Jean-Marie Dufour
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ECONOMETRIC THEORY
EXERCISES 1
MODELS

Reference: Gouriéroux and Monfort (1995, Chapter 1)

1. (a) Define the notion of *statistical model*.
   (b) Explain the distinction between a *dominated statistical model* and a *homogeneous statistical model*.
   (c) When is a model *nested* by another model? What is a *submodel*? What a *nesting model*?
2. (a) Explain what is an *exponential statistical model*.
   (b) Give two examples of exponential statistical models and explain why these models belong to the exponential family.
   (c) Is a linear model always an exponential model?
   (d) Which ones of the following terms apply to exponential models: parametric, nonparametric, semiparametric?
   (e) Which ones of the following terms apply to linear models: parametric, nonparametric, semiparametric?
3. Explain the difference between the Bayesian approach and the empirical Bayesian approach to the introduction of *a priori* information.
4. Let P and P^* be two probability distributions possessing densities with respect to the same measure μ.
   (a) Define the Kullback discrepancy between P and P^*.
   (b) Prove that:
      i. $I(P \mid P^*) \geq 0$;
      ii. $I(P \mid P^*) = 0 \Longleftrightarrow P = P^*$.
5. Let $y = (y_1, \ldots, y_n)'$ be a vector of observations. To explain y, we consider the linear model:

$$y = m + u\,,\ m \in L\,,\ u \sim N\left[0,\, \sigma^2 I_n\right]$$

   where L is a vector $\mathbb{R}^n$ with k. If the true probability distribution of y is $N\left[m_0,\, \sigma_0^2 I_n\right]$, find the pseudo true values m_0^*, σ_0^* of m and σ^2. [I_n represents the identity matrix of order n.]

6. Consider the following simple Keynesian model:

$$
\begin{aligned}
C_t &= aR_t + b + u_t\,, \\
Y_t &= C_t + I_t\,, \\
R_t &= Y_t\,,
\end{aligned}
$$

where C_t represents consumption (at time t), R_t income, Y_t production, I_t investment, and u_t is a random disturbance.

   (a) Find the reduced form of this model.
   (b) Is a coherency condition needed to derive this reduced form? If yes, which one and why?
   (c) Does this model contain *latent* variables? If so, which ones?

   (a) Explain the notion of *exogeneity* with respect to a parameter.

7. Consider the following simplified equilibrium model:

$$
\begin{aligned}
D_t &= \alpha + 2p_t + u_{1t}\,, \\
S_t &= c + u_{2t}\,, \\
Q_t &= D_t = S_t \qquad , \; t = 1, \ldots, T
\end{aligned}
$$

where D_t is the demand for a product, S_t the supply for the same product, and Q_t the quantity produced and sold. We suppose that the vectors $(u_{1t},\, u_{2t})'$, $t = 1, \ldots, T$, are independent and $N[0,\, I_2]$.

   (a) Find the reduced form of this model.
   (b) For which parameters is the vector $Q = (Q_1, \ldots, Q_T)'$ exogenous? Justify your answer.
   (c) For which parameters is the vector $p = (p_1, \ldots, p_T)'$ exogenous? Justify your answer.
   (d) Are the variables Q_t and p_t simultaneous?

8. Prove the equivalence between non-causality in the sense of Granger and non-causality in the sense of Sims. (Define clearly these two notions.)

9. Give a sufficient condition under which *sequential exogeneity* is equivalent to *exogeneity* (for a parameter α) and justify your answer.

10. Consider the following equilibrium model:

$$
\begin{aligned}
&Q_t = a + bp_t + u_{1t}\,, \\
&p_t = c + dp_{t-1} + u_{2t} \qquad , \; t = 1, \ldots, T
\end{aligned}
$$

$$p_0 \text{ is fixed}$$

where the disturbances $(u_{1t},\, u_{2t})'$, $t = 1, \ldots, T$ are independent $N[0, I_2]$, Q_t represents the quantity sold, and p_t the price. For which parameters is the vector $p = (p_1, \ldots, p_T)'$

(a) sequentially exogenous?

(b) exogenous?

(c) strongly exogenous?

(d) Further, does Q_t cause p_t in the sense of Granger?

Justify your answers.

11. Consider the following equilibrium model:

$$\begin{aligned} &Q_t = a + bp_{t+1} + u_{1t}\,, \\ &p_t = c + dp_{t-1} + u_{2t} \qquad , \; t = 1, \ldots, T \\ &p_0 \text{ is fixed} \end{aligned}$$

where the disturbances $(u_{1t},\, u_{2t})'$, $t = 1, \ldots, T$ are independent $N[0, I_2]$, Q_t represents the quantity sold and p_t the price. For which parameters is the vector $p = (p_1, \ldots, p_T)'$

(a) exogenous for (a, b)?

(b) exogenous for (c, d)?

(c) sequentially exogenous for (a, b)?

(d) sequentially exogenous for (c, d)?

(e) strongly exogenous for (a, b)?

(f) strongly exogenous for (c, d)?

Justify your answers.

12. Consider the following equilibrium model:

$$\begin{aligned} &Q_t = a + bp_t + u_{1t}\,, \\ &p_t = c + dQ_{t-1} + u_{2t}\,, \\ &Q_0 \text{ is fixed} \end{aligned}$$

where the disturbances $(u_{1t},\, u_{2t})'$, $t = 1, \ldots, T$ are independent $N[0, I_2]$, Q_t represents the quantity sold, and p_t the price. For which parameters is the vector $p = (p_1, \ldots, p_T)'$

(a) exogenous for (a, b)?

(b) exogenous for (c, d)?

(c) sequentially exogenous for (a, b)?

(d) sequentially exogenous for (c, d)?

(e) strongly exogenous for (a, b)?

(f) strongly exogenous for (c, d)?

Justify your answers.

13. Consider the following equilibrium model:

$$\begin{aligned} D_t &= a + bp_t + u_{1t}\,, \\ S_t &= c + dp_{t-1} + ex_t + fx_{t-1} + u_{2t}\,, \\ Q_t &= D_t = S_t \qquad , \; t = 1, \ldots, T \end{aligned}$$

where D_t is the demand for a product, S_t the supply for the same product, Q_t the quantity produced, x_t is an exogenous variable, p_0 and x_0 are fixed, and $u_t = (u_{1t}, u_{2t})'$ is random vector such that $E(u_t) = 0$.

(a) Give the structural form associated with this model.

(b) Give the reduced form of this model.

(c) Find the short-term multipliers for p_t and Q_t.

(d) Find the final form of the model.

(e) Find the dynamic multipliers for p_t.

(f) Find the long-run form of the model and the long-term multipliers for p_t and Q_t.

References

GOURIÉROUX, C., AND A. MONFORT (1995): *Statistics and Econometric Models, Volumes One and Two*. Cambridge University Press, Cambridge, U.K., Translated by Quang Vuong.