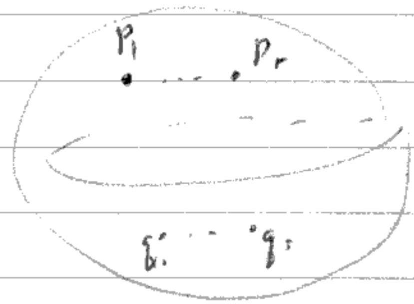
Period m orbit. Let $P=\{p_1,\cdots,p_r\}$ $\quad r+s=m$

$Q=\{q_1,\cdots,q_s\}$



$\exists\, i,j$ s.t. $\phi(p_i)=q_j$

~~[illegible]~~ $(\iota\circ\phi)(p_i)=\iota\circ q_j$

Is it possible to $\iota\circ q_j=p_k$? Yes. $(\iota\circ\phi)(p_i)=p_k$

Maybe divide orbits in two types: ι-inv. and ι-pairs

must come in even #.

ι-inv. $\quad \iota(P\cup Q)=P\cup Q$

$\left(\overset{?}{\Rightarrow}\iota(P)=Q\right)$

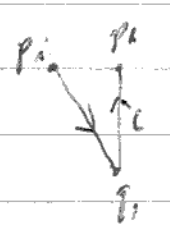
ι-pair: $\iota(P\cup Q)=P'\cup Q'$ $\qquad \iota(P)=Q',\ \iota(Q)=P'$

(not ι-inv.)

$\left(\overset{?}{\Rightarrow} P\cap P'=\phi\ ?\right)$

Assume ι-pair. $\phi(p_i)=q_j$ for some i,j $(s>0)$

Assume $\iota\circ q_j=p_k$. Show $P'=P$, $Q'=Q$, so ι-inv., not pair.

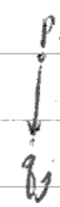


If ~~[illegible]~~ $k=i$, $\phi(p_i)=q_j$

$\iota\circ\phi(p_i)=p_i$

$\phi^2(p_i)=p_i$

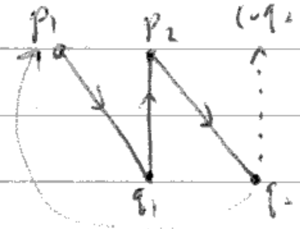
p_i is period 2.



$\iota\circ\phi(p_i)=p_k$

$\phi^2(p_i)=\iota\circ\phi(p_k)$

example:



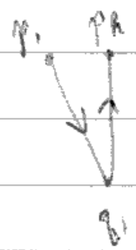
$\phi(p_1) = q_1$, $\iota(q_1) = p_2$

$\iota \circ \phi(p_1) = p_2$, $\phi \circ \iota \circ \phi(p_1) = \phi(p_2) = q_2$

$(\iota\circ\phi)^2(p_1) = \phi^2(p_1) = \iota \circ q_2 = \iota\circ\phi(p_2)$

So $\iota \circ q_2 \in P$. $\boxed{\phi(p_1) = \iota \circ p_2}$

Generally: let $\underset{(WLOG)}{\phi(p_1) = q_1}$. Assume $\iota \circ q_1 = p_k \in P$



$((\iota\circ\phi)(p_1) = \iota \circ q_1 = p_k$

$(\iota\circ\phi)^2(p_1) = \phi^2(p_1) = ((\iota\circ\phi)(p_k) = \phi(\iota\circ p_k)$

$= \iota \circ \phi(p_k)$

$\phi^2(p_1) = \iota\circ\phi(p_k)$ Hence, both p_k and $\iota\circ\phi(p_k)$ are in the orbit.

$\phi(p_1) = \iota \circ p_k$

$\phi(P) = P$, with $\phi(p_i) \neq p_j$ $\forall i \neq j$.

Assume $\iota(P) = Q \neq P$. Let $p_0 \in P \cap \iota(P)$.

$\phi(p) \in P$

$\phi(\iota\circ p)$