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Estimation uncertainty in structural inflation models with real wage rigidities

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ABSTRACT

Real wage rigidities have recently been proposed as a way of building intrinsic persistence in inflation within the context of New Keynesian Phillips Curves. Using two recent structural models, the importance of real wage rigidities in the data, and the extent to which such models provide useful information regarding price stickiness, are evaluated empirically. Structural estimation and testing is carried out using Canadian data and identification-robust methods. Results based on the first model reveal important real wages rigidities and underscore the impact of the price of crude materials. However estimates indicate almost fully-flexible prices and an ambiguous role for unemployment. In contrast, and despite some estimate uncertainty, structural estimations are supportive of the second model. Economically-reasonable ranges for estimates of average frequency of price changes are obtained, as well as significant and correctly-signed implied reduced-form coefficient estimates, showing a trade-off between unemployment and inflation in the NKPC. From a methodological perspective, results derive from the treatment of the productivity term as observable although with error, which seems to capture vital information and improve overall identification. From a substantive perspective, the findings suggest that wage-rigidity based New Keynesian Phillips Curves hold promise empirically and provide interesting research directions.

1. Introduction

The existence of real wage rigidities in labor markets has been the focus of recent work on inflation models; see, for example, Christoffel and Linzert (2006), Rotemberg (2006), Gertler et al. (forthcoming), Blanchard and Gali (2007, 2008) and Krause et al. (2008). One reason for this is that modeling approaches based on real wage rigidities can generate theoretical inflation inertia in Calvo-based New Keynesian Phillips Curve (NKPC) equations. A second reason is that the existence of such frictions allows the New Keynesian framework to present a trade-off between the stabilization of inflation and output by Central Banks, thereby breaking the unrealistic so-called 'divine coincidence' in otherwise standard New Keynesian setups.

In this paper, we consider two illustrative structural inflation models with real wage rigidities which lend themselves well to estimation, and evaluate statistically (i) the extent to which they provide useful information regarding price stickiness, and (ii) the importance of real wage rigidities in the data. Canadian quarterly data are used for the structural estimations. The two studies we consider, namely Blanchard and Gali (2007, 2008), differ in the way they capture labor market conditions

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in the model structure, but they are similar in the Calvo-type assumptions made in firms' price-setting behavior. Examining whether reliable estimates can be obtained for the structural measure of inflation persistence in the data thus amounts to looking at the precision of Calvo parameter estimates from these models, since it is this parameter that is used to calculate the average frequency of price adjustments in the economy. As for the extent of real rigidities present, in both models we look at the value of the parameter estimate representing the real wage rigidity index.

Additional contributions in this paper are that we provide structural estimates of the two models that we examine; to our knowledge, Blanchard and Gali (2007) has so far only been estimated in reduced-form while the structural equation we consider from Blanchard and Gali (2008) has not yet been estimated. As a matter of fact, the latter model is not directly amenable to estimation. The equation we consider from Blanchard and Gali (2008) explains inflation as a function of current and lagged unemployment as well as a productivity shock, where the coefficients on each of these variables, including the productivity shock, is a nonlinear function of the structural parameters. Consequently, vital information on the structural constraints coming from the productivity term needs to be captured for estimation purposes. We address this difficulty by using an observable proxy for productivity and by accounting for the fact that it is observed with error.

A number of econometric challenges arise in our analysis. The chosen NKPC specifications require, among other things, building proxies for some regressors as argued above, finding valid instrumental variables to conduct estimations with, and accounting for specification and estimation uncertainties. Thus, errors-in-variables, underidentification, weak instruments, and specification issues are important concerns. To deal with these difficulties we resort to estimation and testing using identification-robust methods. For comprehensive surveys on accounting for some of these issues in the presence of identification problems, see Stock et al. (2002) and Dufour (2003). Additional references include Dufour (1997), Staiger and Stock (1997), Wang and Zivot (1998), Zivot et al. (1998), Dufour and Jasiak (2001), Kleibergen (2002, 2005), Dufour and Taamouti (2005, 2007), Andrews et al. (2006), Hoogerheide et al. (2007), Joseph and Kiviet (2005), Kiviet and Niemczyk (2007), Bolduc et al. (2008), and Beaulieu et al. (2009). Such methods are valid irrespective of the identification status of the examined model, which is an advantage not shared, for example, by standard method-of-moments-based approaches. As a result, we can find out how well a particular structural parameter is identified, and, what the "true" (i.e., the reliably-assessed) uncertainty associated with its estimate is if the parameter is weakly-identified. In addition, the methods provide further advantages, such as formally accounting for the integration of calibration with estimation, and correcting for errors-in-variables. The latter property is specially appealing in the case of the Blanchard and Gali (2008) model for which estimation in a structural manner is not straightforward, and which, as mentioned, we address by introducing a variable that is observed with error.

Results from the specification based on the Blanchard and Gali (2007) model reveal important real wages rigidities and underscore the impact of changes in the price of crude materials. However, our estimates also indicate almost flexible prices and an ambiguous role for unemployment, which go against the basic NKPC assumptions underlying this model. In contrast, and despite some estimate uncertainty, our structural estimations are supportive of the Blanchard and Gali (2008) model. We obtain economically reasonable ranges for estimates of average frequency of price changes and significant and correctly-signed implied reduced-form coefficient estimates, showing a trade-off between unemployment and inflation in the NKPC.

Recent econometric methods for dealing with weak instruments are gaining credibility in macroeconomics. Studies examining identification issues in inflation models previously include Ma (2002), Mavroeidis (2004), Khalaf and Kichian (2005), Mavroeidis (2005), Dufour et al. (2006), Canova and Sala (2006), Nason and Smith (2008), Kleibergen and Mavroeidis (in press) as well as Dufour et al. (2008). Most of this work suggests that data may be weakly informative on key NKPC parameters, which raises serious concerns. In contrast, our results provide support for the Blanchard and Gali (2008) model that encourages further work with wage-rigidity-based NKPCs. Results come from our treatment of the productivity term in the model as an observable variable with error, which seems to capture vital information from the data and improve overall identification and fit.

In the next section, we present the structural forms of the models that we examine. Section 3 explains the methodology applied. Section 4 presents the empirical results, and Section 5 offers some conclusions.

2. The models

It has recently been suggested that one way of building intrinsic persistence into NKPC models is to allow for stickiness in real wages. A variety of modeling assumptions capturing different aspects of labor market search and matching frictions have been proposed for this purpose. Examples include Christoffel and Linzert (2006), Gertler et al. (forthcoming), Rotemberg (2006), Blanchard and Gali (2007, 2008) as well as Krause et al. (2008). We focus on the models by Blanchard and Gali (2007, 2008) for our analysis, for they lend themselves relatively easily to structural estimation. For instance, the Krause et al. (2008) approach constructs a measure of real marginal costs reflecting underlying labor conditions and thus requires data that is not as readily available.

Blanchard and Gali (2007) propose a Calvo (1983) staggered price setting mechanism where, in any given period, each firm has a probability $1-\theta$ of re-setting its price. That is, a fraction $1-\theta$ of firms can adjust their prices. Another assumption made in the model is that, as a result of some market imperfection, real wages respond sluggishly to labor demand conditions. An index of real wage rigidity, γ_1, is proposed such that the higher its value the more wages depend on lagged wages. Furthermore, an inflation–unemployment relationship is derived implying the following inflation equation:

$$\pi_t = \gamma_{1f} E_t \pi_{t+1} + \gamma_{1b} \pi_{t-1} + \chi_1^u U_t + \chi_1^v \Delta v_t + \epsilon_{1,t+1},$$

where π_t is the inflation rate, U_t is the rate of unemployment, Δv_t is the change in the real price of the non-produced good in the economy, and the error term is an independently identically distributed process.

Under rational expectations, and imposing the structural constraints on the coefficients of the equation, the econometric version of the above model is given by:

$$\pi_t = \frac{\beta}{1+\beta}\pi_{t+1} + \frac{1}{1+\beta}\pi_{t-1} - \frac{\lambda_1(1-\alpha_1)(1-\gamma_1)\phi_1}{\gamma_1(1+\beta)}U_t + \frac{\alpha_1\lambda_1}{(1+\beta)}\Delta v_t + e_{1,t+1}, \tag{2.1}$$

where β is the subjective discount rate, α_1 is the share of the non-produced good in total output, ϕ_1 is the slope of labor supply, the error term now reflects rational expectation error, and

$$\lambda_1 = \frac{(1-\theta)(1-\beta\theta)}{\theta}. \tag{2.2}$$

The second model that we examine is the one proposed in Blanchard and Gali (2008). In this case, staggered price and nominal wage setting is combined with an articulated set of assumptions regarding frictions in the labor market, along the lines of the search and matching model of Diamond–Mortensen–Pissarides. Again, log-linearization around a zero steady-state inflation, and making a theoretical link between inflation and the unemployment rate, yields the following inflation equation:

$$\pi_t = \chi_2^u\hat{U}_t + \chi_2^l\hat{U}_{t-1} + \chi_2^p\overline{a}_t, \tag{2.3}$$

where $\hat{U}_t$ is the unemployment rate in deviation from the steady-state unemployment (U_2) and $\overline{a}_t$ is the log deviations of productivity from its steady-state. It is assumed that $\overline{a}_t$ follows a stationary autoregressive process with a parameter ρ_2. The coefficients χ_2^u, χ_2^l as well as χ_2^p are nonlinear functions [as shown below] of the model's "deep parameters"; these include a Calvo parameter denoted θ [we retain the same notation as in the previous model], and an index of real wage rigidities, denoted γ_2, that intervenes only via χ_2^p. As described, this model is not immediately amenable to estimation. In particular, the specification of the productivity term affects the way in which vital information coming from the structural constraints on the productivity term would be accounted for.

We address this challenge by using an observable proxy, a_t, for the variable $\overline{a}_t$, and by accounting for the fact that it is observed with error. In this new context, all the structural constraints of the model can be imposed and the following econometric model is obtained:

$$\pi_t = -\kappa_2\hat{U}_t + \kappa_2(1-\delta_2)(1-x_2)\hat{U}_{t-1} - \Psi_2\gamma_2 a_t + e_{2,t}. \tag{2.4}$$

In addition, the coefficients κ_2 and Ψ_2 are defined as:

$$\kappa_2 = \frac{\alpha_2\lambda_1 m_2 g_2}{\delta_2(1-U_2)}, \tag{2.5}$$

and

$$\Psi_2 = \frac{\lambda_1\phi_2}{(1-\beta\rho_2)}, \tag{2.6}$$

where $\phi_2 = 1-(1-\beta(1-\delta_2))m_2g_2$ is less than one, δ_2 is an exogenous separation rate in the labor market, x_2 is the job finding rate, α_2 is a parameter related to hiring costs, $m_2 = \epsilon/(\epsilon-1)$ is the gross steady-state mark-up, ϵ is the price elasticity of demand, $g_2 = B_2(x_2{}^{\alpha_2})$, B_2 is a parameter related to the level of hiring costs, the steady-state unemployment rate is given by $U_2 = (\delta_2(1-x_2))/(x_2+\delta_2(1-x_2))$, and $\lambda_1 = [(1-\theta)(1-\beta\theta)]/\theta$. For later reference, we denote the reduced-form coefficients on $\hat{U}_t$, $\hat{U}_{t-1}$, and a_t in the econometric model as χ_2^a, χ_2^b, and χ_2^c, respectively.

3. Weak identification and inference

In this section, we briefly re-visit the intuition for the use of identification-robust methods and present the specific procedure we adopt. An illustration of the application of this method to the Blanchard and Gali (2007) model is also provided. The reader may consult the above cited econometric literature for insights and further references; for macroeconomic applications, see Mavroeidis (2004), Mavroeidis (2005), Dufour et al. (2006), Canova and Sala (2006), Nason and Smith (2008), Dufour et al. (2008), and Kleibergen and Mavroeidis (in press).

When taken to the data, the models described in the previous section (as well as most optimization-based models) are often confronted with two central concerns: (i) endogeneity, entailed by the presence of expectation-based regressors and errors in variables, and (ii) parameter nonlinearity, which follows from the connection between the key parameters of the underlying theoretical model and the parameters of the estimated econometric model. For a discussion of these problems, see, for example, Galí et al. (2005) and Sbordone (2005).

Although the models lead to orthogonality conditions that lend themselves well to instrumental variable (IV) or GMM estimation methods, endogeneity and nonlinear parameter constraints, in conjunction with weak instruments, lead to the eventuality of *weak identification*. The latter causes the breakdown of standard asymptotic procedures such as IV-based t-tests and Wald-type confidence intervals of the form: [estimate $\pm$ (asymptotic standard error) $\times$ (asymptotic critical point)], and a heavy dependence on unknown nuisance parameters. As a result, standard and even bootstrap-based tests and confidence intervals can be unreliable and spurious model rejections can occur even with large data sets. Indeed, non-identification should, in principle, lead to diffuse confidence sets that can alert the researcher to the problem. Unfortunately, if traditional Wald-type methods are applied when estimating weakly-identified parameters, the expected diffuse intervals often do not obtain. Rather, traditional Wald-type methods may yield tight confidence intervals focusing on "wrong" values. For practitioners, this problem is doubly-misleading. On the one hand, estimated intervals severely understate estimation uncertainty. On the other hand, and perhaps more importantly, intervals will fail to cover the true parameter value, which, in view of their tightness, will go unnoticed.

These problems are averted if one applies an inference method that does not require identification. Formally, identification-robust methods are inference procedures where error probabilities [e.g. test size, confidence level] can be controlled in the presence of endogeneity, nonlinear parameter constraints and identification difficulties. From a confidence set perspective, when parameters are not identifiable on a subset of the parameter space, or when the admissible set of parameter values is unbounded (which occurs, for example, with nonlinear parameter constraints such as ratios), it is rarely possible to ensure proper coverage unless the set construction method allows for unbounded outcomes. Our methodology can be described as follows.

Consider a nonlinear equation of the form

$$\mathcal{F}_t(\mathcal{Y}_t,\ \vartheta) = \mathcal{U}_t, \quad t = 1, \ldots, T, \tag{3.1}$$

where $\mathcal{F}_t$, $t = 1, \ldots, T$ are scalar functions that may have a different form for each observation, ϑ is an $m \times 1$ vector of unknown parameters of interest, $\mathcal{Y}_t$ is the $n \times 1$ vector of observed variables and $\mathcal{U}_t$ is a disturbance with mean zero. Conformably with the GMM literature, our notation for $\mathcal{Y}_t$ includes the exogenous and endogenous variables. The objective is to invert an identification-robust test of the hypothesis:

$$H_0 : \vartheta = \vartheta_0.$$

Inverting a test produces the set of parameter values that are not rejected by this test; furthermore, the least-rejected parameters are the so-called Hodges–Lehmann point estimates (see Hodges and Lehmann (1963, 1983), and Dufour et al. (2006)).

Clearly, if H_0 holds true, then $\mathcal{F}_t(\mathcal{Y}_t,\ \vartheta_0) = \mathcal{U}_t$. Thus, if Z_t is a $k \times 1$ vector of exogenous or predetermined variables such that $k \geq m$, then the coefficients of the regression

$$\mathcal{F}_t(\mathcal{Y}_t, \vartheta_0) = Z_t'\varpi + \varepsilon_t \tag{3.2}$$

should be close to zero. Hence, H_0 in the context of (3.1) can be tested by assessing

$$H_0' : \varpi = 0 \tag{3.3}$$

in the context of (3.2). Z_t can be viewed as a vector of instruments, which may include the exogenous variables in $\mathcal{Y}_t$; (3.2) may be viewed as an auxiliary or artificial regression and the test of H_0' in the context of (3.2) an auxiliary or artificial regression test for H_0. Rewriting the latter in matrix form where

$$\mathcal{F}(\mathcal{Y},\ \vartheta_0) = [\mathcal{F}_1(\mathcal{Y}_t, \vartheta_0), \ldots, \mathcal{F}_T(\mathcal{Y}_t, \vartheta_0)]', \tag{3.4}$$

$$\mathcal{Y} = [\mathcal{Y}_1,\ \mathcal{Y}_2, \ldots, \mathcal{Y}_T]', \tag{3.5}$$

$$Z = [Z_1,\ Z_2, \ldots,\ Z_T]', \tag{3.6}$$

the F-statistic for H_0' is given by

$$\mathcal{T}(\vartheta_0) = \frac{\mathcal{F}(\mathcal{Y}, \vartheta_0)'\,(I - M\,[Z])\,\mathcal{F}(\mathcal{Y},\ \vartheta_0)/k}{\mathcal{F}(\mathcal{Y}, \vartheta_0)' M\,[Z]\,\mathcal{F}(\mathcal{Y}, \vartheta_0)/(T-k)} \tag{3.7}$$

$$M\,[Z] = I - Z(Z'Z)^{-1}Z'. \tag{3.8}$$

If Z and $\varepsilon_1,\ \varepsilon_2, \ldots, \varepsilon_T$ are independent, the matrix Z has full column rank and $\varepsilon_1,\ \varepsilon_2, \ldots,\ \varepsilon_T$ are *i.i.d.* homoskedastic normal, under the null (3.3), $\mathcal{T}(\vartheta_0)$ follows a central Fisher distribution with degrees of freedom k and $T-k$. The latter exact result may be relaxed leading to the standard χ^2 based distribution compatible with classical least-squares.

To illustrate the above, consider the model in Eq. (2.1) which is reproduced here for convenience:

$$\pi_t = \frac{\beta}{1+\beta}\pi_{t+1} + \frac{1}{1+\beta}\pi_{t-1} - \frac{\lambda_1(1-\alpha_1)(1-\gamma_1)\phi_1}{\gamma_1(1+\beta)}U_t + \frac{\alpha_1\lambda_1}{(1+\beta)}\Delta v_t + e_{1,t+1}, \tag{3.9}$$

$$\lambda_1 = \frac{(1-\theta)(1-\beta\theta)}{\theta}. \tag{3.10}$$

Our aim is to estimate the structural parameters θ and γ_1, given $\Omega = (\beta, \alpha_1, \phi_1)'$ which we will calibrate, conforming with common practice. The model can be rewritten as in (3.1), with $\mathcal{Y}_t = (y_t, Y_t')'$, $y_t = \pi_t$, $Y_t = (\pi_{t+1}, \pi_{t-1}, U_t, \Delta v_t)'$, $\vartheta = (\theta, \gamma_1)'$,

$$\mathcal{F}_t(\mathcal{Y}_t, \vartheta) = y_t - Y_t'\bar{\Gamma}\,(\vartheta; \Omega)\,, \tag{3.11}$$

and $\bar{\Gamma}(\vartheta;\Omega)$ is the four-dimensional function of ϑ

$$\bar{\Gamma}(\vartheta;\Omega)=\begin{bmatrix}\beta/(1+\beta)\\ 1/(1+\beta)\\ (\lambda_1(1-\alpha_1)(1-\gamma_1)\phi_1)/(\gamma_1(1+\beta))\\ \alpha_1\lambda_1/(1+\beta)\end{bmatrix} \tag{3.12}$$

implied by (3.9) and (3.10). If the *i.i.d.* error hypothesis is maintained, the test associated with (3.7) using (3.11) can be inverted. Depending on whether instruments are strongly or weakly exogenous, the associated procedure will be either exact or asymptotically valid. In the latter case, regular least-squares-based asymptotics would hold, in contrast with usual IV methods that require rank restrictions to identify ϑ. Allowing for departures from the *i.i.d.* error hypothesis, the test we invert is based on a Wald-type statistic with Newey–West autocorrelation-consistent covariance estimator given by:

$$AR\text{-HAC}(\vartheta_0)=\mathcal{F}(\mathcal{Y},\vartheta_0)'Z(Z'Z)^{-1}\hat{Q}^{-1}(Z'Z)^{-1}Z'\mathcal{F}(\mathcal{Y},\vartheta_0), \tag{3.13}$$

$$\hat{Q}=\frac{1}{T}\sum_{t=1}^{T}\hat{u}_t^2Z_tZ_t'+\frac{1}{T}\sum_{l=1}^{L}\sum_{t=l+1}^{T}w_l\hat{u}_t\hat{u}_{t-l}\left(Z_tZ_{t-l}'+Z_{t-l}Z_t'\right),$$

$$w_l=1-\frac{l}{L+1},$$

where $\hat{u}_t$ is the OLS residual associated with the artificial regression (3.2), and L is the number of allowed lags. Although, for simplicity, our notation may not clearly reflect this fact, it is worth emphasizing that $\hat{Q}$ is a function of ϑ_0 so the minimum-distance based test we consider involves continuous updating of the weighting matrix.

It is easy to see [refer e.g. to Dufour (2003)] that if (3.1) is a linear Limited Information Simultaneous Equation, then $\mathcal{T}(\vartheta_0)$ reduces to the test proposed by Anderson and Rubin (1949) for hypotheses specifying the full vector of the left hand side endogenous variables coefficients. In addition, the nonlinear test statistics (3.7) and (3.13) correspond closely to Stock and Wright (2000)'s asymptotic GMM-based test. The statistical foundations which lead to the test identification robustness are the following: whereas traditional set estimation and testing in the context of (3.1) [via GMM or even with regular FIML] is inappropriate and cannot be salvaged under weak identification, inverting the auxiliary regression test of (3.3) in the context of (3.2) translates the problem into the regular regression framework while maintaining its structural foundations. The modification that we perform in this paper in order to correct for non-*i.i.d.* errors exploits the fact that (3.2) is indeed a regular regression where routine heteroskedasticity- and autocorrelation-consistent (HAC) corrections can be applied. Our procedure has two further "built-in" advantages. First, extremely-wide confidence sets reveal identification difficulties. Second, if all economically-sound values of the model's deep parameters are rejected at some chosen significance level, the confidence set will be empty and we can then infer that the model is soundly rejected. This provides an identification-robust alternative to the standard GMM-based J-test.

In practice, test inversion is performed numerically. A confidence set with level $1-\alpha$ is constructed by collecting the couples (θ_0,γ_0) that, given the calibrated Ω_0, are not rejected by the above tests at level α. For this purpose, we conduct a grid search over the economically-meaningful set of values for the structural parameters, sweeping the choices for θ_0, γ_0, given Ω_0. For each parameter combination choice, Eq. (3.12) is used in order to obtain $\bar{\Gamma}(\vartheta_0;\Omega_0)$. The appropriate test statistic is applied, and the associated p-value is calculated from the $\chi^2(k)$ null distribution. Collecting those vector choices for which the p-values are greater than a test level α constitute a joint confidence region with level $1-\alpha$. Individual confidence intervals for each parameter can then be obtained by projecting the latter region (i.e. by computing, in turn, the smallest and largest values for each parameter included in this region). A point estimate can also be obtained from the joint confidence set. This corresponds to the model that is most compatible with the data, or, alternatively, that is least-rejected, and is given by the vector of parameter values with the largest p-value.

4. Empirical results

4.1. Data

We consider quarterly Canadian data from 1982Q2 to 2007Q2. The price level P_t is measured by the GDP deflator, and to obtain the real price of the non-produced good in the economy V_t, the producer price of crude materials is deflated by the GDP deflator.

Taking the log of these series (which we represent by the corresponding small letters), we define inflation, π_t, as gross inflation, and the change in the real price of the non-produced good, Δv_t, as the log difference in V_t. In addition, we use the quarterly unemployment rate for the variable U_t, and define productivity, a_t, as the first difference of the log of the ratio of GDP to employment, where total non-farm employment is used for employment, and where the first difference of the ratio is taken to render the series stationary.

A number of additional variables are used as instruments. These include the yield spread, defined as the 10-year bond yield minus the yield on 3-month Treasury bill, the log difference in total commodity prices, and the log difference of employment. Finally, we use a quadratically-detrended measure of the output gap, defined in a real-time sense so that the gap value at time t does not use information beyond that date. Thus, as in Dufour et al. (2006), we obtain the value of

Table 1
Blanchard and Gali (2007) model, estimation and test results.

γ_1	θ	F_q	χ_1^u	χ_1^v
		$\underline{\alpha_1 = 0.330}$ Max P-val $= 0.2209$		
1.00	0.02	1.02	0.00	7.96
(0.76, 1.00)	(0.02, 0.12)	(1.02, 1.14)	(−5.11, 0.00)	(1.07, 7.96)
		$\underline{\alpha_1 = 0.025}$ Max P-val ≤ 0.050		
		empty set		

Note: Instruments include the second lag of: Inflation, unemployment rate, productivity, change in real price of crude materials, yield spread, change in total commodity price, and change in employment.

Table 2
Blanchard and Gali (2008) model, estimation and test results.

γ_2	θ	F_q	χ_2^a	χ_2^b	χ_2^c
		$\underline{\rho_2 = 0.90}$	Max P-val $= 0.5819$		
0.96	0.66	2.94	−0.14	0.04	−1.55
(0.20, 1.00)	(0.42, 0.74)	(1.72, 3.85)	(−0.65, −0.08)	(0.02, 0.17)	(−2.57, −0.85)
		$\underline{\rho_2 = 0.95}$	Max P-val $= 0.6284$		
0.98	0.74	3.85	−0.08	0.02	−1.53
(0.12, 1.00)	(0.42, 0.80)	(1.72, 5.00)	(−0.65, −0.04)	(0.01, 0.17)	(−2.67, −0.84)

Note: Instruments include the second lag of: inflation, unemployment rate, productivity, change in real price of crude materials, yield spread, change in total commodity price, and change in employment.

the gap at time t by detrending GDP with data ending in t. Then the sample is extended by one observation and the trend is re-estimated. The latter is used to detrend GDP, yielding a value for the gap at time $t + 1$. The process is repeated in this fashion until the end of the sample.

4.2. Estimation results

The test applied in all cases is the AR-HAC test, and significance refers to a five per cent test level. All variables are taken in deviation from the sample mean, which is in accordance with not fixing steady-state values to specific (zero or non-zero) parameters, but allowing them to be free constants. Recall that, in contrast with model (2.3), the unemployment rate in (2.1) is implicitly assumed to be in deviation from a zero steady-state. Nevertheless, we estimate an unrestricted constant for both models. See Sbordone (2007) for a discussion on the importance of doing so in empirical contexts. Four lags are used in the Newey–West heteroskedascticity and autocorrelation-consistent covariance estimator.

Estimation and test results are reported in the Tables 1 and 2. In the case of each model, we report the point estimates of the structural and selected reduced-form parameters, the average frequency of price adjustment, denoted by *Fq* and given by $1/(1-\theta)$, as well as the test *p*-value associated with the vector of point estimates (i.e., the maximal *p*-value). In addition, for each estimated parameter, we report in parentheses its smallest and highest values in the confidence set.

We conduct estimations for each model using a single instrument set comprised of lags of the endogenous variables of the two models as well as some extra-model variables. The set therefore includes the second lag of each of: inflation, unemployment rate, productivity, real price of crude material inflation, the yield spread, total commodity price inflation, and change in employment. Except for change in employment, the extra-model instruments were also used in the original Galí and Gertler (1999) study. Relying on the optimal instrument set which corresponds to Kleibergen (2002)'s method [see also Dufour et al. (2006)] yields qualitatively similar results.

In the case of the Blanchard and Gali (2007) model, we structurally estimate θ and the real wage rigidity index, γ_1, calibrating the remaining parameters. As in the original study, we set the subjective discount rate, β, to 0.99, the Frisch labor supply elasticity, ϕ_1, to 1, and α_1 to 0.025. For sensitivity analysis, we also consider the value 0.33 for the latter parameter, which is in line with the Chari et al. (2000) study. The search space for θ is (0.02, 0.98), and for γ_1 it is (0.02,1.00). For both parameters, the grid increments are 0.02.

Table 1 reports structural estimation and test results for this model. We first note that results differ importantly depending on the calibration for α_1. This parameter intervenes via the coefficients of both unemployment and the inflation rate in the non-produced good; see (2.1). Indeed, the confidence set for θ and γ_1 is empty at the 5% level with $\alpha_1 = 0.025$, which suggests that Canadian data is not compatible with the lower α_1 calibration. In contrast, we find that both θ and γ_1 are hard to precisely pin-down with the 0.33 calibration, in the following sense: while the upper parameter space boundary cannot be ruled out in the case of θ, the lower boundary cannot be ruled out for γ_1. Thus, it can be affirmed that the wage rigidity index is at least as high as 0.76 (implying a dominant backward-looking component in real wages), and that average frequency of price changes (as measured by *Fq*) are at most 1.14 quarters. For all practical purposes, estimates for *Fq* indicate fully-flexible prices, despite the explicit assumption in the model for generating nominal price stickiness in the NKPC. As for the implied reduced-form parameters, while the projection range for the coefficient on unemployment covers zero, the coefficient estimate on the inflation rate in the non-produced good is significant.

Overall, these findings: (i) underscore the key impact calibration assumptions can have on identification and inference; (ii) indicate important sluggishness in real wages (as captured by high values of the index) but little stickiness in nominal prices (given that average prices change fairly frequently), and (iii) suggest that changes in real price of crude materials seem to play an important role for the dynamics of Canadian inflation.

We next turn to the results for the Blanchard and Gali (2008) model. In this case, we estimate structurally the parameters θ (the Calvo parameter) and γ_2 (the real wage rigidity index), calibrating β again to 0.99. The search ranges are again (0.02, 0.98) for θ and (0.02, 1.00) for γ_2, while the grid search increment is 0.02. As explained previously, ρ_2 is also calibrated; we consider two values for this parameter: 0.90 and 0.95.

Table 2 reports the estimation results of this model. Overall, we find that outcomes are fairly similar across the two calibrated values for ρ_2. This parameter intervenes via the coefficients of productivity; see (2.4) and (2.6). Interestingly, and unlike with the previous specification, projections for the Calvo parameter are bounded at both ends with this model and fully-flexible wages are ruled out despite relatively wide projections (with both ρ_2 calibrations). In addition, implied reduced-form parameter estimates are significant and have the right signs.

The above-cited recent literature on the NKPC that caters for weak-instruments problems seems to suggest that macro-data may be weakly informative on the Calvo parameter, which corresponds to *Fq* for the specification at hand. In contrast, we find that the Blanchard and Gali (2008) model can deliver reasonable confidence bounds on this parameter. Indeed, taking a closer look at the estimated projections for *Fq*, our results suggest that prices adjust on average every 1.72 to 3.85 quarters, with $\rho_2 = 0.90$, and up to 5 quarters with $\rho_2 = 0.95$. While the impact of calibration here again is worth noting, such evidence is largely in line with micro-based evidence on price adjustments.

There is more uncertainty regarding the importance of real wages as captured by the rigidity index in the model. Nonetheless, with $\rho_2 = 0.90$, it is possible to affirm that there is at minimum a 20% weight attributable to lags in real wages. Furthermore, both unemployment and productivity are found to play significant roles in the dynamics of Canadian inflation.

The motivation in Blanchard and Gali (2008) for introducing real wage rigidities through a rigidity index into a dynamic stochastic general equilibrium model was twofold: to generate intrinsic inflation persistence and to create inflation-output trade-off. In this sense, the results of our structural estimations are supportive of the model. We find on the one hand that the model implies average frequencies of price changes that are economically-reasonable, and on the other hand, we find that there is a significant trade-off between inflation and unemployment, with higher rates of unemployment leading over two quarters to lower inflation in Canada.

From a statistical perspective, our treatment of the productivity term seems empirically vital for the Blanchard and Gali (2008) model. Indeed, the observable proxy we use for productivity seems to capture crucial information on the structural constraints which improves overall identification yet maintains the structural foundations of the model. Our results underscore the identifying role of this variable which motivates its use beyond our specific setting.

5. Conclusion

We apply identification-robust methods to structurally estimate two recent inflation models based on real wage rigidities, with Canadian quarterly data. Both models (Blanchard and Gali, 2007, 2008) attempt to build intrinsic inflation persistence in inflation and to generate a real-nominal trade-off. To do so, they make similar Calvo assumptions on nominal prices but they model labor market frictions differently. We aim to assess the importance of real wage rigidities in the data and the extent to which such models provide useful information regarding price stickiness.

Results from the Blanchard and Gali (2007) model reveal important real wages rigidities, and that the changes in the price of crude materials have a significant impact on inflation dynamics. These findings are noteworthy for the Canadian economy. However, the effect of unemployment on the dynamics of inflation is less clear. Furthermore, our estimates indicate almost fully flexible prices, which, in addition to the insignificant role for unemployment, does not line up with the assumptions of the underlying theoretical NKPC model.

In contrast, and despite estimation uncertainty with the wage rigidity index, the results of our structural estimations are supportive of the Blanchard and Gali (2008) model. Using our empirical specification for this model, we obtain economically reasonable ranges for average frequency of price changes. The model also yields significant and correct signs on the reduced-form coefficients, in particular showing a trade-off between unemployment and inflation in the New Keynesian Phillips curve. These findings underscore the informational content of the productivity variable which we introduce to formulate the empirically testable implications arising from the Blanchard and Gali (2008) model.

More generally, our findings suggest that wage-rigidity based NKPCs hold promise empirically and provide interesting research directions.

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References

Anderson, T.W., Rubin, H., 1949. Estimation of the parameters of a single equation in a complete system of stochastic equations. Annals of Mathematical Statistics 20, 46–63.
Andrews, D.W.K., Moreira, M.J., Stock, J.H., 2006. Optimal two-sided invariant similar tests for instrumental variables regression. Econometrica 74, 715–752.
Beaulieu, M.-C., Dufour, J.-M., Khalaf, L., 2009. Finite sample multivariate tests of asset pricing models with coskewness. Computational Statistics and Data Analysis 53, 2008–2021.
Blanchard, O., Gali, J., 2007. Real wage rigidities and the new Keynesian model. Journal of Money, Credit and Banking 39, 35–65.
Blanchard, O., Gali, J., 2008. A new Keynesian model with unemployment. Technaical Report, CREI, Uneversitat Pompeu Fabra.
Bolduc, D., Khalaf, L., Moyneur, E., 2008. Identification-robust simulation-based inference in joint discrete/continuous models for energy markets. Computational Statistics and Data Analysis 52, 3148–3161.
Canova, F., Sala, L., 2006. Back to square one: Identification issues in DSGE models. Tecnical Report, Bocconi University, IGIER Working Papers 303.
Chari, V., Kehoe, P., McGrattan, E., 2000. Sticky price models of the business cycle: Can the contract multiplier solve the persistence problem? Econometrica 68 (5), 1151–1180.
Christoffel, K., Linzert, T., 2006. The role of real wage rigidities and labor market frictions for unemployment and inflation dynamics. Discussion Paper 11/2006, Deutsche Bundesbank.
Dufour, J.-M., 1997. Some impossibility theorems in econometrics, with applications to structural and dynamic models. Econometrica 65, 1365–1389.
Dufour, J.-M., 2003. Identification, weak instruments and statistical inference in econometrics. Canadian Journal of Economics 36 (4), 767–808.
Dufour, J.-M., Jasiak, J., 2001. Finite sample limited information inference methods for structural equations and models with generated regressors. International Economic Review 42, 815–843.
Dufour, J.-M., Khalaf, L., Kichian, M., 2006. Inflation dynamics and the new Keynesian Phillips curve: An identification robust econometric analysis. Journal of Economic Dynamics and Control 30, 1707–1728.
Dufour, J.-M., Khalaf, L., Kichian, M., 2008. Structural multi-equation macroeconomic models: Identification-robust estimation and fit. Working Paper, McGill University.
Dufour, J.-M., Taamouti, M., 2005. Projection-based statistical inference in linear structural models with possibly weak instruments. Econometrica 73, 1351–1365.
Dufour, J.-M., Taamouti, M., 2007. Further results on projection-based inference in IV regressions with weak, collinear or missing instruments. Journal of Econometrics 139, 133–153.
Galí, J., Gertler, M., 1999. Inflation dynamics: A structural econometric analysis. Journal of Monetary Economics 44, 195–222.
Galí, J., Gertler, M., Lopez-Salido, J.D., 2005. Robustness of the estimates of the hybrid new Keynesian Phillips curve. Journal of Monetary Economics 52, 1107–1118.
Gertler, M., Sala, L., Trigari, A., 2007. An estimated monetary DSGE model with unemployment and staggered nominal wage bargaining, Journal of Money, Credit and Banking (forthcoming).
Hodges Jr., J.L., Lehmann, E.L., 1963. Estimates of location based on rank tests. Annals of Mathematical Statistics 34, 598–611.
Hodges Jr., J.L., Lehmann, E.L., 1983. Hodges–Lehmann estimators. In: Johnson, N.L., Kotz, S., Read, C. (Eds.), Encyclopedia of Statistical Sciences, vol. 3. John Wiley & Sons, New York, pp. 642–645.
Hoogerheide, L.F., Kaashoek, J.F., van Dijk, H.K., 2007. On the shape of posterior densities and credible sets in instrumental variable regression models with reduced rank: An application of flexible sampling methods using neural networks. Journal of Econometrics 139 (4), 154–180.
Joseph, A.S., Kiviet, J.F., 2005. Viewing the relative efficiency of IV estimators in models with lagged and instantaneous feedbacks. Computational Statistics and Data Analysis 49, 417–444.
Khalaf, L., Kichian, M., 2005. Exact tests of the stability of the Phillips curve: The Canadian case. Computational Statistics and Data Analysis 49.
Kiviet, J.F., Niemczyk, J., 2007. The asymptotic and finite sample distributions of OLS and simple IV in simultaneous equations. Computational Statistics and Data Analysis 51, 3296–3318.
Kleibergen, F., 2002. Pivotal statistics for testing structural parameters in instrumental variables regression. Econometrica 70 (5), 1781–1803.
Kleibergen, F., 2005. Testing parameters in GMM without assuming that they are identified. Econometrica 73, 1103–1123.
Kleibergen, F., Mavroeidis, S., 2008. Weak instrument robust tests in GMM and the new Keynesian Phillips curve. Journal of Business and Economic Statistics (in press).
Krause, M., Lopez-Salido, D., Lubik, T., 2008. Inflation dynamics with search frictions: A structural econometric analysis. Journal of Monetary Economics 55 (5), 892–916.
Ma, A., 2002. GMM estimation of the new Keynesian Phillips curve. Economics Letters 76, 411–417.
Mavroeidis, S., 2004. Weak identification of forward-looking models in monetary economics. Oxford Bulletin of Economics and Statistics 66, 609–635.
Mavroeidis, S., 2005. Identification issues in forward-looking models estimated by GMM with an application ot the Phillips curve. Journal of Money, Credit and Banking 37, 421–449.
Nason, J., Smith, G., 2008. Identifying the new Keynesian Phillips curve. Journal of Applied Econometrics 23, 525–551.
Rotemberg, J., 2006. Cyclical wages in a search-and-bargaining model with large firms. Manuscript.
Sbordone, A., 2005. Do expected marginal costs drive inflation dynamics. Journal of Monetary Economics 52, 1183–1197.
Sbordone, A., 2007. Inflation persistence: Alternative interpretations and policy implications. Journal of Monetary Economics 54, 1311–1339.
Staiger, D., Stock, J.H., 1997. Instrumental variables regression with weak instruments. Econometrica 65 (3), 557–586.
Stock, J.H., Wright, J.H., 2000. GMM with weak identification. Econometrica 68, 1097–1126.
Stock, J.H., Wright, J.H., Yogo, M., 2002. A survey of weak instruments and weak identification in generalized method of moments. Journal of Business and Economic Statistics 20 (4), 518–529.
Wang, J., Zivot, E., 1998. Inference on structural parameters in instrumental variables regression with weak instruments. Econometrica 66 (6), 1389–1404.
Zivot, E., Startz, R., Nelson, C.R., 1998. Valid confidence intervals and inference in the presence of weak instruments. International Economic Review 39, 1119–1144.