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McGill University
ECN 467
Econ 469: Econometrics
Mid-term exam

No documentation allowed
Time allowed: 1.5 hour

30 points 1. Let $\gamma(k)$ the autocovariance function of second-order stationary process on the integers. Prove that:

(a) $\gamma(0) = Var(X_t)$ et $\gamma(k) = \gamma(-k)$, $\forall k \in \mathbb{Z}$;

(b) $|\gamma(k)| \leq \gamma(0)$, $\forall k \in \mathbb{Z}$;

(c) the function $\gamma(k)$ is positive semi-definite.

40 points 2. Consider the following models:

$$X_t = 10 + 0.7\, X_{t-1} - 0.2\, X_{t-2} + u_t \tag{0.1}$$

where $\{u_t : t \in \mathbb{Z}\}$ is an $i.i.d.$ $N(0,1)$ sequence. For each one of these models, answer the following questions.

(a) Is this model stationary? Why?

(b) Is this model invertible? Why?

(c) Compute:

i. $E(X_t)$;

ii. $\gamma(k)\,,\ k = 1,\, \ldots\,,\, 8$;

iii. $\rho(k)\,,\ k = 1,\, 2,\, \ldots\,,\, 8$.

(d) Graph $\rho(k)\,,\ k = 1,\, 2,\, \ldots\,,\ 8$.

(e) Find the coefficients of $u_t,\ u_{t-1},\ u_{t-2},\ u_{t-3}$ and u_{t-4} in the moving average representation of X_t.

(f) Compute the first four partial autocorrelations of X_t.

30 points 3. Let $X_1,\ X_2,\ \ldots\ ,\ X_T$ be a time series.

(a) Define:

i. the sample autocorrelations for this series;

ii. the partial autocorrelations for this series.

(b) Discuss the asymptotic distributions of these two sets of autocorrelations in the following cases:

i. under the hypothesis that $X_1,\ X_2,\ \ldots\ ,\ X_T$ are independent and identically distributed (i.i.d.);

ii. under the hypothesis that the process follows a moving average of finite order.

(c) Describe how you would identify the process described in equation (0.1) in question 2.