Protocol Verification Techniques - Theorem Provers

Design and Verification of Security Protocols and Security Ceremonies

Programa de Pós-Graduacão em Ciências da Computação Dr. Jean Everson Martina

August-November 2016





Attention!

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This topic will be divided into two lectures. One will deal with automatic theorem provers using FOL and the second will deal with theorem provers using HOL

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- Higher-order logic admits quantification over sets that are nested arbitrarily deeply.

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- This is the definition of the set of Natural Numbers.



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Larry's Protocol Verification Time-line

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- Last work published in 2015: "Verifying multicast-based security protocols using the inductive method. Martina, J.E., Paulson, L.C."

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- We prove the theorems inductively to demonstrate correctness.

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 - Use a theorem prover (Isabelle)to write the proofs.

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 - Public.thy which inherits from Event.thy, which despite the narrowly chosen name, accounts for the specification of symmetric and asymmetric cryptographic primitives;
 - We also have some other specialised theories for Smart-Cards, Threshold Cryptography and Multicast communication.



Definition

Agent datatype definition

datatype

Definition

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datatype

• First we have the definition of a friendly agent by the bijection explained above;

Definition

Agent datatype definition

datatype

```
agent = Friend nat
| Server
| Spy
```

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- The second category regards the trusted third party;
- Finally we have the attacker, which is categorised separately.



Definition

```
shrK function definition
```

```
consts
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shrK :: "agent => key"
specification (shrK)
inj_shrK: "inj shrK"
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- A shared key is specified as an injective function taking an agent and returning a key.

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invKey function definition
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invKey :: "key => key"
specification (invKey)
  invKey [simp]: "invKey (invKey K) = K"
  invKey_symmetric: "all_symmetric --> invKey = id"
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symKeys set definition
```

constdefs

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 The symmetric key set is defined as containing all keys where the inverse of the key by the application of the function *invKey* is itself;

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symKeys set definition

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- The symmetric key set is defined as containing all keys where the inverse of the key by the application of the function *invKey* is itself;
- By stating K ∈ symKeys ∧ K ∉ range shrK we can create a second class of symmetric keys that are not long-term so that it represents sessions keys in our protocols being verified.

Definition

Axiom for symmetric usage of shared keys

axioms

```
sym\_shrK [iff]: "shrK X \in symKeys"
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Establishs that our long-term keys are symmetric with an axiom.

```
publicKey definition
datatype
  keymode = Signature | Encryption
consts
  publicKey :: "[keymode,agent] => key"
specification (publicKey)
  injective_publicKey:
   "publicKey b A = publicKey c A' ==> b=c & A=A'"
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privateKey axiom definition
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Definition

Public Key abbreviations

abbreviation

```
pubEK :: "agent => key" where
"pubEK == publicKey Encryption"
pubSK :: "agent => key" where
"pubSK == publicKey Signature"
privateKey :: "[keymode, agent] => key" where
"privateKey b A == invKey (publicKey b A)"
priEK :: "agent => key" where
"priEK A == privateKey Encryption A"
priSK :: "agent => key" where
"priSK A == privateKey Signature A"
```

Inductive Method Details - Compromised Agents

Definition

bad set definition

```
consts
  bad :: "agent set"
specification (bad)
  Spy_in_bad [iff]: "Spy ∈ bad"
  Server_not_bad [iff]: "Server ∉ bad"
```

Inductive Method Details - Messages

Definition

```
msg datatype definition
```

datatype

Inductive Method Details - Events

Definition

```
event datatype definition
```

datatype

Inductive Method Details - Initial Knowledge

Definition

initState definition

consts

initState :: "agent => msg set"

Inductive Method Details - Initial Knowledge

```
Spy agent initial knowledge definition

primrec
  initState_Spy:
    "initState Spy =
    (Key ` invKey ` pubEK ` bad) \(\text{Key} ` invKey ` pubSK ` bad) \(\text{Key} ` invKey ` pubSK ` bad) \(\text{Key} ` shrK ` bad) \(\text{Ley} ` range pubSK) \(\text{Wey} ` range pubSK) \
```

Inductive Method Details - Knows

Definition

consts

```
knows function definition
```

```
knows :: "agent => event list => msg set"
primrec
  knows Nil: "knows A [] = initState A"
  knows_Cons:
    "knows A (ev # evs) =
    (if A = Spy then
    (case ev of
      Says A' B X => insert X (knows Spy evs)
      | Gets A' X => knows Spy evs
      | Notes A' X =>
        if A' \in bad then insert X (knows Spy evs)
```

alsa knows Sny ave)

Inductive Method Details - Knows

Definition

knows function definition

```
consts
    else
    (case ev of
      Says A' B X =>
        if A'=A then insert X (knows A evs) else
knows A evs
      | Gets A' X =>
        if A'=A then insert X (knows A evs) else
knows A evs
      | Notes A' X = >
        if A'=A then insert X (knows A evs) else
knows A evs))"
```

```
parts inductive set definition
```

```
inductive_set
```

```
parts :: "msg set => msg set"
for H :: "msg set"
where
    Inj [intro]: "X ∈ H ==> X ∈ parts H"
    | Fst: "{|X,Y|} ∈ parts H ==> X ∈ parts H"
    | Snd: "{|X,Y|} ∈ parts H ==> Y ∈ parts H"
    | Body: "Crypt K X ∈ parts H ==> X ∈ parts H"
```

Definition

analz inductive set definition

 $X \in analz H''$

```
inductive_set
```

Definition

synth inductive set definition

 $\{|X,Y|\} \in \text{synth } H''$

```
inductive_set
  synth :: "msg set => msg set"
  for H :: "msg set"
  where
       Inj [intro]: "X \in H \Longrightarrow X \in \text{synth } H"
        | Agent [intro]: "Agent agt ∈ synth H"
        | Number [intro]: "Number n \in synth H"
        | Hash [intro]: "X \in \text{synth } H \Longrightarrow \text{Hash } X \in \mathbb{R}
synth H"
        | MPair [intro]: "[|X \in synth H; Y \in synth H]
==>
```

Definition

used function definition

```
consts
```

Dummy Protocol

```
      1. A \rightarrow B :
      \{|A, B, Na|\}_{K_B}

      2. B \rightarrow A :
      \{|Na, Nb, K_{AB}|\}_{K_A}

      3. A \rightarrow B :
      \{|Nb\}_{K_{AB}}
```

Figure: Example protocol

Definition

inductive definition of example protocol

inductive_set example :: "event list set"

```
where
    Nil: "[] \in example"
    |Fake:"[|evsf \in example; X \in synth(analz (knows
Spy evsf))|]
     \Rightarrow Says Spy B X # evsf \in example"
    |EX1: "[|evs1 ∈ example; Nonce NA ∉ used evs1|]
     \RightarrowSays A B (Crypt (pubK B){|Agent A, Agent B,
Nonce NAI}) #
     evs1 \in example"
```

```
|EX3: "[|evs3 \in example;
       Says A B (Crypt (pubK B) { | Agent A, Agent B,
Nonce NAI})
       \in set evs3:
       Says B A (Crypt (pubK A) { | Nonce NA, Nonce
NB, Key AB()
       ∈ set evs3 |1
     \RightarrowSays A B (Crypt (Key AB){| Nonce NB|}) # evs3
∈ example"
```

```
|Oops: "[|evso ∈ example;
Says B A (Crypt (pubK A) {| Nonce NA, Nonce NB,
Key AB|})
∈ set evso|]
⇒ Notes Spy {|Nonce NA, Nonce NB, Key AB|}#
evso ∈ example"
```

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- Proof tactics are known for most of the usual goals;
- The problem starts when you want to prove a new goal....

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- Understanding Isabelle cryptic messages is painful
- The person carrying the verification need to be clever and seasoned with the tool, otherwise there will be pain;
- Too much freedom and power are sometimes difficult to use.

Discussion

What else can you foresee modelled using this strategy?

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- Can this be extended?

Discussion

- What else can you foresee modelled using this strategy?
- Can this be extended?
- What this strategy can not do?

Questions????



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