## McGill University Department of Economics Econ 467D2: Econometrics Final exam (deferred)

No documentation allowed Time allowed: 3 hours

30 points

- 1. Answer by TRUE, FALSE or UNCERTAIN to each one of the following statements, and justify briefly your answers (maximum: 1 page per statement).
  - (a) In the classical linear model, disturbances (errors) are correlated but least squares residuals are uncorrelated.
  - (b) By Studentizing least squares residuals, outliers are introduced.
  - (c) The Durbin-Watson test is a test meant to detect autocorrelated errors.
  - (d) The generalized least squares method is a special case of the instrumental variables method.
  - (e) Maximum likelihood estimators can be obtained by setting the score function to one.
  - (f) The Ljung-Box statistic is always smaller than the Box-Pierce statistic.

25 points

2. Consider the following models:

$$X_t = 0.5 X_{t-1} + u_t - 0.5 u_{t-1}$$

where  $\{u_t: t \in \mathbb{Z}\}$  is an i.i.d. N(0,1) sequence. For each one of these models, answer the following questions.

- (a) Is this model stationary? Why?
- (b) Is this model invertible? Why?
- (c) Compute:

i. 
$$E(X_t)$$
;

ii. 
$$\gamma(k), k = 1, ..., 8;$$

iii. 
$$\rho(k)$$
,  $k = 1, 2, ..., 8$ .

(d) Graph  $\rho(k)$ , k = 1, 2, ..., 8.

(e) Find the coefficients of  $u_t$ ,  $u_{t-1}$ ,  $u_{t-2}$ ,  $u_{t-3}$  and  $u_{t-4}$  in the moving average representation of  $X_t$ .

10 points

3. Describe a case where the uses of the seemingly unrelated regression technique does not yield an improvement over ordinary least squares applied to each equation and justify your answer.

15 points

4. Consider the model

$$y_t = x_t' \beta + u_t , \quad t = 1, \dots, T \tag{1}$$

where

$$u_t = \rho u_{t-1} + \varepsilon_t , \quad t = \dots, 0, 1, 2, \dots$$
 (2)

$$|\rho| < 1 \,, \tag{3}$$

$$\{\varepsilon_t\}_{t=1}^T$$
 is a sequence of i.i.d. disturbances, (4)

$$\mathsf{E}\left(\varepsilon_{t}\right) = 0 \;, \quad \mathsf{V}\left(\varepsilon_{t}\right) = \sigma^{2} \;, \forall t \;. \tag{5}$$

- (a) Explain how the above linear regression could be transformed to make the disturbances i.i.d. (when  $\rho$  is unknown).
- (b) Discuss how  $\rho$  could be estimated in the above model.
- (c) Discuss how  $\beta$  cold be estimated in the above model.

20 points

5. Consider the following demand and supply model:

$$q_t = a_1 + b_1 p_t + c_1 Y_t + u_{t1}$$
, (demand function) (6)

$$q_t = a_2 + b_2 p_t + c_2 R_t + u_{t2}$$
, (supply function) (7)

where

 $q_t = \text{ quantity (at time } t), p_t = \text{ price}, Y_t = \text{ income}, R_t = \text{ rain volume},$ 

 $u_{t1}$  and  $u_{t2}$  are random disturbances.

- (a) Derive the reduced form of this model.
- (b) Explain why applying least squares to the equations (6)-(7) may not be an appropriate method to estimate the parameters of these two equations.
- (c) Are the parameters of equations (6)-(7) identified? Explain your answer.
- (d) Propose an estimation method for the parameters of equations (6)-(7) and discuss its properties.