2\epsilon}\right)\int_{s^*-\epsilon}^{s^*+\epsilon} \left(\frac{1}{2} + \frac{1}{2\epsilon}\left(\theta - s^*\right)\right)^{n} \left(\frac{1}{2} - \frac{1}{2\epsilon}\left(\theta - s^*\right)\right)^{m-n} d\theta + \frac{1}{2\epsilon}\int_{s^*-\epsilon}^{s^*+\epsilon} \theta \left(\frac{1}{2} + \frac{1}{2\epsilon}\left(\theta - s^*\right)\right)^{n} \left(\frac{1}{2} - \frac{1}{2}\right. \\
=&\left(\frac{1}{2} - \frac{s^*}{2\epsilon}\right)2\epsilon\frac{n!\,(m-n)!}{(m+1)!} + \frac{1}{2\epsilon}\int_{s^*-\epsilon}^{s^*+\epsilon} \theta \left(\frac{1}{2} + \frac{1}{2\epsilon}\left(\theta - s^*\right)\right)^{n} \left(\frac{1}{2} - \frac{1}{2\epsilon}\left(\theta - s^*\right)\right)^{m-n} d\theta
\end{aligned}
$$

re-arraigning gives

$$
\begin{aligned}
&\frac{1}{2\epsilon}\int_{s^*-\epsilon}^{s^*+\epsilon} \theta \left(\frac{1}{2} + \frac{1}{2\epsilon}\left(\theta - s^*\right)\right)^{n} \left(\frac{1}{2} - \frac{1}{2\epsilon}\left(\theta - s^*\right)\right)^{m-n} d\theta \\
=&2\epsilon\frac{(n+1)!\,(m-n)!}{(m+2)!} - \left(\frac{1}{2} - \frac{s^*}{2\epsilon}\right)2\epsilon\frac{n!\,(m-n)!}{(m+1)!}
\end{aligned}
$$

the conditional mean can then be derived as

$$\begin{aligned}
\mathbb{E}\left[\theta \mid piv_b\right] &= \int_{-\infty}^{\infty} \theta f\left(\theta \mid piv_i\right) d\theta \\
&= \frac{m+1}{2n+1-m}\frac{1}{2\epsilon}\sum_{i=m-n}^{n}\int_{s^*-\epsilon}^{s^*+\epsilon}\theta\begin{pmatrix} m \\ i \end{pmatrix}\left(\frac{1}{2}+\frac{1}{2\epsilon}\left(\theta-s^*\right)\right)^{i}\left(\frac{1}{2}-\frac{1}{2\epsilon}\left(\theta-s^*\right)\right)^{m-i} d\theta \\
&= \frac{m+1}{2n+1-m}\sum_{i=m-n}^{n}\begin{pmatrix} m \\ i \end{pmatrix}\frac{1}{2\epsilon}\int_{s^*-\epsilon}^{s^*+\epsilon}\theta\left(\frac{1}{2}+\frac{1}{2\epsilon}\left(\theta-s^*\right)\right)^{i}\left(\frac{1}{2}-\frac{1}{2\epsilon}\left(\theta-s^*\right)\right)^{m-i} d\theta \\
&= \frac{m+1}{2n+1-m}\sum_{i=m-n}^{n}\begin{pmatrix} m \\ i \end{pmatrix}\left[2\epsilon\frac{(i+1)!\,(m-i)!}{(m+2)!}-\left(\frac{1}{2}-\frac{s^*}{2\epsilon}\right)2\epsilon\frac{i!\,(m-i)!}{(m+1)!}\right] d\theta \\
&= \frac{m+1}{2n+1-m}\sum_{i=m-n}^{n}\frac{\left((2i-m)\,\epsilon+(m+2)\,s^*\right)}{(m+2)\,(m+1)} \\
&= s^* \\
&< 0
\end{aligned}$$

The blockholder has lowered his expectations about the project's quality. This downwards-revision, together with the private benefit he receives if the proposal is rejected, leads him to conclude that the proposal should be rejected. To ensure its rejection, he must instruct all her representatives to vote against it. This is necessary because he cannot tell exactly how many inside board members voted yes or no from the pivotal event. For instance, if n out of m inside board members have voted yes and he would be pivotal as a consequence of this configuration, he needs every one of her representatives to vote no; otherwise, the proposal would be approved.

Proof of Proposition 2

The first step of the proof proceeds by confirming that the proposed voting strategy is an equilibrium in the voting sub-game. Suppose all inside board members vote according to the threshold strategy s^*. The likelihood of an inside board member being pivotal in state θ is then the likelihood that n of the $m-1$ other inside board members are voting in favor of the proposal.

$$P\left(piv_i \mid \theta\right) = \binom{m-1}{n}\left(\frac{1}{2}+\frac{1}{2\epsilon}\left(\theta-s^*\right)\right)^n\left(\frac{1}{2}-\frac{1}{2\epsilon}\left(\theta-s^*\right)\right)^{m-n-1}\mathbb{1}_{\{\theta\in[s^*-\epsilon,s^*+\epsilon]\}}$$

The posterior probability of θ in the event of being pivotal and having observed a signal exactly equal to the cutoff value of the other inside board members s^* can then be derived as

$$g\left(\theta \mid s^*, piv_i\right) = \frac{f\left(s^* \mid \theta\right)P\left(piv_i \mid \theta\right)f\left(\theta\right)}{\int_{-\infty}^{\infty} f\left(s^* \mid \theta\right)P\left(piv_i \mid \theta\right)f\left(\theta\right)d\theta} \tag{4}$$

$$= \frac{\frac{1}{2\epsilon}\mathbb{1}_{s^*\in\{\theta-\epsilon,\theta+\epsilon\}}P\left(piv_i \mid \theta\right)\frac{1}{2\omega}\mathbb{1}_{\{\theta\in[-\omega,\omega]\}}}{\int_{-\infty}^{\infty}\frac{1}{2\epsilon}\mathbb{1}_{s^*\in\{\theta-\epsilon,\theta+\epsilon\}}P\left(piv_i \mid \theta\right)\frac{1}{2\omega}\mathbb{1}_{\{\theta\in[-\omega,\omega]\}}d\theta} \tag{5}$$

$$= \frac{\left(\frac{1}{2}+\frac{1}{2\epsilon}\left(\theta-s^*\right)\right)^n\left(\frac{1}{2}-\frac{1}{2\epsilon}\left(\theta-s^*\right)\right)^{m-n-1}\mathbb{1}_{\{\theta\in[s^*-\epsilon,s^*+\epsilon]\}}}{\int_{s^*-\epsilon}^{s^*+\epsilon}\left(\frac{1}{2}+\frac{1}{2\epsilon}\left(\theta-s^*\right)\right)^n\left(\frac{1}{2}-\frac{1}{2\epsilon}\left(\theta-s^*\right)\right)^{m-n-1}d\theta} \tag{6}$$

The integral in the denominator can be evaluated by change of variable

$$p = \frac{1}{2}+\frac{1}{2\epsilon}\left(\theta-s^*\right)$$

$$\frac{dp}{d\theta} = \frac{1}{2\epsilon}$$

and noting again that $\int_0^1 p^n\left(1-p\right)^{m-n-1}dp$ is the Euler integral of the first kind with the value $\frac{n!(m-n-1)!}{m!}$, we have

$$\begin{aligned}
&\int_{s^*-\epsilon}^{s^*+\epsilon}\binom{m-1}{n}\left(\frac{1}{2}+\frac{1}{2\epsilon}\left(\theta-s^*\right)\right)^n\left(\frac{1}{2}-\frac{1}{2\epsilon}\left(\theta-s^*\right)\right)^{m-n-1}d\theta \\
&= \binom{m-1}{n}2\epsilon\int_0^1 p^n\left(1-p\right)^{m-n-1}dp \\
&= \binom{m-1}{n}2\epsilon\frac{n!\left(m-n-1\right)!}{m!} \\
&= \frac{2\epsilon}{m}
\end{aligned}$$

So, the posterior density is thus given by

$$g(\theta \mid s^*, piv_i) = \frac{1}{2\epsilon} \frac{m!}{n!(m-n-1)!} \left(\frac{1}{2} + \frac{1}{2\epsilon}(\theta - s^*)\right)^n \left(\frac{1}{2} - \frac{1}{2\epsilon}(\theta - s^*)\right)^{m-n-1} \mathbb{1}_{\theta \in [s^*-\epsilon, s^*+\epsilon]}$$

The expected conditional mean can now be derived using the fact that

$$\begin{aligned}
&\frac{1}{2\epsilon} \int_{s^*-\epsilon}^{s^*+\epsilon} \theta \left(\frac{1}{2} + \frac{1}{2\epsilon}(\theta - s^*)\right)^n \left(\frac{1}{2} - \frac{1}{2\epsilon}(\theta - s^*)\right)^{m-n-1} d\theta \\
&= 2\epsilon \frac{(n+1)!(m-n-1)!}{(m+1)!} - \left(\frac{1}{2} - \frac{s^*}{2\epsilon}\right) 2\epsilon \frac{n!(m-n-1)!}{m!}
\end{aligned}$$

Thus

$$\begin{aligned}
\mathbb{E}[\theta \mid s^*, piv_i] &= \int_{-\infty}^{\infty} \theta g(\theta \mid s^*, piv_i)\, d\theta \\
&= \frac{1}{2\epsilon} \frac{m!}{n!(m-n-1)!} \int_{s^*-\epsilon}^{s^*+\epsilon} \theta \left(\frac{1}{2} + \frac{1}{2\epsilon}(\theta - s^*)\right)^n \left(\frac{1}{2} - \frac{1}{2\epsilon}(\theta - s^*)\right)^{m-n-1} d\theta \\
&= \frac{m!}{n!(m-n-1)!} \left[2\epsilon \frac{(n+1)!(m-n-1)!}{(m+1)!} - \left(\frac{1}{2} - \frac{s^*}{2\epsilon}\right) 2\epsilon \frac{n!(m-n-1)!}{m!}\right] \\
&= 2\epsilon \frac{n+1}{m+1} - (\epsilon - s^*) \\
&= \frac{2n+1-m}{m+1}\epsilon + s^*
\end{aligned}$$

An inside board member who observes exactly s^* has to be indifferent between accepting and rejecting, so

$$\frac{2n+1-m}{m+1}\epsilon + s^* + l = 0$$

or

$$s^* = -\frac{2n+1-m}{m+1}\epsilon - l$$

To show uniqueness, I proceed to show that the best response is strictly increasing. Let $s^{*,-i}$ denote the cutoff signal of all other inside board members. Then, the conditional expectation of an inside board member who observes a signal s_i in the event of being pivotal is given by

$$\mathbb{E}\left[\theta \mid s_i, piv; s^{*,-i}\right] = \int_{\max\{s_i, s^*\}-\epsilon}^{\min\{s_i, s^*\}+\epsilon} \theta \left(\frac{1}{2} + \frac{1}{2\epsilon}(\theta - s^*)\right)^n \left(\frac{1}{2} - \frac{1}{2\epsilon}(\theta - s^*)\right)^{m-n-1} d\theta$$

Suppose the other inside board members increase their cutoff from s^* to $\hat{s}^*$, then

$$\min\{s^*, \hat{s}^*\} + \epsilon = s^* + \epsilon$$
$$\max\{s^*, \hat{s}^*\} - \epsilon = \hat{s}^* - \epsilon$$

Thus,

$$\mathbb{E}[\theta \mid s^*, piv; \hat{s}^*] = \int_{\hat{s}^*-\epsilon}^{s^*+\epsilon} \theta \left(\frac{1}{2} + \frac{1}{2\epsilon}(\theta - \hat{s}^*)\right)^n \left(\frac{1}{2} - \frac{1}{2\epsilon}(\theta - \hat{s}^*)\right)^{m-n-1} d\theta$$

Using the Leibniz rule, we get the derivative with respect to $\hat{s}^*$

$$\frac{\partial}{\partial \hat{s}^*}\mathbb{E}[\theta \mid s^*, piv; \hat{s}^*]$$
$$= \int_{\hat{s}^*-\epsilon}^{s^*+\epsilon} \frac{1}{2\epsilon}\theta \left(\frac{1}{2} + \frac{1}{2\epsilon}(\theta - s^*)\right)^{n-1} \left(\frac{1}{2} - \frac{1}{2\epsilon}(\theta - s^*)\right)^{m-n-2} \left(\frac{1}{2} + \frac{1}{2\epsilon}(\theta - s^*)\right)\left(\underbrace{m-n-1}_{\geq 0}\right) d\theta$$
$$> 0$$

Thus, after an increase in $\hat{s}^*$,

$$\mathbb{E}[\theta \mid s^*, piv; \hat{s}^*] + l > 0$$

and the inside board member will strictly increase her cutoff point where she is indifferent. The best response is thus strictly increasing.

Proof of Lemma 3

The proof consists of integrating over each summand and then adding up the sum. The proof uses the result that

$$\sum_{i=n+1}^{m} \binom{m}{i} \left[\underbrace{\frac{1}{2} + \frac{1}{2\epsilon}(\theta - s^*)}_{P(v_i=1|\theta)}\right]^i \left[\underbrace{\frac{1}{2} - \frac{1}{2\epsilon}(\theta - s^*)}_{P(v_i=0|\theta)}\right]^{m-i}$$

$$\int_{s^*-\epsilon}^{s^*+\epsilon} (\theta - b) \left(\begin{array}{c} m \\ i \end{array}\right) \left[\frac{1}{2} + \frac{1}{2\epsilon}(\theta - s^*)\right]^{m-i} \left[\frac{1}{2} - \frac{1}{2\epsilon}(\theta - s^*)\right]^i d\theta = \frac{\epsilon(m-2i) + (s^*-b)(m+2)}{(m+1)(m+2)} 2\epsilon$$

$$
\begin{aligned}
&v\left(m\right)\\
&=\frac{1}{2\omega}\int_{-\omega}^{\omega}\left(\theta-b\right)P_{n,m}\left(d=1\mid\theta\right)d\theta\\
&=\frac{1}{2\omega}\left\{\sum_{i=n+1}^{m}\int_{s^*-\epsilon}^{s^*+\epsilon}\left(\theta-b\right)\binom{m}{i}\left[\frac{1}{2}+\frac{1}{2\epsilon}\left(\theta-s^*\right)\right]^{i}\left[\frac{1}{2}-\frac{1}{2\epsilon}\left(\theta-s^*\right)\right]^{m-i}d\theta+\int_{s^*+\epsilon}^{\omega}\left(\theta-b\right)d\theta\right\}\\
&=\frac{1}{2\omega}\left\{\sum_{i=0}^{m-n-1}\int_{s^*-\epsilon}^{s^*+\epsilon}\left(\theta-b\right)\binom{m}{i}\left[\frac{1}{2}+\frac{1}{2\epsilon}\left(\theta-s^*\right)\right]^{m-i}\left[\frac{1}{2}-\frac{1}{2\epsilon}\left(\theta-s^*\right)\right]^{i}d\theta+\int_{s^*+\epsilon}^{\omega}\left(\theta-b\right)d\theta\right\}\\
&=\frac{1}{2\omega}\left\{\sum_{i=0}^{m-n-1}\frac{\epsilon\left(m-2i\right)+\left(s^*-b\right)\left(m+2\right)}{\left(m+1\right)\left(m+2\right)}2\epsilon+\frac{1}{2}\left(\omega^2-\left(s^*+\epsilon\right)^2\right)+b\left(\left(s^*+\epsilon\right)-\omega\right)\right\}\\
&=\frac{1}{2\omega}\left[\frac{\left(m-n\right)\left[\left(n+1\right)\epsilon+\left(m+2\right)\left(s^*-b\right)\right]}{\left(m+1\right)\left(m+2\right)}2\epsilon+\frac{1}{2}\left(\omega^2-\left(s^*+\epsilon\right)^2\right)+b\left(\left(s^*+\epsilon\right)-\omega\right)\right]\\
&=\frac{1}{2\omega}\left[\frac{2\epsilon\left(m-n\right)}{\left(m+1\right)\left(m+2\right)}\left[\left(n+1\right)\epsilon-\left(m+2\right)\left(b-s^*\right)\right]+\frac{1}{2}\left(\omega^2-\left(s^*+\epsilon\right)^2\right)+b\left(\left(s^*+\epsilon\right)-\omega\right)\right]\\
&=\frac{1}{2\omega}\left[\frac{2\epsilon\left(m-n\right)}{\left(m+1\right)\left(m+2\right)}\left[\left(n+1\right)\epsilon-\left(m+2\right)\left(b-s^*\right)\right]+\frac{1}{2}\left(\left(\omega-s^*-\epsilon\right)\left(\omega+s^*+\epsilon-2b\right)\right)\right]\\
&=\frac{1}{2\omega}\left[\frac{2\epsilon\left(m-n\right)}{\left(m+1\right)\left(m+2\right)}\left[\left(n+1\right)\epsilon-\left(m+2\right)\left(b-s^*\right)\right]+\frac{1}{2}\left[\left(\omega-b\right)^2-\left(b-s^*-\epsilon\right)^2\right]\right]
\end{aligned}
$$

Proof of Proposition4

$$v\left(m\right)=\frac{1}{2\omega}\left\{\frac{2\epsilon\left(m-n\right)}{\left(m+1\right)\left(m+2\right)}\left[\left(n+1\right)\epsilon-\left(m+2\right)\left(b-s^*\right)\right]+\frac{1}{2}\left(\left(\omega-s^*-\epsilon\right)\left(\omega+s^*+\epsilon-2b\right)\right)\right\}$$

- FOC wrt s^*

  $$\frac{1}{2\omega}\left(b-s^*+\frac{m-2n-1}{m+1}\epsilon\right)$$

  SOC wrt s^*

  $$-\frac{1}{2\omega}$$

  thus there is a maximum at

  $$s^*=b-\frac{2n+1-m}{m+1}\epsilon$$

  in equilibrium

  $$-l=\frac{2n+1-m}{m+1}\epsilon+s^*=b$$

however for

$$b > -l$$

$$s^* < b - \frac{2n+1-m}{m+1}\epsilon$$

so that in the FOC

$$\frac{1}{2\omega}\left(b - s^* + \frac{m-2n-1}{m+1}\epsilon\right) > \frac{2}{2\omega}\left(\frac{2n+1-m}{m+1}\epsilon\right) > 0$$

thus in total we have that for a bias $b > -l$ or equivalently $s^* < b - \frac{2n+1-m}{m+1}\epsilon$ the value function is increasing in s^*

- at maximum

$$v\left(s^* = b - \frac{2n+1-m}{m+1}\epsilon\right) = \frac{1}{2\omega}\left[\frac{1}{2}(\omega - b)^2 - 2\epsilon^2\frac{(n+1)(m-n)}{(m+1)^2(m+2)}\right]$$

so that if

$$(\omega - b)^2 - 4\epsilon^2\frac{(n+1)(m-n)}{(m+1)^2(m+2)} < 0$$

the value function is always negative

- positivity constraint is thus

$$\frac{1}{2\omega}\left\{\frac{2\epsilon(m-n)}{(m+1)(m+2)}\left[(n+1)\epsilon - (m+2)(b - s^*)\right] + \frac{1}{2}\left((\omega - s^* - \epsilon)(\omega + s^* + \epsilon - 2b)\right)\right\} = 0$$

(need positive solution for b>-l, but how about out of equilibrium values?)

- need that at optimum value fct is positive otherwise no delegation is possible
- or for

$$b - \sqrt{(\omega - b)^2 - \underbrace{\frac{4(n+1)(m-n)}{(m+1)^2(m+2)}}_{<1}\epsilon^2} \leq s^* + \frac{2n+1-m}{m+1}\epsilon \leq b + \sqrt{(\omega - b)^2 - \underbrace{\frac{4(n+1)(m-n)}{(m+1)^2(m+2)}}_{<1}\epsilon^2}$$

(RHS values of b below -l (not considered here), LHS values of b above -l) alternatively the value fct is positive

- condition for VF to me negative at at $m = n+1$

$$
\begin{aligned}
s^* \leq & b - \frac{n}{n+2}\epsilon - \sqrt{(\omega - b)^2 - \frac{4(n+1)}{(n+2)^2(n+3)}\epsilon^2} \\
< & b - \frac{n}{n+4}\epsilon
\end{aligned}
$$

which is equivalent to

$$
n+1 > \frac{\epsilon(2n+1) - 3(b - s^*)}{\epsilon + b - s^*}
$$

so that the value function being negative at $m = n+1$ implies that it is even lower for all other m and thus negative everywhere.

Proof of Proposition 9

First note that Bayes law gives

$$
f_{\theta|s_i,piv}(\theta) = \frac{f_{s_i|\theta}(s_i)\, P(piv \mid \theta)\, f_\theta(\theta)}{P(piv_i \mid s_i)\, f_{s_i}(s_i)} \mathbb{1}_{\theta \in [\max\{s_i - \epsilon^{*,-i}, s_i - \epsilon\}, \min\{s^* + \epsilon^{*,-i}, s_i + \epsilon\}]}
$$

Using this result we can write

$$
\begin{aligned}
& \int_{s^*}^{\omega+\epsilon} \mathbb{E}\left[(\theta + l) \mid s_i, piv_i\right] P(piv \mid s_i)\, f(s_i)\, ds_i \\
= & \int_{s^*}^{\omega+\epsilon} \int_{\max\{s_i - \epsilon, s^* - \epsilon^{*,-i}\}}^{\min\left\{s_i + \epsilon, s^* + \epsilon^{*,-i}\right\}} (\theta + l)\, f_{\theta|s_i,piv}(\theta)\, d\theta P(piv \mid s_i)\, f(s_i)\, ds_i \\
= & \int_{s^*}^{\omega+\epsilon} \int_{\max\{s_i - \epsilon, s^* - \epsilon^{*,-i}\}}^{\min\left\{s_i + \epsilon, s^* + \epsilon^{*,-i}\right\}} (\theta + l)\, \frac{f_{s_i|\theta}(s_i)\, P(piv \mid \theta)\, f_\theta(\theta)}{P(piv_i \mid s_i)\, f_{s_i}(s_i)} d\theta P(piv \mid s_i)\, f(s_i)\, ds_i \\
= & \frac{1}{2\omega}\left[\int_{s^* - \epsilon^{*,-i}}^{s^* + \epsilon^{*,-i}} P(piv \mid \theta)\,(\theta + l) \int_{\theta - \epsilon}^{\theta + \epsilon} \mathbb{1}_{s_i \geq s^*} \frac{1}{2\epsilon} ds_i d\theta\right] \\
= & \frac{1}{2\omega}\left[\int_{s^* - \min\{\epsilon^{*,-i}, \epsilon\}}^{s^* + \min\left\{\epsilon^{*,-i}, \epsilon\right\}} P(piv \mid \theta)\,(\theta + l)\, \frac{\theta + \epsilon - s^*}{2\epsilon} d\theta + \int_{s^* + \min\{\epsilon^{*,-i}, \epsilon\}}^{s^* + \epsilon^{*,-i}} P(piv \mid \theta)\,(\theta + l)\, d\theta\right]
\end{aligned}
$$

upward deviation (less info acq then others)

$$
\epsilon > \epsilon^{*,-i}
$$

$$\frac{1}{2\omega}\left[\int_{s^*-\epsilon^{*,-i}}^{s^*+\epsilon^{*,-i}}\frac{\partial}{\partial\epsilon}P\left(piv\mid\theta\right)\left(\theta+l\right)\frac{\theta+\epsilon-s^*}{2\epsilon}d\theta\right]$$

- downward deviation (more info acq then others)

$$\epsilon<\epsilon^{*,-i}$$

$$\frac{1}{2\omega}\left[\int_{s^*-\epsilon}^{s^*+\epsilon}P\left(piv\mid\theta\right)\left(\theta+l\right)\frac{\theta+\epsilon-s^*}{2\epsilon}d\theta+\int_{s^*+\epsilon}^{s^*+\epsilon^{*,-i}}P\left(piv\mid\theta\right)\left(\theta+l\right)d\theta\right]$$

$$\begin{aligned}
&\frac{\partial}{\partial\epsilon}\frac{1}{2\omega}\left[\int_{s^*-\epsilon}^{s^*+\epsilon}\frac{1}{2\epsilon}\left(\theta+\epsilon-s^*\right)P\left(piv\mid\theta\right)\left(\theta+l\right)d\theta+\int_{s^*+\epsilon}^{s^*+\epsilon^{*,-i}}P\left(piv\mid\theta\right)\left(\theta+l\right)d\theta\right]\\
=&\frac{1}{2\omega}\left[P\left(piv\mid s^*+\epsilon\right)\left(s^*+\epsilon+l\right)+\frac{1}{2\epsilon}\left(s^*-\epsilon+\epsilon-s^*\right)P\left(piv\mid s^*-\epsilon\right)\left(s^*-\epsilon+l\right)\right]\\
&+\frac{1}{2\omega}\int_{s^*-\epsilon}^{s^*+\epsilon}\frac{\partial}{\partial\epsilon}\frac{1}{2\epsilon}\left(\theta+\epsilon-s^*\right)P\left(piv\mid\theta\right)\left(\theta+l\right)d\theta\\
&-\frac{1}{2\omega}P\left(piv\mid s^*+\epsilon\right)\left(s^*+\epsilon+l\right)\\
=&\frac{1}{4\epsilon\omega}\int_{s^*-\epsilon}^{s^*+\epsilon}\frac{\partial}{\partial\epsilon}P\left(piv\mid\theta\right)\left(\theta+l\right)\left(\theta+\epsilon-s^*\right)d\theta
\end{aligned}$$

so derivatives are the same

$$\begin{aligned}
&\frac{\partial}{\partial\epsilon}\int_{s^*-\epsilon}^{s^*+\epsilon}\left(\theta+l\right)\left(\underbrace{\frac{1}{2}+\frac{1}{2\epsilon}\left(\theta-s^*\right)}_{P(v=1|piv,\theta;\epsilon)}\right)P\left(piv\mid\theta\right)f\left(\theta\right)d\theta\\
=&-\frac{1}{2\epsilon^2}\int_{s^*-\epsilon}^{s^*+\epsilon}\left(\theta+l\right)\left(\theta-s^*\right)P\left(piv\mid\theta\right)f\left(\theta\right)d\theta\\
=&-\frac{1}{\epsilon^2}\int_{s^*-\epsilon}^{s^*+\epsilon}\frac{1}{2}\left(\theta+l\right)\left(\theta-s^*\right)\left(\frac{1}{2}+\frac{1}{2\epsilon}\left(\theta-s^*\right)\right)^n\left(\frac{1}{2}-\frac{1}{2\epsilon}\left(\theta-s^*\right)\right)^{m-n-1}d\theta\\
=&-\frac{1}{\epsilon^2}\left[\frac{\epsilon^2n!\left(m-n-1\right)!\left\{\left(m+2\right)\left(l+s\right)\left(2n+1-m\right)+\epsilon\left[\left(2n+1-m\right)^2+m+1\right]\right\}}{\left(m+2\right)!}\right]\\
=&-\frac{n!\left(m-n-1\right)!\left\{\left(m+2\right)\left(l+s\right)\left(2n+1-m\right)+\epsilon\left[\left(2n+1-m\right)^2+m+1\right]\right\}}{\left(m+2\right)!}
\end{aligned}$$

The FOC then becomes

$$-\frac{n!\,(m-n-1)!}{(m+2)!}\left((m+2)\,(l+s)\,(2n+1-m)+\epsilon\left((2n+1-m)^2+m+1\right)\right)=-c$$

$$\epsilon\left((2n+1-m)^2+m+1\right)=\frac{(m+2)!}{n!\,(m-n-1)!}c-(m+2)\,(l+s)\,(2n+1-m)$$

$$\begin{aligned}
\epsilon &= \frac{c-\frac{n!(m-n-1)!}{(m+2)!}\,(m+2)\,(l+s)\,(2n+1-m)}{\frac{n!(m-n-1)!}{(m+2)!}\left((2n+1-m)^2+m+1\right)} \\
&= \frac{c}{\frac{n!(m-n-1)!}{(m+2)!}\left((2n+1-m)^2+m+1\right)}-\frac{(m+2)\,(2n+1-m)}{\left((2n+1-m)^2+m+1\right)}\,(l+s) \\
&= \frac{m+2}{(2n+1-m)^2+m+1}\left[\binom{m+1}{n}c-(2n+1-m)\,(l+s)\right]
\end{aligned}$$