Blockholder Influence

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Abstract

Blockholder influence has attracted recent interest, not only in the context of corporate boards but also in the context of decentralized autonomous organizations (DAOs). I analyze a model of project choice with dispersed information. I focus on the question whether a blockholder should delegate control to a set of delegates. I assume that agents' preferences are not aligned, in the sense that delegates derive private benefits from the acceptance of the proposal while the blockholder derives benefits from the rejection of the proposal. The blockholder chooses composition of the board between delegates and direct representatives of her own interests. I find that when private interests are low, or signals are imprecise there can be multiple equilibria. The most preferred equilibrium for the blockholder is the one with minimum meaningful delegation. This equilibrium also turns out to be the second best and in particular, the equilibrium value of the firm is higher compared to full delegation. This gives rise to a blockholder premium. Furthermore, I solve the information acquisition problem of the committee in closed form.

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1 Introduction

Boards of independent directors are often proposed as an effective means to protect shareholder interests, addressing the agency problem that arises from the separation of ownership and control. However, the significant influence of blockholders over board composition has raised questions about the alignment of these so-called independent boards with the interests of small shareholders. Similarly, the rise of decentralized autonomous organizations (DAOs), initially praised for their democratic and decentralized governance structures, has raised similar concerns. In DAOs, large tokenholders, such as venture capital firms, have significant voting power, and recent cases suggest that their objectives may not always align with those of smaller tokenholders[2]. Interestingly, in both corporate boards and DAOs, blockholders often refrain from exercising their full control, opting instead to delegate some authority to a committee ex-ante, even though they anticipate potential disagreements ex-post. Despite the significant implications of blockholder influence, particularly regarding the protection of small stakeholders, the theoretical literature has paid relatively little attention to the influence of blockholders on boards.

In this paper, I analyze a model of project choice with dispersed information. I focus on the question of whether a blockholder should delegate control to a set of delegates. Each delegate observes a signal about the quality of the project. In addition, they can exert costly effort to improve the quality of their signal. I assume that the agents' preferences are not aligned: delegates derive private benefits from the acceptance of the project, while the blockholder derives benefits from its rejection. The blockholder strategically chooses the composition of the board between delegates and her own representatives.

Building on the literature on strategic voting, which examines how informed committee members make decisions by considering their pivotality and the information contained in such event, this paper extends the analysis to settings where the number of informed voters is endogenously determined by the blockholder. Unlike previous studies that fix the proportion of informed to uninformed voters or assume heterogeneous preferences among voters, this model allows the blockholder to balance her private interests with information aggregation

[2]see for example https://tokeninsight.com/en/news/why-did-a16z-vote-against-uniswap-s-latest-expansion

by choosing the number of informed delegates versus representatives.

In this environment, I derive conditions under which the blockholder voluntarily limits her influence on the board by delegating control to a meaningful number of delegates. The intuition is that she trades off the benefits of better information aggregation against the loss of direct control. Delegates, on the other hand, also adjust their voting strategies based on anticipated delegation levels. The more board seats they expect the blockholder to fill with her direct representatives, the more strongly they vote in the opposite direction to offset her biased decision making. This strategic interplay can lead to multiple equilibria where the blockholder and delegates' decisions counterbalance each other. The blockholder-preferred equilibrium, which in some cases is also the unique equilibrium, involves minimal meaningful delegation where the blockholder delegates just enough seats to lose her majority. This equilibrium is also the second best, as the blockholder internalizes the information aggregation decision. Consequently, the equilibrium value of the firm is higher under minimal delegation compared to full delegation from the perspective of all shareholders, including small investors who do not derive private benefits. This results in a control premium.

In equilibrium, the blockholder always rejects the proposal, and by selecting the number of delegates, she effectively sets the voting rule. Her decision to delegate to the minimal meaningful number of informed voters leads to a scenario where all informed committee members must agree to pass a proposal - effectively implementing a unanimity requirement. This finding that unanimity among informed board members leads to optimal information aggregation differs sharply from previous studies where the number of informed board members is fixed. In those settings, unanimity rules tend to result in poor information aggregation because an individual informed voter becomes pivotal only if all other informed voters accept the proposal. The informational content of this event is so overwhelming that the voter may disregard their private signal entirely and simply follow others. In contrast, in my setting, the blockholder's rejection motivates informed voters to accept proposals more frequently to counterbalance her influence. This behavior offsets the tendency to rely solely on being pivotal, thereby enhancing information aggregation compared to scenarios where all informed voters vote and the blockholder has no voice.

Furthermore, I solve the committee's information acquisition problem in closed form, finding that lower levels of delegation lead to higher precision, as fewer informed delegates increase the probability of being pivotal and the investment thus having effect. Thus, scenarios with blockholder representation on the committee are superior to full delegation to inside board members also in terms of providing the right incentives for delegates to become better informed.

Related Literature

This work is rooted in property rights theory, as established by Grossman and Hart (1986) and Hart and Moore (1990), which provides a framework for understanding control as the residual right of owners to make decisions in contingencies not covered by contracts due to contractual incompleteness. When control is delegated from owners to managers, agency problems arise due to the separation of ownership and control. These issues are exacerbated in firms with dispersed ownership, where collective action problems among shareholders hinder effective monitoring of management Jensen and Meckling (1976).

Blockholders—shareholders with significant stakes in a firm—can mitigate these problems. Their substantial holdings provide them with the right incentives to monitor management, thereby overcoming the collective action problem inherent in dispersed ownership. For a comprehensive review of the literature on blockholders, see Edmans and Holderness (2017).

However, the impact of blockholders is not always entirely positive. Empirical studies by Zingales (1994), Bianco and Casavola (1999), and La Porta et al. (1998) provide evidence that blockholders may pursue private interests not aligned with those of minority shareholders, extracting benefits at their expense—a phenomenon known as entrenchment. From a theoretical perspective, Burkart, Gromb, and Panunzi (1997) argue that while blockholders can reduce agency costs through monitoring, excessive oversight can undermine managerial initiative and deter firm-specific investments.

Despite these concerns, blockholder-controlled firms remain prevalent, particularly in Europe. Studies by La Porta, Lopez-de-Silanes, and Shleifer (1999), Claessens, Djankov, and Lang (2000), and Barca and Becht (2001) document the widespread existence of large shareholders in European firms. Similarly, in the world of Decentralized Autonomous Organizations (DAOs)—blockchain-based entities governed collectively by their members through smart contracts—blockholders are omnipresent (Rossello (2024); Fritsch, Müller, and Wattenhofer (2022); Barbereau et al. (2023)). These observations suggest to reevaluate the role of the blockholder, suggesting that it may not always be detrimental and can, in fact, play a positive role in corporate governance even for minority shareholders.

While much of the literature focuses on the relationships between blockholders and managers, less attention has been paid to the interaction between blockholders and boards of directors. One exception is Krüger, Limbach, and Voss (2022), who analyze the conditions under which blockholders choose to have board representation. They find that blockholders weigh the benefits of enhanced monitoring against the potential market impact of perceived agency problems.

The present work explores how blockholders may voluntarily cede control over the board of directors to capitalize on the superior information held by other board members. By stepping back, blockholders can allow for better information aggregation within the board, leading to more informed decision-making that benefits the firm as a whole. This voluntary delegation contrasts with the traditional view of blockholders primarily exerting control through board representation.

This work also contributes to the understanding of information aggregation in strategic voting. Seminal works by Austen-Smith and Banks (1996) and Feddersen and Pesendorfer (1997) explore how private information is aggregated in voting settings and the implications for rational decision-making. These studies focus on strategic voting behavior, where informed committee members consider their pivotal role in the outcome. In these models, voters decide whether to support a proposal based not only on their private information but also by anticipating that they will influence the outcome only if they are pivotal. This leads them to consider what information other voters may have received. Such strategic behavior can result in voters disregarding their private signals in favor of inferred collective information, potentially hindering optimal information aggregation. For a review of the literature on voting in committees, see Gerling et al. (2005).

Contrasting with traditional models, the current work examines a scenario where the number of informed voters is endogenously determined by a blockholder who balances private interests and information aggregation. In equilibrium, the blockholder always rejects proposals. By choosing the number of informed delegates, the blockholder implicitly sets the proportion of informed delegates required to accept a proposal—effectively determining the voting rule.

The blockholder's privately and socially optimal decision is to delegate to the minimal meaningful number of informed voters so that they no longer hold a majority. Consequently, all informed committee members must agree to pass a proposal, implementing a unanimity requirement. This mechanism aligns the blockholder's incentives with optimal information aggregation, ensuring that only proposals supported by all informed members are accepted.

The finding that unanimity among informed board members leads to optimal information aggregation deviates from previous studies where the number of informed board members is fixed. In traditional models, such as Feddersen and Pesendorfer (1998), the unanimity rule results in poor information aggregation because an individual informed voter becomes pivotal only if all other informed voters accept the proposal. The informational content of this event is so significant that the voter may disregard their own private signal and follow the expected collective decision.

In the current setting, however, the blockholder's consistent rejection incentivizes informed voters to accept proposals more frequently to counterbalance their influence. This dynamic offsets the tendency of a voter to accept based solely on information inferred from being pivotal. As a result, the inferiority of the unanimity rule observed in earlier studies no longer holds, and information aggregation improves compared to scenarios where all informed voters participate and the blockholder has no voice.

Another strand of the voting literature examines how committee size influences information acquisition. Persico (2004) analyzes how the size of a committee affects members' incentives to acquire information. When committee members perceive that they are less likely to be pivotal and that their individual vote is unlikely to decide the outcome, they have less motivation to incur the costs associated with obtaining accurate information. While larger committees have the potential to aggregate more information due to the higher number of members, the decreased individual incentives can lead to less overall information being acquired. In smaller groups, each member's vote carries more weight, increasing the likelihood that they will invest in acquiring information. Similar themes are explored by Malenko and

Malenko (2019) and Meirowitz and Pi (2021).

Most closely related to this work is the study by Bar-Isaac and Shapiro (2020), who examine how blockholders behave in sharholder voting and find that they may choose not to vote with all their shares to prevent crowding out of information from smaller shareholders, thus enhancing decision efficiency. In my work blockholders have no information at all and vote only to implement their private interests.

This paper proceeds as follows: First the model is described. Then a numerical example is provided to highlight the intuition of the model. Next, the equilibrium is derived for the case of no information acquisition. Finally, the information acquisition problem is solved.

2 Model

An organization has the opportunity to adopt a valuable project proposal. A committee is appointed to decide on whether or not the proposed project is to be implemented. A blockholder holds the absolute majority of voting power and can thus choose the composition of the committee. The committee can be thought of as the board of directors of a corporation or as the set of delegates of a DAO. For simplicity, I will use the board terminology in what follows; however, the mechanisms are equally applicable to delegation by blockholders in the context of DAOs.

The project can either increase or decrease the value of the organization and is represented by a random variable θ, which is distributed uniformly over an interval symmetric around zero.

$$\tilde{\theta} \sim \mathcal{U}\left[-\omega, \omega\right]$$

where $\omega > 0$. Whether the project is accepted or not is determined by simple majority voting of the committee. We say $d = 1$ if the outcome of the vote is for the project to be accepted and $d = 0$ if it is to be rejected.

Information Structure Each inside board member i privately observes a noisy signal about the realization of θ of $\tilde{\theta}$

$$s_i = \theta + \sigma_i$$

where the noise term u_i is uniformly distributed over the interval $[-\epsilon, \epsilon]$

$$\sigma_i \sim \mathcal{U}\left[-\epsilon, \epsilon\right]$$

and ϵ represents the precision of the signal: the smaller the value of ϵ, the more precise the signal.

Each inside board member can reduce the noise of their signal by incurring a linear cost. However, inside board members are not allowed to communicate the signal - an assumption that at first may seem demanding. As sharing information undermines the motive for acquiring it, one could reinterpret this assumption as the ability to commit not to share information ex-post.

Preferences I assume that the agents' preferences are not aligned: the blockholder derives private benefits from the rejection of the proposal. She values the outcome of the project as $\theta - b$ if it is accepted and 0 if it is rejected, where b is a positive constant representing her bias. Her preferences can be represented by the utility function:

$$u_b\left(d, \theta\right) = \mathbb{1}_{\{d=1\}}\left(\theta - b\right)$$

where $\mathbb{1}_{\{d=1\}}$ is an indicator function that equals 1 if the decision is to accept the project $(d = 1)$ and 0 if it is rejected $(d = 0)$. This formulation ensures that, without delegation, the blockholder always votes against the project as the expected utility of accepting the proposal is negative. There are $2n + 1$ board members in total, who can be of two different types: Blockholder representatives directly represent the blockholder's interests and are thus assumed to have the same preferences as her. Inside board members, on the other hand, derive private benefits from the acceptance of the project. They can be thought of as employees of the company such as members of the management team in the context of corporations or developers in the context of DAO. As they are closely related to who proposes the project, they are more favourable towards its acceptance, but because of their proximity to the operations and expert knowledge also possess superior knowledge. They value the outcome of the project by $\theta + l$ if it is accepted and 0 if it is rejected where l is a positive constant. Their preferences can be represented by

$$u_i(d, \theta) = \mathbb{1}_{\{d=1\}}(\theta + l)$$

The project choice is, however, not contractible. Otherwise, the blockholder could just compensate all inside board members by paying a side payment of $b+l$ in case of rejection to perfectly align incentives - if feasible and worthwhile. The blockholder can choose the number m of inside board members, the remaining $2n+1-m$ board members are consequently blockholder representatives.

Parametric Assumptions It is assumed that the noise in the signal is not too large, compared to the noise in the underlying project value, specifically $2\epsilon < \omega$. This assumption is necessary for the existence of an equilibrium in the voting sub-game. Furthermore, the potential upside of the project for the blockholder, represented as $\omega - b$, needs to be large relative to the difference in private benefits $b+l$ in order to allow for a positive surplus of delegation: $b + l < \omega - b$.

Timing is as follows: (i) the blockholder chooses the composition of the committee, (ii) inside board members decide on the signal quality, (iii) signals realize, and (iv) inside board members observe their signal vote. Even though the delegation decision of the blockholder happens chronologically before voting and effort decisions of the committee, it is assumed that committee members cannot observe the blockholders choice of board composition. Those actions are thus strategically simultaneous. I focus on equilibria in monotone strategies where the likelihood of voting in favor of the project is increasing in the signal received as well as the private benefits.

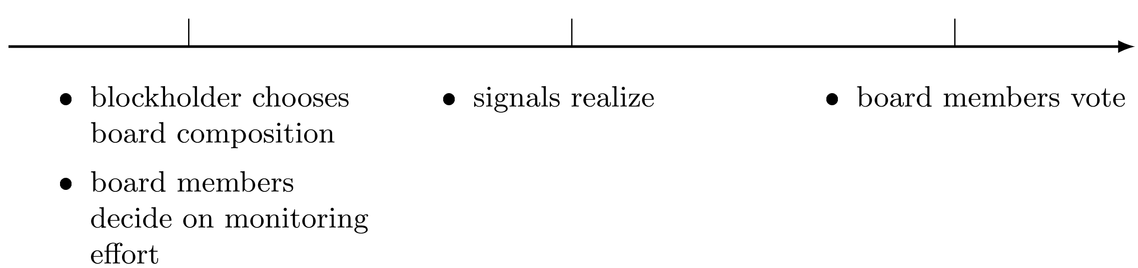

Figure 1: Timeline

The equilibrium concept is that of a Bayes Nash Equilibrium. An equilibrium consists of a set of inside board members' voting decision rules, the blockholder's board composition m^*, and the inside board members' signal precision choice ϵ^* such that

1. Board Composition: Given the noise ϵ_i^* and voting strategy s^* of inside board members, the blockholder chooses the number of inside board members

$$m^* \in \arg\max \mathbb{E}_\theta \left[P\left(d=1 \mid \theta\right)\left(\theta - b\right)\right]$$

2. Signal Precision: Given the board composition m^*, the noise of other inside board members $\epsilon^{*,-i}$, and the voting strategy of other inside board members, a board member i chooses ϵ_i^* such that

$$\epsilon_i^* \in \arg\max \mathbb{E}_s \left[\mathbb{E}_{\theta|s,piv}\left[(\theta + l)\right] P\left(piv_i \mid s; \epsilon^{*,-i}\right) \mid d=1\right] + c\epsilon$$

3. Voting Decision: Given the board composition m^*, the monitoring effort ϵ^*, and the voting strategy of other board members, a board member i accepts the project whenever

$$\mathbb{E}\left[\theta \mid piv_i, s_i\right] + l \geq 0$$

where piv_i stands for the event of being pivotal

4. beliefs are consistent.

3 Numerical Example

Before proceeding to a detailed analysis of the previously described model, I first present a numerical example to intuitively explain the mechanisms at play. This example deviates from the general setup only in that the state space is assumed to be discrete rather than continuous. I thus assume that θ can take only two values: 1 if the project succeeds and -1 if it fails, with each outcome being equally likely. Furthermore, I assume $n = 1$, which implies that the board has only three members. The blockholder, as well as her direct representatives, have a bias of $b = \frac{1}{8}$ so that the utility in case of acceptance of the proposal is given by $\theta - \frac{1}{8}$. For illustrative purposes and for arithmetic simplicity, I assume that inside board members have no private benefits ($l = 0$) and therefore evaluate the proposal purely based on its fundamental value, θ. The noise in the signal is assumed to be $\epsilon = 2$. In the good state, the signal is then uniformly distributed over the interval $[-1, 3]$ and in the bad state over $[-3, 1]$.

The blockholder decides on the composition of the board, considering three possible scenarios:

1. **No delegation.** The blockholder could fill all board seats with her representatives, resulting in decisions that fully reflect the blockholder's preferences. In this scenario, no inside information can affect the decision. As the expected value of θ equals zero, all blockholder representatives have a negative expected utility of $-\frac{1}{8}$ in case the proposal is accepted and therefore always reject the proposal.
2. **Partial delegation.** The blockholder could fill two of the board seats with inside board members and one with a blockholder representative. This arrangement allows the inside board members some degree of control but does not eliminate the blockholder's influence.
3. **Full delegation.** The blockholder could choose to fill all board seats with inside board members, thereby completely abstaining from influence. In this case, the outcome mirrors that of a firm without a blockholder, as all decisions reflect the views of the inside board members. The latter two scenarios require a more detailed analysis, which I will address in the remainder of this example.

Partial Delegation

In the partial delegation scenario, two inside board members and one blockholder representative vote on the proposal. We will now see that there is an equilibrium where the blockholder representative always rejects the proposal, and inside board members accept whenever their signal is greater than -1. First, consider the inside board members. When deciding their voting strategy, they only consider contingencies where their vote is pivotal so that their vote could determine the outcome. With 2 inside members and 1 blockholder representative, an inside member is pivotal if the other inside member accepts, since the blockholder representative always rejects.

From being pivotal, the inside board member then infers that the other inside board member must have had a signal greater than -1. If an inside board member observes his own signal of less than -1, he concludes that this can only have occurred if the project is bad and thus reject the proposal. If, on the other hand, he observes a signal greater than 1, he similarly concludes that the proposal must be good and accepts it. If he receives a signal in the interval $[-1, 1]$, the signal provides no information since it is equally likely under both states. However, being pivotal implies that the other inside board member had a signal greater than -1, which happens with probability $\frac{1}{2}$ if the proposal is bad and with probability 1 if the proposal is good.

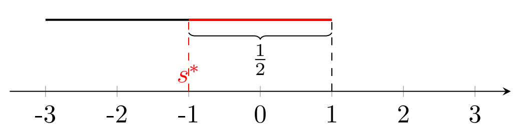

Figure 2: individual acceptance probability in low state

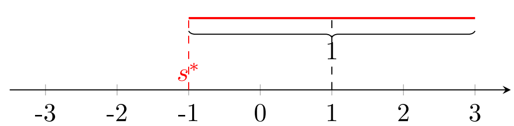

Figure 3: individual acceptance probability in high state

By Bayes' law, his revised beliefs about the proposal being good becomes:

$$P\left(\theta = 1 \mid piv\right) = \frac{1}{1 + \frac{1}{2}} = \frac{2}{3}$$

. Given these revised beliefs, it is optimal for the inside board member to accept the proposal for any signal in the interval $[-1, 1]$, so that -1 is indeed his optimal cutoff level. The blockholder representative does not receive a signal but can still infer information from being pivotal. For the representative, being pivotal means that one inside board member accepted the proposal and the other rejected it. Given the voting strategy of the inside board members, this can only occur if the project is bad, as for a good project, both inside board members would always accept. Thus, the blockholder representative concludes that the proposal is bad and rejects it.

The proposed strategies form an equilibrium. Similarly, it can be shown that an alternative equilibrium exists in which the blockholder always accepts, while inside board members use a signal cutoff of 1. However, such an equilibrium is implausible due to its lack of monotonicity with respect to the private benefits.

A proposal is accepted by majority when at least two of the three board members vote in favor. Given that the blockholder representative always rejects, the proposal can only be accepted if both inside board members vote in favor. Based on the previously derived voting strategies, a good proposal is accepted with probability 1^2, while a bad proposal is accepted with probability $\left(\frac{1}{2}\right)^2$. The ex-ante utility of inside board members and minority shareholders is then:

$$1 \times \frac{1}{2} \times 1 + \frac{1}{4} \times \frac{1}{2} \times (-1) = 0.375$$

while the ex-ante utility of the blockholder is given by

$$1 \times \frac{1}{2} \times \left(1 - \frac{1}{8}\right) + \frac{1}{4} \times \frac{1}{2} \times \left(-1 - \frac{1}{8}\right) \simeq 0.297$$

Full Delegation

In the full delegation scenario, only inside board members vote on the proposal. When determining their voting strategy, they again consider only contingencies where they are pivotal.

The equilibrium voting strategy for inside board members in the full delegation case is to accept the proposal whenever the signal is greater than 0. In the good state, the probability of accepting for each individual inside board member is $\frac{3}{4}$ while

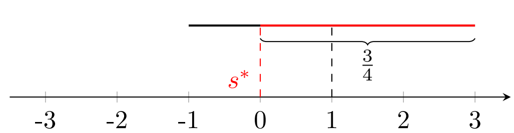

Figure 4: individual acceptance probability in high state

in the bad state, it is now $\frac{1}{4}$.

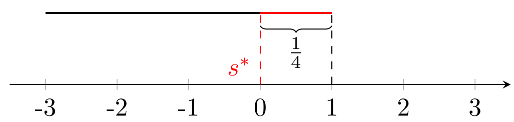

Figure 5: individual acceptance probability in low state

An inside board member is pivotal when exactly one of the other two inside board members accepts and one rejects the proposal. Due to the symmetry of this example with a cutoff value of 0, the probability of being pivotal is the same in both the high and low state.

$$P\left(piv \mid \theta=1\right)=2\times\frac{1}{4}\times\frac{3}{4}=P\left(piv \mid \theta=-1\right)$$

Thus, no additional information can be inferred from being pivotal, and each inside board member bases their decision entirely on their own signal.

A signal greater than 1 can only occur if the project is good and is therefore fully revealing, leading the inside board member to always accept. A signal less than -1 can only occur if the project is bad and is also fully revealing, leading to rejection. For signals in the range of $[-1,1]$, each realization is equally likely under both outcomes, providing no additional information. Given the symmetry of the uniform distribution around 0, no updating takes place, so that for $s_i \in [-1,1]$

$$P\left(\theta=1 \mid s_i, piv\right)=P\left(\theta=1\right)=\frac{1}{2}$$

At the cutoff signal of 0, the expected utility of an inside board member is 0, making them indifferent as required for this to be the equilibrium cutoff.

A proposal is accepted if at least two of the three inside board members vote in favor. The probability that all three accept is given by $\left(\frac{3}{4}\right)^3$ in the good state and $\left(\frac{1}{4}\right)^3$ in the bad state, while the probability that exactly two accept is $3\left(\frac{3}{4}\right)^2\left(\frac{1}{4}\right)$ in the good state and respectively $3\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right)$ in the bad state. The probability that the proposal is accepted is consequently $\left(\frac{3}{4}\right)^3+3\left(\frac{3}{4}\right)^2\left(\frac{1}{4}\right)\simeq 0.844$ in the good state and $\left(\frac{1}{4}\right)^3+3\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right)\simeq 0.156$ in the bad state.

This leads to an ex-ante utility for inside board members and minority shareholders of:

$$0.844 \times \frac{1}{2} \times 1 + 0.156 \times \frac{1}{2} \times (-1) = 0.344$$

(compared to 0.375 in the partial delegation scenario) while the ex-ante utility of the blockholder is given by

$$0.844 \times \frac{1}{2} \times \left(1 - \frac{1}{8}\right) + 0.156 \times \frac{1}{2} \times \left(-1 - \frac{1}{8}\right) \simeq 0.282$$

(compared to 0.297 in the partial delegation scenario).

In this example, all agents are better off in the partial delegation scenario. The presence of the blockholder has increased welfare by better aggregating information, even though fewer signals have contributed to the decision.

4 No Information Acquisition

This section derives and analyzes the equilibrium given a specific level of noise, ϵ, in the inside board members' signals. The next section will endogenize the noise. First, the voting behavior of the blockholder's representatives and inside board members is examined individually, before determining the blockholder's optimal delegation decision. From there, the full equilibrium is derived. This approach allows me to obtain the equilibrium value of delegation and, consequently, the control premium that a blockholder gains by determining the board's composition. Furthermore, the impact of the blockholder's control on small shareholders is analyzed.

4.1 Blockholder Representatives' Voting Decision

Although the blockholder does not receive any private signals about the project's value θ, she votes strategically by extracting information from the event of being pivotal. From this event, the blockholder infers which signals the inside board members are likely to have received and, consequently, updates her belief about the underlying state θ. Since the inside board members are biased toward accepting the project due to their positive bias l, they tend to accept the project even when they receive slightly negative signals.

In order to understand the blockholder's voting behavior, the concept of being pivotal first needs to be extended to a voter who controls multiple votes: Suppose there are $2n+1$ delegates in total, consisting of m inside board members and $2n+1-m$ blockholder representatives. For delegation to be meaningful, the blockholder must delegate at least $n+1$ inside board members ($m \geq n+1$); otherwise, her own representatives retain the majority, and she can never be pivotal. As a consequence, in all cases where $m < n+1$, the blockholder votes according to her prior and thus rejects the project with a majority, resulting in all of those choices being outcome-equivalent.

In all that follows, we thus mean by delegation that at least $m \geq n+1$ of the delegates are inside board members. Assume now that, out of the m inside board members, i vote in favor of the project, and $m-i$ vote against. If $i \geq n+1$ inside board members vote in favor of the project, there is a majority in favor of the project among the inside board members alone, and the blockholder cannot be pivotal, so that the project will pass regardless of her votes. Conversely, if there are $m-i \geq n+1$ inside board members rejecting the project, there is a majority against the project among the inside board members, and again the blockholder cannot be pivotal; the project will be rejected regardless of her votes. Therefore, the blockholder can be pivotal only when the number of votes in favor of the project among inside board members satisfies $i \in \{m-n, \ldots, n\}$.

The probability that the blockholder is pivotal, given the state θ, can then be written as:

$$P\left(piv_b \mid \theta\right) = \sum_{i=m-n}^{n} \binom{m}{i} \left[P\left(v_i = 1 \mid \theta\right)\right]^i \left[P\left(v_i = 0 \mid \theta\right)\right]^{m-i} \mathbb{1}_{\{\theta \in [s^* - \epsilon, s^* + \epsilon]\}} \tag{1}$$

Proposition 1. *A blockholder revises her expectation downwards in the event of being pivotal and thus always rejects the project.*

The proof proceeds by showing that the blockholder always instructs her own representatives to reject the project independently of the number of inside board members she has chosen. This is because she is already biased against the project through her private benefits, and the more she delegates to blockholder representatives, the more she becomes pessimistic about the project by extracting information from the event of being pivotal as discussed in the previous section.

4.2 Inside board members' Voting Decision

Inside board members decide whether to vote in favor or against a project based on their private signal and the conjectured presence of blockholder representatives, conditioning on the event of being pivotal. A stronger bias toward the acceptance of the project results in a lower acceptance threshold, thereby increasing the likelihood of supporting the project. The number of blockholder representatives influences the information extracted from the event of being pivotal as it affects the proportion of yes and no votes that are required for a split vote. A single inside board member rejects the project after observing signal s_i whenever

$$s_i \leq s^*$$

The probability of a single inside board member rejecting the project in the state θ is given by

$$\begin{aligned} P\left(v_i = 0 \mid \theta\right) &= P\left(s_i \leq s^* \mid \theta\right) \\ &= \begin{cases} 0 & s^* \leq \theta - \epsilon \\ \frac{1}{2} - \frac{1}{2\epsilon}\left(\theta - s^*\right) & s^* \in \left(\theta - \epsilon, \theta + \epsilon\right) \\ 1 & s^* \geq \theta - \epsilon \end{cases} \end{aligned} \tag{2}$$

and likewise

$$P\left(v_i = 1 \mid \theta\right) = \begin{cases} 1 & s^* \leq \theta - \epsilon \\ \frac{1}{2} + \frac{1}{2\epsilon}\left(\theta - s^*\right) & s^* \in \left(\theta - \epsilon, \theta + \epsilon\right) \\ 0 & s^* \geq \theta - \epsilon \end{cases} \tag{3}$$

An inside board member is pivotal when the other inside board members' votes are split such that exactly n vote yes and n vote no. Given that the blockholder always votes no with her $2n+1-m$ votes, only another $m-n-1 < n$ no votes are required from the other inside board members in order to obtain a total of n no votes. The probability of being pivotal in state θ in the case of m inside board members is thus:

$$P\left(piv_i \mid \theta\right) = \binom{m-1}{n} \left(\frac{1}{2} + \frac{1}{2\epsilon}\left(\theta - s^*\right)\right)^n \left(\frac{1}{2} - \frac{1}{2\epsilon}\left(\theta - s^*\right)\right)^{m-n-1} \mathbb{1}_{\{\theta \in [s^* - \epsilon, s^* + \epsilon]\}}$$

The event of being pivotal implies proportionally fewer no votes compared to yes votes in the presence of blockholder. An inside board member thus revises his beliefs upwards in the event of being pivotal and is willing to accept the project for lower values of his signal s_i. In effect, the inside board member votes as if he would counterbalance the blockholder's opposition by accepting the project more often than he would without the presence of the blockholder.

This adjustment is stronger when the signals are less precise (i.e., when ϵ is larger), as the inside board members pay less attention to the signal and rely more on their original bias due to the higher uncertainty. To illustrate this intuition, consider the following two scenarios:

In the case of full delegation ($m = 2n+1$), all delegates are inside board members. Since all inside board members have the same bias and use the same cutoff, being pivotal does not provide additional information, and the inside board member bases his decision solely on his own signal and bias.

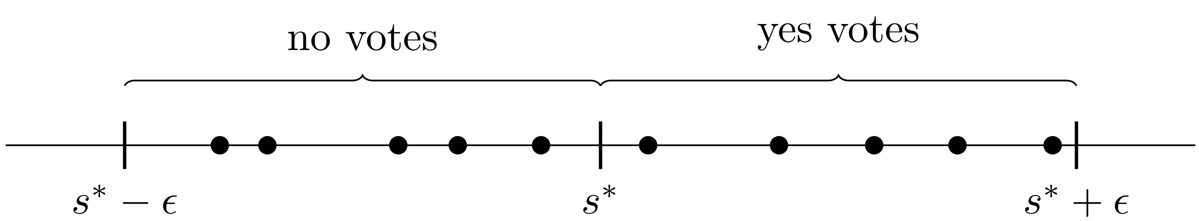

Figure 6: Full delegation: $n = 5$, $m = 11$

In contrast, with partial delegation ($2n+1 > m > n+1$), some delegates are blockholder representatives who always vote in opposition. When an inside board member is pivotal, he infers that the votes against the project include both blockholder representatives and some inside board members. This implies that fewer inside board members voted against and more voted in favor of the project, suggesting that the signals among inside board members are higher on average. A larger value of ϵ suggests a higher potential upside on the possible

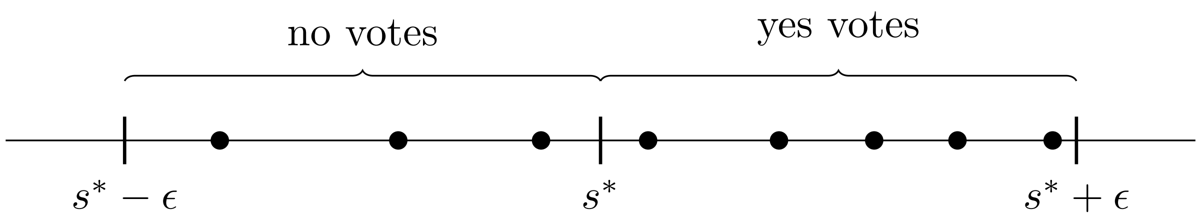

Figure 7: Partial delegation: $n = 5$, $m = 9$

values of the signals. This leads the pivotal inside board member to revise his belief about θ upwards and become more inclined to accept the project by lowering his acceptance threshold further. More noise or a larger presence of blockholder representatives strengthens this effect.

The following proposition formalizes this intuition and shows that there is a unique equilibrium in the voting sub-game in cutoff strategies.

Proposition 2. *For*

$$-l \in [-\omega + 2\epsilon, \omega - 2\epsilon]$$

there exists a unique equilibrium in cutoff strategies of the voting sub-game where an inside board member accepts the project whenever

$$s_i \geq s^*$$

with

$$s^* = -\frac{2n + 1 - m}{m + 1}\epsilon - l.$$

Proof. See Appendix. □

The sub-game is unique because the best response functions of the inside board members are strictly increasing in the cutoff s^*. An increase in the cutoff used by other inside board members leads each inside board member to increase their own cutoff, as their posterior belief about θ upon being pivotal improves. This strategic complementarity ensures that there is a unique intersection point where all inside board members' best responses coincide.

4.3 The Blockholder's Delegation Decision

The blockholder aims to choose the optimal number of inside board members m^* to maximize her expected utility. This involves balancing the benefit of improved decision-making through better information aggregation against the cost of the inside board members' bias toward accepting the project. The blockholder's expected utility is given by:

$$v(m) = \frac{1}{2\omega} \int_{-\omega}^{\omega} (\theta - b) P_{n,m}(d = 1 \mid \theta) \, d\theta$$

where $P_{n,m}(d = 1 \mid \theta)$ is the probability that the project is accepted given θ, with n and m specifying the number of blockholder and inside board members, respectively. This can be written as the probability of at most $m - n - 1$ inside board members rejecting, plus the $2n+1-m$ blockholder representatives, so in total at most n inside board members rejecting.

$$P_{n,m}(d = 1 \mid \theta) = \sum_{i=0}^{m-n-1} \binom{m}{i} \Bigg[\underbrace{\frac{1}{2} + \frac{1}{2\epsilon}(\theta - s^*)}_{P(v_i = 1 \mid \theta)} \Bigg]^{m-i} \Bigg[\underbrace{\frac{1}{2} - \frac{1}{2\epsilon}(\theta - s^*)}_{P(v_i = 0 \mid \theta)} \Bigg]^{i}$$

By integrating over θ, we immediately obtain the value of delegation for the blockholder.

Lemma 3. *The blockholder's value of delegation is given by*

$$v_b(m) = \begin{cases} \frac{1}{2\omega} \left\{ \frac{2\epsilon(m-n)}{(m+1)(m+2)} \left[(n+1)\epsilon - (m+2)(b-s) \right] + \frac{1}{2} \left((\omega - s - \epsilon)(\omega + s + \epsilon - 2b) \right) \right\} & \text{for } m \geq n+1 \\ 0 & \text{otherwise} \end{cases}$$

Proof. See Appendix. □

From here, the benefit of adding an additional inside board member is readily derived as

$$v_b(m+1) - v_b(m) = \frac{\epsilon}{\omega} \frac{(n+1)}{(m+1)(m+2)} \left[\frac{2n+1-m}{m+3} \epsilon - (b - s^*) \right]$$

This expression has the following interpretation. If the difference between the blockholder's bias and the inside board members' threshold value is relatively large compared to the noise term, adding an additional inside board member increases the chance that the project is accepted in states where the blockholder has negative utility. The loss of control is thus relatively large compared to the gain in precision due to an additional signal. If, on the other hand, the signal is less precise (large ϵ), the benefit of adding an additional inside board member is considerable because the existing inside board members' aggregated information is relatively noisy. This argument becomes clearer when considering the limiting case of almost perfect information. In this case, the m inside board members already know the state very precisely, and adding an additional inside board member disproportionately increases the risk of unfavorable decisions from the blockholder's perspective. The factor $\frac{2n+1-m}{m+3}$ in front of the noise implies that the benefit of information aggregation is decreasing with the number of inside board members.

The blockholder thus increases the number of inside board members as long as $v_b(m+1) - v_b(m) > 0$ or

$$m < \frac{\epsilon(2n+1) - 3(b-s^*)}{\epsilon + b - s^*}.$$

The optimal m is thus when, for the first time,

$$m \geq \frac{\epsilon(2n+1) - 3(b-s^*)}{\epsilon + b - s^*}.$$

If, on the other hand,

$$n+1 \geq \frac{\epsilon(2n+1) - 3(b-s^*)}{\epsilon + b - s^*},$$

the blockholder does not delegate above the minimal meaningful level of delegation and might or might not prefer even no delegation at all. It then depends on the surplus of delegation whether minimal delegation is attractive or not. These findings are formalized in the following proposition.

Proposition 4. *If either*

$$(\omega - b)^2 < \frac{4(n+1)}{(n+2)^2(n+3)}\epsilon^2$$

or

$$b > s^* + \frac{n}{n+1}\epsilon + \sqrt{(\omega - b)^2 - \frac{4(n+1)}{(n+2)^2(n+3)}\epsilon^2},$$

delegation is never optimal ($m^* = 0$). Otherwise, the optimal level of delegation is given by

$$m^* = \begin{cases} n+1 & \textit{if } b - s^* \geq \frac{n}{n+4}\epsilon \\ \left\lceil \frac{(2n+1)\epsilon - 3(b-s^*)}{\epsilon + b - s^*} \right\rceil & \textit{if } b - s^* \in \left(\frac{n}{n+4}\epsilon, \frac{1}{2n+3}\epsilon\right) \\ 2n+1 & \textit{if } b - s^* \leq \frac{1}{2n+3}\epsilon \end{cases}$$

The proposition states that if the maximal upside on the project value ω is low relative to the blockholder's bias b, or her bias is large with respect to the cutoff value of the inside board members s^*, then the blockholder prefers a board only composed of her own representatives. If none of these conditions are satisfied, there is value in delegation, and the number of inside board members decreases with the difference in bias. Furthermore, the optimal m^* is increasing in noise or decreasing in precision. This is because if information is relatively precise (small ϵ), a lower number of inside board members aggregates information comparatively well, so that adding an extra inside board member brings relatively little advantage compared to the cost of shifting the bias against in the direction of acceptance.

4.4 Equilibrium

From the individual optimization problems, we have seen that the inside board members revise their threshold value s^* downwards the more blockholder representatives there are, while the blockholder increases delegation to her own representatives the lower s^*. If the signal noise is relatively large (but within the limits of a positive surplus of delegation), this counterbalancing behavior of both strategies can lead to a situation of multiple equilibria where any level of delegation can be supported as an equilibrium. This is because, at high levels of noise, the benefit of adding an additional inside board member is large as we have just seen in the previous section, which in turn allows to sustain high levels of delegation. If, on the other hand, signals are relatively precise, this benefit is too small so that large numbers of inside board members are not sustainable. Either scenario is, of course, subject to delegation having a positive surplus. These observations are formalized in the following proposition.

Proposition 5. *For a large bias of the blockholder in respect to ω*

$$(\omega - b)^2 < \frac{4(n+1)}{(n+2)^2(n+3)}\epsilon^2$$

or a large difference in biases

$$(b+l)^2 > (\omega - b)^2 - \frac{4(n+1)}{(n+2)^2(n+3)}\epsilon^2,$$

no delegation can be supported as an equilibrium. Otherwise, if also

$$b + l \geq \frac{1}{2n+3}\epsilon,$$

the only equilibrium that can be supported is that of minimal delegation

$$m^* = n + 1,$$

while if

$$b + l < \frac{1}{2n+3}\epsilon,$$

there is a multiplicity of equilibria, and any

$$m^* \in \{n+1, \ldots, 2n+1\}$$

can be supported as an equilibrium.

From the expressions above, we can also derive the equilibrium value of the firm.

Proposition 6. *The equilibrium value for the blockholder is given by*

$$v_b^*(m) = \frac{(b-\omega)^2 - (b+l)^2 - 4\frac{(n+1)(m-n)}{(m+1)^2(m+2)}\epsilon^2}{2(2\omega)}$$

We can see that the optimal level of delegation is not dependent on either the private benefits or the distribution of the project or the level of noise.

Corollary 7. *For $n \geq 3$, the blockholder prefers the equilibrium with minimal delegation ($m^* = n + 1$).*

This alignment occurs because the blockholder internalizes the effects of delegation on information aggregation and takes into account the impact on the project's acceptance probability.

4.5 Second Best

Suppose that a social planner would choose the optimal number of inside board members with the objective of minimizing the error of falsely accepting a project with negative value or falsely rejecting a project with positive value. The social planner does not have any bias in favor of or against the project but takes into account the inside board members' voting behavior. His objective function is thus

$$v_0\left(m\right) = \frac{1}{2\omega}\int_{-\omega}^{\omega} \theta P_{n,m}\left(d = 1 \mid \theta\right) d\theta$$

As the following proposition will show, the presence of a blockholder improves the welfare of small shareholders. This improvement is not caused by enhanced monitoring by either the inside board members or the blockholder, as is usually argued in the blockholder literature. Instead, it results from a more balanced decision-making body. The blockholder voluntarily restrains her power because she internalizes the benefits of aggregating information.

The equilibrium with the least possible delegation provides the best expected outcome for small shareholders because it requires that for a project to be accepted, all inside board members vote in favor of the project using the lowest possible threshold level and thus a very high probability of voting yes. This is the most efficient way of aggregating information in this biased environment.

Proposition 8. *The value of the firm for a small shareholder under blockholder influence is given by*

$$v_0^*\left(m\right) = \frac{\omega^2 - l^2 - 4\frac{(n+1)(m-n)}{(m+1)^2(m+2)}\epsilon^2}{2\left(2\omega\right)}$$

For $n \geq 3$, the equilibrium with minimal delegation $m = n+1$ delivers the highest expected utility, and the blockholder equilibrium is thus constrained efficient.

4.6 Blockholder Premium

The benefit the blockholder adds to the value of the firm can be derived by comparing the value of the firm at the socially optimal level of delegation $(m^* = n + 1)$

$$v_0(n+1) = \frac{\omega^2 - l^2 - 4\frac{(n+1)}{(n+2)^2(n+3)}\epsilon^2}{4\omega}$$

to the value that would prevail if there was no blockholder, i.e., at full delegation

$$v_0(2n+1) = \frac{\omega^2 - l^2 - 4\frac{(n+1)^2}{(2n+2)^2(2n+3)}\epsilon^2}{4\omega}.$$

The difference is given by

$$\frac{n\left((n-1)n-4\right)\epsilon^2}{4\left(n+2\right)^2\left(n+3\right)\left(2n+3\right)\omega}$$

This expression is increasing in the noise term, does not depend on any bias term, and is in fact the same from the perspective of the blockholder. This confirms that the welfare improvement is purely due to an improvement in information aggregation.

5 Information Acquisition

This section extends the previous analysis by giving inside board members the opportunity to choose the signal precision at a cost. Each inside board member decides how much to invest in his signal, considering only the contingency in which he can use his signal, which is the case when he is pivotal. Suppose an inside board member acquires a signal with precision ϵ while all other inside board members acquire signals with precision $\epsilon^{*,-i}$. His expected profit from acquiring the signal is then

$$\max_{\epsilon} \int_{s^*}^{\omega+\epsilon} \mathbb{E}\left[\theta + l \mid s_i, piv_i\right] P\left(piv \mid s_i\right) f\left(s_i\right) ds_i - c\epsilon$$

Proposition 9. *The inside board members' optimal level of noise is given by*

$$\epsilon^* = \frac{m+2}{(2n+1-m)^2+m+1}\left[\binom{m+1}{n}c-(2n+1-m)(l+s^*)\right]$$

Proof. See Appendix. □

The optimal noise is increasing in the cost and decreasing in the difference in biases.

5.1 Equilibrium with Information Acquisition

Inserting the equilibrium value of s^*, we can derive the equilibrium level of noise

$$\epsilon = \frac{(m+2)(m+1)^2\binom{m}{n+1}}{4(m-n)}c$$

which is increasing in m since

$$\begin{aligned}\epsilon(m+1)-\epsilon(m) &= c\frac{(m(n+3)+n+5)\Gamma(m+3)}{4\Gamma(n+2)\Gamma(m-n+2)}\\ &> 0\end{aligned}$$

An equilibrium with more delegation to inside board members leads to more noise, while fewer inside board members improve signal precision. This further increases the advantage of minimal delegation and thus blockholder delegation.

6 Conclusion

We have seen that blockholder influence on boards is not necessarily a bad thing, and in cases of opposing private benefits of the boards, blockholder influence can lead to a welfare improvement over pure board decision-making. However, there are also situations where the blockholder has private benefits of the same sign as the boards', and one can expect that the same is no longer true in this scenario. Furthermore, we have left out the issue of communication, which might also be impacted by the presence of blockholder representatives on the board. The issue of communication or deliberation in such scenarios would be an interesting topic for future research.

7 References

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Appendix

Proof of Proposition 1

Substituting the individual inside board members acceptance and rejection probabilities (2) and (3) into equation (1) The probability of being pivotal in state θ can then be expressed as

$$P\left(piv_b \mid \theta\right)=\sum_{i=m-n}^{n}\binom{m}{i}\left[\frac{1}{2}+\frac{1}{2\epsilon}\left(\theta-s^*\right)\right]^i\left[\frac{1}{2}-\frac{1}{2\epsilon}\left(\theta-s^*\right)\right]^{m-i}\mathbb{1}_{\{\theta\in[s^*-\epsilon,s^*+\epsilon]\}}$$

. The posterior probability of θ in the event of being pivotal can then be derived as

$$\begin{aligned}
f\left(\theta \mid piv_b\right) &= \frac{P\left(piv_b \mid \theta\right) f\left(\theta\right)}{\int_{-\infty}^{\infty} P\left(piv_b \mid \theta\right) f\left(\theta\right) d\theta} \\
&= \frac{\sum_{i=m-n}^{n}\left(\begin{array}{c} m \\ i \end{array}\right)\left[\frac{1}{2}+\frac{1}{2\epsilon}\left(\theta-s^*\right)\right]^i\left[\frac{1}{2}-\frac{1}{2\epsilon}\left(\theta-s^*\right)\right]^{m-i}\mathbb{1}_{\{\theta\in[s^*-\epsilon,s^*+\epsilon]\}}}{\int_{s^*-\epsilon}^{s^*+\epsilon}\sum_{i=m-n}^{n}\left(\begin{array}{c} m \\ i \end{array}\right)\left[\frac{1}{2}+\frac{1}{2\epsilon}\left(\theta-s^*\right)\right]^i\left[\frac{1}{2}-\frac{1}{2\epsilon}\left(\theta-s^*\right)\right]^{m-i}\mathbb{1}_{\{\theta\in[s^*-\epsilon,s^*+\epsilon]\}}d\theta} \\
&= \frac{1}{2\epsilon}\frac{m+1}{2n+1-m}\sum_{i=m-n}^{n}\left(\begin{array}{c} m \\ i \end{array}\right)\left[\frac{1}{2}+\frac{1}{2\epsilon}\left(\theta-s^*\right)\right]^i\left[\frac{1}{2}-\frac{1}{2\epsilon}\left(\theta-s^*\right)\right]^{m-i}\mathbb{1}_{\{\theta\in[s^*-\epsilon,s^*+\epsilon]\}}
\end{aligned}$$

where the integrals in the denominator are evaluated by change of variable

$$p=\frac{1}{2}+\frac{1}{2\epsilon}\left(\theta-s^*\right)$$

$$\frac{dp}{d\theta}=\frac{1}{2\epsilon}$$

and noting that $\int_0^1 \left(p\right)^n\left(1-p\right)^{m-n}dp$ is the The Euler integral of the first kind with the value $\frac{n!(m-n)!}{(m+1)!}$ so that

$$\int_{s^*-\epsilon}^{s^*+\epsilon}\left(\begin{array}{c} m \\ n \end{array}\right)\left(\frac{1}{2}+\frac{1}{2\epsilon}\left(\theta-s^*\right)\right)^n\left(\frac{1}{2}-\frac{1}{2\epsilon}\left(\theta-s^*\right)\right)^{m-n}d\theta$$

the expected conditional mean can now be derived byso that

$$
\begin{aligned}
\mathbb{E}\left[\theta \mid piv_b\right] &= \int_{-\infty}^{\infty} \theta f\left(\theta \mid piv_i\right) d\theta \\
&= \frac{1}{2\epsilon}\frac{m+1}{2n+1-m}\sum_{i=m-n}^{n}\int_{s^*-\epsilon}^{s^*+\epsilon}\theta\left(\begin{array}{c} m \\ i \end{array}\right)\left[\frac{1}{2}+\frac{1}{2\epsilon}\left(\theta-s^*\right)\right]^{i}\left[\frac{1}{2}-\frac{1}{2\epsilon}\left(\theta-s^*\right)\right]^{m-i} d\theta \\
&= \frac{1}{2\epsilon}\frac{m+1}{2n+1-m}\sum_{i=m-n}^{n} 2\epsilon\frac{\left(\left(2i-m\right)\epsilon+\left(m+2\right)s^*\right)}{\left(m+2\right)\left(m+1\right)} \\
&= s^* \\
&< 0
\end{aligned}
$$

Proof of Proposition 2

The first step of the proof proceeds by confirming that the proposed voting strategy is indeed an equilibrium in the voting sub-game. Suppose all inside board members vote according to the threshold strategy s^*. The likelihood of an inside board member being pivotal in state θ is then the likelihood that n of the $m-1$ other inside board members are voting in favor of the proposal.

$$
P\left(piv_i \mid \theta\right) = \binom{m-1}{n}\left(\frac{1}{2}+\frac{1}{2\epsilon}\left(\theta-s^*\right)\right)^{n}\left(\frac{1}{2}-\frac{1}{2\epsilon}\left(\theta-s^*\right)\right)^{m-n-1}\mathbb{1}_{\{\theta\in[s^*-\epsilon,s^*+\epsilon]\}}
$$

The posterior probability of θ in the event of being pivotal and having observed s^* can then be derived as

$$
\begin{aligned}
g\left(\theta \mid s^*, piv_i\right) &= \frac{f\left(s^* \mid \theta\right)P\left(piv_i \mid \theta\right)f\left(\theta\right)}{\int_{-\infty}^{\infty} f\left(s^* \mid \theta\right)P\left(piv_i \mid \theta\right)f\left(\theta\right)d\theta} && (4) \\
&= \frac{\frac{1}{2\epsilon}\mathbb{1}_{s^*\in\{\theta-\epsilon,\theta+\epsilon\}}P\left(piv_i \mid \theta\right)\frac{1}{2\omega}\mathbb{1}_{\{\theta\in[-\omega,\omega]\}}}{\int_{-\infty}^{\infty}\frac{1}{2\epsilon}\mathbb{1}_{s^*\in\{\theta-\epsilon,\theta+\epsilon\}}P\left(piv_i \mid \theta\right)\frac{1}{2\omega}\mathbb{1}_{\{\theta\in[-\omega,\omega]\}}d\theta} && (5) \\
&= \frac{\left(\frac{1}{2}+\frac{1}{2\epsilon}\left(\theta-s^*\right)\right)^{n}\left(\frac{1}{2}-\frac{1}{2\epsilon}\left(\theta-s^*\right)\right)^{m-n-1}\mathbb{1}_{\{\theta\in[s^*-\epsilon,s^*+\epsilon]\}}}{\int_{s^*-\epsilon}^{s^*+\epsilon}\left(\frac{1}{2}+\frac{1}{2\epsilon}\left(\theta-s^*\right)\right)^{n}\left(\frac{1}{2}-\frac{1}{2\epsilon}\left(\theta-s^*\right)\right)^{m-n-1}d\theta} && (6)
\end{aligned}
$$

The integral in the denominator can be evaluated by change of variable

$$
p = \frac{1}{2}+\frac{1}{2\epsilon}\left(\theta-s^*\right)
$$

$$\frac{dp}{d\theta} = \frac{1}{2\epsilon}$$

and noting that $\int_0^1 p^n (1-p)^{m-n-1}\, dp$ is the Euler integral of the first kind with the value $\frac{n!(m-n-1)!}{m!}$, we have

$$\begin{aligned}
&\int_{s^*-\epsilon}^{s^*+\epsilon} \binom{m-1}{n} \left(\frac{1}{2} + \frac{1}{2\epsilon}(\theta - s^*)\right)^n \left(\frac{1}{2} - \frac{1}{2\epsilon}(\theta - s^*)\right)^{m-n-1} d\theta \\
&= \binom{m-1}{n} 2\epsilon \int_0^1 p^n (1-p)^{m-n-1}\, dp \\
&= \binom{m-1}{n} 2\epsilon \frac{n!\,(m-n-1)!}{m!} \\
&= \frac{2\epsilon}{m}
\end{aligned}$$

So, the posterior density is thus given by

$$g(\theta \mid s^*, piv_i) = \frac{1}{2\epsilon} \frac{m!}{n!\,(m-n-1)!} \left(\frac{1}{2} + \frac{1}{2\epsilon}(\theta - s^*)\right)^n \left(\frac{1}{2} - \frac{1}{2\epsilon}(\theta - s^*)\right)^{m-n-1} \mathbb{1}_{\theta \in [s^*-\epsilon, s^*+\epsilon]}$$

The expected conditional mean can now be derived using the fact that

$$\begin{aligned}
&\frac{1}{2\epsilon} \int_{s^*-\epsilon}^{s^*+\epsilon} \theta \left(\frac{1}{2} + \frac{1}{2\epsilon}(\theta - s^*)\right)^n \left(\frac{1}{2} - \frac{1}{2\epsilon}(\theta - s^*)\right)^{m-n-1} d\theta \\
&= 2\epsilon \frac{(n+1)!\,(m-n-1)!}{(m+1)!} - \left(\frac{1}{2} - \frac{s^*}{2\epsilon}\right) 2\epsilon \frac{n!\,(m-n-1)!}{m!}
\end{aligned}$$

Thus

$$\begin{aligned}
\mathbb{E}[\theta \mid piv_i] &= \int_{-\infty}^{\infty} \theta g(\theta \mid s^*, piv_i)\, d\theta \\
&= \frac{1}{2\epsilon} \frac{m!}{n!\,(m-n-1)!} \int_{s^*-\epsilon}^{s^*+\epsilon} \theta \left(\frac{1}{2} + \frac{1}{2\epsilon}(\theta - s^*)\right)^n \left(\frac{1}{2} - \frac{1}{2\epsilon}(\theta - s^*)\right)^{m-n-1} d\theta \\
&= \frac{m!}{n!\,(m-n-1)!} \left[2\epsilon \frac{(n+1)!\,(m-n-1)!}{(m+1)!} - \left(\frac{1}{2} - \frac{s^*}{2\epsilon}\right) 2\epsilon \frac{n!\,(m-n-1)!}{m!}\right] \\
&= 2\epsilon \frac{n+1}{m+1} - (\epsilon - s^*) \\
&= \frac{2n+1-m}{m+1}\epsilon + s^*
\end{aligned}$$

An inside board member who observes exactly s^* has to be indifferent between accepting and rejecting, so

$$\frac{2n+1-m}{m+1}\epsilon + s^* + l = 0$$

or

$$s^* = -\frac{2n+1-m}{m+1}\epsilon - l$$

To show uniqueness, I proceed to show that the best response is strictly increasing. Let $s^{*,-i}$ denote the cutoff signal of all other inside board members. Then, the conditional expectation of an inside board member who observes a signal s_i in the event of being pivotal is given by

$$\mathbb{E}\left[\theta \mid s_i, piv; s^{*,-i}\right] = \int_{\max\{s_i,s^*\}-\epsilon}^{\min\{s_i,s^*\}+\epsilon} \theta \left(\frac{1}{2} + \frac{1}{2\epsilon}\left(\theta - s^*\right)\right)^n \left(\frac{1}{2} - \frac{1}{2\epsilon}\left(\theta - s^*\right)\right)^{m-n-1} d\theta$$

Suppose the other inside board members increase their cutoff from s^* to $\hat{s}^*$, then

$$\min\left\{s^*, \hat{s}^*\right\} + \epsilon = s^* + \epsilon$$
$$\max\left\{s^*, \hat{s}^*\right\} - \epsilon = \hat{s}^* - \epsilon$$

Thus,

$$\mathbb{E}\left[\theta \mid s^*, piv; \hat{s}^*\right] = \int_{\hat{s}^*-\epsilon}^{s^*+\epsilon} \theta \left(\frac{1}{2} + \frac{1}{2\epsilon}\left(\theta - \hat{s}^*\right)\right)^n \left(\frac{1}{2} - \frac{1}{2\epsilon}\left(\theta - \hat{s}^*\right)\right)^{m-n-1} d\theta$$

Using the Leibniz rule, we get the derivative with respect to $\hat{s}^*$

$$\begin{aligned}
&\frac{\partial}{\partial \hat{s}^*}\mathbb{E}\left[\theta \mid s^*, piv; \hat{s}^*\right] \\
&= \int_{\hat{s}^*-\epsilon}^{s^*+\epsilon} \frac{1}{2\epsilon}\theta \left(\frac{1}{2} + \frac{1}{2\epsilon}\left(\theta - s^*\right)\right)^{n-1} \left(\frac{1}{2} - \frac{1}{2\epsilon}\left(\theta - s^*\right)\right)^{m-n-2} \left(\frac{1}{2} + \frac{1}{2\epsilon}\left(\theta - s^*\right)\right) \left(\underbrace{m-n-1}_{\geq 0}\right) d\theta \\
&> 0
\end{aligned}$$

Thus, after an increase in $\hat{s}^*$,

$$\mathbb{E}\left[\theta \mid s^*, piv; \hat{s}^*\right] + l > 0$$

and the inside board member will strictly increase her cutoff point where she is indifferent. The best response is thus strictly increasing.

Proof of Lemma 3

The proof consists of integrating over each summand and then adding up the sum. The proof uses the result that

$$\int_{s^*-\epsilon}^{s^*+\epsilon} (\theta - b) \begin{pmatrix} m \\ i \end{pmatrix} \left[\frac{1}{2} + \frac{1}{2\epsilon}(\theta - s^*)\right]^{m-i} \left[\frac{1}{2} - \frac{1}{2\epsilon}(\theta - s^*)\right]^{i} d\theta = \frac{\epsilon (m-2i) + (s-b)(m+2)}{(m+1)(m+2)} 2\epsilon$$

$$\begin{aligned}
&v(m) \\
&= \frac{1}{2\omega} \int_{\underline{\theta}}^{\omega} (\theta - b) P_{n,m}(d = 1 \mid \theta)\, d\theta \\
&= \frac{1}{2\omega} \left\{ \sum_{i=0}^{m-n-1} \int_{s^*-\epsilon}^{s^*+\epsilon} (\theta - b) \begin{pmatrix} m \\ i \end{pmatrix} \left[\frac{1}{2} + \frac{1}{2\epsilon}(\theta - s^*)\right]^{m-i} \left[\frac{1}{2} - \frac{1}{2\epsilon}(\theta - s^*)\right]^{i} d\theta + \int_{s^*+\epsilon}^{\omega} (\theta - b)\, d\theta \right\} \\
&= \frac{1}{2\omega} \left\{ \sum_{i=0}^{m-n-1} \frac{\epsilon (m-2i) + (s^*-b)(m+2)}{(m+1)(m+2)} 2\epsilon + \frac{1}{2}\left(\omega^2 - (s^*+\epsilon)^2\right) + b\left((s^*+\epsilon) - \omega\right) \right\} \\
&= \frac{1}{2\omega} \left[\frac{(m-n)\left[(n+1)\epsilon + (m+2)(s-b)\right]}{(m+1)(m+2)} 2\epsilon + \frac{1}{2}\left(\omega^2 - (s^*+\epsilon)^2\right) + b\left((s^*+\epsilon) - \omega\right) \right] \\
&= \frac{1}{2\omega} \left\{ \frac{2\epsilon (m-n)}{(m+1)(m+2)} \left[(n+1)\epsilon - (m+2)(b-s)\right] + \frac{1}{2}\left(\omega^2 - (s^*+\epsilon)^2\right) + b\left((s^*+\epsilon) - \omega\right) \right\} \\
&= \frac{1}{2\omega} \left[\frac{2\epsilon (m-n)}{(m+1)(m+2)} \left[(n+1)\epsilon - (m+2)(b-s)\right] + \frac{1}{2}\left((\omega - s - \epsilon)(\omega + s + \epsilon - 2b)\right) \right]
\end{aligned}$$

Proof of Proposition4

- $$\frac{1}{2\omega} \left\{ \frac{2\epsilon (m-n)}{(m+1)(m+2)} \left[(n+1)\epsilon - (m+2)(b-s)\right] + \frac{1}{2}\left((\omega - s - \epsilon)(\omega + s + \epsilon - 2b)\right) \right\}$$

  just above

- FOC wrt s^*

$$\frac{1}{2\omega}\left(b - s + \frac{m-2n-1}{m+1}\epsilon\right)$$

SOC wrt s^*

$$-\frac{1}{2\omega}$$

thus there is a max at

$$s^* = b - \frac{2n+1-m}{m+1}\epsilon$$

in equilibrium

$$-l = \frac{2n+1-m}{m+1}\epsilon + s^* = b$$

however for

$$b > -l$$

$$s^* < b - \frac{2n+1-m}{m+1}\epsilon$$

so that in the FOC

$$\frac{1}{\omega-\underline{\theta}}\left(b - s + \frac{m-2n-1}{m+1}\epsilon\right) > \frac{2}{\omega-\underline{\theta}}\left(\frac{2n+1-m}{m+1}\epsilon\right) > 0$$

thus in total we have that for a bias $b > -l$ or equivalently $s^* < b - \frac{2n+1-m}{m+1}\epsilon$ the value fct is increasing in s^*

- at maximum

$$v\left(s^* = b - \frac{2n+1-m}{m+1}\epsilon\right) = \frac{1}{\omega-\underline{\theta}}\left[\frac{1}{2}(\omega-b)^2 - 2\epsilon^2\frac{(n+1)(m-n)}{(m+1)^2(m+2)}\right]$$

so that if

$$(\omega-b)^2 - 4\epsilon^2\frac{(n+1)(m-n)}{(m+1)^2(m+2)} < 0$$

the value function is always negative

- positivity constraint is thus

$$\frac{1}{2\omega}\left\{\frac{2\epsilon(m-n)}{(m+1)(m+2)}\left[(n+1)\epsilon - (m+2)(b-s)\right] + \frac{1}{2}\left((\omega-s-\epsilon)(\omega+s+\epsilon-2b)\right)\right\} = 0$$

(need positive solution for b>-l, but how about out of equilibrium values?)

- need that at optimum value fct is positive otherwise no delegation is possible
- or for

$$b-\sqrt{(\omega-b)^2-\underbrace{\frac{4(n+1)(m-n)}{(m+1)^2(m+2)}}_{<1}\epsilon^2}\leq s^*+\frac{2n+1-m}{m+1}\epsilon\leq b+\sqrt{(\omega-b)^2-\underbrace{\frac{4(n+1)(m-n)}{(m+1)^2(m+2)}}_{<1}\epsilon^2}$$

(RHS values of b below -l (not considered here), LHS values of b above -l) alternatively the value fct is positive

- condition for VF to me negative at at $m=n+1$

$$\begin{aligned} s^* \leq & b-\frac{n}{n+2}\epsilon-\sqrt{(\omega-b)^2-\frac{4(n+1)}{(n+2)^2(n+3)}\epsilon^2} \\ < & b-\frac{n}{n+4}\epsilon \end{aligned}$$

which is equivalent to

$$n+1>\frac{\epsilon(2n+1)-3(b-s^*)}{\epsilon+b-s^*}$$

so that the value function being negative at $m=n+1$ implies that it is even lower for all other m and thus negative everywhere.

Proof of Proposition 9

First note that Bayes law gives

$$f_{\theta|s_i,piv}(\theta)=\frac{f_{s_i|\theta}(s_i)\,P(piv\mid\theta)\,f_\theta(\theta)}{P(piv_i\mid s_i)\,f_{s_i}(s_i)}\mathbb{1}_{\theta\in[\max\{s_i-\epsilon^{*,-i},s_i-\epsilon\},\min\{s^*+\epsilon^{*,-i},s_i+\epsilon\}]}$$

Using this result we can write

$$
\begin{aligned}
&\int_{s^*}^{\omega+\epsilon} \mathbb{E}\left[(\theta+l) \mid s_i, piv_i\right] P\left(piv \mid s_i\right) f\left(s_i\right) ds_i \\
=&\int_{s^*}^{\omega+\epsilon} \int_{\max\left\{s_i-\epsilon, s^*-\epsilon^{*,-i}\right\}}^{\min\left\{s_i+\epsilon, s^*+\epsilon^{*,-i}\right\}} (\theta+l) f_{\theta|s_i,piv}(\theta)\, d\theta P\left(piv \mid s_i\right) f\left(s_i\right) ds_i \\
=&\int_{s^*}^{\omega+\epsilon} \int_{\max\left\{s_i-\epsilon, s^*-\epsilon^{*,-i}\right\}}^{\min\left\{s_i+\epsilon, s^*+\epsilon^{*,-i}\right\}} (\theta+l) \frac{f_{s_i|\theta}\left(s_i\right) P\left(piv \mid \theta\right) f_\theta(\theta)}{P\left(piv_i \mid s_i\right) f_{s_i}\left(s_i\right)} d\theta P\left(piv \mid s_i\right) f\left(s_i\right) ds_i \\
=&\frac{1}{2\omega}\left[\int_{s^*-\epsilon^{*,-i}}^{s^*+\epsilon^{*,-i}} P\left(piv \mid \theta\right)(\theta+l) \int_{\theta-\epsilon}^{\theta+\epsilon} \mathbb{1}_{s_i \geq s^*} \frac{1}{2\epsilon} ds_i d\theta\right] \\
=&\frac{1}{2\omega}\left[\int_{s^*-\min\left\{\epsilon^{*,-i},\epsilon\right\}}^{s^*+\min\left\{\epsilon^{*,-i},\epsilon\right\}} P\left(piv \mid \theta\right)(\theta+l) \frac{\theta+\epsilon-s^*}{2\epsilon} d\theta + \int_{s^*+\min\left\{\epsilon^{*,-i},\epsilon\right\}}^{s^*+\epsilon^{*,-i}} P\left(piv \mid \theta\right)(\theta+l)\, d\theta\right]
\end{aligned}
$$

upward deviation (less info acq then others)

$$
\epsilon > \epsilon^{*,-i}
$$

$$
\frac{1}{2\omega}\left[\int_{s^*-\epsilon^{*,-i}}^{s^*+\epsilon^{*,-i}} \frac{\partial}{\partial\epsilon} P\left(piv \mid \theta\right)(\theta+l) \frac{\theta+\epsilon-s^*}{2\epsilon} d\theta\right]
$$

- downward deviation (more info acq then others)

$$
\epsilon < \epsilon^{*,-i}
$$

$$
\frac{1}{2\omega}\left[\int_{s^*-\epsilon}^{s^*+\epsilon} P\left(piv \mid \theta\right)(\theta+l) \frac{\theta+\epsilon-s^*}{2\epsilon} d\theta + \int_{s^*+\epsilon}^{s^*+\epsilon^{*,-i}} P\left(piv \mid \theta\right)(\theta+l)\, d\theta\right]
$$

$$
\begin{aligned}
&\frac{\partial}{\partial\epsilon} \frac{1}{2\omega}\left[\int_{s^*-\epsilon}^{s^*+\epsilon} \frac{1}{2\epsilon}\left(\theta+\epsilon-s^*\right) P\left(piv \mid \theta\right)(\theta+l)\, d\theta + \int_{s^*+\epsilon}^{s^*+\epsilon^{*,-i}} P\left(piv \mid \theta\right)(\theta+l)\, d\theta\right] \\
=&\frac{1}{2\omega}\left[P\left(piv \mid s^*+\epsilon\right)\left(s^*+\epsilon+l\right) + \frac{1}{2\epsilon}\left(s^*-\epsilon+\epsilon-s^*\right) P\left(piv \mid s^*-\epsilon\right)\left(s^*-\epsilon+l\right)\right] \\
&+\frac{1}{2\omega} \int_{s^*-\epsilon}^{s^*+\epsilon} \frac{\partial}{\partial\epsilon} \frac{1}{2\epsilon}\left(\theta+\epsilon-s^*\right) P\left(piv \mid \theta\right)(\theta+l)\, d\theta \\
&-\frac{1}{2\omega} P\left(piv \mid s^*+\epsilon\right)\left(s^*+\epsilon+l\right) \\
=&\frac{1}{4\epsilon\omega} \int_{s^*-\epsilon}^{s^*+\epsilon} \frac{\partial}{\partial\epsilon} P\left(piv \mid \theta\right)(\theta+l)\left(\theta+\epsilon-s^*\right) d\theta
\end{aligned}
$$

so derivatives are the same

$$\begin{aligned}
&\frac{\partial}{\partial \epsilon}\int_{s^*-\epsilon}^{s^*+\epsilon}(\theta+l)\left(\underbrace{\frac{1}{2}+\frac{1}{2\epsilon}(\theta-s^*)}_{P(v=1|piv,\theta;\epsilon)}\right)P(piv\mid\theta)\,f(\theta)\,d\theta\\
&=-\frac{1}{2\epsilon^2}\int_{s^*-\epsilon}^{s^*+\epsilon}(\theta+l)(\theta-s^*)\,P(piv\mid\theta)\,f(\theta)\,d\theta\\
&=-\frac{1}{\epsilon^2}\int_{s^*-\epsilon}^{s^*+\epsilon}\frac{1}{2}(\theta+l)(\theta-s^*)\left(\frac{1}{2}+\frac{1}{2\epsilon}(\theta-s^*)\right)^n\left(\frac{1}{2}-\frac{1}{2\epsilon}(\theta-s^*)\right)^{m-n-1}d\theta\\
&=-\frac{1}{\epsilon^2}\left[\frac{\epsilon^2 n!\,(m-n-1)!\left\{(m+2)(l+s)(2n+1-m)+\epsilon\left[(2n+1-m)^2+m+1\right]\right\}}{(m+2)!}\right]\\
&=-\frac{n!\,(m-n-1)!\left\{(m+2)(l+s)(2n+1-m)+\epsilon\left[(2n+1-m)^2+m+1\right]\right\}}{(m+2)!}
\end{aligned}$$

The FOC then becomes

$$-\frac{n!\,(m-n-1)!}{(m+2)!}\left((m+2)(l+s)(2n+1-m)+\epsilon\left((2n+1-m)^2+m+1\right)\right)=-c$$

$$\epsilon\left((2n+1-m)^2+m+1\right)=\frac{(m+2)!}{n!\,(m-n-1)!}c-(m+2)(l+s)(2n+1-m)$$

$$\begin{aligned}
\epsilon&=\frac{c-\frac{n!(m-n-1)!}{(m+2)!}(m+2)(l+s)(2n+1-m)}{\frac{n!(m-n-1)!}{(m+2)!}\left((2n+1-m)^2+m+1\right)}\\
&=\frac{c}{\frac{n!(m-n-1)!}{(m+2)!}\left((2n+1-m)^2+m+1\right)}-\frac{(m+2)(2n+1-m)}{\left((2n+1-m)^2+m+1\right)}(l+s)\\
&=\frac{m+2}{(2n+1-m)^2+m+1}\left[\binom{m+1}{n}c-(2n+1-m)(l+s)\right]
\end{aligned}$$