Statistical inference for calibrated parameters in computable general equilibrium models *

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Abstract

We consider the problem of assessing the uncertainty of calibrated parameters (as opposed to free parameters) in computable general equilibrium (CGE) models. A general projection technique is proposed which allows the construction of confidence sets (or intervals) for the calibrated parameters given a confidence set for the model free-parameter vector. We discuss how this approach can be applied to the parameters of CES (Armington-type) functions frequently used in CGE models. The proposed method is applied to a model of the Moroccan economy.

Keywords: computable general equilibrium model, nonlinear model, calibration, sensitivity analysis, confidence set, confidence interval, projection, Morocco.
J.E.L. classification codes: D50, C52, C63, C68.

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Résumé

Nous considérons le problème de la prise en compte de l'incertitude sur certains paramètres calibrés des modèles calculables d'équilibre général (MCEG) en construisant des régions (ou des intervalles) de confiance pour ces paramètres. Nous étudions en détail une méthode qui permet de ce faire. Cette approche est une extension des travaux de Abdelkhalek et Dufour (1998) et repose sur une technique de projection qui permet de construire des régions de confiance pour les paramètres calibrés à partir de régions de confiance pour les paramètres libres d'un MCEG déterministe. Nous discutons en détail comment cette approche peut être appliquée aux paramètres d'une fonction CES (de type Armington) d'usage fréquent dans les MCEG et nous l'illustrons sur des modèles de l'économie marocaine.

Mots clés: modèles calculables d'équilibre général, calibration, région de confiance, intervalle de confiance, projection, analyse de sensibilité, Maroc
Classification J.E.L.: D50; C52, C63, C68.

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1 Introduction

Computable general equilibrium (CGE) models have come into extensive use for analyzing and simulating the effects of economic policy changes in developing and industrialized countries. Presentations and overviews of this policy analysis tool may be found in Shoven and Whalley (1984, 1992), Decaluwé and Martens (1988), Gunning and Keyzer (1995), and Ginsburgh and Keyzer (1997). These models are generally non-stochastic and strongly nonlinear. Results obtained by simulating such models rely on various assumptions pertaining to the behavior of agents, technology, the choice of exogenous variables (the *"closure"* of the model), etc. The nature and quality of the data used (such as a social-accounting matrix), as well as the values assigned to the parameters of behavioral functions, which underlie the *"calibration"* of the model, are also crucial.

Calibration requires less time and effort than econometric estimation. Further, the data required to perform such an estimation are typically not available, especially in developing countries. So CGE model designers use calibration quite widely. This process relies on a (largely arbitrary) distinction between *"free parameters"*, which can be obtained from external sources or simply assigned on the basis of subjective judgements, and *"calibrated parameters"* which are derived (*"estimated"*) from the former to reproduce reference data (*e.g.,* the data for a base-year). Calibration may be viewed as a two-stage estimation procedure by which, starting from the values of the free parameters and the reference-year data, values are assigned to the calibrated parameters. The uncertainty of the free parameters clearly induces uncertainty on the inferences drawn from the model.

Since CGE models are rarely estimated using econometric methods [except in the notable work of Jorgenson (1984) and his associates], it is difficult to perform tests or build confidence regions on the calibrated parameters and the endogenous variables of the model. Even if the general specification of the model is not questioned, the reliability of the conclusions is of course affected by the choice of parameter values. For an early discussion of the key role played by the selection of these parameters in determining economic policy simulations as well as the difficulties encountered by researchers during calibration, the reader may consult Mansur and Whalley (1984) and Shoven and Whalley (1984).

Recognizing the importance of these problems, several authors have proposed approaches to translate free-parameter uncertainty into a measure of uncertainty on the results of the simulations; see Pagan and Shannon (1985), Harrison (1989), Bernheim, Scholz and Shoven (1989), Wigle (1991), Harrison and Vinod (1992), Harrison, Jones, Kimbell and Wigle (1993) and Abdelkhalek and Dufour (1998). The problem of assessing the uncertainty of calibrated parameters is not however considered by the above authors. Like free parameters, the calibrated parameters [such as factor share parameters in production functions] may be interesting from an economic perspective. Calibration has the advantage of being much less demanding than traditional econometric methods, both from the perspective of data requirements and numerical procedures. However, this process has received surprisingly little attention in the CGE literature [for a notable exception, see Mansur and Whalley (1984)].

In this paper, we study the problem of assessing the uncertainty of calibrated parameters which is due to free-parameter uncertainty. For that purpose, we consider a general setup similar to the one studied in Pagan and Shannon (1985) and Abdelkhalek and Dufour (1998), and we propose an extension of the projection technique described in Abdelkhalek and Dufour (1998) for measuring simulation uncertainty to the problem of measuring calibrated parameter uncertainty. The technique suggested allows one to translate any confidence set for the free-parameter vector into a confidence set (with the same level) for the calibrated parameters. An important advantage of this method is that its reliability is not affected by the nonlinear features of the model. The procedure is applicable irrespective whether the confidence set for the free parameters is a standard sampling confidence set [e.g., one obtained from an econometric study] or a subjective (Bayesian) confidence set derived from a prior (or posterior) distribution. Further, it can implemented easily using the same optimization algorithms as the ones which are used to solve CGE models.

The general CGE setup studied is described in section 2. We present the general projection technique for calibrated parameters in section 3. For illustration purposes, the method is applied to the calibrated parameters of CES production functions used in two simple CGE models of the Moroccan economy: some essential elements of the theoretical framework of the models are explained in section 4, while the numerical results obtained are given in section 5. We conclude in Section 6.

2 Theoretical framework

In its most general form, a CGE model may be represented by a function M such that:

$$Y = M\left(X,\, \beta,\, \gamma\right) \tag{2.1}$$

where Y is an m-dimensional vector of endogenous variables, M is a (generally nonlinear) function which may be analytically complicated (but remains computable), X is a vector of exogenous or economic policy variables, β is a p-dimensional vector of free parameters belonging to a subset Ω of $\mathbb{R}^p$, and γ is a vector with k elements containing the parameters to be calibrated. From a theoretical viewpoint, β and γ are not fundamentally different. However, they play very different roles in these models. While the elements of β are parameters (*e.g.*, *elasticities*) of the behavioral equations of the model (utility/demand, production/supply, imports, exports, etc.), those of γ are generally scale or share parameters. The calibration procedure thus consists of setting the vector of parameters γ to *exactly* reproduce the data of a reference year, given a point estimate of the free parameters β of the model. Thus, it is not surprising that the choice of these parameters has a large influence on the simulation results.

More formally, consider the equation:

$$Y_0 = M\left(X_0,\, \beta,\, \gamma\right) \tag{2.2}$$

where Y_0 and X_0 are vectors of endogenous and exogenous variables respectively for a given base year. We then solve for γ (assuming that the solution exists and is unique) and eliminate γ from the model:

$$\gamma = H\left(Y_0, X_0, \beta\right) = h\left(\beta\right)\ ,\ Y = M\left(X,\, \beta,\, h\left(\beta\right)\right) = \overline{M}\left(X,\, \beta\right)\ . \tag{2.3}$$

When an estimate $\hat{\beta}$ of β is available, the vector γ is estimated by replacing β with its estimate in equations (2.3) and (2.2). Furthermore, one can usually decompose γ into two subvectors γ_1 and γ_2, where γ_1 (of dimension $0 \leq k_1 < k$) does not depend on β :

$$\gamma_1 = h_1\left(Y_0, X_0\right)\ ,\ \gamma_2 = \bar{h}_2\left(Y_0, X_0, \beta\right) = h_2\left(\beta\right)\ . \tag{2.4}$$

This feature may considerably simplify both theoretical dwerivations and the numerical solution of the model. Ideally, β would be estimated econometrically, so usual measures of uncertainty (standard deviations, confidence regions) could be obtained. However, this type of information is generally ignored in appraisals of the reliability of the results.

Note also that the difficulties associated with the calibration of CGE models are not explicitly considered by usual methods for sensitivity analysis. These methods only deal with the estimations of the vector β, not γ. In CGE models, the dimension of the joint vector $\left(\beta', \gamma'\right)'$ may be quite large and econometric estimation difficult, if not impossible. In fact, the number of parameters of a CGE model tends to increase rapidly with the number of sectors and consumers. Statistical data for high levels of disaggregation are frequently not available. The number of parameters to estimate may easily surpass the size of the sample. Thus, calibration may be viewed as an estimation procedure for γ.

3 Projection-based confidence sets for calibrated parameters

In this section, we describe an approach that allows one to evaluate the uncertainty associated with the subvector of calibrated parameters, γ_2, once a confidence region for the vector of free parameters β has been formulated. As in Abdelkhalek and Dufour (1998), we assume that we have a confidence region C with level $1-\alpha$ for the parameter β. In other words, C is a subset of $\mathbb{R}^p$ such that

$$P\left[\beta \in C\right] \geq 1-\alpha \tag{3.1}$$

where $0 \leq \alpha < 1$. Two different interpretations may be put forward for C. First, we can assume that C is a sampling (frequentist) confidence region based on previous statistical studies and observations, *i.e.* $C = C\left(Z\right)$ is a random subset of $\mathbb{R}^p$ generated by a sample Z such that the probability that a given vector β is contained within $C\left(Z\right)$ is greater than or equal to $1-\alpha$. Second, in other situations we may treat the parameter β as stochastic and consider that $\beta \in C$ is a Bayesian confidence region for β. The arguments developed below are applicable under anyone of these two interpretations. The region C of $\mathbb{R}^p$ may be discrete, compact, connected or continuous.

Let $h_2\left(C\right)$ represent the image of C over a calibration function h_2 defined in equation (2.4):

$$h_2\left(C\right) = \left\{\gamma_2 \in \mathbb{R}^{k_2} : \gamma_2 = h_2\left(\beta_0\right) \text{ for at least one } \beta_0 \in C\right\}. \tag{3.2}$$

Clearly, the implication $\beta \in C \Rightarrow h_2\left(\beta\right) \in h_2\left(C\right)$ holds, hence

$$P\left[\gamma_2 \in h_2\left(C\right)\right] \geq P[\beta \in C] \geq 1-\alpha\,. \tag{3.3}$$

Thus $h_2\left(C\right)$ is a confidence region for γ_2, with level greater than or equal to $1-\alpha$. In particular, when C is a sampling confidence region for β, we have:

$$P\left[h_2\left(\beta\right) \in h_2\left(C\right)\right] \geq P[\beta \in C] \geq 1-\alpha\,, \quad \forall \beta \in \Omega\,. \tag{3.4}$$

Given C, we can also obtain individual confidence intervals for the elements $\gamma_{2i} = h_{2i}\left(\beta\right)$ of the vector $h_2(\beta) = \big(h_{21}(\beta), \ldots, h_{2k_2}(\beta)\big)'$. Since

$$h_2(\beta) \in h_2(C) \Rightarrow [h_{2i}(\beta) \in h_{2i}(C), \text{ for } i = 1, \ldots, k_2], \tag{3.5}$$

we have, for $j = 1, \ldots, k_2$,

$$P\left[\gamma_{2j} \in h_{2j}(C)\right] \geq P\left[\gamma_{2i} \in h_{2i}(C), \ i = 1, \ldots, k_2\right] \geq P\left[\gamma_2 \in h_2(C)\right] \geq 1 - \alpha. \tag{3.6}$$

As the function h_2 is generally nonlinear, the set $h_2(C)$ may be difficult to determine or visualize. In particular, it is not usually an interval or an ellipse. However, we can always construct simultaneous confidence intervals for the different elements of $h_2(\beta)$. Consider the extreme values:

$$h_{2i}^L(C) = \inf\{h_{2i}(\beta) : \beta \in C\}, \ h_{2i}^U(C) = \sup\{h_{2i}(\beta) : \beta \in C\} \tag{3.7}$$

where $-\infty \leq \gamma_{2i}^L < \infty$ and $-\infty < \gamma_{2i}^U \leq \infty$, $i = 1, \ldots, k_2,$. Since $[h_{2i}(\beta) \in h_{2i}(C), \ i = 1, \ldots, k_2] \Rightarrow [h_{2i}^L(C) \leq h_{2i}(\beta) \leq h_{2i}^U(C), \ i = 1, \ldots k_2]$, we have, for $j = 1, \ldots, k_2$:

$$\begin{aligned} P\left[h_{2j}^L(C) \leq \gamma_{2j} \leq h_{2j}^U(C)\right] &\geq P\left[h_{2i}^L(C) \leq \gamma_{2i} \leq h_{2i}^U(C), i = 1, \ldots, k_2\right] \\ &\geq P\left[\gamma_{2i} \in h_{2i}(C), i = 1, \ldots, k_2\right] \geq 1 - \alpha. \end{aligned} \tag{3.8}$$

It is thus sufficient to minimize and maximize each element of $\gamma_2 = h_2(\beta)$ subject to the constraint $\beta \in C$ to obtain (simultaneous) confidence intervals with level $1 - \alpha$ for the components of γ_2. For further discussion of projection techniques in econometrics, see Dufour (1989, 1990, 1997) and Dufour and Kiviet (1998).

Using these results we can construct confidence regions for each component of the subvector γ_2 of the calibrated parameter from those of the vector β of free parameters. This result allows one to substantially simplify the numerical procedures to construct confidence regions for the endogenous variables of CGE models, especially when the dimension of vector γ_2 is large. We will outline the process of building confidence regions for calibrated parameters using two relatively simple examples. A more detailed discussion of these examples and the models considered are available in Abdelkhalek and Dufour (1998, 2000).

4 Calibrated parameters for CES and CET functions

To illustrate the approach proposed above, we will now analyze in some detail the case of an *Armington-type* import function commonly used in CGE models. This general form can be used to model sectorial production, exports, portfolio composition (models with financial flows), etc. In other words, this example covers a large number of cases of calibration in the presence of free parameters (elasticities) in CGE models. This function is linearly homogenous in its arguments, the number of which depends upon the model (inputs or factors of production, origin of imports, markets for exports, substitutable financial assets). In our example we have an import model in which a consumer derives utility from consuming a composite good denoted Q. This good combines imported goods M and domestic goods D. The consumer's problem consists in choosing quantities M and D that minimize overall expenditure, given the two prices p_M and p_D and the level Q. The Armington form of this CES function is given by

$$Q = B\left[\delta M^{(\sigma-1)/\sigma} + (1-\delta)\, D^{(\sigma-1)/\sigma}\right]^{\sigma/(\sigma-1)} \tag{4.1}$$

where B is a constant, δ is a share parameter, and σ is the (constant) elasticity of substitution between imported and domestic goods. In our terminology, given the deterministic calibration procedures applied to this type of function in CGE models [see Mansur and Whalley (1984)], B and δ are *calibrated* parameters while σ is a *free* parameter estimated or borrowed from outside the model, independent of the data from the social-accounting matrix for the reference year. The first-order condition associated with this problem is given by the equality between the price ratio for the two types of good and the marginal rate of substitution between imported and domestic goods:

$$M/D = [\delta/(1-\delta)]^{\sigma}\,(p_D/p_M)^{\sigma}\,. \tag{4.2}$$

This method of modelling imports, examined in detail by de Melo and Robinson (1989) and by Devarajan, Lewis and Robinson (1990), is extensively used in CGE models; for a review of general equilibrium studies having used these forms, see Decaluwé and Martens (1988). This seems more realistic than the classic formulation with perfect substitutability between goods. The CES function is sufficiently tractable for the analytical derivations and the calibration of parameters, despite the fact that it introduces a free parameter.

To calibrate the parameters of this type of function in CGE models different techniques have been used (estimates, literature reviews, international comparisons, or arbitrary fixing) to assign a value $(\hat{\sigma})$ to the free parameter — the elasticity of substitution (σ) in this case. This value is crucial and constitutes the first step of the calibration process. Replacing σ by its estimate $\hat{\sigma}$ and using data from a base-year (say Q_0, M_0, D_0, p_{D_0}, p_{M_0}),[1] we can solve equations (4.1)-(4.2) for σ and B, which yields the estimates:

$$\begin{aligned} \hat{\delta} &= \frac{p_{M_0}}{p_{D_0}}\left(\frac{M_0}{D_0}\right)^{1/\hat{\sigma}} \Bigg/ \left[1 + \frac{p_{M_0}}{p_{D_0}}\left(\frac{M_0}{D_0}\right)^{1/\hat{\sigma}}\right] = h_{21}(\hat{\sigma})\ , && (4.3)\\ \hat{B} &= Q_0/\left[\hat{\delta} M_0^{(\hat{\sigma}-1)/\hat{\sigma}} + \left(1-\hat{\delta}\right) D_0^{(\hat{\sigma}-1)/\hat{\sigma}}\right]^{\hat{\sigma}/(\hat{\sigma}-1)} = h_{22}(\hat{\sigma})\ . && (4.4) \end{aligned}$$

In equations (4.3) and (4.4) the essential role played by the free parameter in determining the values of the other parameters appears clearly. From this deterministic approach to calibration, we seek to construct confidence intervals for the two calibrated parameters, δ and B, given that of the free parameter σ. To achieve this we work not with a point estimate for σ, $\hat{\sigma}$, but rather with a set estimate.

Moving from the definition for a continuous function of a confidence interval $C \subset \mathbb{R}$ for σ given in expression (4.3) towards a subset $h_{21}(C) \subset \mathbb{R}$, we analytically illustrate the construction of what is to

[1] As done usually in CGE models, all prices, except those which include taxes or subsidies, are normalized to one for the base year, which means we use price indices whose values are one for the base-year.

become a confidence interval for δ. To simplify notation, let us write:

$$\delta = h_{21}(\sigma) = \frac{N(\sigma)}{1+N(\sigma)}\,,\ N(\sigma) = \frac{p_{M_0}}{p_{D_0}}\left(\frac{M_0}{D_0}\right)^{1/\sigma} = \frac{p_{M_0}}{p_{D_0}}\exp\left(\sigma^{-1}\ln\left(M_0/D_0\right)\right)\,. \tag{4.5}$$

We now examine the behavior of the function h_{21}, especially within the confidence interval C. From equation (4.5) we see that:

$$\frac{d\delta}{d\sigma} = \frac{N'(\sigma)\left[1+N(\sigma)\right] - N'(\sigma)\,N(\sigma)}{\left[1+N(\sigma)\right]^2} = \frac{N'(\sigma)}{\left[1+N(\sigma)\right]^2}\,, \tag{4.6}$$

$$N'(\sigma) = \frac{dN}{d\sigma} = \frac{p_{M_0}}{p_{D_0}}\left[-\frac{1}{\sigma^2}\ln\left(\frac{M_0}{D_0}\right)\right]\left(\frac{M_0}{D_0}\right)^{1/\sigma}. \tag{4.7}$$

So it is clear that the sign of $d\delta/d\sigma$ is the same as that of $dN/d\sigma$, *i.e.,*

$$sgn\left(\frac{d\delta}{d\sigma}\right) = sgn\left(\frac{dN}{d\sigma}\right) = sgn\left[-\ln\left(\frac{M_0}{D_0}\right)\right] = sgn\left[\ln\left(\frac{D_0}{M_0}\right)\right] \tag{4.8}$$

where $sgn(x) = 1$ if $x > 0$, $sgn(x) = -1$ if $x < 0$, and $sgn(x) = 0$ if $x = 0$. If $D_0 > M_0$, then $(d\delta/d\sigma) > 0$ and vice versa. This result, which we have never encountered in the literature on CGE models, is quite surprising and, depending on the context, it may have interesting economic interpretations. We see that the function h_{21} is continuous and strictly monotone. If we assume that the confidence interval for $\sigma\,(C)$ is a closed bounded set of the form $[\hat{\sigma}_L, \hat{\sigma}_U]$, with level $1-\alpha$, then one of the two intervals $[h_{21}(\hat{\sigma}_L), h_{21}(\hat{\sigma}_U)]$ and $[h_{21}(\hat{\sigma}_U), h_{21}(\hat{\sigma}_L)]$ is a confidence interval with level $1-\alpha$ for the calibrated parameter δ. So, we see that $P(\sigma \in [\hat{\sigma}_L, \hat{\sigma}_U]) \geq 1-\alpha$ entails:

$$P\left(\min\{h_{21}(\hat{\sigma}_L), h_{21}(\hat{\sigma}_U)\} \leq \delta \leq \max\{h_{21}(\hat{\sigma}_L), h_{21}(\hat{\sigma}_U)\}\right) \geq 1-\alpha\,. \tag{4.9}$$

In addition to the share parameter δ, a similar analysis may be performed on the scale parameter B. This work is analytically not as simple as that on δ, but remains feasible numerically (see Section 5).

5 Application to a model of Morocco

In this section, we apply the projection method described in Section 3 to build confidence sets for the calibrated parameters (which depend on free parameters) in the context of two different models for Morocco. The first one is a submodel of a type 1-2-3 CGE model [Devarajan et al. (1990)] considered in Abdelkhalek and Dufour (1998). The second one is a submodel of a two-sector model (agriculture and industry) used by Abdelkhalek and Martens (1996). Both models include imported goods (M) and locally produced goods (D), which are aggregated through an Armington-type CES function. The Moroccan reference year data come from 1985 for the first model and from 1990 for the second model. Calculations and optimizations were performed using the GAMS-MINOS program [see Brooke, Kendrick and Meeraus (1988)]. The program used is described in detail in Abdelkhalek and Dufour (2000).

Given the reference-year values Q_0, M_0, D_0, p_{M_0} and p_{D_0} and a level $1-\alpha$ confidence region C

TABLE 1: Moroccan data used in the calibrations [a]

Variables	SAM 1985	SAM 1990 Agriculture	SAM 1990 Industry
Q_0	252653	69589.32	317195.92
M_0	42806	4248	59327.9
D_0	209847	65341.32	257868.02
T_{M_0}	9046.7	-391.79	10048.1
t_m	0.211	-0.0922	0.16936
p_{D_0}	1	1	1
p_{w_0}	1	1	1
E_0	1	1	1
p_{M_0}	1.211	0.9078	1.16936
σ	[0.7838, 2.0809]	[0.5, 4.5]	[0.5, 4.5]

[a] SAM: social accounting matrix. T_{M_0} is the net amount of import taxes. Data for Q_0, M_0, D_0 and T_{M_0} are in millions of dirhams and were obtained from Groupe de recherche en économie internationale (G.R.E.I.) (1992) for 1985 and from Abdelkhalek and Martens (1996) for 1990. The confidence intervals for σ are econometric estimates for 1985 from Abdelkhalek and Dufour (1998). Those for 1990 are subjectively determined although consistent with the values reported by Reinert and Roland-Holst (1992) for 163 sectors of the U.S. economy (ranging from 0.14 to 3.49).

for the free parameter σ, the confidence intervals are obtained by minimizing and maximizing the values of the calibrated parameters subject to the restriction that the free parameter remains in its confidence region. Note the confidence set C for σ may be truncated to only contain values in a set C_0 of economically admissible values; the resulting smaller confidence set $C \cap C_0$ has the same level as the original set C [see Abdelkhalek and Dufour (1998)]. The set $C \cap C_0$ to which σ is restricted is usually specified through a set of nonlinear inequalities. More precisely, we solve the following problems:

$$\begin{aligned}&(1) \text{ minimize and maximize } \delta = h_{21}(\sigma) \equiv \frac{(p_{M_0}/p_{D_0})\,(M_0/D_0)^{1/\sigma}}{1+(p_{M_0}/p_{D_0})\,(M_0/D_0)^{1/\sigma}} \\ &\qquad \text{subject to} \quad \sigma \in C \cap C_0\,; \end{aligned} \tag{5.1}$$

$$\begin{aligned}(2) \text{ minimize and maximize } B &= h_{22}(\sigma) = Q_0 / \left[\delta M_0^{(\sigma-1)/\sigma} + (1-\delta)\, D_0^{(\sigma-1)/\sigma}\right]^{\sigma/(\sigma-1)} \\ \text{subject to} \quad \delta &= h_{21}(\sigma) \quad \text{and} \quad \sigma \in C \cap C_0\,. \end{aligned} \tag{5.2}$$

It is also useful to remember that the price of the imported goods is given by the equation $p_{M_0} = p_{w_0}(1+t_m)E_0$, where p_{w_0} is the international price of imports, t_m is the tariff on imports and E_0 is the nominal exchange rate, evaluated at the reference year.

The Moroccan data used in these simulations are summarized in Table 1, while the confidence intervals for the calibrated parameters δ and B appear in Table 2. For the one-sector model calibrated on

TABLE 2: Confidence intervals for δ and B

Parameter	δ			B		
Confidence bounds	Lower	Upper	Range	Lower	Upper	Range
1985	0.137	0.361	0.224	1.568	1.862	0.294
Agriculture 1990	0.004	0.331	0.327	1.010	1.658	0.648
Industry 1990	0.058	0.458	0.400	1.470	1.988	0.518

the reference year 1985, we used for the free parameter σ the 95% confidence interval $[0.7838, 2.0809]$, which is based on the estimations presented in Abdelkhalek and Dufour (1998) on σ gets translated into the intervals $[0.137, 0.361]$ and $[1.568, 1.862]$ for δ and B respectively. These intervals show there is a non-negligible uncertainty on the calibrated parameters even though the confidence intervals remain remarkably tight and informative. For the two-sector model (calibrated on 1990 data), we used the wider interval $[0.5, 4.5]$. The latter was a subjectively determined, although quite consistent with the range of values reported by Reinert and Roland-Holst (1992) for similar elasticities. Not surprisingly, we find in this case wider (although still informative) intervals for the sectorial parameters δ and B associated with agriculture and industry: $\delta \in [0.004, 0.331]$ and $B \in [1.010, 1.658]$ for the agricultural sector, $\delta \in [0.058, 0.458]$ and $B \in [1.470, 1.988]$ for the industrial sector.

6 Conclusion

In this paper we have proposed a method for assessing the uncertainty of calibrated parameters in CGE models through the construction of confidence sets or intervals. The approach suggested is based on a projection technique that translates confidence sets on the free parameters into confidence sets on the calibrated parameters. The validity of the method is unaffected by the nonlinearity of the model and can be implemented easily through standard numerical algorithms used for solving CGE models. After discussing the relevant numerical methods, the technique was applied to draw inference on the calibrated parameters of CES functions in models of the Moroccan economy.

Of course, further applications and extensions are desirable. For example, it would be important to allow for perturbations (or errors) in the data used for calibration and, more generally, in the equations of the model. But the proposed method of statistical inference for calibrated parameters in CGE models goes part way to solving one problem associated with these parameters and allows one to explicitly manage the issue of uncertainty in the calibration of CGE models. Even though the illustration presented deals with a specific example, it is clear from the discussion in sections 2 and 3 that the proposed approach is in no way limited to the special functional form considered and can be applied to much more general and complicated models. Finally, it would certainly be of interest to apply similar methods in the context of calibrated stochastic models, such as real business cycle models and other similar models used in macroeconomics [for a discussion of the related statistical problems, see Gregory and Smith (1993)].

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